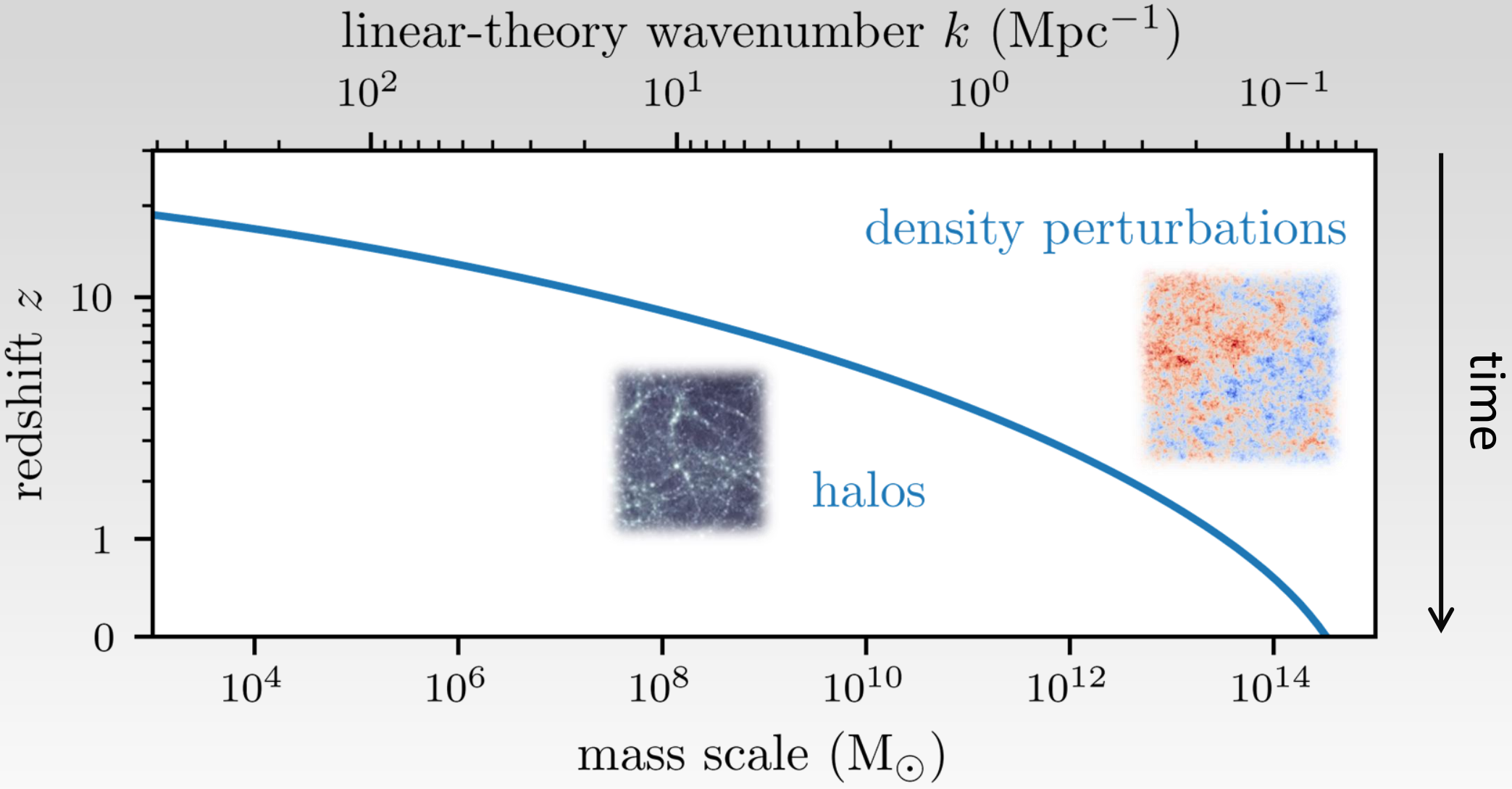
Cosmology with prompt cusps

M. Sten Delos (Carnegie Observatories)

Cosmology 2025 @ Elba Island
10 September 2025

CARNEGIE SCIENCE | Observatories

Phases of cosmic structure



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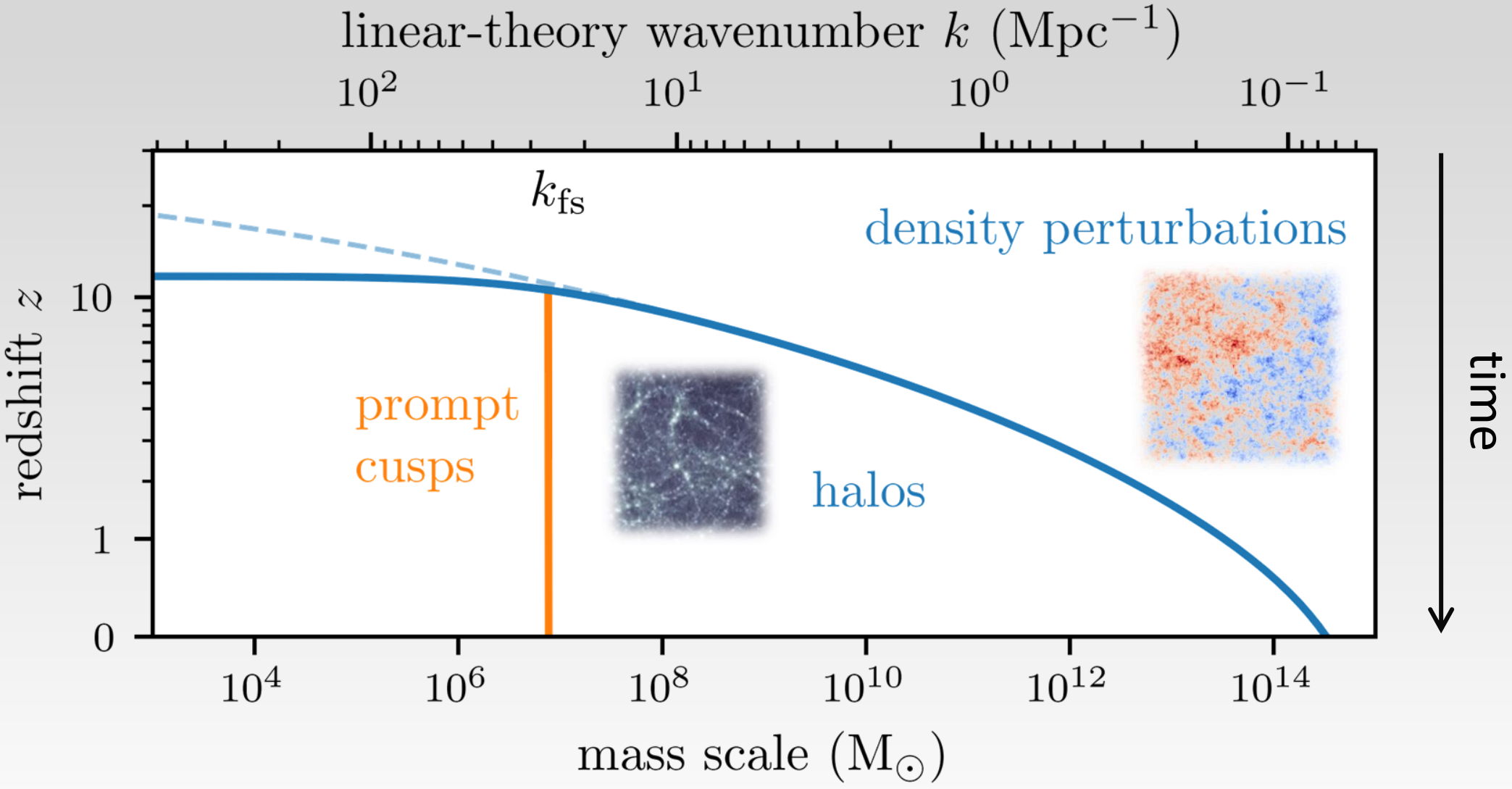
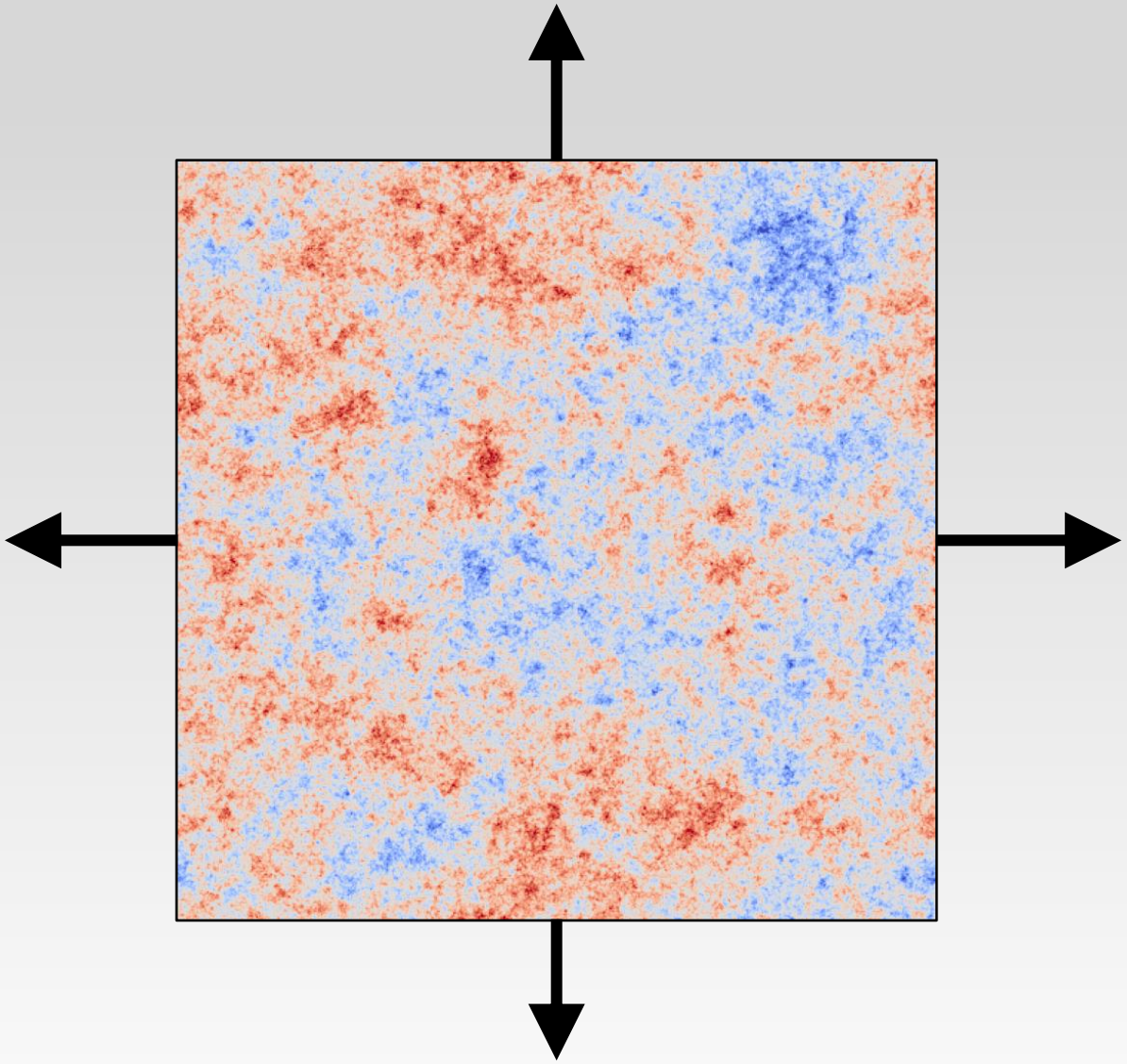


The cosmological initial conditions



A random density field

- Expanding over time
- Gravitationally amplified over time

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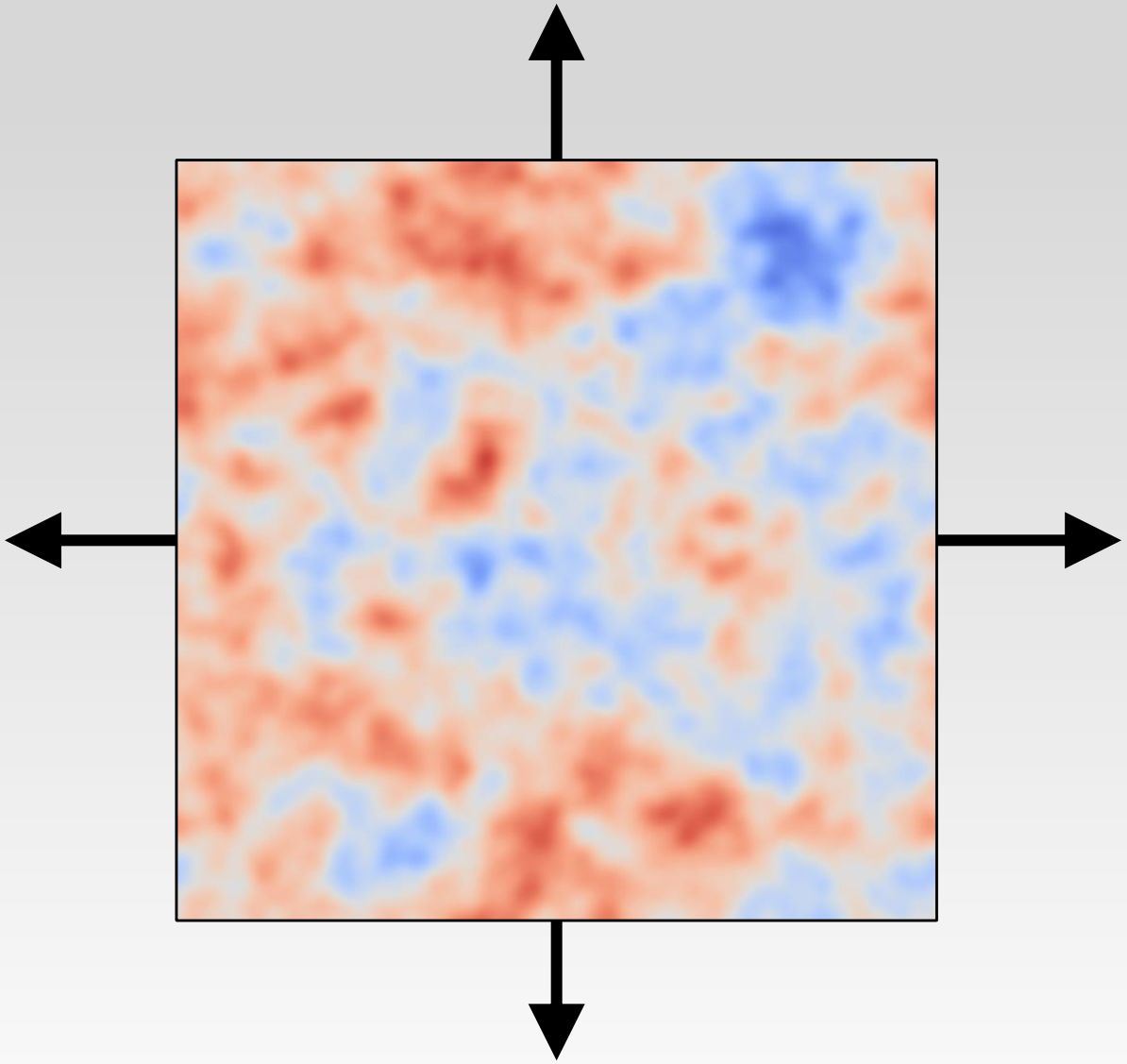

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Smooth on sufficiently small scales

e.g., due to thermal motion of the dark matter

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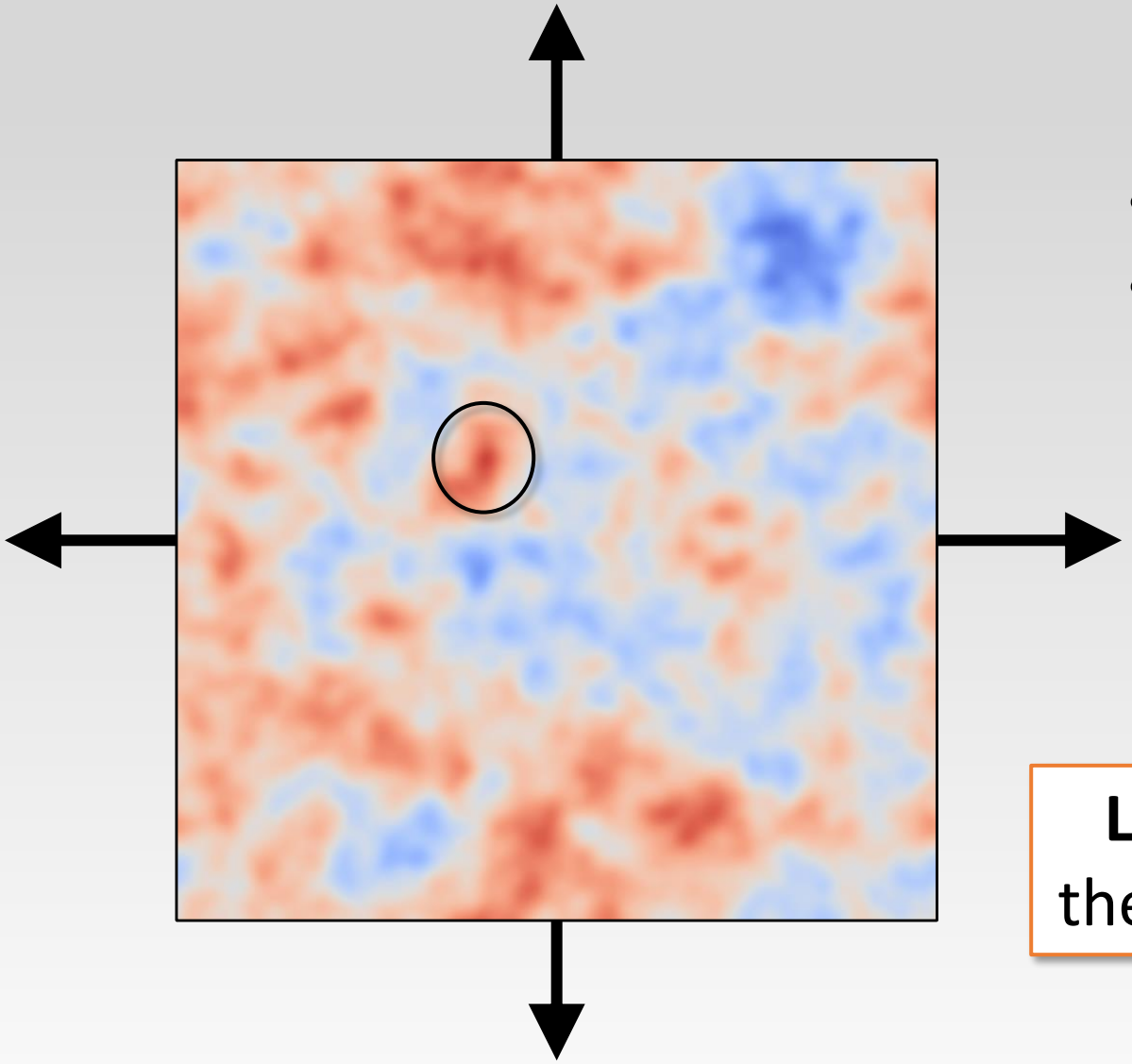


A random density field

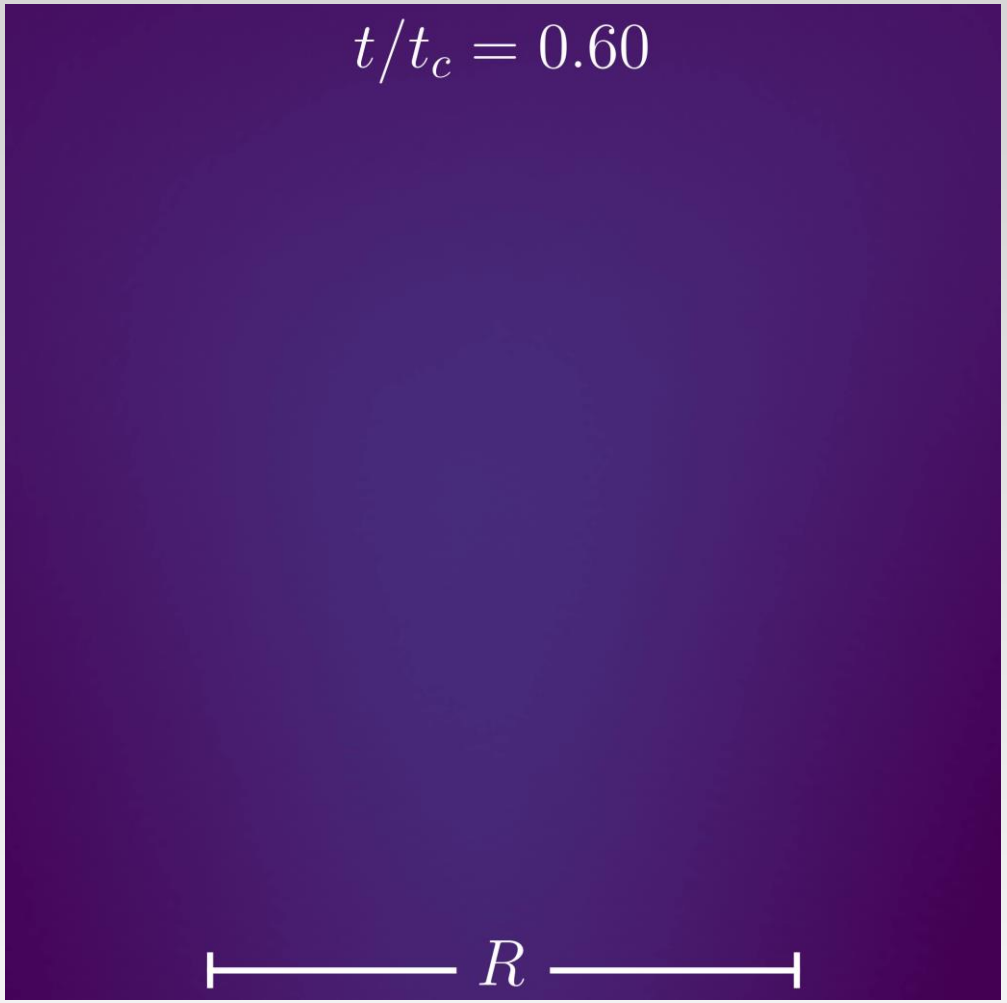
- Expanding over time
- Gravitationally amplified over time

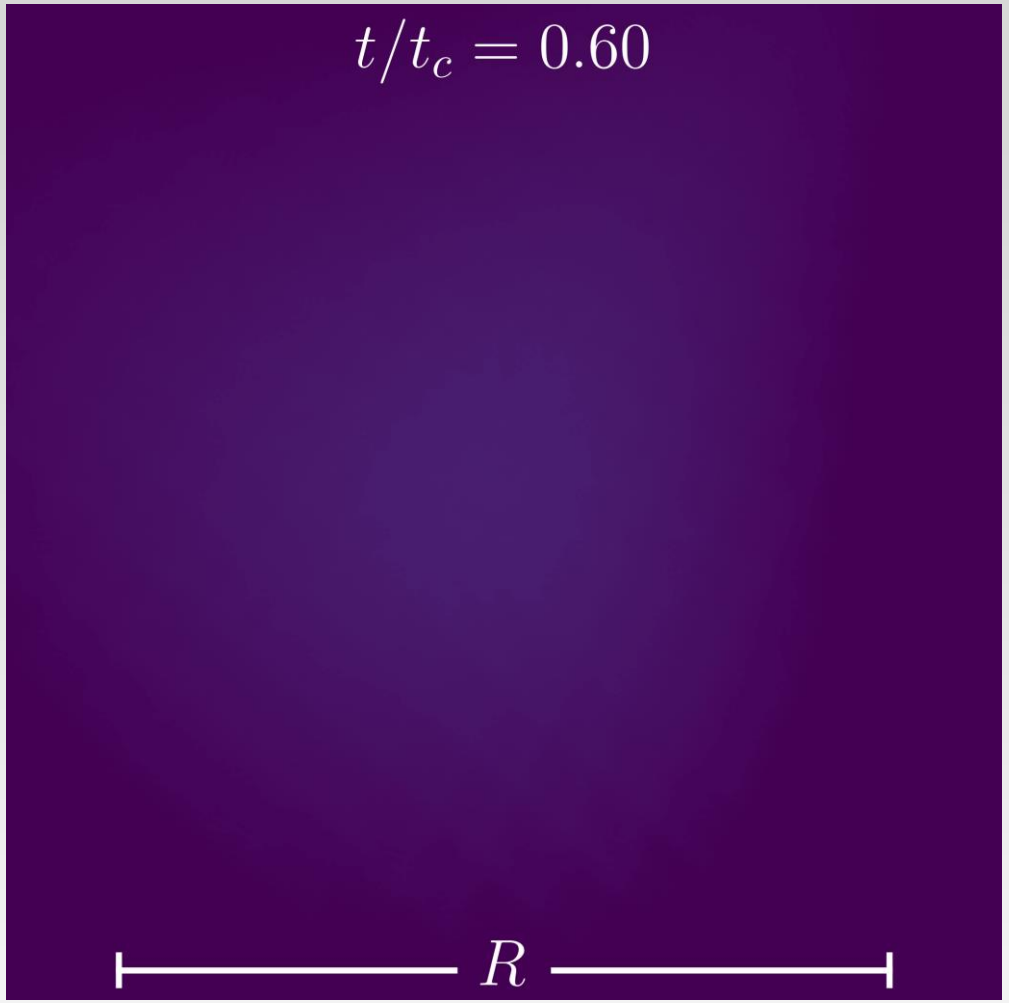
Smooth on sufficiently small scales

e.g., due to thermal motion of the dark matter

Local maxima in the density field are the first places to gravitationally collapse

Collapse of density peaks





t_c = ellipsoidal collapse time

$R = \sqrt{-\delta/\nabla^2\delta}$

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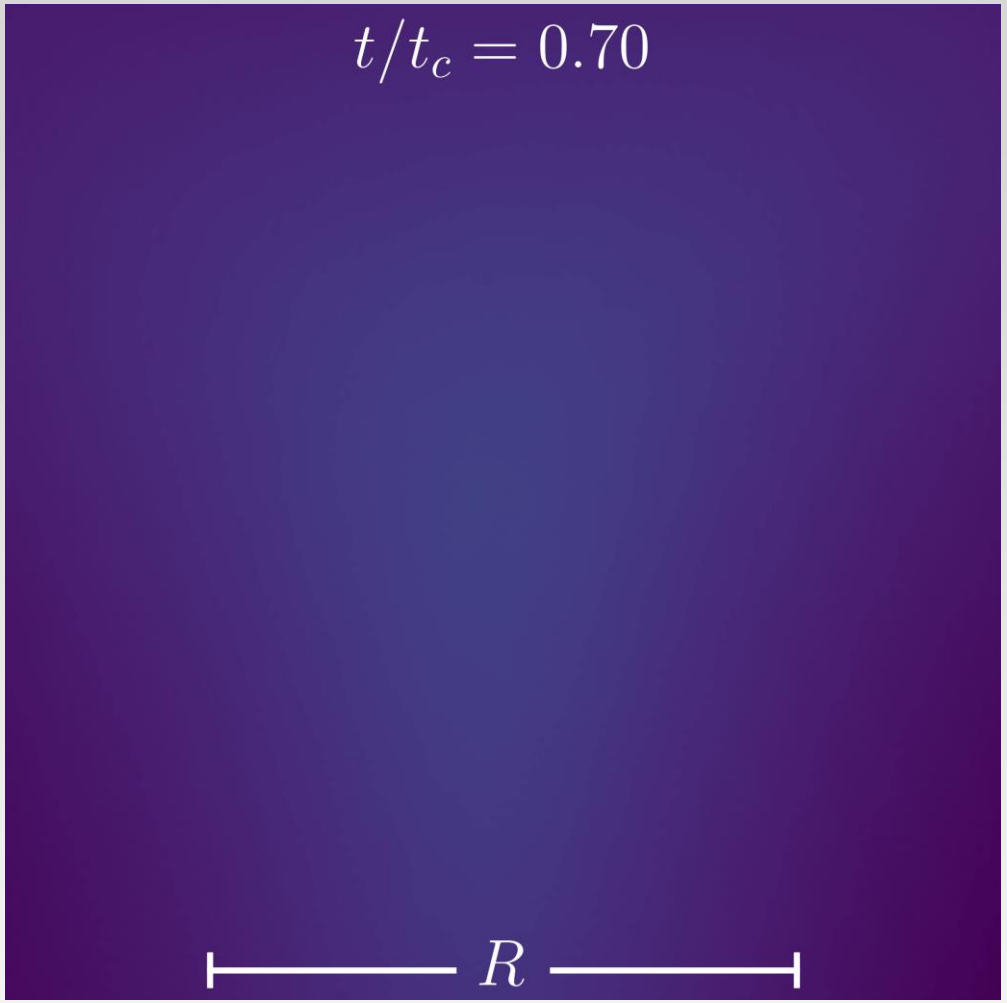

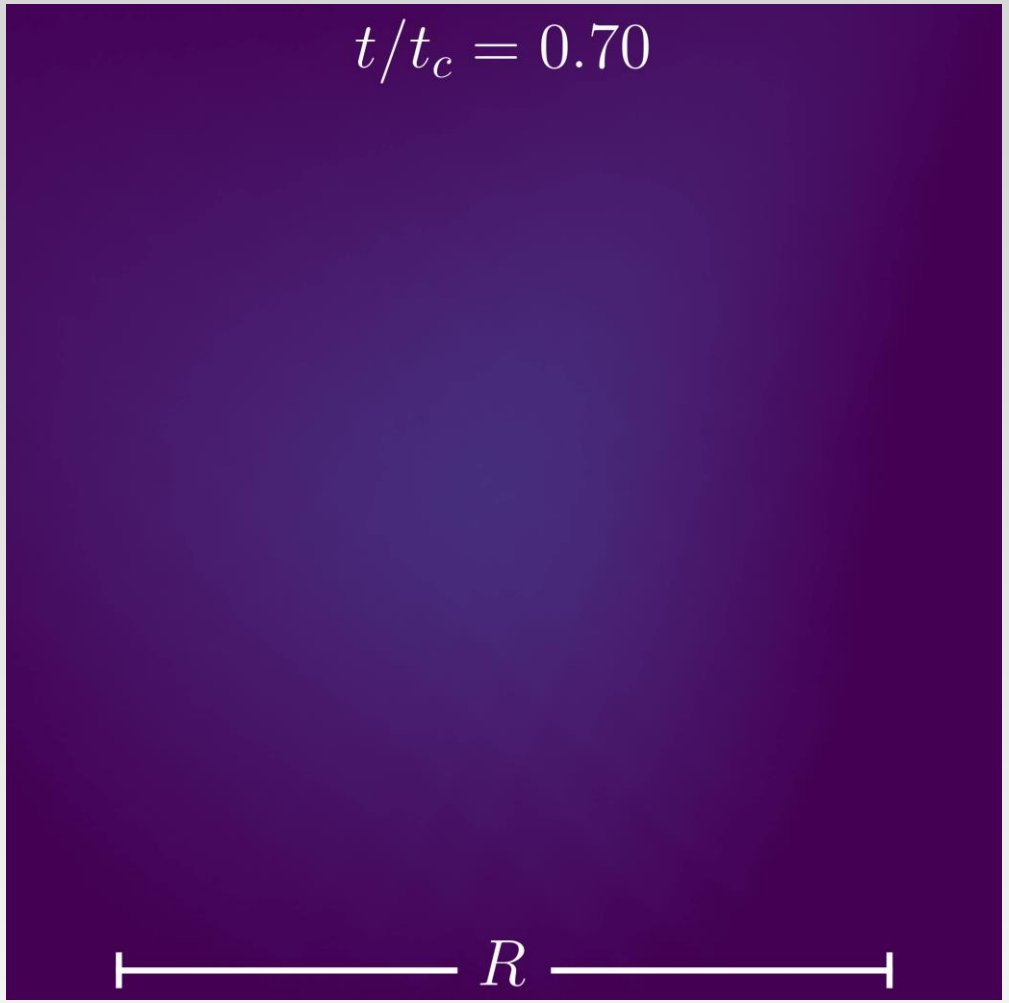

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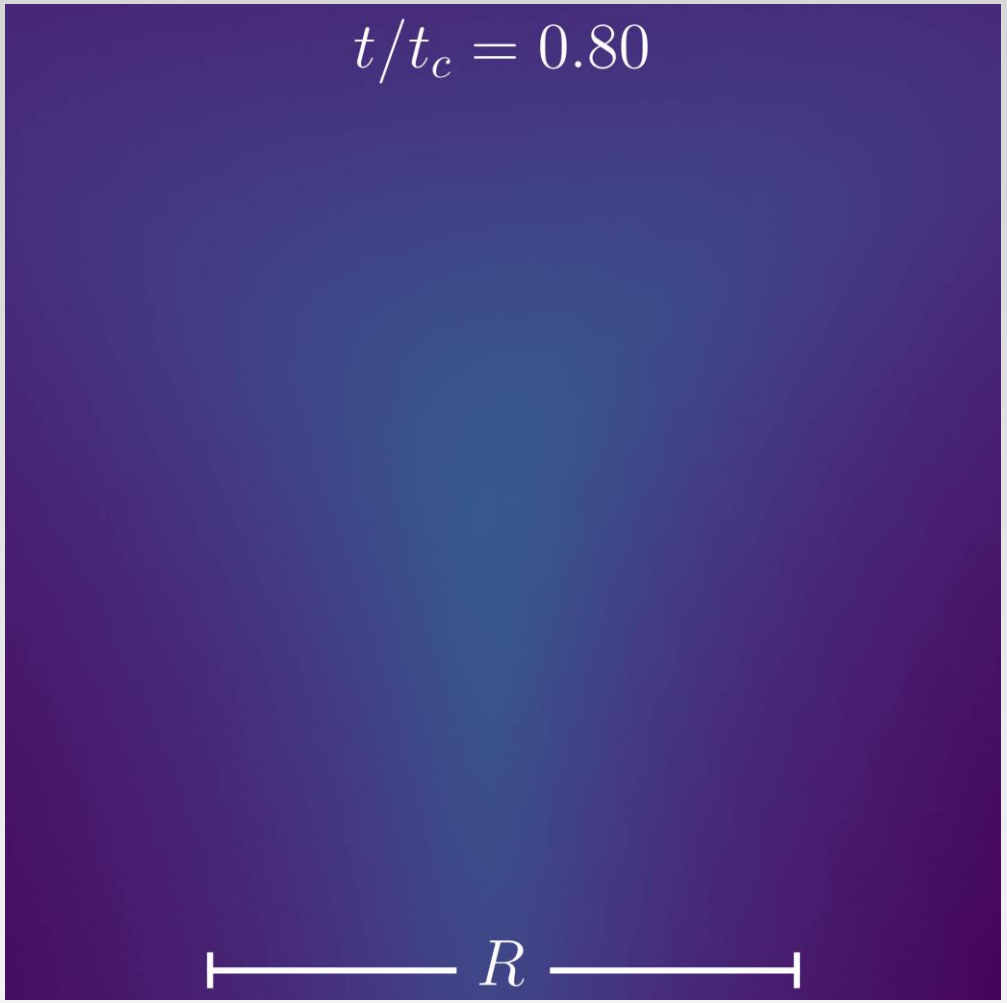

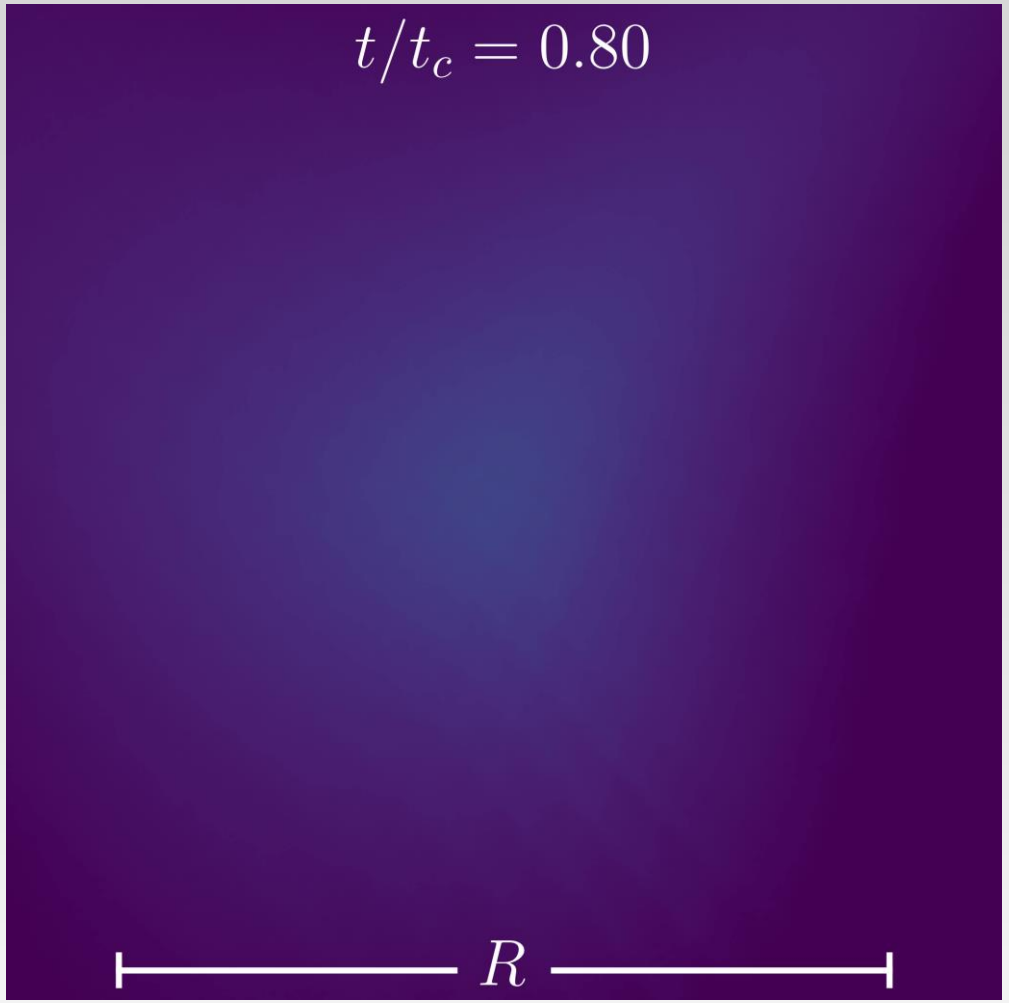

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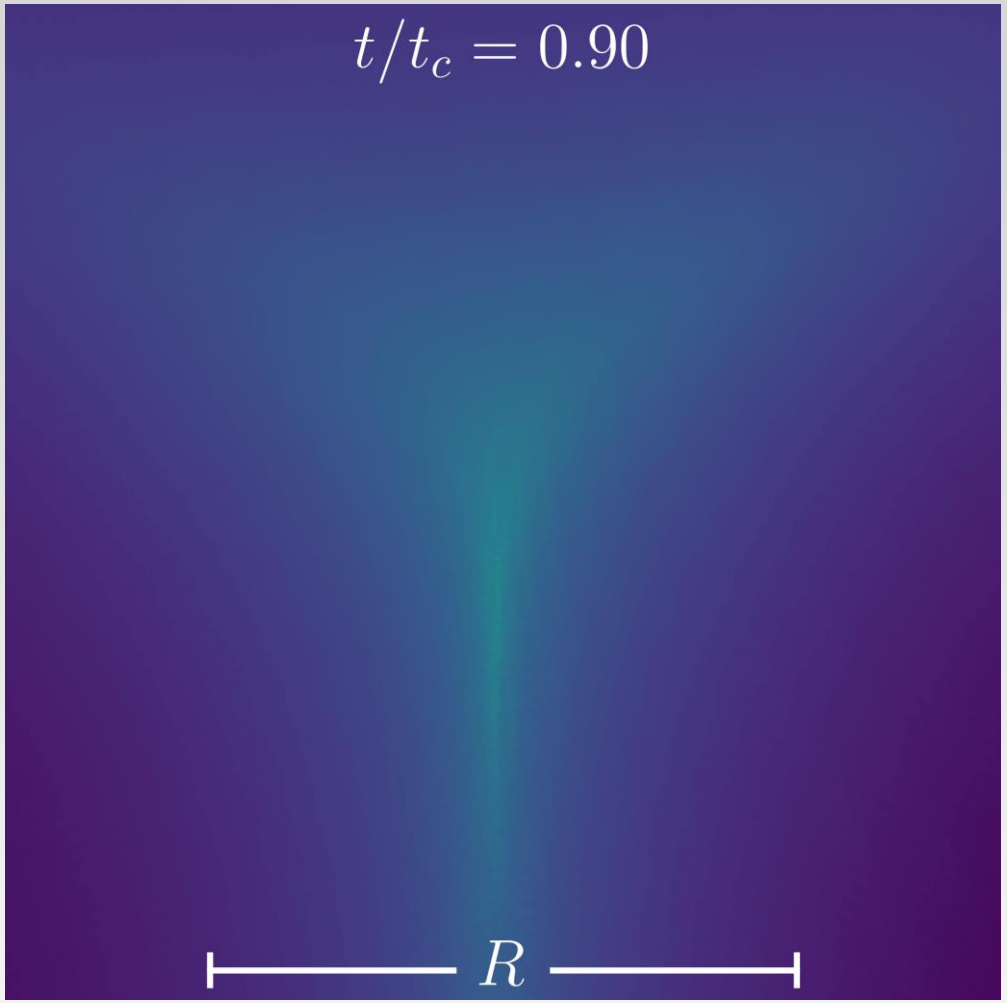

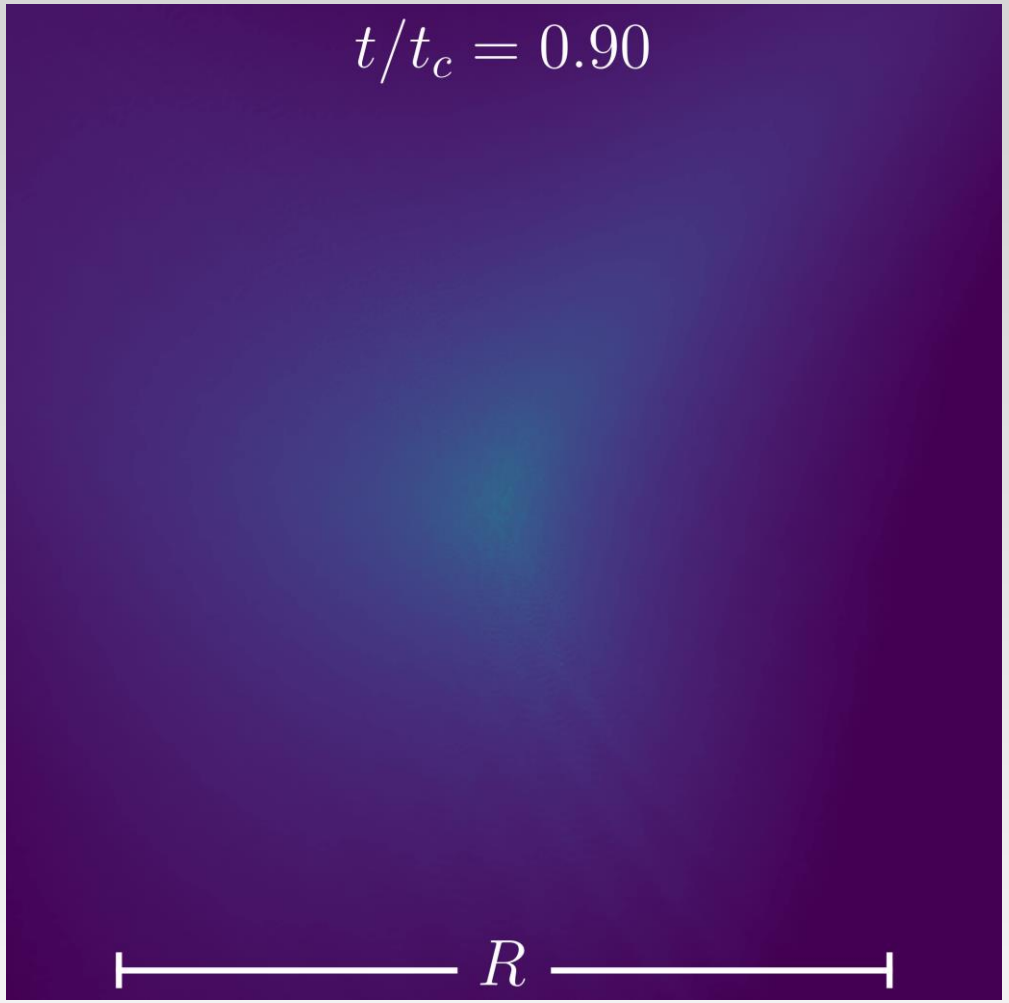

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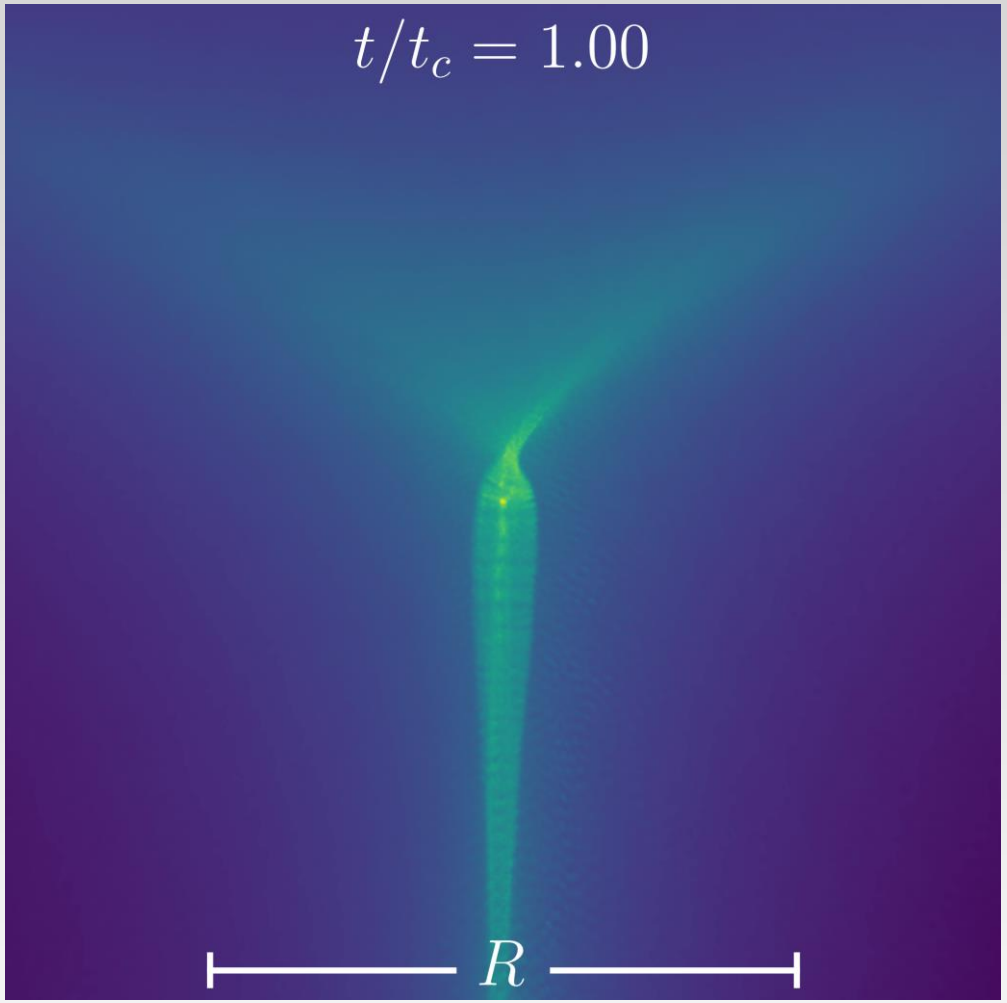

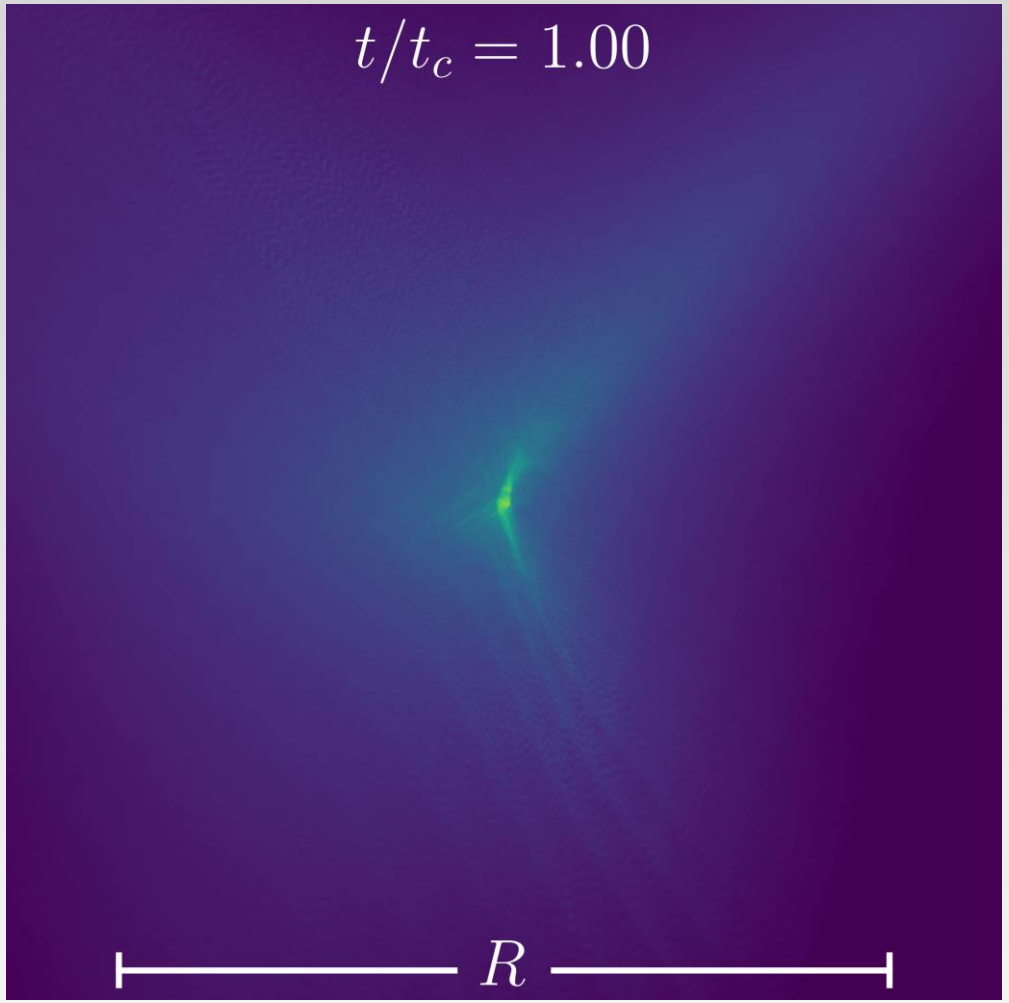

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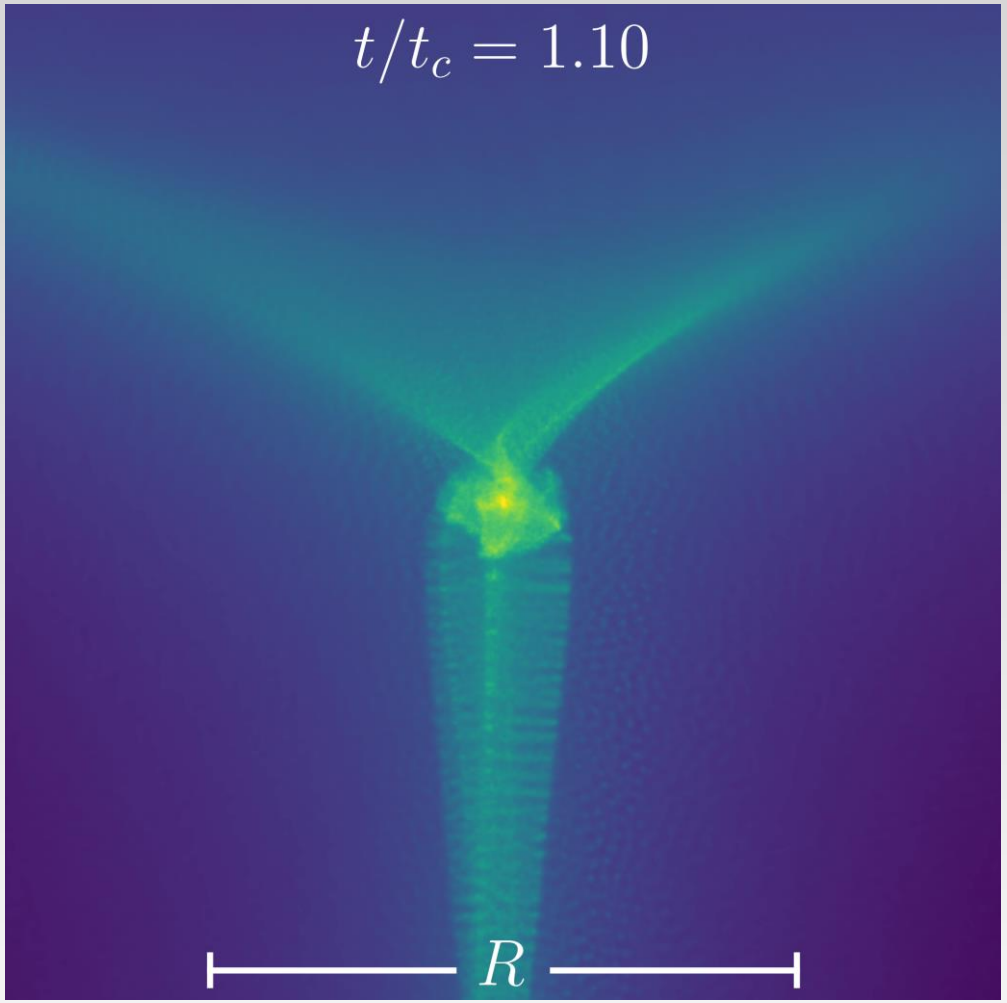

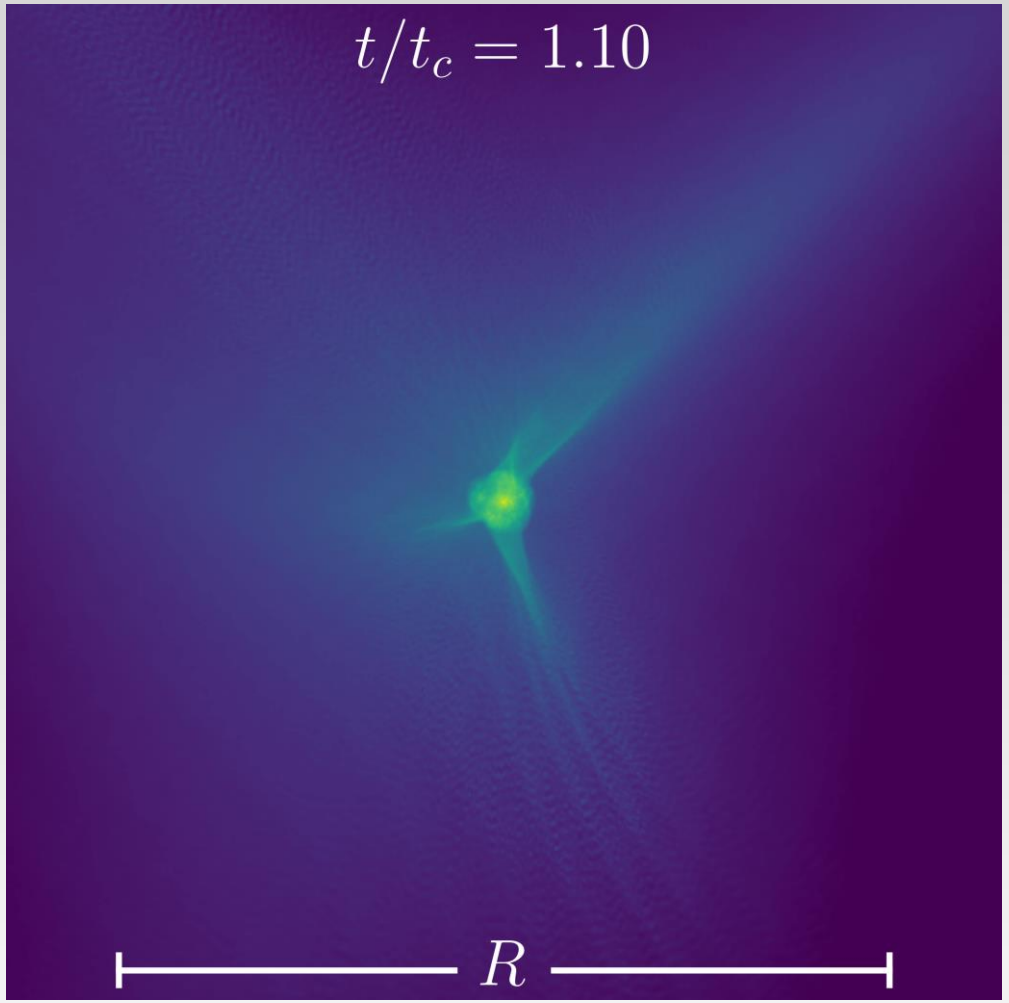

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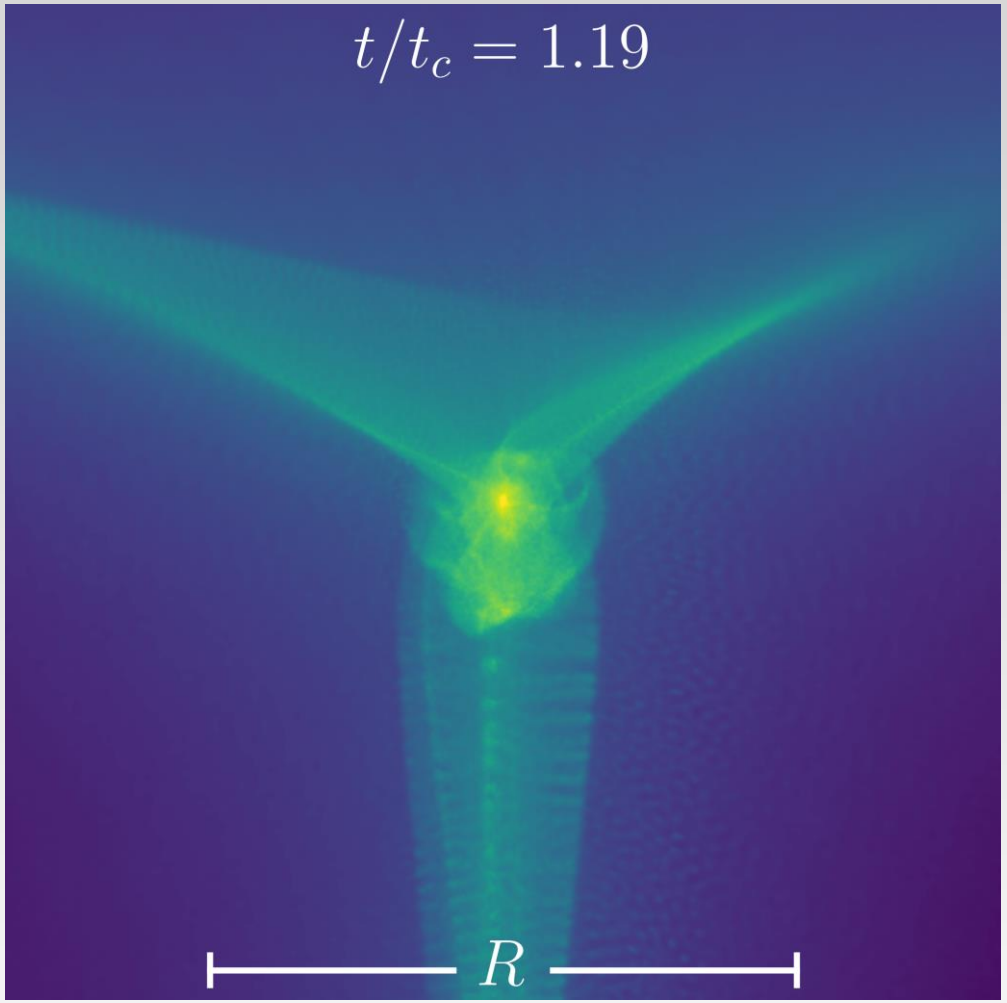

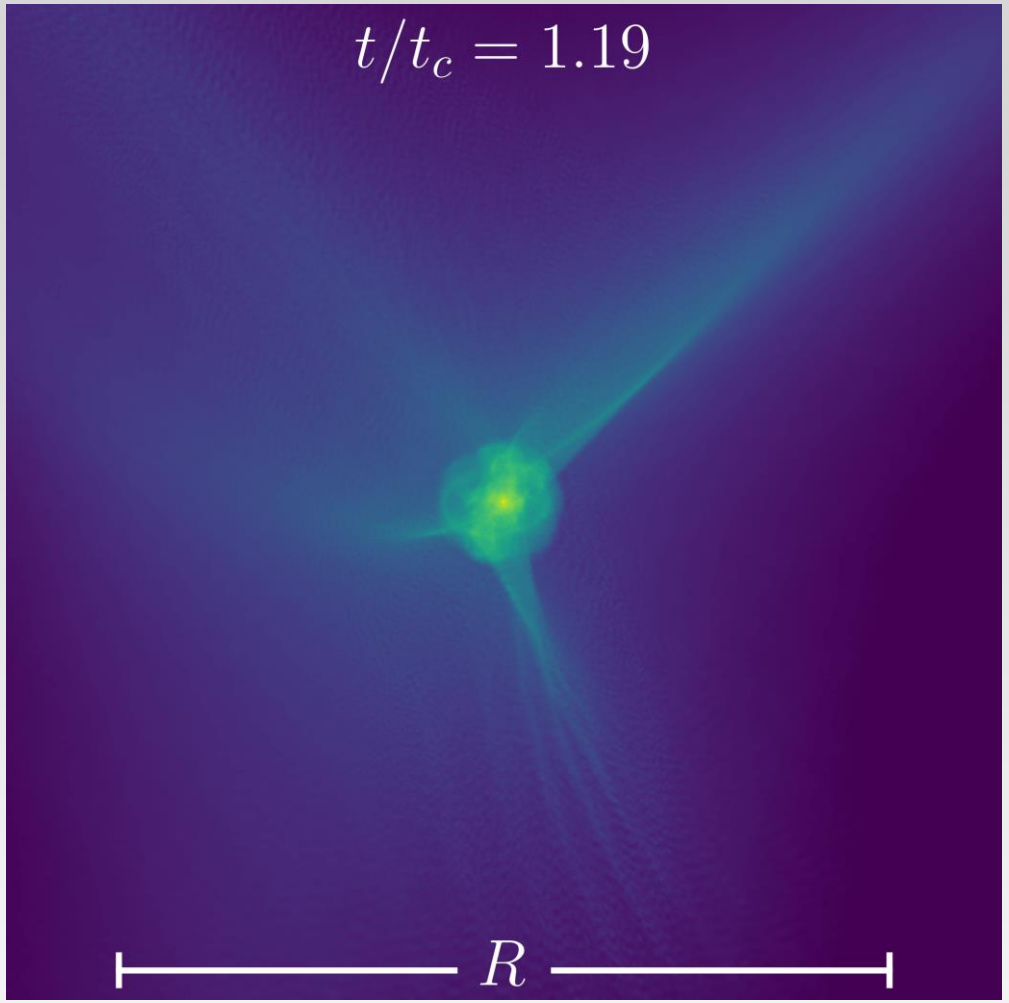

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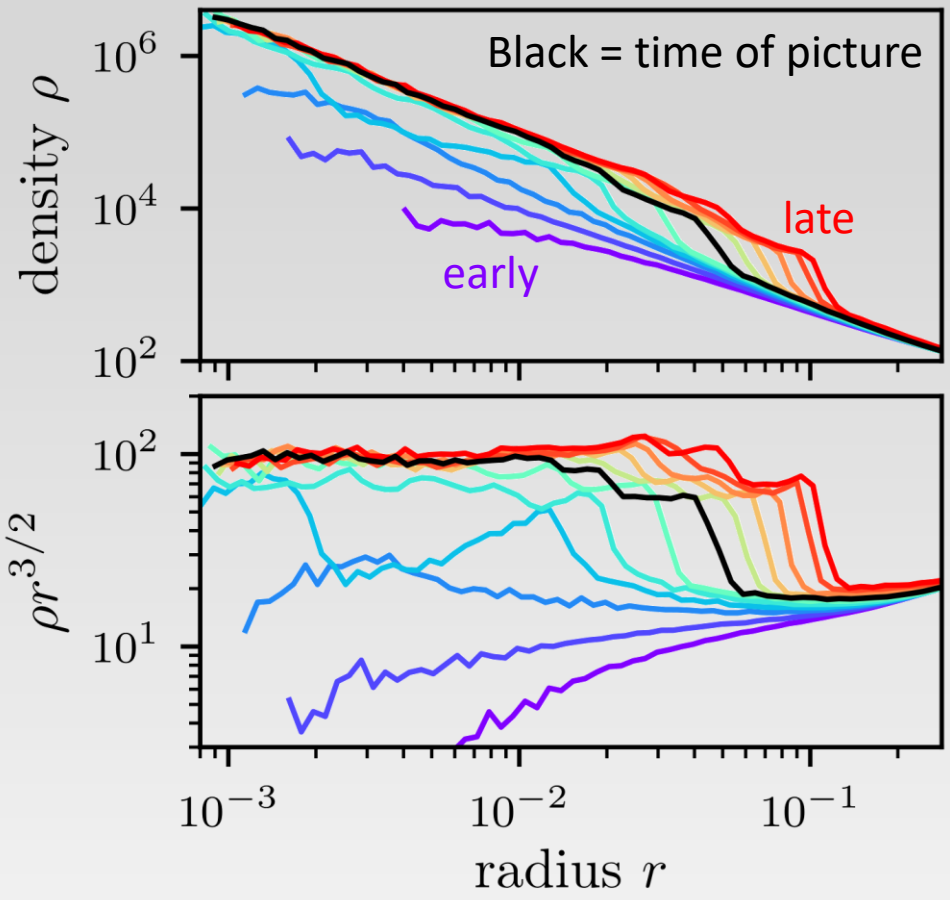
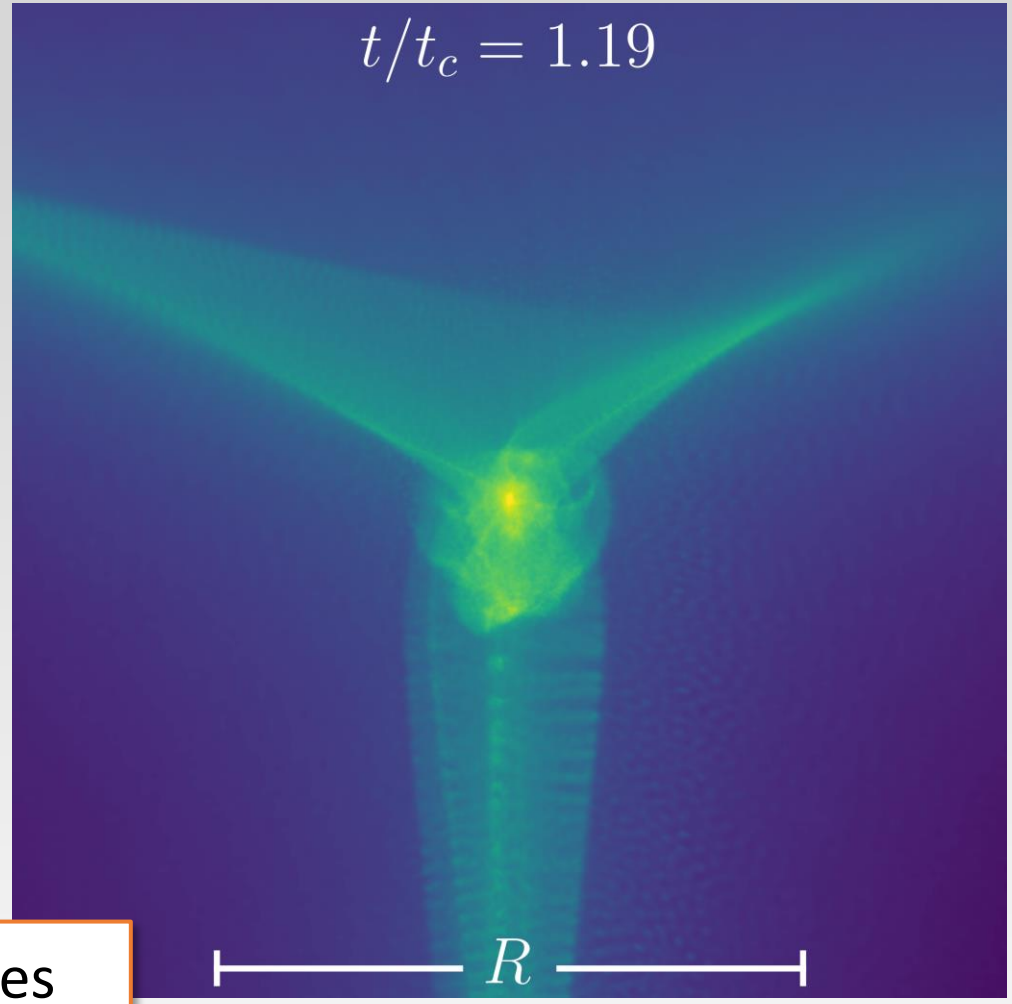
“Prompt cusp”





$\rho \propto r^{-3/2}$ cusp stabilizes immediately after formation

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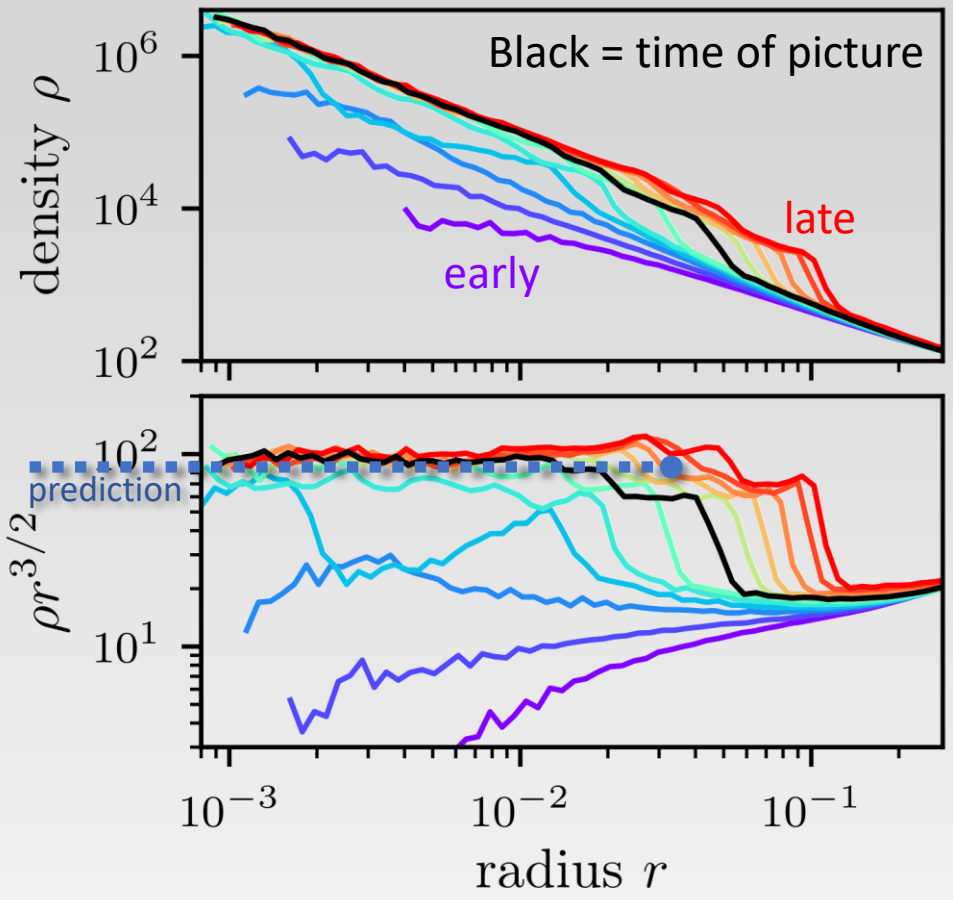
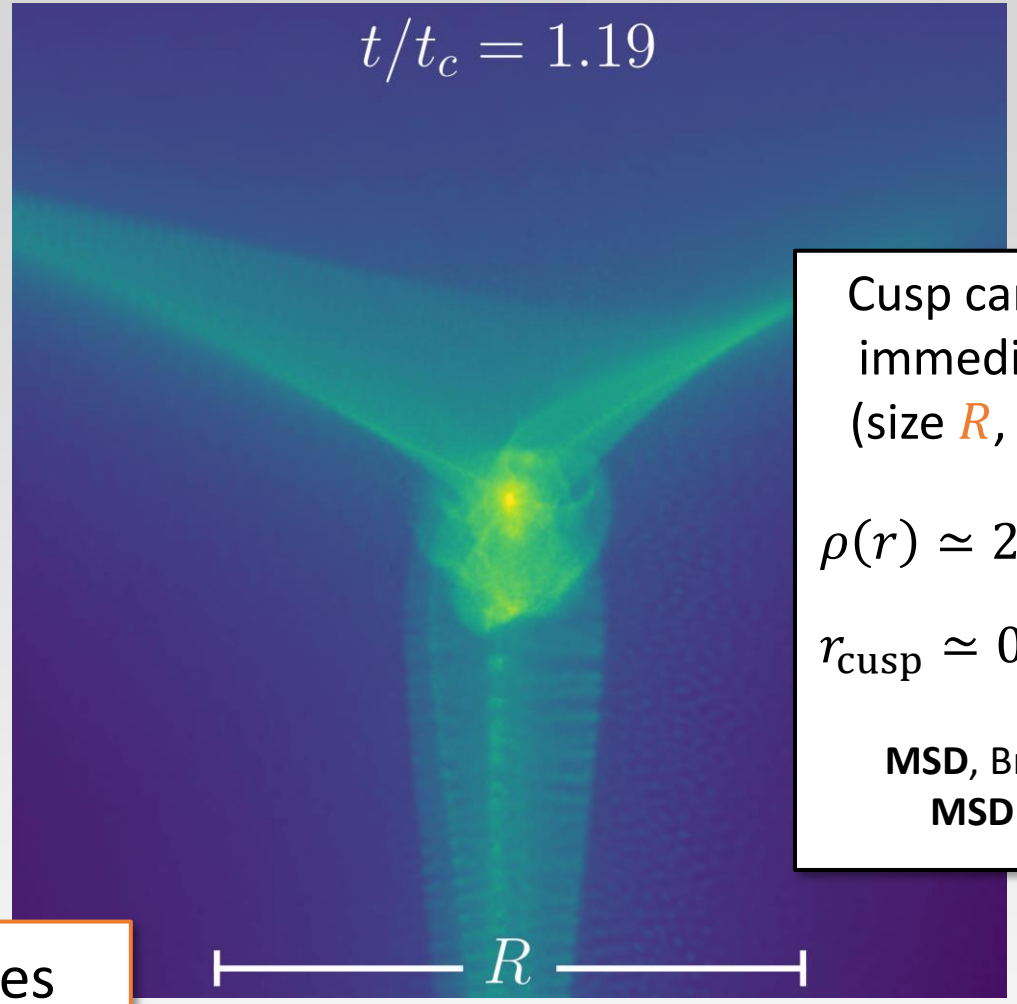




Cusp can only depend on immediate density peak (size R, collapse time a_c)

$$\rho(r) \simeq 24\,\bar{\rho}(a_c)\left(\frac{a_c R}{r}\right)^{3/2}$$

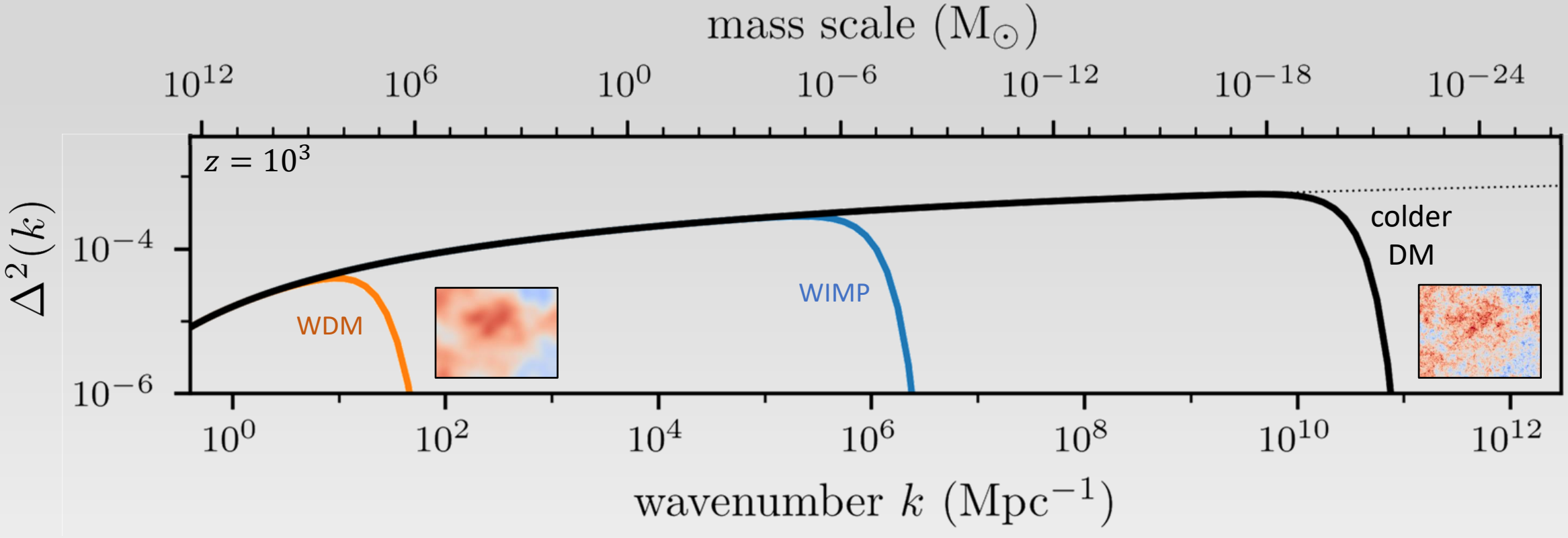
$$r_{\rm cusp} \simeq 0.11 a_c R$$

MSD, Bruff, Erickcek (2019)
MSD & White (2023a)

$\rho \propto r^{-3/2}$ cusp stabilizes immediately after formation

“prompt”

Statistics of prompt cusps



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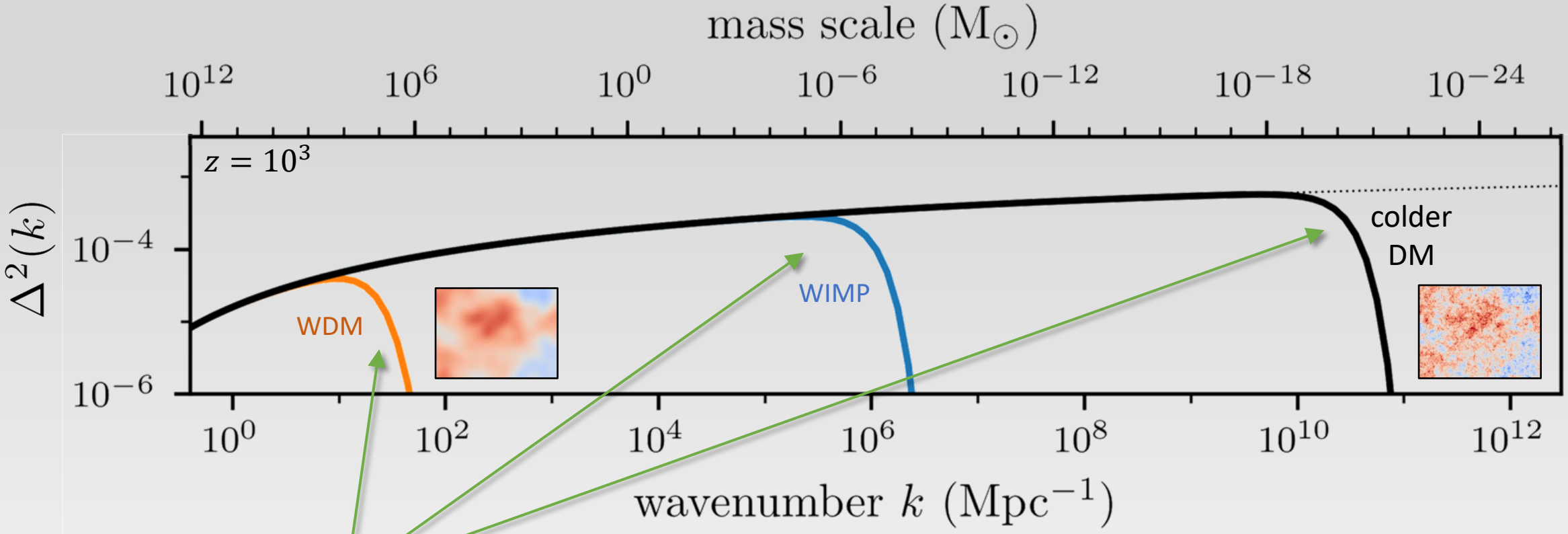

prompt cusps form at the smoothing scale

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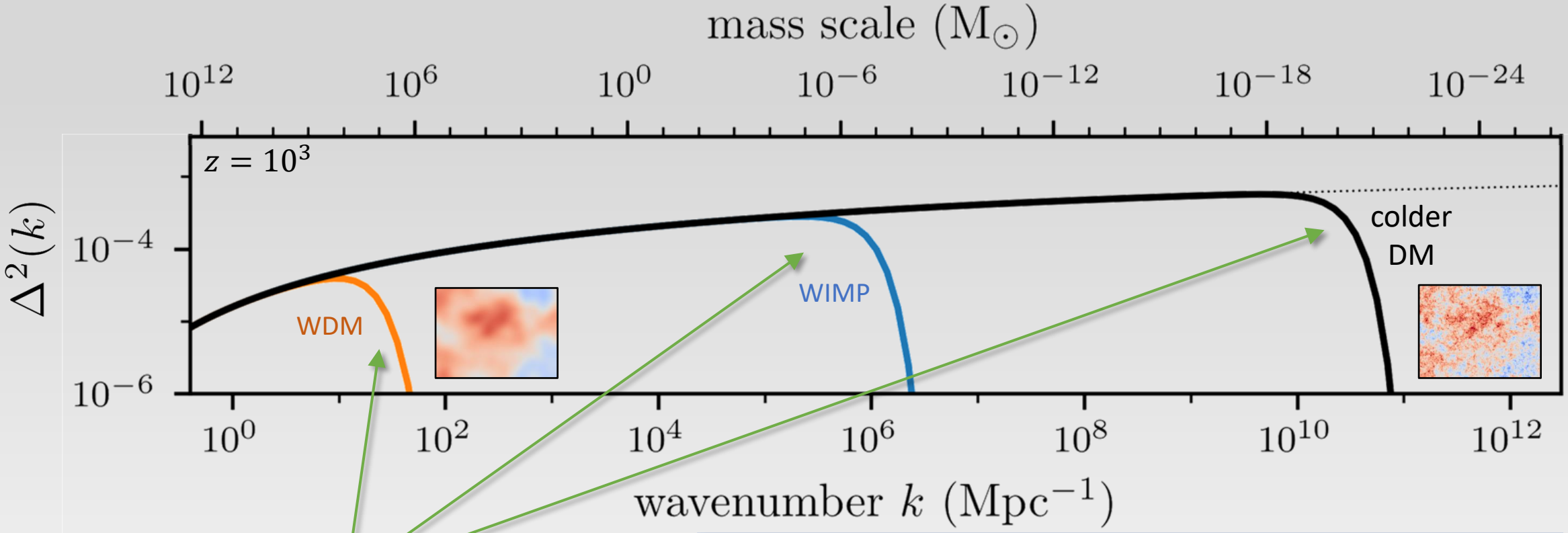
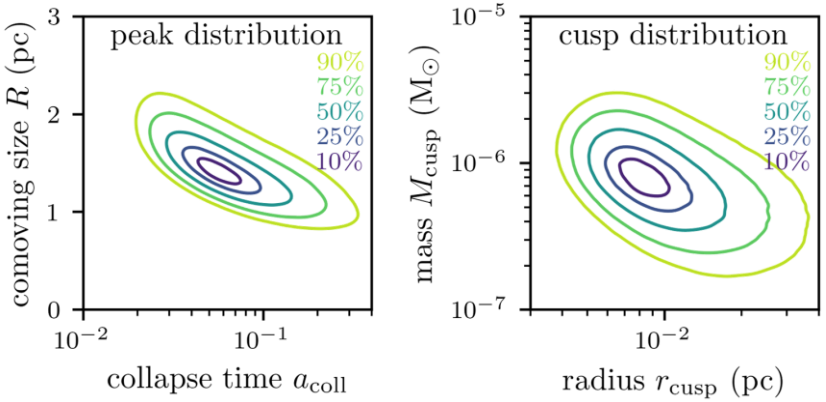


prompt cusps form at the smoothing scale

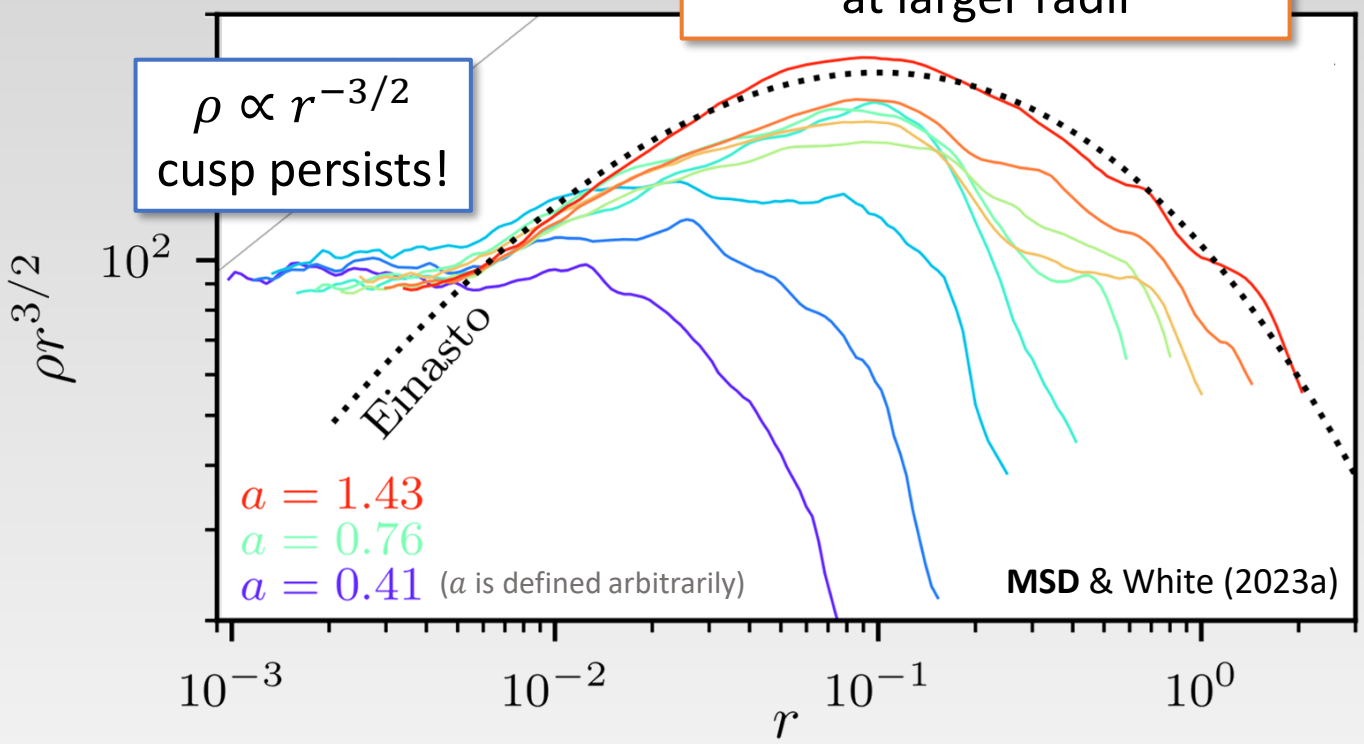
Can evaluate prompt cusp distribution directly from statistics of peaks:



Cusps and halos

Buildup of material makes **standard halo profile** at larger radii

$\rho \propto r^{-3/2}$ cusp persists!



- Most new material has too much energy and angular momentum to reach the central cusp
- Major mergers can affect the center, but effect appears to be small

Every halo/subhalo should have a central prompt cusp!

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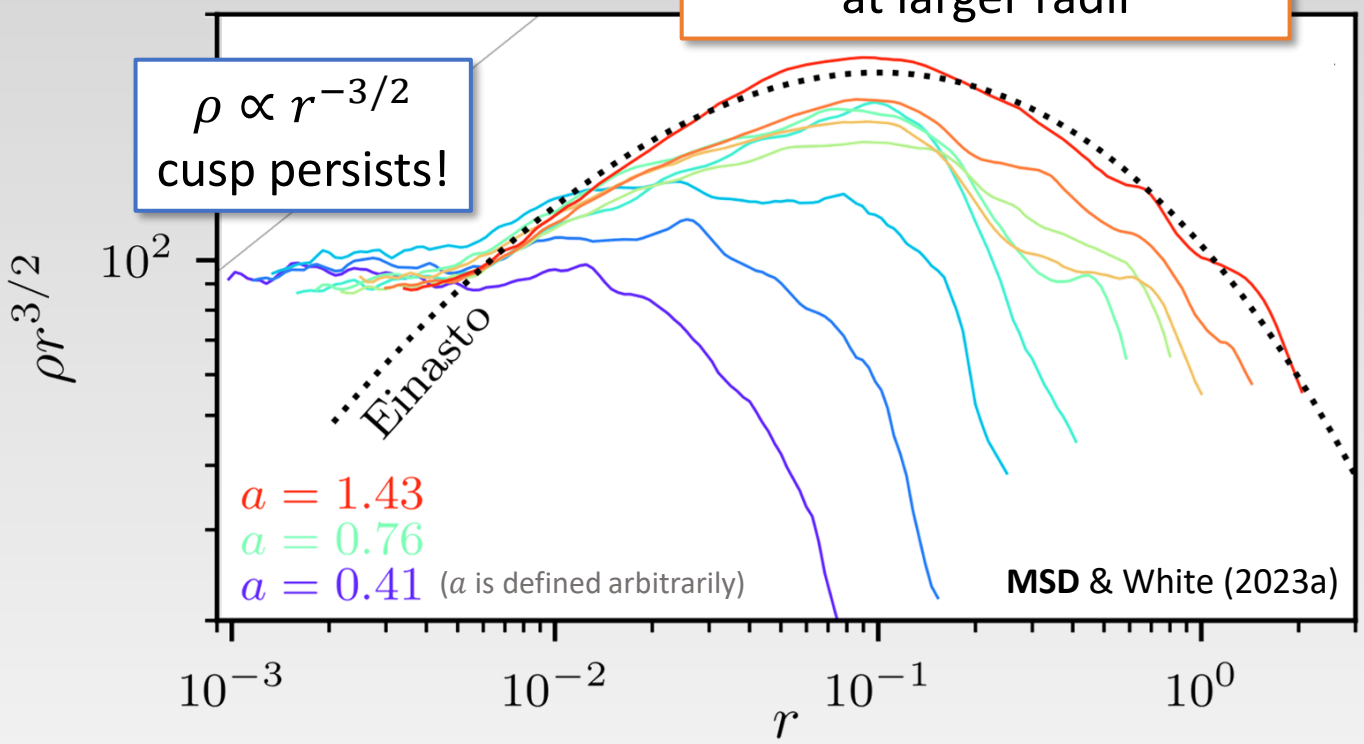

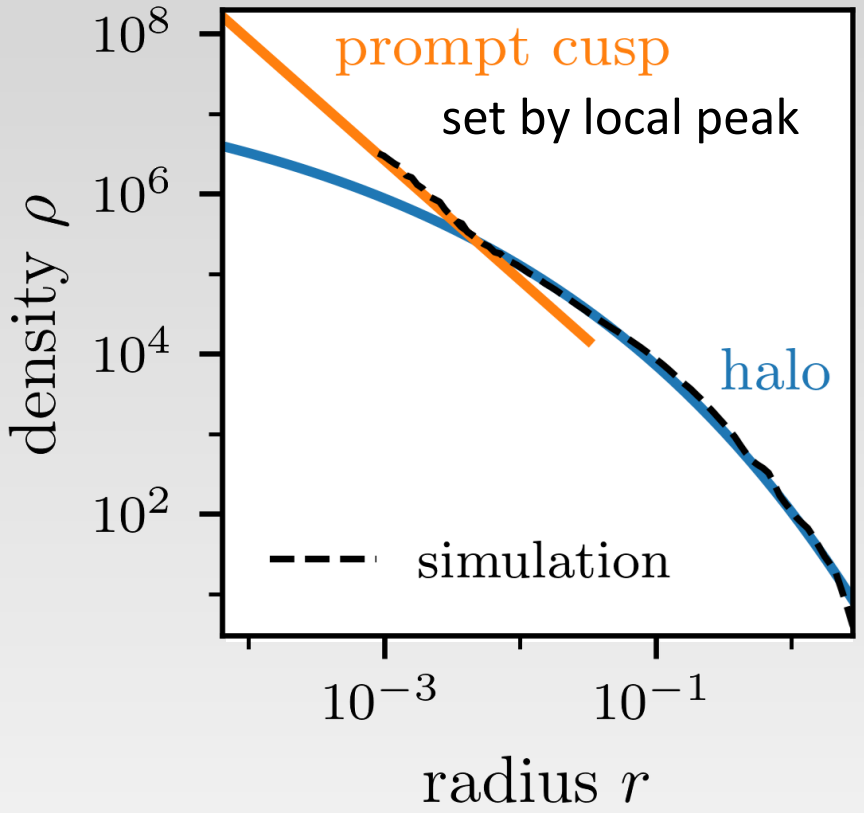

set by growth over time

accreted "atmosphere" of initial cusp

- Most new material has too much energy and angular momentum to reach the central cusp
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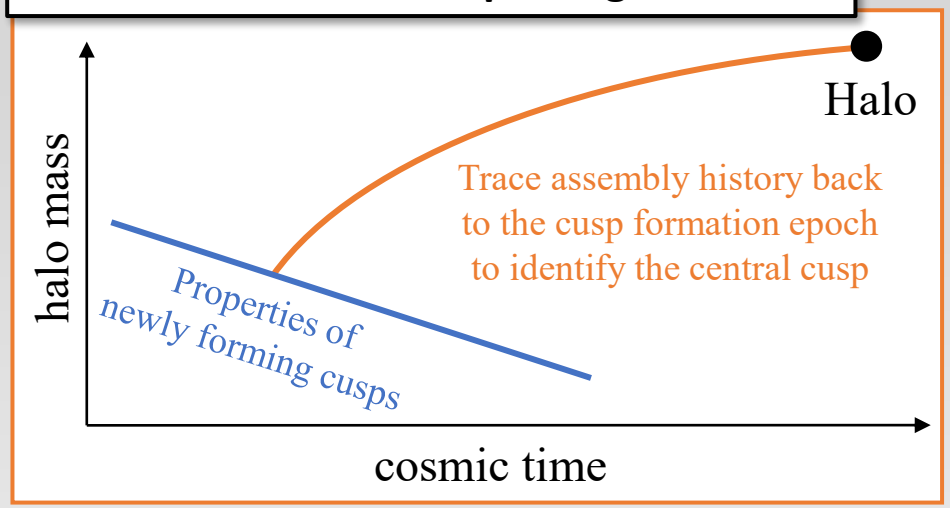
Every halo/subhalo should have a central prompt cusp!

The cusp-halo relation

What is the central cusp of a given halo?



Full prescription for influence of smoothing scale on halo structures:

- arXiv:2506.03240
- https://github.com/delos/cusp-halo-relation

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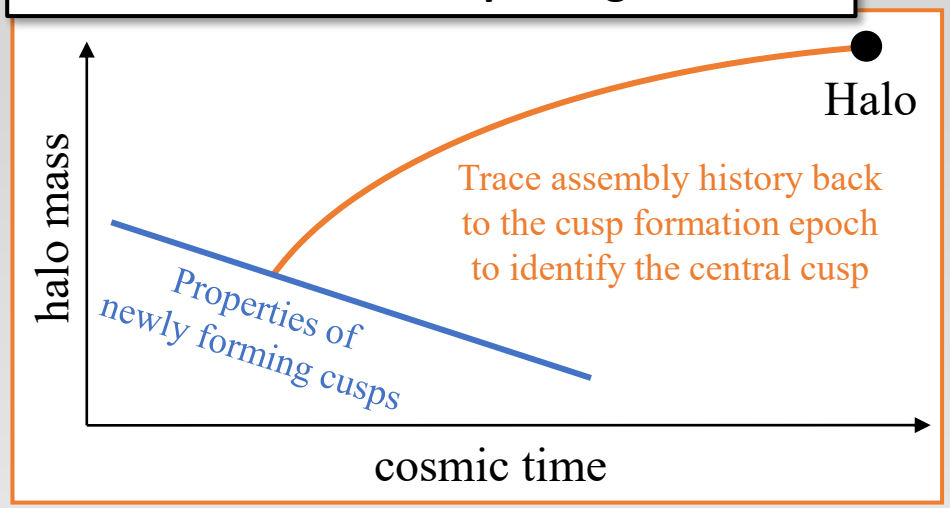

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Prediction from:

- Initial peak → cusp (Delos et al. 2019)
- Growth history → halo (Ludlow et al. 2013)

What is the density profile of a halo with a central prompt cusp?

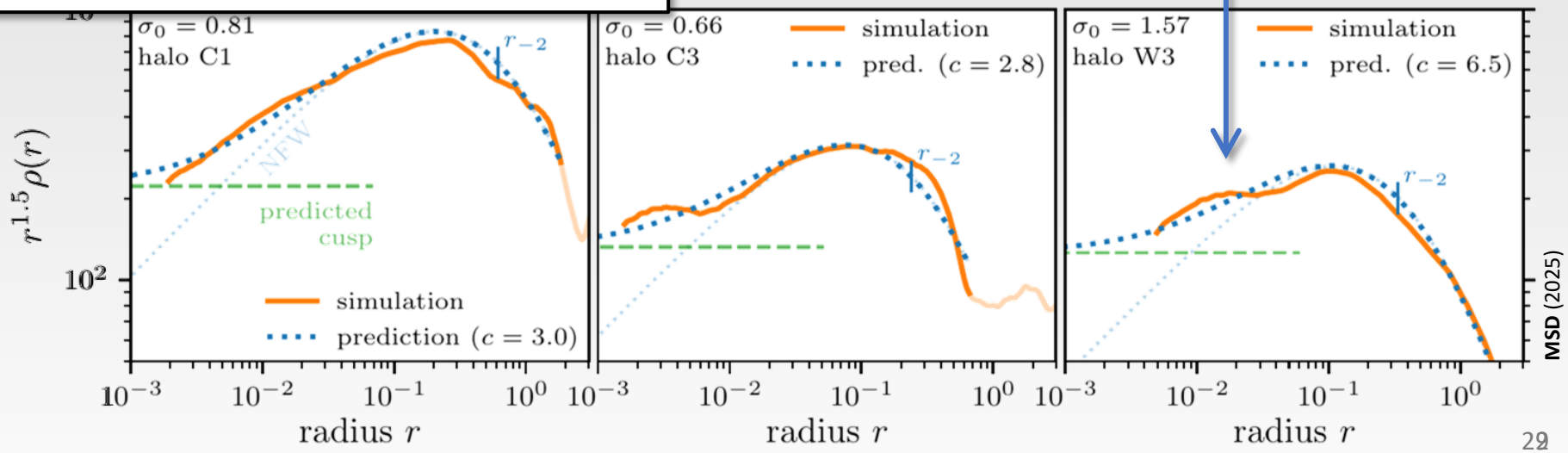
"cusp-NFW"

$$\rho(r) = \frac{\sqrt{y^2 + x}}{x^{1.5}(1+x)^2}\rho_s,$$

$$x = r/r_s, \qquad y = A/(\rho_s r_s^{1.5})$$

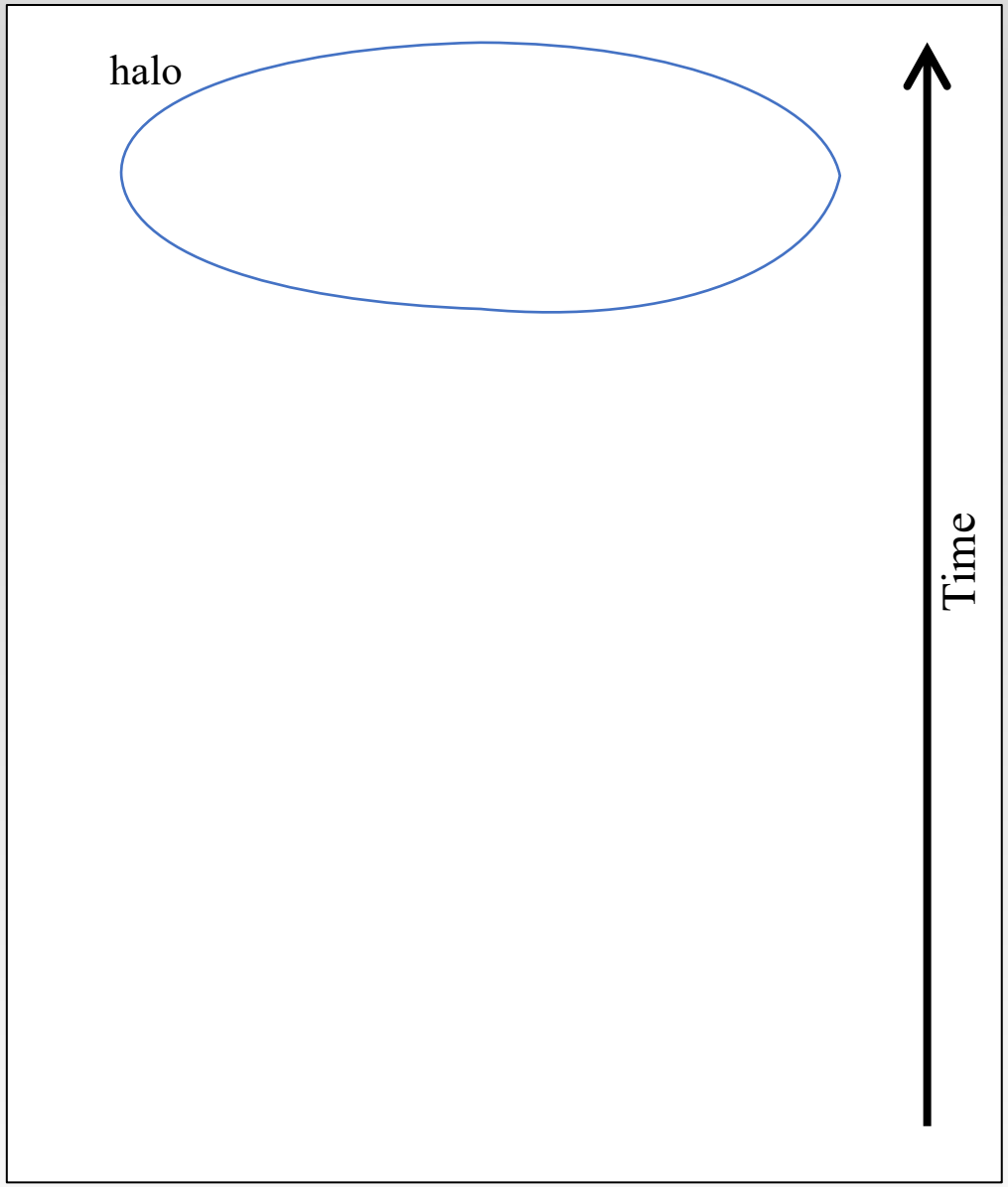
$$0 \le y \le 1$$

NFW — pure cusp



Hierarchical assembly of cosmic structure

Halos grow by merging with other halos



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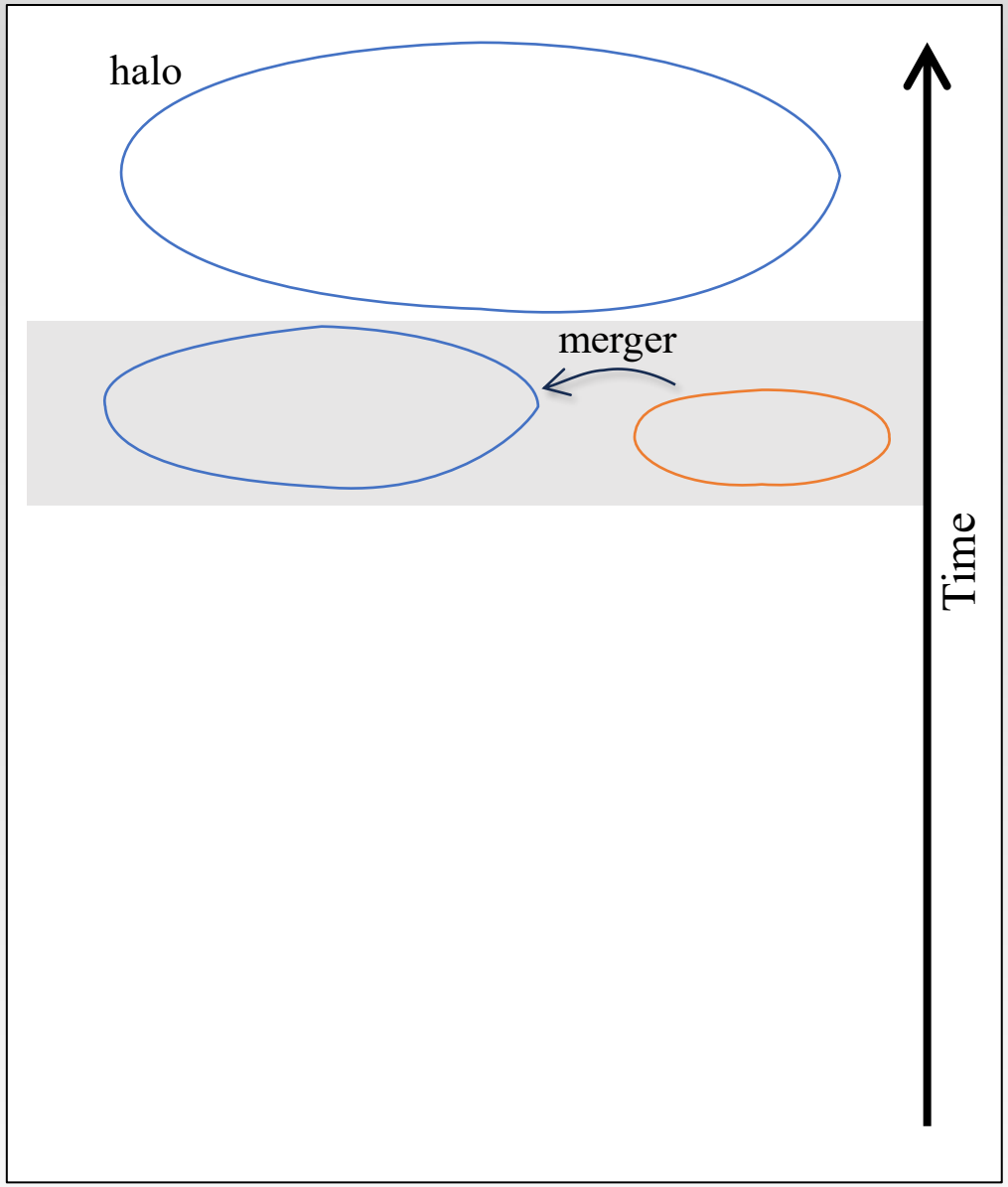

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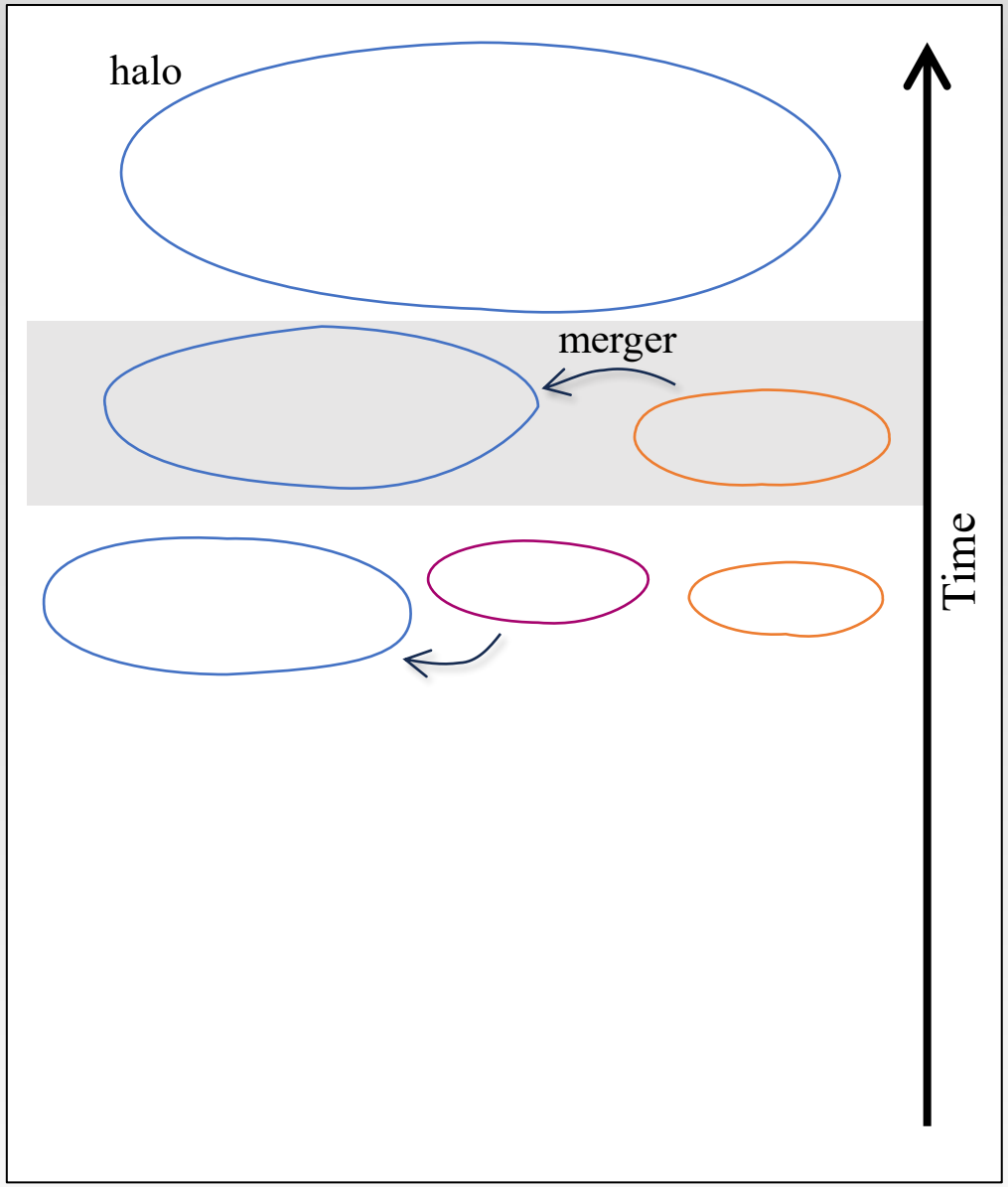

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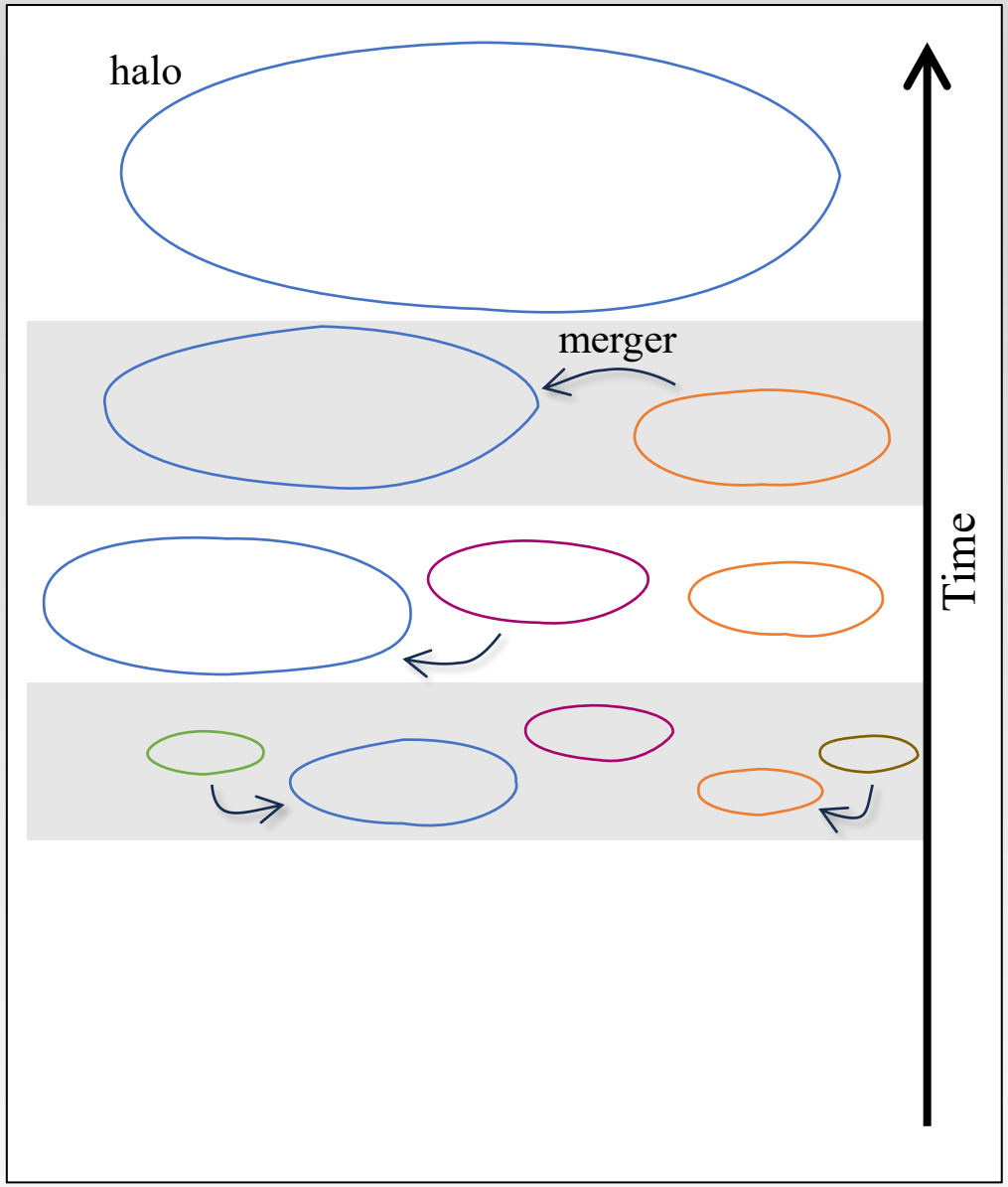

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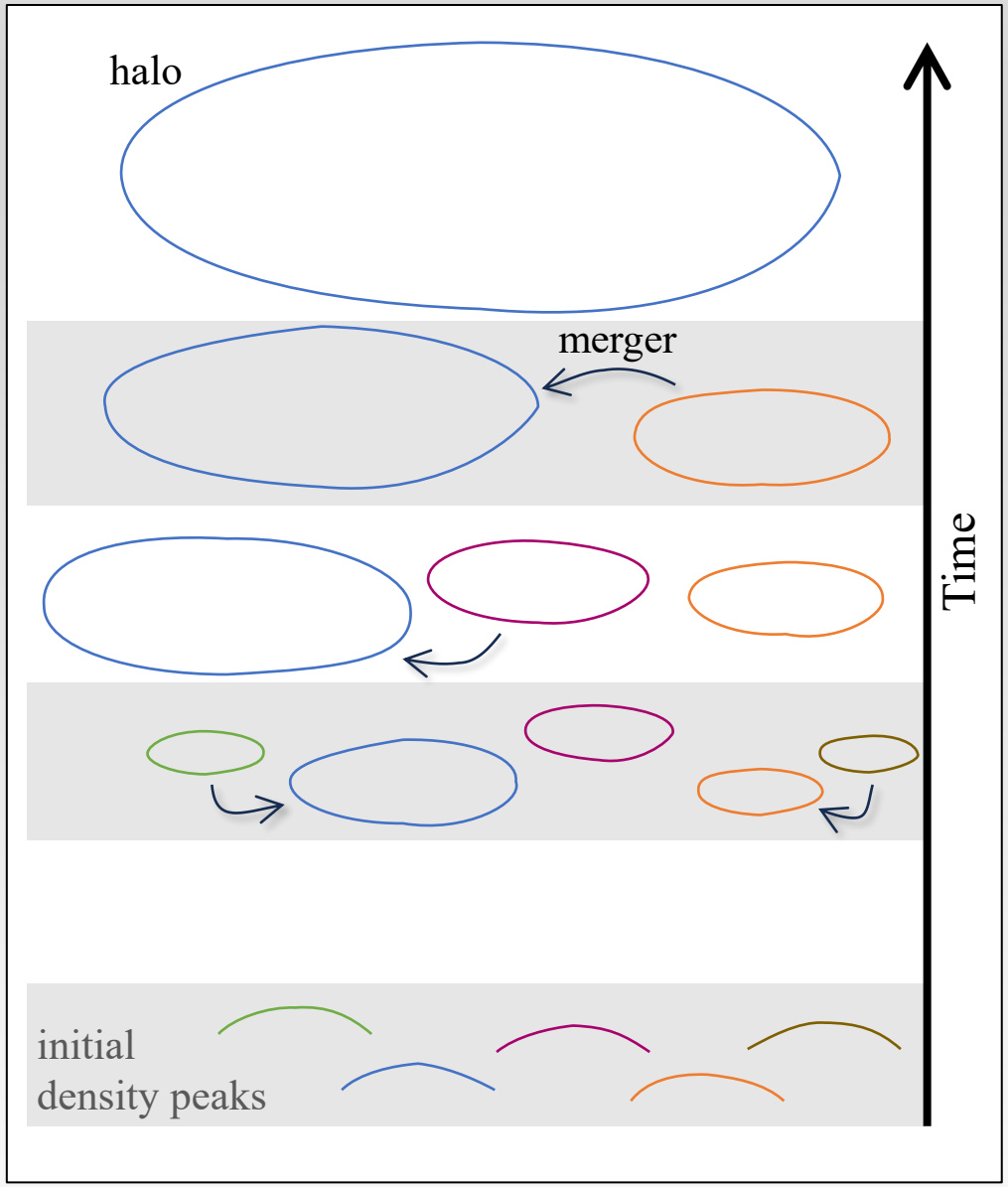
Halos grow by merging with other halos

- Early universe was smooth (no halos)



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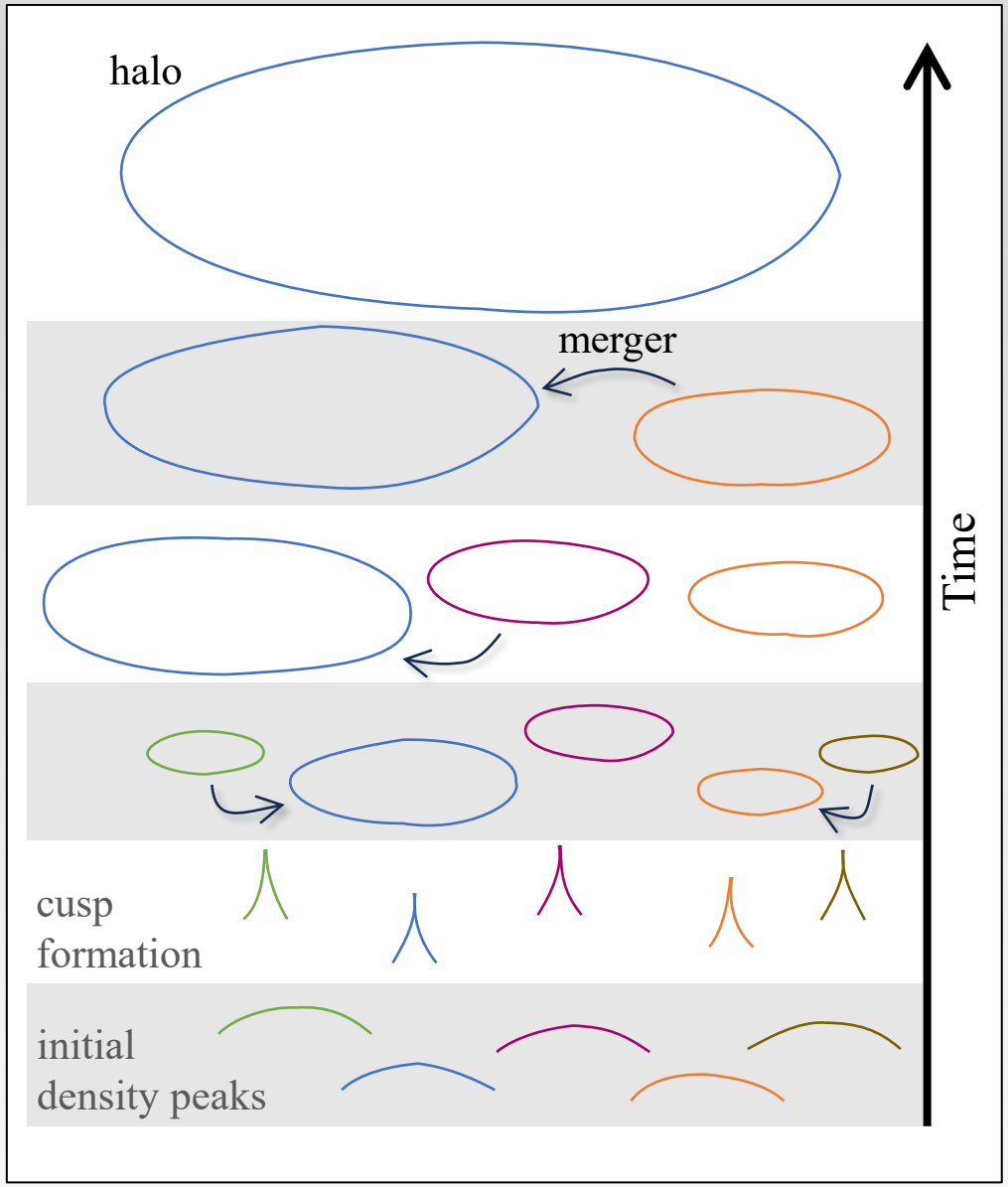
- Early universe was smooth (no halos)
- Prompt cusps: 1st stage of cosmic structure



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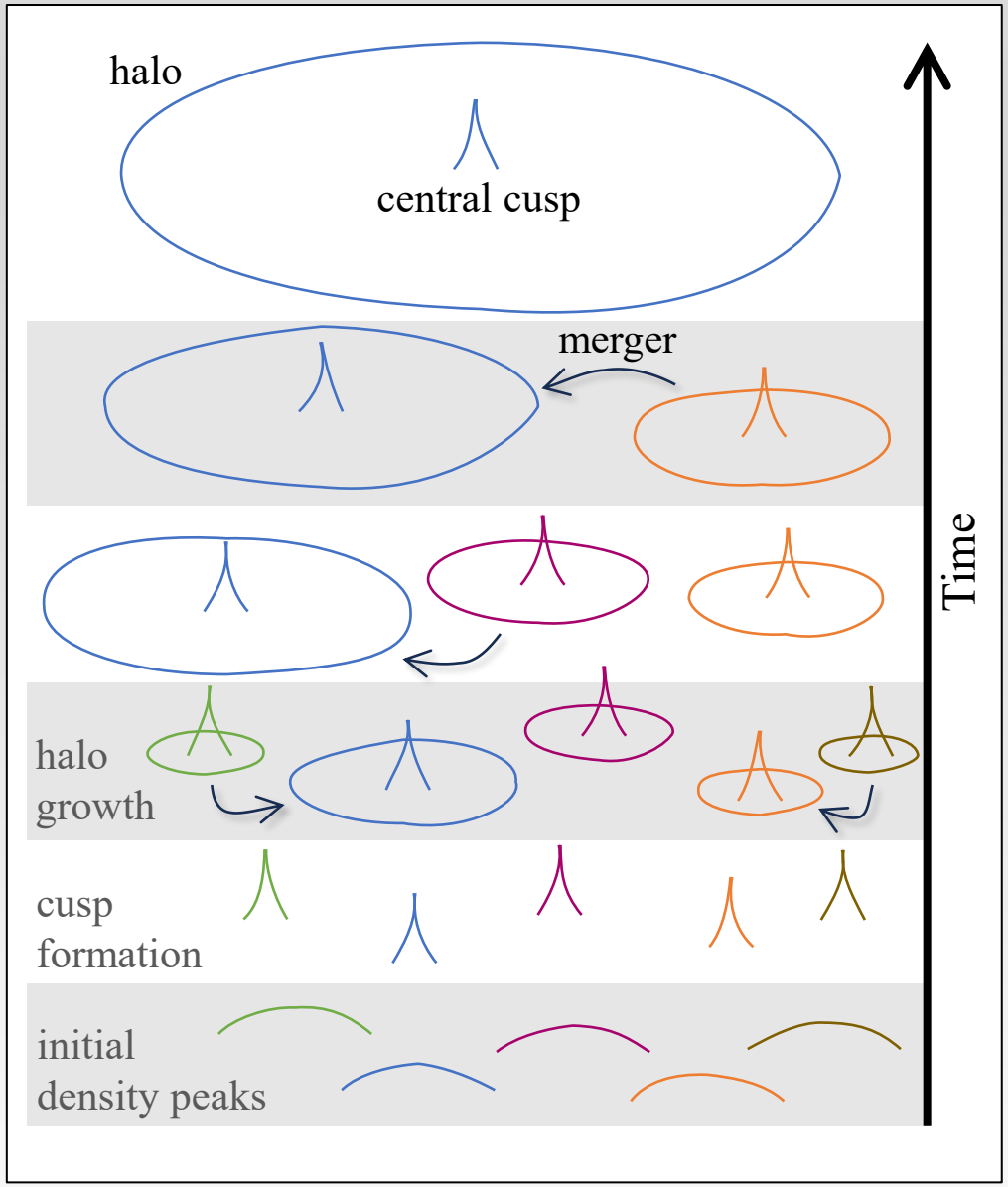
  (but halo can grow much bigger than the initial cusp)



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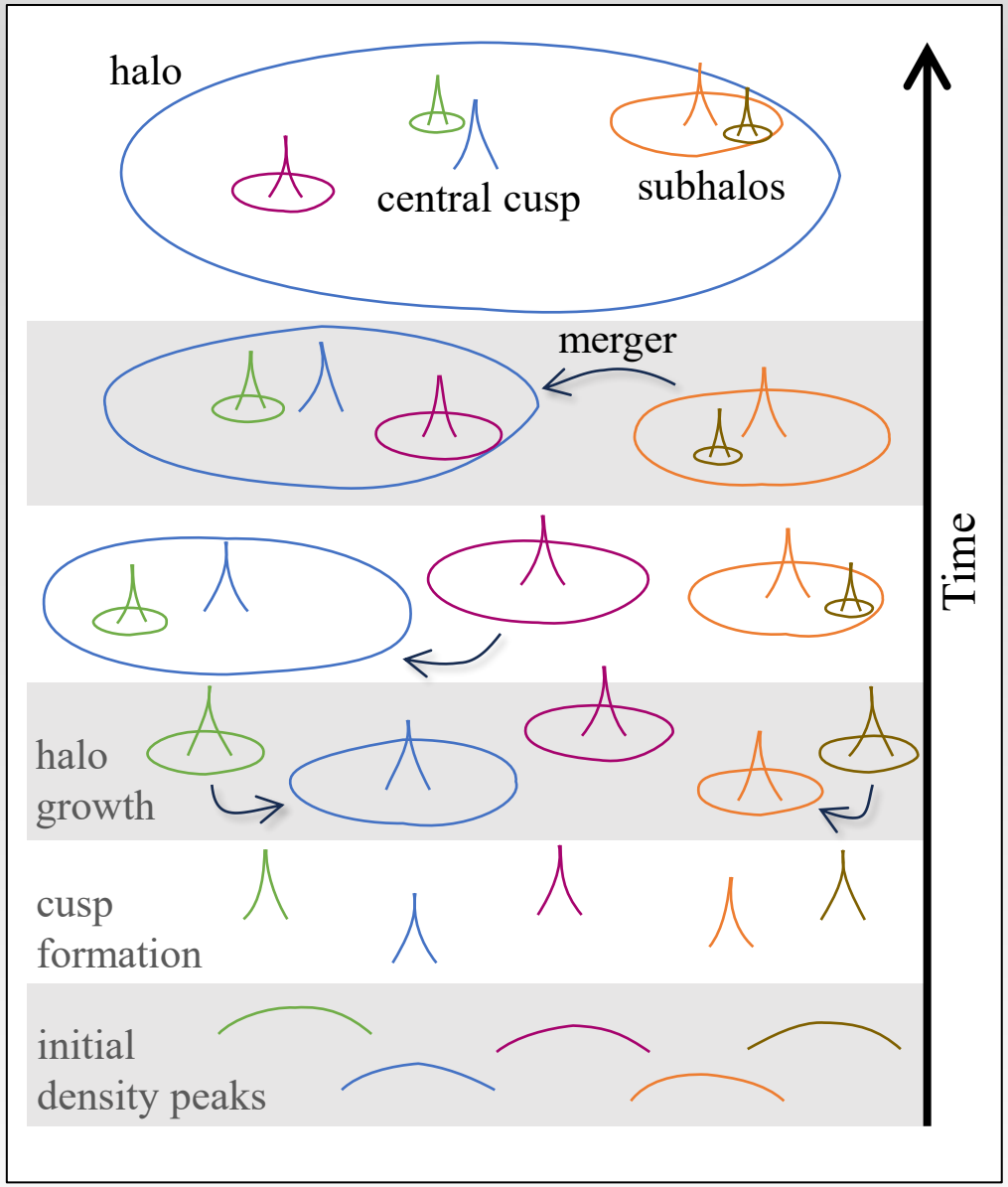
- Early universe was smooth (no halos)
- Prompt cusps: 1st stage of cosmic structure
- Prompt cusps persist at halo centers
  (but halo can grow much bigger than the initial cusp)
- Most prompt cusps survive hierarchical assembly
  (as substructures)



Inner slopes of density profiles

1996 NFW profile

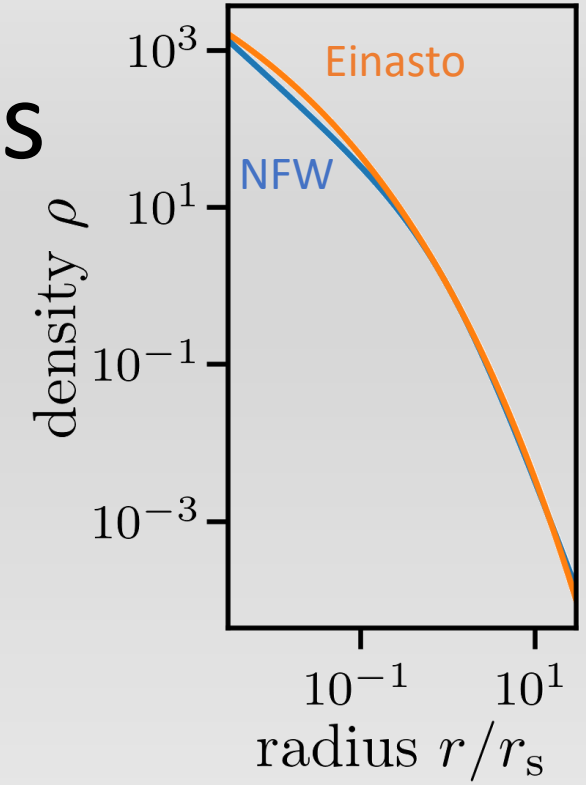
Disagreement about inner slope
(e.g. Moore et al. 1999: $\rho \propto r^{-1.5}$)

2004 Einasto profile

Navarro et al. (including F, W):
"Our density profiles show no sign of converging to a well-defined asymptotic inner power law."

Halo:
no power law

CDM does not predict $\rho \propto r^{-1}$.



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2004	Einasto profile Navarro et al. (including F, W): **"Our density profiles show no sign of converging to a well-defined asymptotic inner power law."**
2006-2020	Diemand Ishiyama Ogiya Angulo etc. $\rho \propto r^{-1.5}$ (or similar) for the smallest halos Softened in hierarchical assembly? (Ogiya, Angulo)
2019-2023	Connection to initial peaks Survival in halo growth Survival in hierarchical assembly } prompt cusps

Halo: no power law

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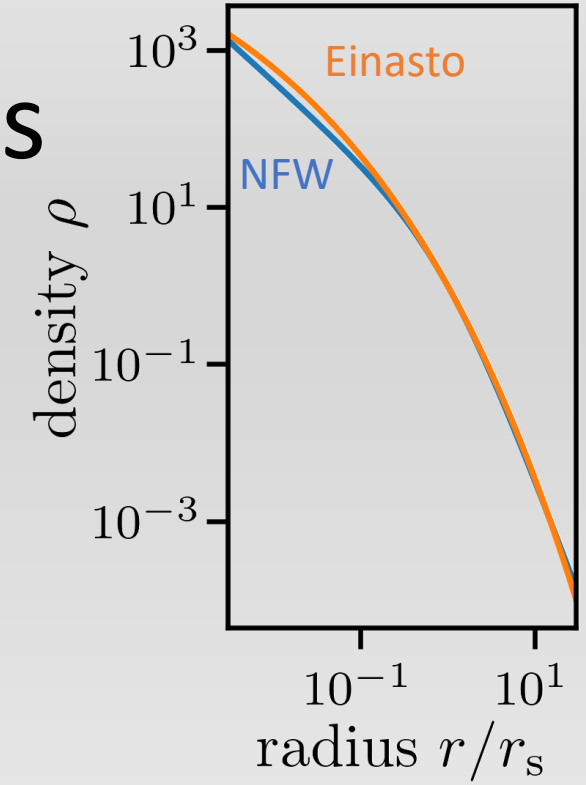
Prompt cusp: pure power law



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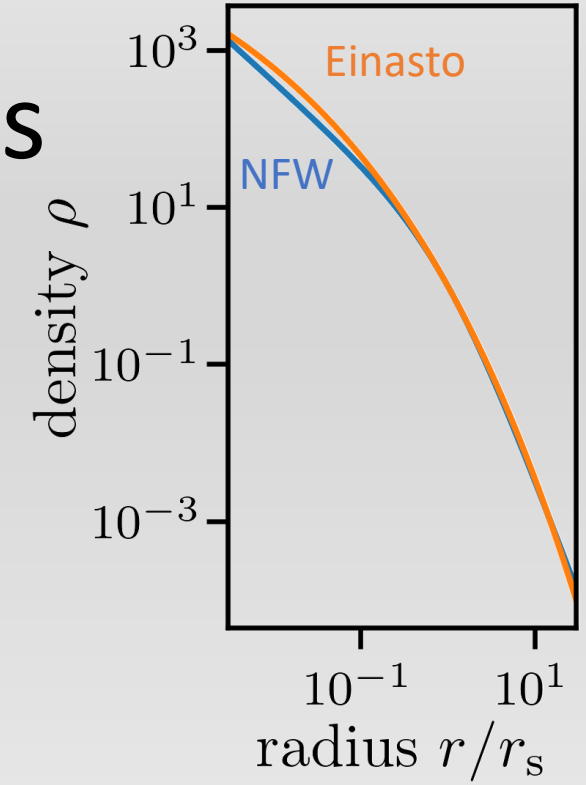

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CDM does not predict $\rho \propto r^{-1}$.

2006-2020 — Diemand, Ishiyama, Ogiya, Angulo, etc.

$\rho \propto r^{-1.5}$ (or similar) for the smallest halos

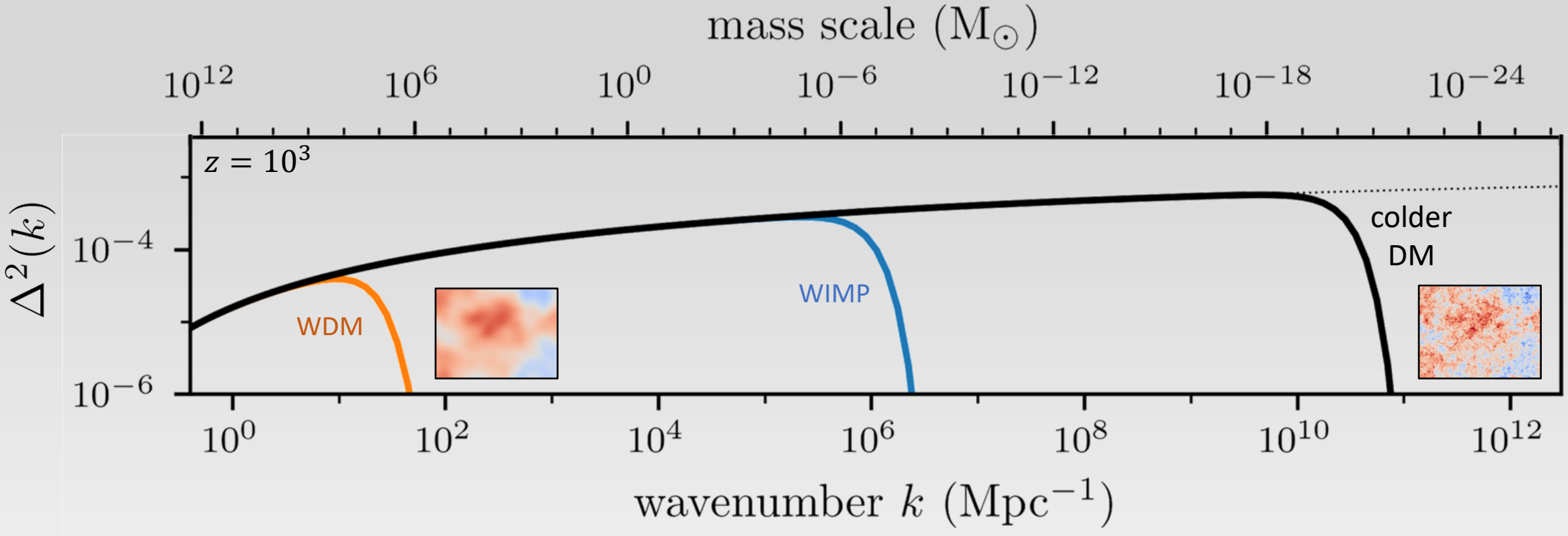
Softened in hierarchical assembly? (Ogiya, Angulo)

2019-2023 — Connection to initial peaks
Survival in halo growth
Survival in hierarchical assembly
} prompt cusps

Prompt cusp:
pure power law

$\rho \propto r^{-1.5}$ confirmed in phase-space tessellation simulations (Colombi 2021; Ondaro-Mallea et al. 2024)
more robustly predicted than Einasto/NFW, which come from N-body only!

Cosmology with prompt cusps



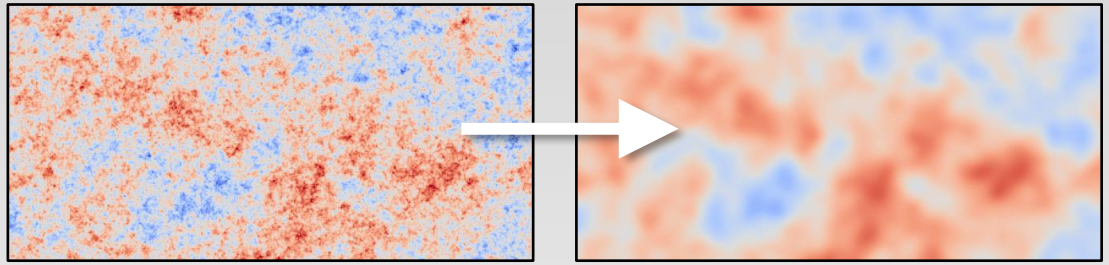
Prompt cusps are a signature of the smoothing scale (e.g., from free streaming)

Prompt cusps of warm dark matter would affect dwarf galaxy structures

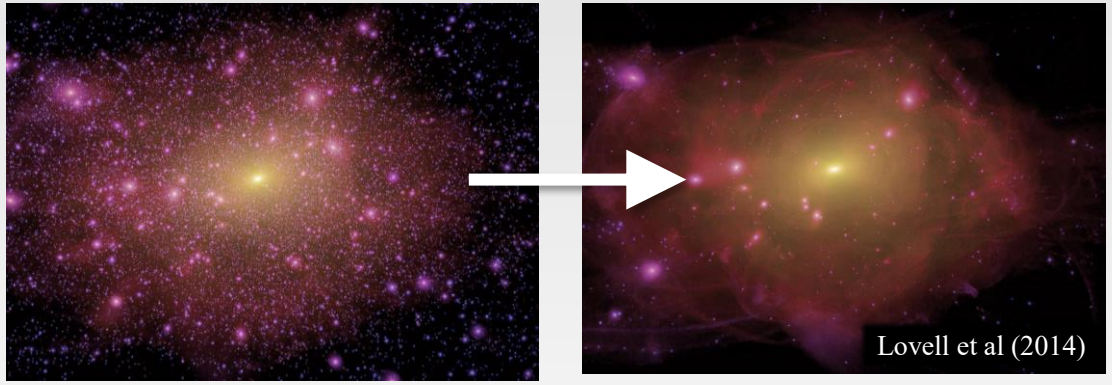
Warm or interacting dark matter

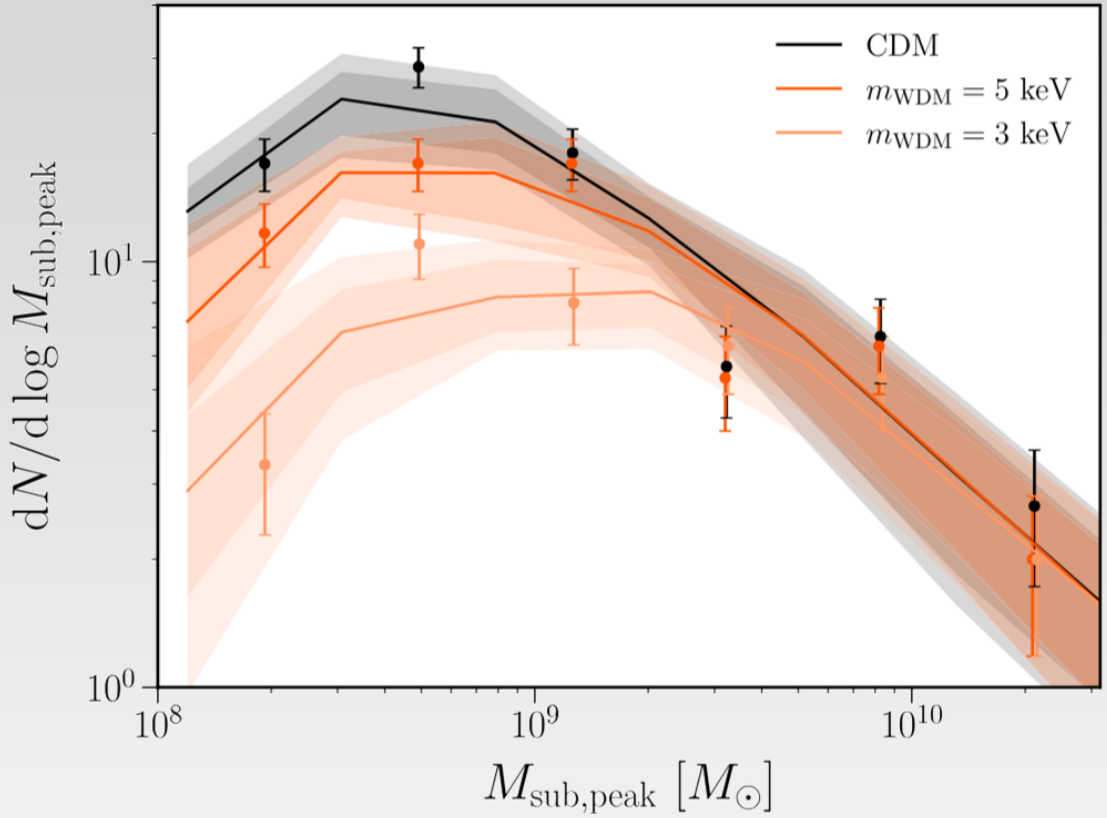
Random particle motion smooths initial conditions

- DM production mechanism?
- Interactions with Standard Model?



which suppresses the abundance of low-mass halos:





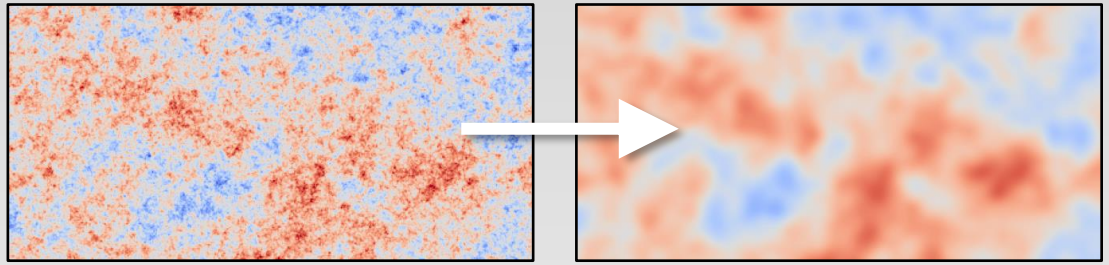
Nadler, An, Gluscevic, Benson, Du (2025)

$m_{\rm WDM} > 5.9$ keV (95% confidence).

Warm or interacting dark matter

Random particle motion smooths initial conditions

- DM production mechanism?
- Interactions with Standard Model?



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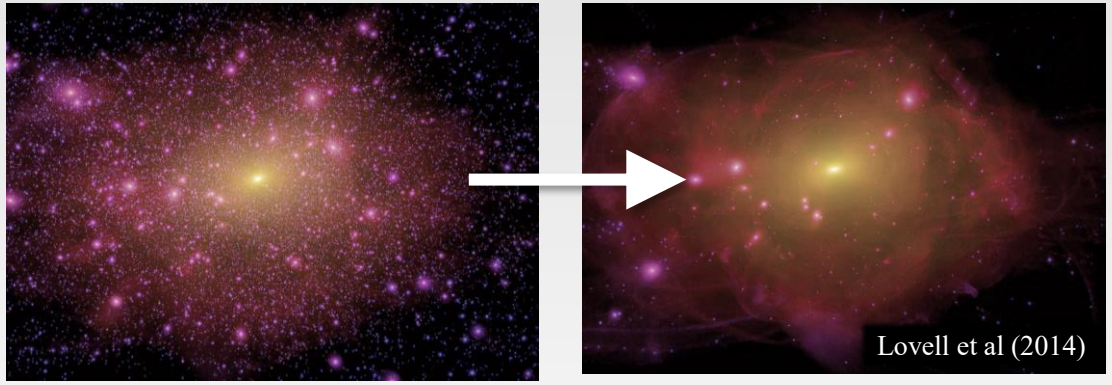
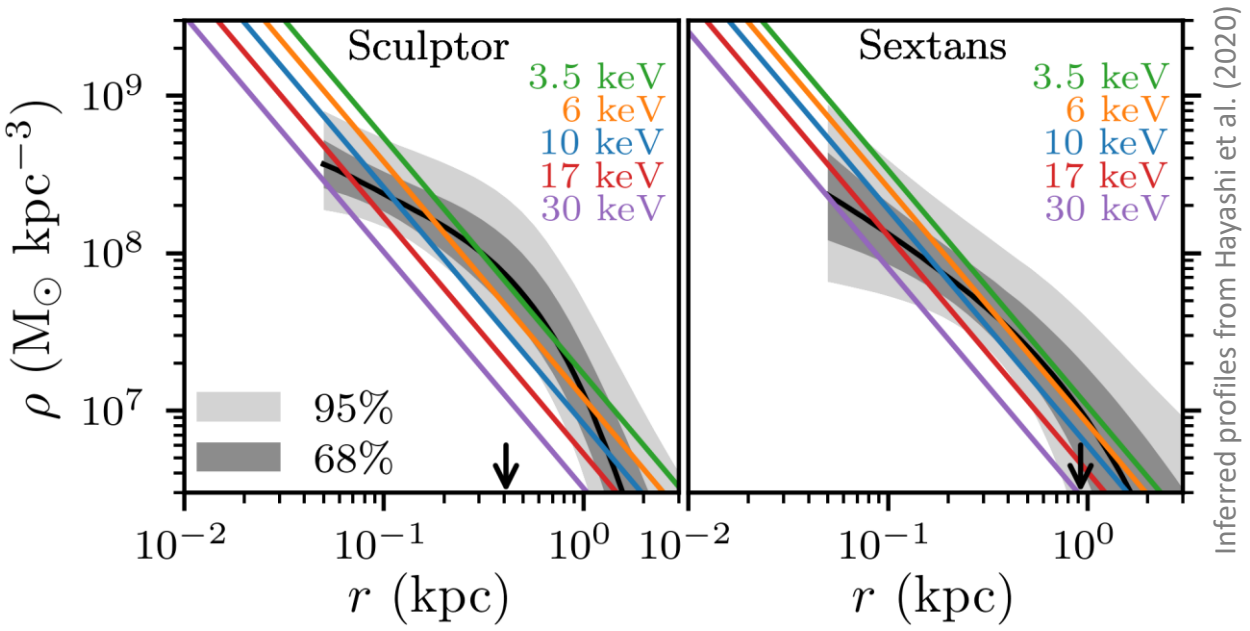


Initial density peaks are much larger
→ Prompt cusps are much more massive



Interpretation: $\rho > \rho_{\rm cusp}$ can be explained by halo growth, but $\rho < \rho_{\rm cusp}$ is more difficult to explain

Uncertainty about

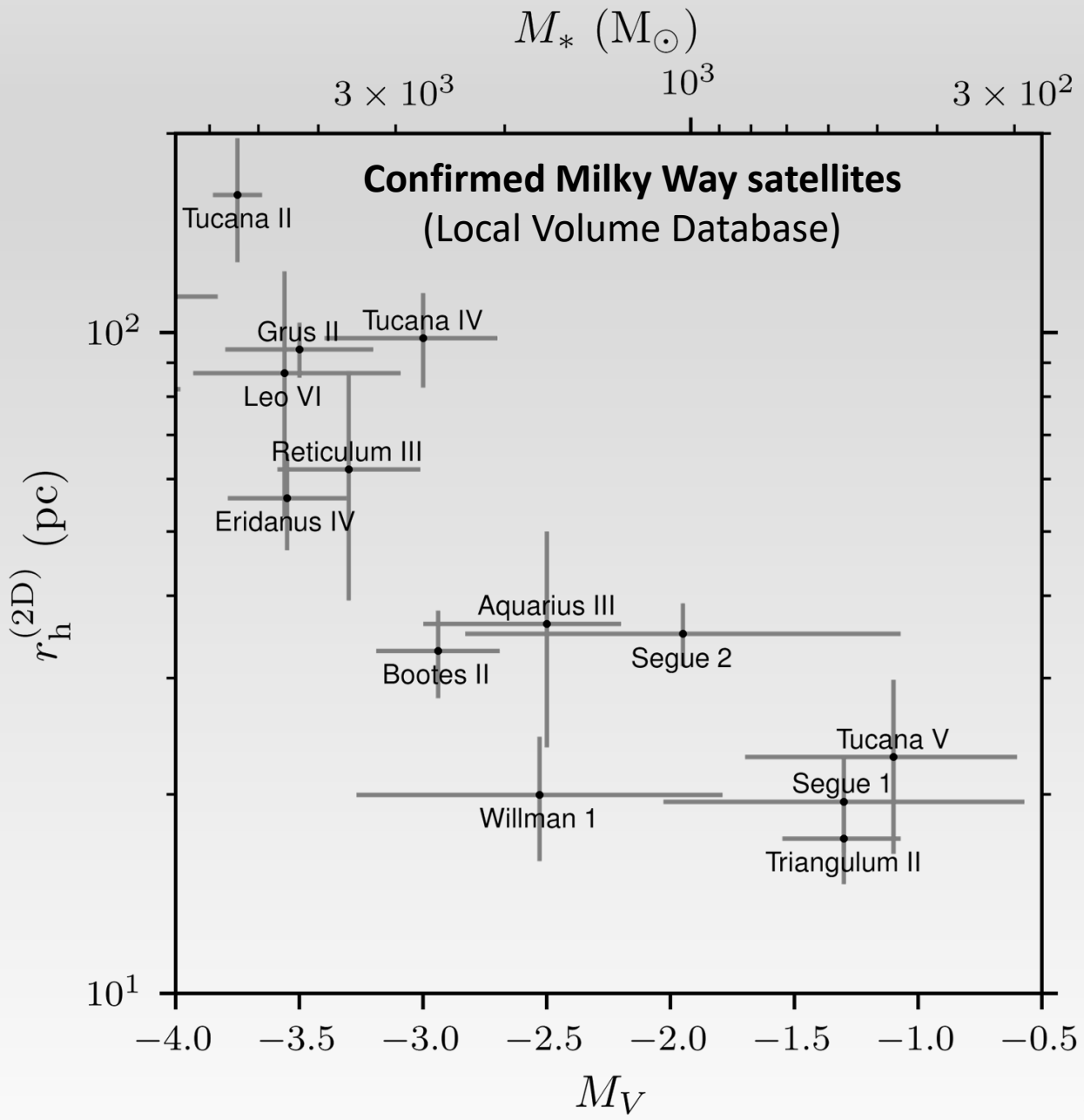
- $r \ll$ half-light radius
- Baryonic feedback

Can do better with smaller galaxies…

Prompt cusps of ultrafaint dwarfs

We want:

- Low half-light radius
  (to measure profile at small r)
- Low brightness
  (so low likelihood of significant feedback)

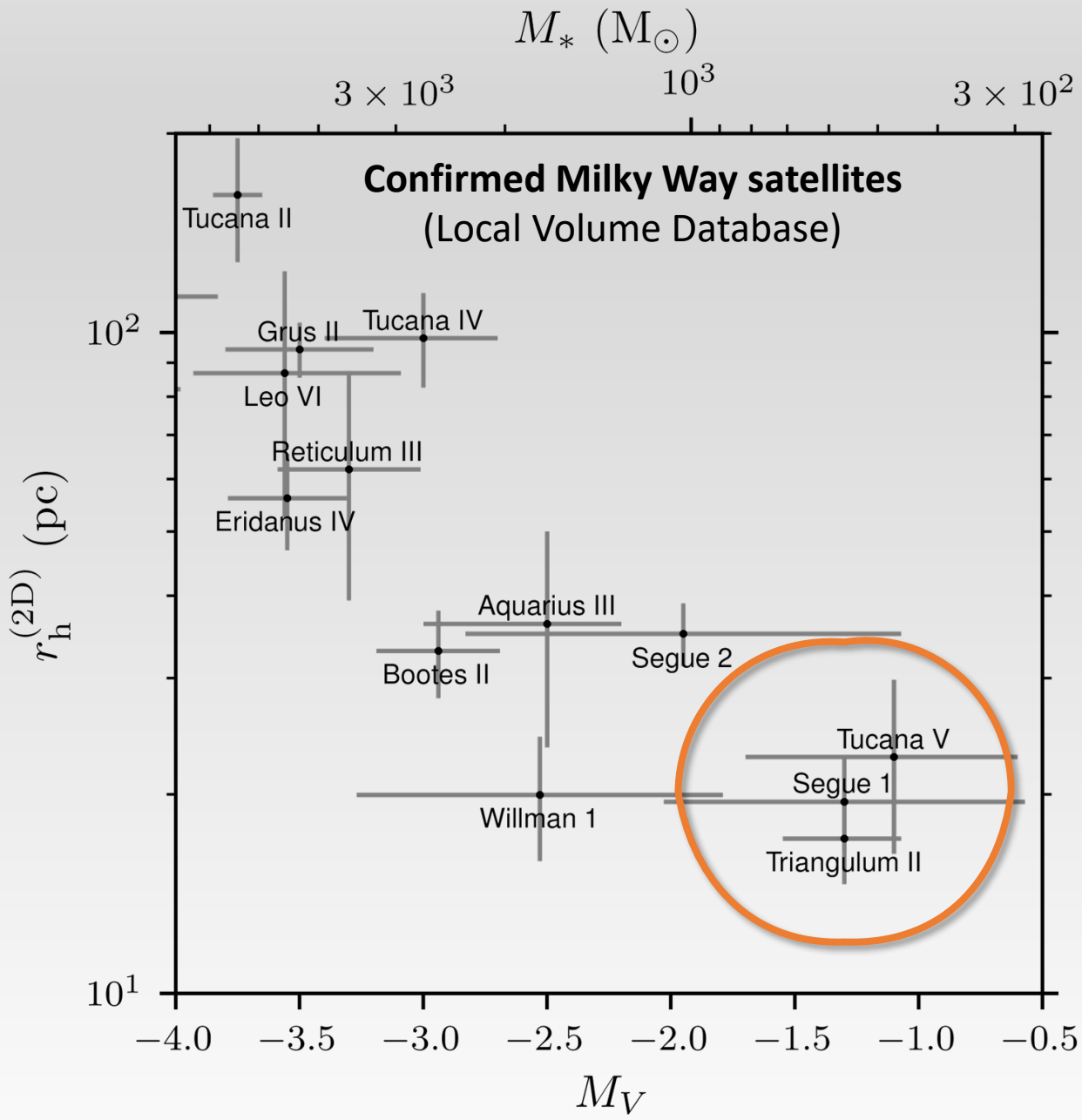


Prompt cusps of ultrafaint dwarfs

We want:

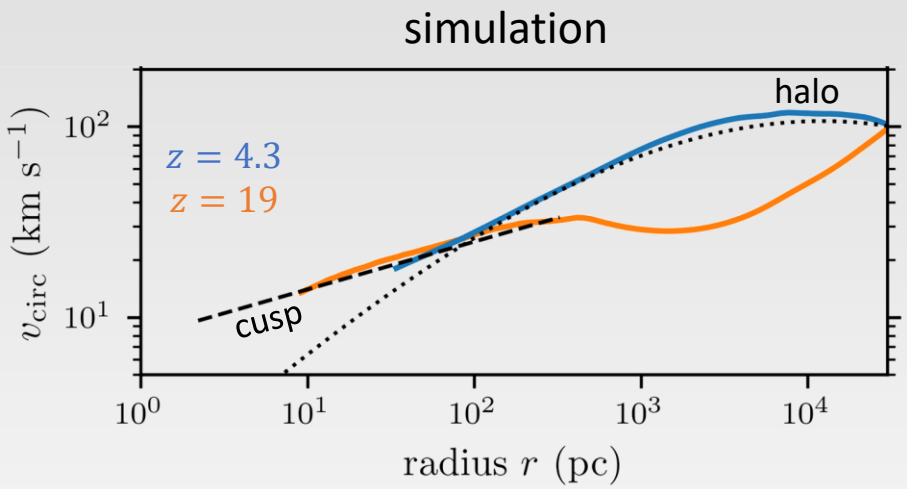
- Low half-light radius
  (to measure profile at small r)
- Low brightness
  (so low likelihood of significant feedback)

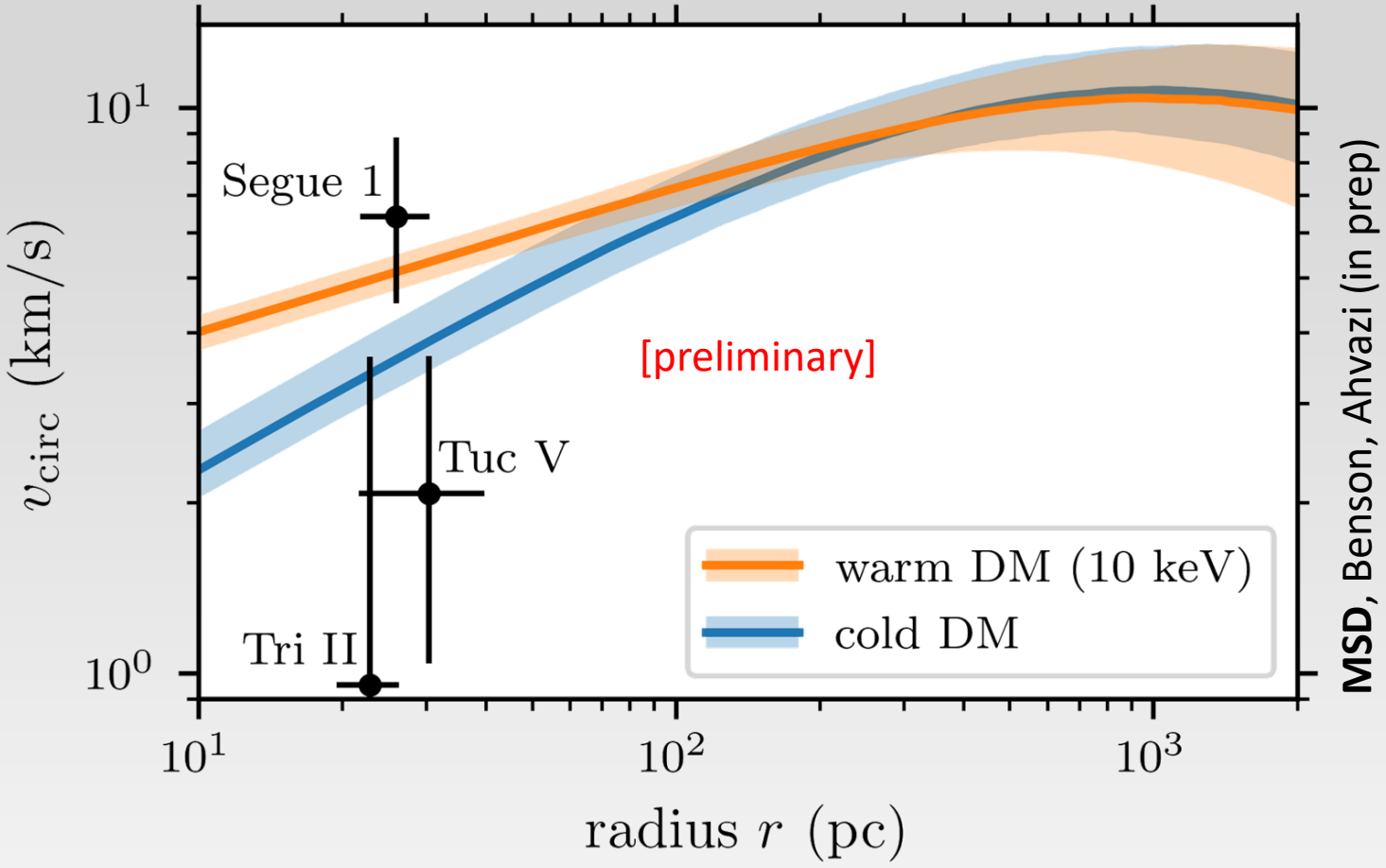
1. Implement prompt cusps in *Galacticus* semianalytic galaxy/subhalo model
   (Benson 2012; calibrated to observed dwarfs by Ahvazi et al. 2024)
2. Identify satellite galaxies similar to Tuc V/Segue 1/Tri II
   (in luminosity/size/orbit)



Prompt cusps of ultrafaint dwarfs

Compare $v_{\rm circ}$ at half-light radius for the 3 faintest confirmed dwarf galaxies

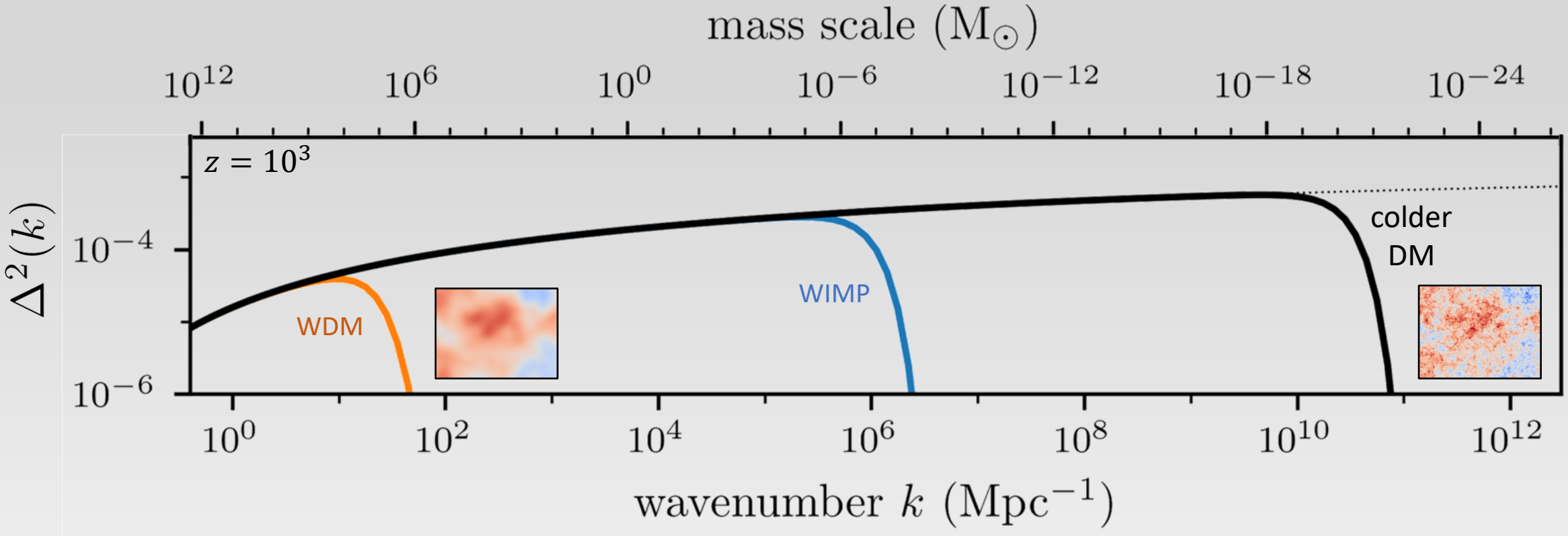




Tuc V and Tri II are $\sim 98\%$ outliers for 10 keV WDM [preliminary]

Segue 1 is consistent with WDM and CDM

Cosmology with prompt cusps



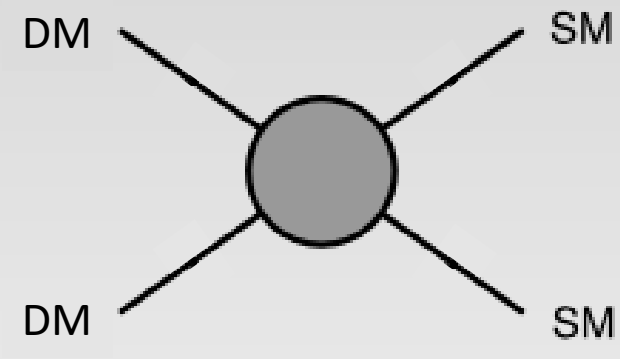
Prompt cusps are a signature of the smoothing scale (e.g., from free streaming)

Prompt cusps of warm dark matter would affect dwarf galaxy structures

Prompt cusps boost dark matter annihilation by a factor of ~ 10

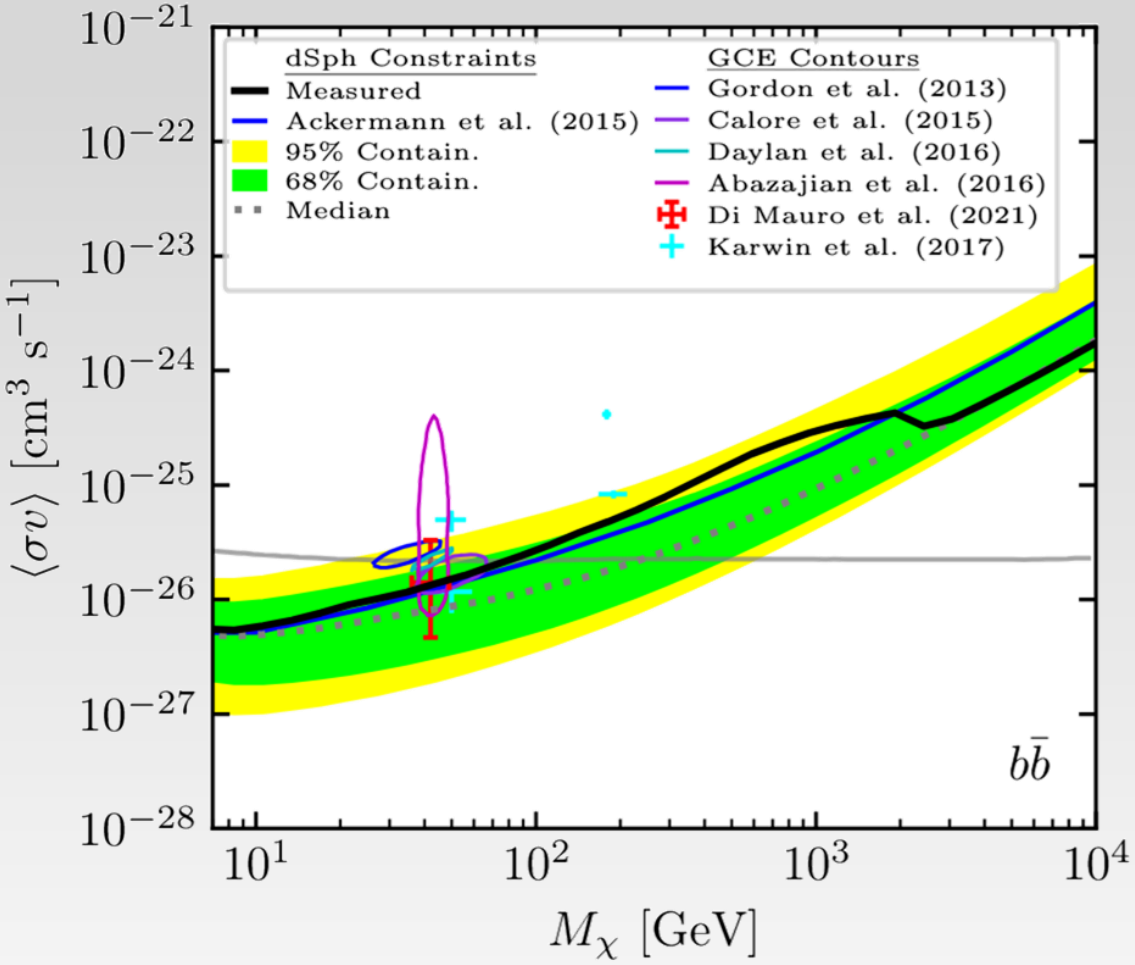
Annihilating dark matter

dark matter particle pair-produced
in the early universe



can annihilate into detectable
SM particles today

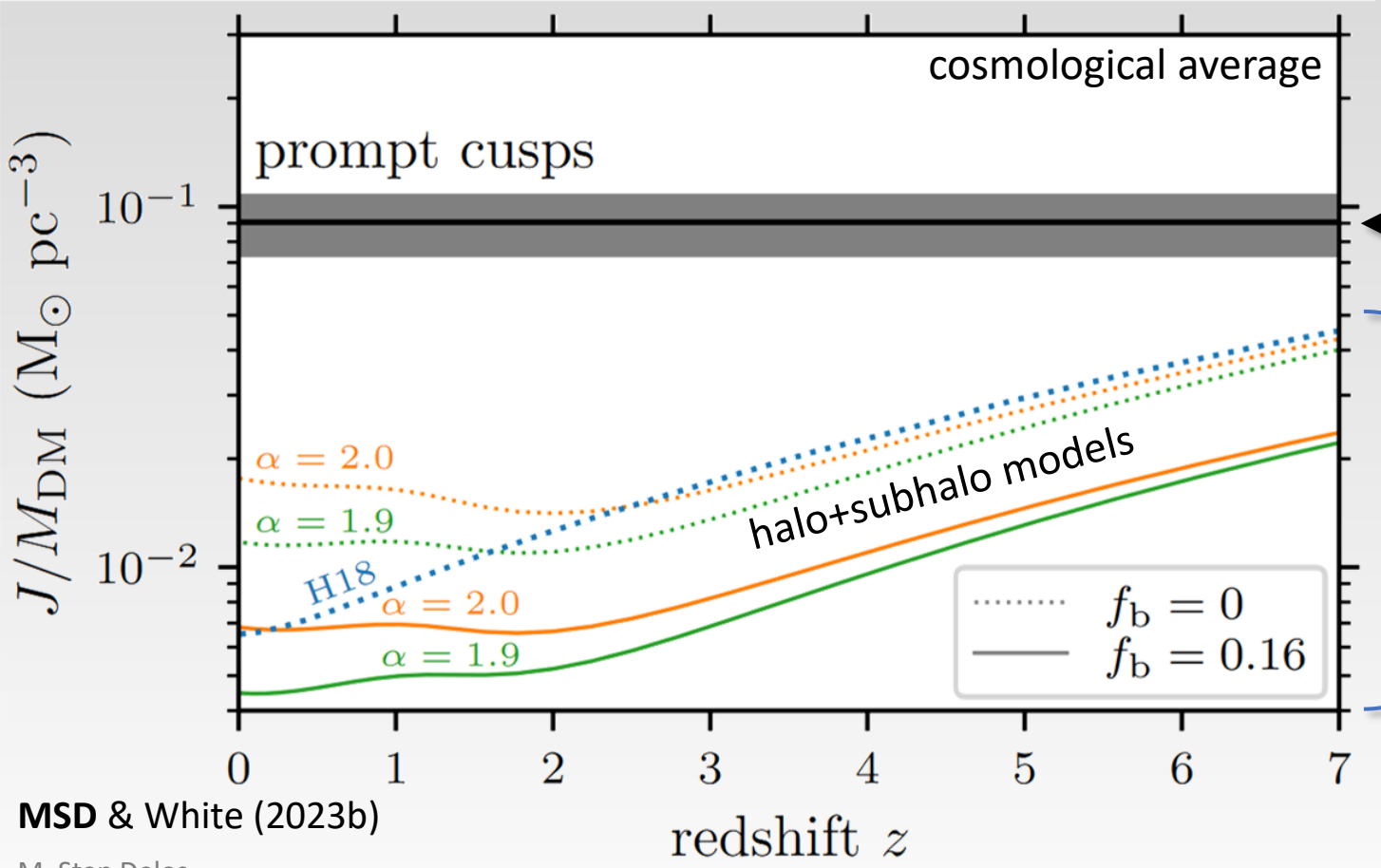
Annihilation rate $\propto \rho^2$



McDaniel, Ajello, Karwin, Di Mauro,
Drlica-Wagner, Sànchez-Conde (2024)

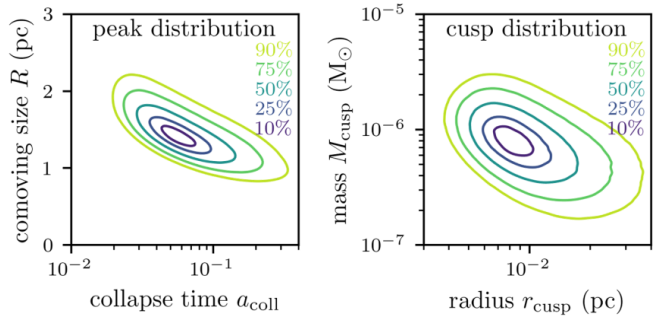
Annihilation in prompt cusps

Abundance and internal density of prompt cusps greatly boost the annihilation rate



MSD & White (2023b)

Directly from statistics of peaks & peak-cusp connection*



Previous predictions: halo & subhalo models

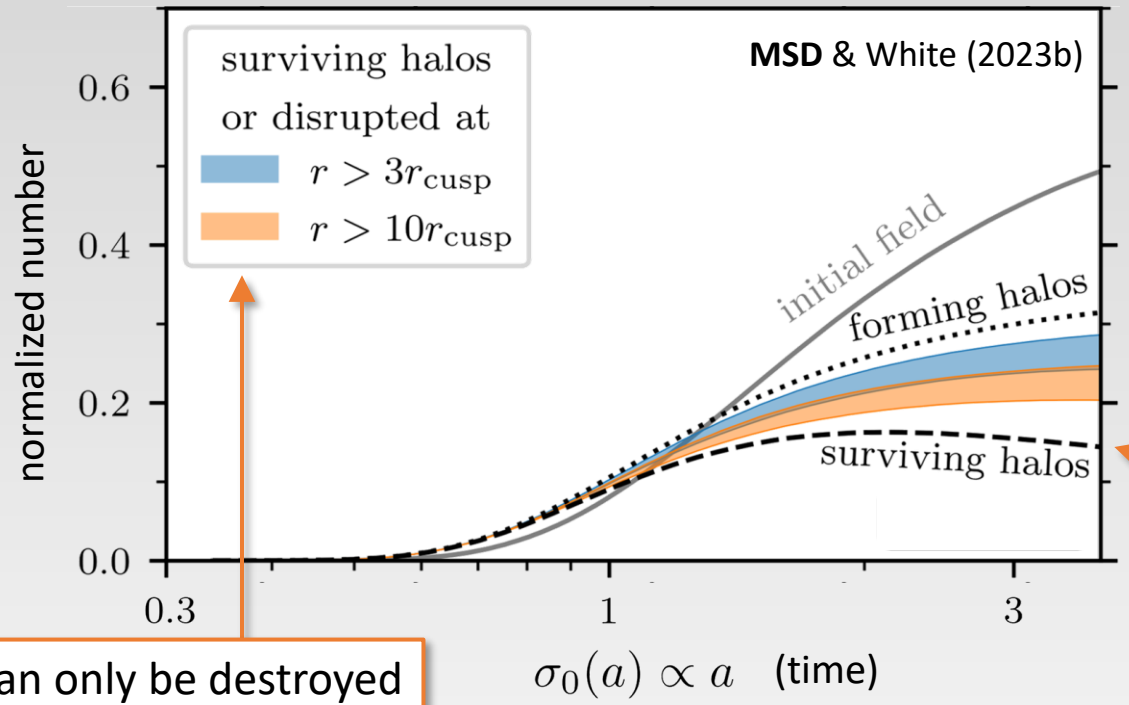
- Extrapolate from much larger scales: $\frac{dN}{dm} \propto m^{-\alpha}$
- Semianalytic modeling (neglected baryons!)

*about 1/2 of initial peaks can be associated with cusps due to halo clustering

Cusp survival in halo clustering

What fraction of initial peaks can be associated with prompt cusps?

Simulation estimate:



Simulated subhalos are artificially fragile
(and difficult to algorithmically identify)

Real subhalos can only be destroyed if they sink to the center of the host
(e.g. Errani & Peñarrubia 2019)

Outcome: $\sim 1/2$ of collapsed peaks can be associated with prompt cusps.

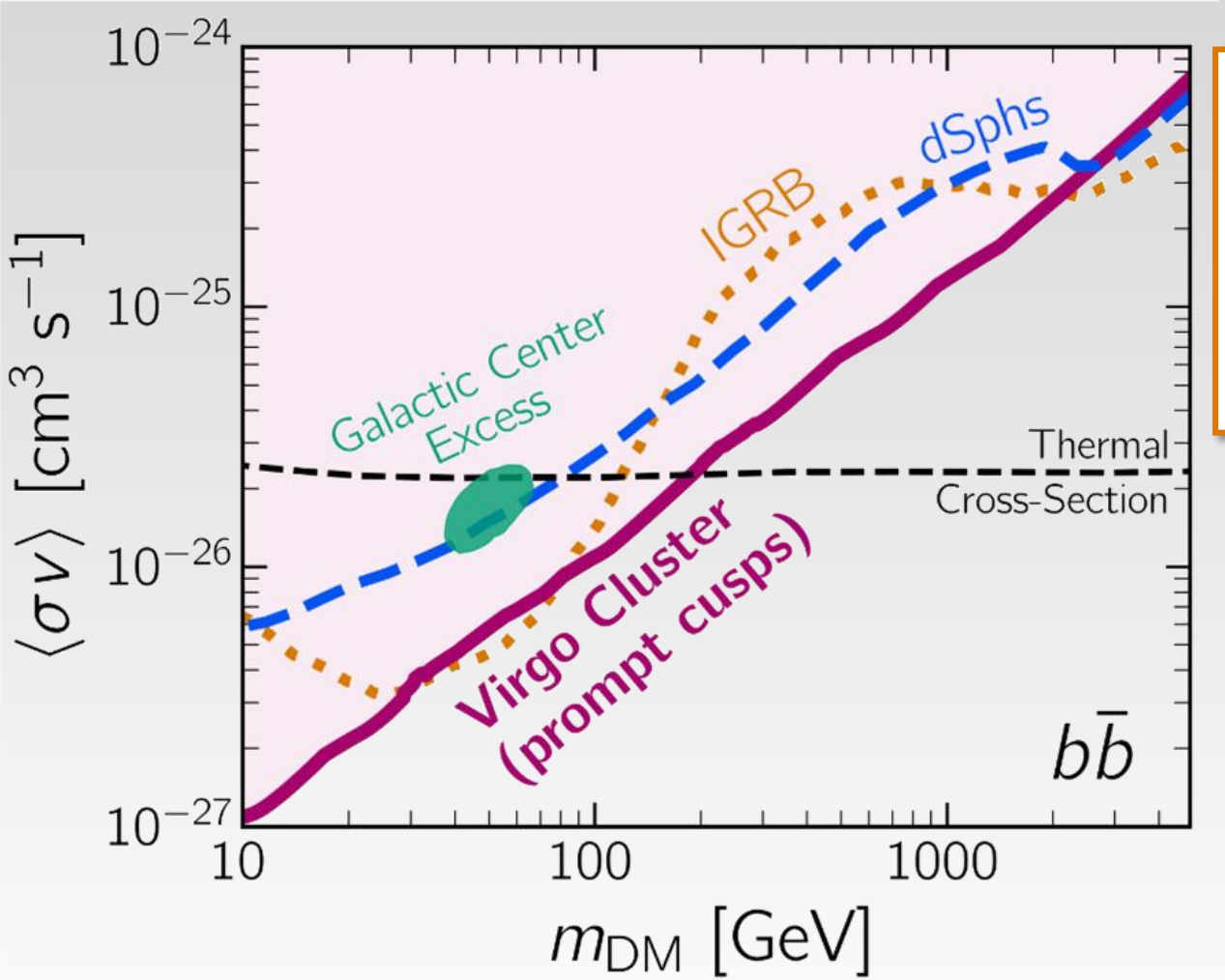
but this depends mildly on $\Delta^2(k)$ (Ganjoo & **MSD** 2025)

Limits on dark matter annihilation

Annihilation rate ∝ number of cusps ∝ total mass of DM

No large annihilation boost in Galactic Center & nearby satellite galaxies:

- Density already high in smooth halo
- Substructure tidally disrupted



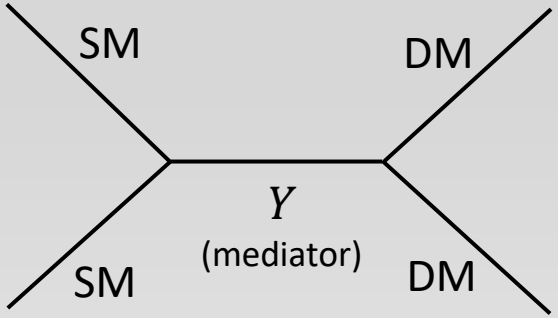
from prompt cusps contributing to isotropic γ-ray background (IGRB)

(**MSD**, Korsmeier, Widmark, Blanco, Linden, White 2024)

from prompt cusps in nearby galaxy clusters

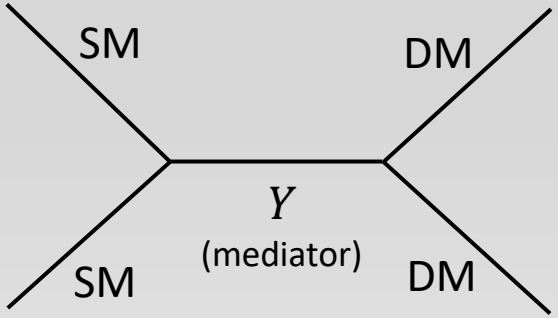
(Crnogorčević, **MSD**, Kuritzén, Linden 2025)

Limits on hidden-sector dark matter



Idea: produce DM thermally while evading colliders/direct detection
(e.g. Berlin, Hooper, Krnjaic 2016)

Limits on hidden-sector dark matter

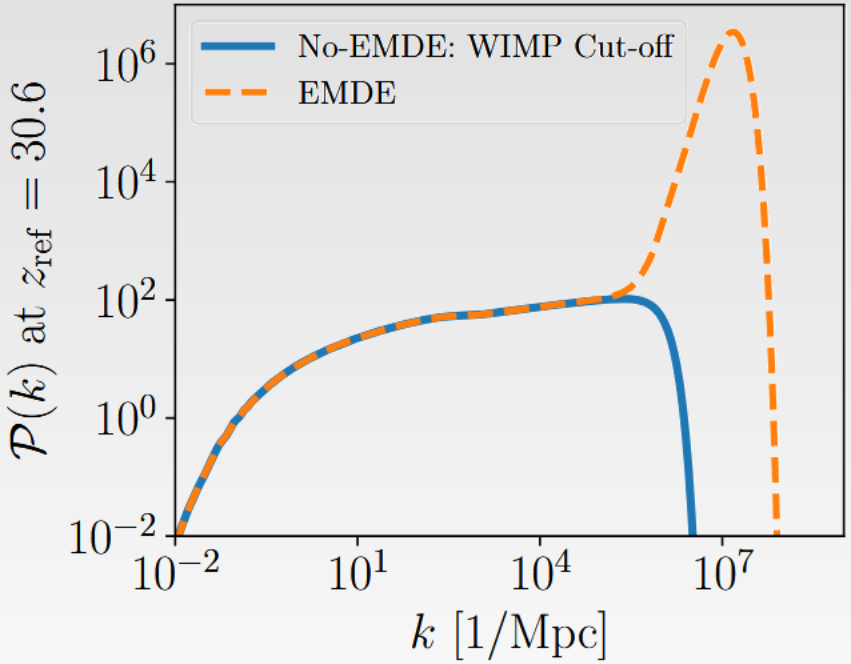


Idea: produce DM thermally while evading colliders/direct detection (e.g. Berlin, Hooper, Krnjaic 2016)

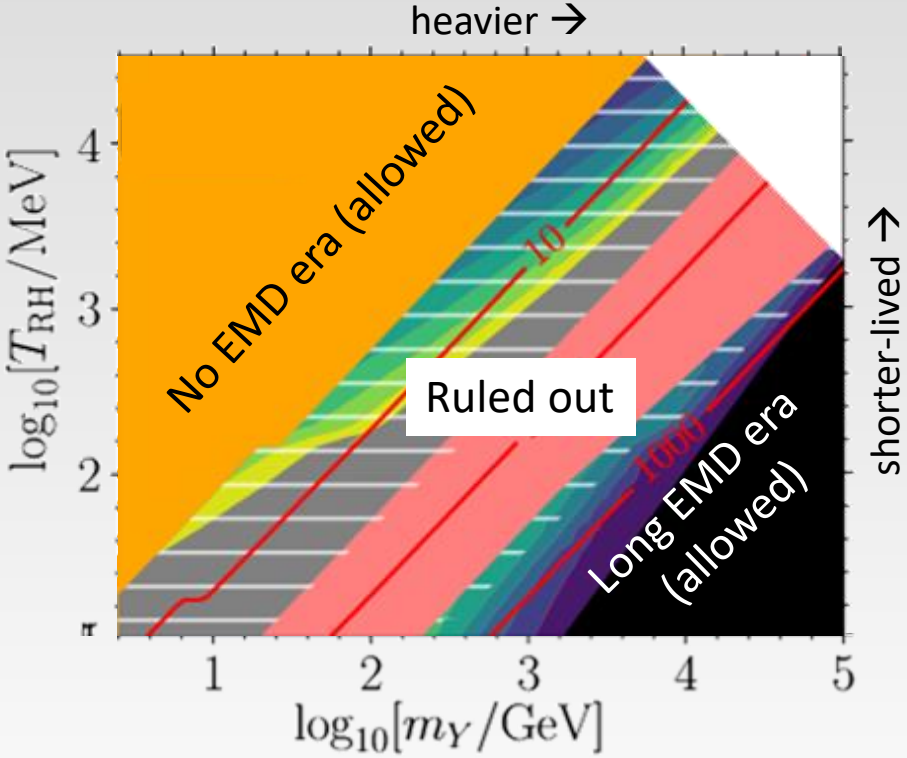
Himanish Ganjoo

Mediator Y can drive early matter-dominated era depending on mass/lifetime

- hugely boosted annihilation rates
- **strong indirect-detection constraints**



Ganjoo & **MSD** (2025)



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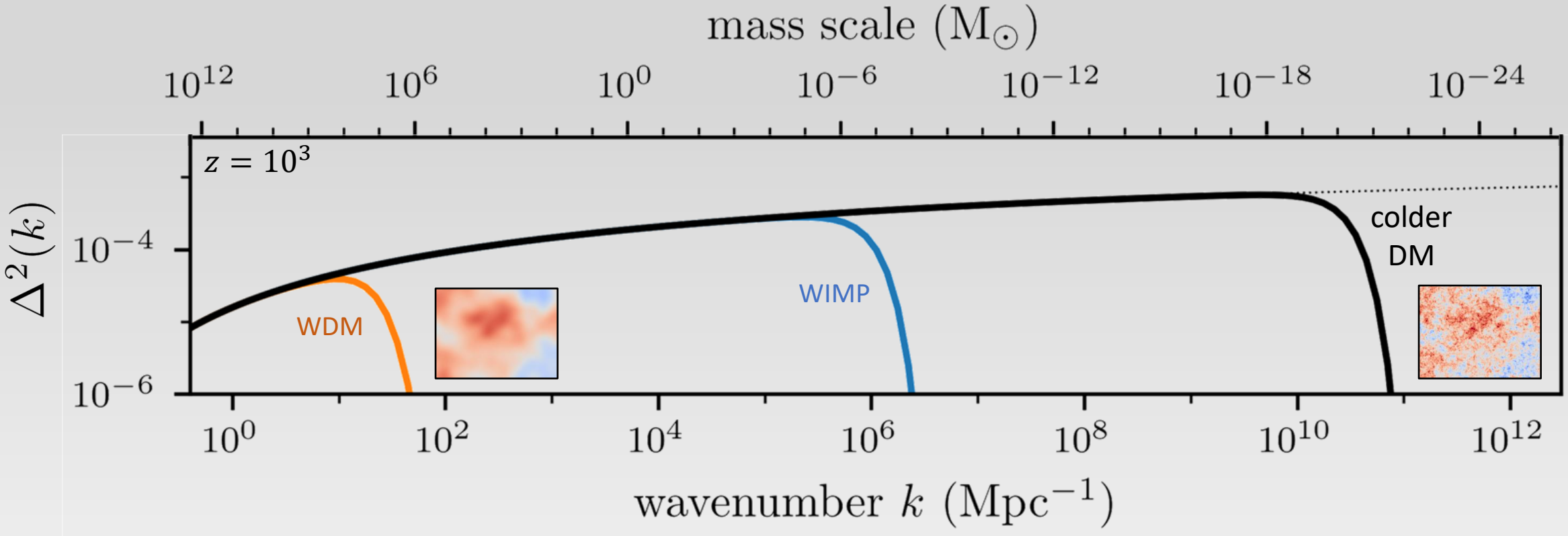
Cosmology with prompt cusps



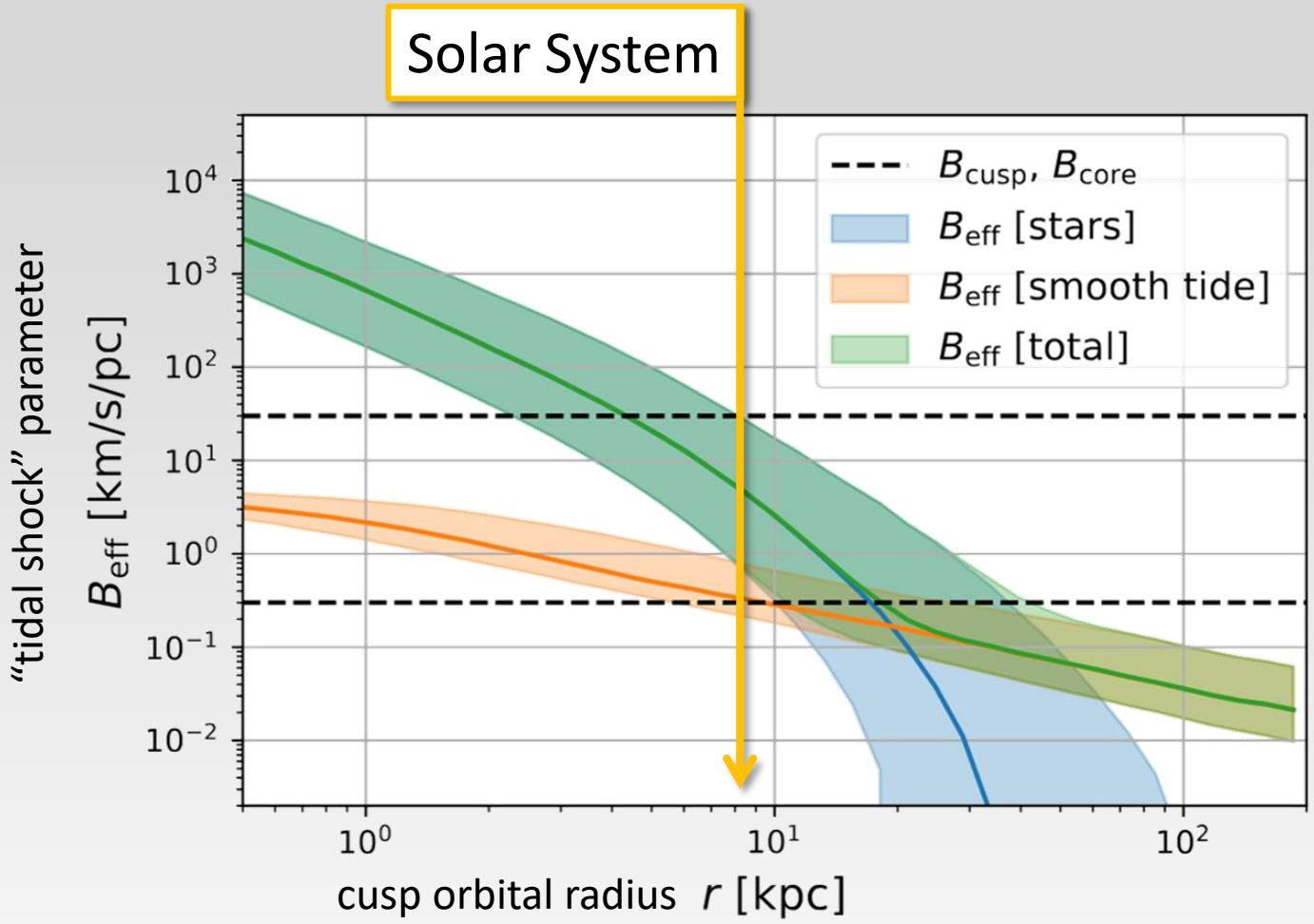
Prompt cusps are a signature of the smoothing scale (e.g., from free streaming)

Prompt cusps of warm dark matter would affect dwarf galaxy structures

Prompt cusps boost dark matter annihilation by a factor of ~ 10

For very cold DM, prompt cusps would pass continuously through the Solar System

Stellar encounters



Average density of subhalo

$\sim 10^6\ \mathrm{M}_\odot/\mathrm{pc}^3$

$\sim 10^3\ \mathrm{M}_\odot/\mathrm{pc}^3$

$\sim 10^0\ \mathrm{M}_\odot/\mathrm{pc}^3$

$\sim 10^{-3}\ \mathrm{M}_\odot/\mathrm{pc}^3$

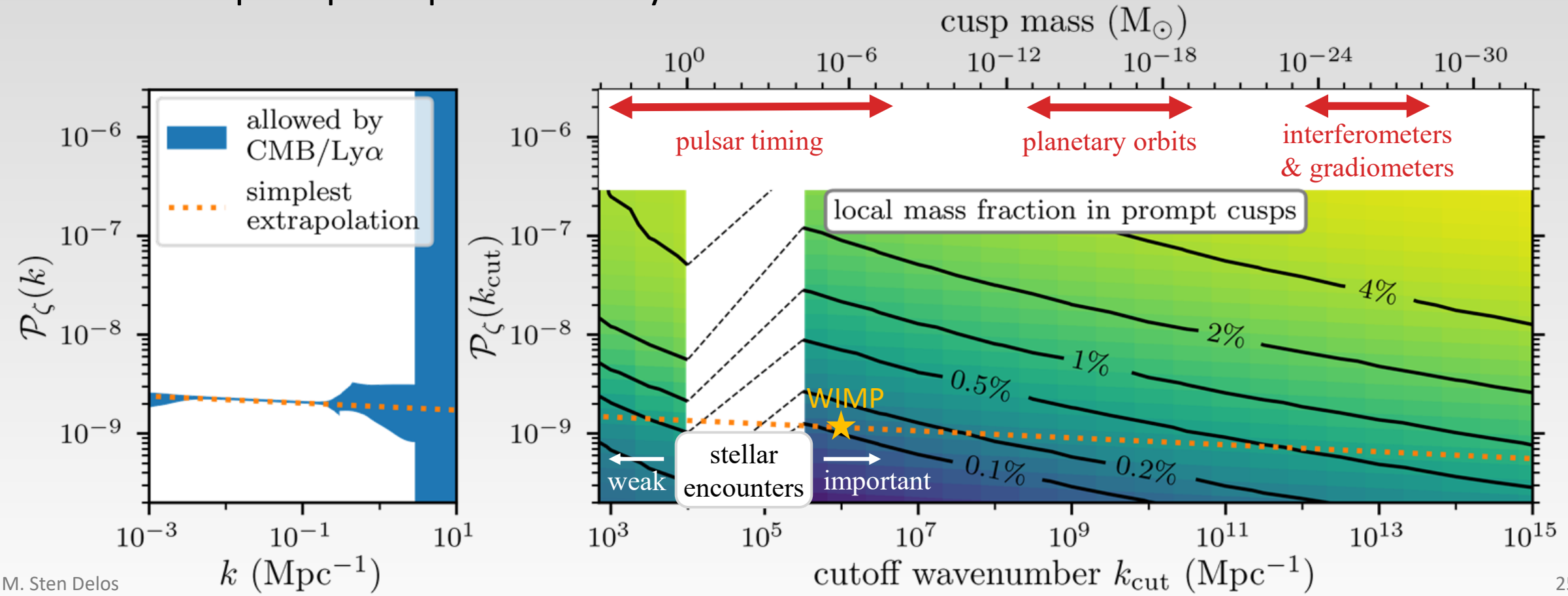
Prompt cusps of WIMP DM with standard $\mathcal{P}_\zeta(k)$

Stücker, Ogiya, White, Angulo (2023)

Local substructure

- Prompt cusps are simple! (peak-cusp connection)
- Stellar encounters are simple! (Stücker et al. 2023b)

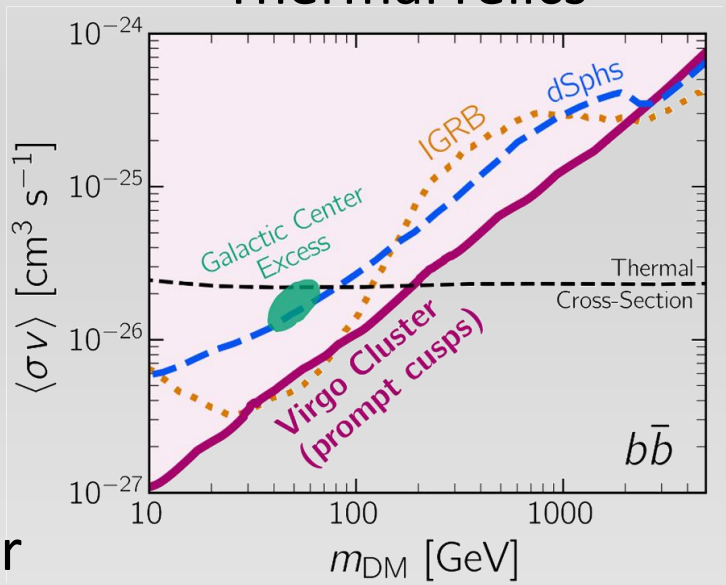
→ Local prompt cusps are an easy calculation:



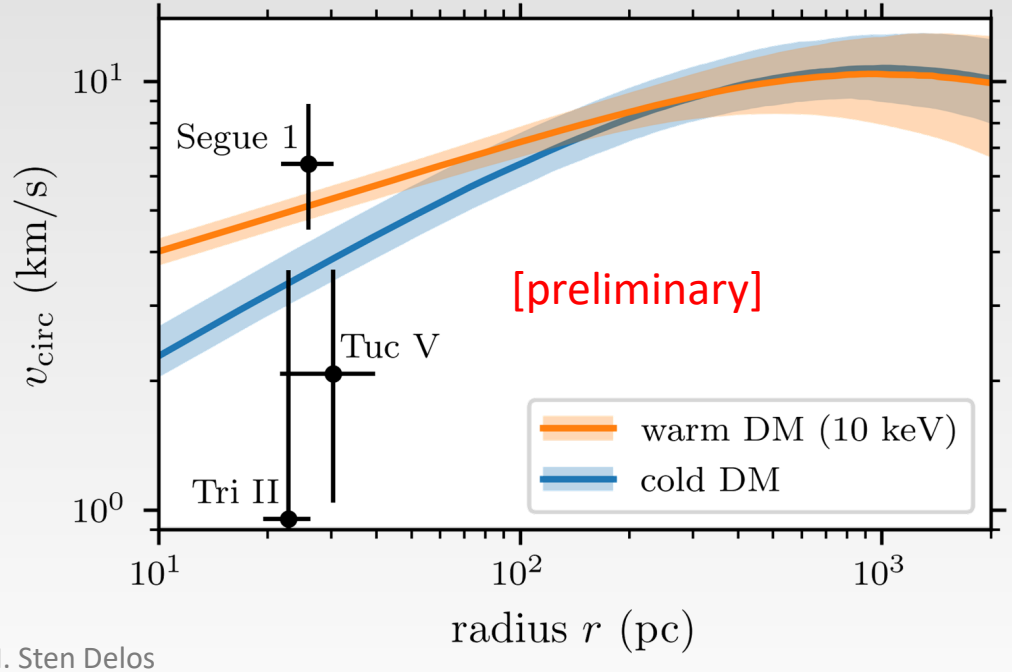
Prompt cusps: summary

Prompt $\rho \propto r^{-1.5}$ cusps
form at the onset of structure formation
and persist at halo centers

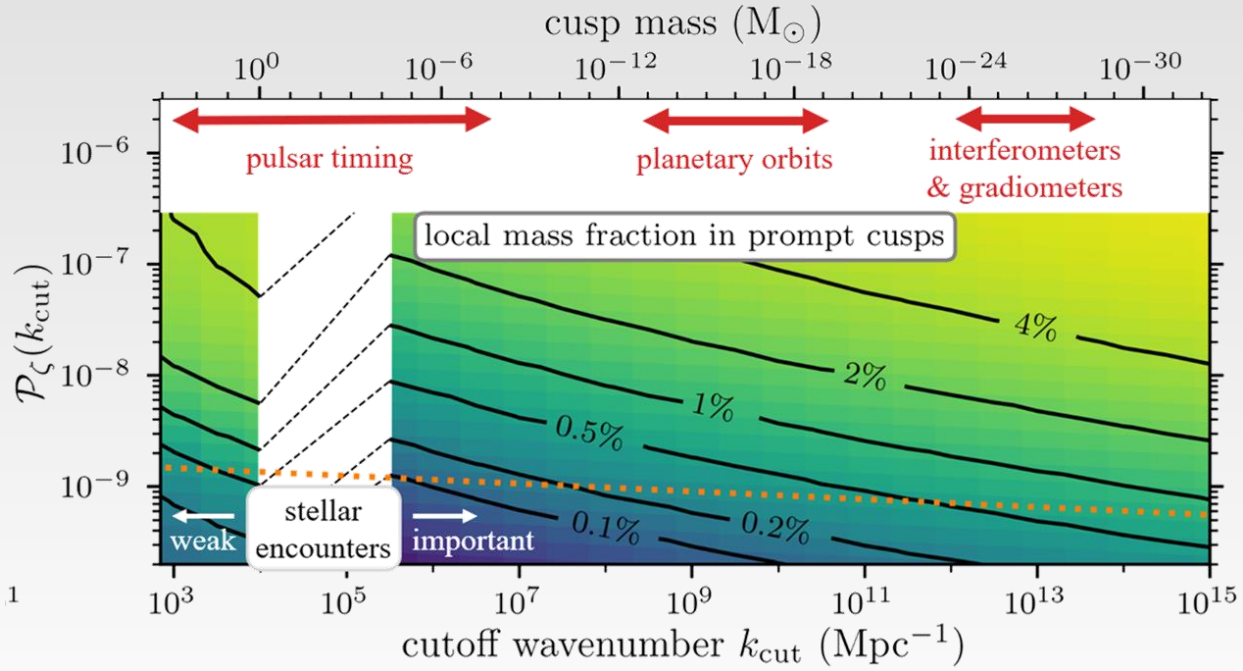
Thermal relics



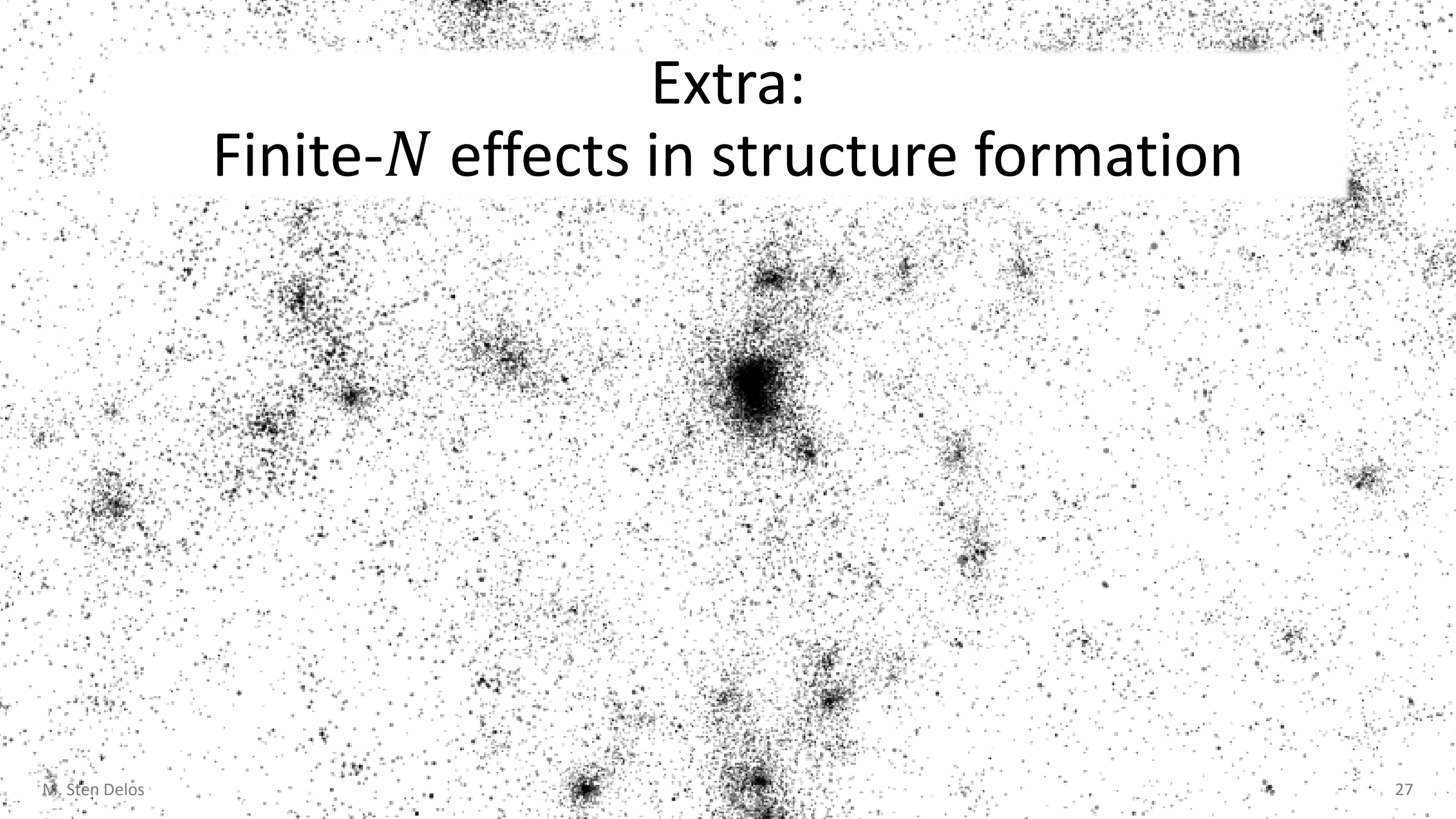
Warm dark matter



Cold dark matter



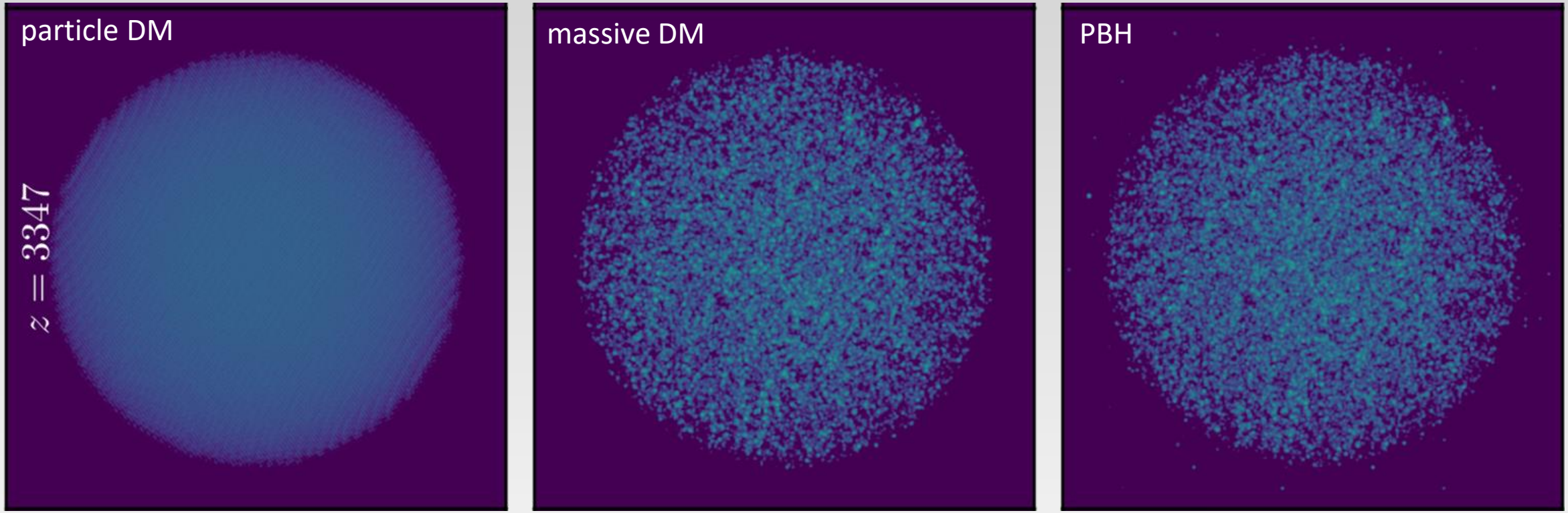
Extra:
Finite-N effects in structure formation



Primordial black hole dark matter

First cosmological simulation with PBH DM that fully resolves collisional+relativistic dynamics

$\sim$ 40 kpc cosmological volume



MSD, Rantala, Young, Schmidt (2024)

Poisson noise due to independently distributed particles ($\sim 10\ \mathrm{M}_\odot$)

interparticle dynamics:

- large-angle scattering
- binaries
- relativistic gravity
- etc.

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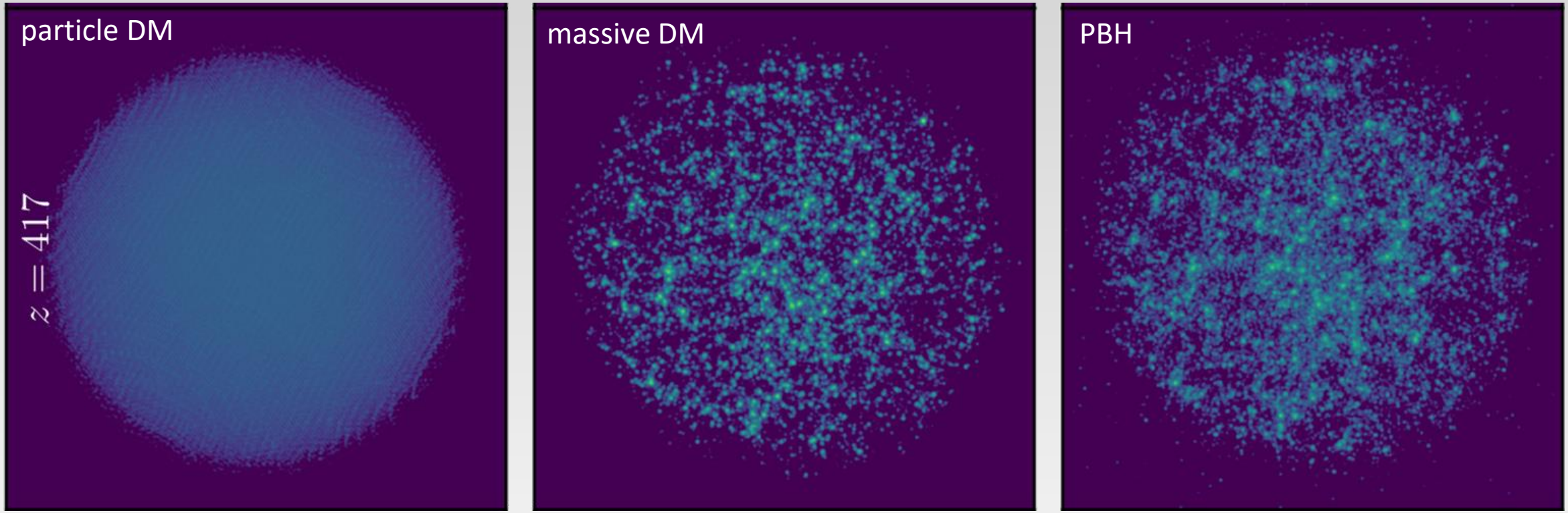

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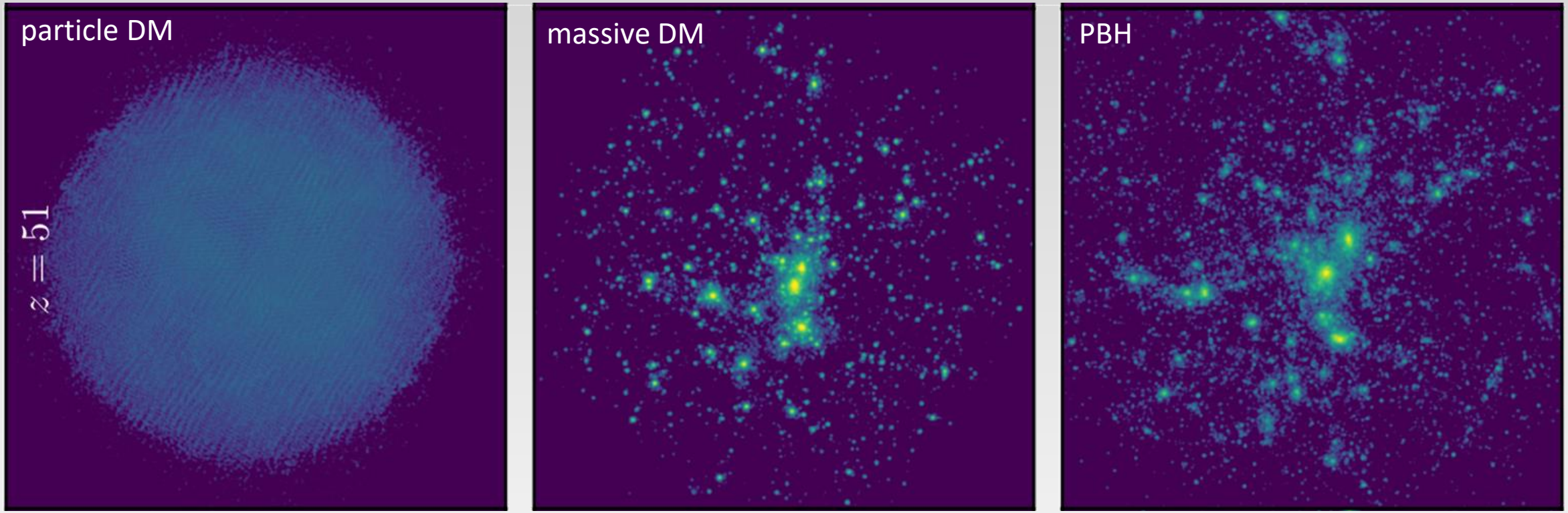

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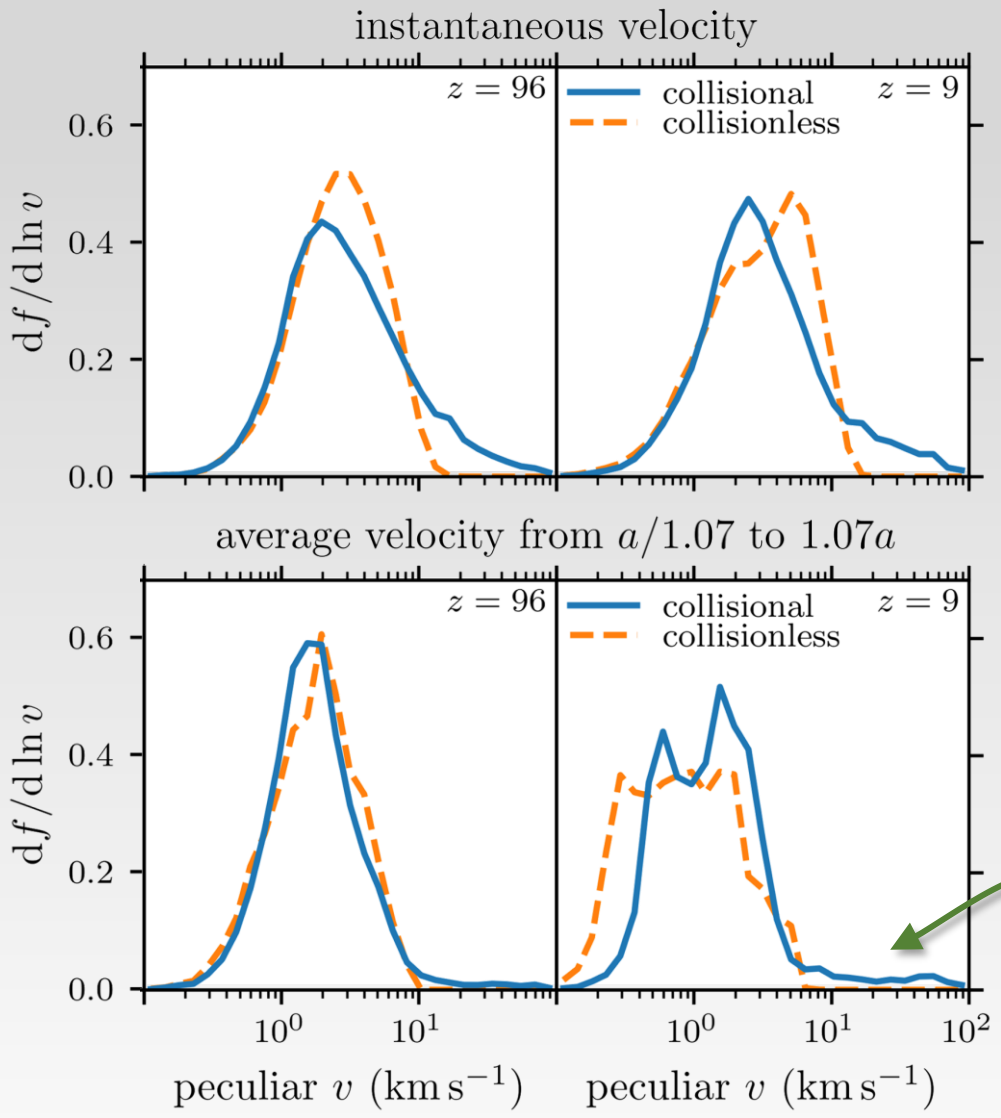
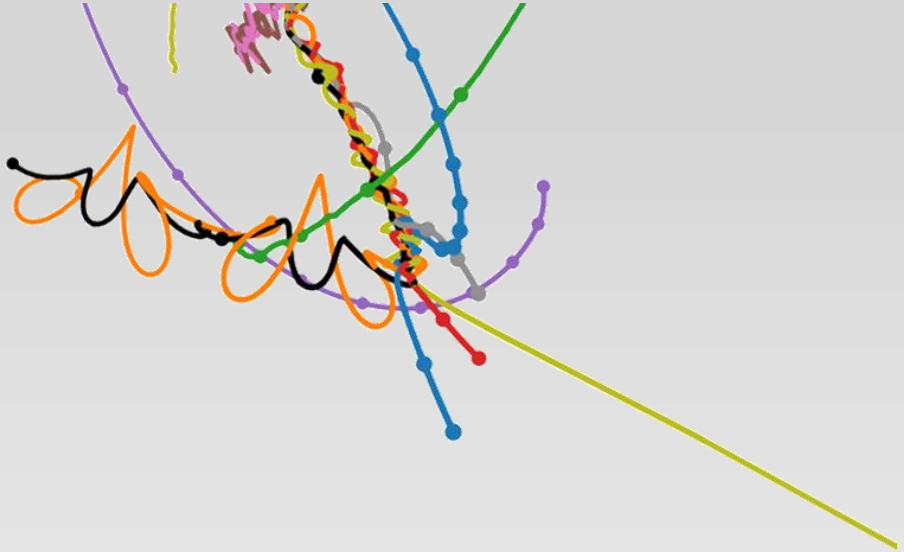
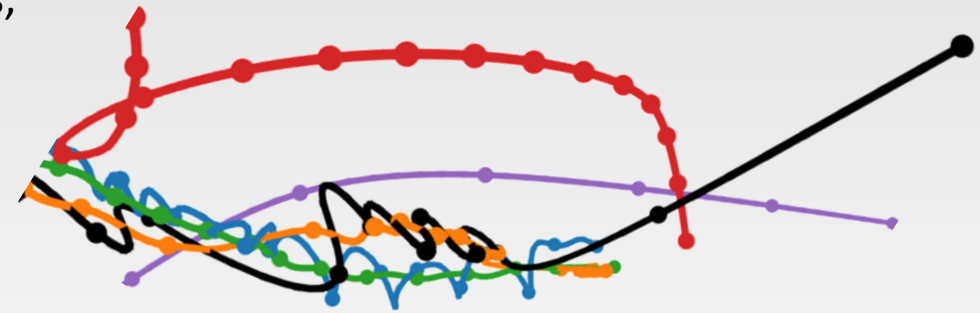
Collisional heating



Binary orbital motion contributes to velocity distribution



Through interactions, this can turn into streaming motion



Hot dark matter component, which would suppress structure growth up to billion-PBH scales

MSD, Rantala, Young, Schmidt (2024)

Binary mergers

Pairs of PBHs can merge, producing detectable gravitational radiation

Predictions for GWs from PBHs consider **merge times from relativistic orbital decay**

$$\tau = \frac{3}{85}\frac{c^5}{G^3}\frac{r_{\rm a}^4 j^7}{m_1 m_2 (m_1 + m_2)}$$

(for $j \to 0$ limit; j is normalized angular momentum)

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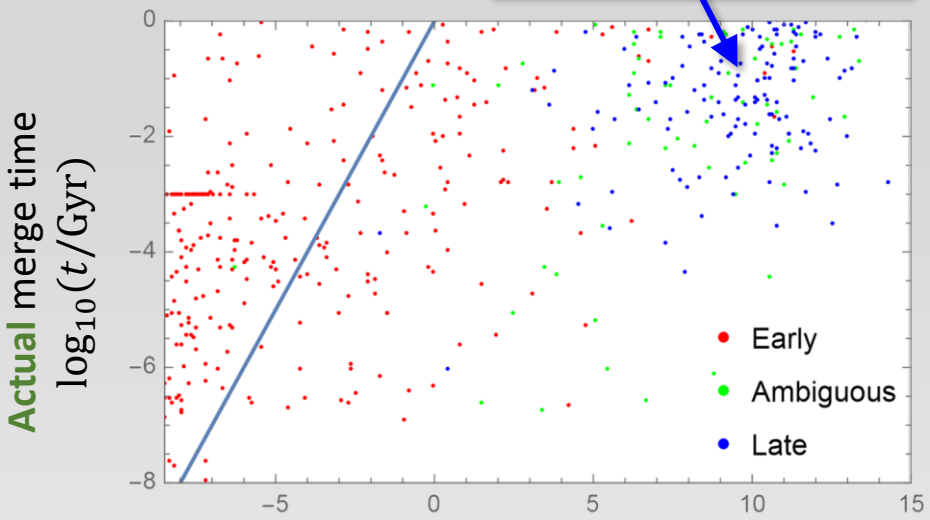
In a cosmological simulation we find

- no correlation between **initial prediction** and **actual** merge time
- certain populations not predicted to merge soon, but they do

Mergers are highly stochastic due to randomization of j in collisions?
(already recognized by the star cluster community)

At no point are these predicted to merge soon



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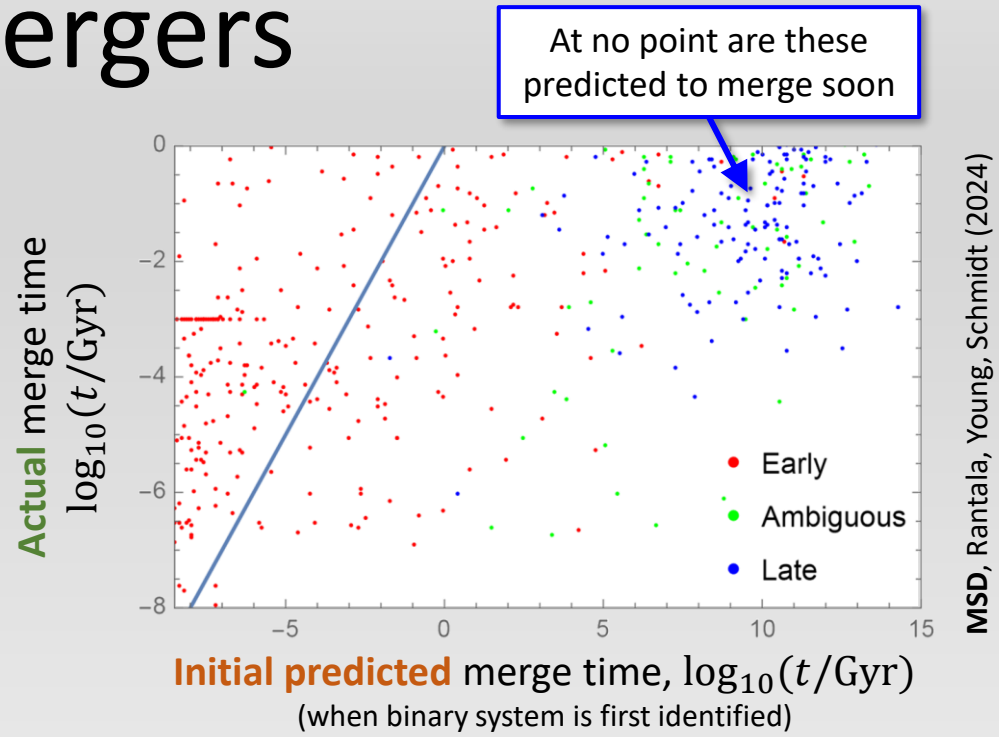

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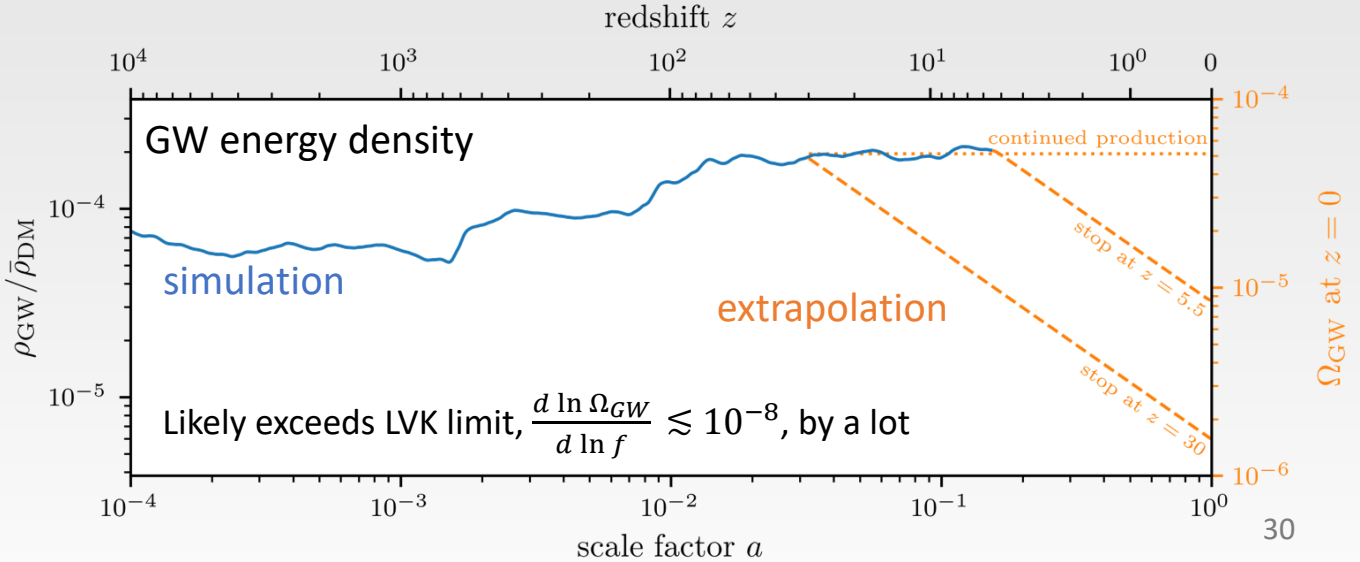
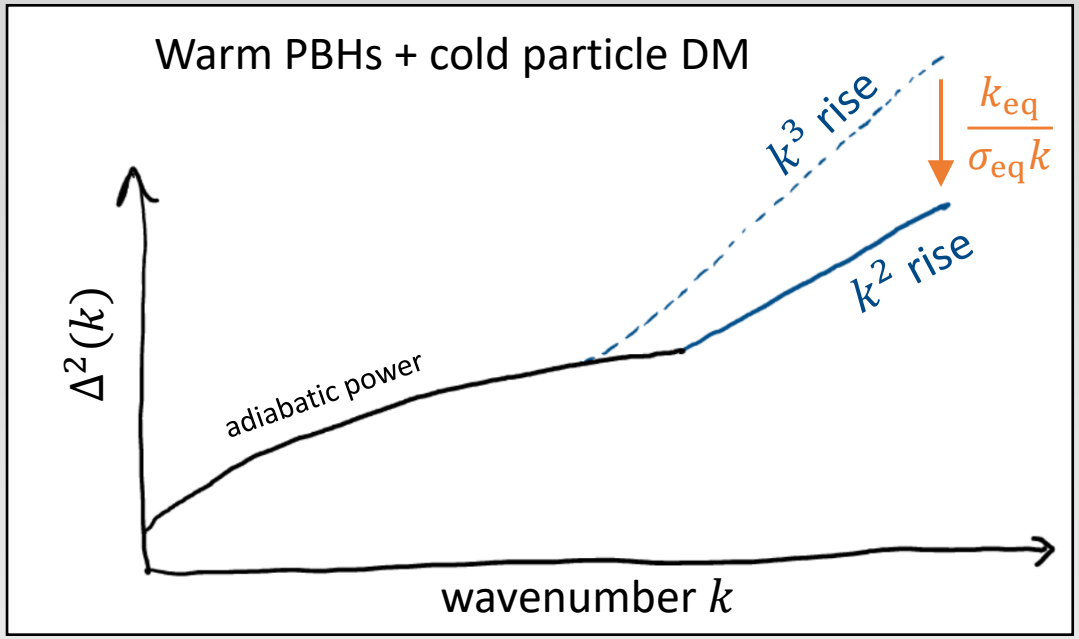
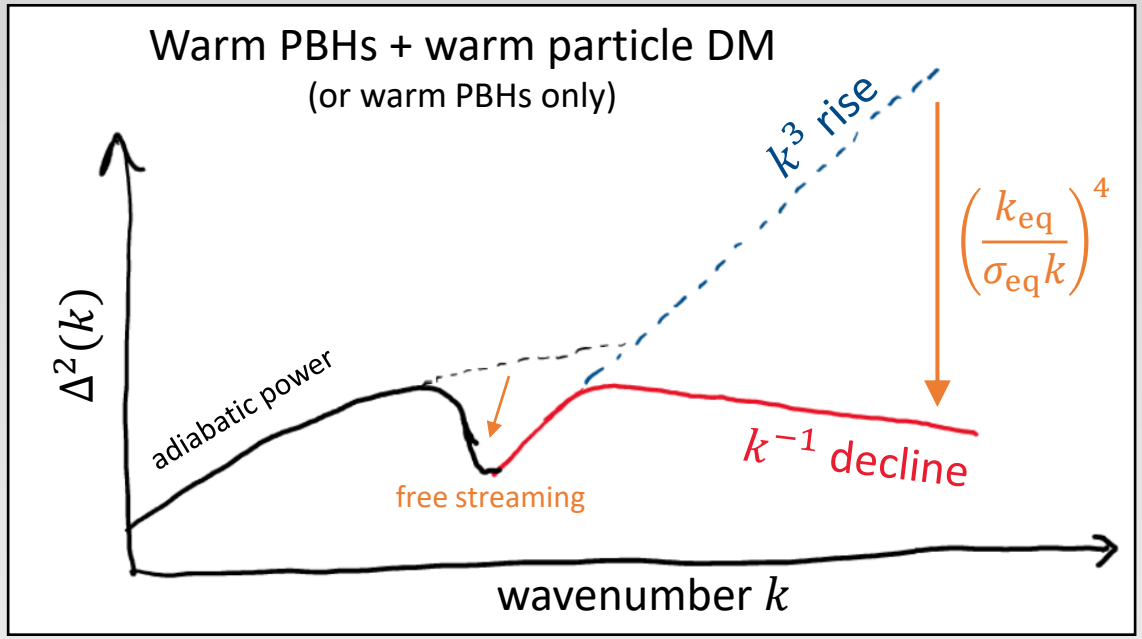


Poisson noise with velocity dispersion

($\Delta^2 \propto k^3$ noise due to independently distributed particle positions)



Amin, **MSD**, Yang (in prep)



Amin, **MSD**, Mirbabayi (2025)

Velocity dispersion suppresses PBH → CDM seeding of perturbations

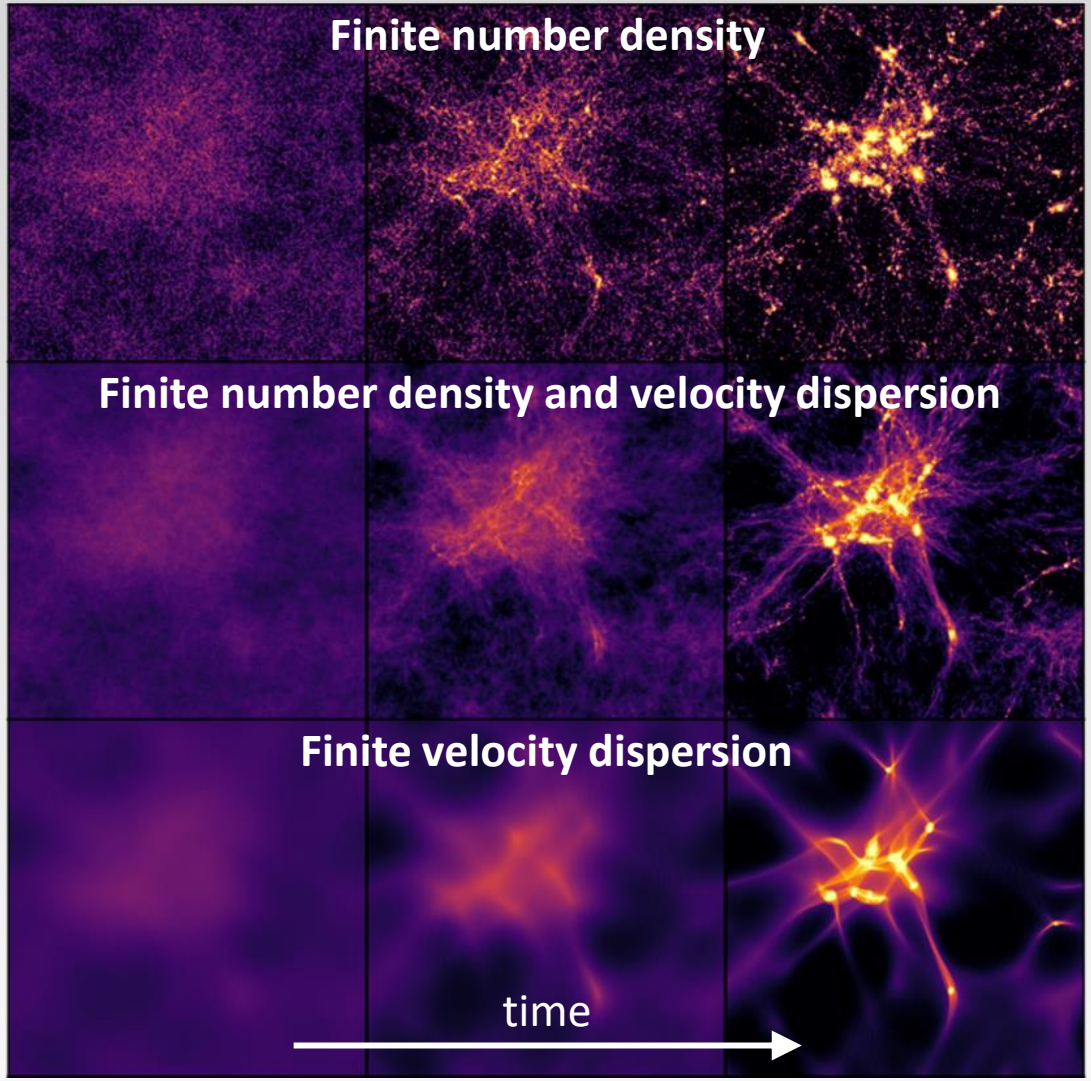
Velocity dispersion delays growth of perturbations

Based on a new formulation of cosmological perturbation theory using BBGKY hierarchy
(Amin, **MSD**, Mirbabayi 2025)

$$P_\delta(y,k) = \underbrace{P_{\delta_{\rm ad}}(y_0,k)\left[\mathcal{T}_k^{\rm ad}(y,y_0)\right]^2}_{\text{adiabatic IC + evolution}} + \underbrace{\frac{1}{\bar{n}}\left[1 + 3\int_{y_0}^{y}\frac{dy'}{\sqrt{1+y'}}\mathcal{T}_k^{(a)}(y,y')\mathcal{T}_k^{(b)}(y,y')\right]}_{\text{white noise IC + evolution}}$$

($\mathcal{T}$ = growth function × free-streaming cutoff)

Finite number density and velocity dispersion



Amin, **MSD**, Mirbabayi (2025)

$\simeq$ primordial black holes

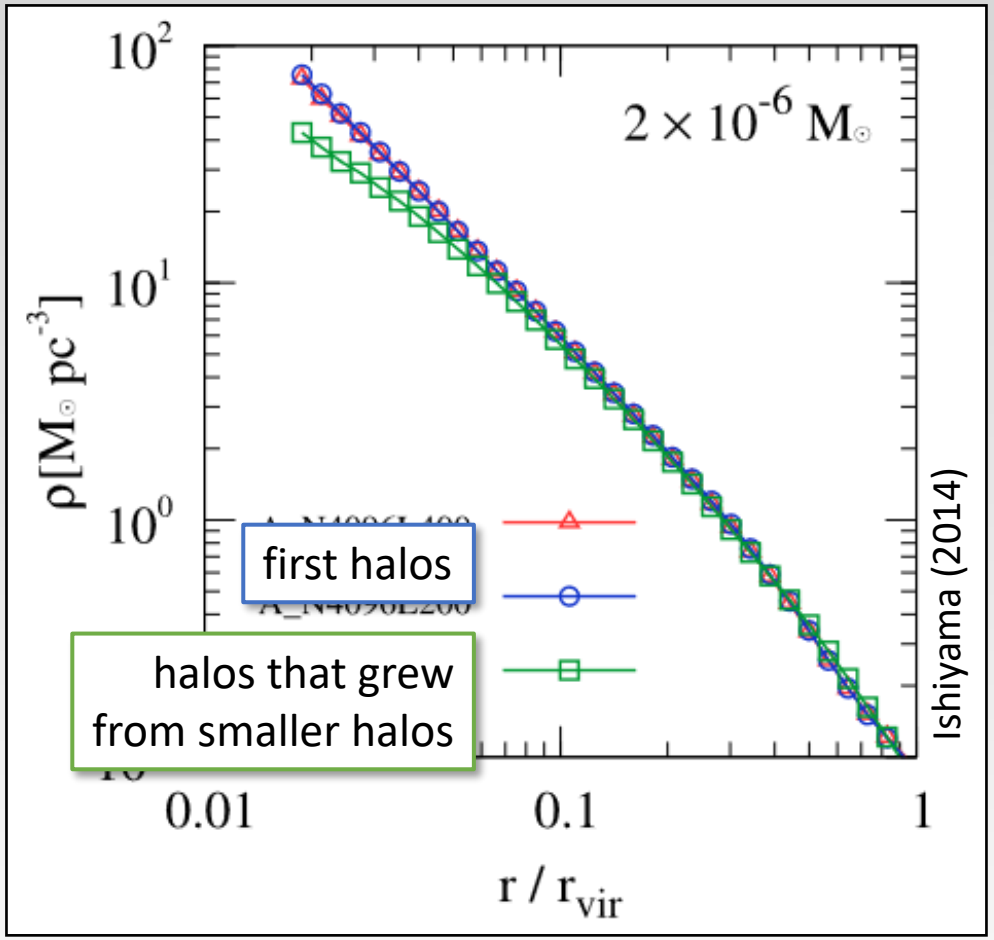
Difference between PBHs and particle DM:
Both particle mass and velocities

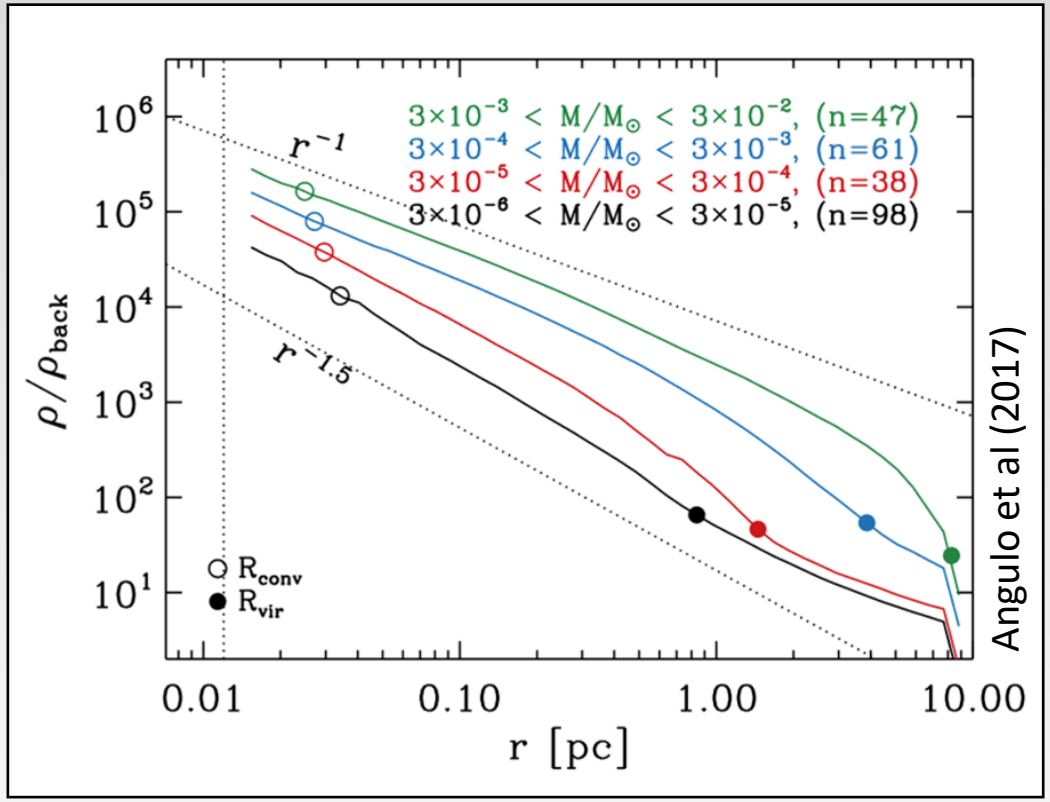
$\simeq$ particle dark matter

Supplementary

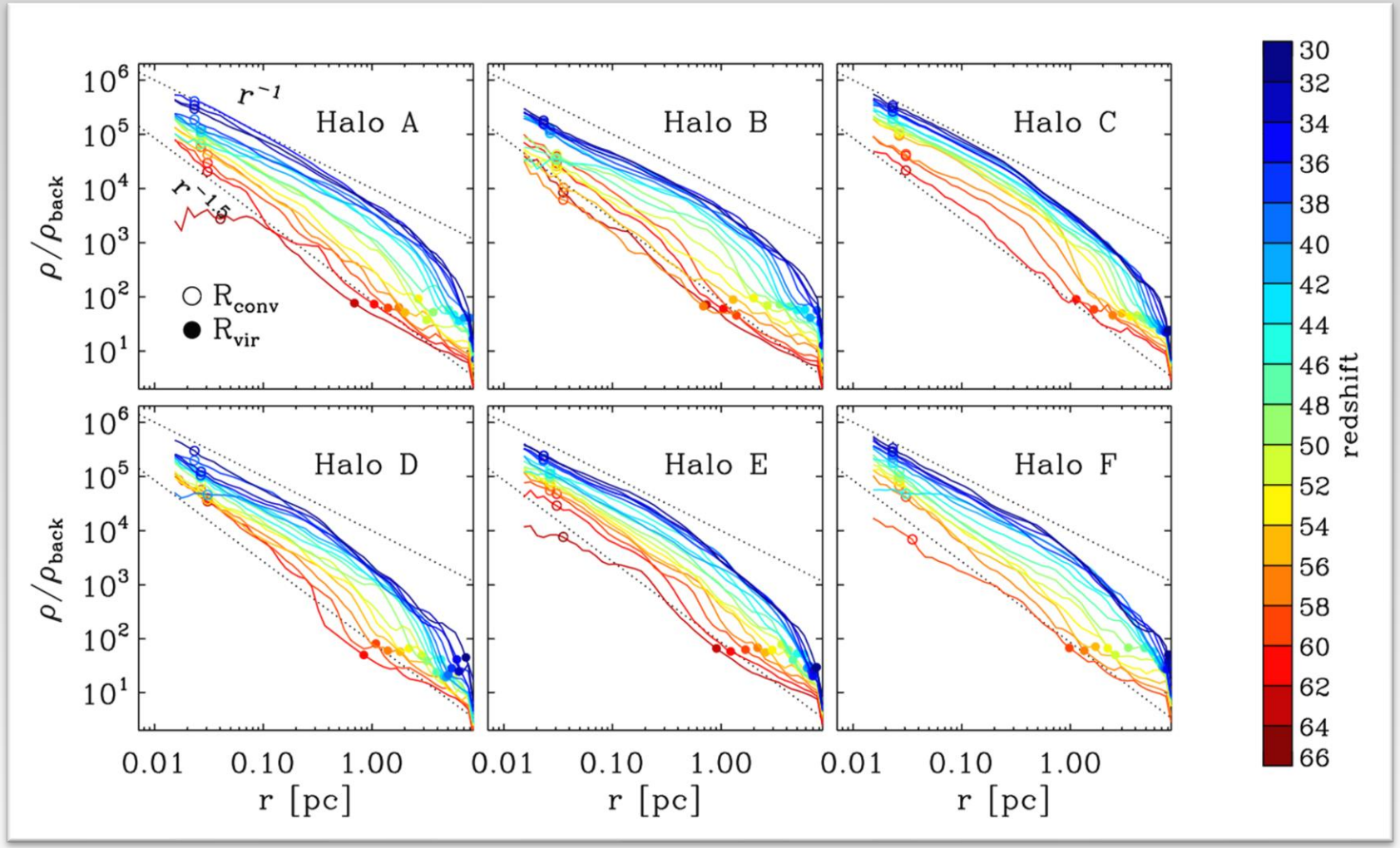
Prompt cusps

Prompt cusps: previous studies





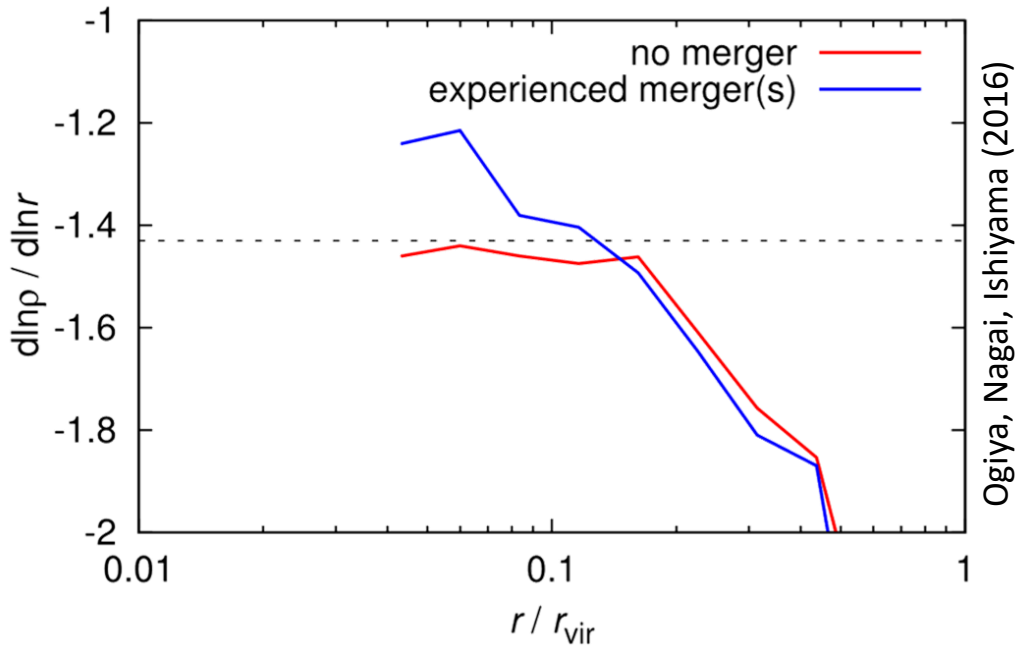
Prompt cusps: previous studies



Angulo et al (2017)

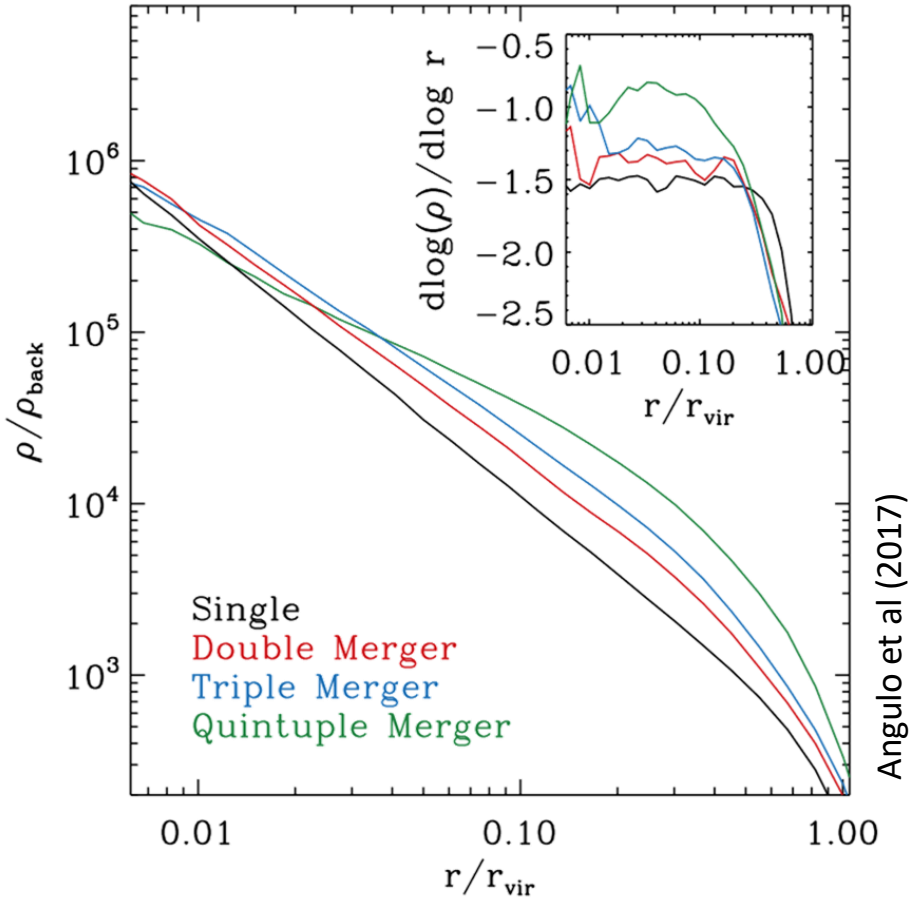
Prompt cusps: previous studies

Correlation between mergers and shallow cusps



Ogiya, Nagai, Ishiyama (2016)

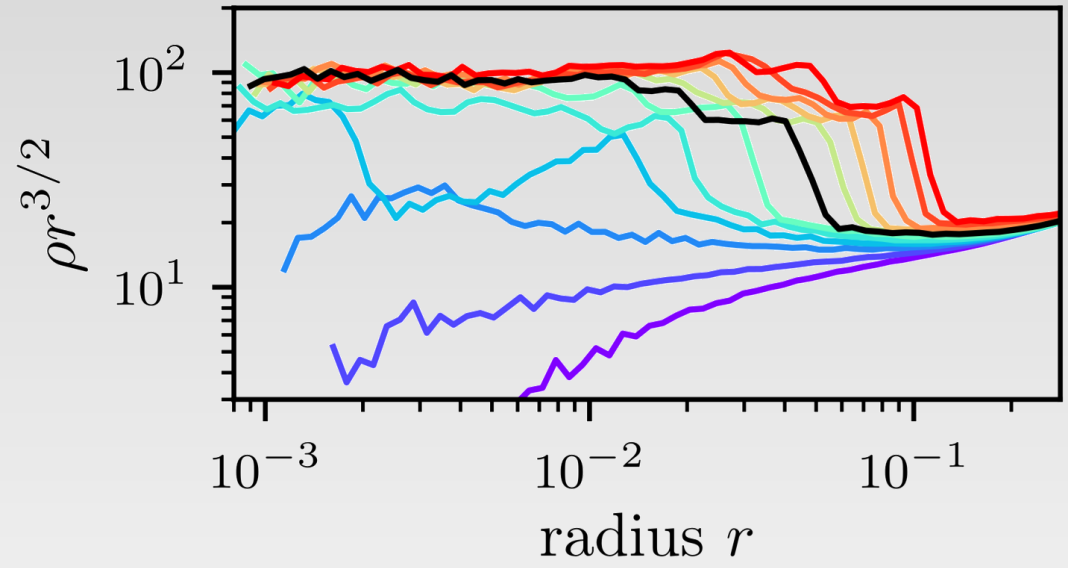
Shallowing in idealized merger simulations



Angulo et al (2017)

Universal density profiles?

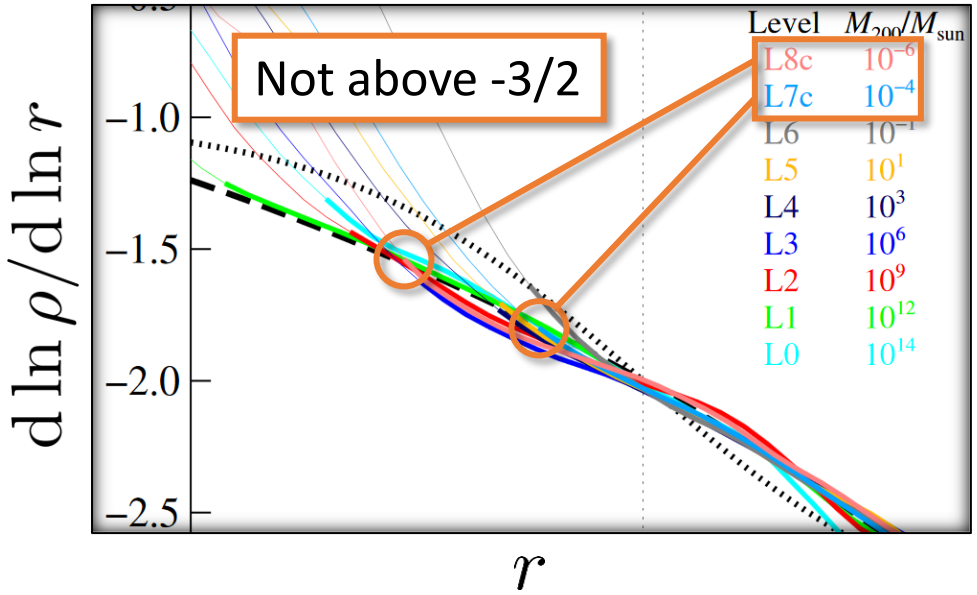
$\rho \propto r^{-3/2}$ cusp



Consistent with:

Universality in the structure of dark matter haloes over twenty orders of magnitude in halo mass

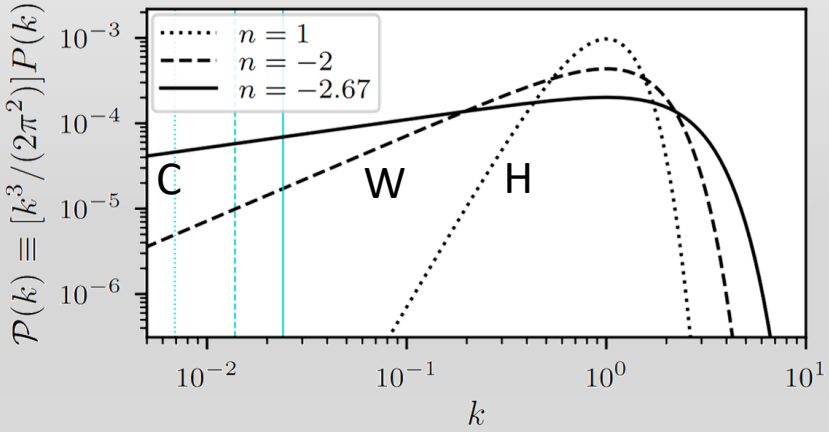
Wang, J.[1], Bose, S.[2], Frenk, C. S.[3], Gao, L.[1], Jenkins, A.[3], Springel, V.[4] & White, S. D. M.[4]

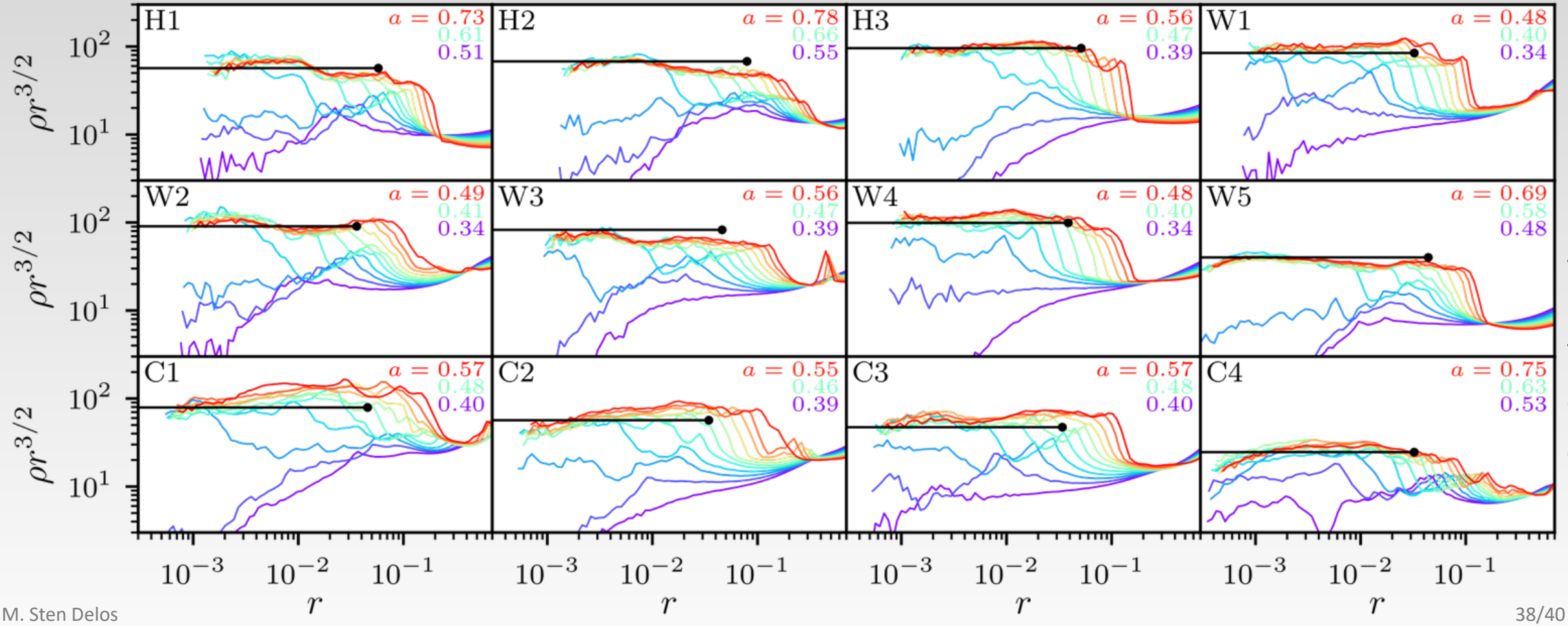


Peak-cusp connection

Twelve high-resolution halos from three cosmologies:

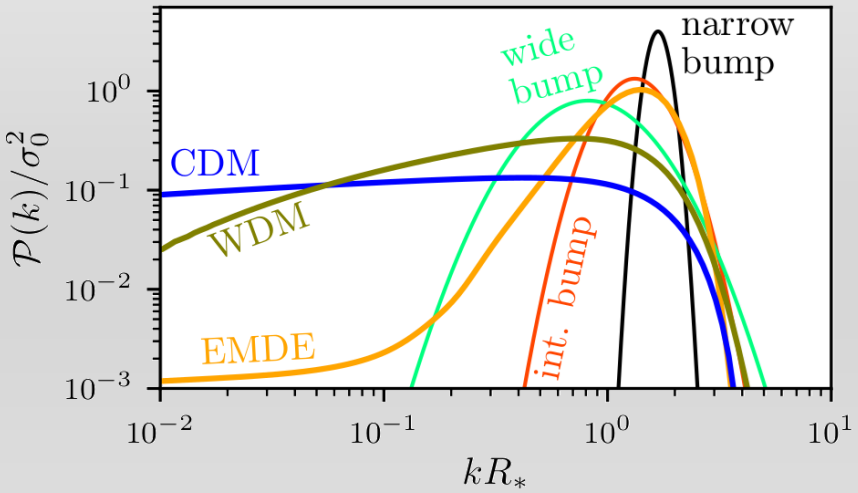
Predictions [black] work well!



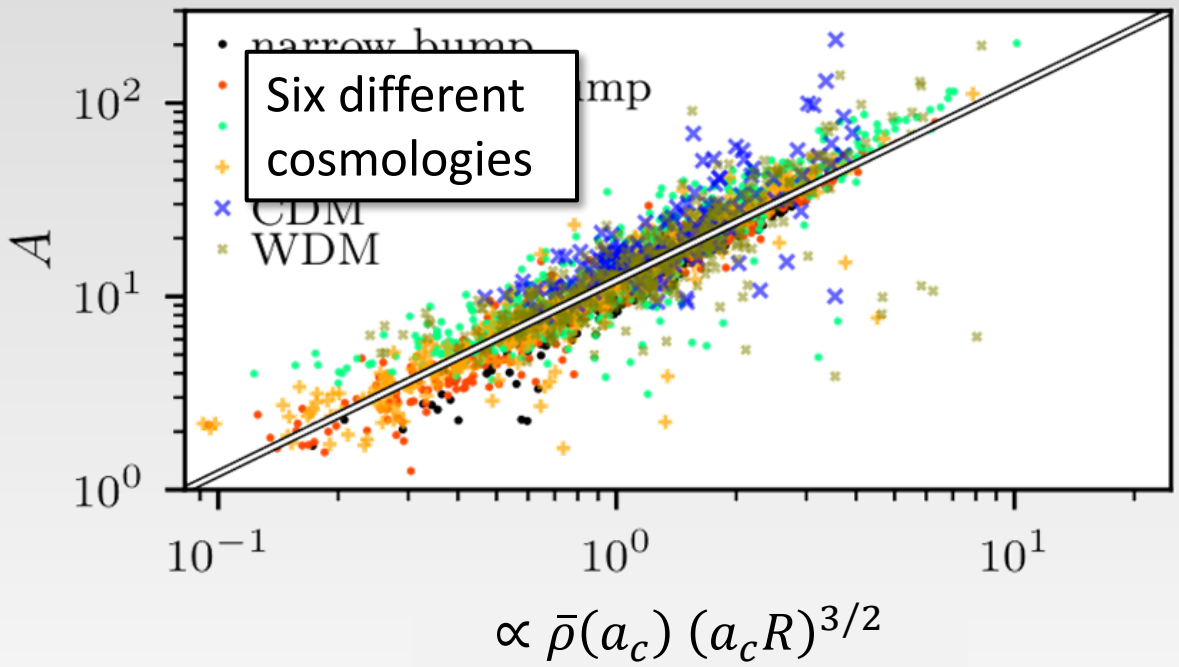


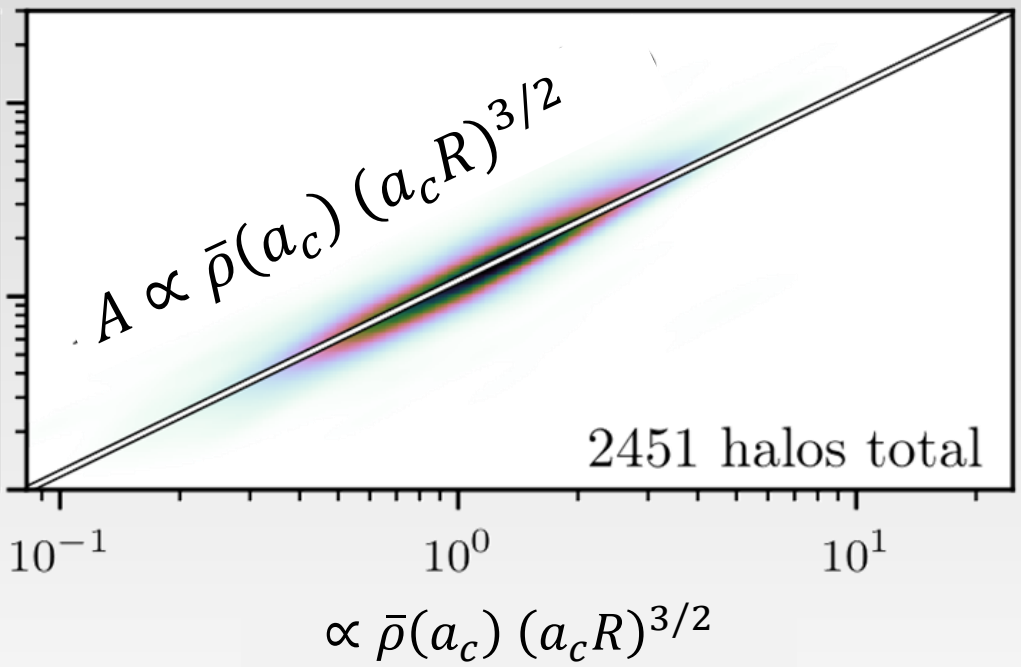
MSD & White (2022a)

Peak-cusp connection



Prediction for $A = \rho r^{3/2}$ also validated using a much larger halo sample



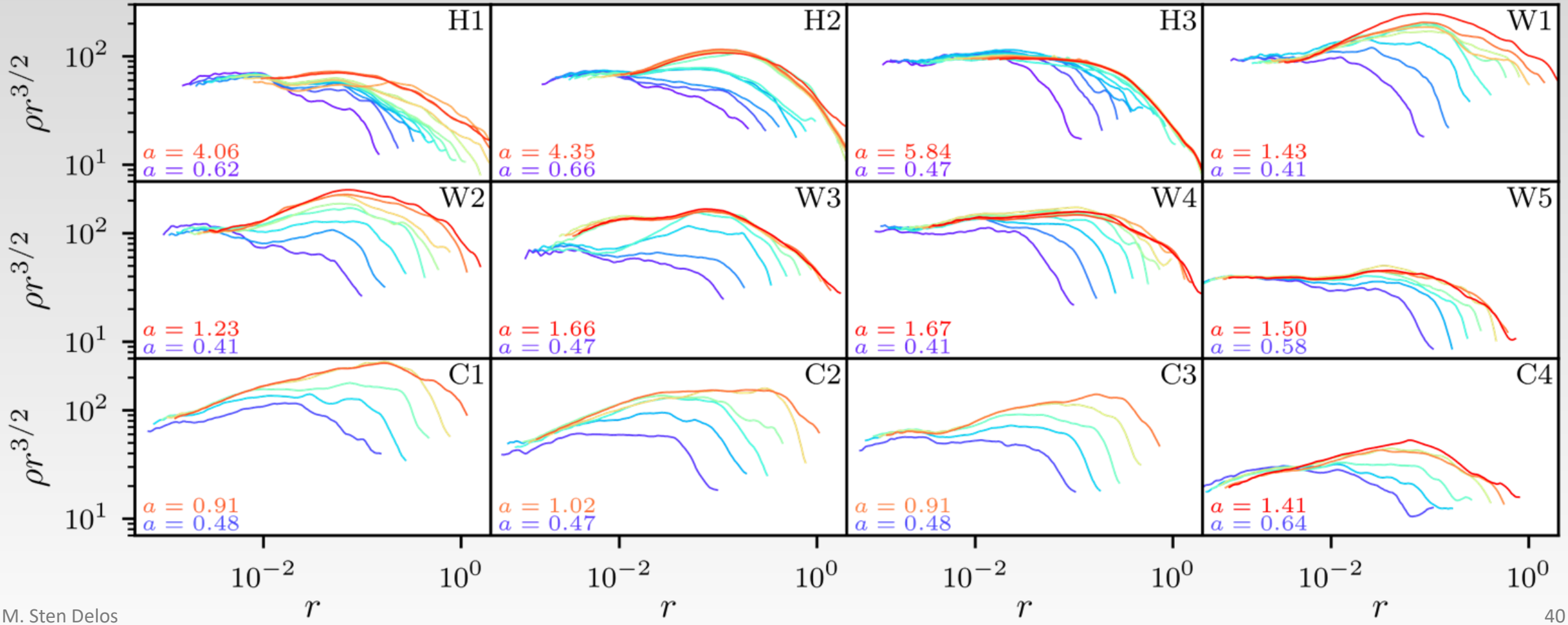


MSD, Bruff, Erickcek (2019)

Prompt cusp survival

Twelve high-resolution halos from three cosmologies:

Prompt cusp forms at collapse; no evidence for disruption



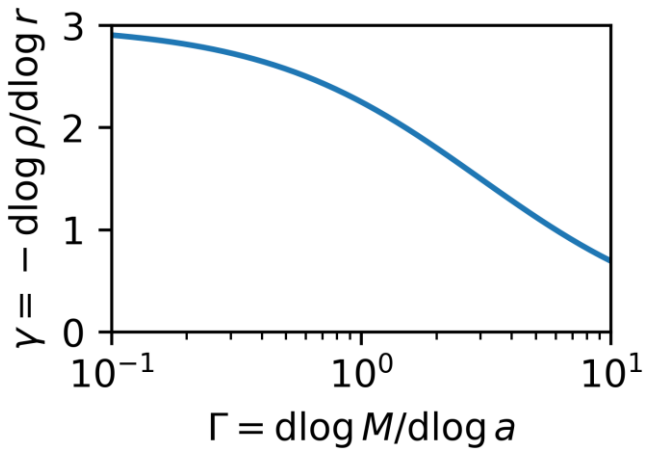
MSD & White (2023a)

Rapid accretion

Shallow NFW/Einasto profiles follow from the accretion history

Self-similar models predict

faster accretion → shallower profile



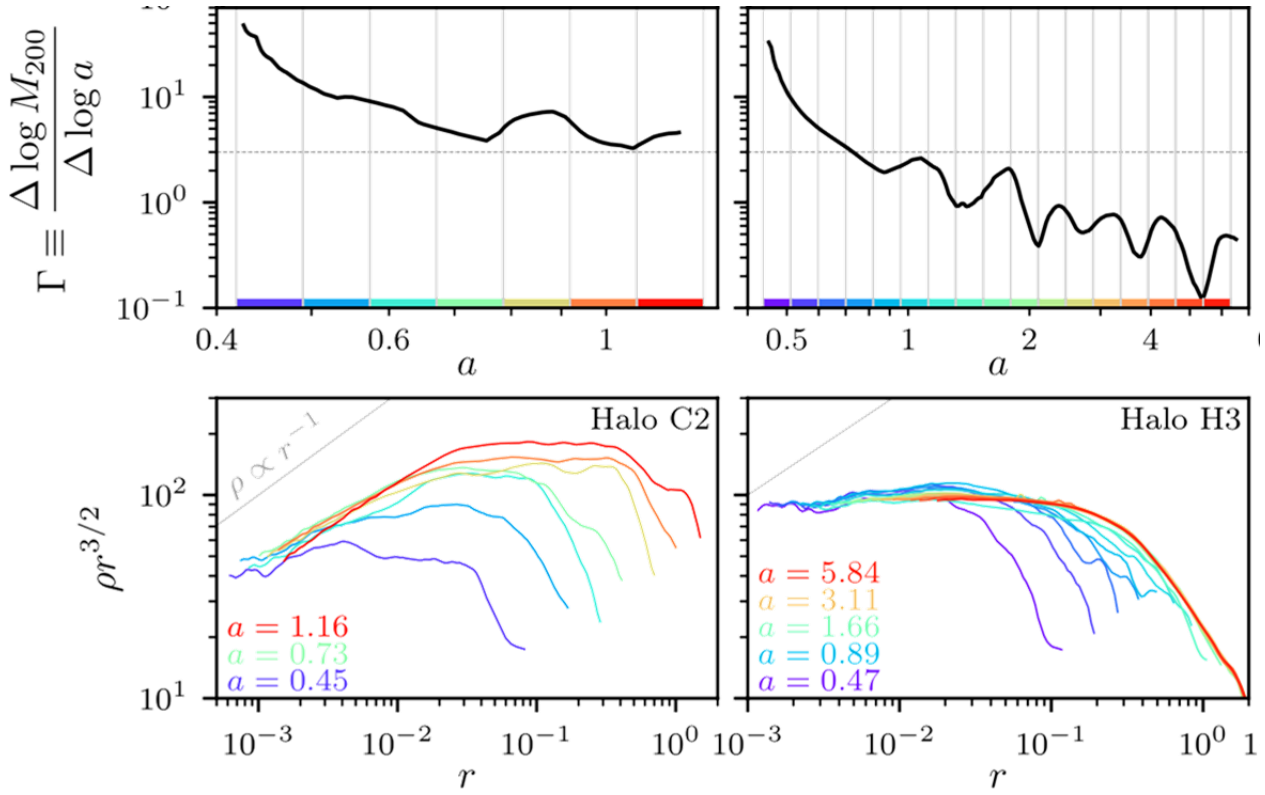
Physically, new material

- builds up density at large radii
- contributes little to smaller radii

Same idea holds in more realistic models

(Ludlow et al 2013, Dalal et al 2010)

Behavior borne out in the first halos



MSD & White (2023)

Rapid accretion builds up large radii without disrupting smaller radii:

No destruction of prompt cusps

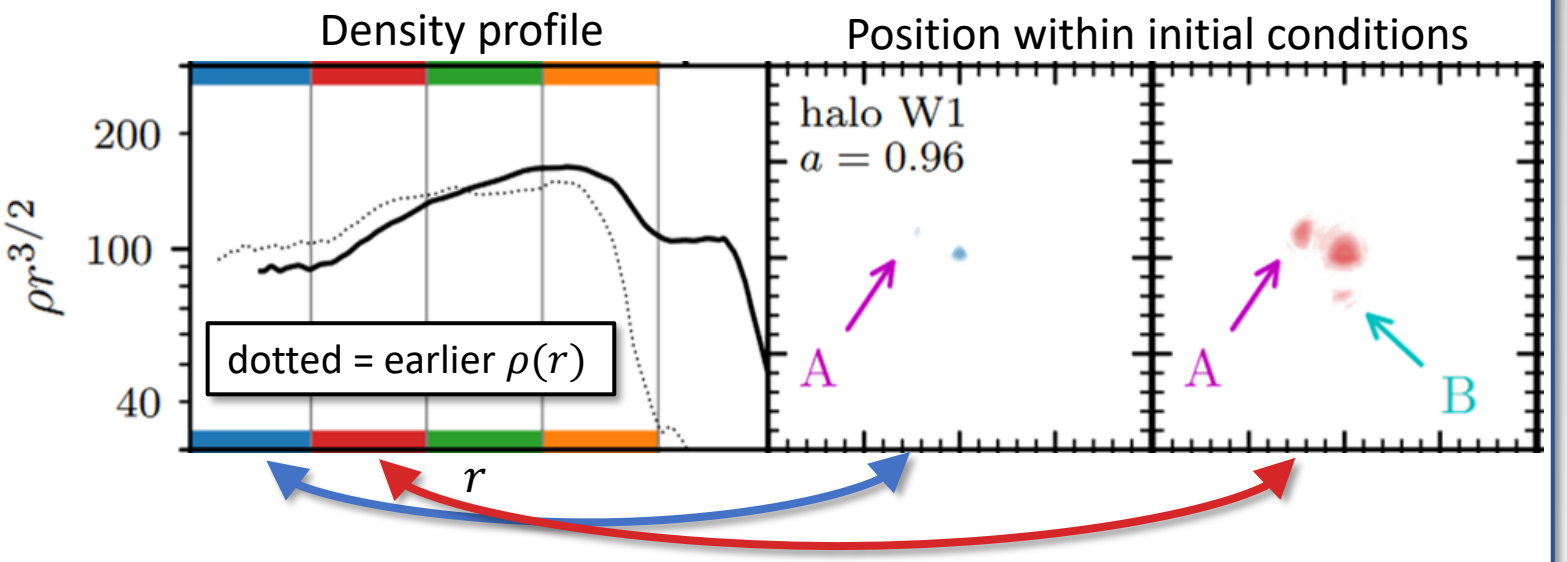
Mergers

Mergers can disturb central cusps:

A massive subhalo sinks due to dynamical friction and can thus disrupt the structure at small radii.

However, the disruption is minimal.

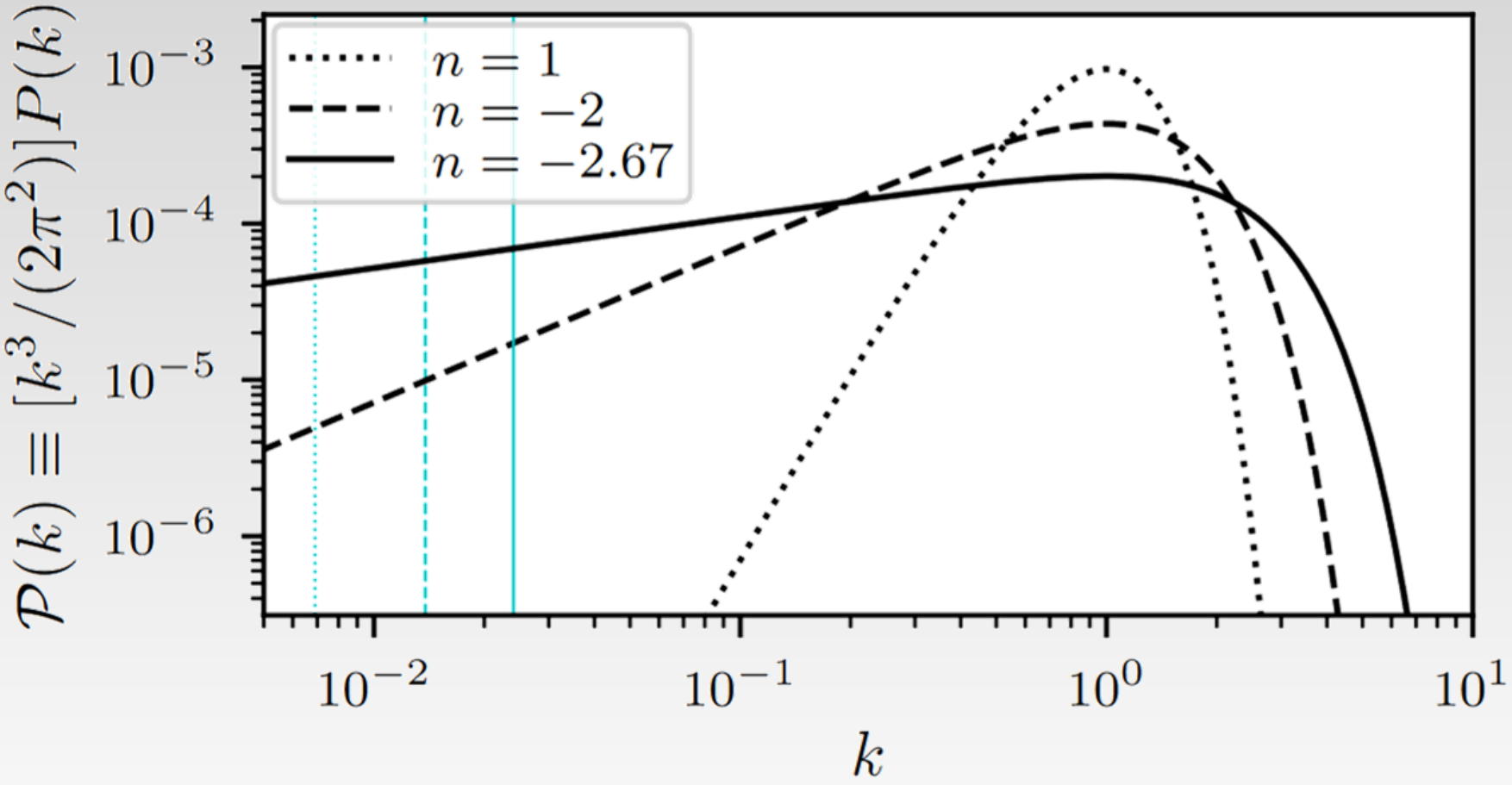
Merging halos 'A' and 'B' deposited material deep inside this halo…



…which caused some disruption of the $\rho \propto r^{-3/2}$ cusp.

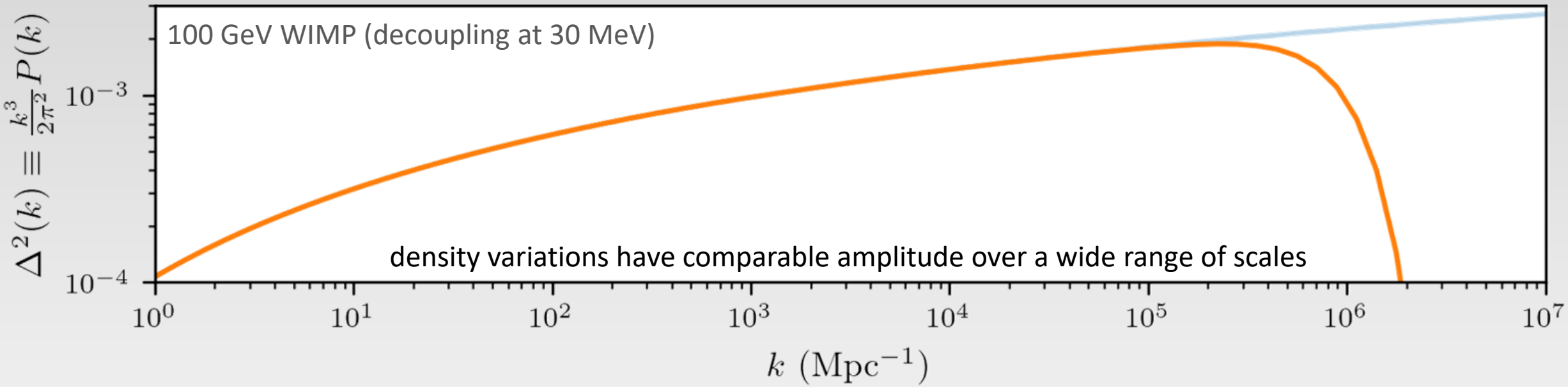
MSD & White (2023)

Power spectra for prompt cusp simulations



Can all peaks be associated with cusps?

Do isolated initial peaks make sense with a power spectrum like this?

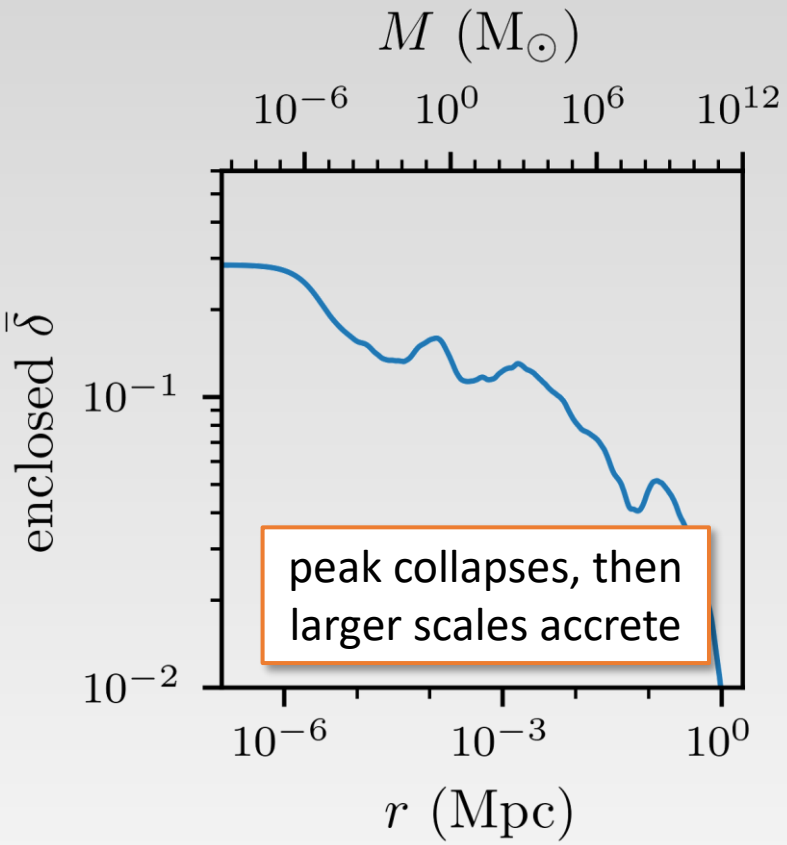


Fluctuations at different scales are uncorrelated.

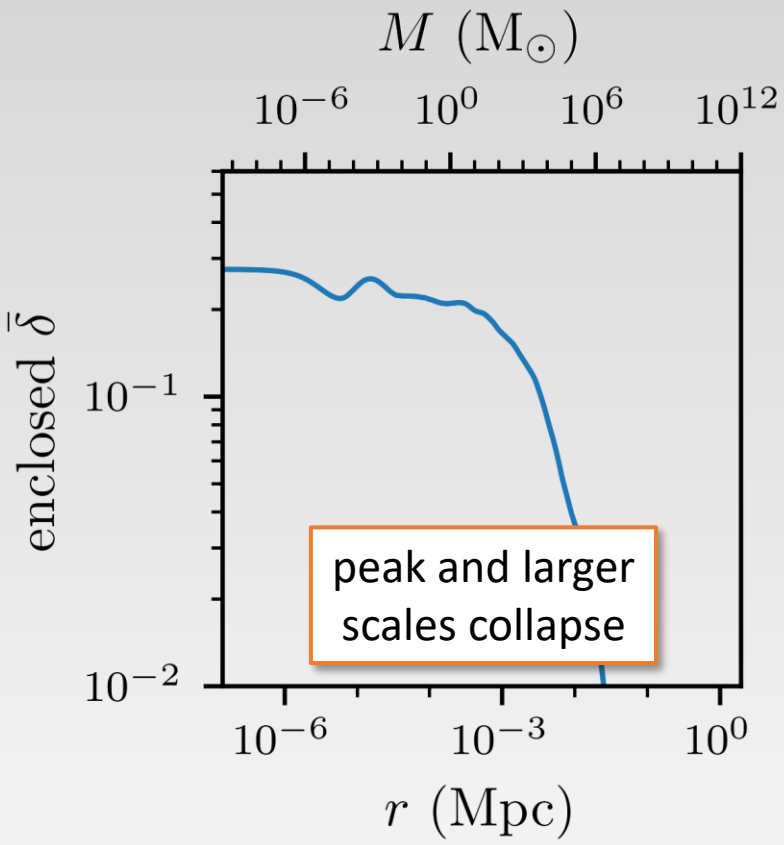
Isolated peaks arise at the smoothing scale and are built on top of larger-scale fluctuations.

Can all peaks be associated with cusps?

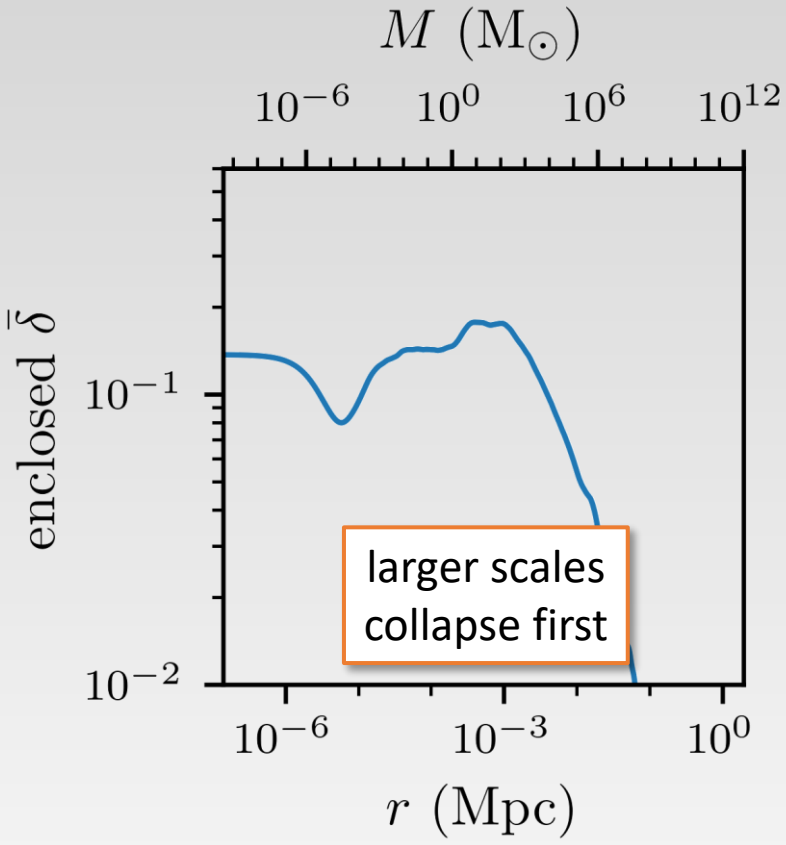
Collapse time scales inversely with density



~ 85%



~ 9%



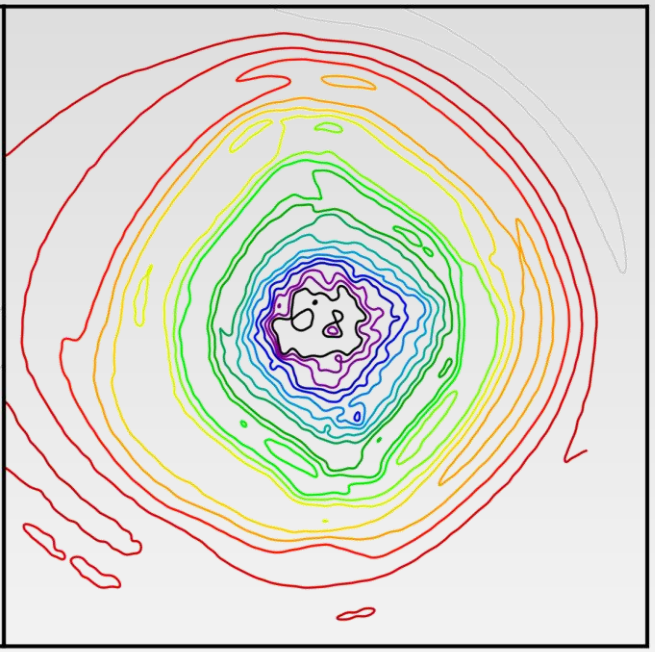
~ 6%

Most peaks produce cusps!

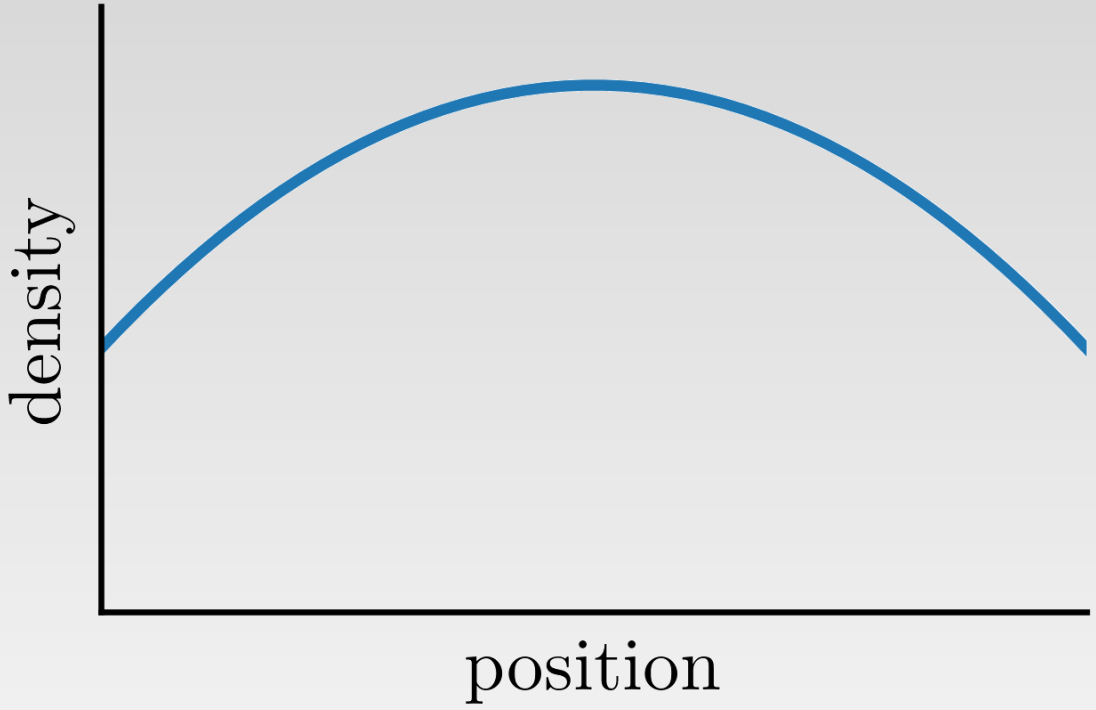
Prompt collapse

Among the simplest realistic initial conditions you can imagine…

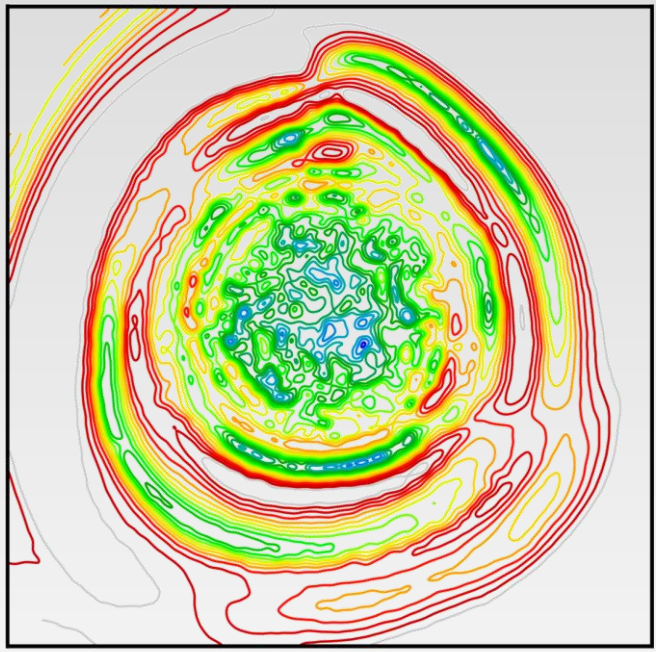
Lagrangian distribution of final energy



little relaxation



Lagrangian distribution of final energy

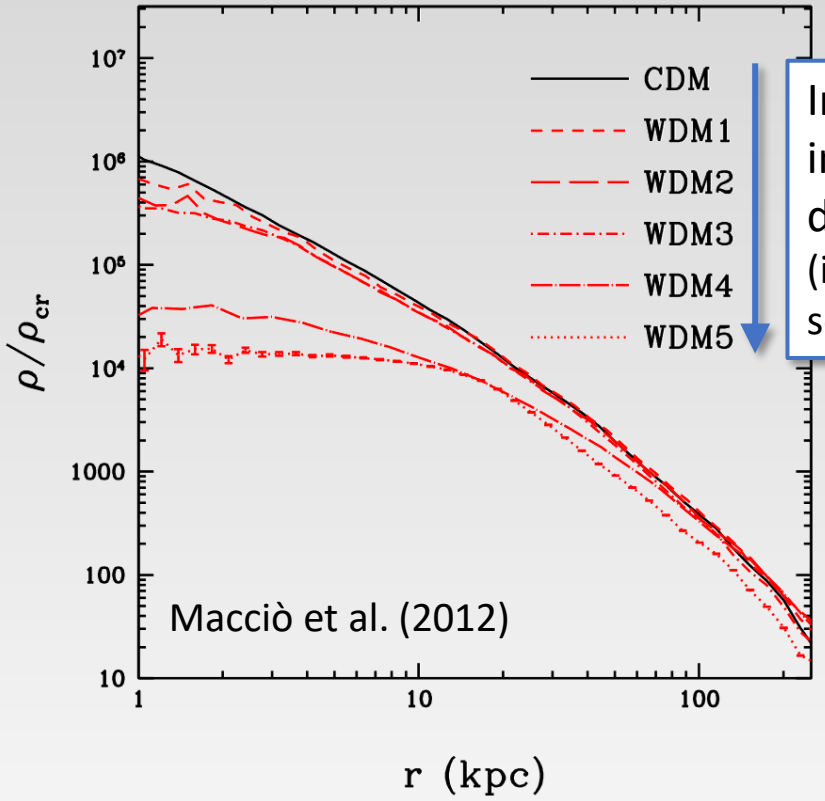


lots of relaxation

Scaling argument predicts $\rho \propto r^{-12/7}$? (White 2022)

Central cores

Initial thermal motion → finite-density core at small radii



Increasing initial velocity dispersion σ (initial smoothing scale not varied)

Use conservation of phase-space density (Liouville's theorem)

$f_{\max}$ = maximum phase-space density of the early universe

$$\sim \bar{\rho}(a)\sigma(a)^{-3}$$

Leads to core radius r_c and density ρ_c

$$\rho_c r_c^6 \simeq 3 \times 10^{-5} G^{-3} f_{\max}^{-2}$$

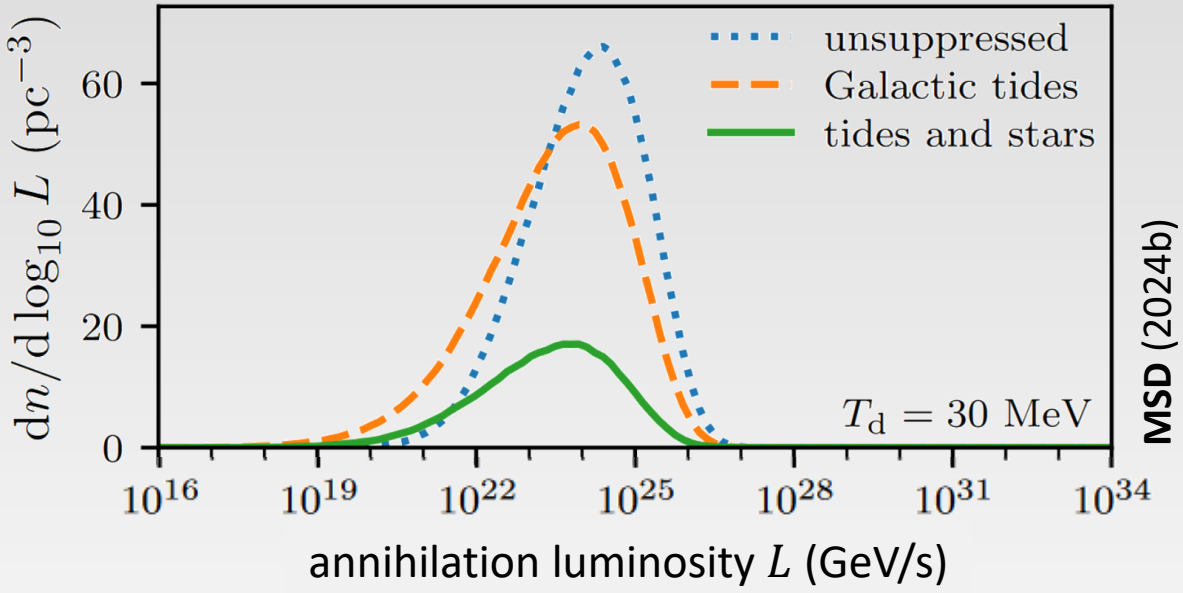
(analytic result from **MSD** & White 2023a)

Outcome: $\rho \propto r^{-3/2}$ cusps cover a factor of $r_{\rm cusp}/r_c \sim 500$ in radius for typical cosmologies

Cusp survival near us

Despite Galactic tides and encounters with stars,
CDM prompt cusps are sufficiently dense and compact that
a significant fraction survive at ~ 8 kpc from the Galactic center

e.g., a common WIMP model:



MSD (2024b)

Tides: mild impact on all cusps

Stars: destroy $\sim 2/3$ of cusps but leave the rest unaffected

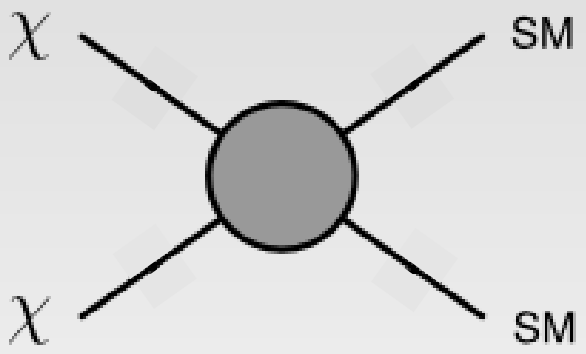
Models of tidal forces & stellar encounters from Stücker et al. (2023a,b)

Supplementary

Annihilation in prompt cusps

Dark matter freeze-out

Well motivated possibility:
thermal relic dark matter particle χ,
pair-produced in the early universe.



Thermal relic cross section:
$\langle \sigma v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$

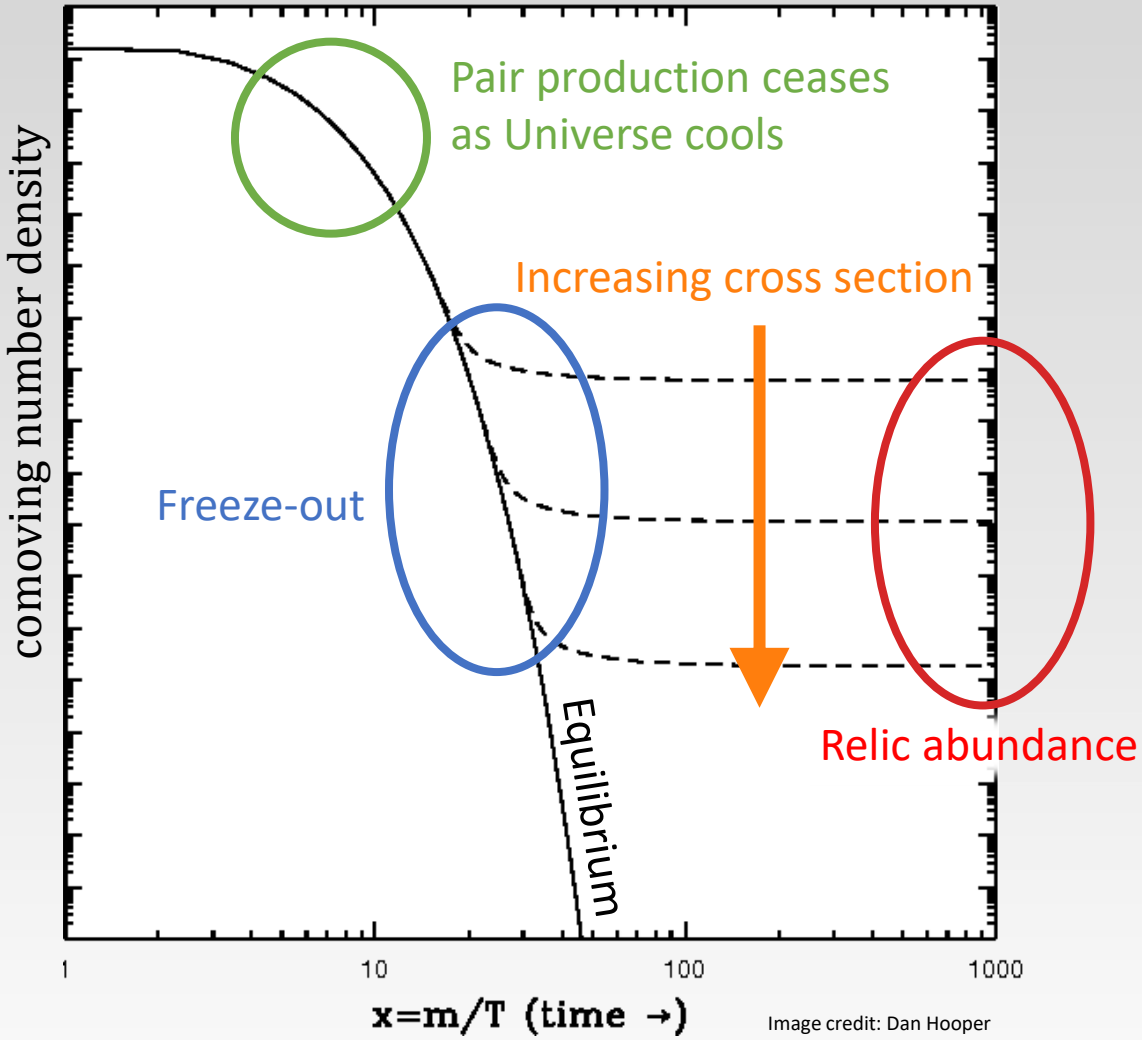
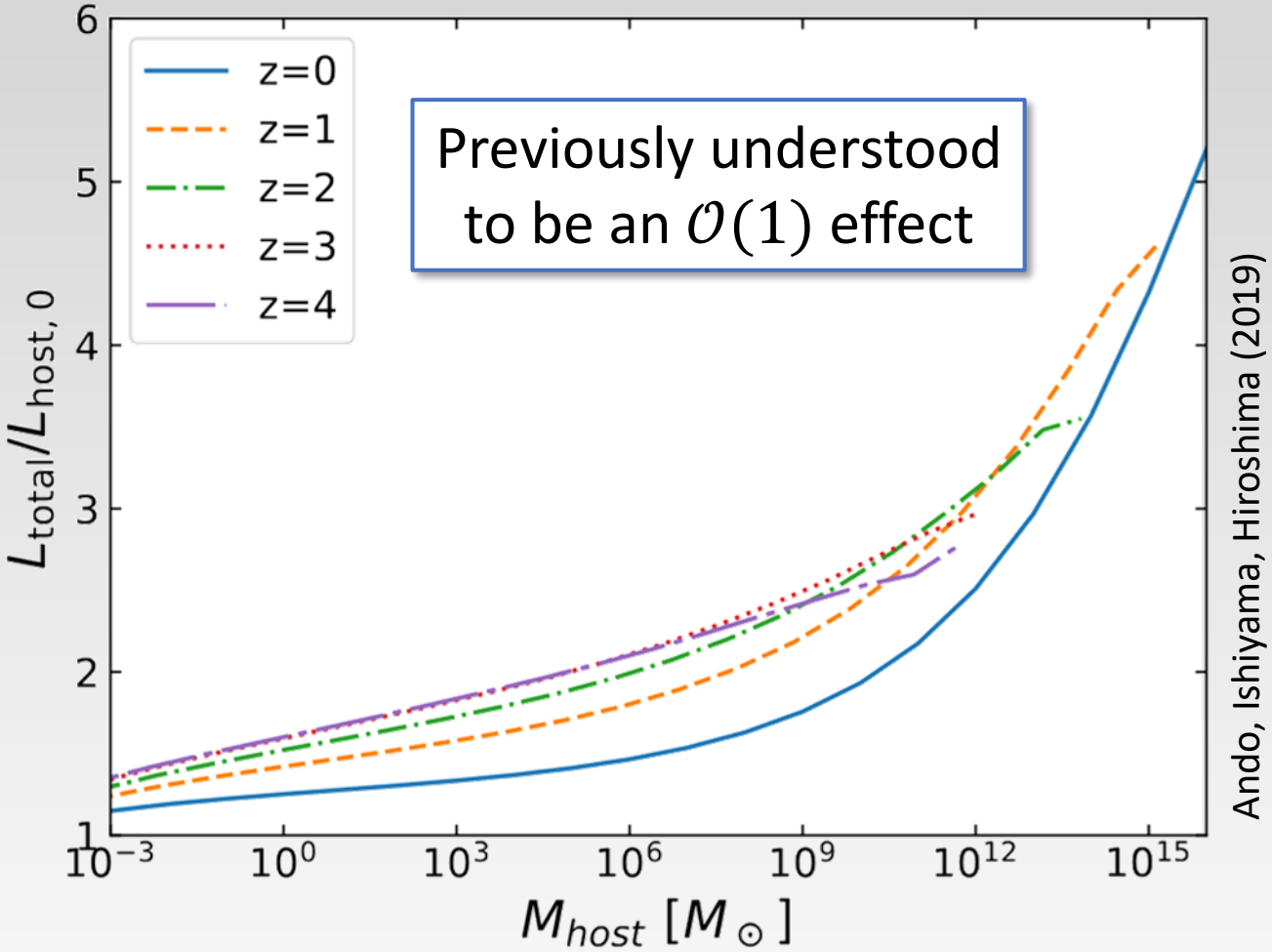


Image credit: Dan Hooper

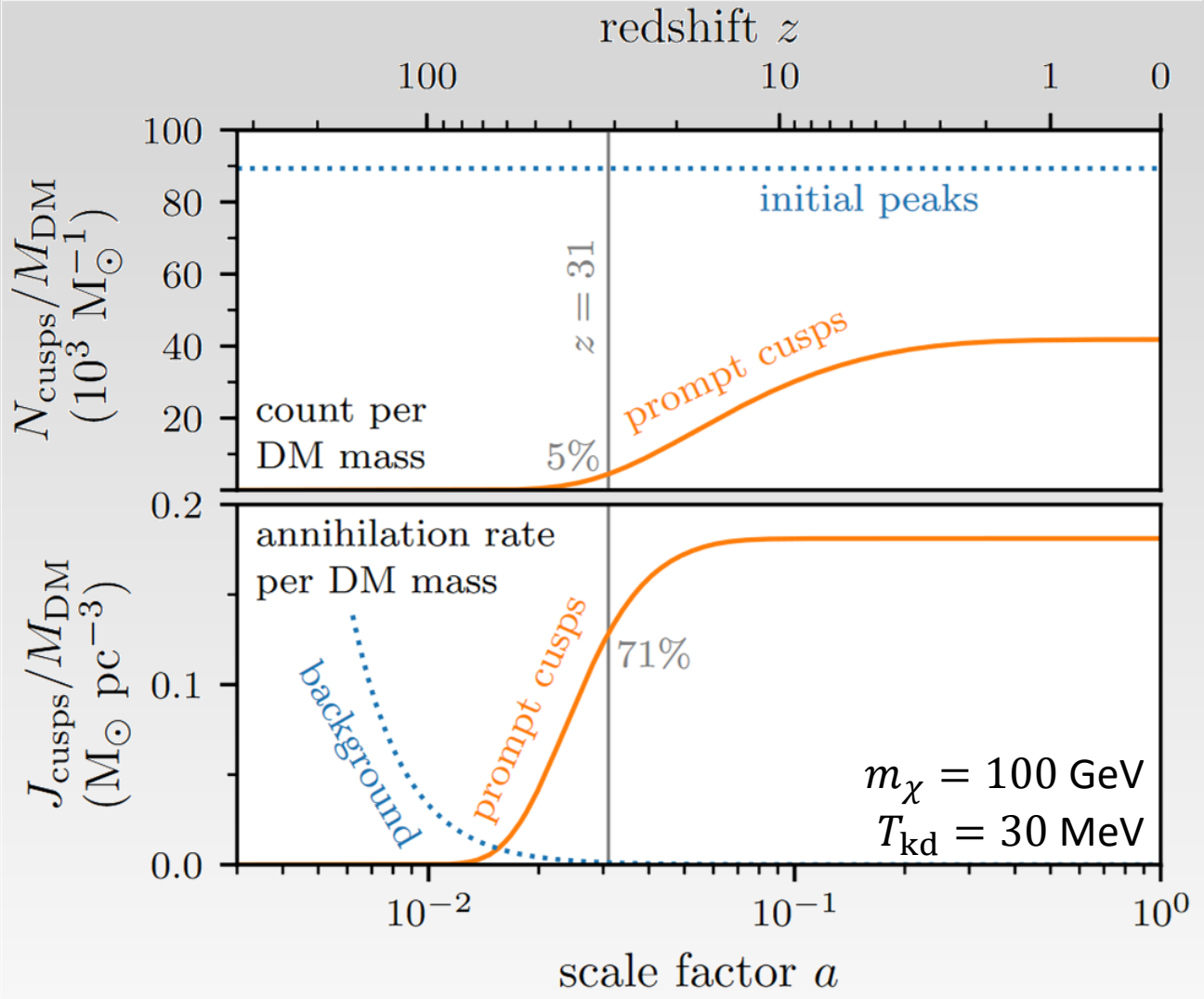
Substructure boost

The annihilation rate inside a halo is boosted by the presence of subhalos

(due to ρ^2 scaling)



Annihilation in prompt cusps



Well fit by

$$\frac{J_{\rm cusps}}{M_{\rm DM}} = 0.08\left[\log\left(\frac{m_\chi}{\rm GeV}\frac{T_{\rm kd}}{\rm GeV}\right) + 36\right]^5 \bar{\rho}_0$$

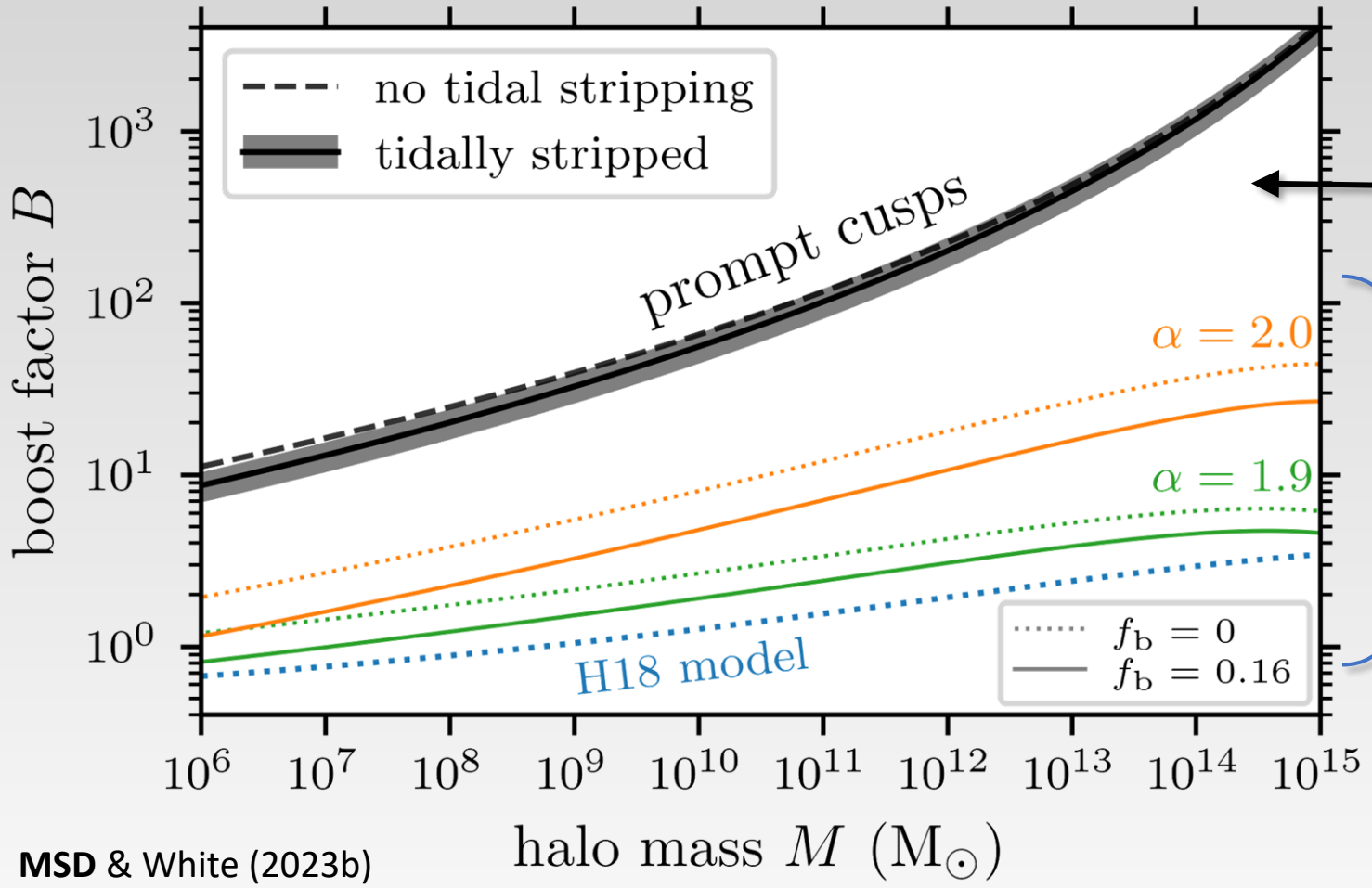
as a function of DM model;

m_χ = DM particle mass
$T_{\rm kd}$ = DM kinetic decoupling temperature
$\bar{\rho}_0$ = mean DM density today

Boost factor from prompt cusps

Annihilation boost inside larger halos:

Same DM model as earlier:
$m_\chi = 100$ GeV, $T_{\rm kd} = 30$ MeV



MSD & White (2023b)

Directly from statistics of peaks & peak-cusp connection

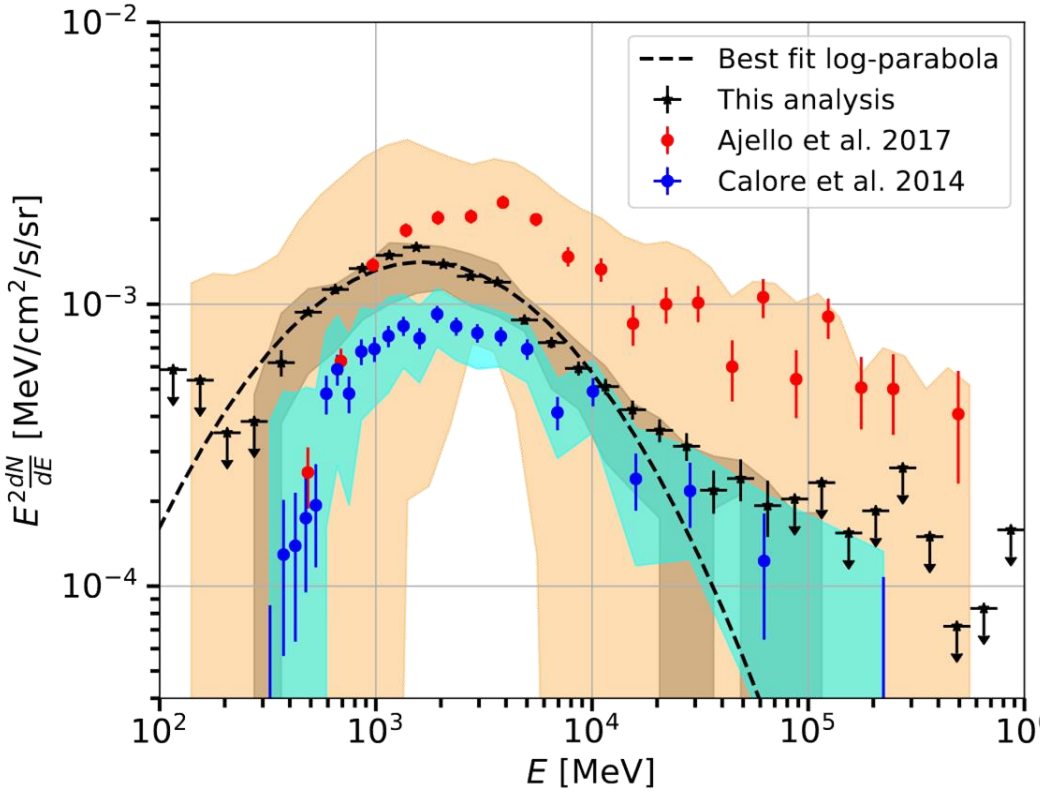
Previous predictions: halo & subhalo models

- Extrapolate from much larger scales: $\frac{dN}{dm} \propto m^{-\alpha}$
- Semianalytic modeling (neglected baryons!)

The Galactic Center gamma-ray excess

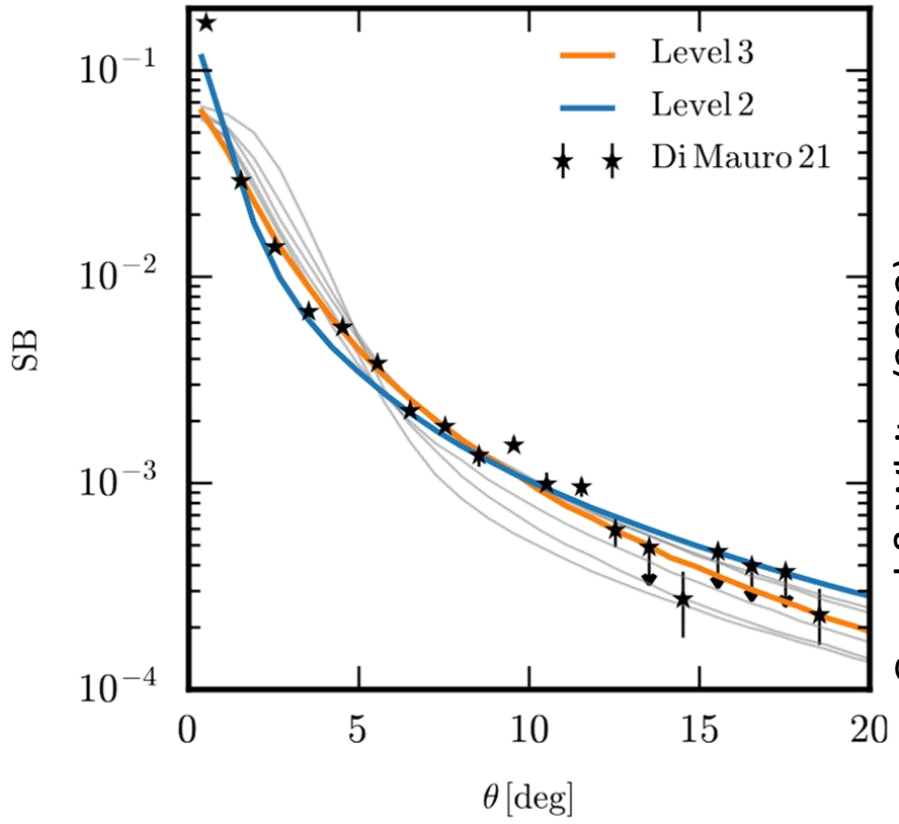
Spatial distribution of annihilation signal changes. Consider GCE as illustration:

Excess of γ rays detected by *Fermi* in the 1-10 GeV energy range



Di Mauro (2021)

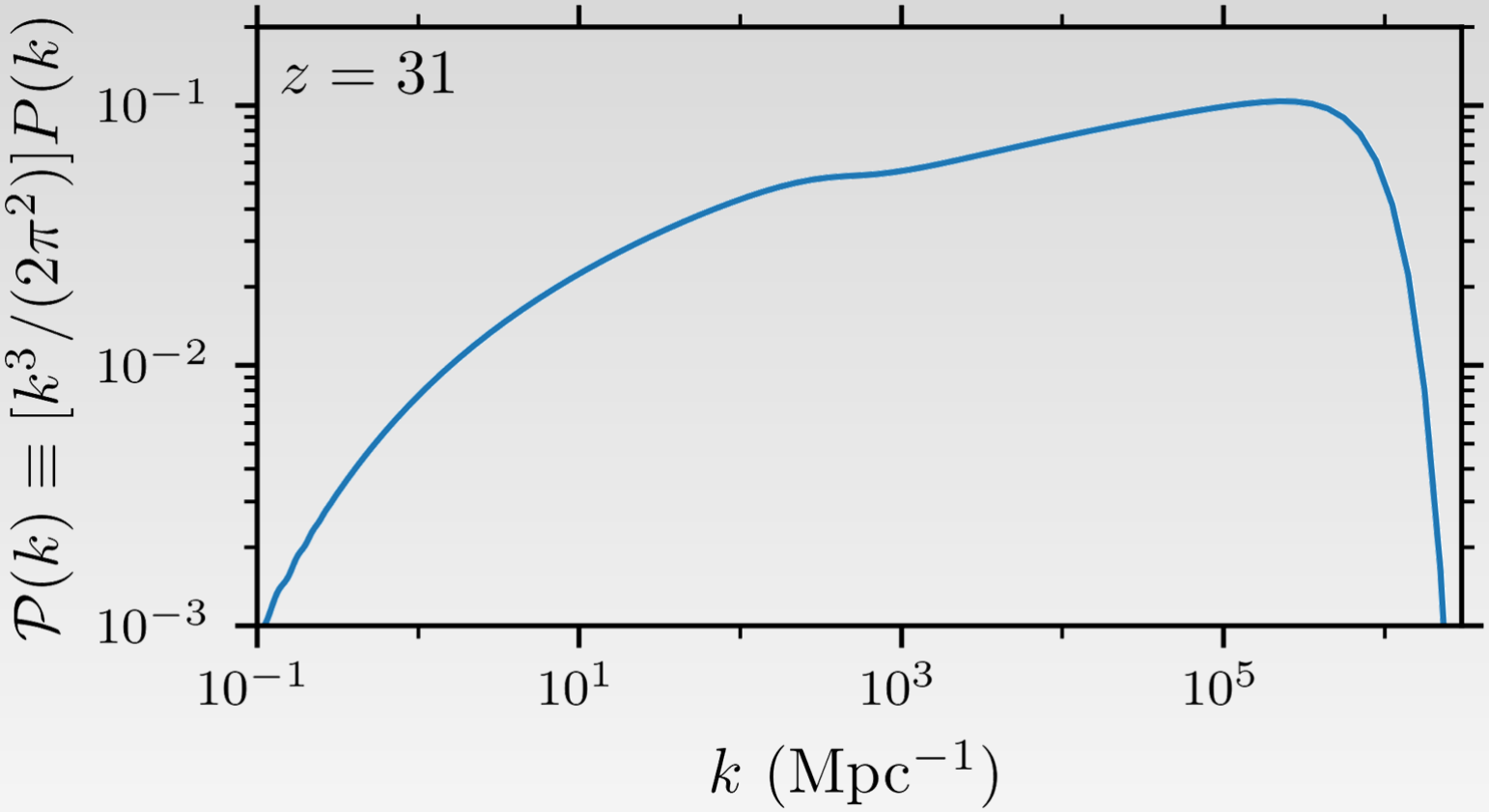
Morphology consistent with DM annihilation from a smooth Galactic halo



Grand & White (2022)

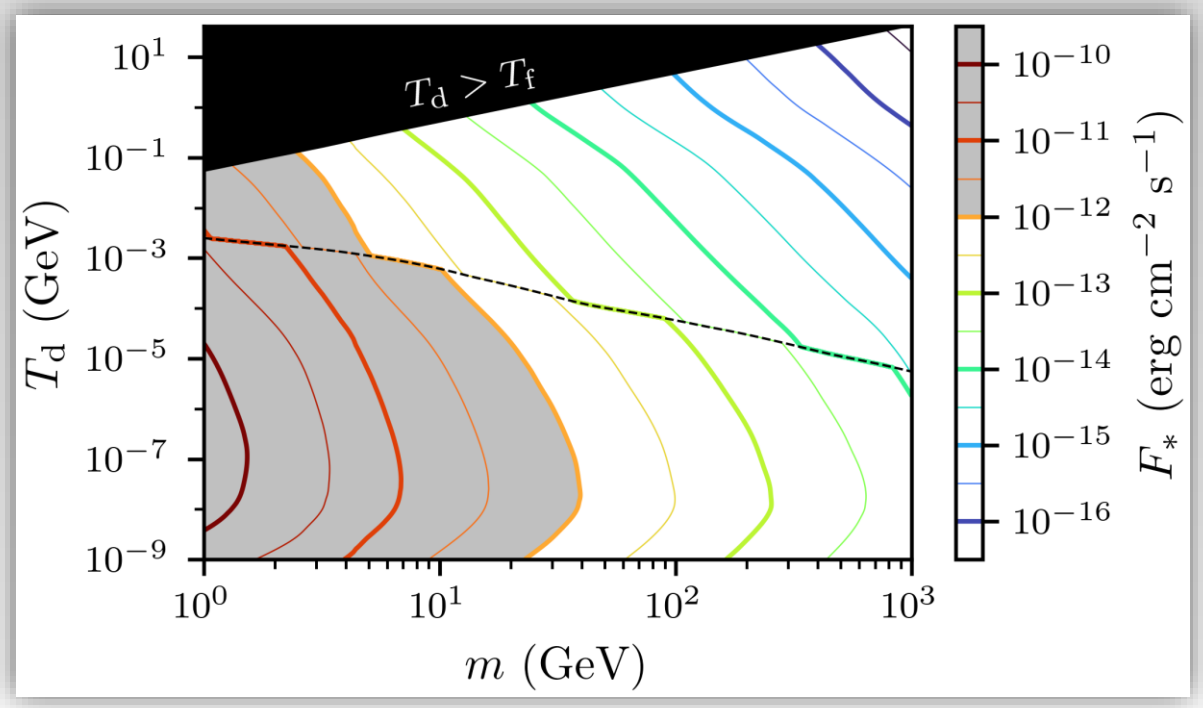
Power spectrum for WIMP DM

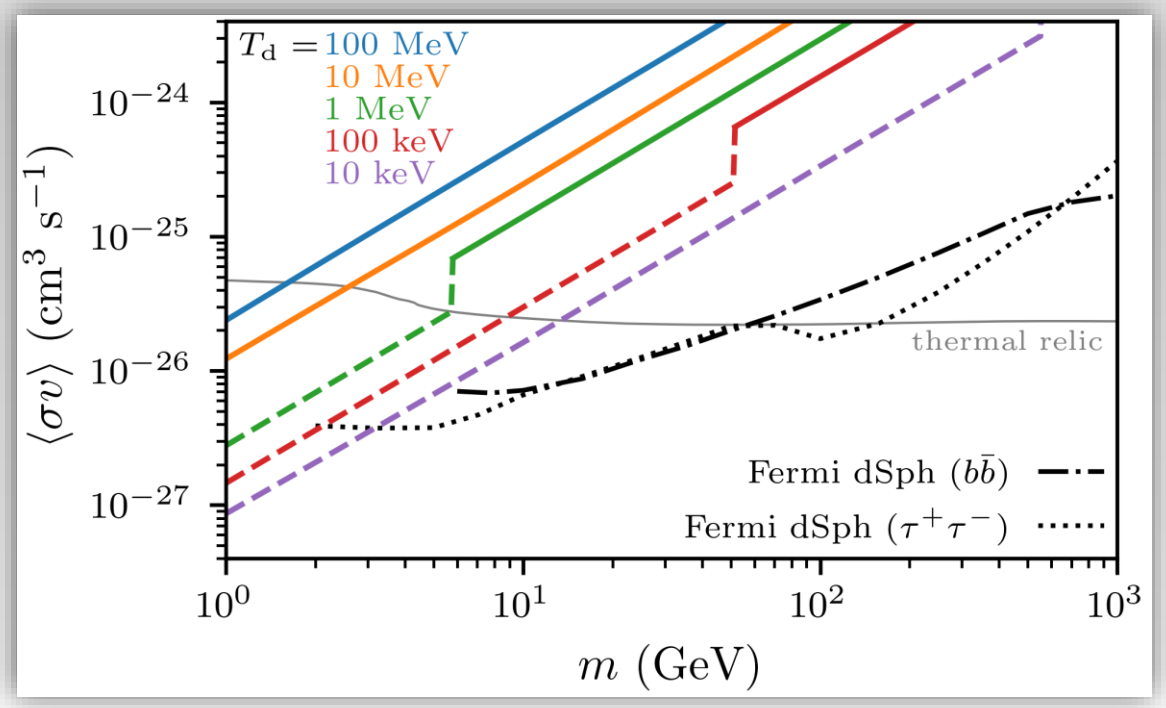
$k_{\rm fs} \simeq 1$ pc



Individual prompt cusps

Only significant annihilation sources if DM is relatively warm



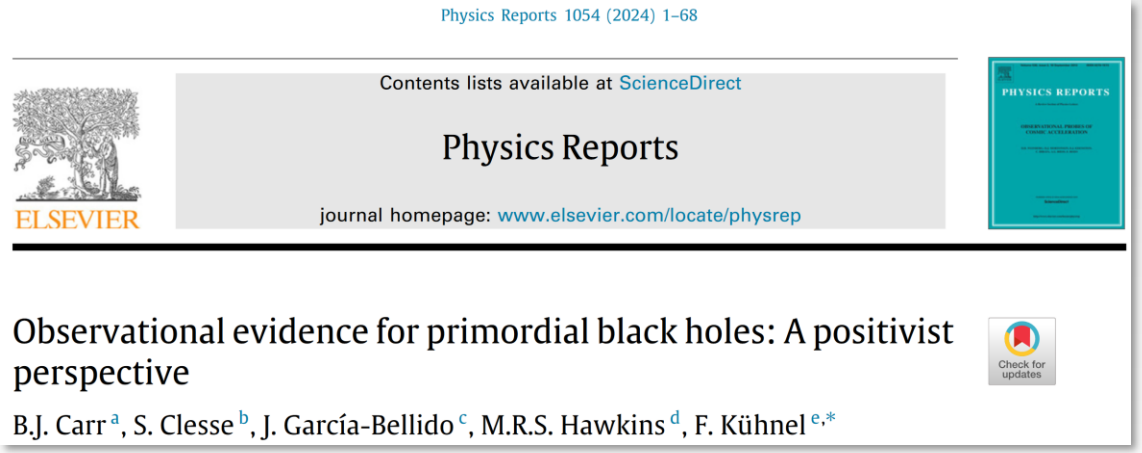


Supplementary

Primordial black holes

Motivation for primordial black holes

- Black holes are fun
- Gravitational waves are popular
- Explain the dark matter without any new species
  (however, requires highly fine-tuned initial conditions)
- Possibly explain other observations?


Physics Reports 1054 (2024) 1–68

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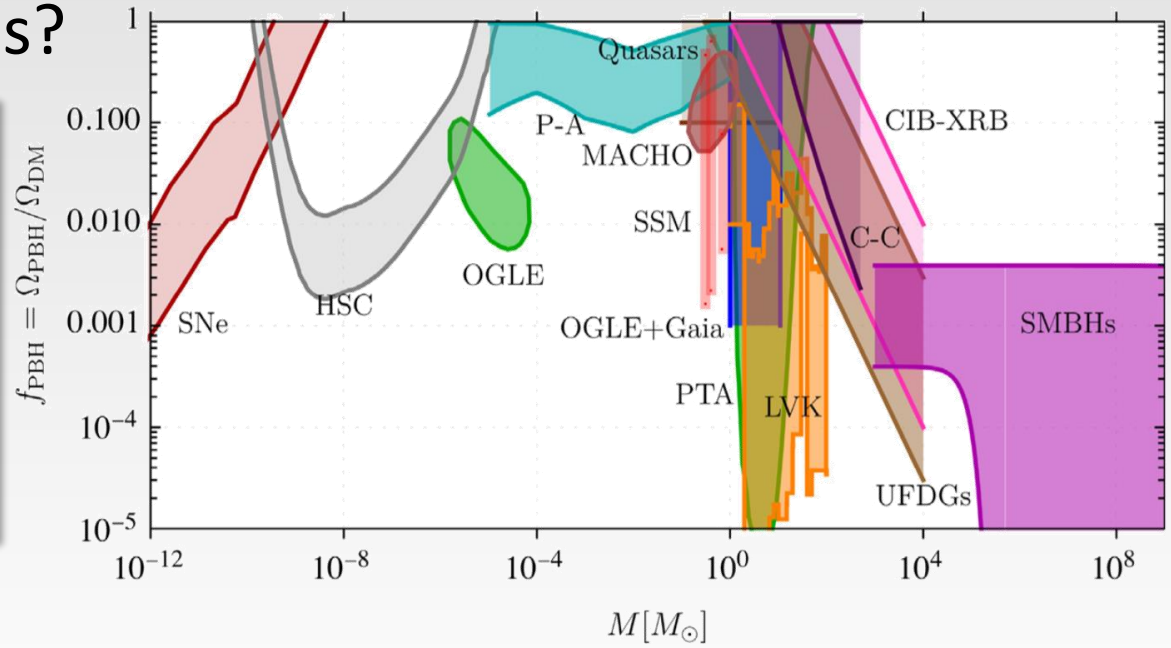
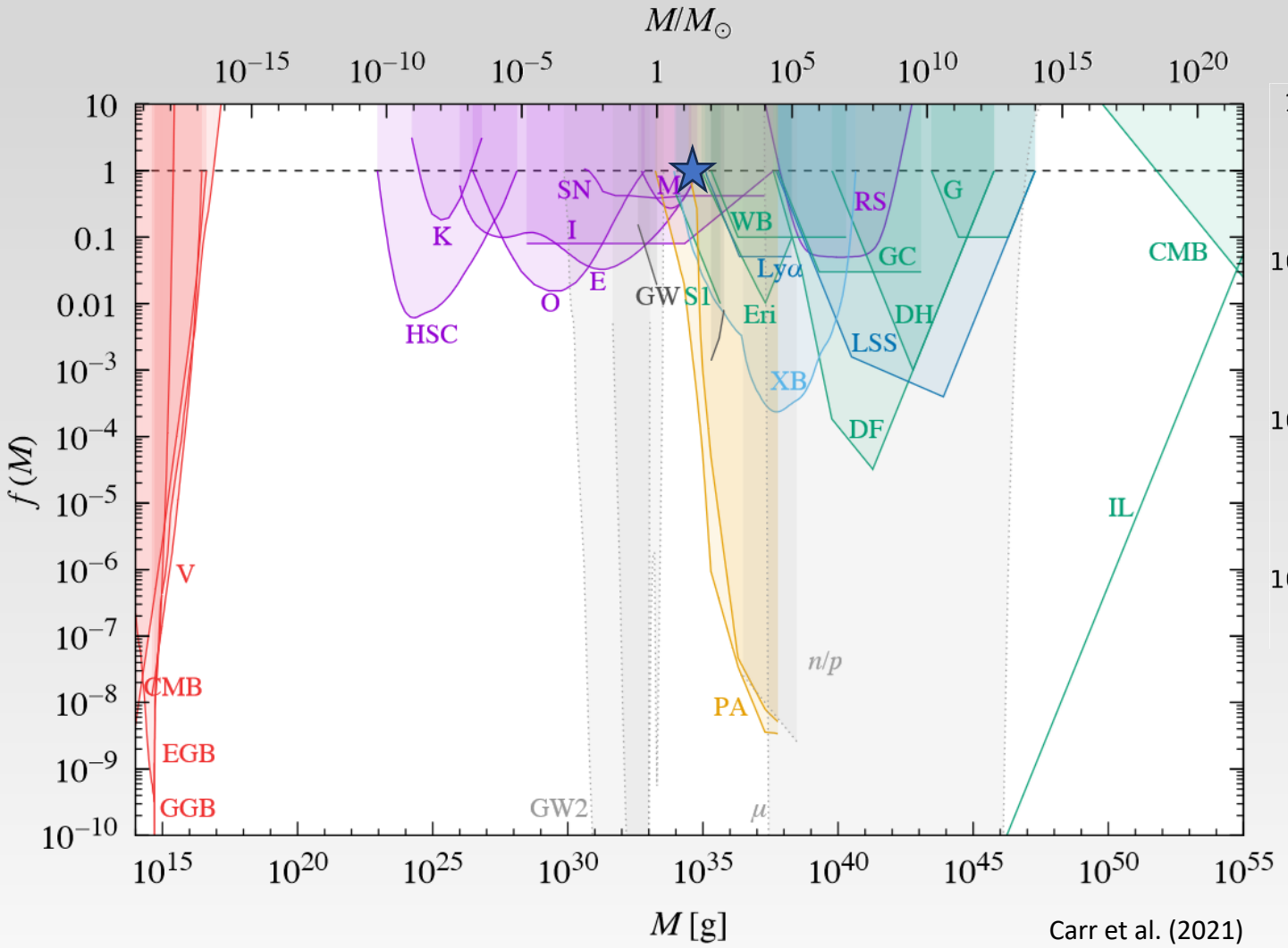
Observational evidence for primordial black holes: A positivist perspective

B.J. Carr [a], S. Clesse [b], J. García-Bellido [c], M.R.S. Hawkins [d], F. Kühnel [e,*]

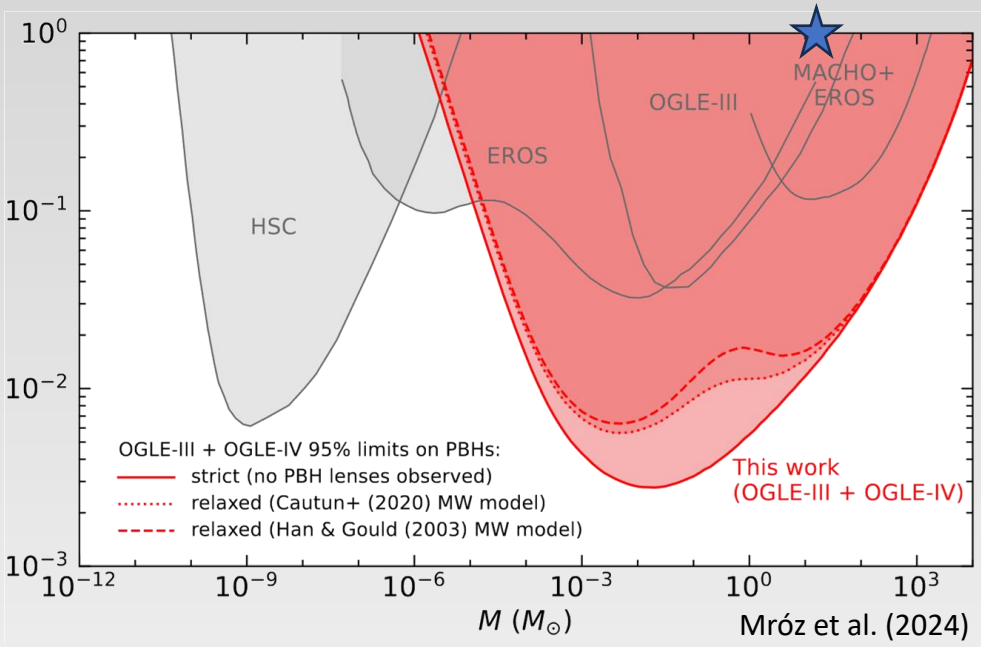


Constraints on PBHs

★ = we are here



Carr et al. (2021)



Mróz et al. (2024)

$f_{\rm PBH} = 1$ is likely ruled out
(although there are uncertainties)

Main purpose is to establish how N-body effects influence structure formation & properties of PBH binaries

Simulating PBHs with a collisional code

BIFROST: simulating compact subsystems in star clusters using a hierarchical fourth-order forward symplectic integrator code

Antti Rantala, Thorsten Naab, Francesco Paolo Rizzuto, Matias Mannerkoski, Christian Partmann, Kristina Lautenschütz

For weakly perturbed binaries, evolve orbital elements directly

Well behaved for close encounters

d ccelerated hierarchical fourth-order forward symplectic integrator code O olve collisional stellar systems with arbitrary binary fractions up to $f_{\rm bin} = 100\%$ by using secular and regularised integration for binaries, triples, multiple systems or small clusters around black holes within the fourth-order forward integrator framework. Post-Newtonian (PN) terms up to order PN3.5 are included in the equations of motion of compact subsystems with optional three-body and spin-dependent terms. PN1.0 terms for interactions with black holes are computed everywhere in the simulation domain. The code has several merger criteria (gravitational-wave inspirals, tidal disruption events and stellar and compact object collisions) with the addition of relativistic recoil kicks for compact object mergers. We show that for systems with N pa ins good up to $N_{\rm GPU} \sim 40 \times N/10^6$ GPUs and that the increasing binary fractions up to de running time (less than a factor ~ 1.5). We also validate the numerical accuracy of BIFROST by presenting a number of star clusters simulations the most extreme ones ediate mass black holes with a relativistic recoil kick.

Relativistic dynamics including mergers and GW recoil kicks

(Currently) not in comoving coordinates:

- **Cannot simulate periodic volume**
- **Not suitable for very high redshifts**

(Currently) cannot include collisionless material: must have $f_{\rm PBH} = 1$

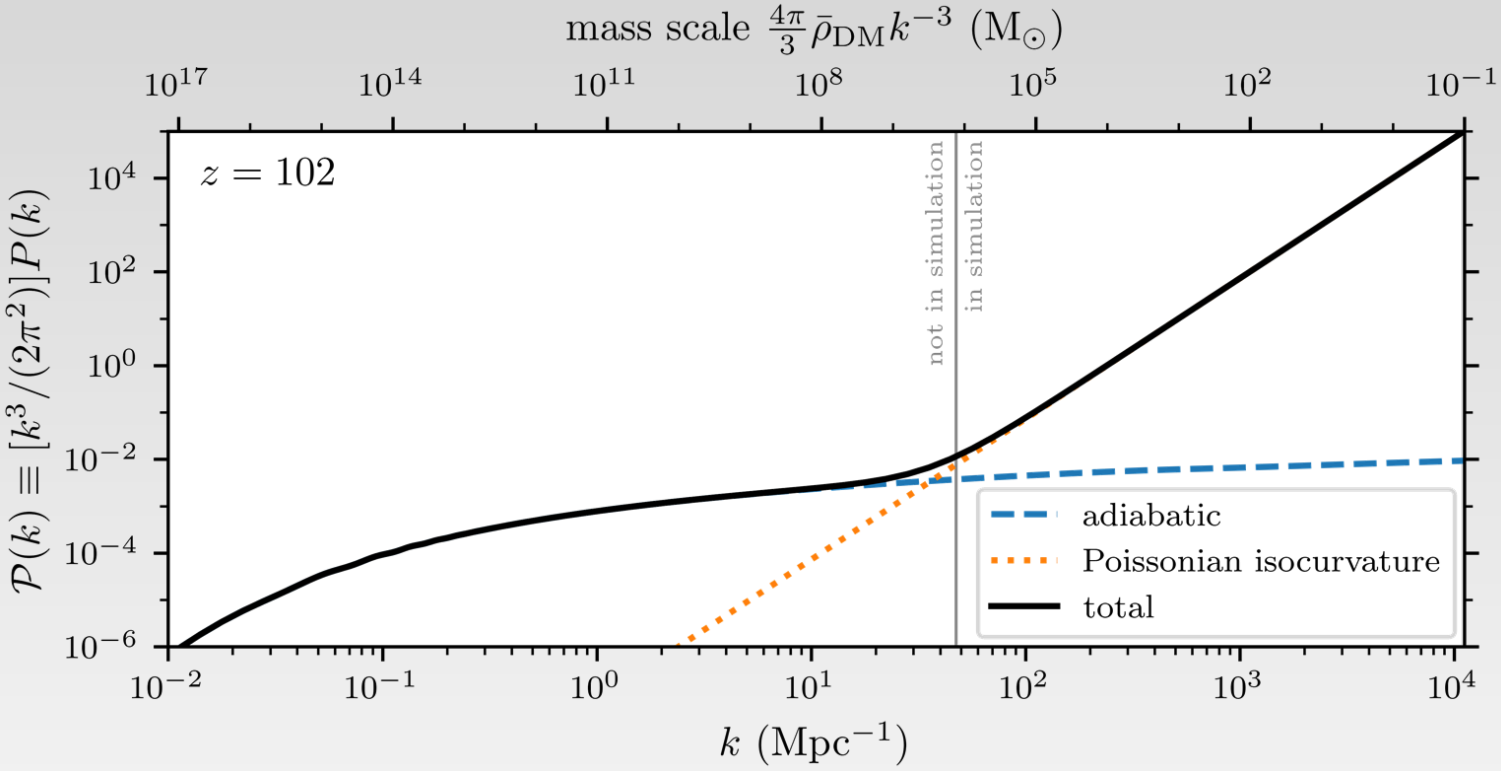
PBH setup

Cosmological scenario:

- Lognormal PBH mass distribution, mean mass 16.5 $M_\odot$
- Poisson spatial distribution
- $\Omega_{\rm PBH} = 0.26$ (dynamical)
- $\Omega_{\rm b} = 0.05$, $\Omega_{\rm r} = 9 \times 10^{-5}$ (background)

Simulation setup:

- $\sim$ 80 000 PBHs
- Sphere of comoving radius 21 kpc, average density, vacuum bounded
- Simulate from $z \sim 10^{12}$ to $z \sim 10^5$ with standard collisionless code (custom Gadget 4)
  - When particles get too close, record their Kepler orbit and restore it later
  - Newtonian (scales on which peculiar gravity is important are subhorizon)
- BIFROST sim starts at $z = 10^5$

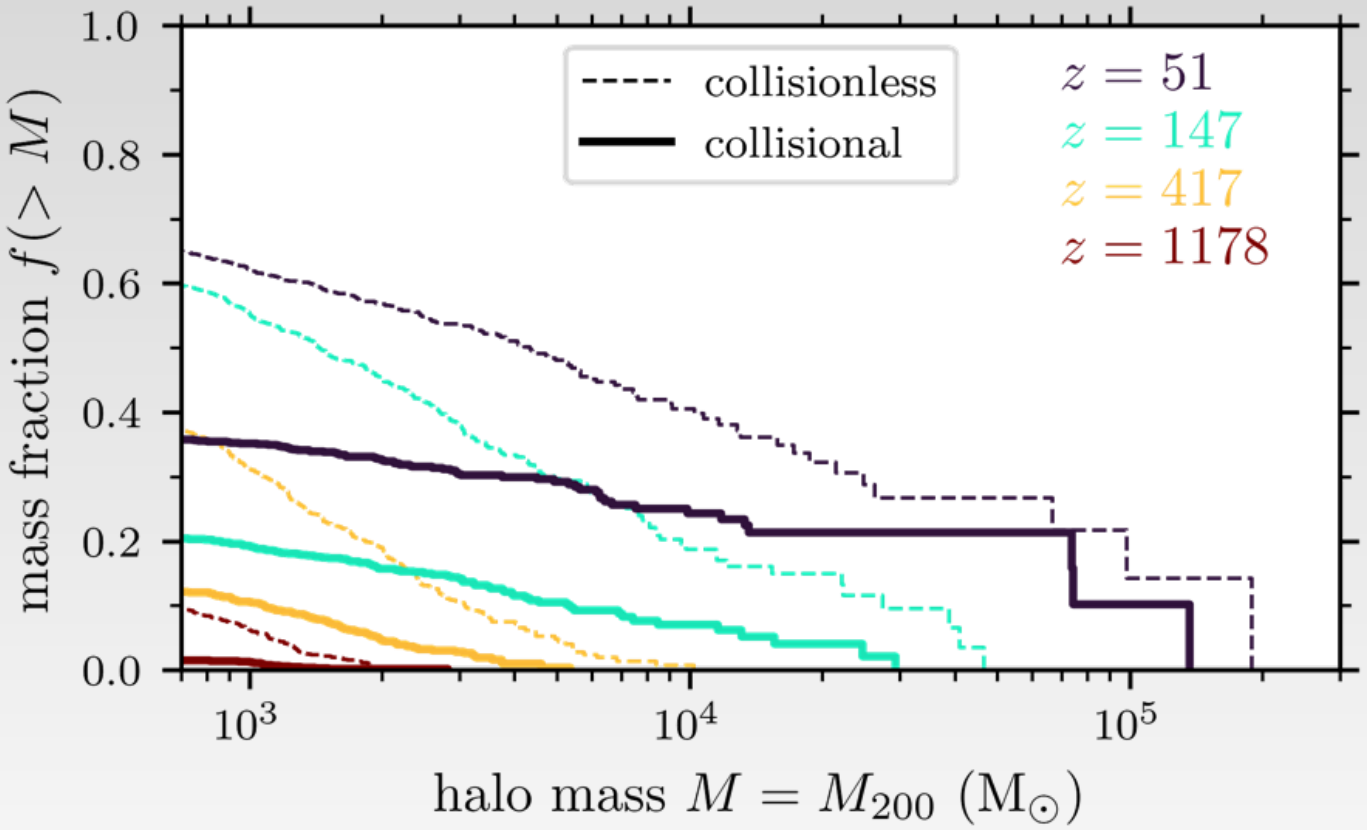


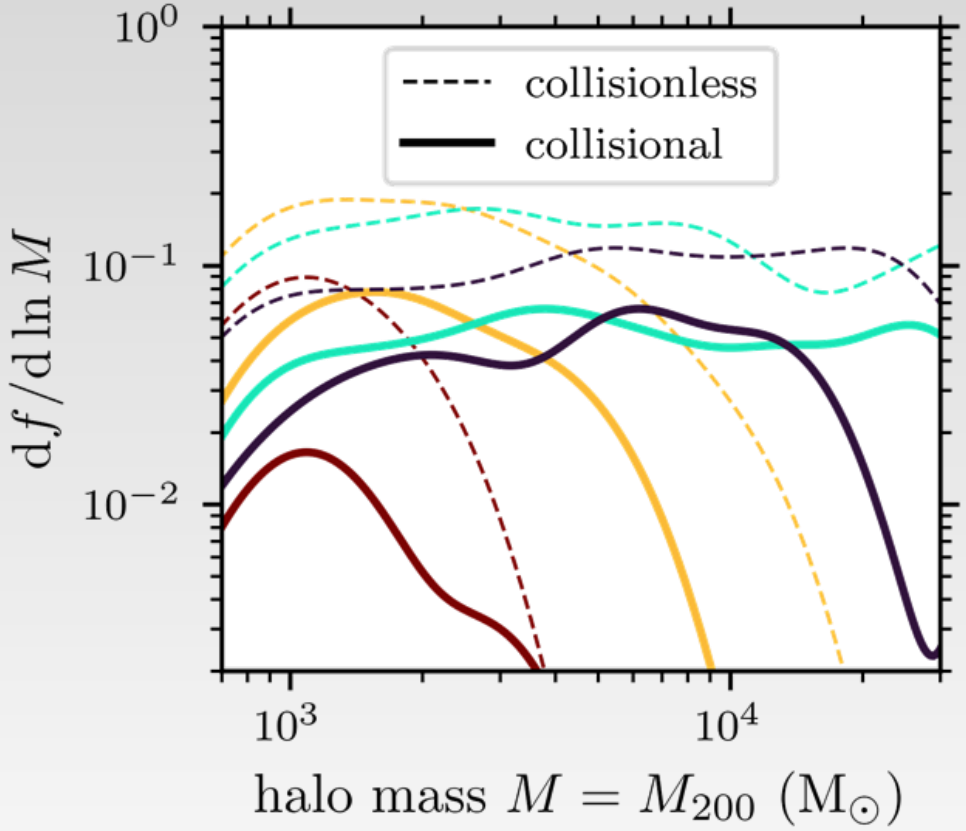
We only resolve the early-forming PBH clusters and not larger-scale systems

(Simulation includes adiabatic modes, but they are negligible on the scale of the simulation volume.)

Collisional dynamics suppress halo abundance

Halos of $N \lesssim 1000$ PBHs are less abundant by a factor of about 2

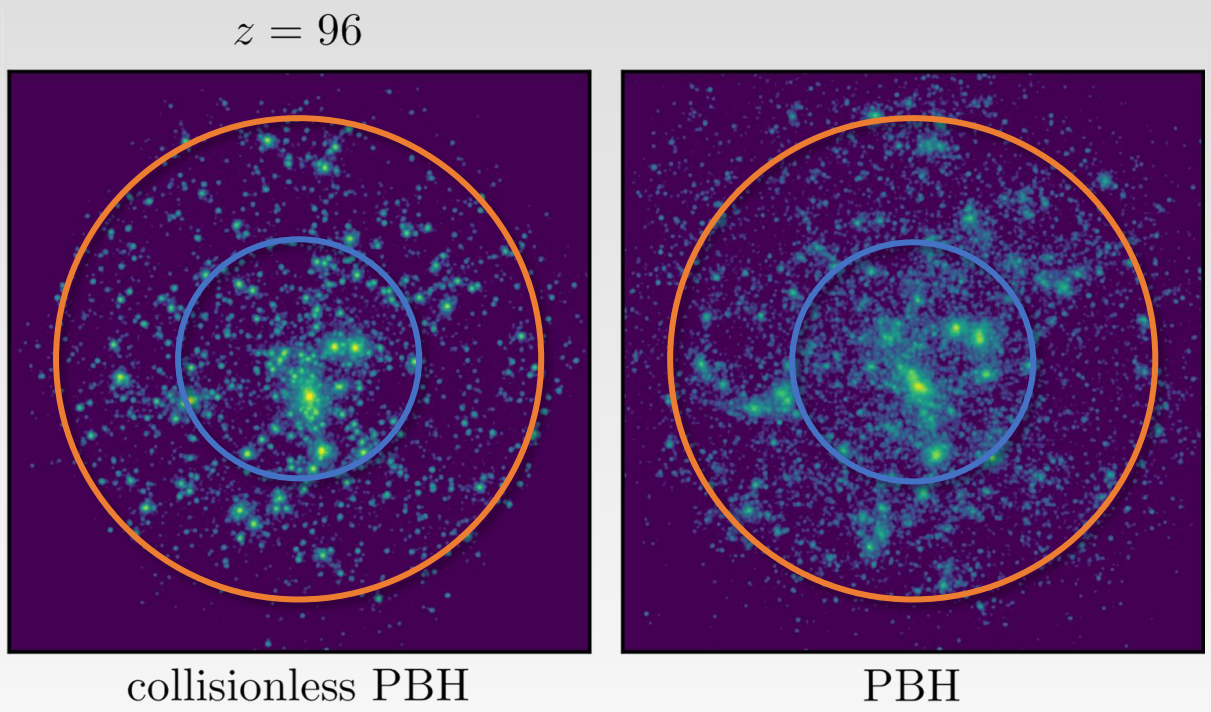


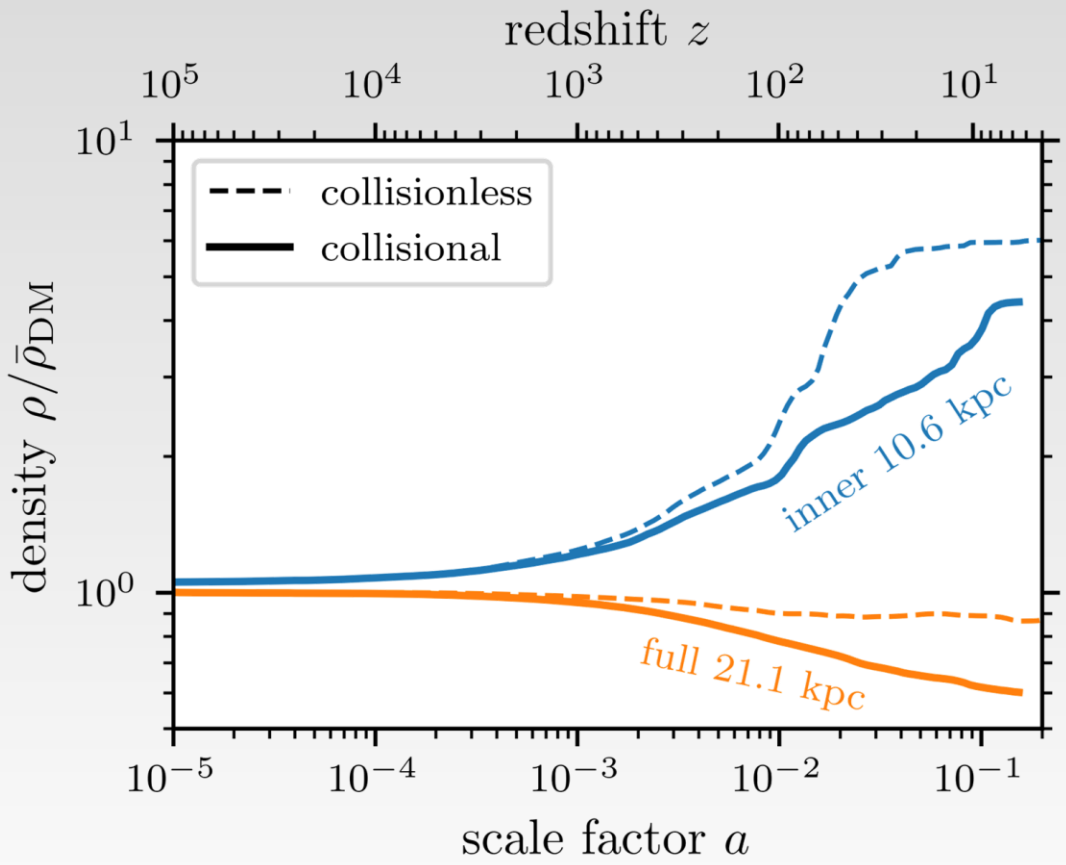


Collisional ejection

High-speed PBHs:

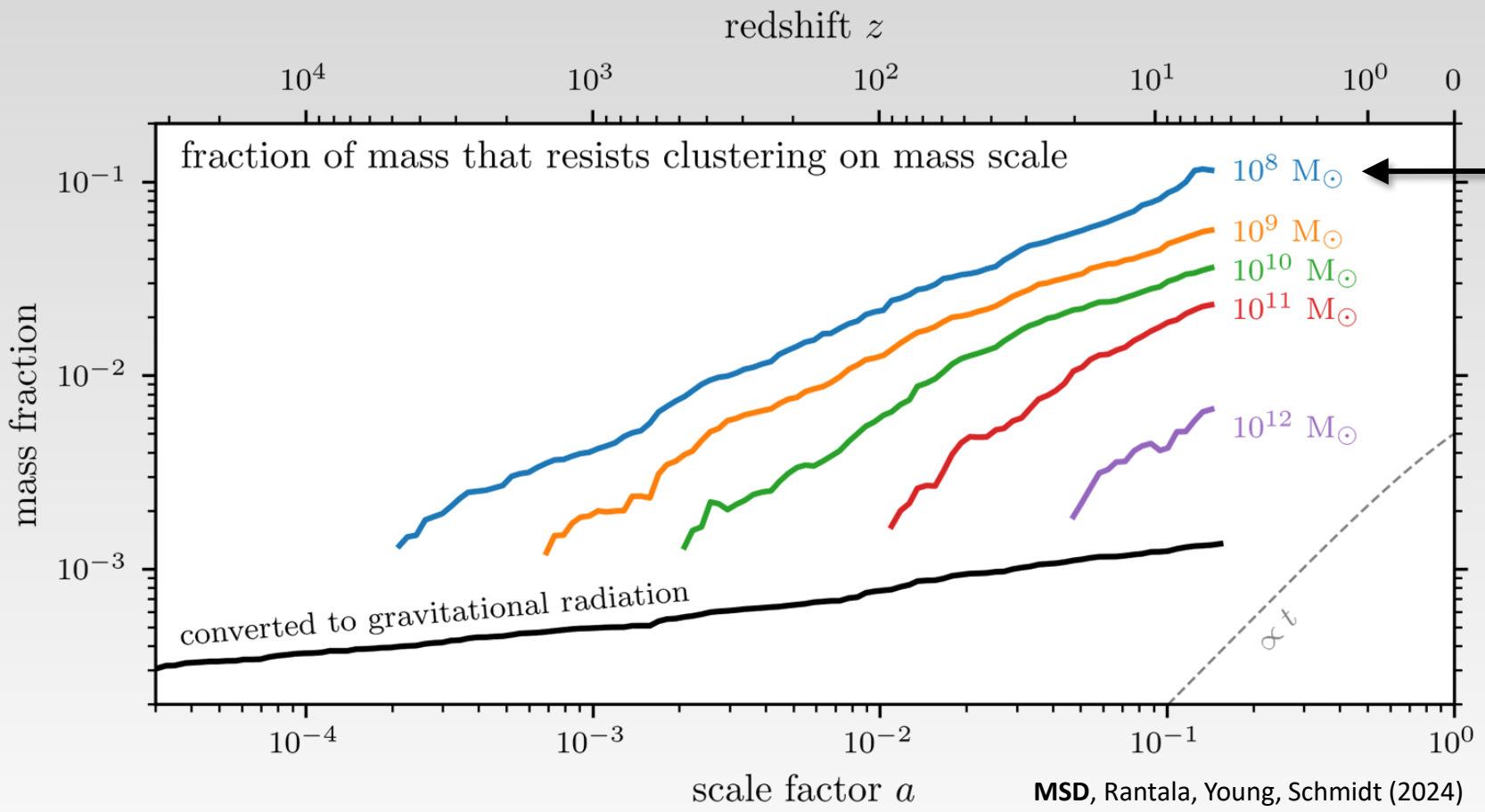
- **fly out of the simulation volume** **(artificial; they should be replenished by other volumes)**
- do not contribute to structure formation (real effect)





Collisional backreaction

High-speed PBHs do not contribute to structure formation (similar to massive ν)



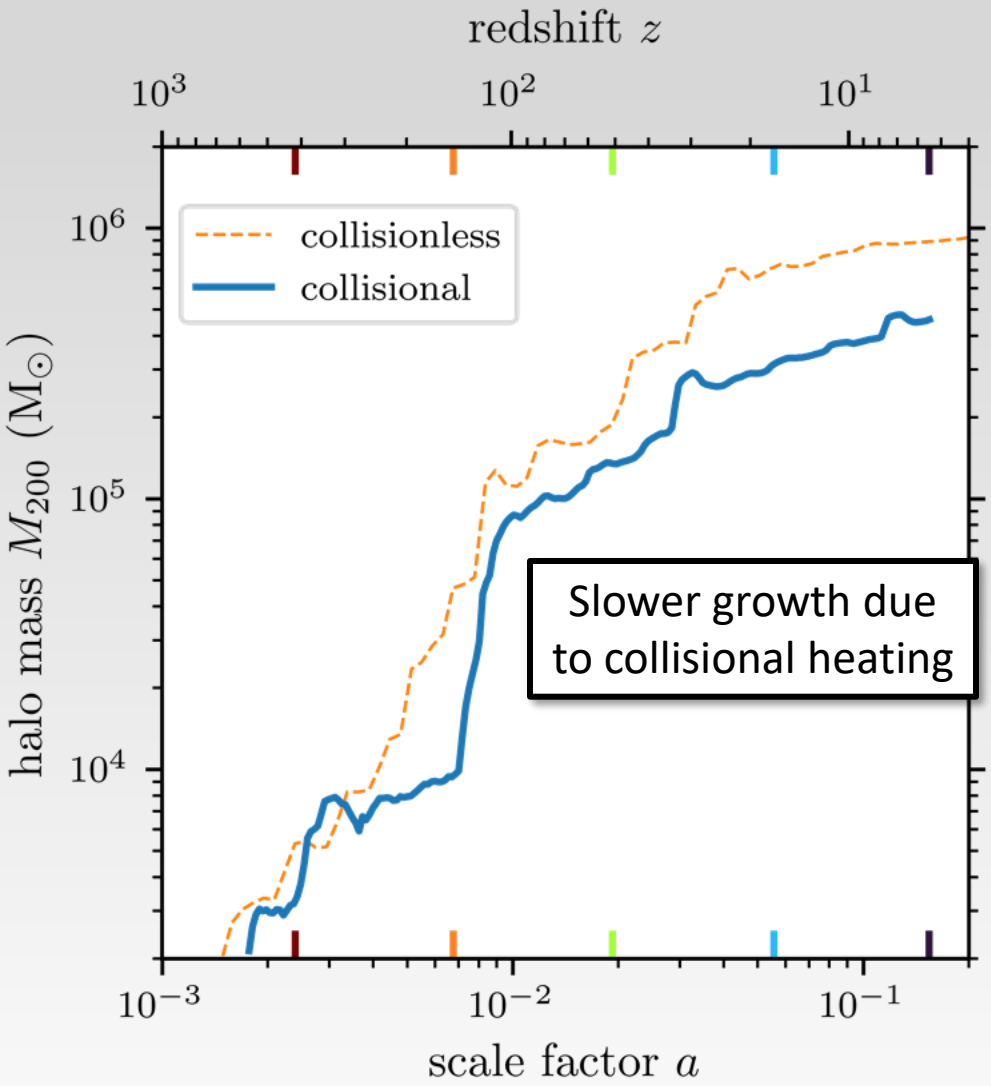
MSD, Rantala, Young, Schmidt (2024)

Jeans mass based on streaming velocities

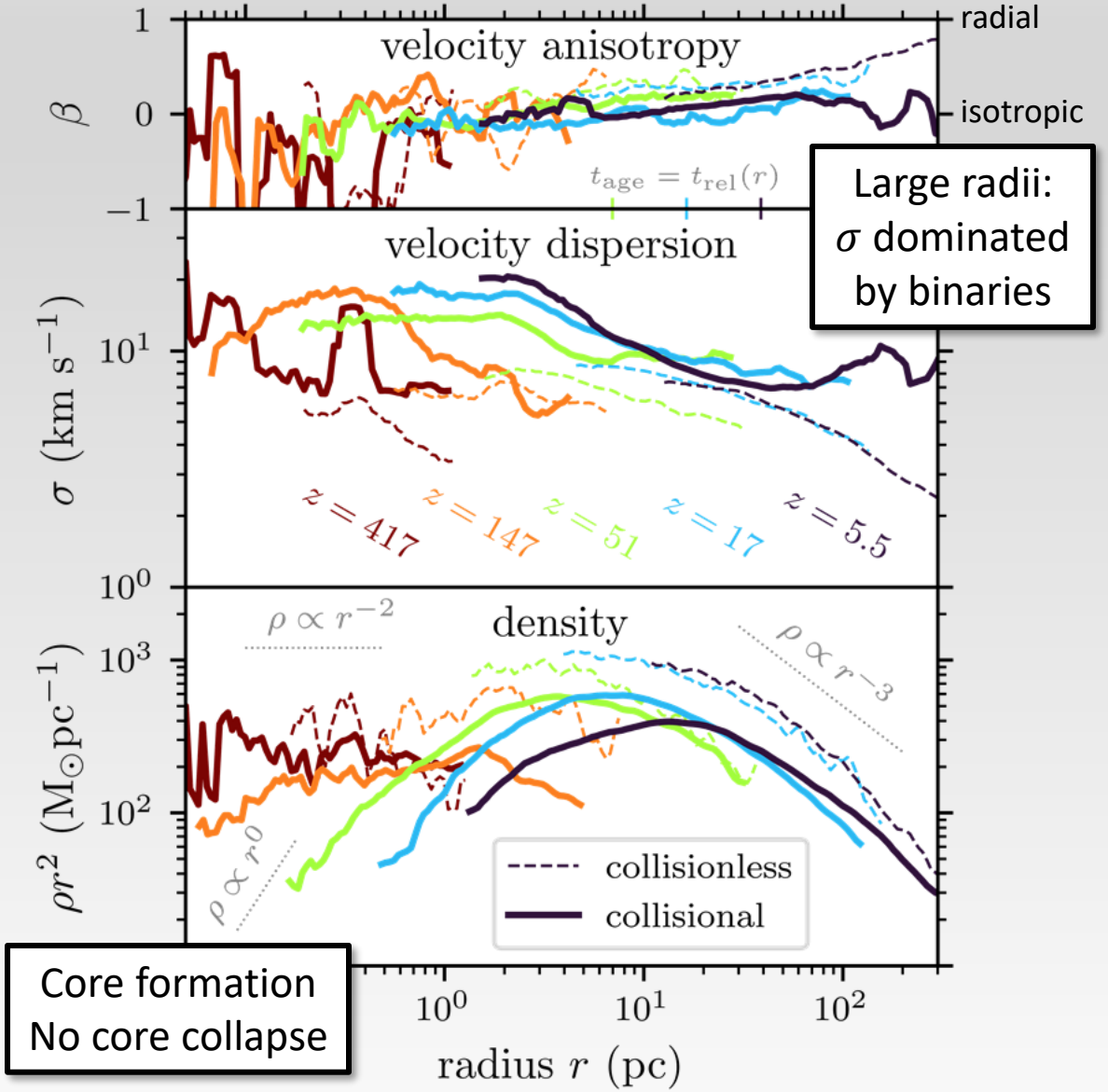
Structure formation on galactic mass scales should be suppressed significantly!

(not represented in the simulation)

Collisional vs collisionless halos



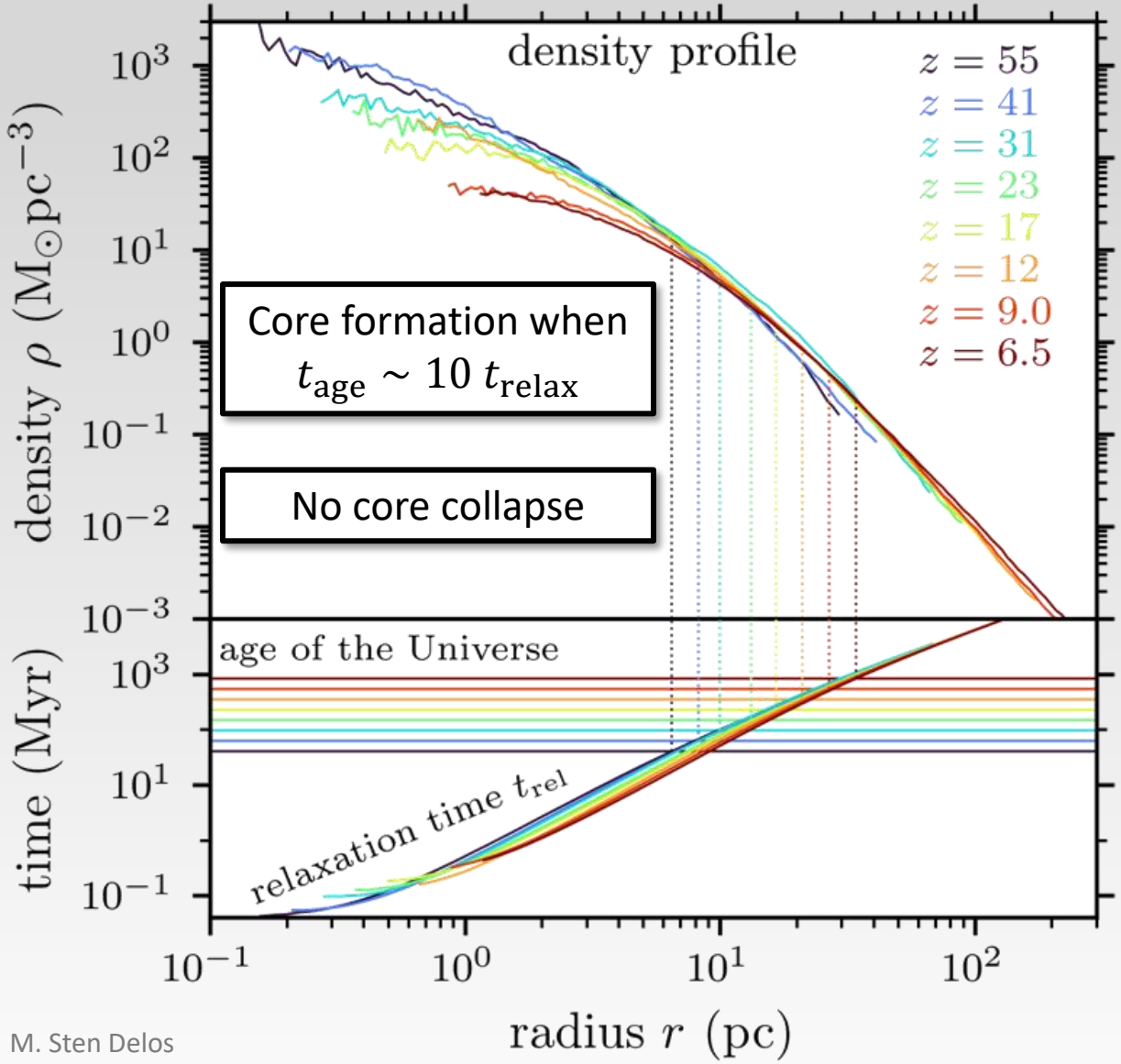
Slower growth due to collisional heating

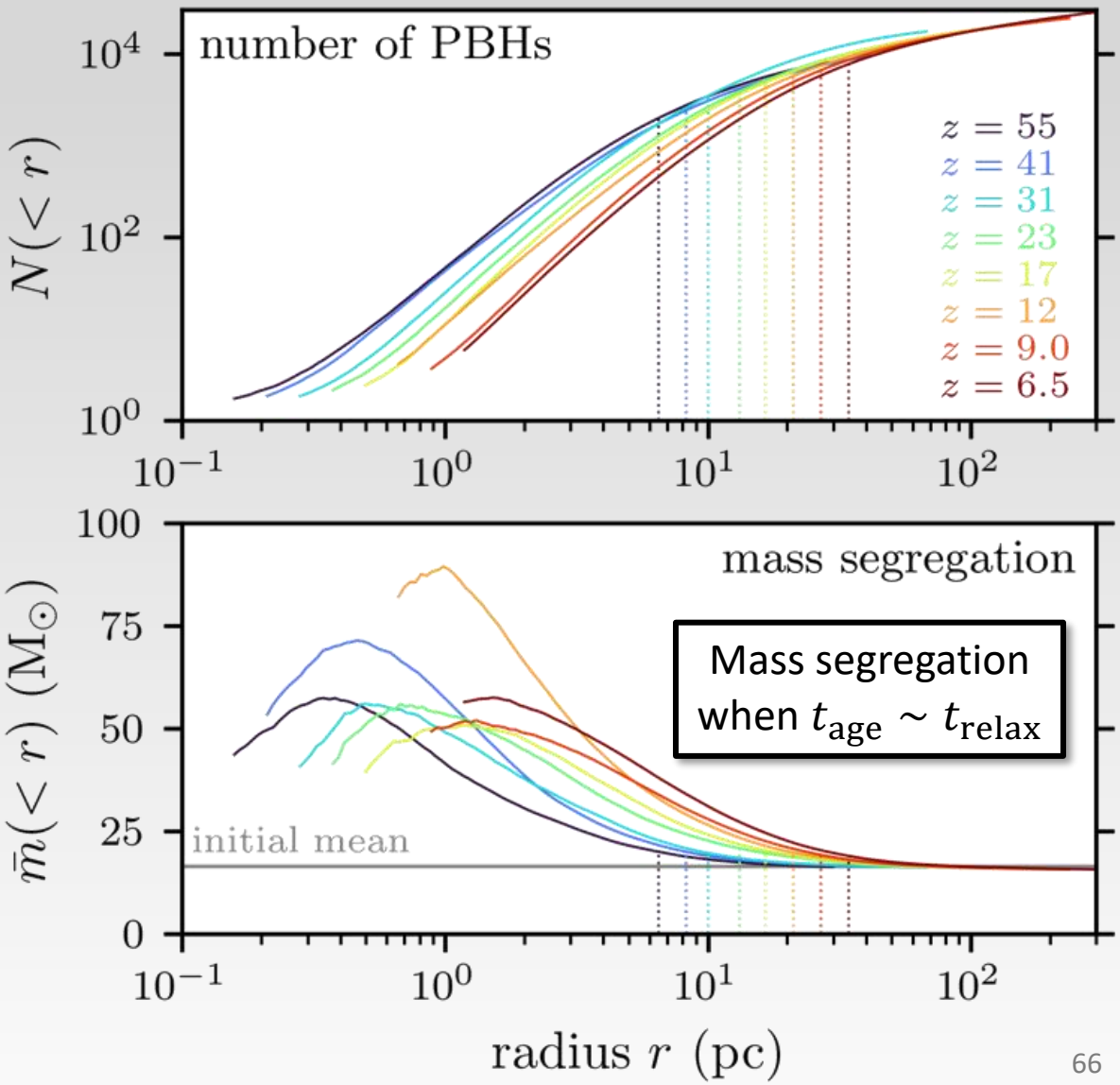


Large radii: σ dominated by binaries

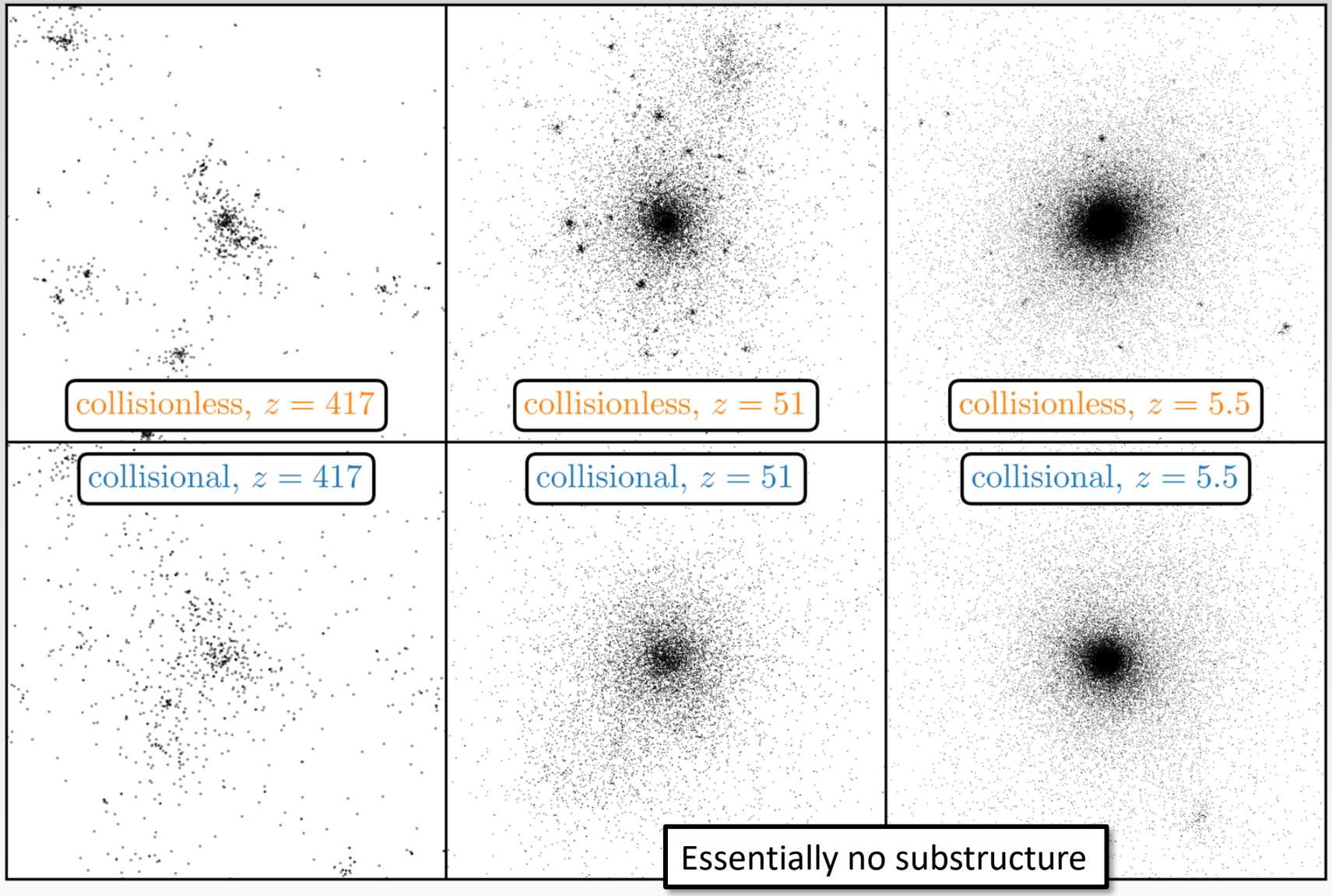
Core formation
No core collapse

Collisional halo evolution





Collisional dynamics suppress subhalos

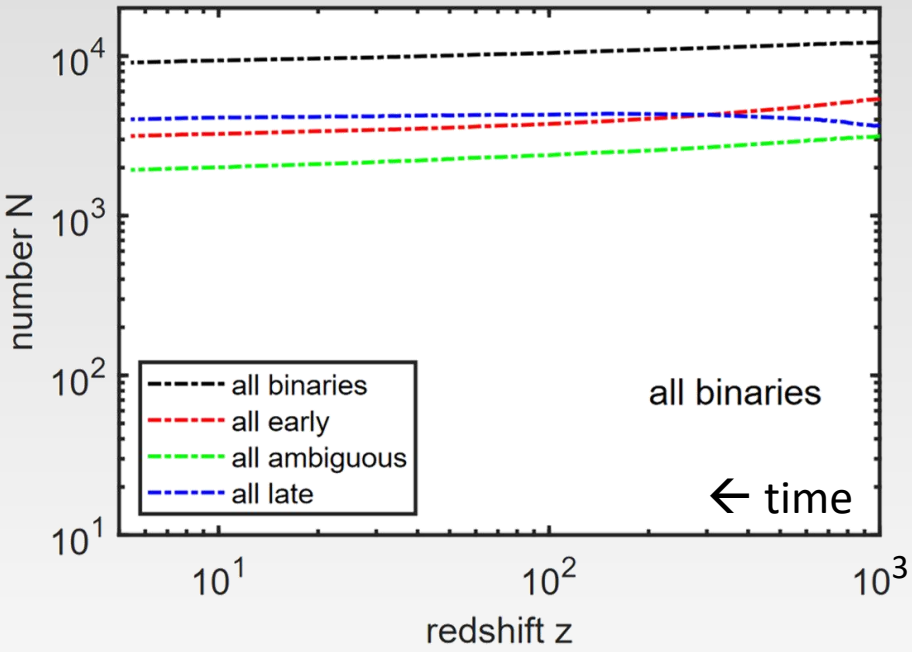


Binaries

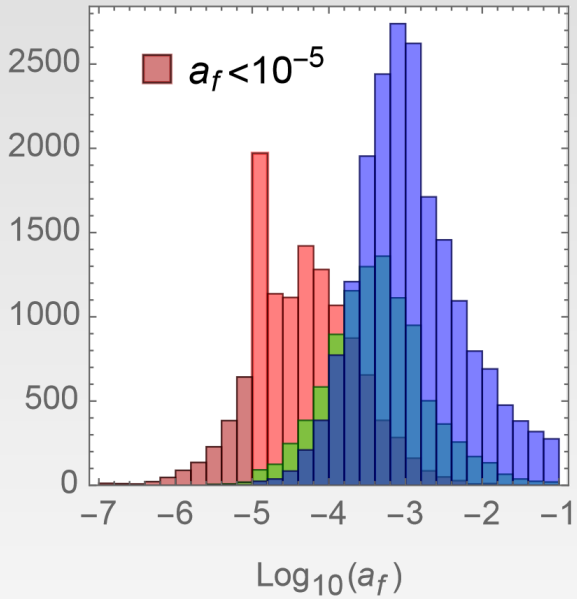
Binary = PBH pair with nearest 3rd body contributing only weak perturbation

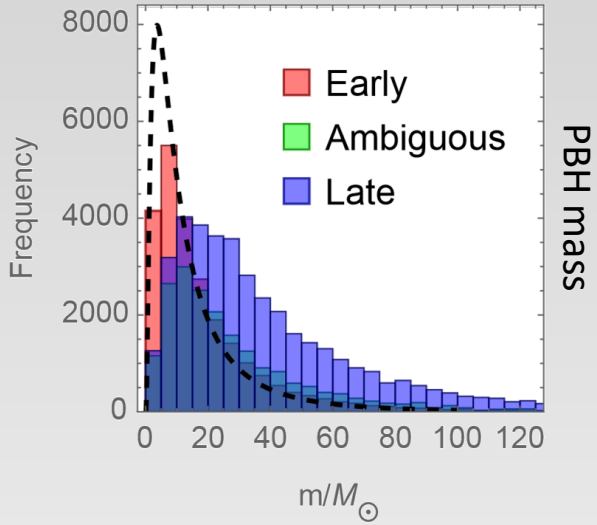
$$\gamma_{\rm pert} = \frac{m_3}{r_3^3}\left\{\frac{m_1 m_2}{(m_1+m_2)\left[r_{\rm a}(1+e)\right]^3}\right\}^{-1} < 0.3$$

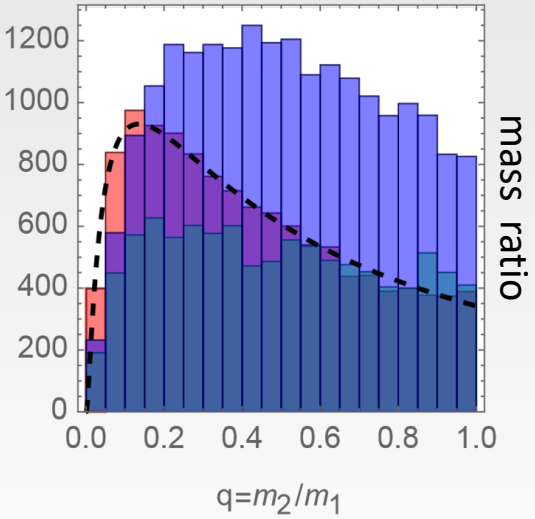
~ 10 000 binaries (25% of all PBHs)



identification time



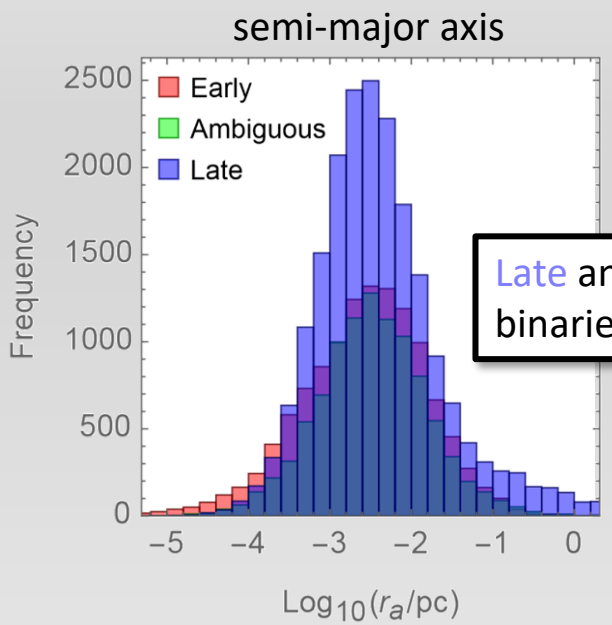




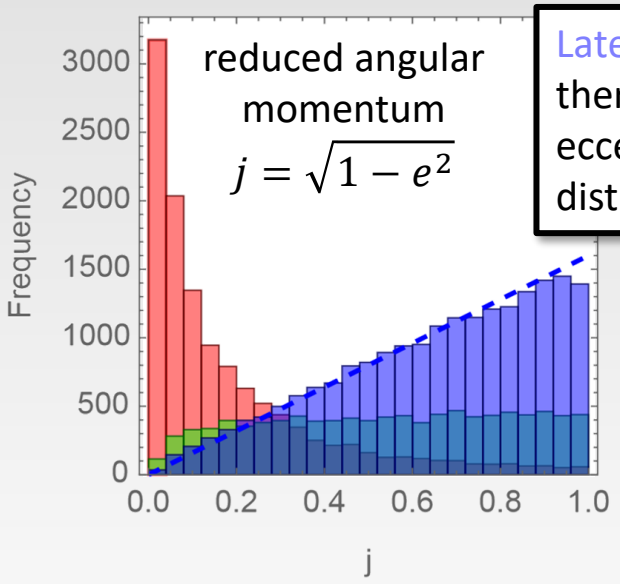
"Early binaries": nearest neighbors in the ICs
"Late binaries": not nearest neighbors in the ICs

Initial binary orbits

(evaluated when binary is first identified)



Late and early binaries: ~ same r_a?



Late binaries: thermal eccentricity distribution

Formalism of Raidal et al. (2019):

$$\delta_b = \frac{m_1 + m_2}{2\rho_{\rm M} V(x_0)}$$

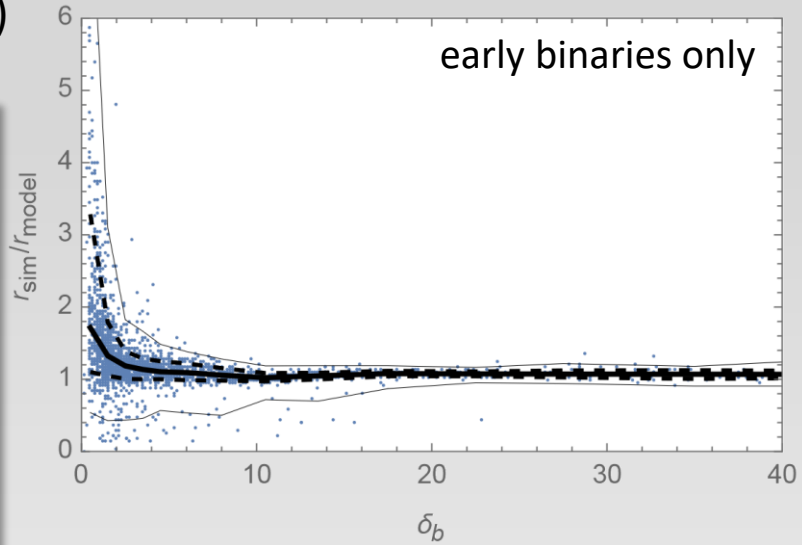
$$r_{\rm a} \approx 0.1 \frac{a_{\rm eq} x_0}{\delta_b}$$

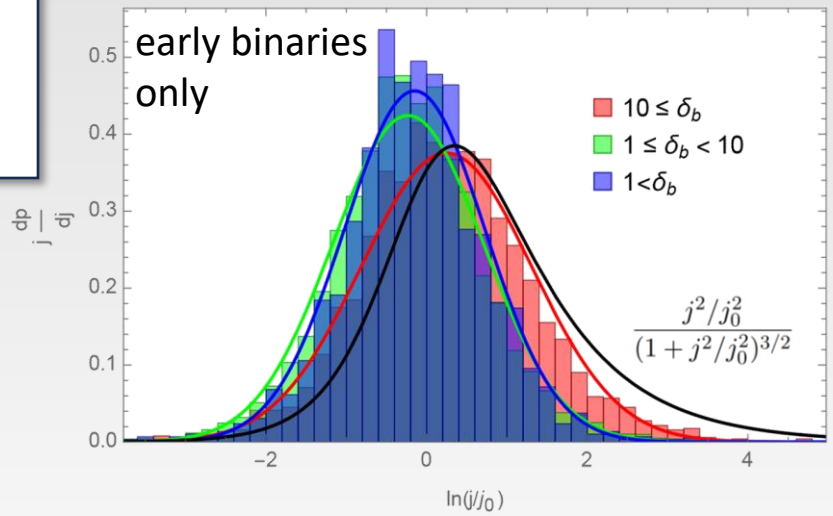
$$j_0 \approx 0.4 \frac{f_{\rm PBH}}{\delta_b}$$

$$j\frac{{\rm d}P}{{\rm d}j} = \frac{j^2/j_0^2}{(1+j^2/j_0^2)^{3/2}}$$

(in the high-δ_b limit)

Is accurate for early binary initial orbits, in the high-δ_b limit:





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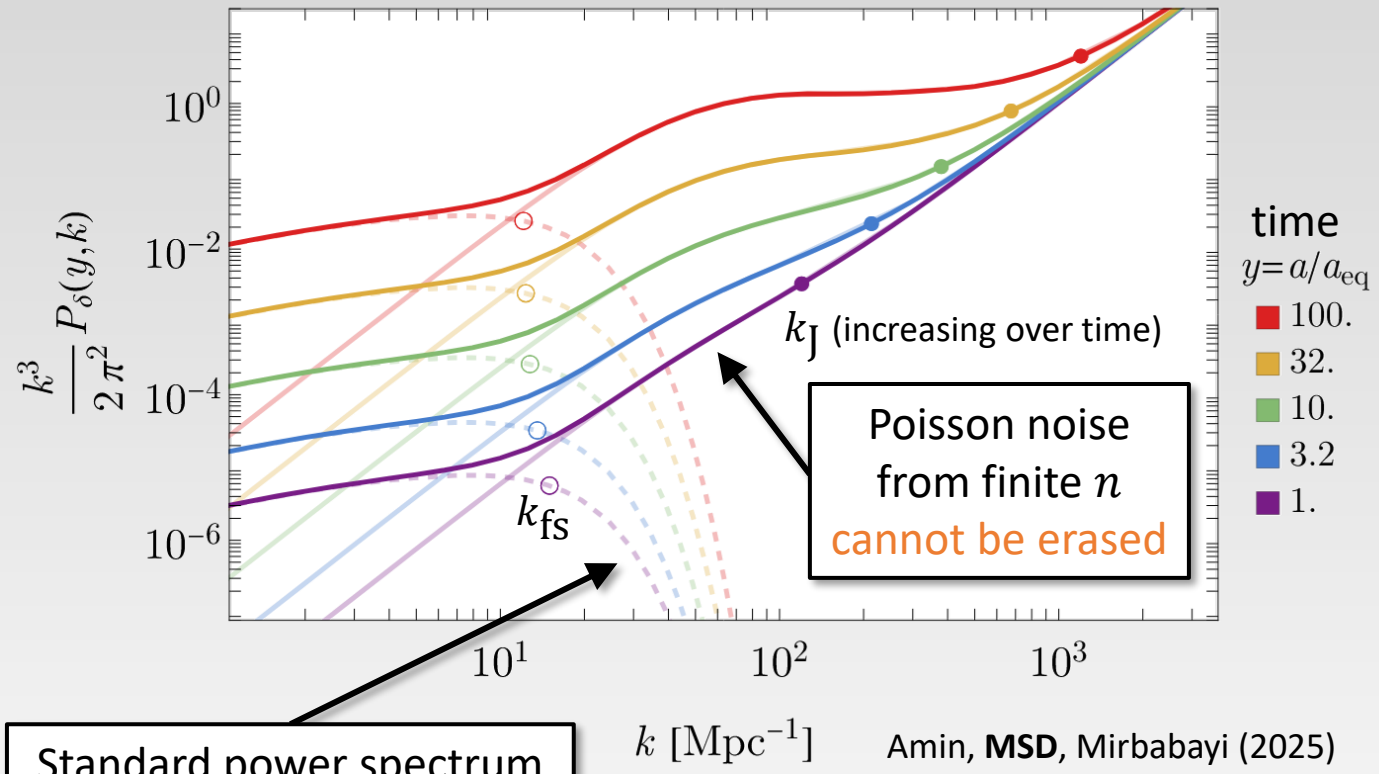
Supplementary

Finite N

Structure formation with finite n and σ

Scale-dependent growth of the matter power spectrum:
only for $k < k_{\rm J}$ (Jeans scale of DM)



Poisson noise from finite n cannot be erased

Standard power spectrum with free-streaming cutoff

Amin, **MSD**, Mirbabayi (2025)

$$P_\delta(y,k) = \underbrace{P_{\delta_{\rm ad}}(y_0,k)\left[\mathcal{T}_k^{\rm ad}(y,y_0)\right]^2}_{\text{adiabatic IC + evolution}} + \underbrace{\frac{1}{\bar{n}}\left[1 + 3\int_{y_0}^{y} \frac{dy'}{\sqrt{1+y'}} \mathcal{T}_k^{(a)}(y,y')\mathcal{T}_k^{(b)}(y,y')\right]}_{\text{white noise IC + evolution}}$$

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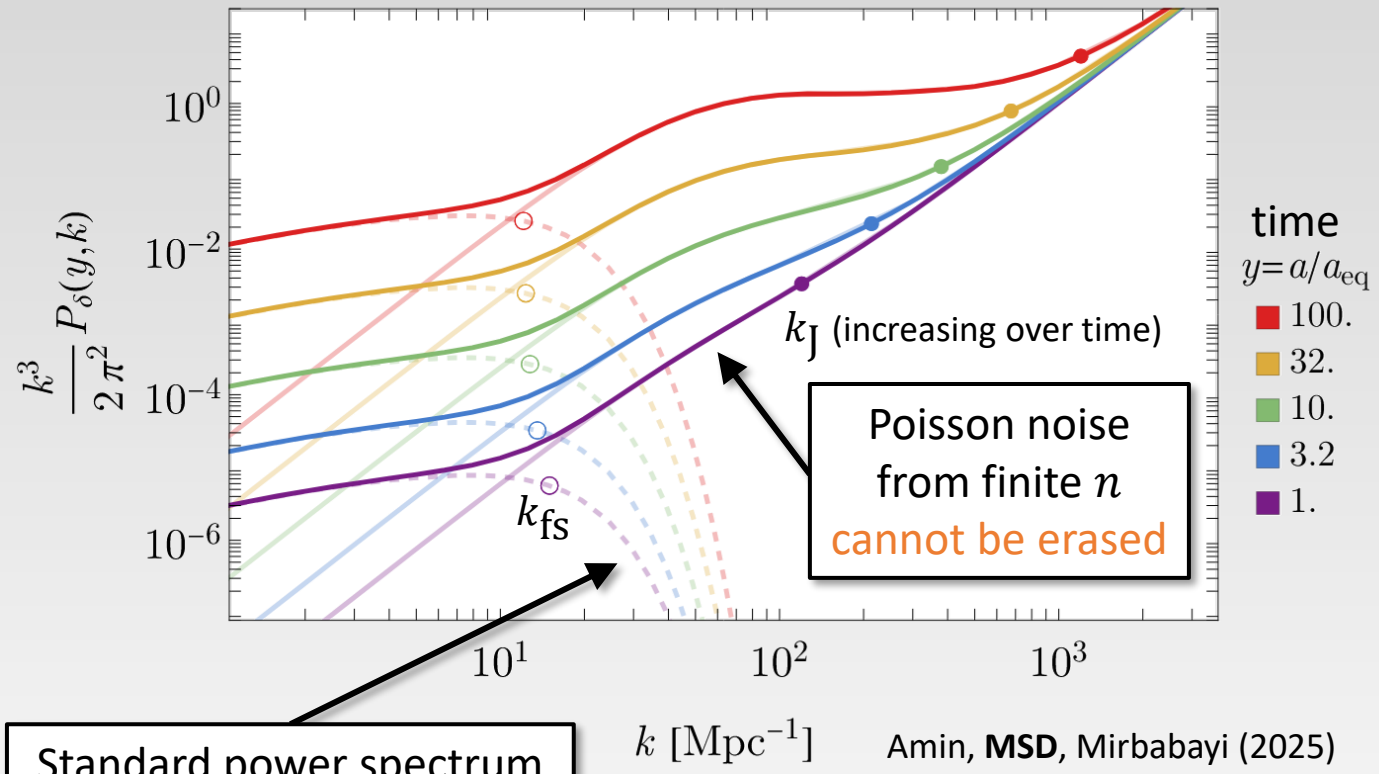

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Calculation requires cosmological perturbation theory beyond the 1-particle distribution function

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($\mathcal{T}$ = growth function × free-streaming cutoff)

Accurately predicts matter power spectrum & halo mass function in simulations

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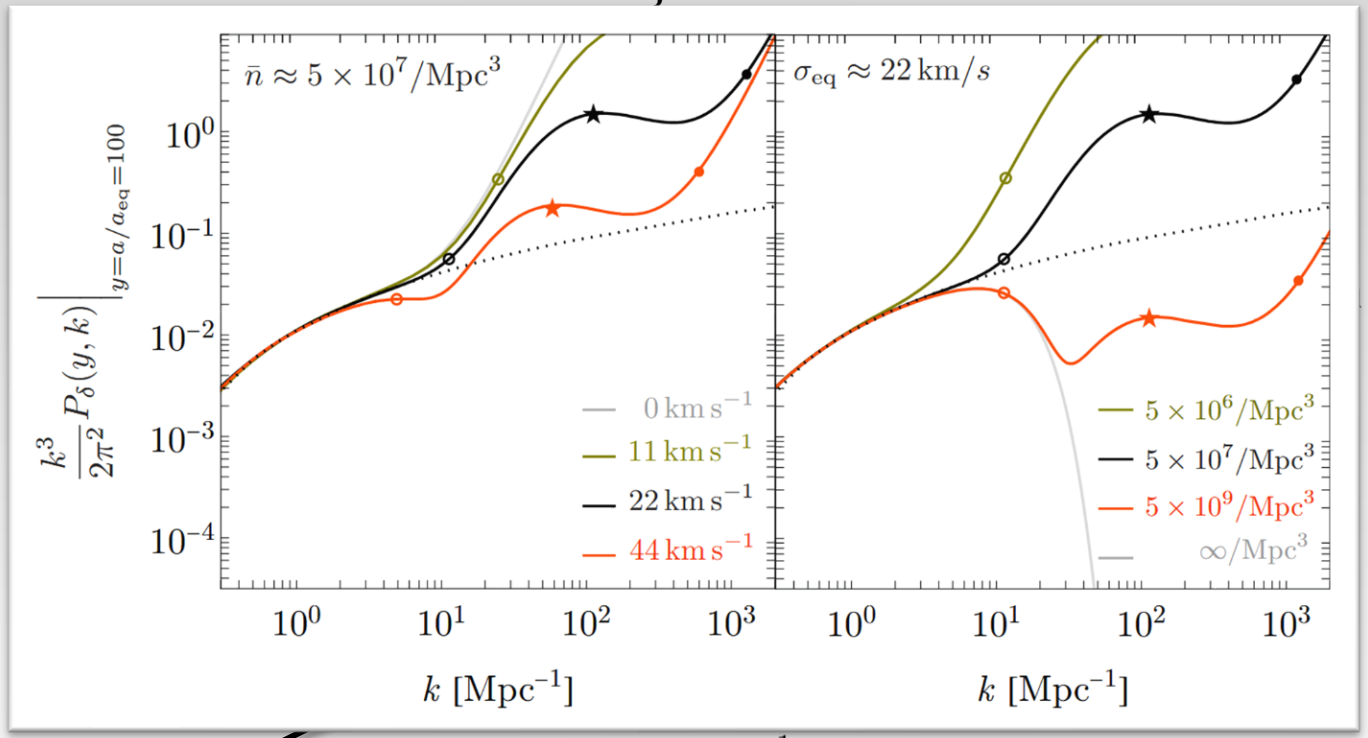

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$k\ [\mathrm{Mpc}^{-1}]$

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