Addressing the Hubble tension problem …

… and a proposal to increase the accuracy of cosmological observables

Astronomy &Astrophysics (doi: 10.1051/0004-6361/202348955) | Horst Foidl and Tanja Rindler-Daller

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VISESS

The Hubble Tension in ΛCDM

…denotes discrepant values for H_0 derived from CMB and local measurements

Measurements of H_0

- **CMB-based measurements**
  - CMB data is extrapolated to the present by applying a cosmological model
- **Local measurements**
  - are not depending on cosmological models
  - yield higher values than the CMB-based measurements

The Hubble Tension in ΛCDM

…the customary computational procedure for the expansion history in major cosmological codes

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the customary "fixed H_0" approach

- Densities are evolved back in time, using the energy conservation equation, starting with $\rho_{\mathrm{crit},0}$ determined by the provided value of H_0
- … used in the Friedmann equation yielding the expansion history
- Modified model parameters vary the early densities, but H_0 remains unmodified
- … by construction it cannot provide a solution to the Hubble tension
  … relegates it to measurements

The Hubble Tension in ΛCDM

…the limitation of the customary computation procedure possibly biases the analysis of data

Planck and DES-Y3 claimed to have confirmed the cosmological constant, by comparing it to a CPL-model of dynamical DE

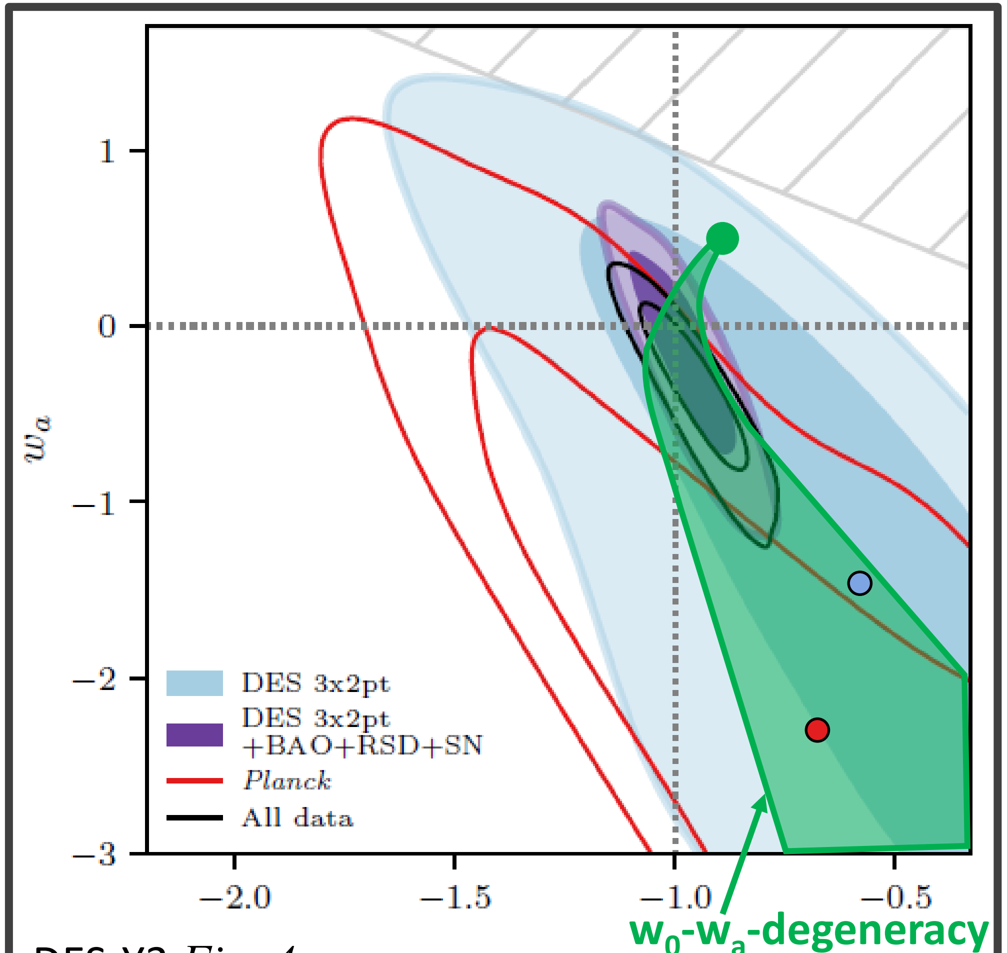
$$w = p/\rho$$

$$w(a) = w_0 + (1 - a)\, w_a$$

w_0-w_a-degeneracy in the computation procedure!

→ an infinite number of models in the green region agree to ΛCDM's results!

… they seem to be affected by this issue



DES-Y3 *Fig. 4*

The Hubble Tension in ΛCDM

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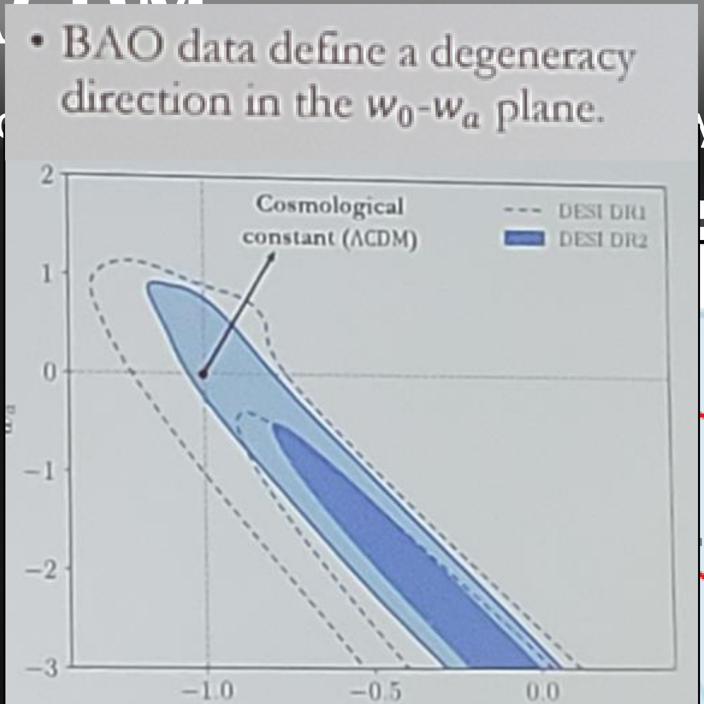
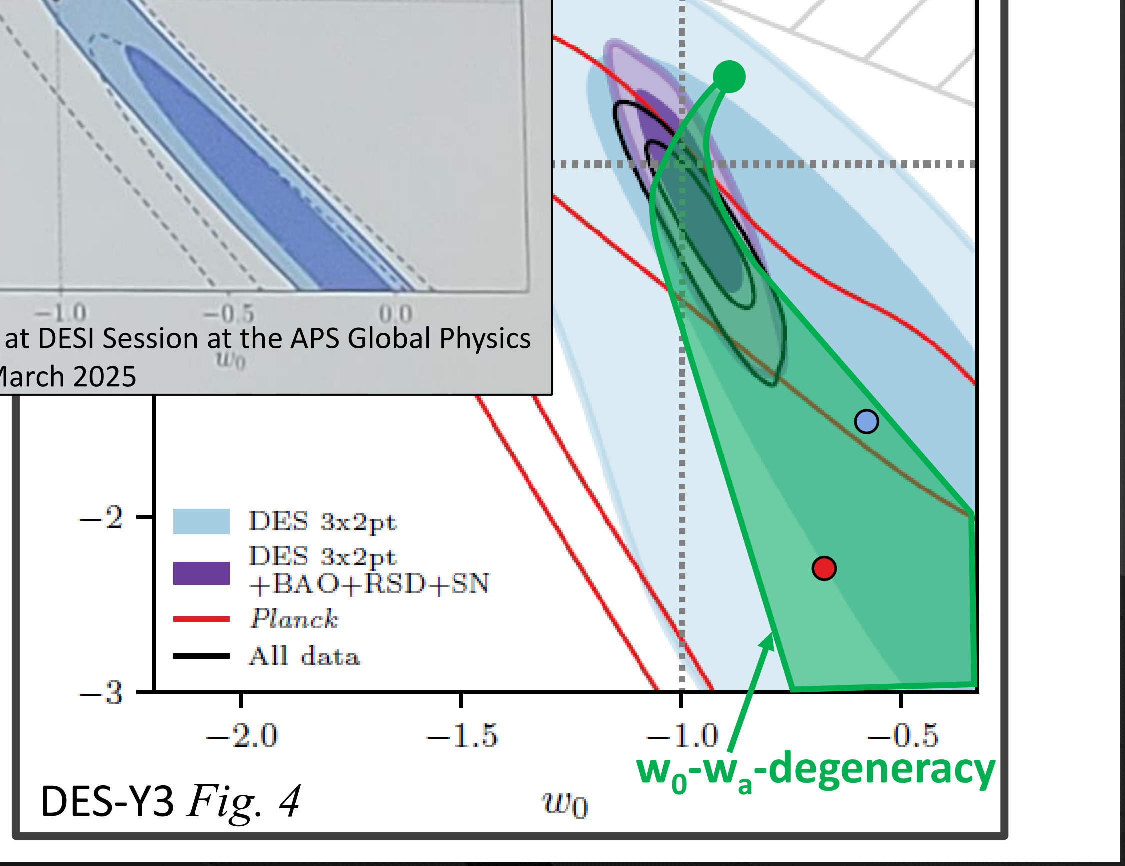
… DESI reported a similar degeneracy in the CPL-plane, attributed to their data



presented at DESI Session at the APS Global Physics Summit, March 2025



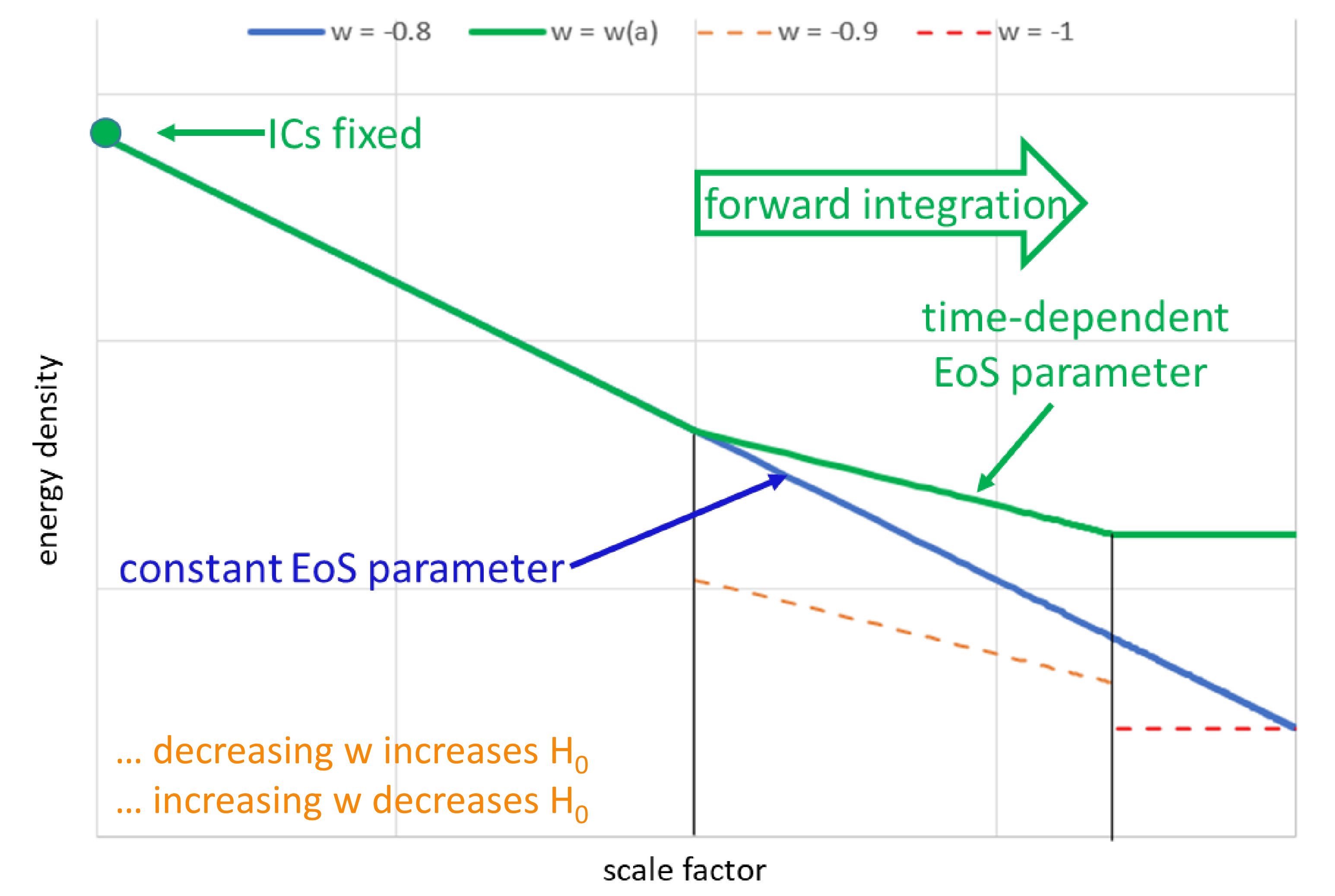
DES-Y3 *Fig. 4*

The Hubble Tension in ΛCDM

…the complementary computational procedure

the complementary "fixed early densities" approach

- can provide a solution to the Hubble tension problem
- the resulting H_0 can be checked to observations in the local Universe

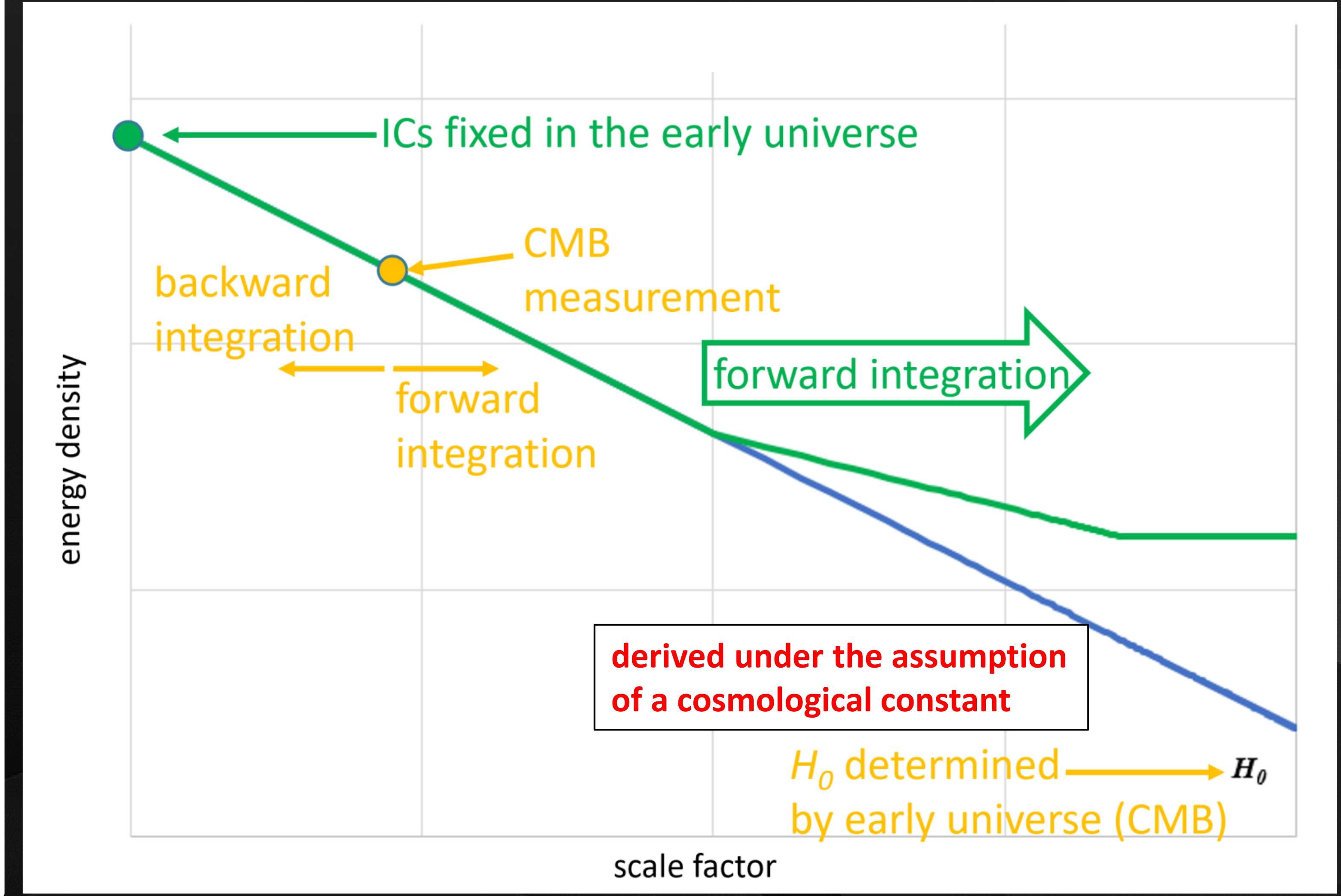


The Hubble Tension in ΛCDM

…the complementary computational procedure

the complementary
“fixed early densities” approach

- can provide a solution to the Hubble tension problem
- the resulting H_0 can be checked to observations in the local Universe
- early densities are determined by CMB measurements



The Hubble Tension in ΛCDM

…the complementary computational procedure

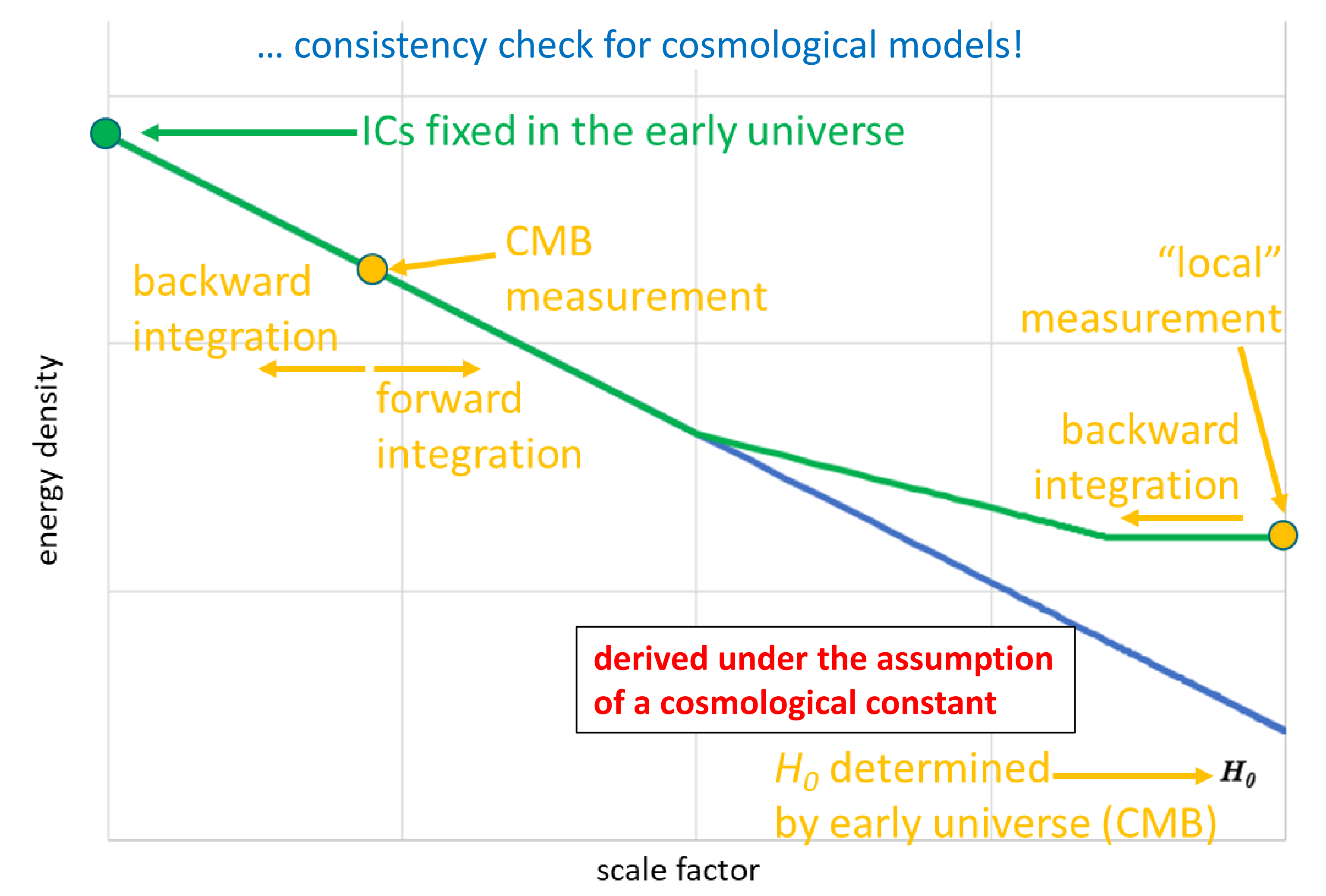
Planck based value of H_0

- ✓ Computation of EDs with concordance parameters
- ✓ Forward-integration, starting at computed EDs
- ✓ …using the model parameters

“local” value of H_0

- ✓ Backward-integration, starting at H_0, using the model parameters

… for any locally measured H_0 we can find a DDE model, yielding identical results in forward and backward computation



MCMC Analysis – Improving the Accuracy

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Extended MCMC procedure for determining model parameters

Step 1

- Perform backward evolution of the sampled cosmological model using the sampled H_0

Step 2

- Forward evolution of the sampled cosmological model using Planck based H_0 to determine the early densities

Step 3

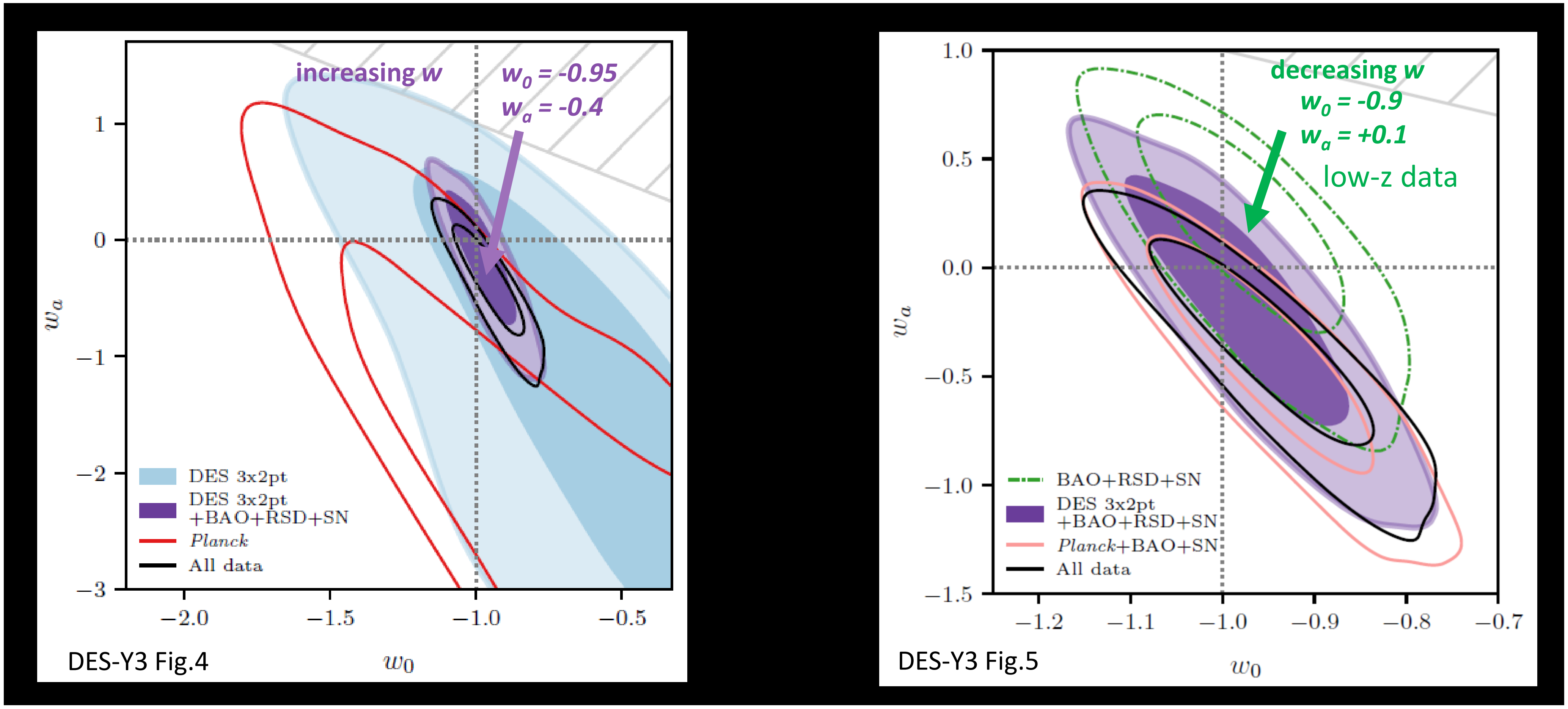
- Compare expansion histories, CMB temperature spectra and matter power spectra for consistency

- Only consistent parameter sets are considered in the computation of the probability distribution

MCMC Analysis – Improving the Accuracy

…exemplary MCMC analysis of two sampled models

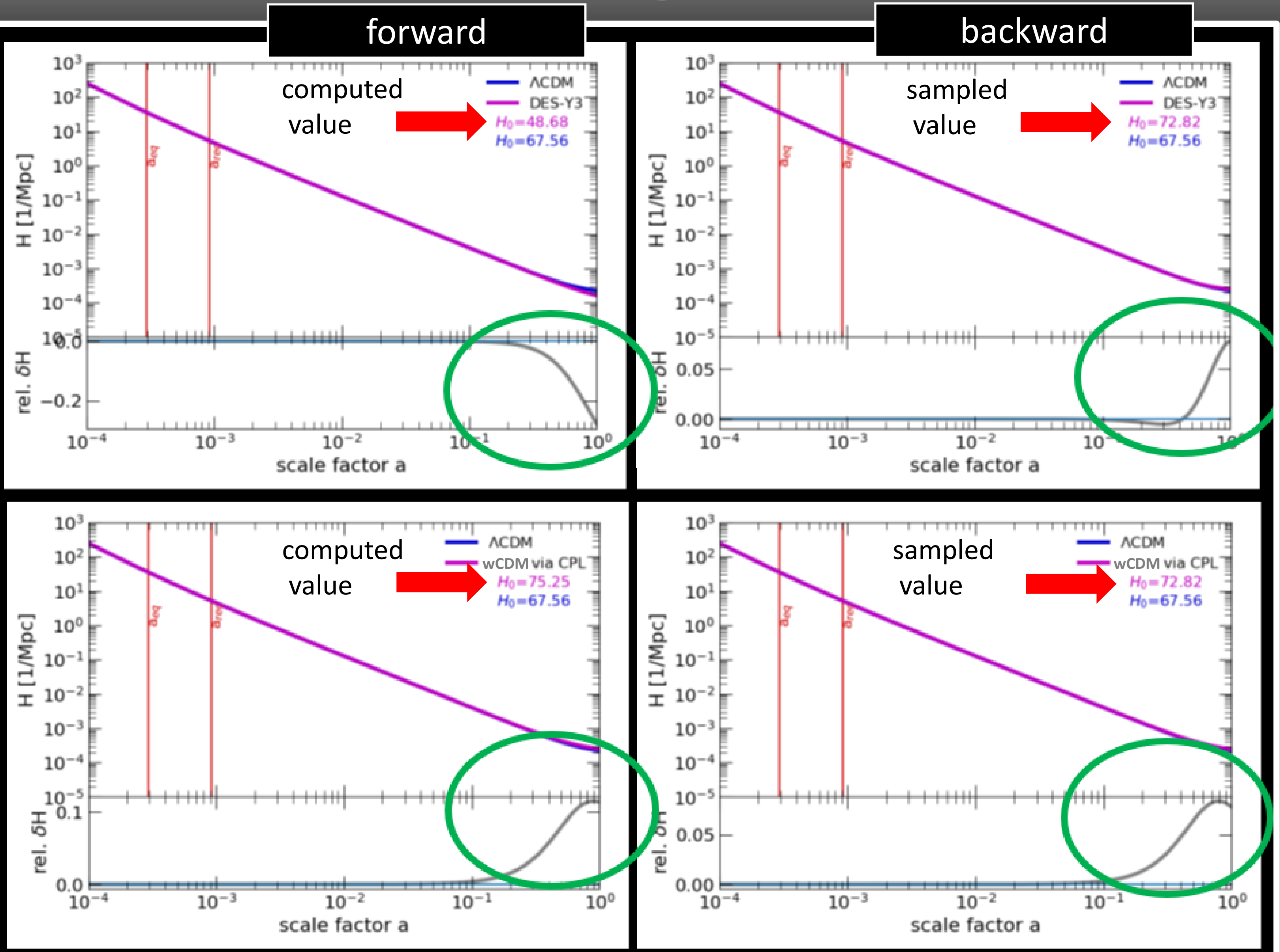
CPL parameterization $w(a) = w_0 + (1 - a)\, w_a$



MCMC Analysis – Improving the Accuracy

DES-Y3
w_0 = - 0.95
w_a = - 0.4

low-z BAO data based
w_0 = - 0.90
w_a = + 0.10



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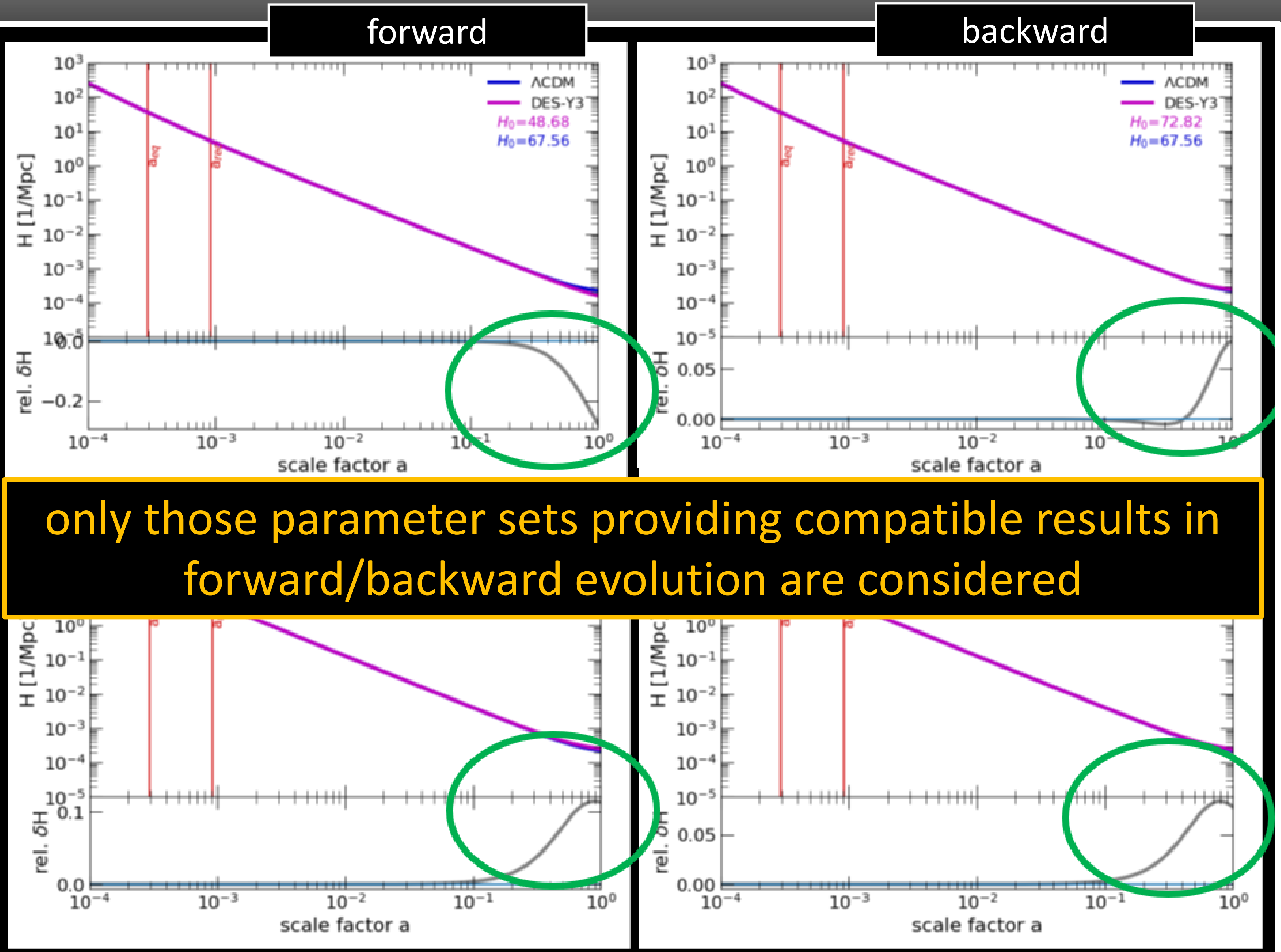

only those parameter sets providing compatible results in forward/backward evolution are considered

low-z BAO data based
w_0 = - 0.90
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MCMC Analysis – Improving the Accuracy

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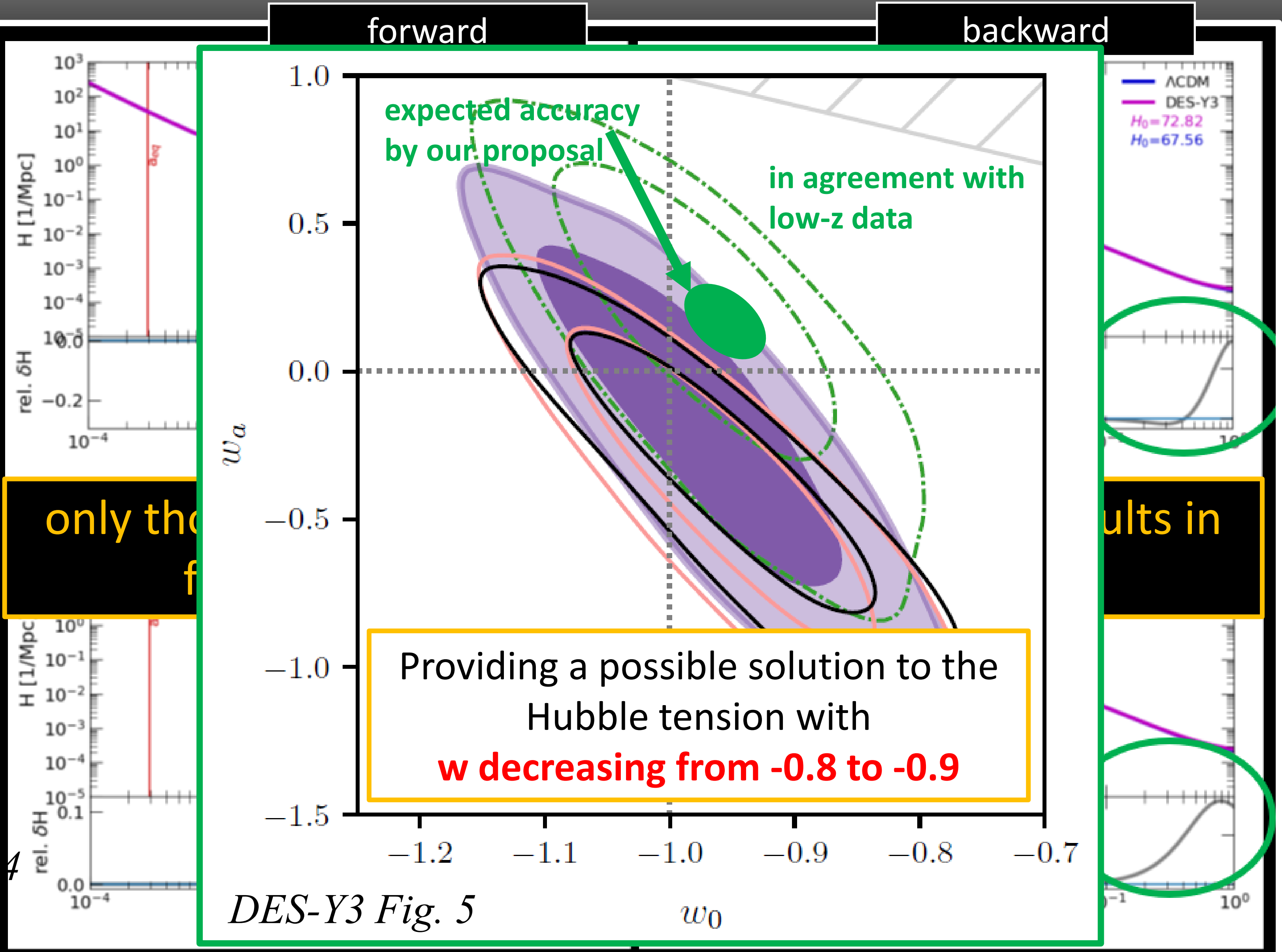

DES-Y3 Fig. 5

Comment on Dynamical Models of DE

Comments on the results of Planck, DES and Dark Energy Spectroscopic Instrument (DESI)

they claimed to have ruled out DDE models and confirmed the cosmological constant

they favor a model of DDE

presented at DESI Session at the APS Global Physics Summit, March 2025

… with **increasing** EOS parameter ($w_a < 0$)

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… with **increasing** EOS parameter ($w_a < 0$)

Our result is **not** in agreement with their DDE models!

… we favor a **decreasing** EOS parameter ($w_a > 0$)

Comment on Dynamical Models of DE

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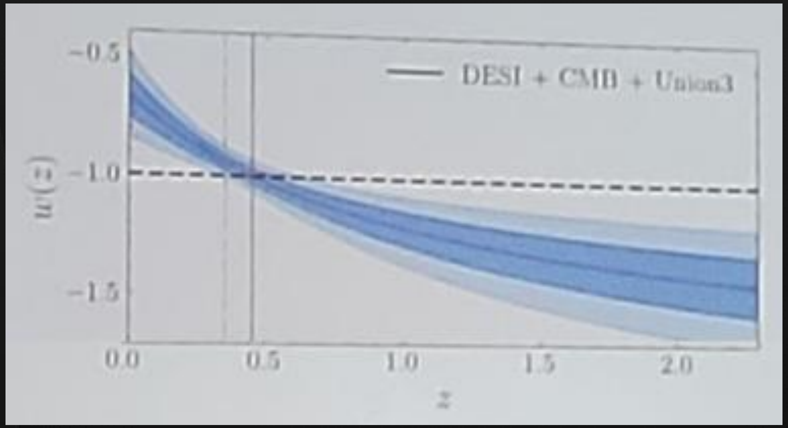
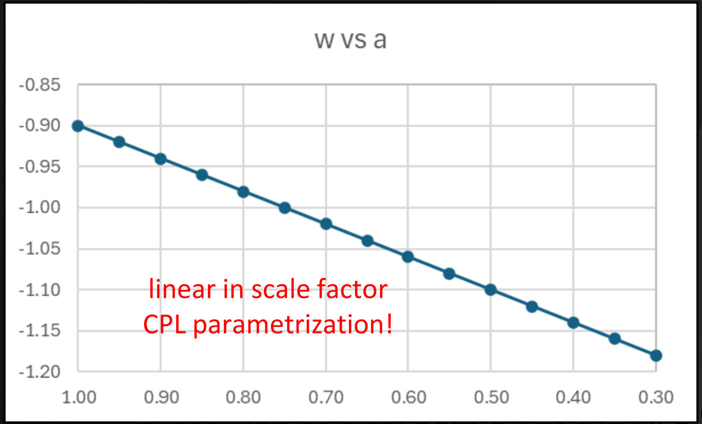
Their DDE models are phantom energy in the early Universe …

… also, in the late stages of the evolution their DDE models have an increasing EOS parameter

… increasing w_{de} from -1 to -0.827 since $z \simeq 0.45$ (e.g. DESI)



presented at the DESI Session
APS Global Physics Summit, March 2025



Global
Physics Summit, March 2025

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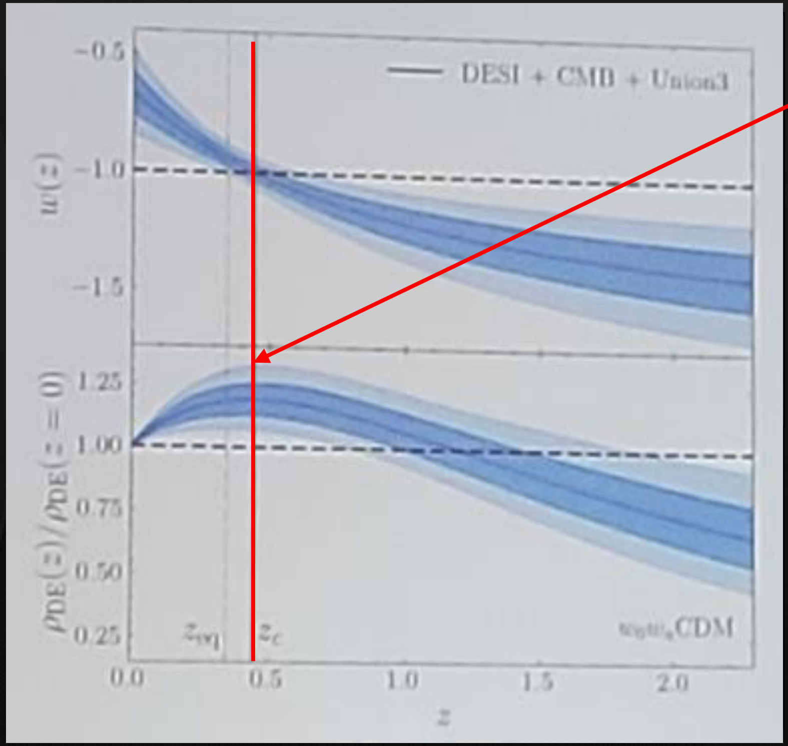
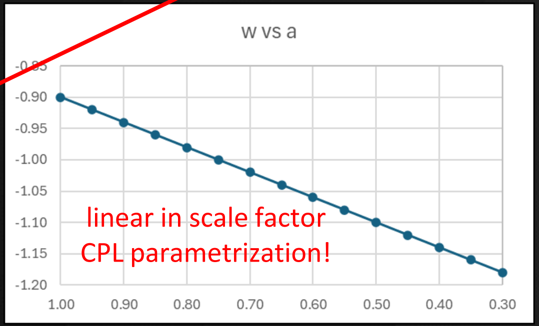


$$\rho \propto a^{-3\cdot(1+w)}$$



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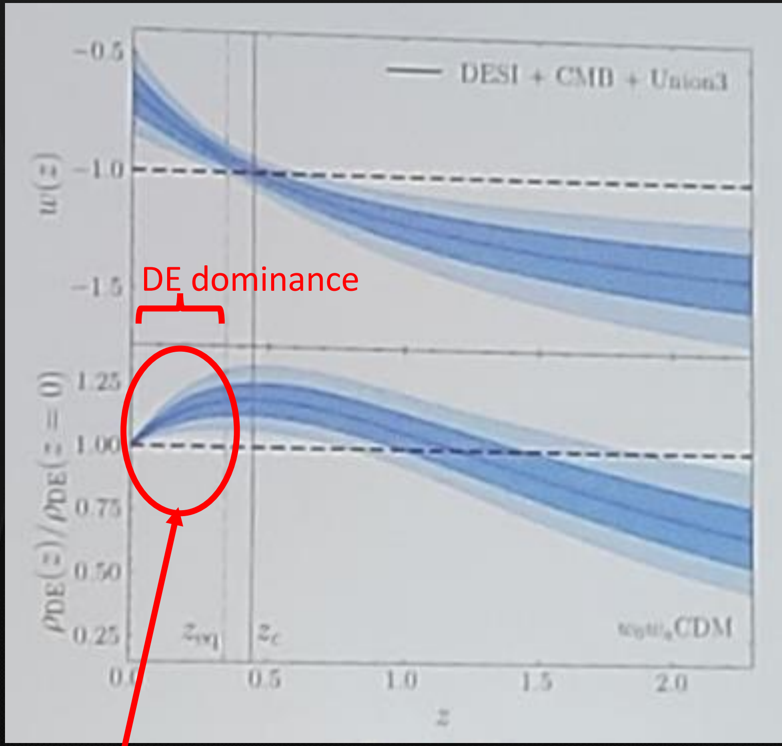
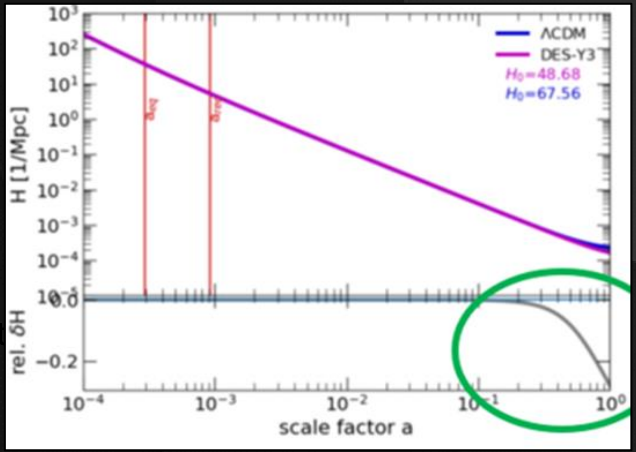


→ Let us consider the physics of the Friedmann equation in the late stages of the evolution of the Universe!

$$H^2(t) = \frac{8\pi G}{3c^2}\left[\cancel{\rho_r(t)} + \cancel{\rho_b(t)} + \cancel{\rho_{\mathrm{CDM}}(t)} + \rho_{\mathrm{de}}(t)\right]$$



→ **this is in accordance with our computational approach**

Comment on Dynamical Models of DE

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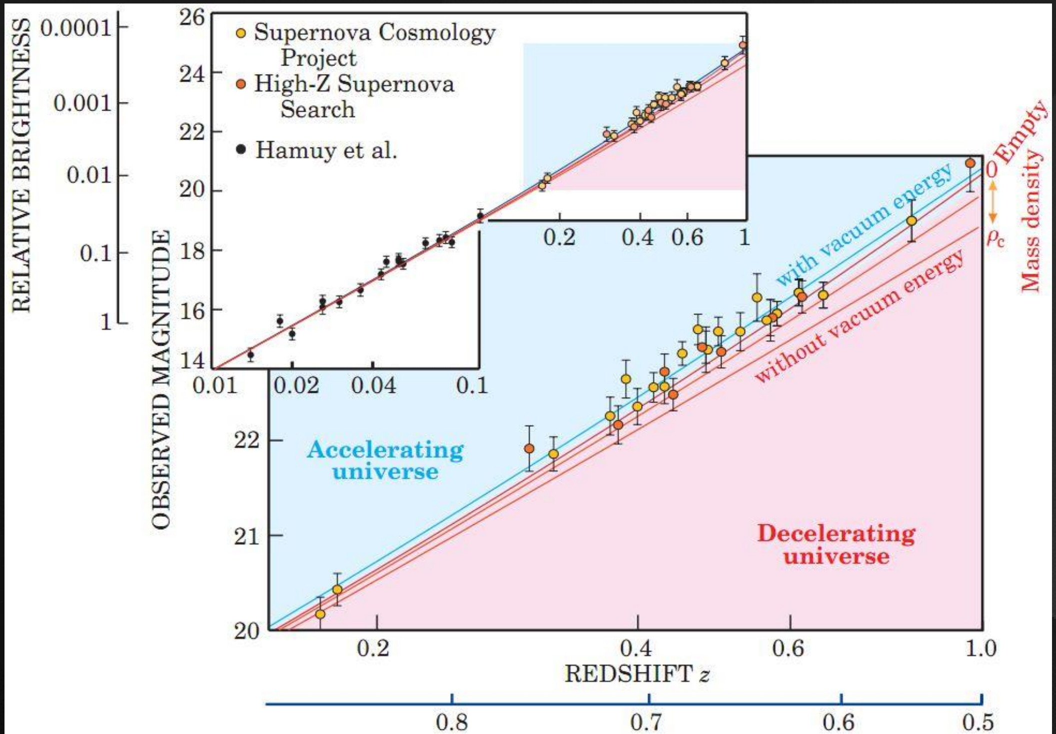
… increasing w_{de} from −1 to −0.827 since $z \simeq 0.45$ (e.g. DESI)

… this means, increasing deceleration

A decreasing expansion rate is not compatible with the accelerated expansion in the late stages of the evolution discovered by Perlmutter, Schmidt and Riess!



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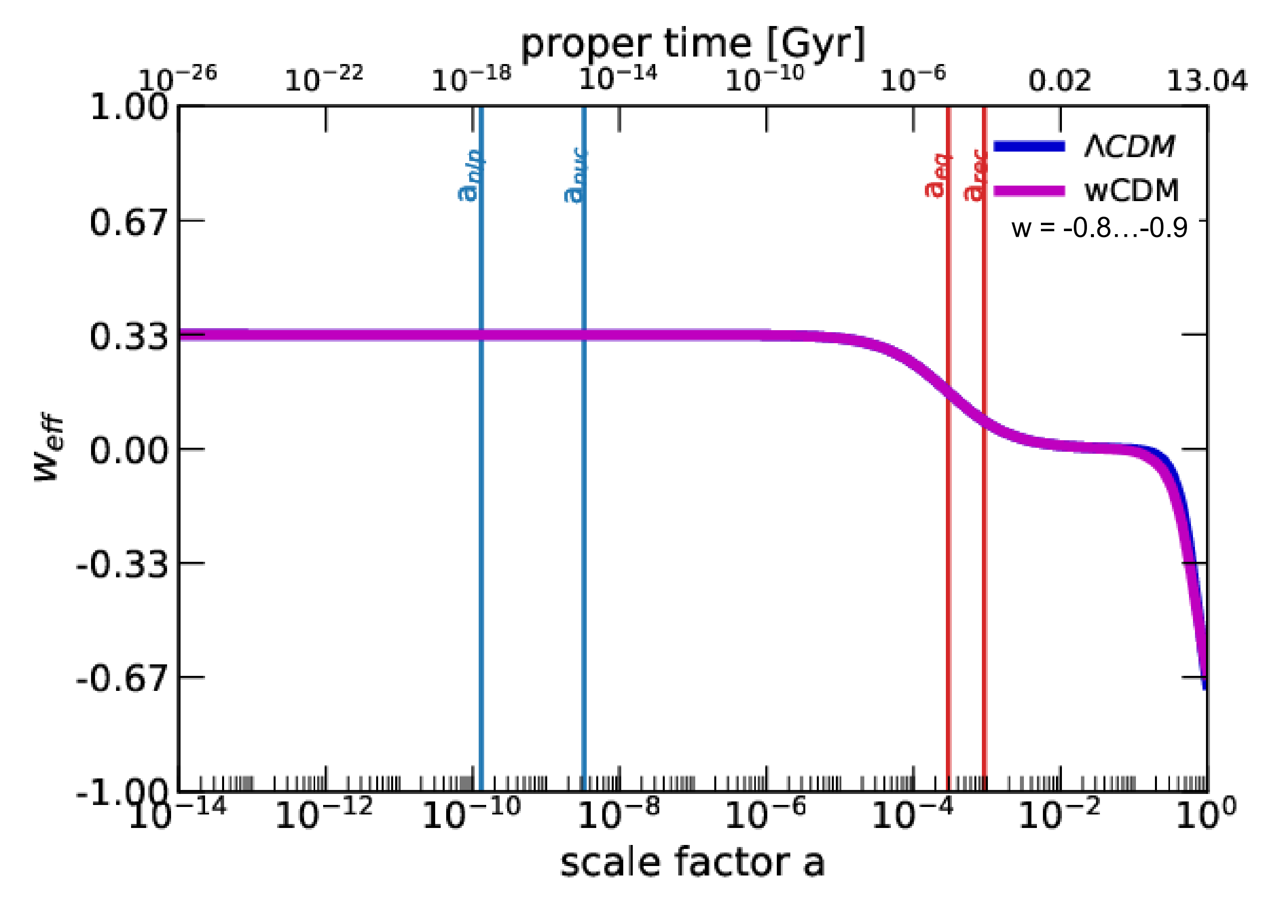
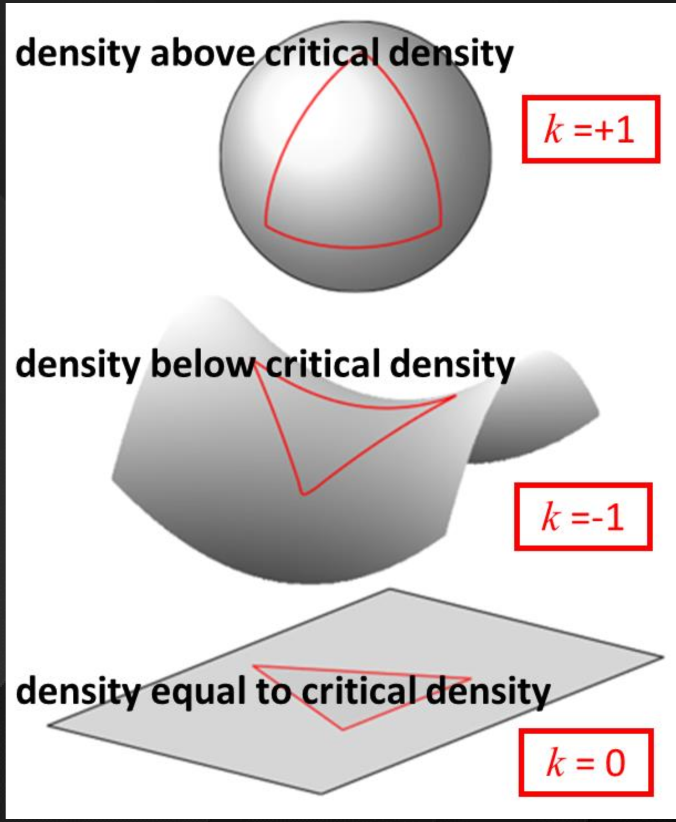
A decreasing expansion rate is not compatible with the accelerated expansion in the late stages of the evolution discovered by Perlmutter, Schmidt and Riess!

… decreasing w_{eff} from 0 to $\simeq -0.7$
… this means, decreasing deceleration



A physically motivated model explaining our empirical results



The FLRW metric

- The Robertson-Walker metric brings Milne's concept of a kinematically determined Universe to GR
- Geometry is flat, open or closed (k)
- **The expansion is described in the reference frame of a comoving observer – moving on a geodesic (i.e. free-falling)**
- We are considered comoving observers

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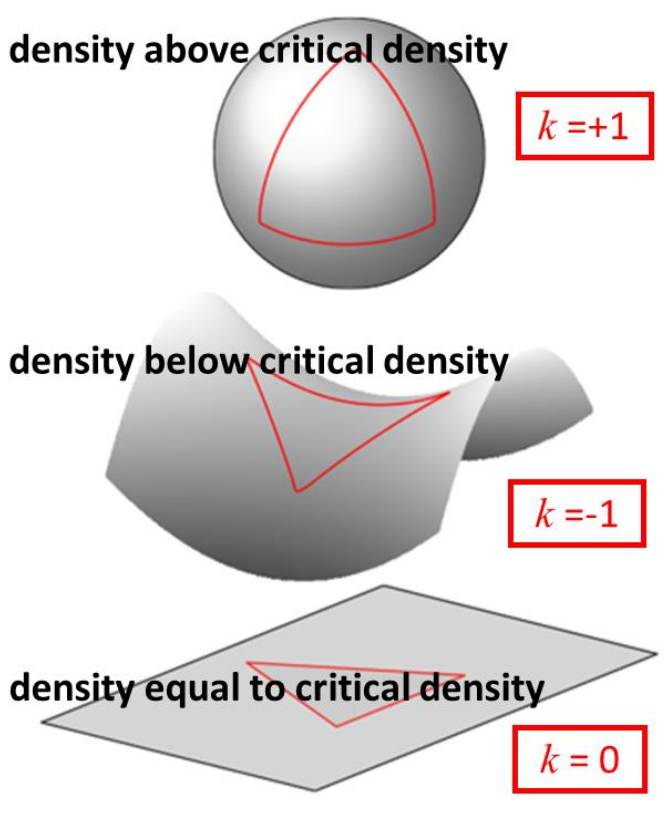

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- **Irrespective of the geometry → Ω_k = 0**
- the initial conditions are defined by the **initial density** in relation to the **initial expansion rate H_{ini}**!

A physically motivated model explaining our empirical results

…the wCDM model

incorporate H_{ini} into ΛCDM formalism

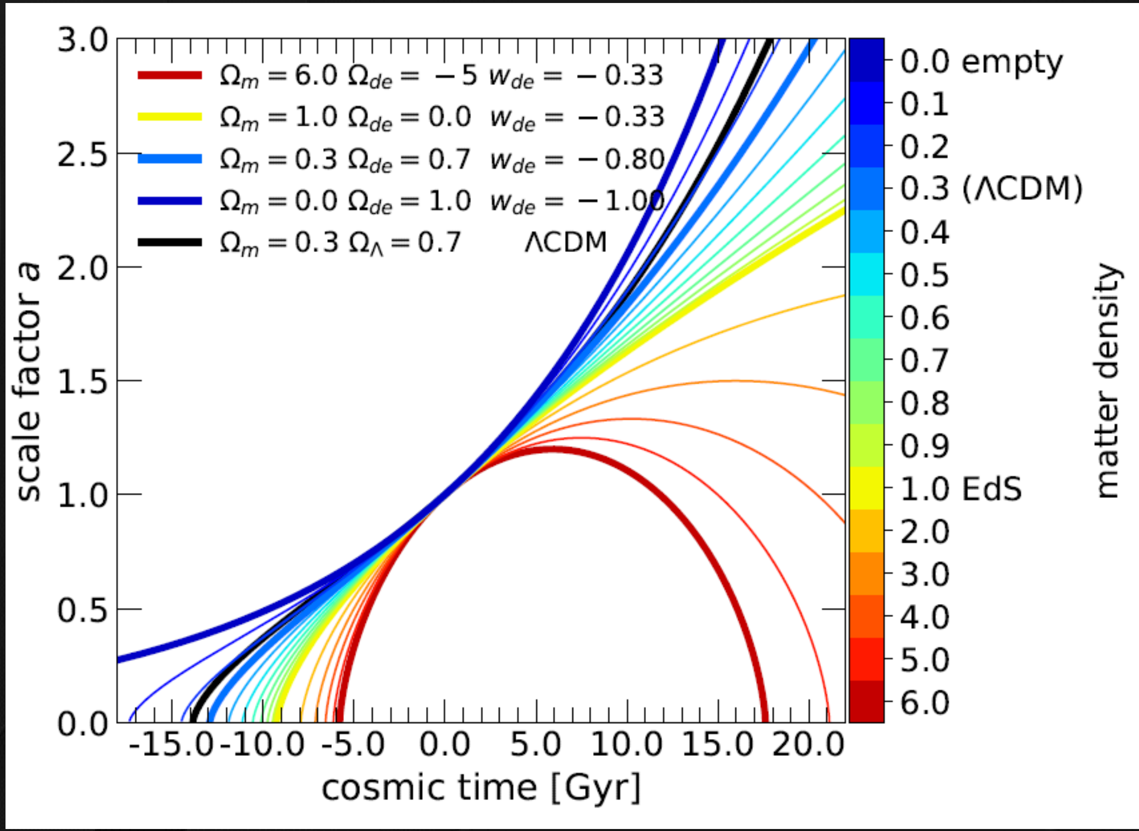
$\Omega_k=0$ we **perceive** flat space

geometry term

$$1 = \Omega_{r,0} + \underbrace{\Omega_{b,0} + \Omega_{\mathrm{CDM},0}}_{\Omega_{\mathrm{phys},0}} + \Omega_{k,0} + \Omega_{\mathrm{de},0}$$

$$w_{\mathrm{de}} = \frac{2}{3}\Omega_{\mathrm{phys},0} - 1$$

empty	$\Omega_m = 0$	$w_{de} = -1$
wCDM	**$\Omega_m = 0.3$**	**$w_{de} = -0.8$**
EdS	$\Omega_m = 1$	$w_{de} = -1/3$



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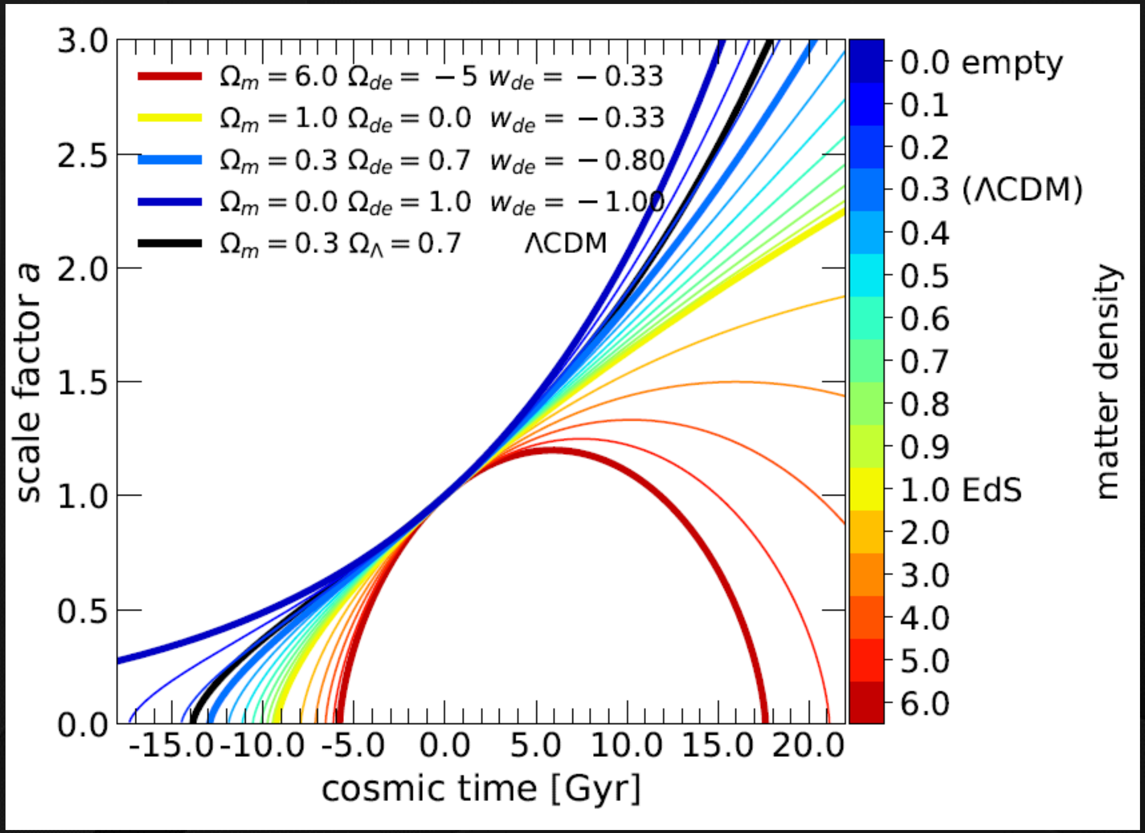

- ✓ the DE component describes a kinematic effect **by the curvature of an auxiliary Riemannian space**
  Robertson & Walker (1936)
- ✓ relativistic adaptation term from Newtonian to general relativistic kinematics
  Robertson (1936)
- ✓ it does not contribute to the energy momentum tensor
- ✓ the Universe has subcritical density

A physically motivated model explaining our empirical results

…the wCDM model in the non-linear regime of structure formation

components of our model

$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0} + \Omega_{\mathrm{de},0}$$

$$w_{\mathrm{de}} = \frac{2}{3}\Omega_{\mathrm{phys},0} - 1$$

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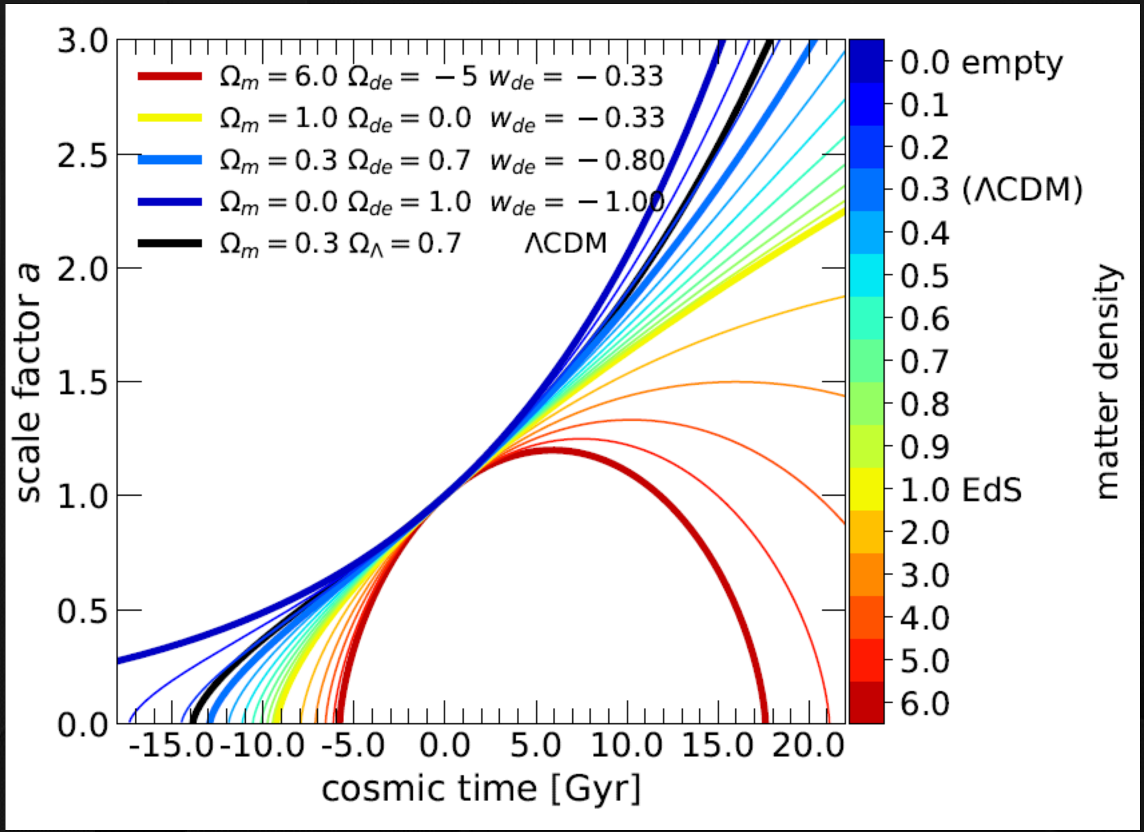

in the non-linear regime of structure formation, we found a minor backreaction onto the expansion history from voids **dominating the volume** of the Universe

$$w_{\mathrm{de}}(a) = \begin{cases} w_{\mathrm{de,early}} & \text{when } a \leq 1/6 \\ w_{\mathrm{de,early}} + \left[\frac{(1/a-1)-5}{5}\right](w_{\mathrm{de,early}} + 0.9) & \\ \quad \text{when } a > 1/6 & \end{cases}$$

using data from Cautun et al. 2014 who analyzed results of the Millenium simulation, we found

→ $w_{de} = -0.8 \ldots -0.9$

asymptotically converging towards – 1

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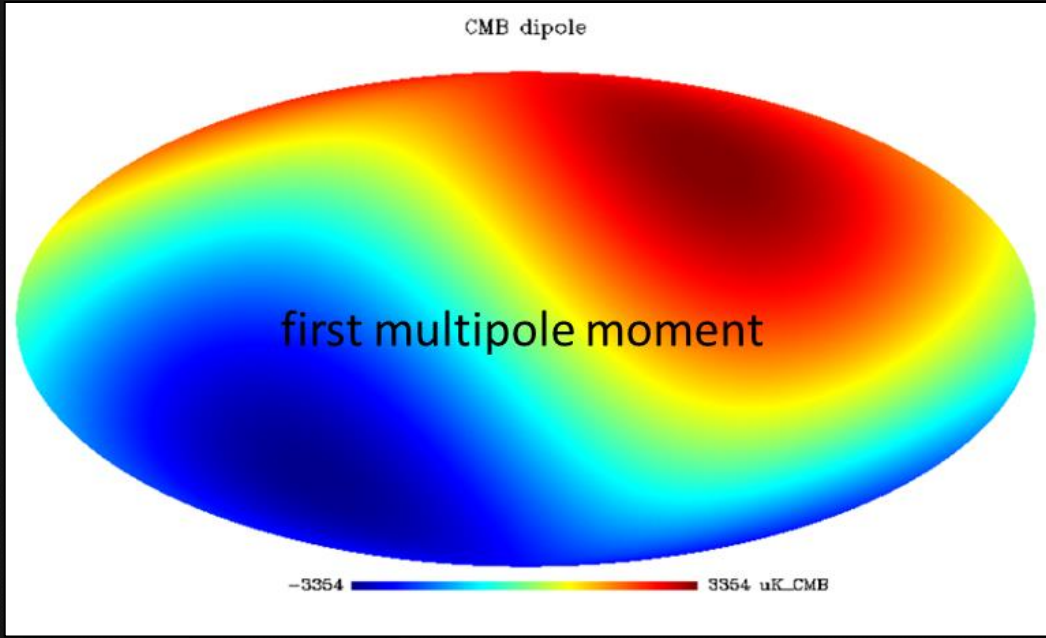
wCDM



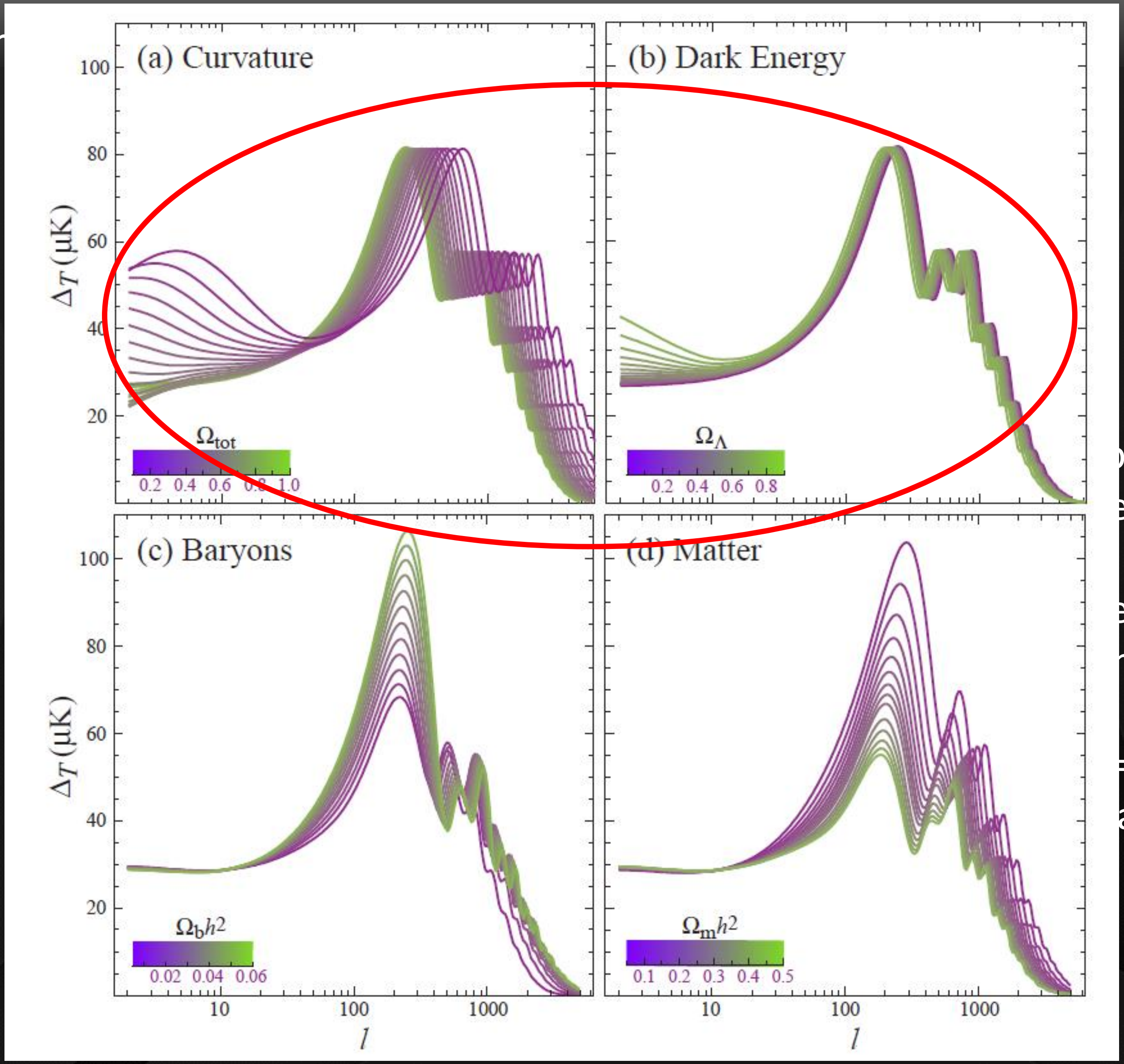
- dipole due to our peculiar motion against the CMB
- the dipole **offsets** us from a perfectly comoving observer
- Ω_k=0 is only valid in the reference frame of a perfectly comoving observer

A physically motivated model explaining our empirical results

…the wCDM m

components of
our model

wCDM



Hu and Dodelson 2002

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pole due to our peculiar motion against
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→ w_{de}= -0.8 … -0.9

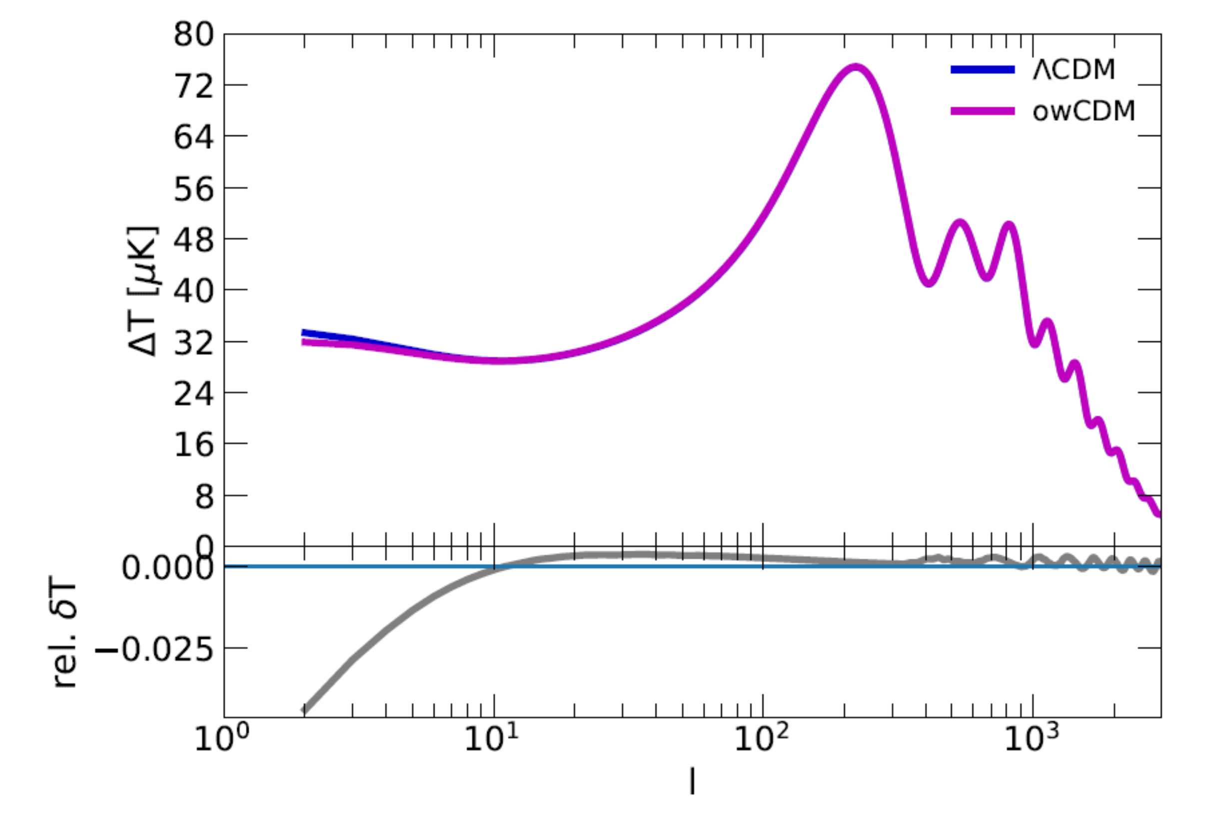
$\Omega_k = -0.0197$

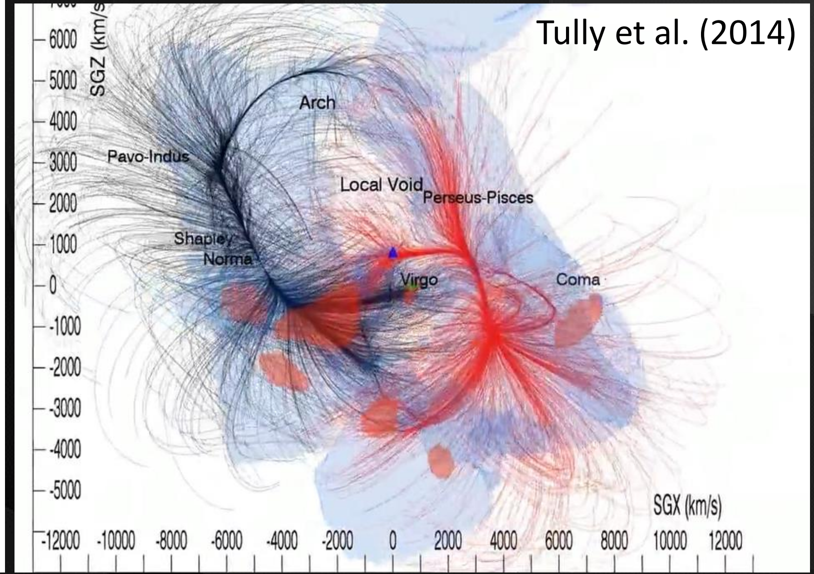
Planck PR4: Ω_k = -0.012 ± 0.010

we fit wCDM to the **ΛCDM spectrum** and

to **H_0=73.04** inferred from measurements in the local Universe

→ owCDM





compatible with local attractive environment

A physically motivated model explaining our empirical results

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we fit wCDM to the **ΛCDM spectrum** and

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→ **owCDM**

This model is probably not the final answer to DE and the Hubble tension, since we derived the parameterization of the DDE component from simple approximations and the results of the Millennium simulation.

It could be seen as a first step in a promising direction.

Future observations will provide high-precision data for reconstructing the EoS of DE

and falsify/confirm our approach.

Articles

The Hubble tension

Astronomy & Astrophysics
doi: 10.1051/0004-6361/202348955
A proposal to improve the accuracy of cosmological observables and address the Hubble tension problem

The physically motivated model

Frontiers in Astronomy and Space Sciences
doi:10.3389/fspas.2025.1627777
The importance of GR's principle of equivalence for understanding kinematically determined FLRW Universes

arXiv:2412.04126
A ΛCDM Extension Explaining the Hubble Tension and the Spatial Curvature $\Omega_{k,0} = -0.012 \pm 0.010$ Measured by the final PR4 of the Planck Mission

Summing up

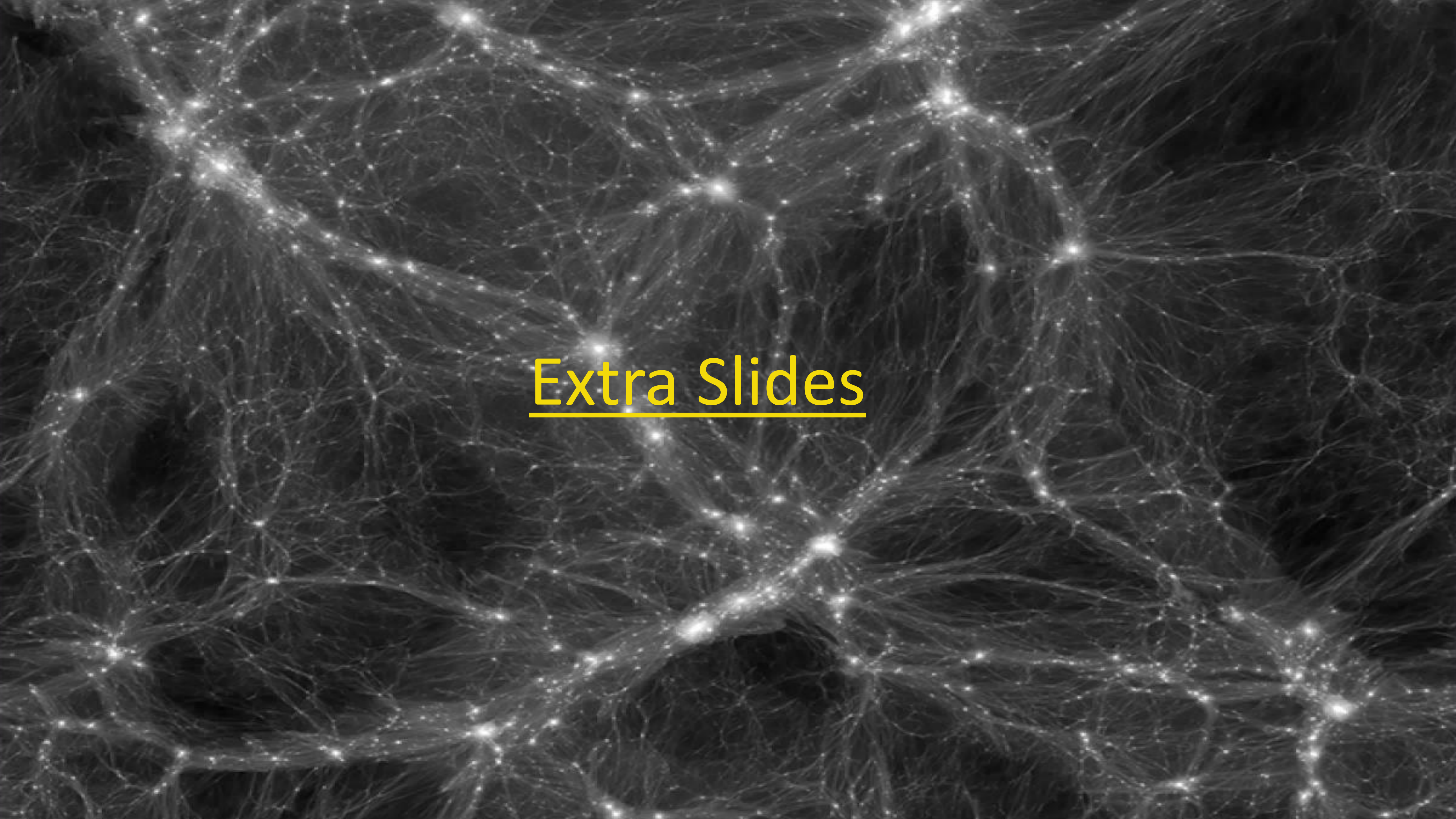
a) The novel complementary two-step forward integration procedure for cosmological models enables to increase the accuracy of MCMC derived parameters of DE.

b) DDE models with a decreasing EoS parameter can provide a solution to the Hubble tension. DDE models with an increasing EoS parameter seem to be not compatible with the observed late-time accelerated expansion of the Universe.

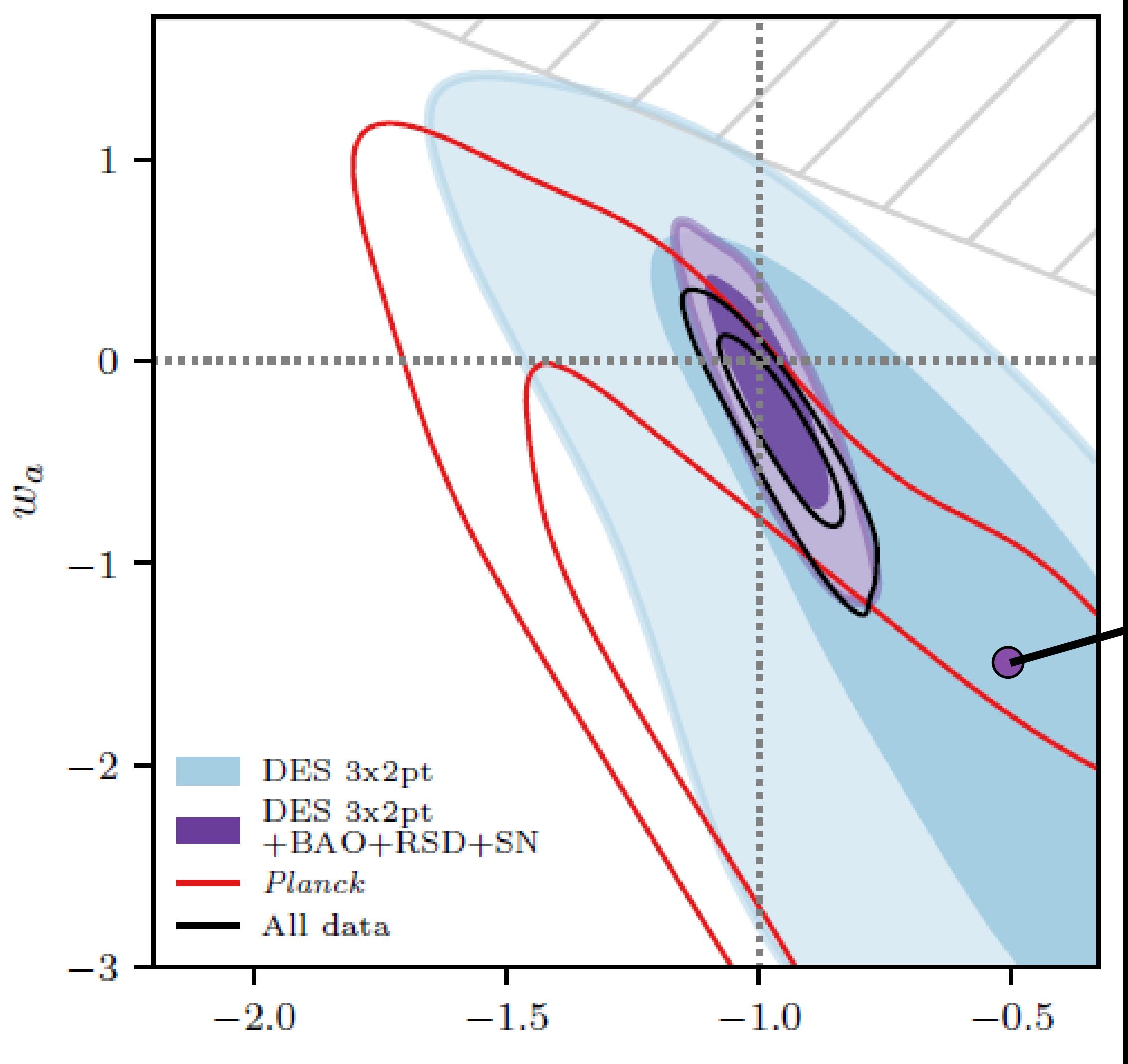
c) Our owCDM model, based on the idea of a kinematically determined universe, describes DE as a kinematic effect, due to the initial conditions consisting of ρ_{ini} and H_{ini}. The model provides a possible solution to the Hubble tension.

d) The time-dependence of DE's EoS and the observed "local" curvature of space are both effects of the non-linear structure formation.

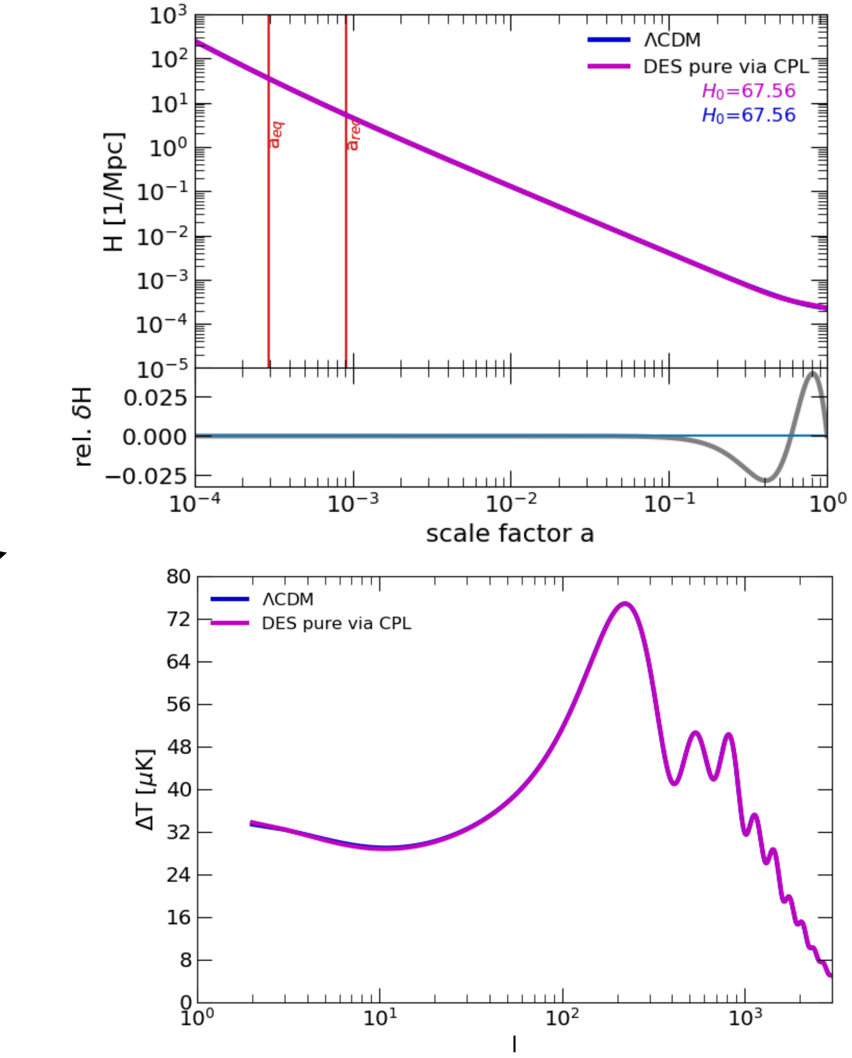
Extra Slides



...The degeneracy in the CPL-plane

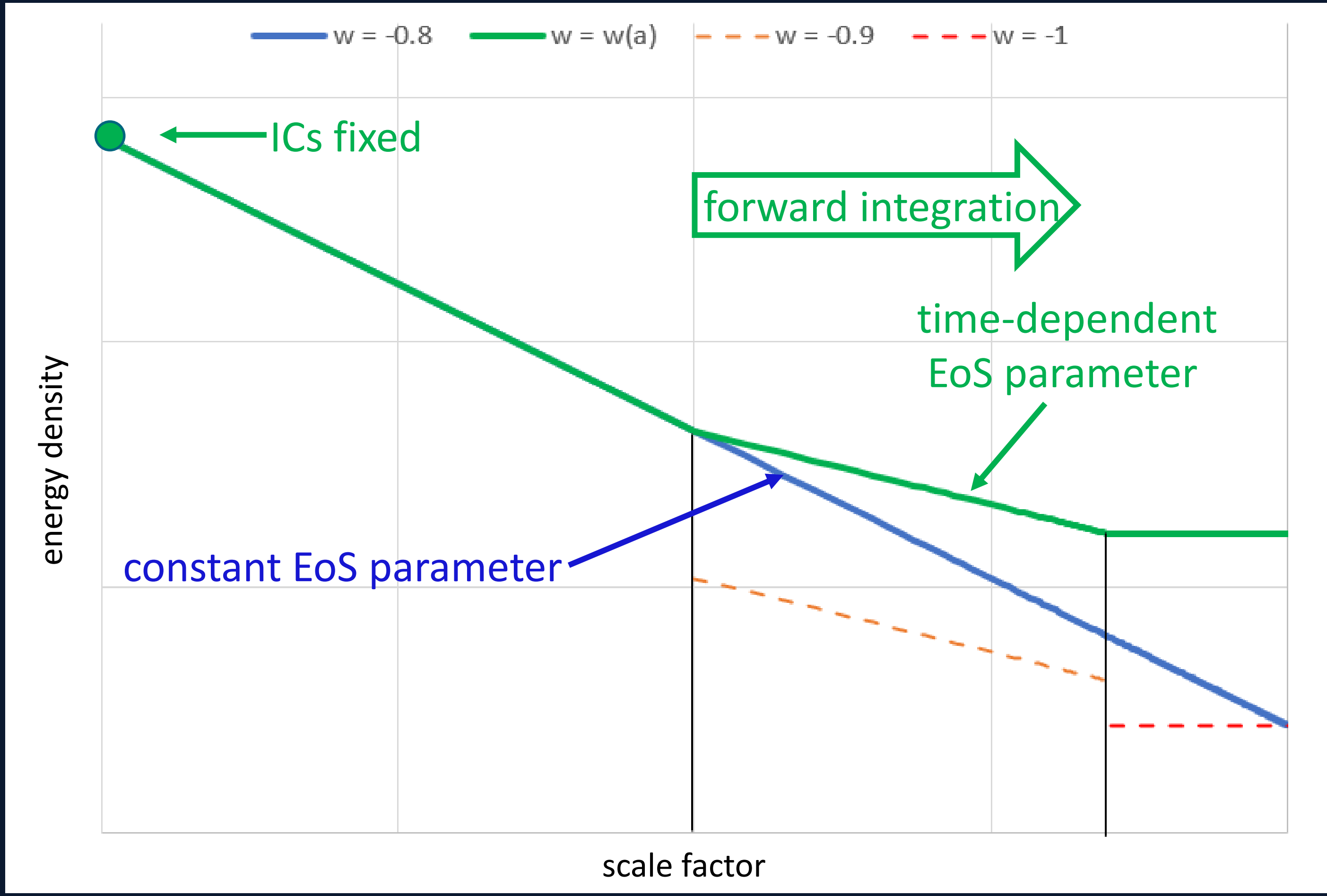


DES-Y3 Fig. 4



Consequences of a time-dependent EOS

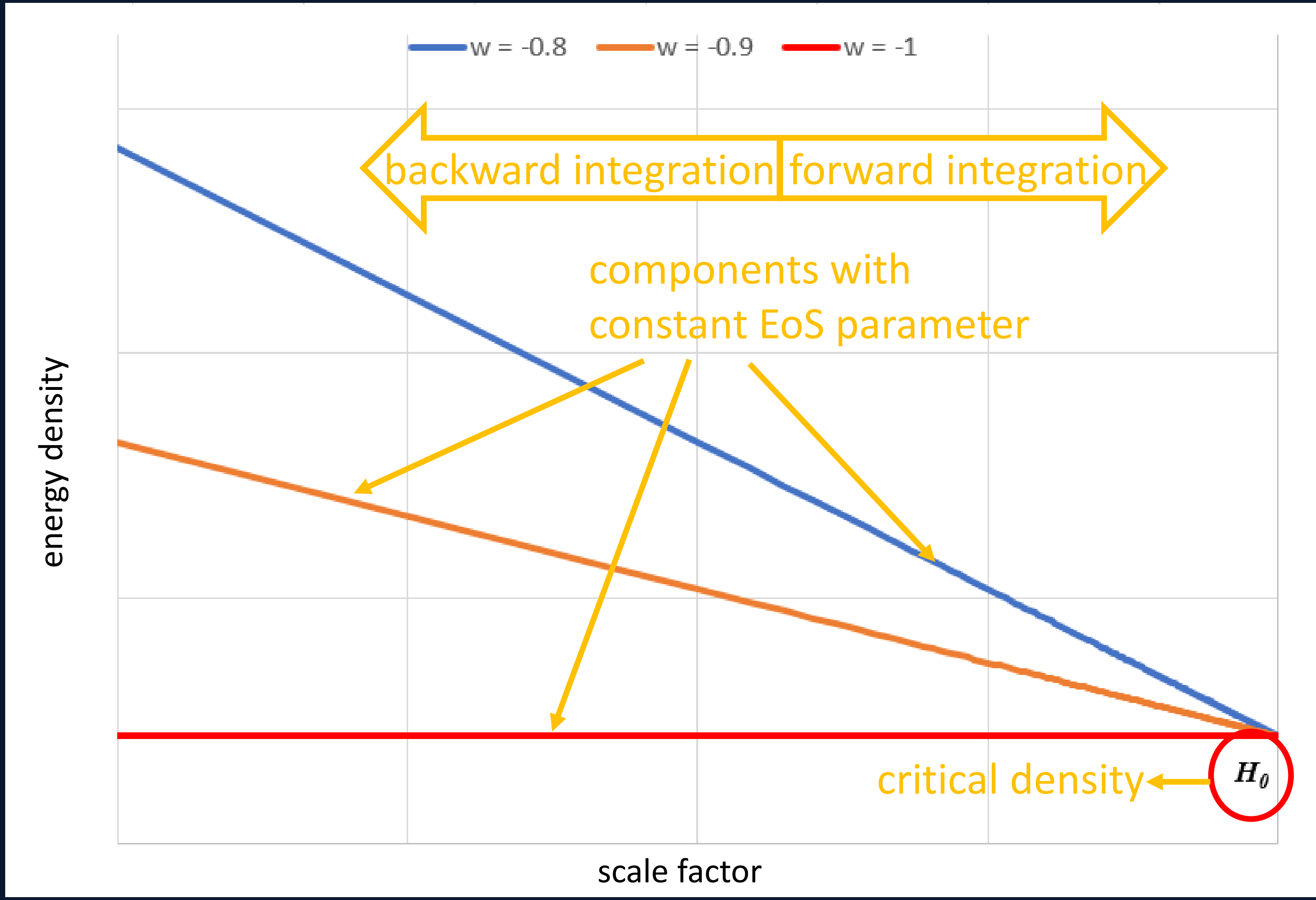
...Schematic for comparing time-dependent vs constant EoS parameter



- ✓ $\rho_i \propto a^{-3(1+w_i)}$ is valid only, if w_i is constant
- ✓ Otherwise, **forward integration** of energy conservation equation is necessary
- ✓ A **decreasing** EoS parameter shifts the value of H_0 to a **higher** $\boldsymbol{H_0}$
- ✓ An **increasing** EoS parameter shifts the value of H_0 to a **lower** $\boldsymbol{H_0}$

Consequences of a time-dependent EOS

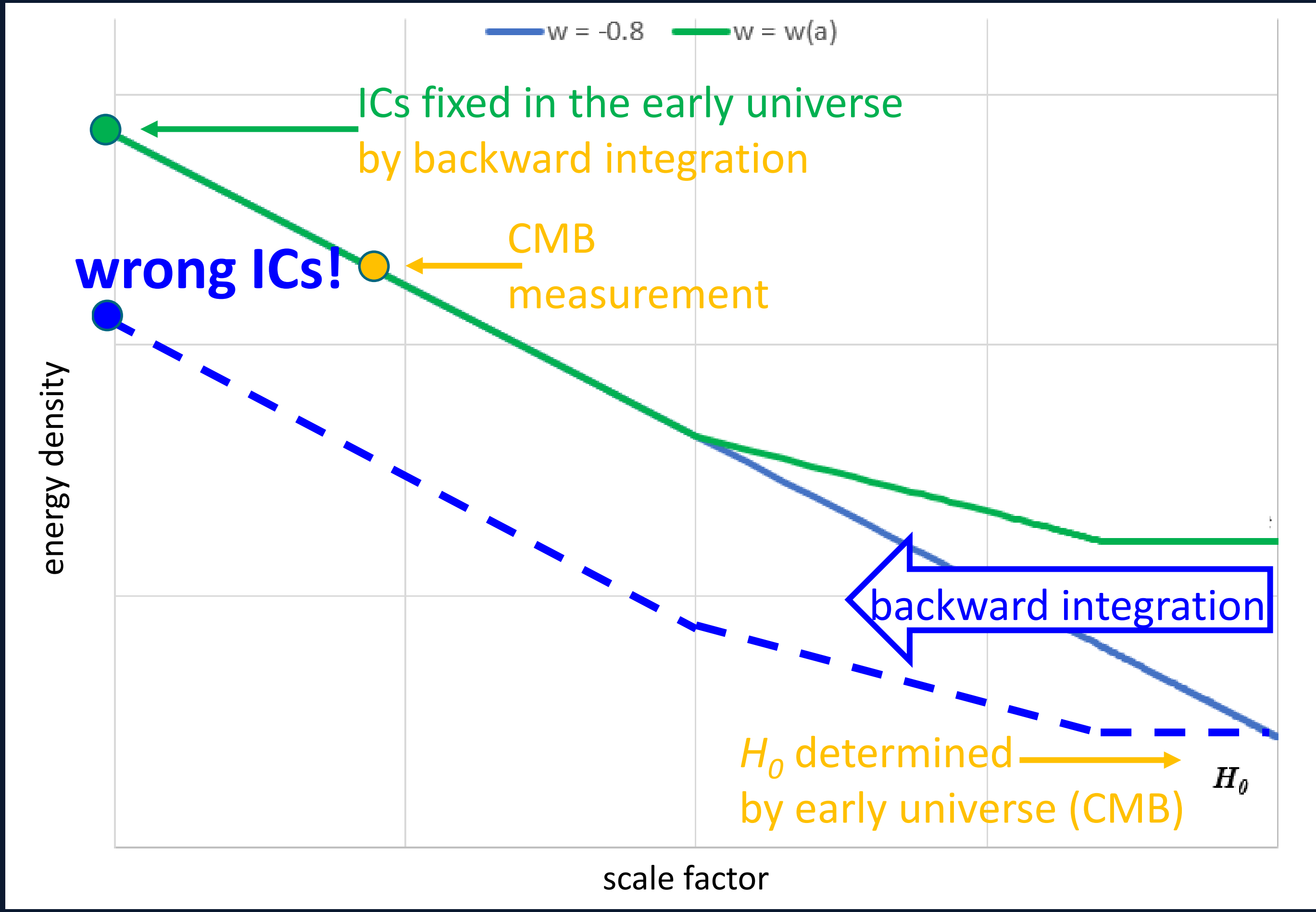
...No shift of H_0 by constant EoS parameter



- ✓ $\rho_i \propto a^{-3(1+w_i)}$ is valid only, if w_i is constant
- ✓ For a constant EoS parameter forward and backward integration yield identical results
- ✓ **Used in ΛCDM! Results do not differ**
- ✓ **We can determine ICs by backward integration!**

Consequences of a time-dependent EOS

...the customary procedure with concordance H_0



✓ $\rho_i \propto a^{-3(1+w_i)}$ is valid only, if w_i is constant

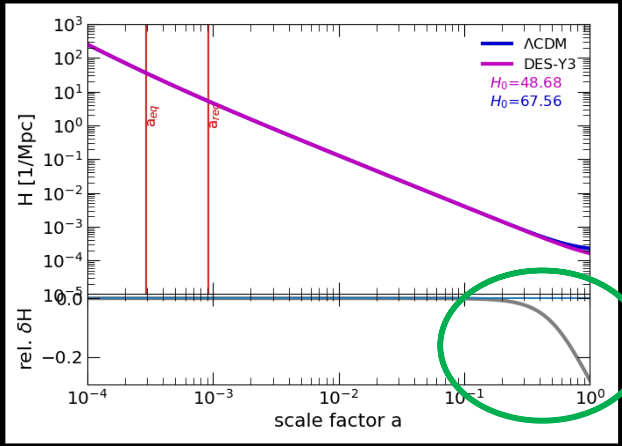
→ $\rho_{cpl} \propto \exp^{3((1+w_0+w_a)\cdot log(1/a)+w_a(a-1))}$

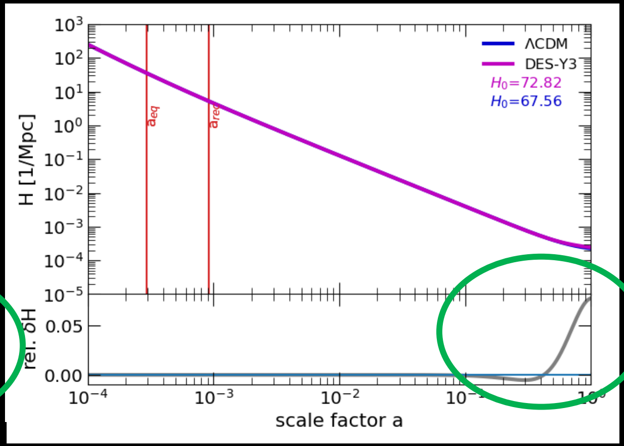
✓ **Backward integration gives wrong ICs!**

MCMC Analysis – Improving the Accuracy

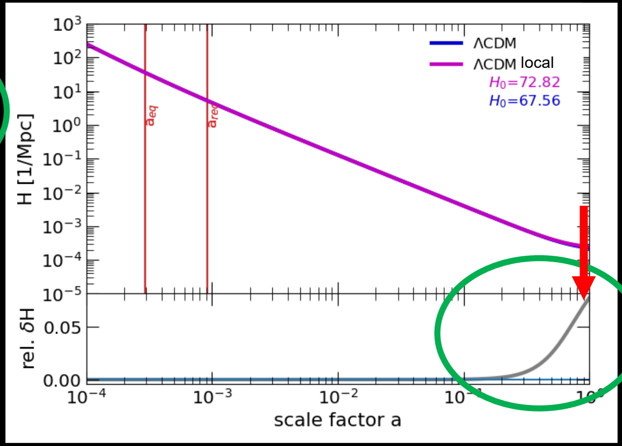
For ongoing and future observation programs….

DES-Y3
w_0 = - 0.95
w_a = - 0.4



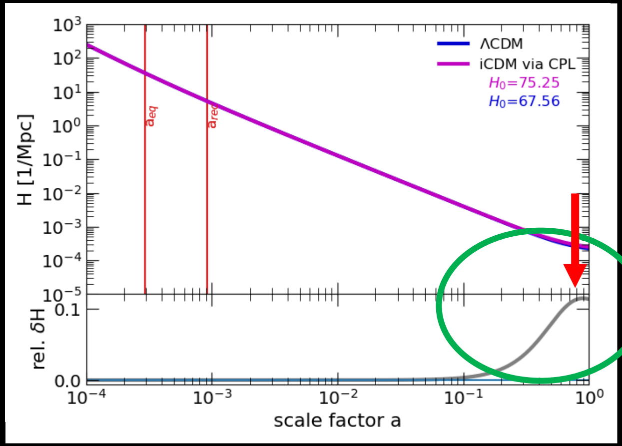


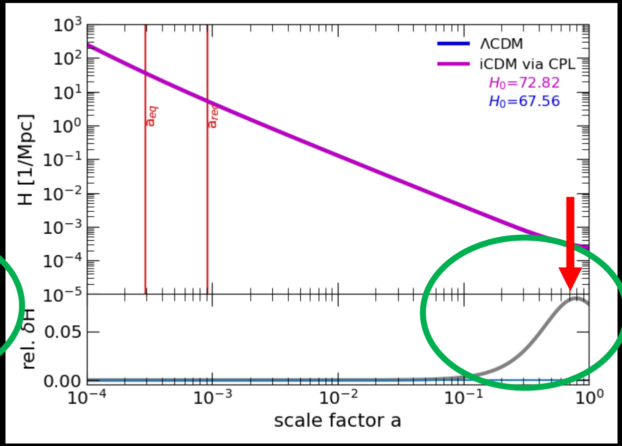
Characteristic signatures



$w_a = 0$

low-z BAO data based
w_0 = - 0.90
w_a = + 0.10

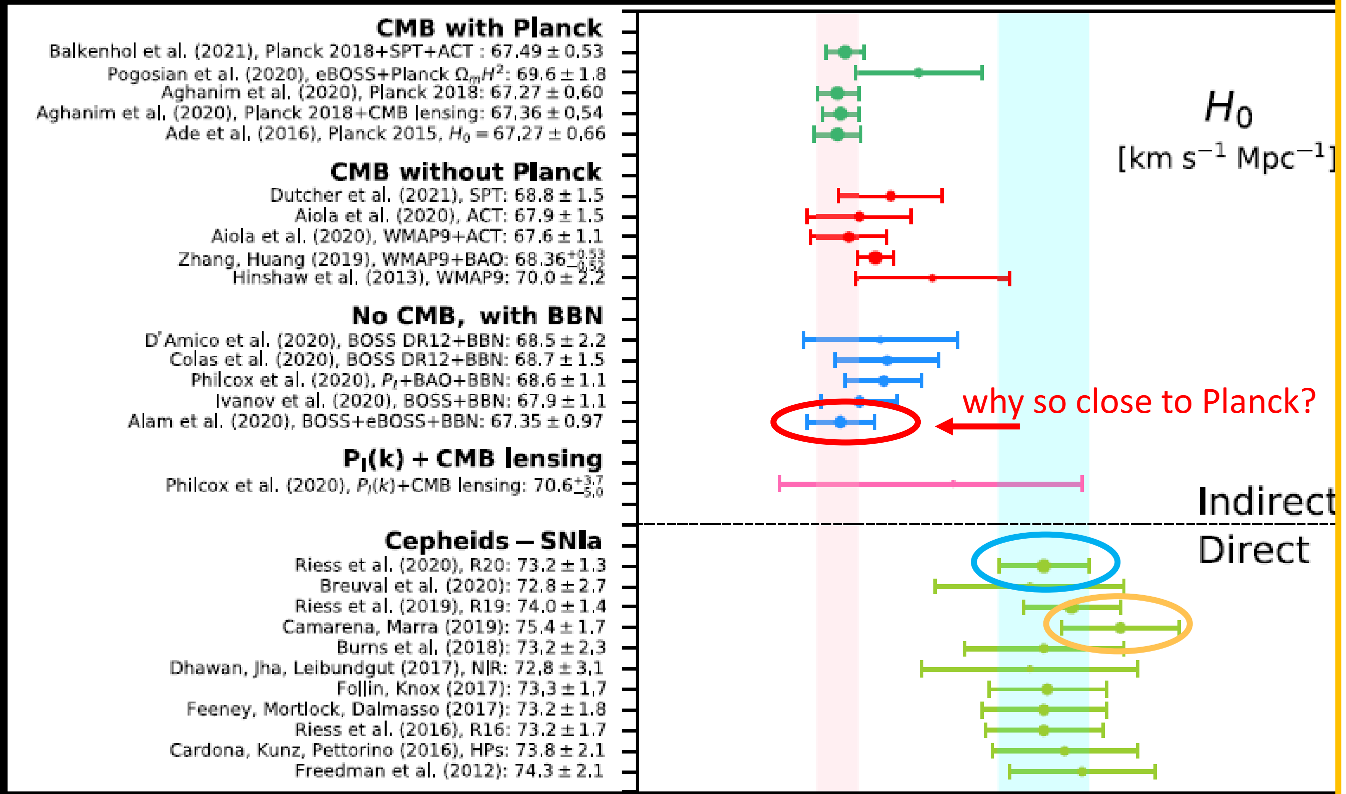




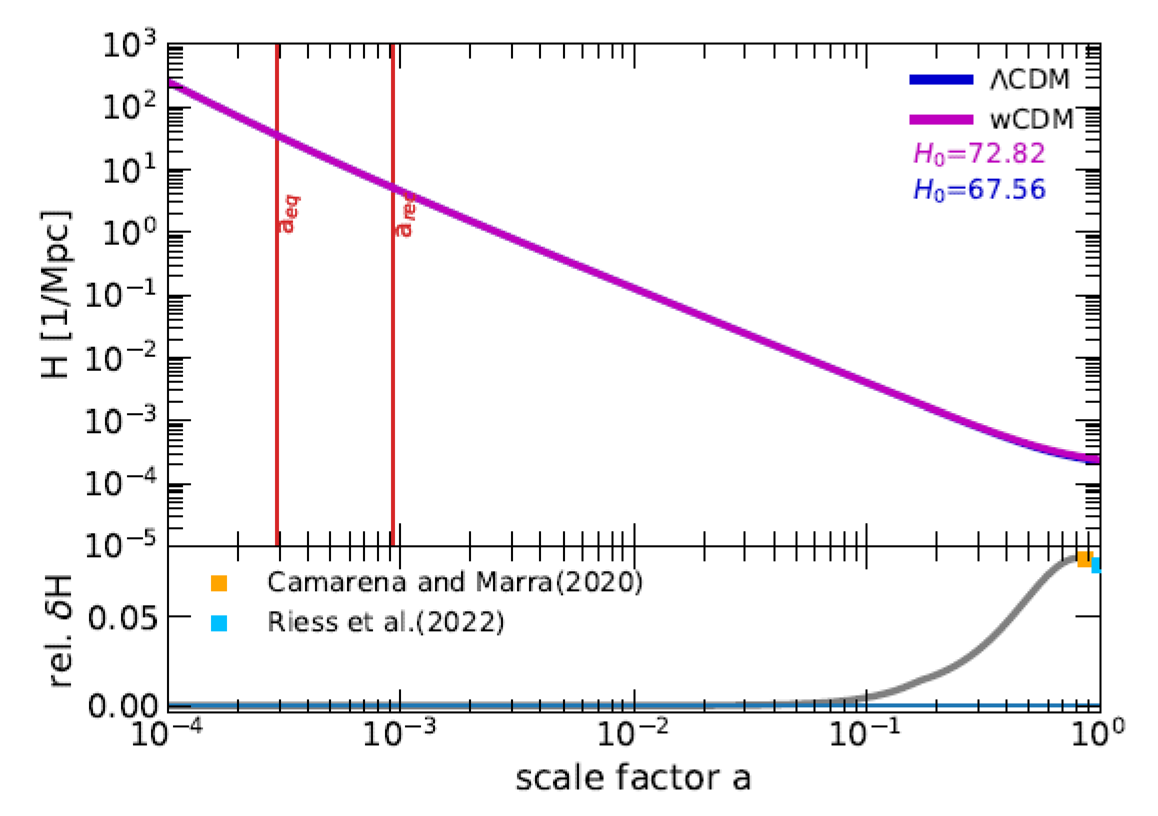
Providing a possible solution to the Hubble tension (w = -0.8 … -0.9)

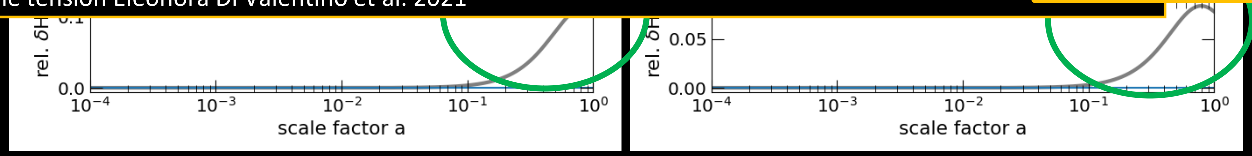
These characteristic signatures provide a tool to discriminate individual dynamic models of DE and a cosmological constant

MCMC Analysis – Improving the Accuracy



In the realm of the Hubble tension Eleonora Di Valentino et al. 2021





Providing a possible solution to the Hubble tension (w = -0.8 … -0.9)

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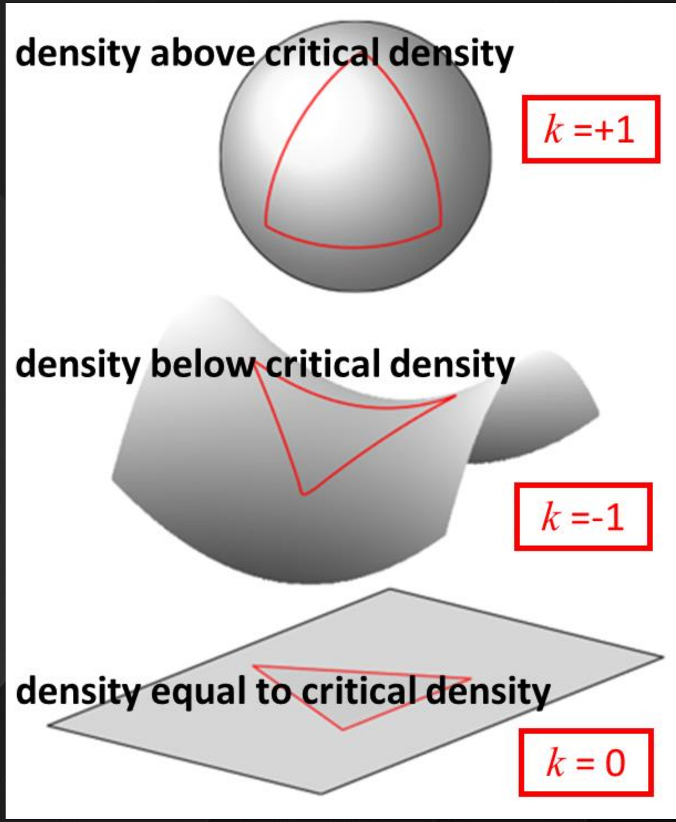
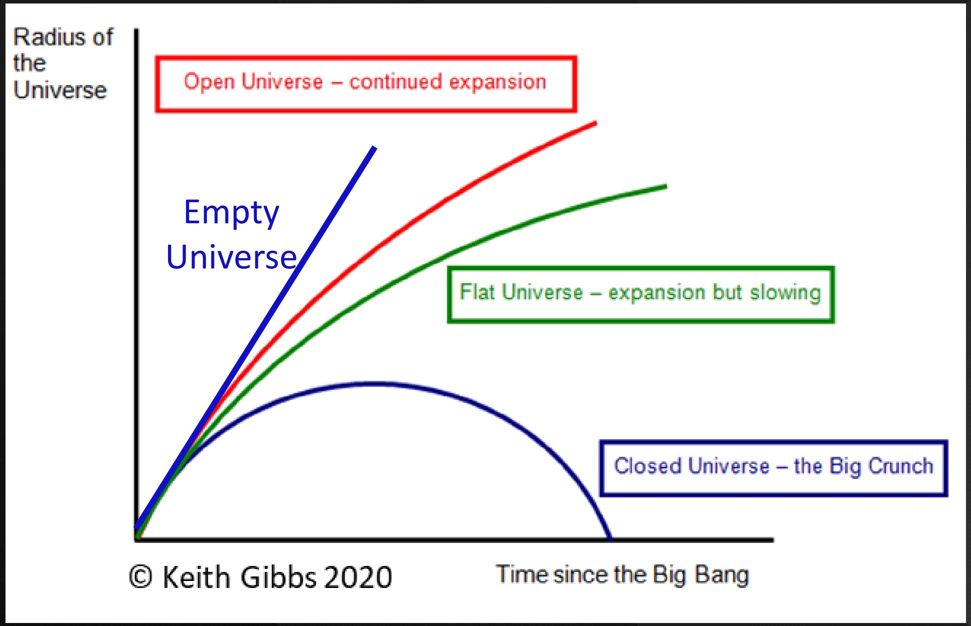
A physically motivated model explaining our empirical results



$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0} + \Omega_{\Lambda,0}$$

does not avoid physically implausible universes

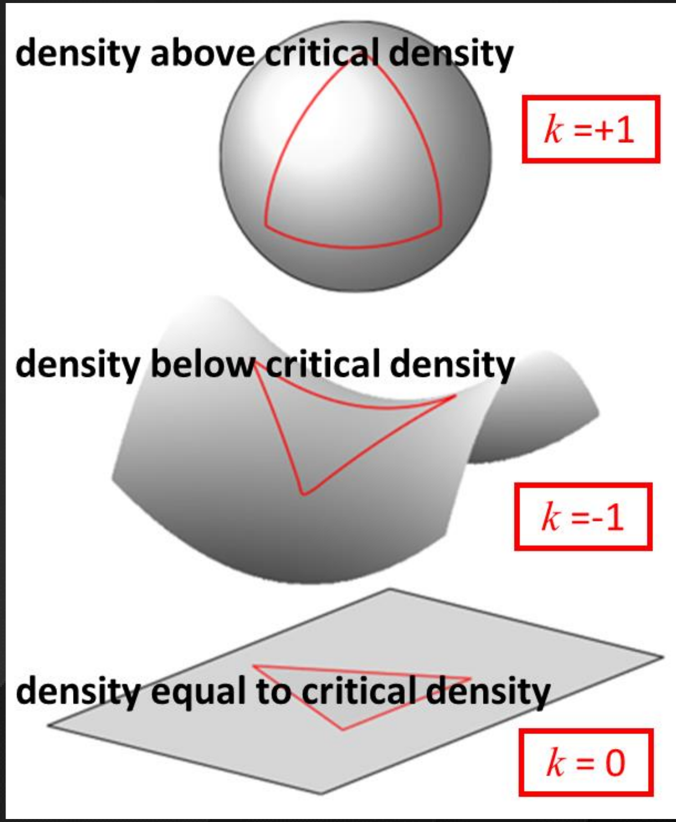
- empty universe



$$H^2(t) = \frac{8\pi G}{3c^2}\cancel{\rho} - \frac{kc^2}{a^2(t)}$$

curvature and deceleration without gravity!

A physically motivated model explaining our empirical results



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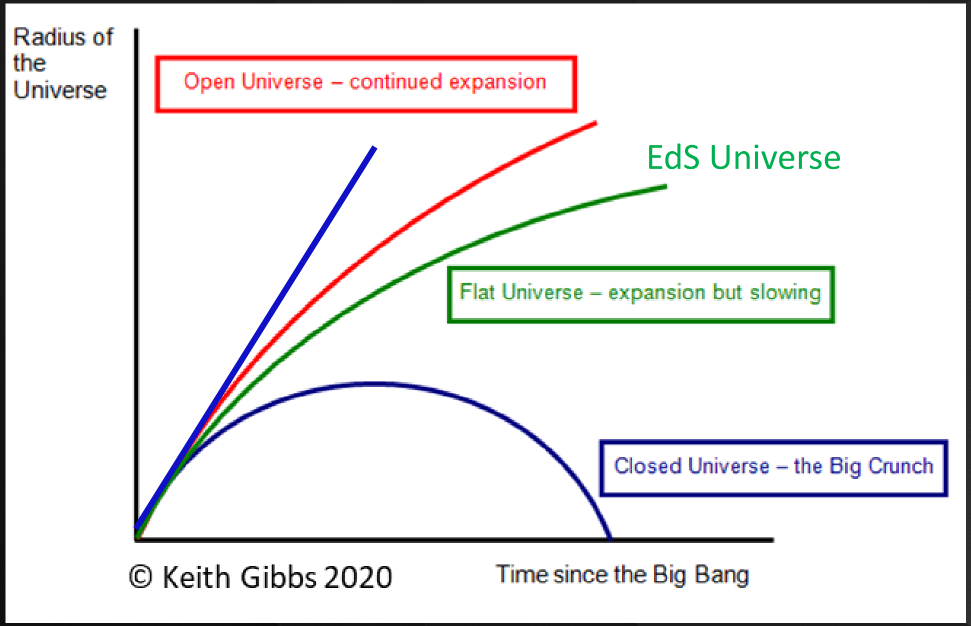
does not avoid physically implausible universes

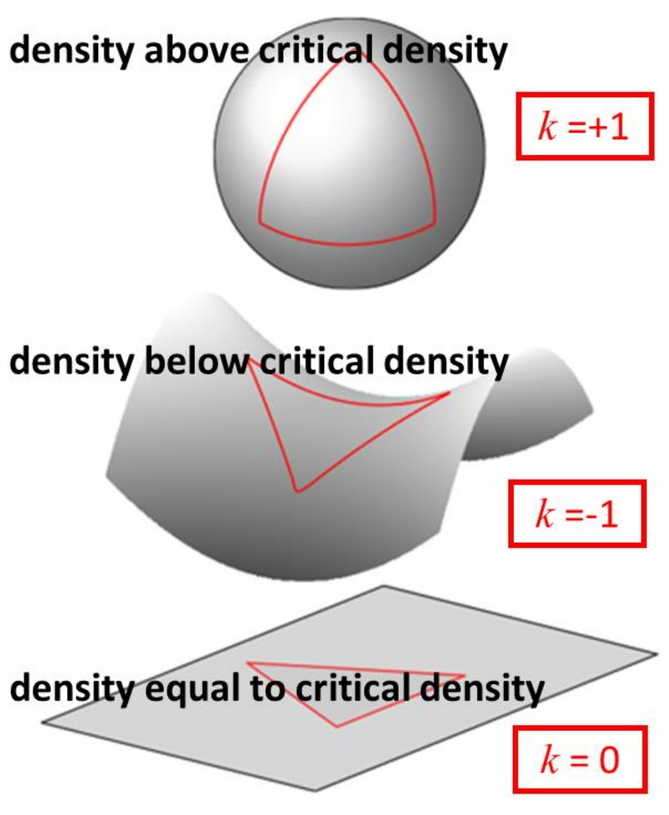
- empty universe
- Einstein de Sitter universe



$$H^2(t) = \frac{8\pi G}{3c^2}\rho - \cancel{\frac{kc^2}{a^2(t)}}$$

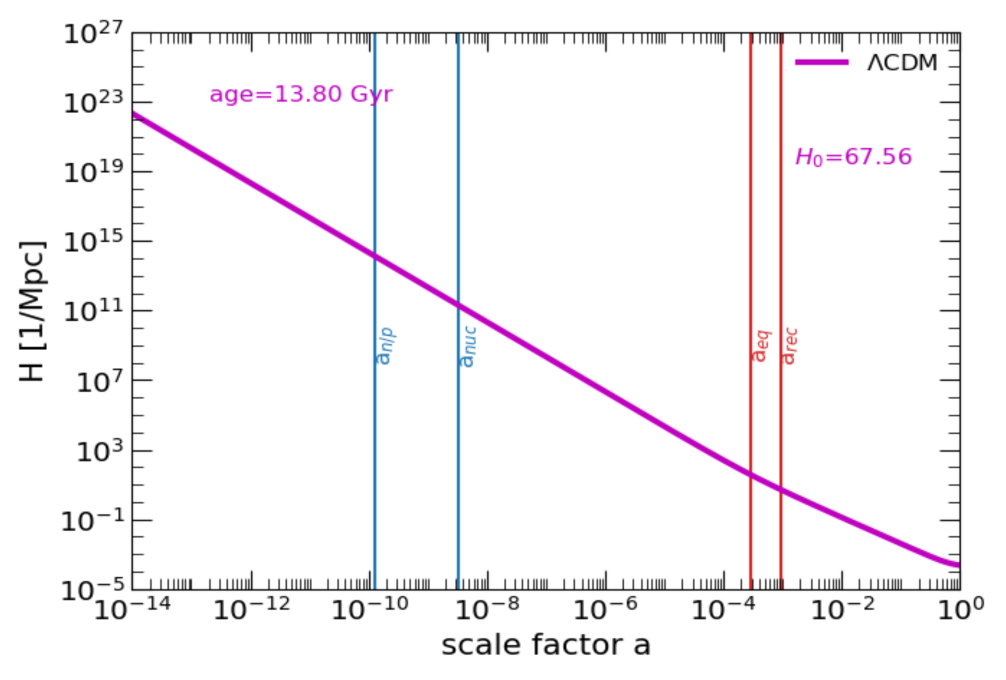
There is decelera-tion due to gravity without curvature!

A physically motivated model explaining our empirical results



We do observe a flat geometry of space!

ΛCDM – a flat Universe



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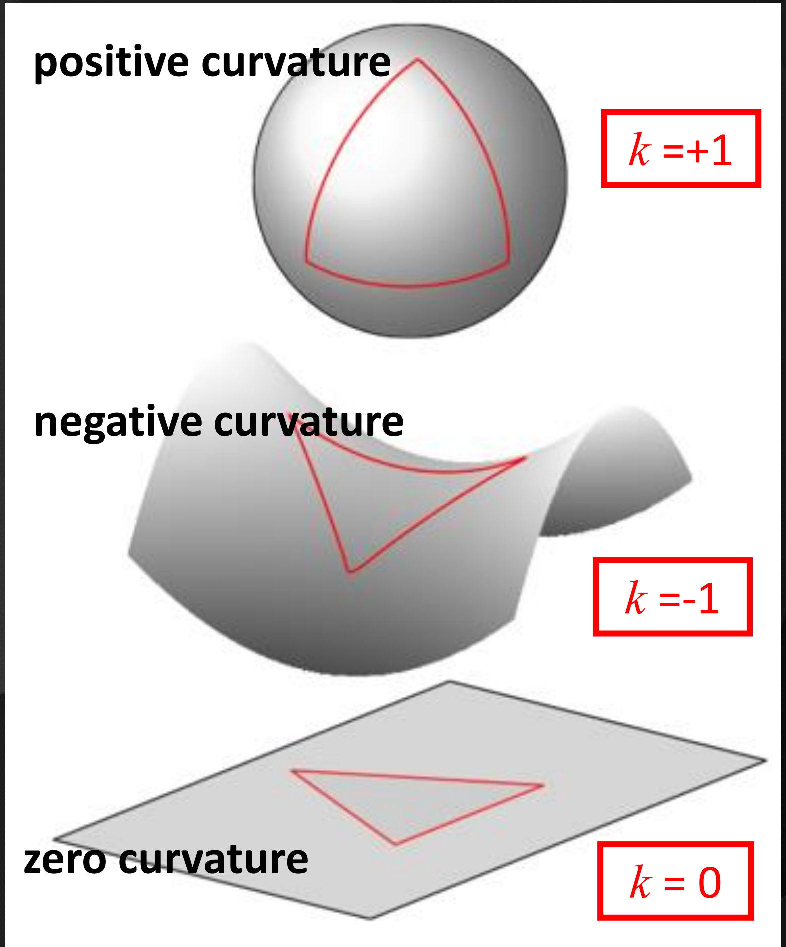
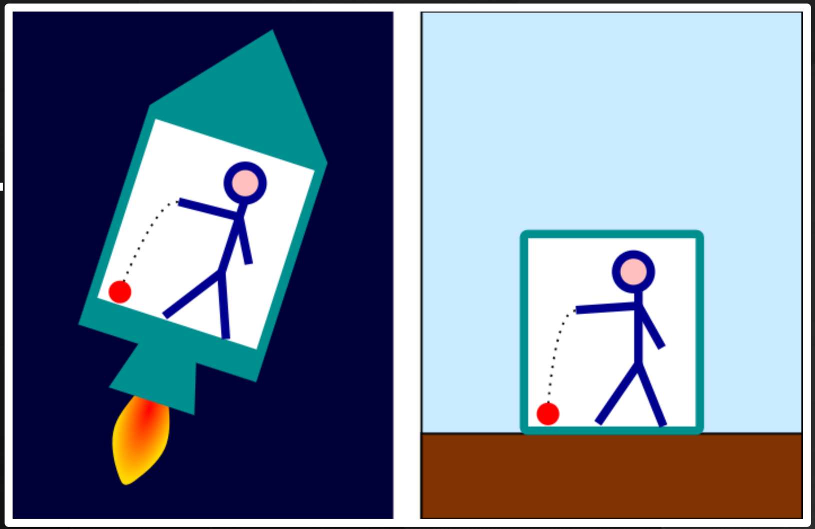
A physically motivated model explaining our empirical results



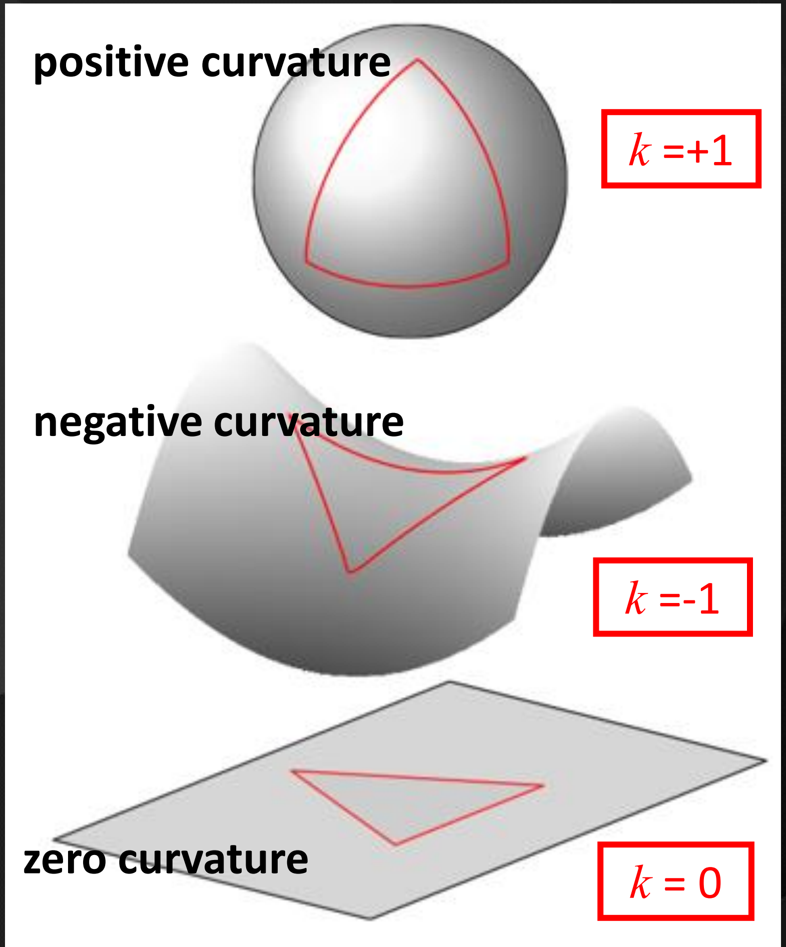
It is the key-concept the FLRW metric is based on

GR's equivalence principle



- The equivalence of the gravitational and the inertial mass
- Acceleration is equivalent to gravity
- Free-falling observers feel no gravity
- … they move on geodesics and **reside in a local inertial frame**

A physically motivated model explaining our empirical results

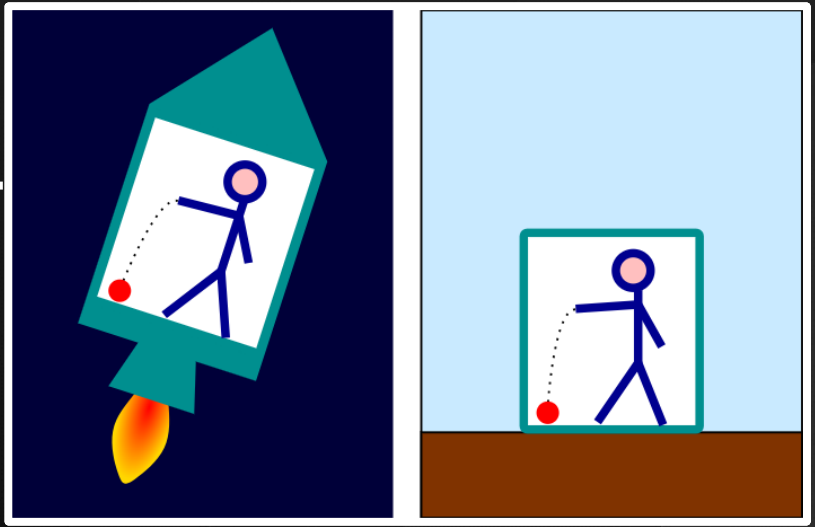


It is the key-concept the FLRW metric is based on

we are comoving observers

Comoving observers **perceive** flat space in a non-flat Universe!

GR's equivalence principle



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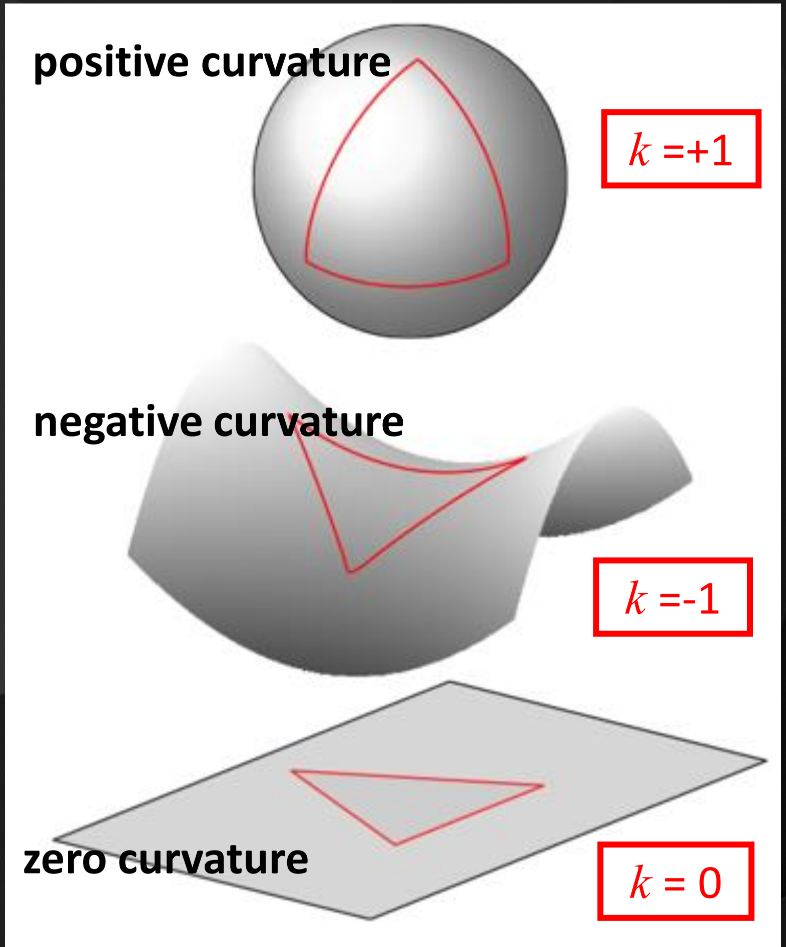
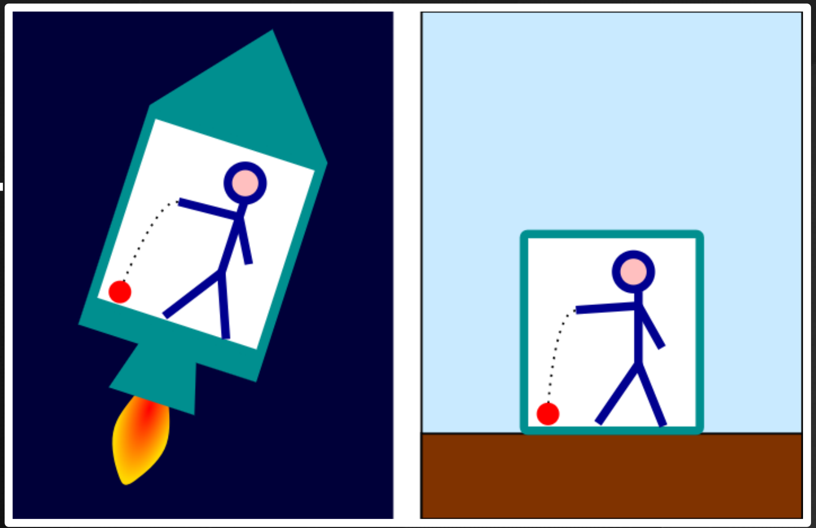


GR's equivalence principle



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Irrespective of the geometry → Ω_k = 0

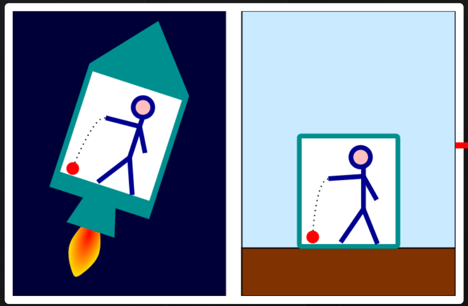
A physically motivated model explaining our empirical results

...determine the geometry term

retaining the ΛCDM formalism

$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0}$$

we have broken this connection!



we **perceive** flat space

Ω_k=0

~~space is flat~~

$$H^2(t) = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$

?

The Curvature of Space and the Geometry of the Universe

…determine the geometry term

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}\left(\rho(t) + 3p(t)\right)$$

integration

$$H^2(t) = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$

definition of k

k =-1 if $\rho < \rho_{\text{crit}}$

k = 0 if $\rho = \rho_{\text{crit}}$

k =+1 if $\rho > \rho_{\text{crit}}$

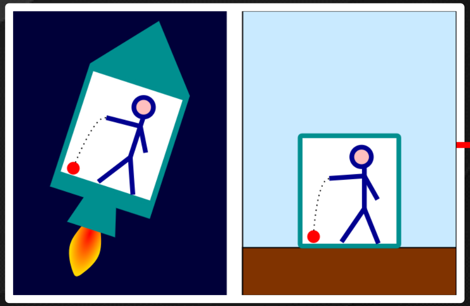
at any point in time

$$H^2(t) = \frac{8\pi G}{3c^2}\rho_{\text{crit}}(t)$$

$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\text{CDM},0} + \Omega_{k,0} + C$$

integration constant

determined by initial conditions!



we **perceive** flat space

Ω_k=0

The Curvature of Space and the Geometry of the Universe

…determine the geometry term

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}\left(\rho(t) + 3p(t)\right)$$

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integration constant

determined by initial conditions!

definition of k

k =-1 if $\rho < \rho_{\mathrm{crit}}$

k = 0 if $\rho = \rho_{\mathrm{crit}}$

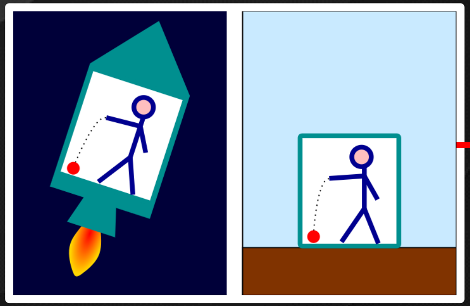
k =+1 if $\rho > \rho_{\mathrm{crit}}$

at an initial point in time

$$\rho_{\mathrm{crit}} = \frac{3H_{\mathrm{ini}}^2 c^2}{8\pi G}$$

$$H_{\mathrm{crit}}^2 = \frac{8\pi G}{3c^2}\rho_{\mathrm{ini}}$$

$H_{\mathrm{ini}} = H_{\mathrm{crit}} \rightarrow k=0$



we **perceive** flat space
Ω_k=0

A physically motivated model explaining our empirical results

…determine the geometry term

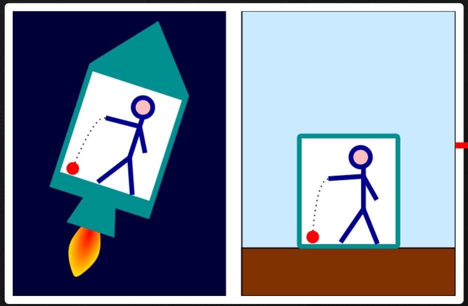
geometry term

retaining the ΛCDM formalism

$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0} + \mathrm{C}$$

integration constant

the initial conditions are defined by the **initial density** in relation to the **initial expansion rate** H_{ini}!



we **perceive** flat space

$\Omega_k=0$

$$H^2(t) = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$

integration

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}\left(\rho(t) + 3p(t)\right)$$

A physically motivated model explaining our empirical results

…transforming Newtonian kinematics to relativistic kinematics

Newtonian kinematics

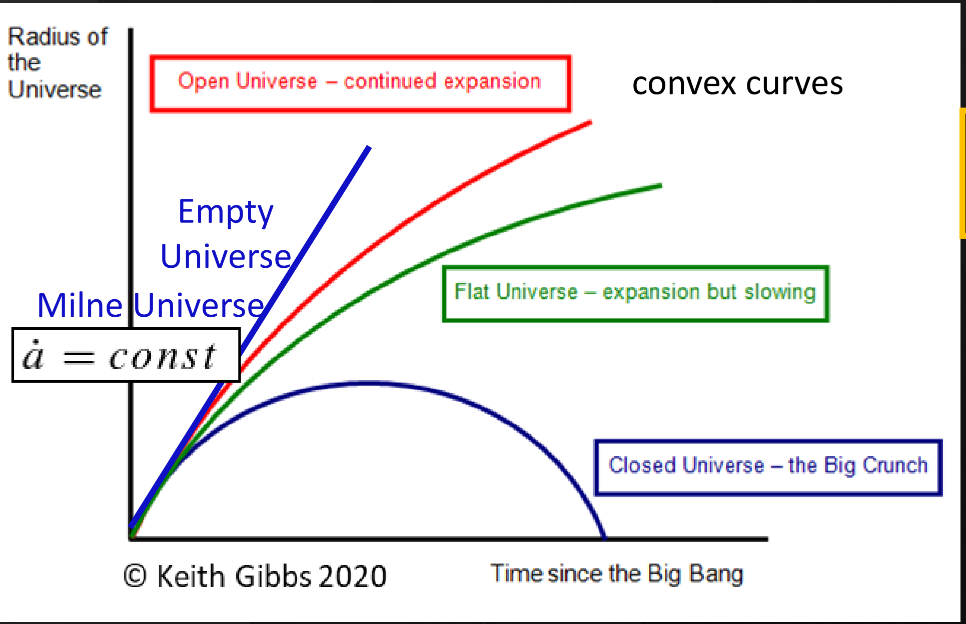
$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0}$$

$w_k = -1/3$

relativistic kinematics

$\Omega_k = 0$

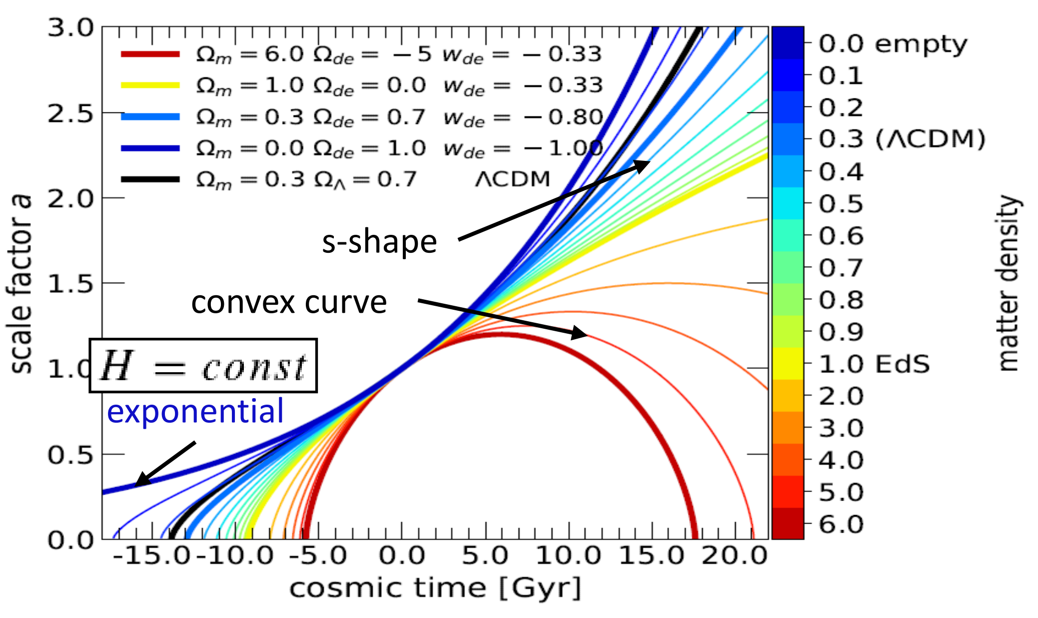
$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0} + \Omega_{\mathrm{de},0}$$



Ω_{de}

relativistic adaptation term to Newtonian kinematics

Robertson (1936)



- **corresponds to Newtonian picture**
- center of gravity
- homogeneous distribution of energy density
- the curvature term can be interpreted as E_{pot}

- **corresponds to relativistic picture**
- no center of gravity (Lemaitre, 1927)
- **homogeneous gravitational field**

A physically motivated model explaining the empirical results

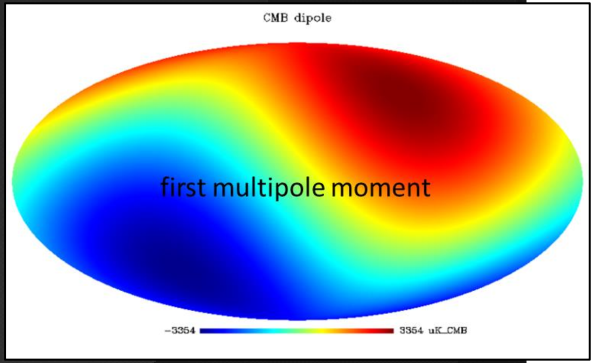
…the owCDM model

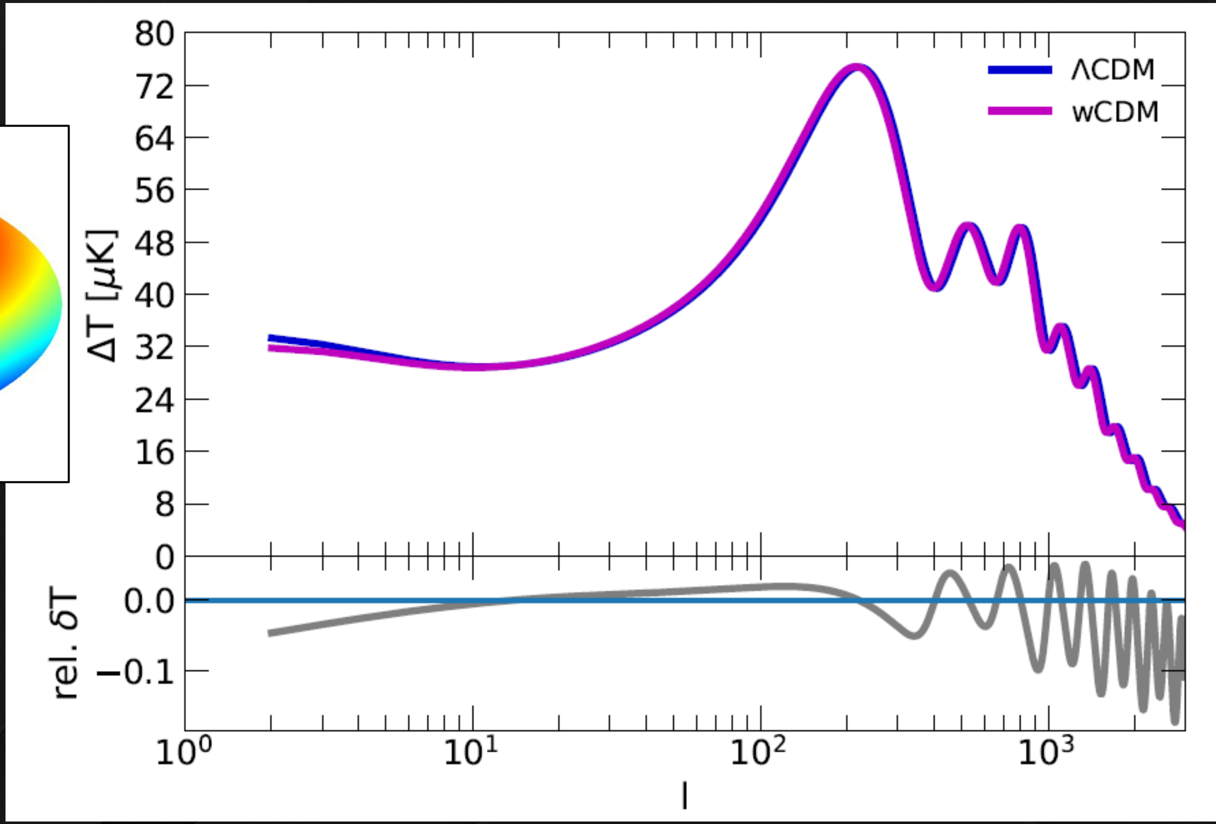
components of our model

$$1 = \Omega_{r,0} + \Omega_{b,0} + \Omega_{\mathrm{CDM},0} + \Omega_{k,0} + \Omega_{\mathrm{de},0}$$

→ w_{de}= -0.8 … -0.9

wCDM





- dipole due to our peculiar motion against the CMB background is not considered in the spectrum
- Ω_k=0 in the reference frame of a perfectly comoving observer
- the dipole **offsets** us from a perfectly comoving observer