“Cold” Freeze-out of Nonthermal Superheavy Dark Matter, Hubble Tension, and Ultra-high Energy Cosmic Rays (UHECRs)

Cosmology 2025 @ Elba Island

Zhijie (Jay) Xu

Pacific Northwest National Laboratory (PNNL)
Richland, Washington, USA
Zhijie.xu@pnnl.gov; zhijiexu@hotmail.com

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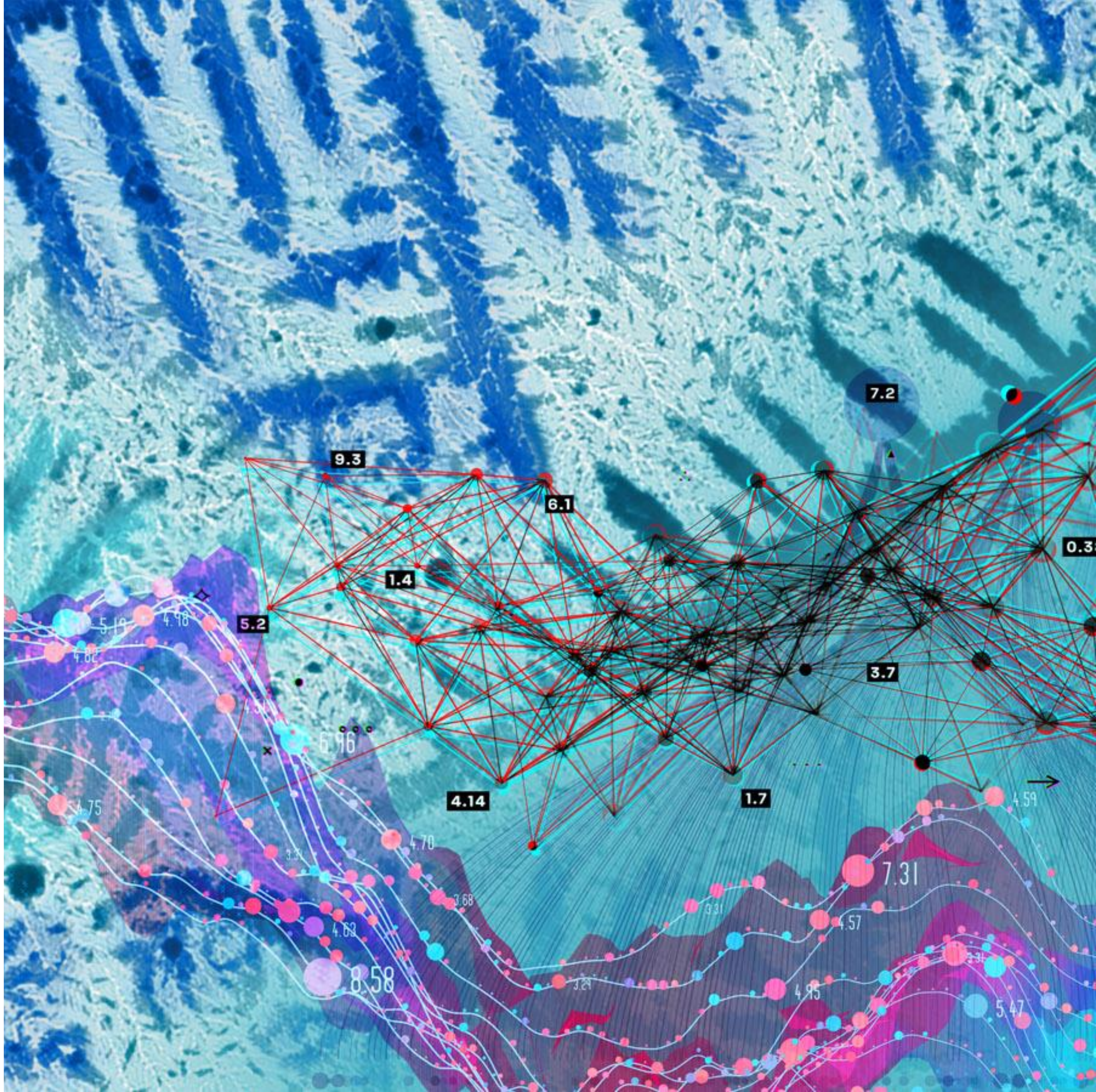
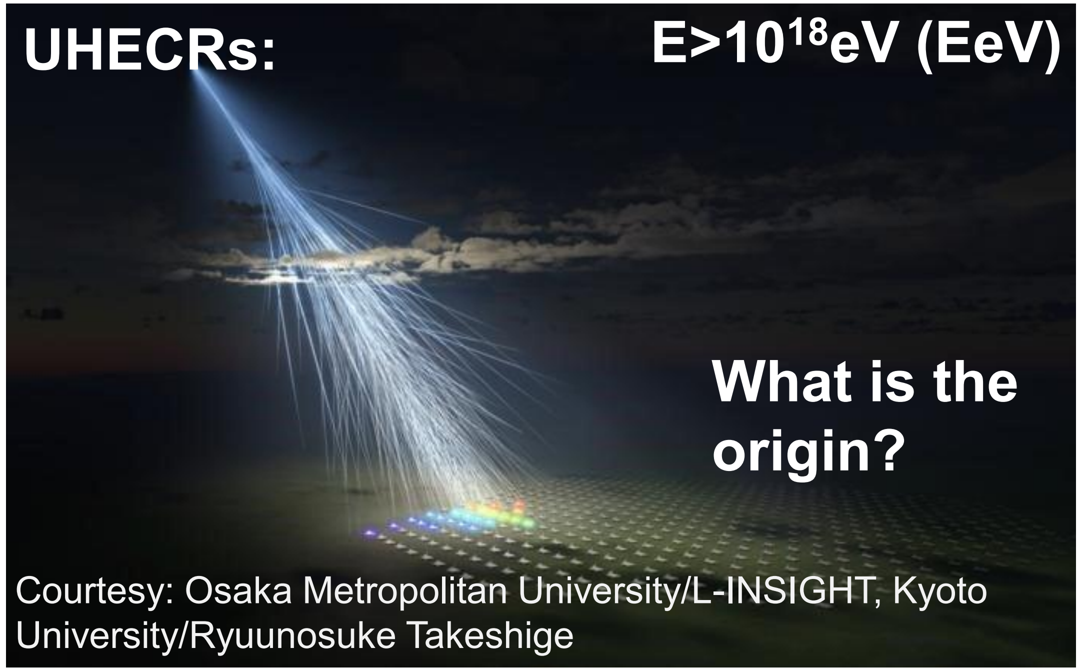


One Solution for Three Unsolved Problems?

Dark Matter

Ultra-high Energy Cosmic Rays (UHECRs)

Hubble Tension



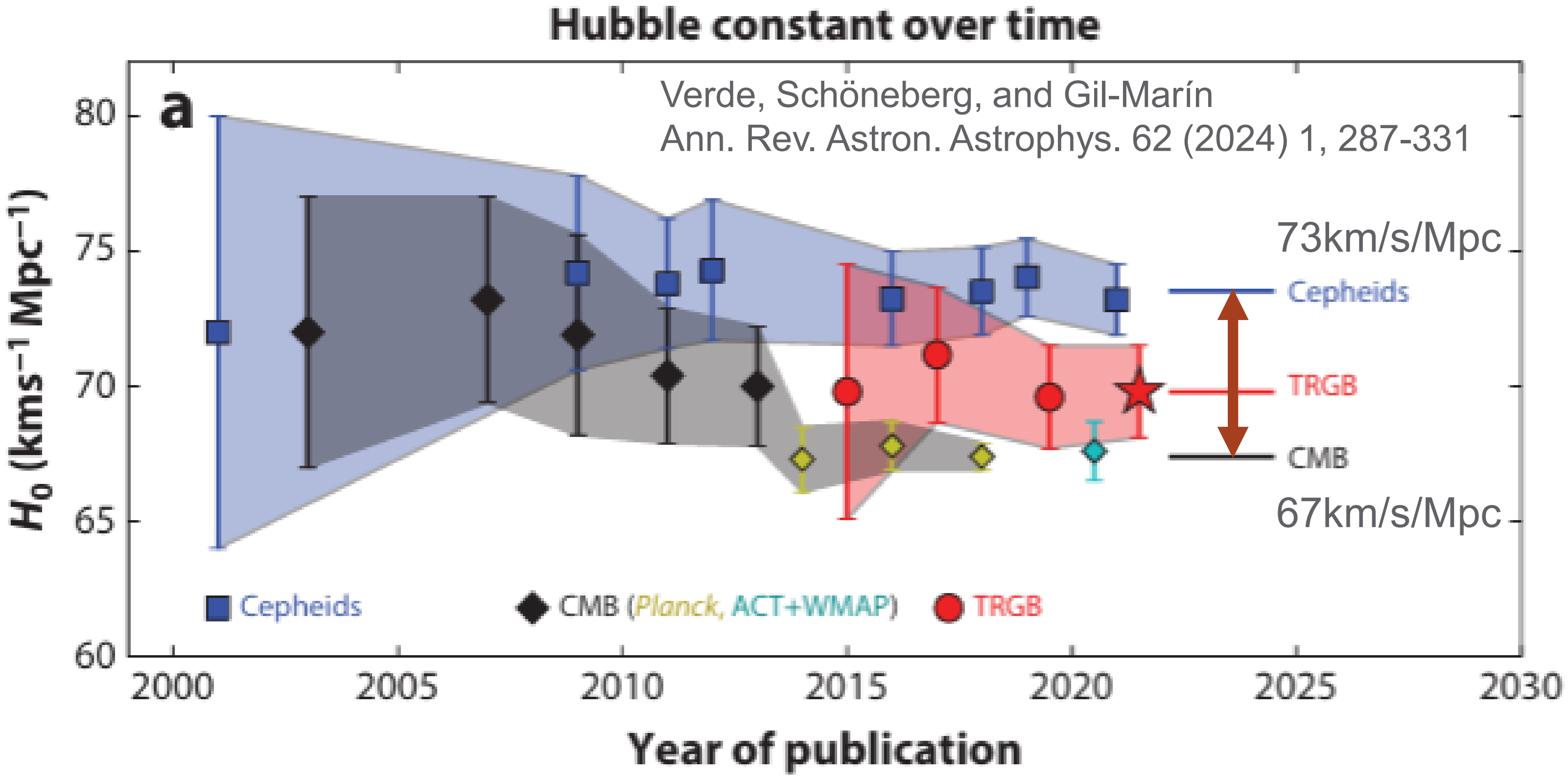
UHECRs:

E>10^{18}eV (EeV)

What is the origin?

Courtesy: Osaka Metropolitan University/L-INSIGHT, Kyoto University/Ryuunosuke Takeshige

A shower of particles as it entered the atmosphere above the Telescope Array in Utah



Thermal Relics	**Nonthermal Relics**
“Hot” freeze-out	**“Cold” freeze-out**
WIMP miracle	**“X” miracle**
100 GeV	**10^{12} GeV**
	Hubble Tension? UHECRs?

arXiv:2202.07240 **Zhijie (Jay) Xu PNNL**

Setting the Stage: Nonthermal X Particles Vs. WIMPs

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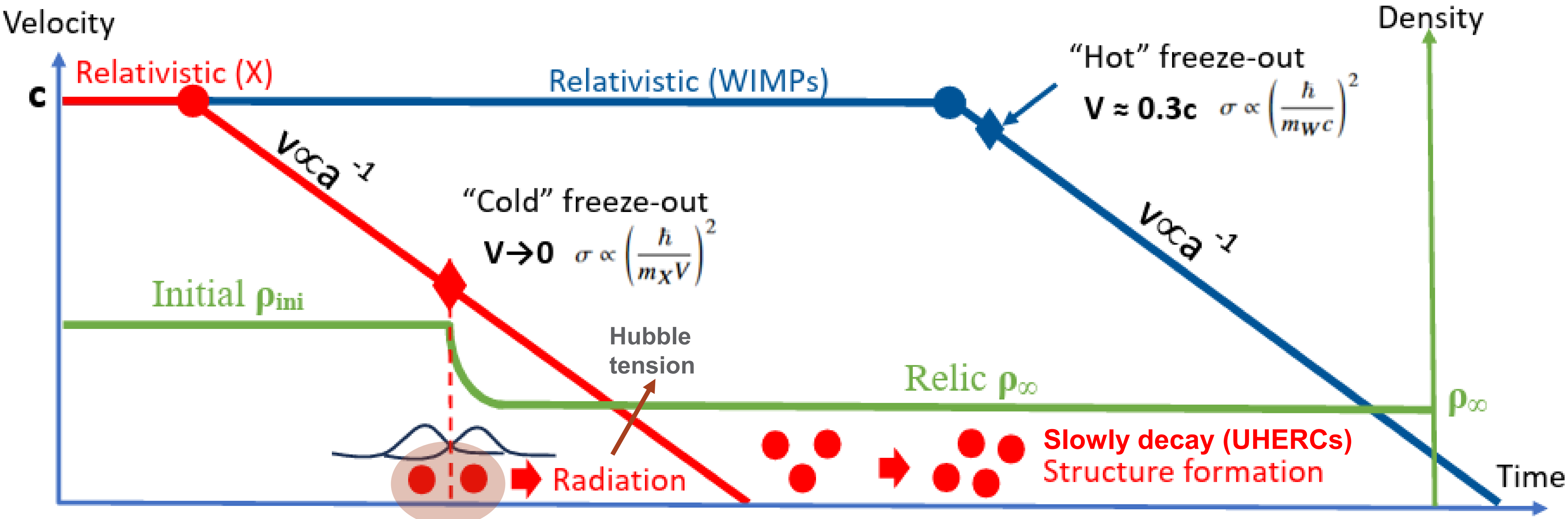


Blue: WIMPs;

Red: Nonthermal X particles;

Green: X particle density;

"X miracle"

- **Nonthermal, heavy, particle DM**
- **Overproduced with $\rho_{ini} >> \rho_\infty$**
- **Become nonrelativistic early**
- **Extremely cold at freeze-out**
- **Gravitationally bound structure**
- **Particle annihilation/decay into radiation**

$$m_X = 10^{12} GeV$$

$$\langle \sigma_X V_{||} \rangle = 3 \times 10^{-22} \mathrm{m}^3/\mathrm{s}$$

Free-streaming Scale of Dark Matter

Three key scales:

- Horizon size (r_H~100 Mpc)
- Largest halo (r_t~1 Mpc, m^*_h~10^{13} M_{sun})
- Free streaming scale (The distance particles travel without interacting)
  - Highly relevant to particle mass m_X

Large scales (> r_t)

- Linear perturbation theory applies

$\delta \propto log(a)$ → $\delta \propto a$

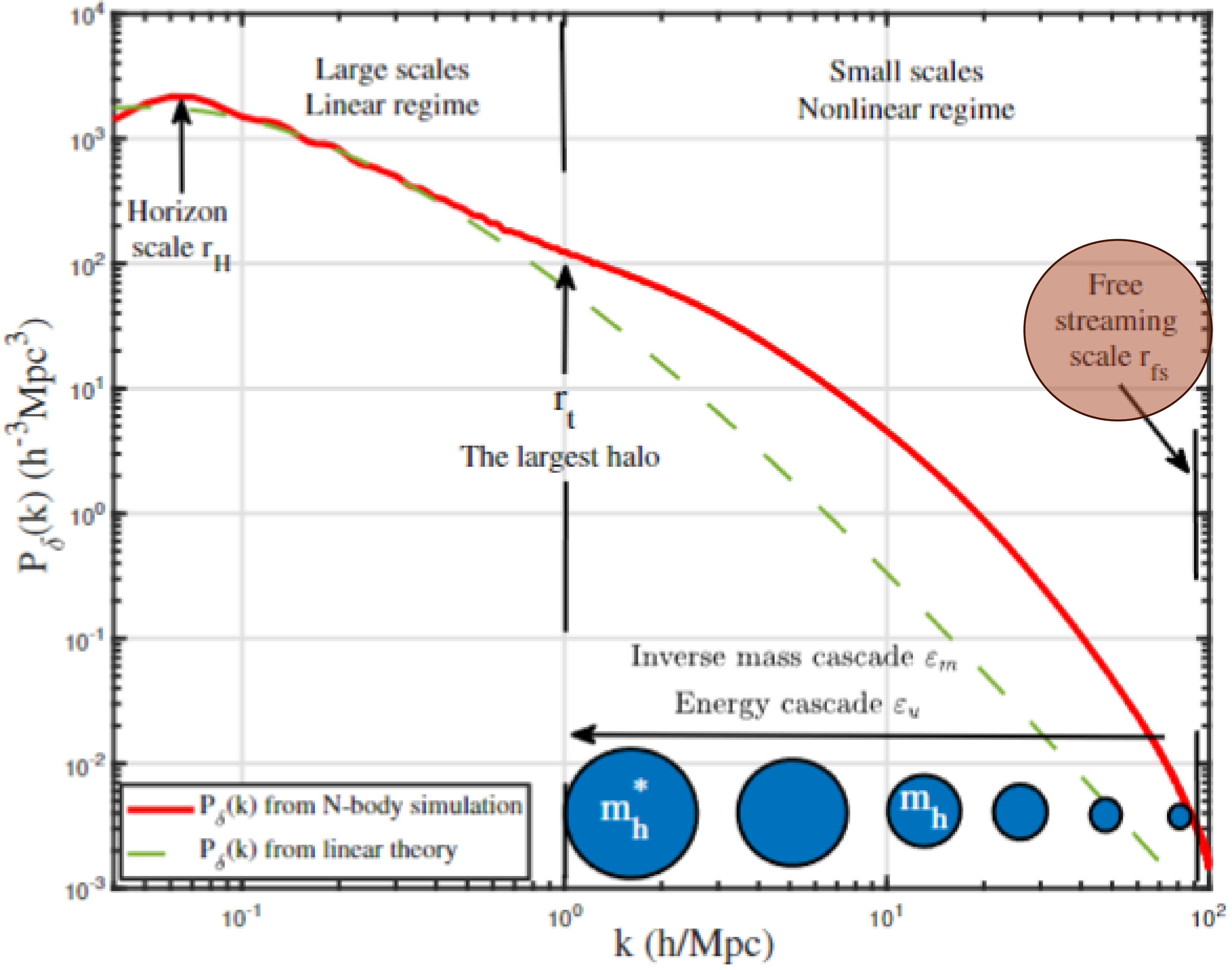
Radiation era — Matter era

Frozen Perturbation — Structure grow

- Only applies in linear regime

Small scales (< r_t)

- Highly nonlinear
- Linear theory does not apply



Matter density power spectrum

Critical Mass (10^{12} GeV) Identified from Free-streaming

The smallest DM structure:

$$M_S=\max(M_{fs}, m_X) \qquad M_{fs}=\frac{\pi}{6}\rho_{DM0}\lambda_{fs}^3$$

Free streaming mass

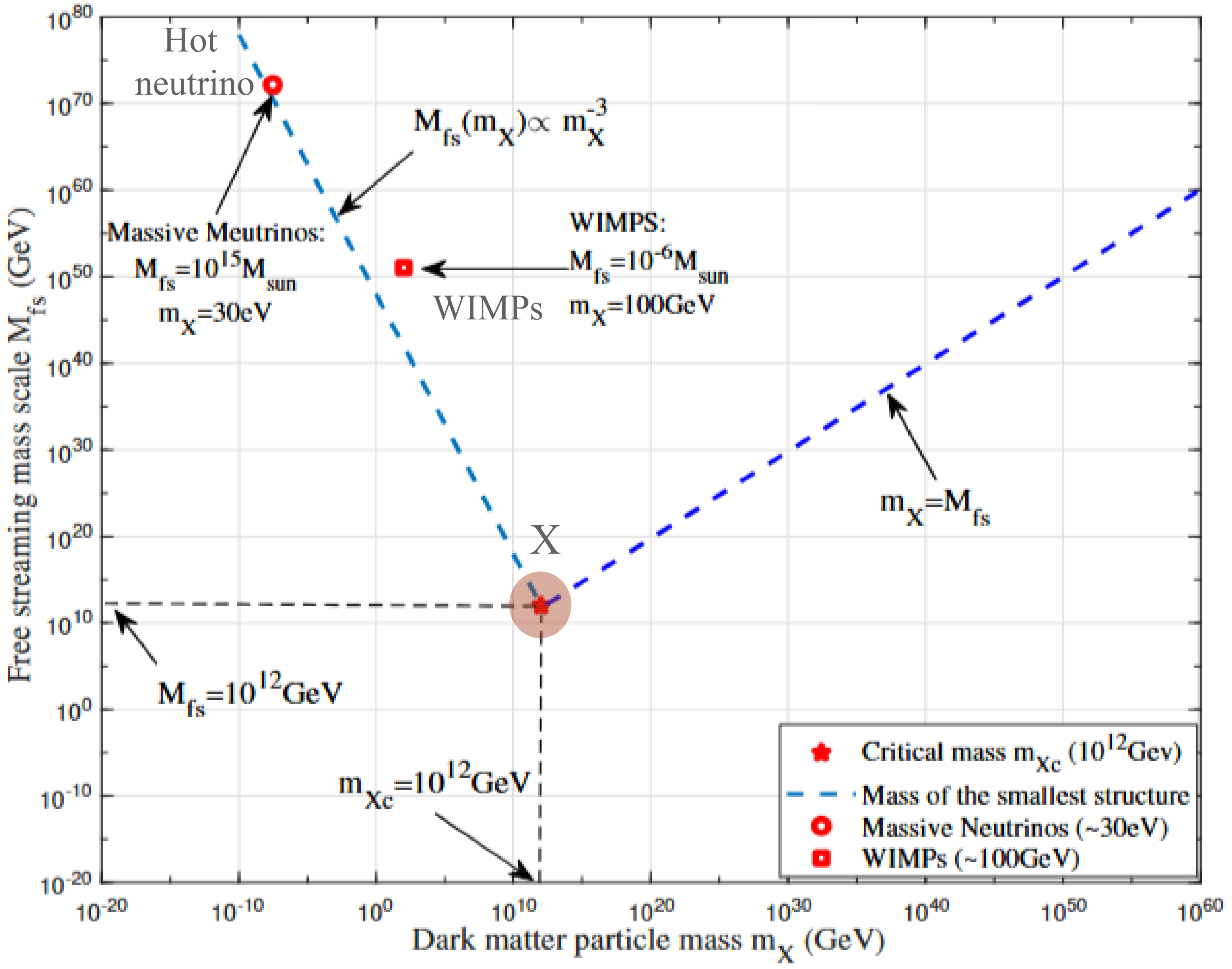
Particle mass

Critical particle mass: $M_{fs}(m_{Xc})=m_{Xc}$

$$m_{Xc}=\frac{1}{2}(3\beta_p)^{\frac{3}{4}}\left(\frac{3}{8\pi\alpha_p}\right)^{\frac{3}{16}}\left(\frac{\Omega_{DM}^2}{\Omega_{rad}^{3/2}}\right)^{\frac{1}{8}}M_{pl}(H_0t_p)^{\frac{1}{8}}$$

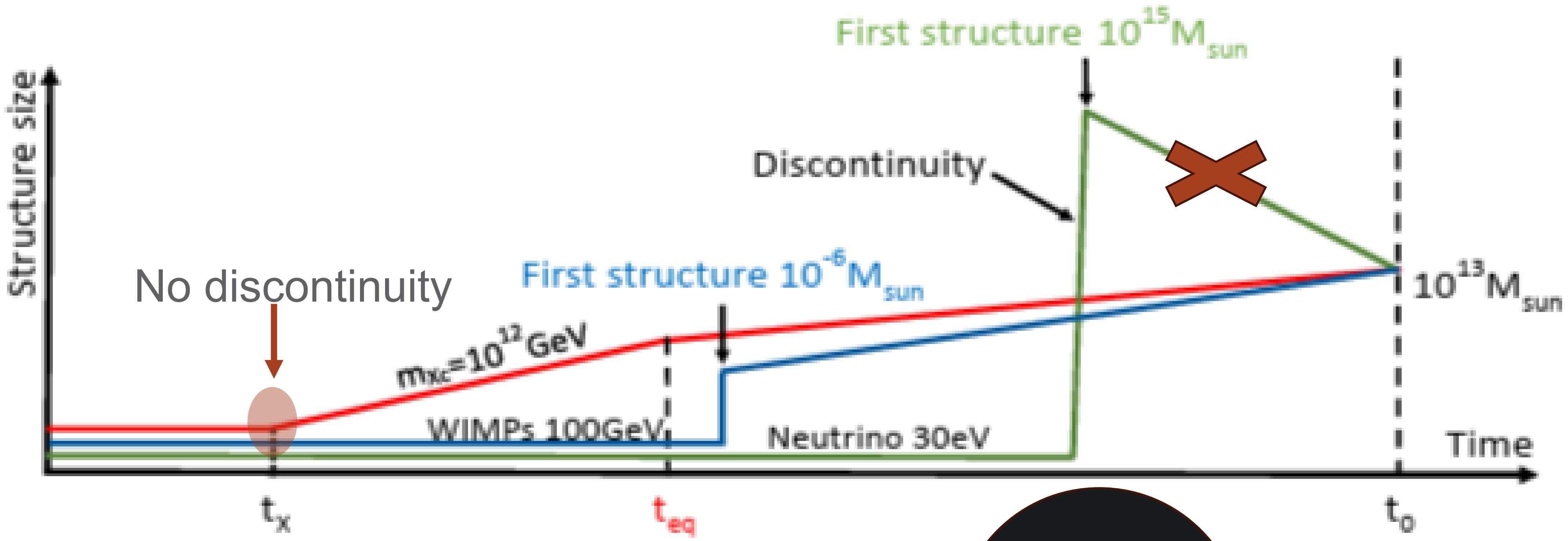
$$\approx 1.5\times10^{-15}kg\ \boxed{\approx 0.8\times10^{12}\text{GeV}.}$$

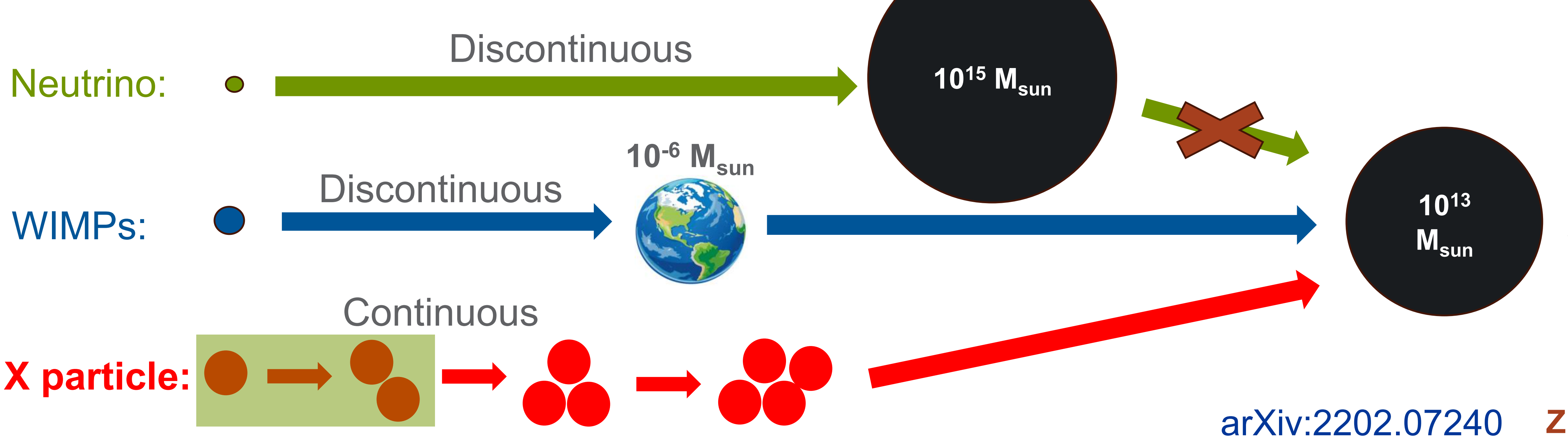
#1: Particles of critical mass $m_{Xc}=10^{12}$ GeV can form the smallest possible structure among all possible m_X, or the earliest possible structure.



Free streaming mass M_{fs} vs. DM particle mass m_X

Structure Evolution for DM Particles of Different Masses





Neutrino: Discontinuous → 10^{15} M_{sun} → 10^{13} M_{sun}

WIMPs: Discontinuous → 10^{-6} M_{sun} → 10^{13} M_{sun}

X particle: Continuous → 10^{13} M_{sun}

Depending on particle mass, the smallest DM structure can have only two DM particles.

arXiv:2202.07240

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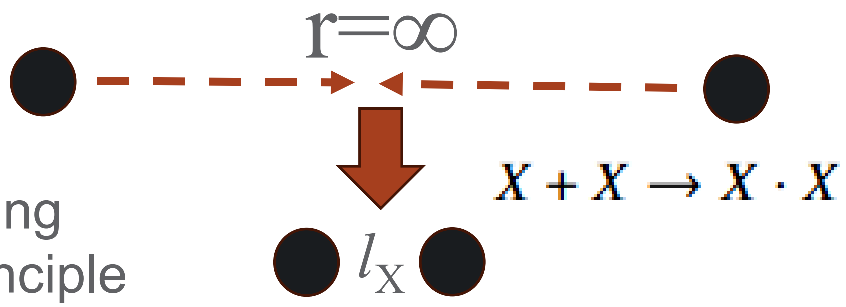
The Smallest Structure from Newtonian Quantum Gravity

The formation of the smallest structure:

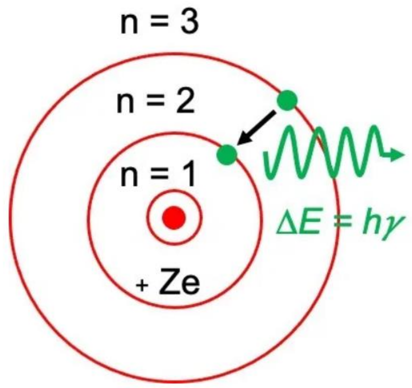


Assuming:

- Gravity only
- Fermions following

Pauli exclusion principle

Bohr model of atom:



Virial theorem:	Uncertainty principle:
$v_X^2 = \frac{Gm_{Xc}}{4r_X}$	$m_{Xc}V_X \cdot r_X = n\hbar$
Gravity	**Quantum**

Analogy

Virial theorem:	Uncertainty principle:
$\frac{e^2}{4\pi\varepsilon_0 r_e} = m_e v_e^2$	$m_e v_e r_e = \hbar$
Coulomb force	**Quantum**

With $m_{Xc} = 10^{12}$GeV:

Particle Size (cross section): $l_X = 2r_X = \frac{8\hbar^2}{G(m_{Xc})^3} \approx 3\times10^{-13}m,$

Velocity: $v_X = \frac{G(m_{Xc})^2}{4\hbar} \approx 4\times10^{-7}m/s,$ **Newtonian**

Structure Formation time: $t_X = \pi\frac{l_X}{v_X} = \frac{32\pi\hbar^3}{G^2(m_{Xc})^5} \approx 2\times10^{-6}s,$

Just like the Bohr model of atom:

A quantum rule

The angular momentum $L = m_e vr$ is an integer multiple of $\hbar$:

$m_e vr = n\hbar.$ n=1 is the ground state

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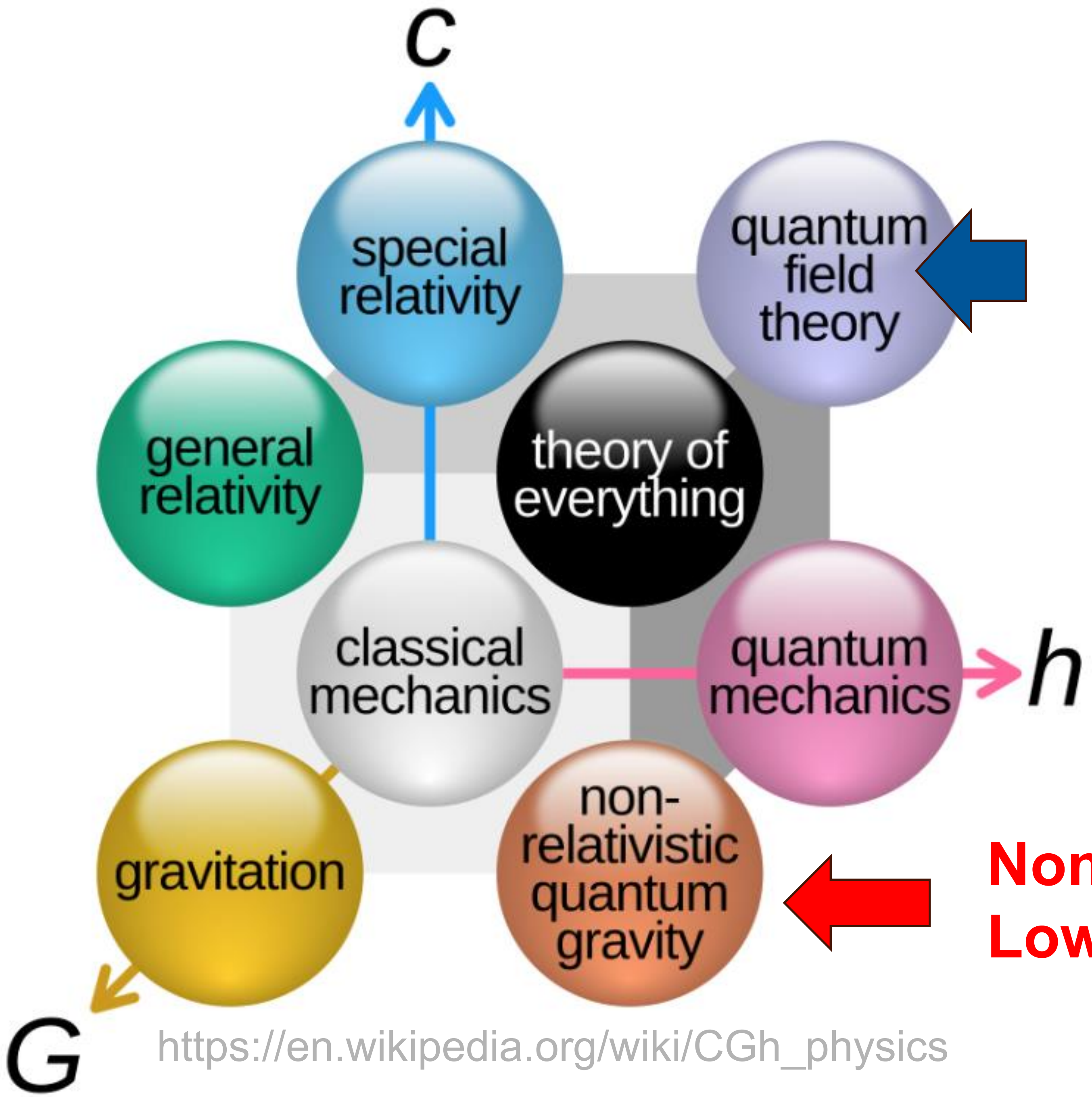
cGh Physics for WIMPs and X Particles



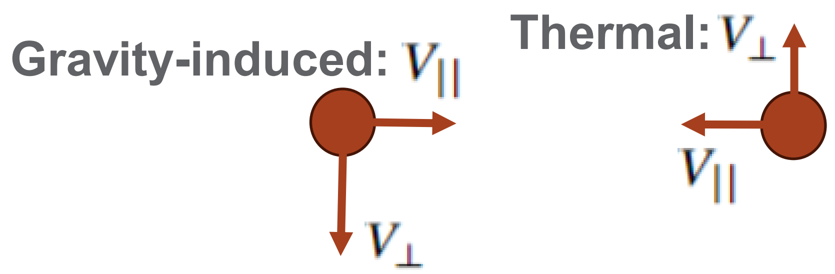
https://en.wikipedia.org/wiki/CGh_physics

Thermal WIMPs (C, h, G=0):
High speed,
Gravitational interaction between particles is neglected;

Nonthermal X Particles (C=0, h, G):
Low speed, Gravity + Quantum

arXiv:2202.07240

DM Thermal Velocity vs. Gravity-induced Velocity



Comoving thermal:

$$\dot{\mathbf{u}}_{\perp} + 2H\mathbf{u}_{\perp} = 0.$$

Peculiar thermal:

$$V_{\perp} = u_{\perp} a \propto a^{-1}$$

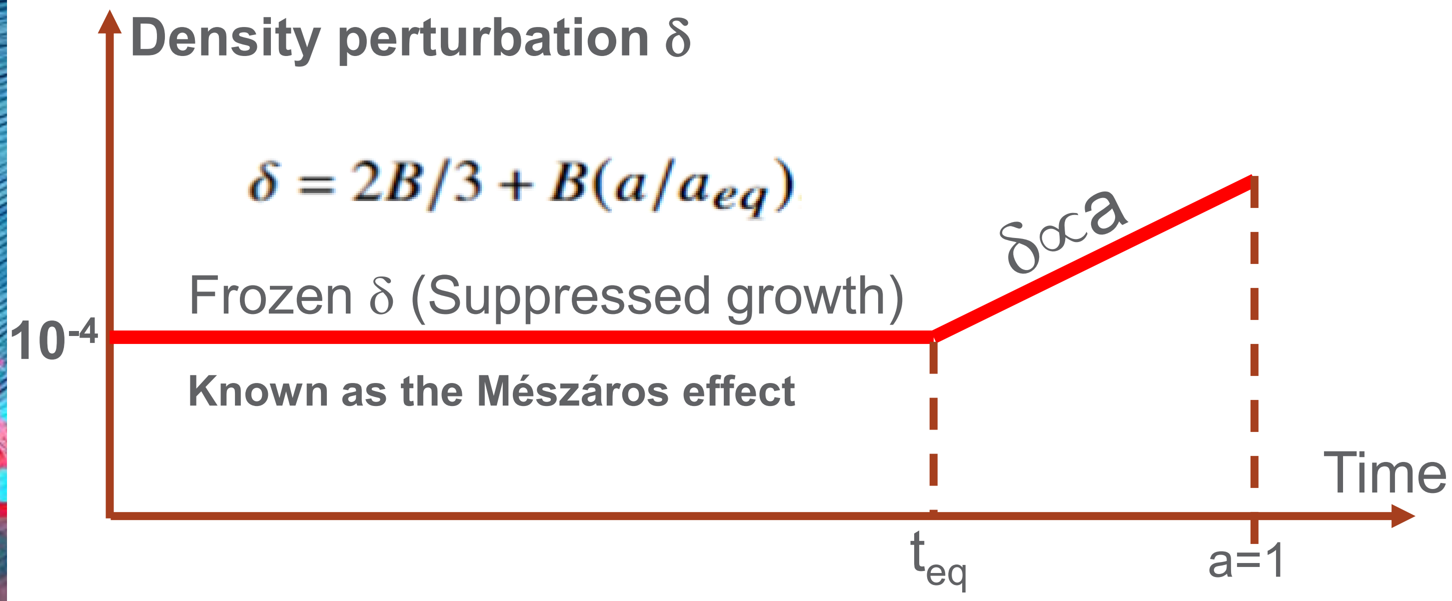
Gravity-induced velocity:

Continuity Eq. : $\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_{DM}\delta,$

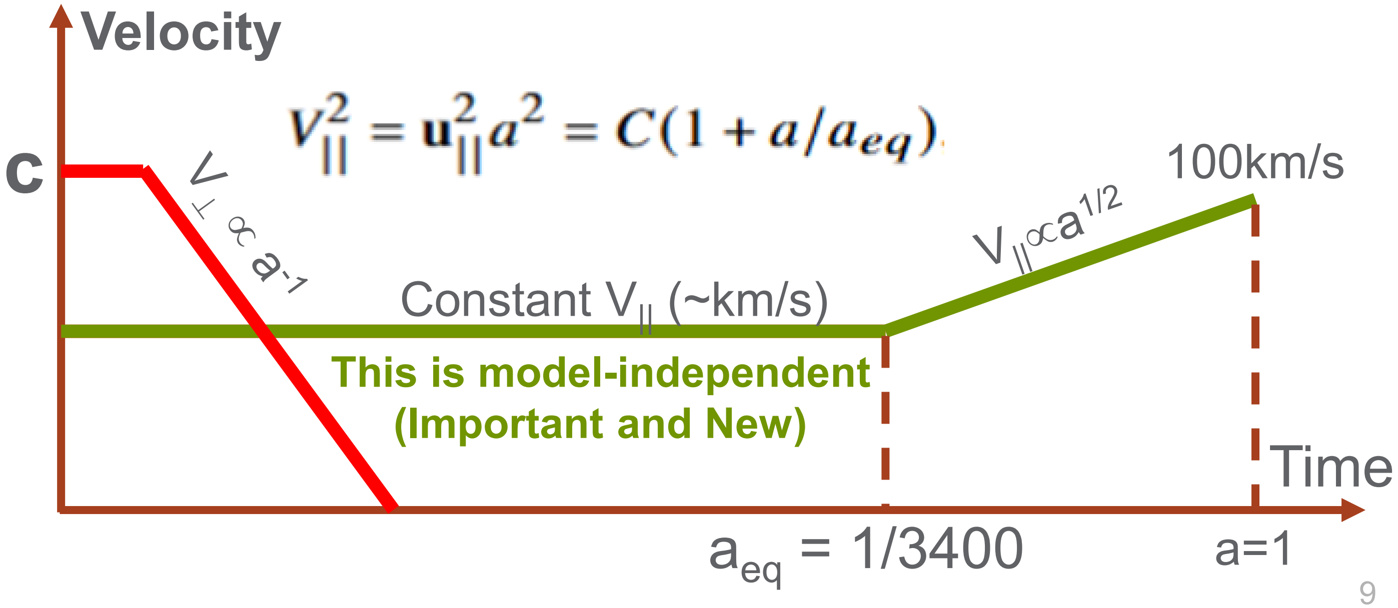
Momentum Eq. : $\dot{\mathbf{u}}_{||} + 2H\mathbf{u}_{||} = 4\pi G\rho_{DM}\frac{\delta}{\dot{\delta}}\mathbf{u}_{||}.$

$$\delta = 2B/3 + B(a/a_{eq})$$

$$V_{||}^2 = \mathbf{u}_{||}^2 a^2 = C(1 + a/a_{eq})$$

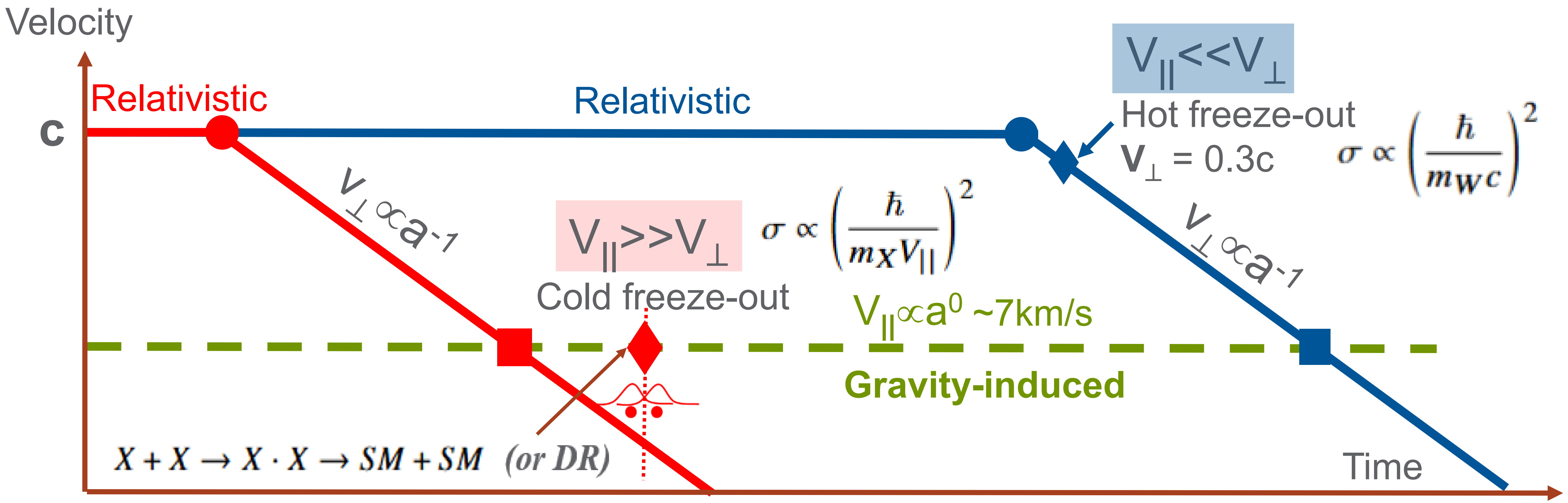


Known as the Mészáros effect



This is model-independent (Important and New)

Cold Freeze-out of Nonthermal X Particles Vs. WIMPs (Hot)



WIMPs (100GeV) velocity evolution (blue):

- Become nonrelativistic much later.
- Semi-relativistic at freeze-out.
- Gravity neglected at freeze-out.

Point-like particles.
Compton wavelength is small.
High-energy scattering.

$$\lambda_c = \frac{\hbar}{m_X c}$$

Superheavy X particles 10^{12} GeV (red):

- Become nonrelativistic much earlier.
- Extremely cold at freeze-out
- The smallest structure (Newtonian Quantum Gravity)
- Cannot neglect gravity

Non-point-like (finite size).
Low-energy scattering.

De Broglie wavelength

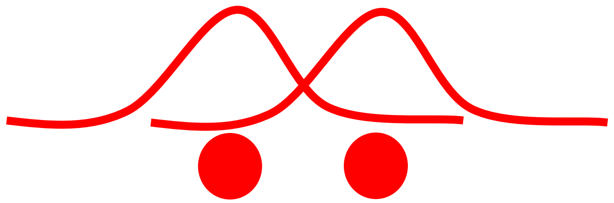
$$\lambda_d = \frac{\hbar}{m_X V_{||}}$$

Freeze-out of Nonthermal X Particles (Cold) Vs. WIMPs (Hot)

I. Initial stage with constant initial overproduced density $\rho_{ini} >> \rho_{\infty}$ (relic) (Out-of-equilibrium)

II. "Cold" freeze-out by gravity

$$X + X \to X \cdot X \to SM + SM \quad (or\ DR)$$



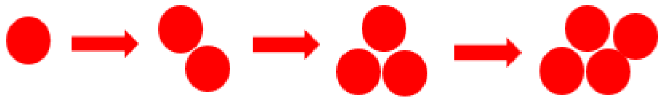
Overlapping wavefunction

Accelerated Particle annihilation/decay

(Relevant to Hubble tension)

III. Relic density and structure formation

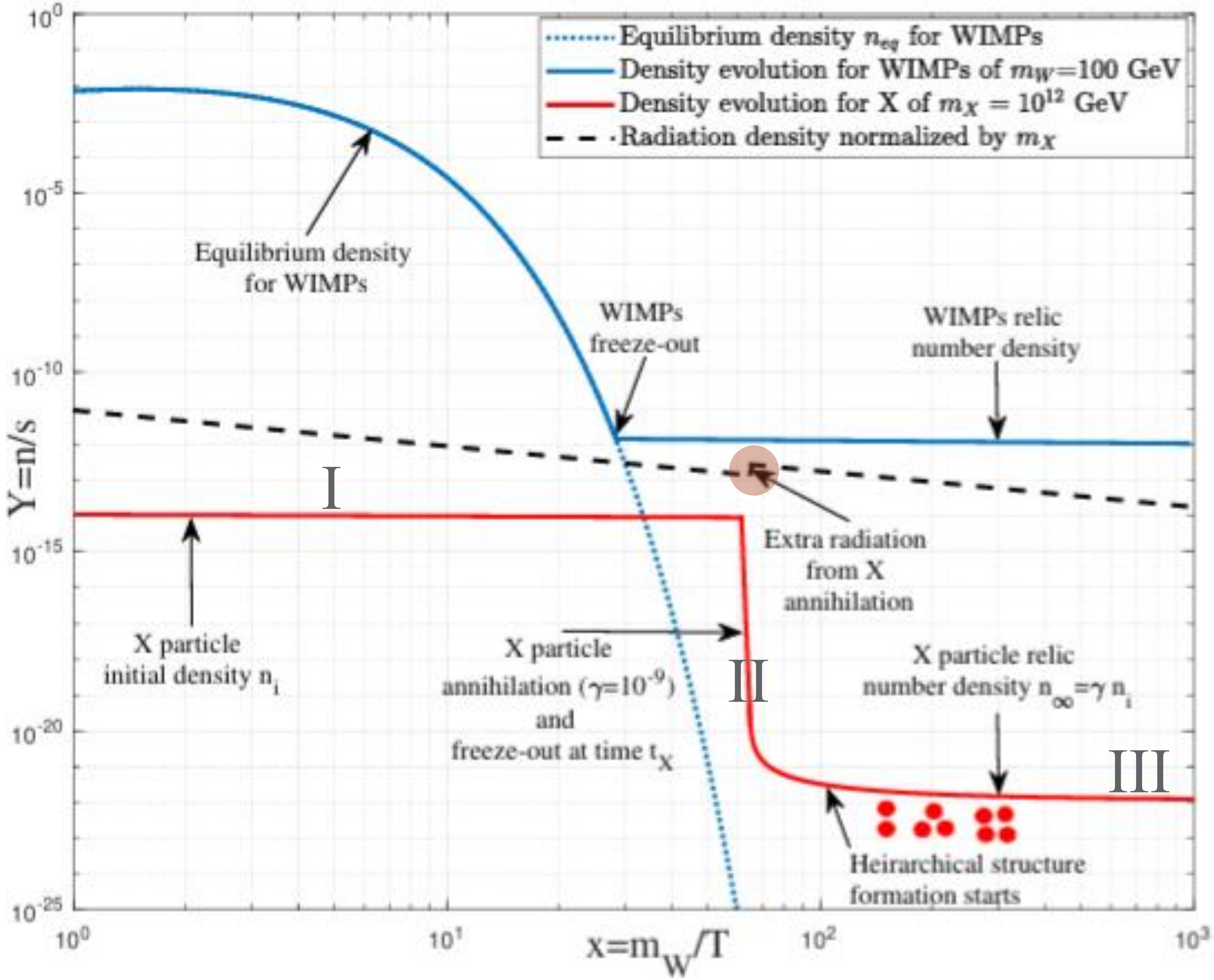
Continuous



$X + X \to SM + SM$ Annihilation

$X \to SM$ Decay

→ UHECRs



Time evolution of WIMP and X particle Density

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WIMP Miracle and Unitarity Bound

Boltzmann Equation: $$\frac{dn}{dt} = -3Hn - \langle \sigma_W v \rangle \left(n^2 - n_{eq}^2\right)$$

Freeze out: interaction rate ~ expansion

$$n_f \langle \sigma_W v \rangle \sim H_f$$

↓

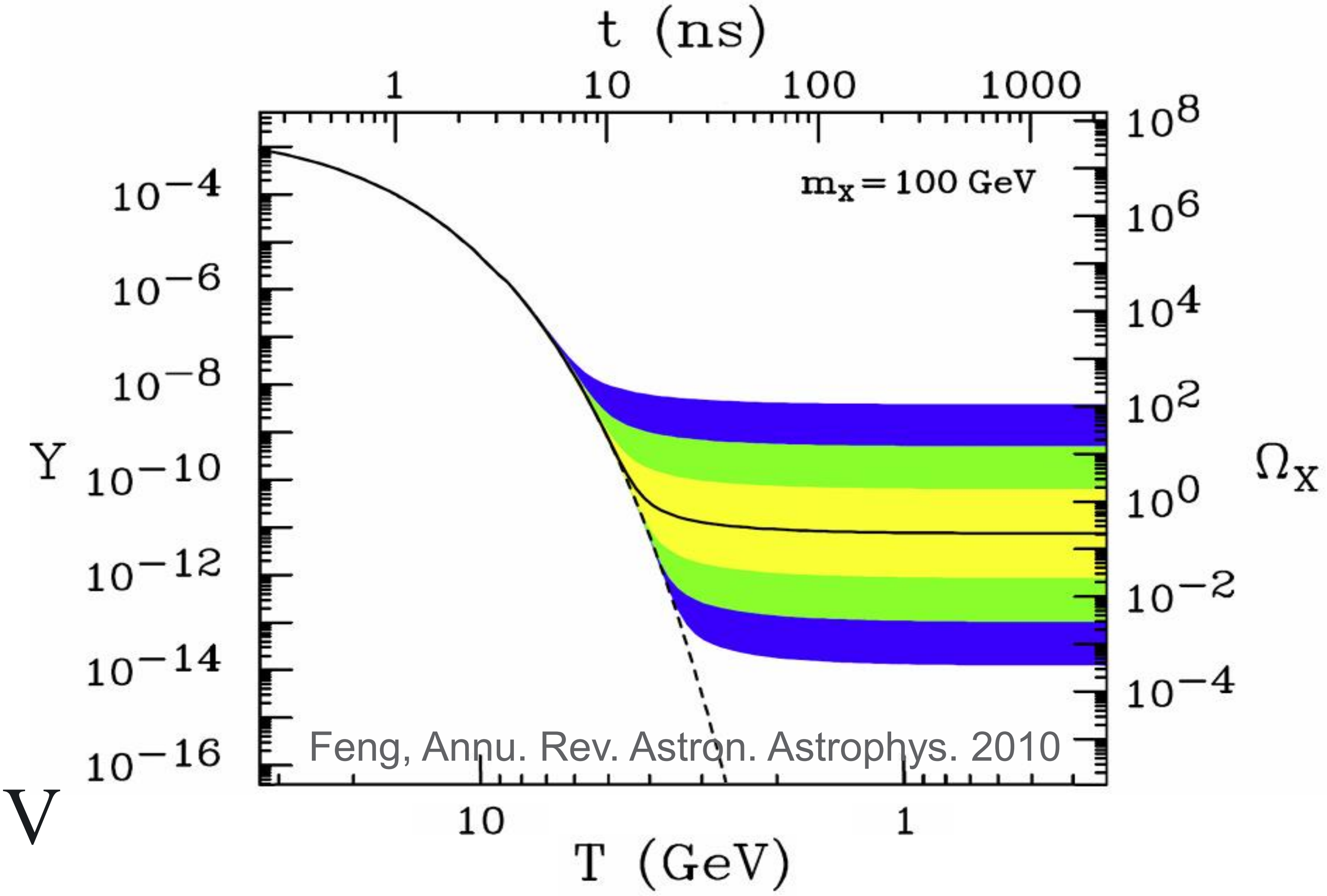
$$\langle \sigma_W v \rangle \approx 3 \times 10^{-32} \frac{m^3}{s}$$

\+ → m_W~100 GeV

Weak interaction

$$\langle \sigma_W v \rangle \propto \left(\frac{\hbar}{m_W c}\right)^2 \frac{g_w^4}{16\pi^2} c$$

Unitarity Bound:
m_W < 100 TeV



Feng, Annu. Rev. Astron. Astrophys. 2010

- **Thermal relics**
- **Gravity is neglected**
- **Point-like and high-energy**

arXiv:2202.07240

From WIMP Miracle to “X Miracle” for Nonthermal Heavy X

$$W+W \rightleftarrows SM+SM$$

Boltzmann Equation:

$$\frac{dn}{dt} = -3Hn - \langle \sigma_W v \rangle \left(n^2 - n_{eq}^2 \right) \qquad n_{eq} \propto e^{-m/T}$$

Freeze out: $n_f \langle \sigma_W v \rangle \sim H_f$

Thermal Equilibrium

$$\langle \sigma_W v \rangle \sim \frac{H_f}{n_f} \sim \frac{m_W a_f^3 H_f}{\bar{\rho}_0 \Omega_{DM}}$$

$$\langle \sigma_W v \rangle \sim \frac{k_B T_{\gamma 0}}{c^2 M_{pl}} \cdot \frac{x_f k_B T_{\gamma 0}}{c^2 \bar{\rho}_0 \Omega_{DM}} \cdot \frac{k_B T_{\gamma 0}}{\hbar}, \quad \boxed{x_f = \frac{m_W c^2}{k_B T_{\gamma f}} \approx 20}$$

Point-like particles;
No Gravity;

$$\langle \sigma_W v \rangle \propto \left(\frac{\hbar}{m_W c} \right)^2 c$$

$m_W \sim 100$ GeV

$$\langle \sigma_W v \rangle \approx 3 \times 10^{-32} \frac{m^3}{s}$$

$X+X \rightarrow SM+SM$ or $X+X \rightarrow DR$?

$$\frac{dn}{dt} = -3Hn - \langle \sigma_X V_{||} \rangle n^2$$

No equilibrium n_{eq}

$$t_X(m_X) = \frac{2^{7/2} \pi \hbar^3}{G^2 (m_X)^5}$$

$$r_X = \frac{4\hbar^2}{G m_X^3}$$

Newtonian Quantum Gravity

$$n_f \langle \sigma_X V_{||} \rangle \sim H_X$$

$$\langle \sigma_X V_{||} \rangle \sim \frac{1}{n_f t_X} \sim \frac{m_X}{\bar{\rho}_0 \Omega_{DM}} \frac{a_X^3}{t_X}$$

$$\langle \sigma_X V_{||} \rangle \sim \frac{k_B T_{\gamma 0}}{c^2 M_{pl}} \cdot \frac{x_f^* k_B T_{\gamma 0}}{c^2 \bar{\rho}_0 \Omega_{DM}} \cdot \frac{k_B T_{\gamma 0}}{\hbar}, \quad \boxed{x_f^* = \frac{m_X c^2}{k_B T_X} \propto m_X^{-3/2}}$$

Finite size $\langle \sigma_X V_{||} \rangle = \pi r_X^2 V_{||}$

Involving Gravity

$$m_X = \left(\frac{9 \cdot 2^{3/2}}{64\pi} \frac{H_0 \Omega_{DM}^2}{\Omega_{rad}^{3/2}} \frac{V_{||}^2 \hbar^5}{G^4} \right)^{1/9} \approx m_{Xc} \sim 10^{12} GeV$$

$$\langle \sigma_X V_{||} \rangle = 3 \times 10^{-22} \mathrm{m}^3/\mathrm{s}$$

Ten orders larger than WIMP’s cross-section

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"X Miracle" for Nonthermal Particle Dark Matter

Freeze out: $n_f \langle \sigma_X V_{||} \rangle \sim H_X$

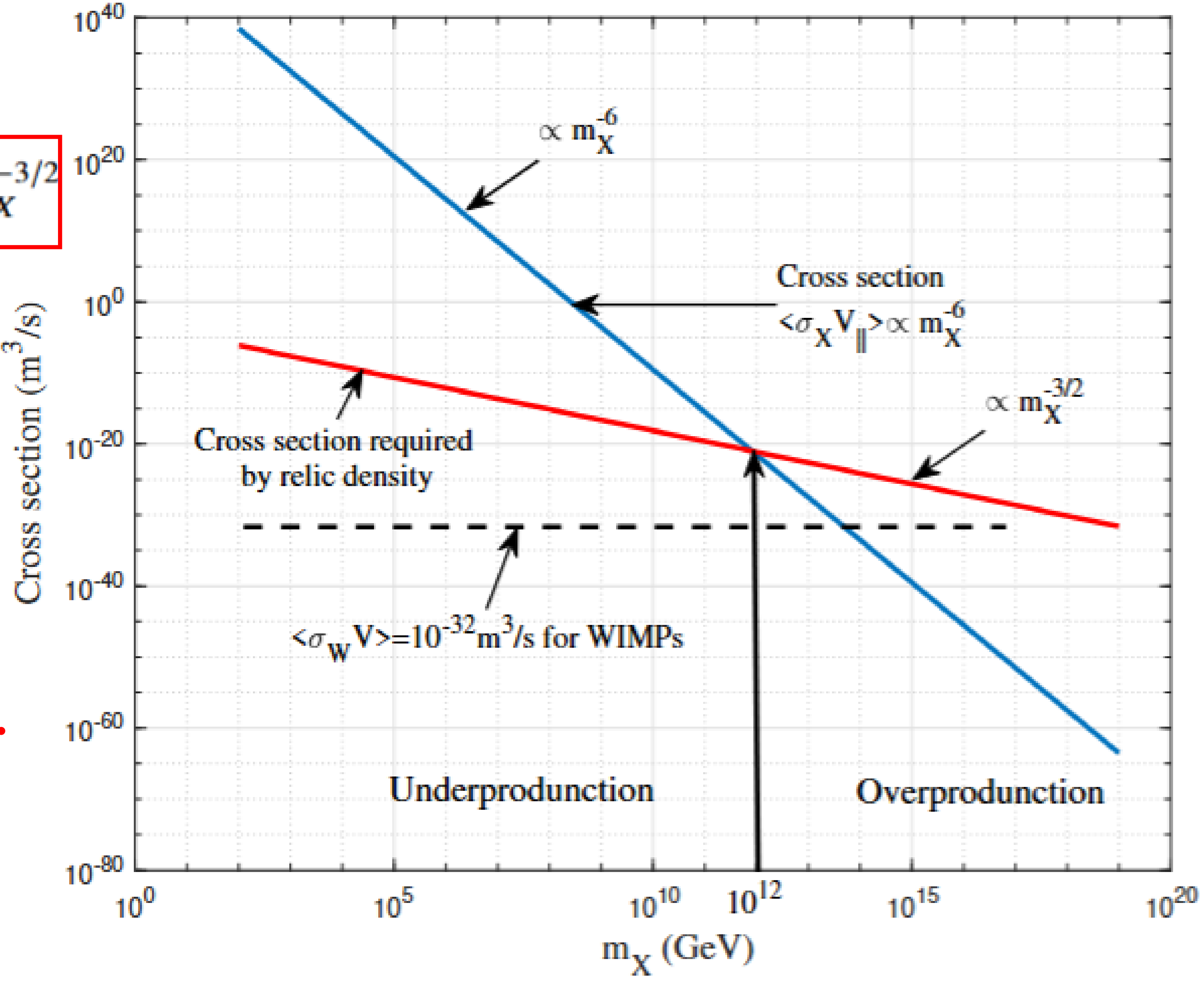
$$\underbrace{\boxed{\langle \sigma_X V_{||} \rangle}}_{\text{Blue Line}} \sim \underbrace{\boxed{\frac{k_B T_{\gamma 0}}{c^2 M_{pl}} \cdot \frac{x_f^* k_B T_{\gamma 0}}{c^2 \bar{\rho}_0 \Omega_{DM}} \cdot \frac{k_B T_{\gamma 0}}{\hbar}}}_{\text{Red Line}}, \quad \boxed{x_f^* = \frac{m_X c^2}{k_B T_X} \propto m_X^{-3/2}}$$

Particle size: $r_X = \frac{l_X}{2} = \frac{4\hbar^2}{G(m_X)^3}$

Cross section: $\sigma_X \propto r_X^2$

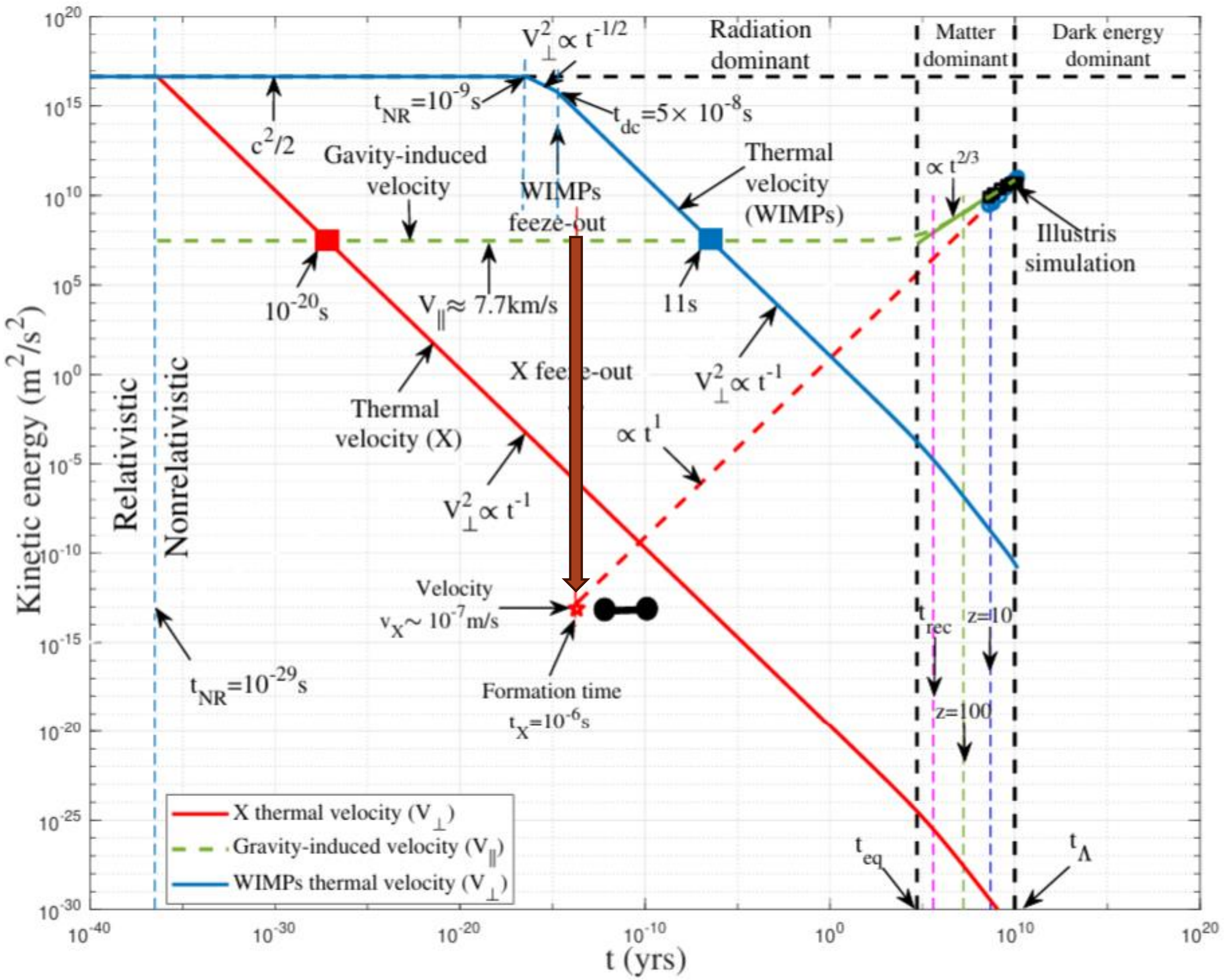
X miracle for nonthermal relics:
Gravitational interaction and particle mass of 10^{12} GeV also yield the correct relic abundance.

WIMP miracle for thermal relics:
Weak-scale interaction and particle mass of 100 GeV yield the correct relic abundance.

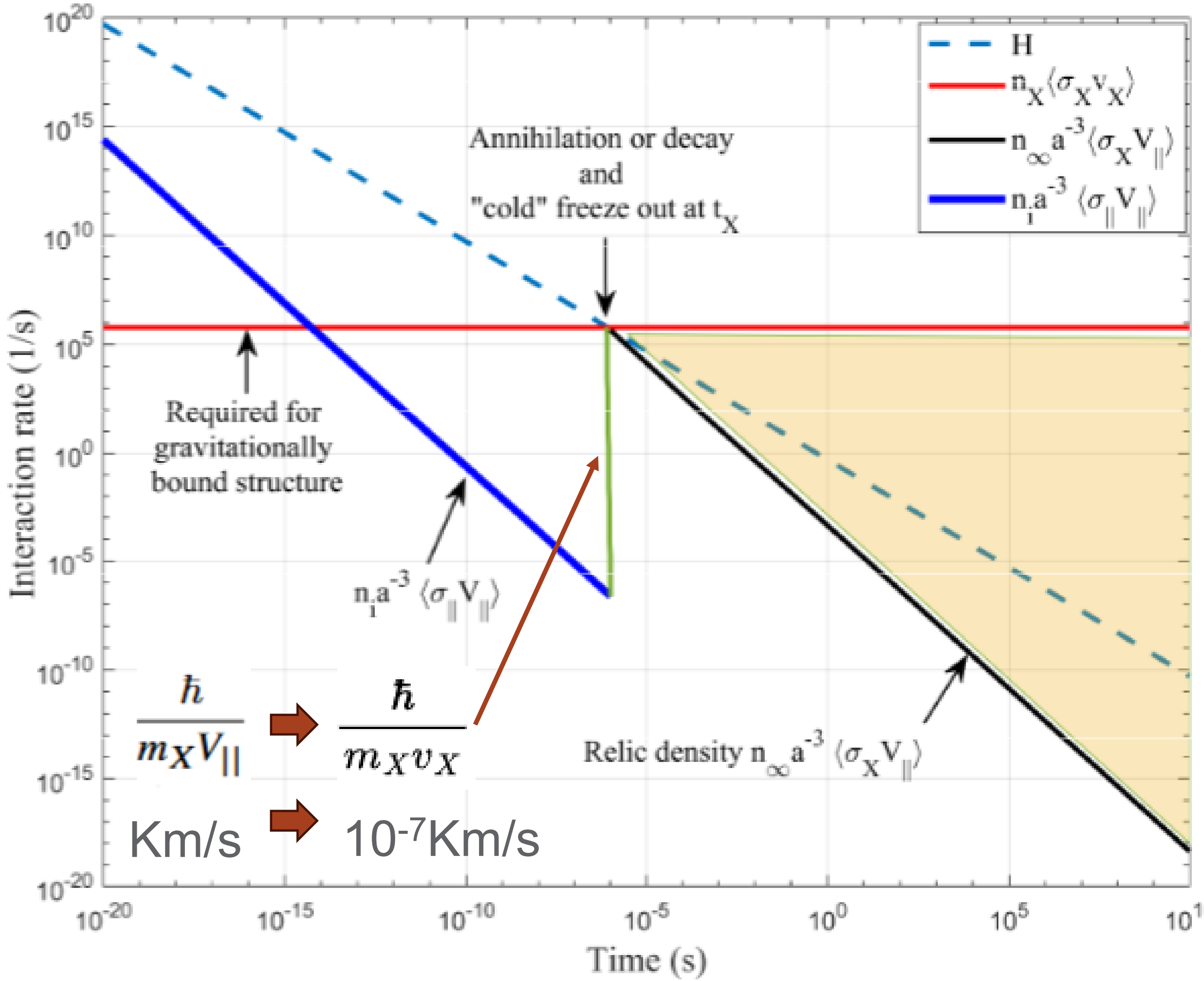


Cross Section vs. Particle Mass

Velocity Evolution and Interaction Rate $\Gamma(t)$ Vs. H



Time Evolution of Particle Velocity



Interaction Rates $n_\infty\langle\sigma_X V_\|\rangle$ and Hubble Parameter H

Extra Radiation: Analytical Solutions of the Boltzmann Equation

$X + X \rightarrow SM + SM$ or $X + X \rightarrow DR$

Boltzmann Equation: $$\frac{dn}{dt} = -3Hn - \langle \sigma_X V_{||} \rangle n^2$$

Analytical Solution:

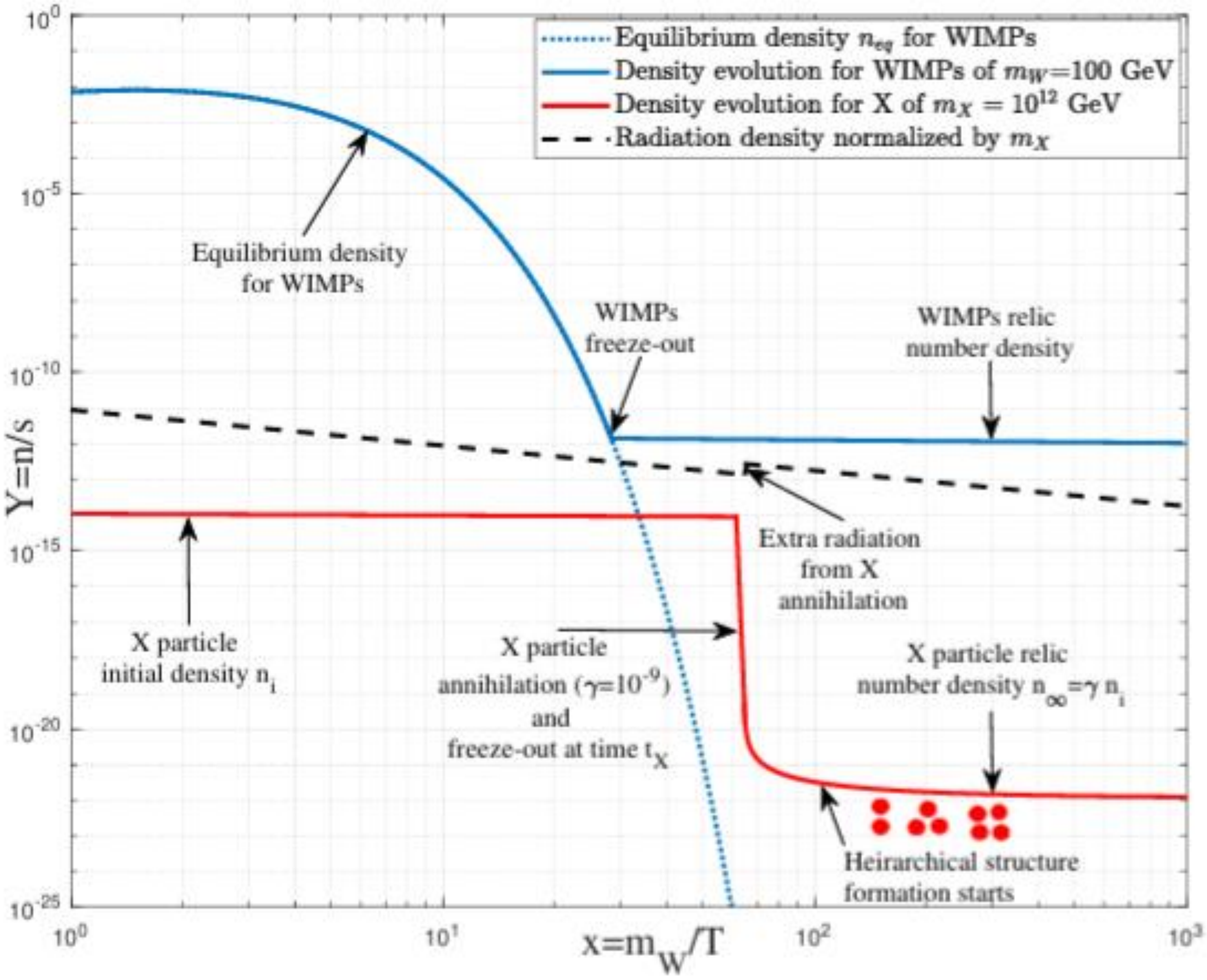
$$\frac{na^3}{n_\infty} = \frac{1}{1-(1-\gamma)\frac{a_i}{a}} \qquad \gamma = \frac{n_\infty}{n_i}$$

$$\gamma \propto \left(\frac{G^{16} m_X^{34}}{\hbar^{18} H_0^2 V_{||}^{12}} \cdot \frac{\Omega_{rad}^3}{\Omega_{DM}^4} \right)^{1/8} \approx 10^{-9}$$

The analytical solution suggests roughly **one in every billion** X particles survived as the relic dark matter today. The rest are converted into radiation.

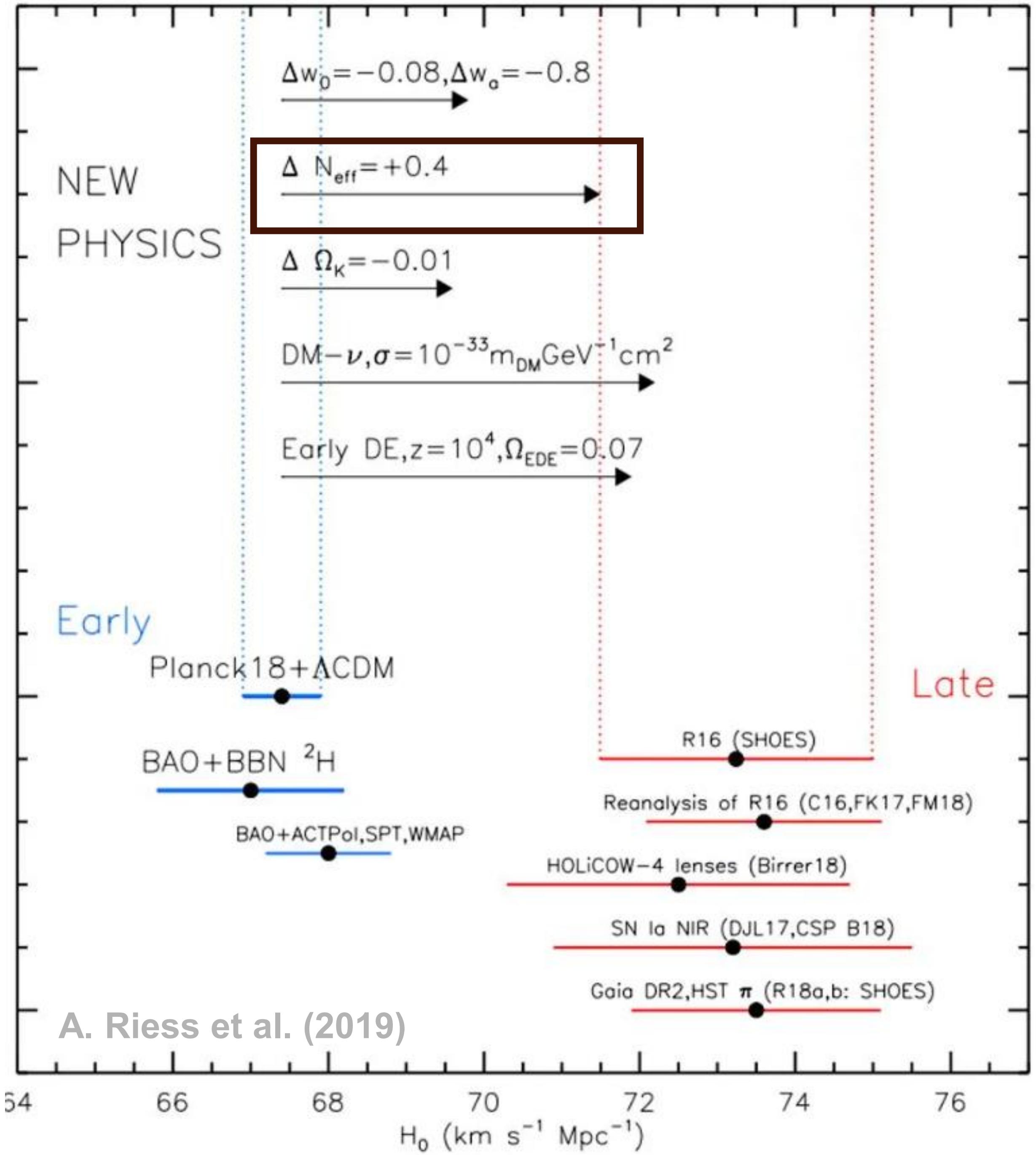
$$\delta_r \approx 0.05$$

5% Extra Radiation than **ΛCDM**

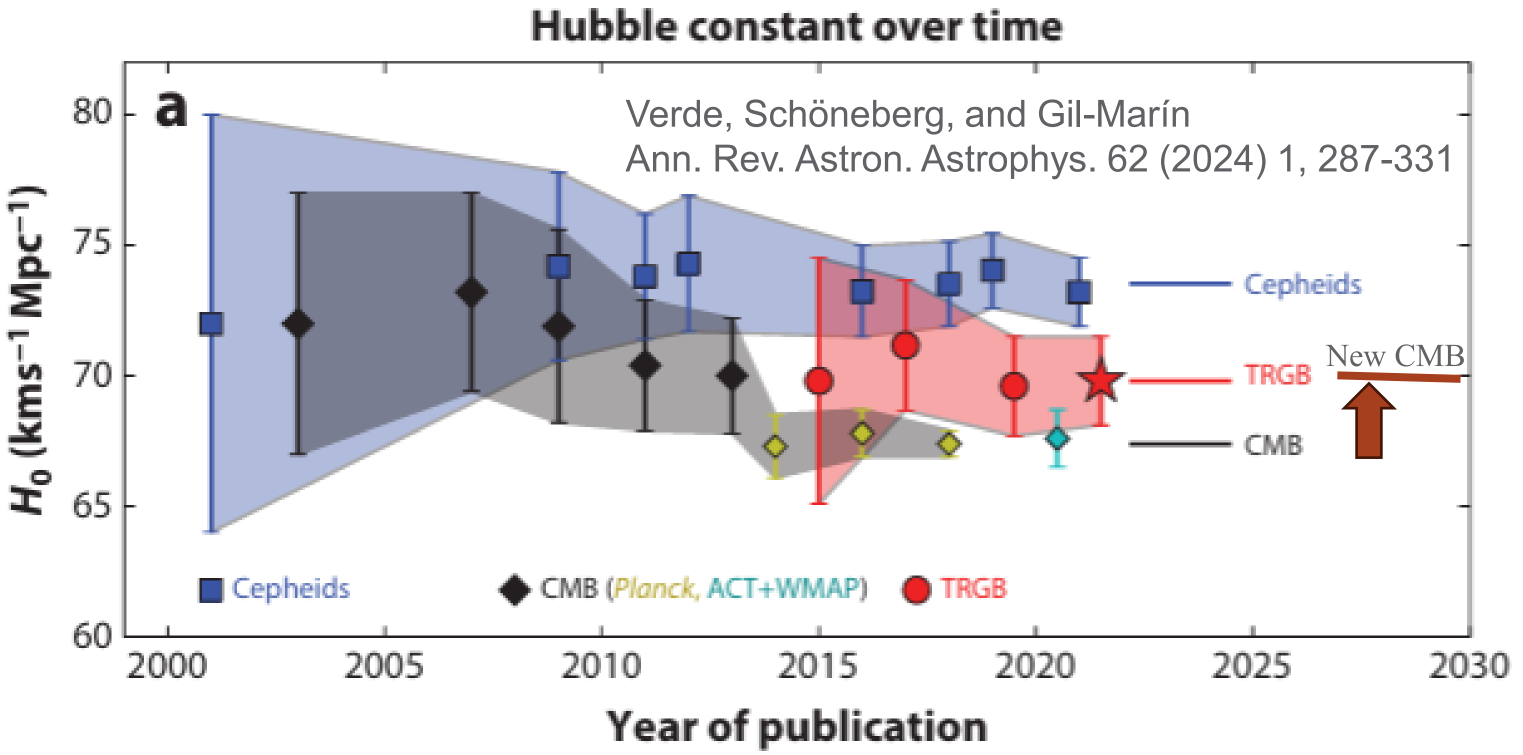


Time evolution of X particle Density

Extremely Early Extra Radiation Alleviating Hubble Tension



A. Riess et al. (2019)



Verde, Schöneberg, and Gil-Marín
Ann. Rev. Astron. Astrophys. 62 (2024) 1, 287-331

$X + X$ → **Radiation (at ~10^{-6}s)**

Extremely Early Extra Radiation compared to ΛCDM: $\delta_r \approx 0.05$

Relativistic DOF: $\Delta N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \delta_r \frac{\Omega_{rad}}{\Omega_\gamma} \approx 0.4$

Change in H_0: **67 km/s/Mpc to 71 km/s/Mpc**

UHERCs From Annihilation of Superheavy DM (10^{12}GeV)

If $X + X \rightarrow SM + SM$

$$\langle \sigma_X V_{||} \rangle = 3 \times 10^{-22} \mathrm{m}^3/\mathrm{s}$$

Annihilation Rate:

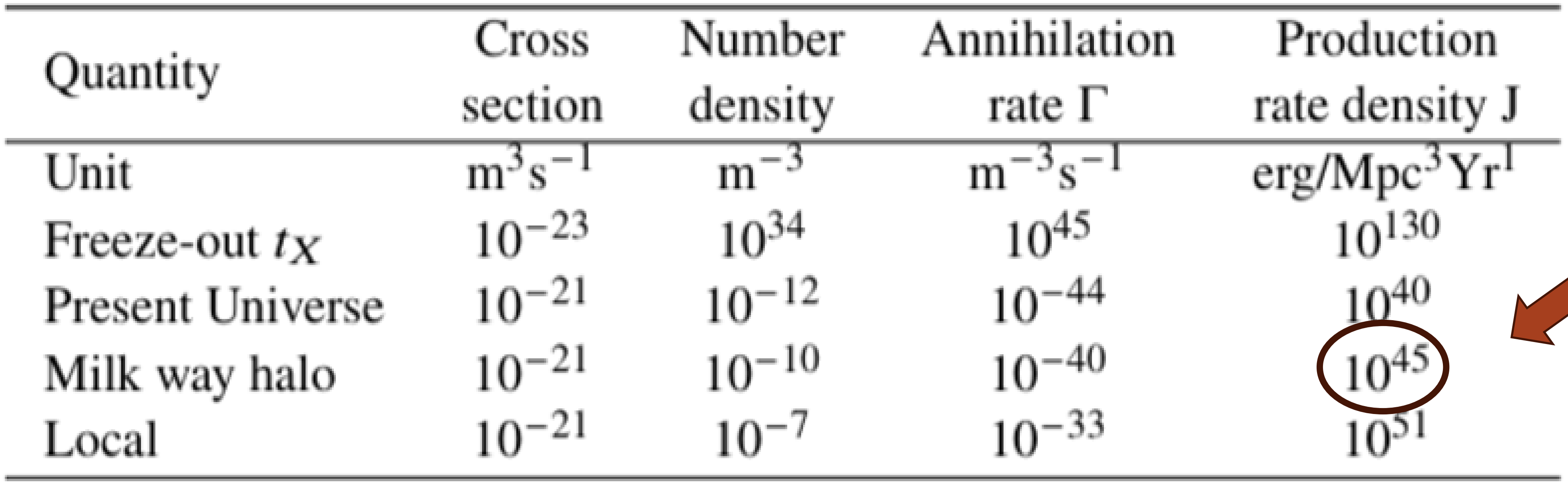
$$\Gamma(t) = n_\infty^2 a^{-6} \langle \sigma_X V_{||} \rangle$$

Particles may continuously annihilate after freeze-out, but at a much lower rate and produce ultra-high energy cosmic rays (UHECRs) as indirect signatures of dark matter.

Energy production rate density:

$$J = m_X c^2 \cdot \Gamma(t)$$

Table 2. Cross section, annihilation rate, and UHECR production rate



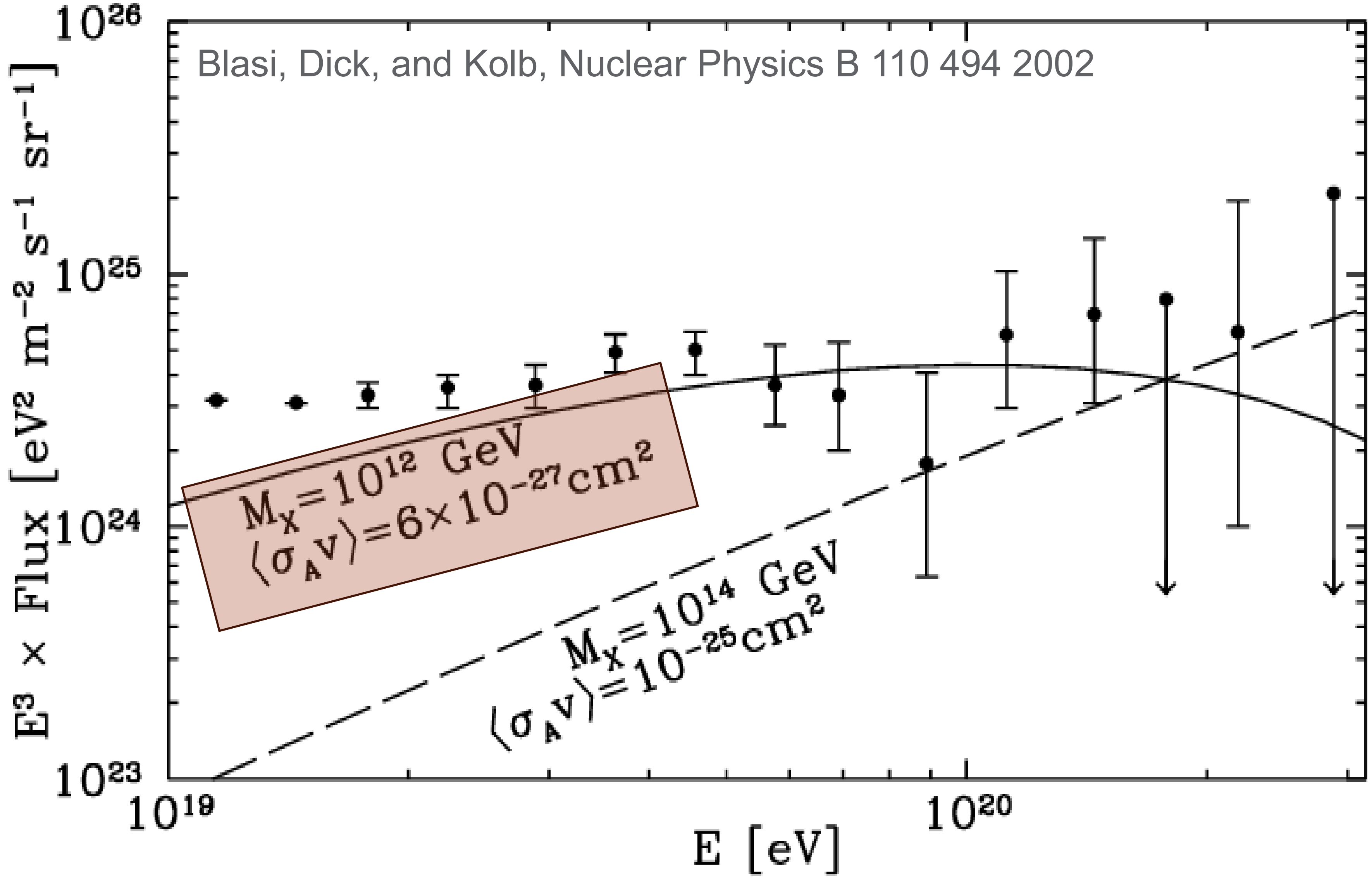
Quantity	Cross section	Number density	Annihilation rate Γ	Production rate density J
Unit	$\mathrm{m}^3\mathrm{s}^{-1}$	m^{-3}	$\mathrm{m}^{-3}\mathrm{s}^{-1}$	$\mathrm{erg/Mpc}^3\mathrm{Yr}^1$
Freeze-out t_X	10^{-23}	10^{34}	10^{45}	10^{130}
Present Universe	10^{-21}	10^{-12}	10^{-44}	10^{40}
Milk way halo	10^{-21}	10^{-10}	10^{-40}	10^{45}
Local	10^{-21}	10^{-7}	10^{-33}	10^{51}

Integrated above 0.63EeV, the total energy production rate is found to be:

10.8×10^{44} erg Mpc^{-3} Yr^{-1}

Quentin Luce et al 2022 ApJ 936 62

UHERCs From Annihilation of Superheavy DM (10^{12}GeV)



UHECR spectra from AGASA (Akeno Giant Air Shower Array) compared to superheavy particle annihilation

The cross section required by the UHECR spectra matched the predicted cross section

$1.8\text{x}10^{-22}$ m^3/s

↕

Our predicted cross-section from X miracle:

$$\langle\sigma_X V_{||}\rangle = 3\times10^{-22}\text{m}^3/\text{s}$$

UHERCs From Decay of Superheavy DM (10^{12} GeV)

The particle decay models are less severely constrained by the observed isotropy of UHECRs.

$$\Lambda_X = m_X v_X^2 = \frac{G^2 m_{Xc}^5}{16\hbar^2}$$

Effective decay energy scale from Newtonian Quantum Gravity

For instanton-induced non-perturbative decay of X particles (similar Eqs. as QCD asymptotic freedom):

$$\alpha_X = \frac{2\pi}{\ln\left(\frac{m_X}{\Lambda_X}\right)} \Rightarrow \Lambda_X = m_X \exp\left(-\frac{2\pi}{\alpha_X}\right) \Rightarrow \tau_X = \frac{\hbar e^{4\pi/\alpha_X}}{m_X c^2}$$

Coupling constant of the hidden gauge interaction

Effective energy scale of the non-perturbative instanton effects

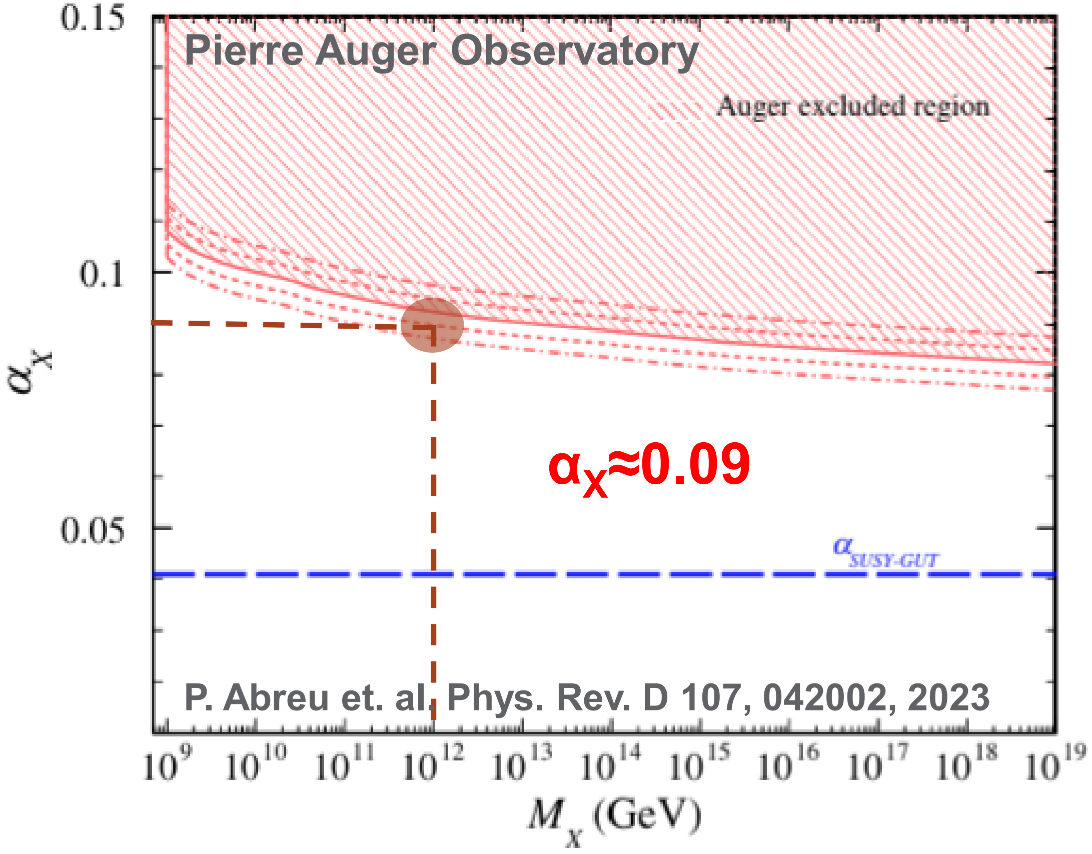
X particle lifetime

Our prediction:

Coupling constant: $\alpha_X = 0.092$

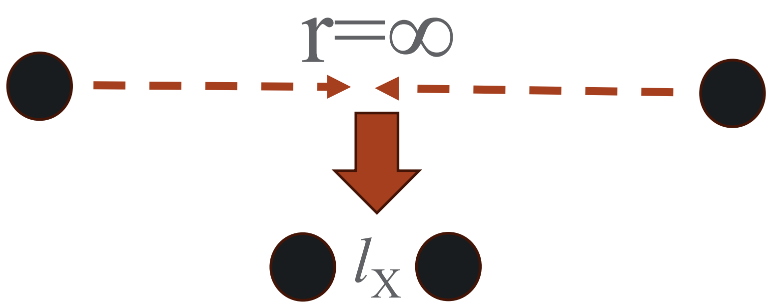
Particle lifetime: $\tau_X = 10^{23}\mathrm{s} = 10^{16}\mathrm{Yrs}$

Constraints from UHERCs



P. Abreu et. al, Phys. Rev. D 107, 042002, 2023

Bonus: Connecting DM Microphysics to Halo Density Profile



Newtonian Quantum Gravity

Size: $$l_X = 2r_X = \frac{8\hbar^2}{G(m_{X_c})^3} \approx 3\times10^{-13} m$$

Velocity: $$v_X = \frac{G(m_{X_c})^2}{4\hbar} \approx 4\times10^{-7} m/s$$

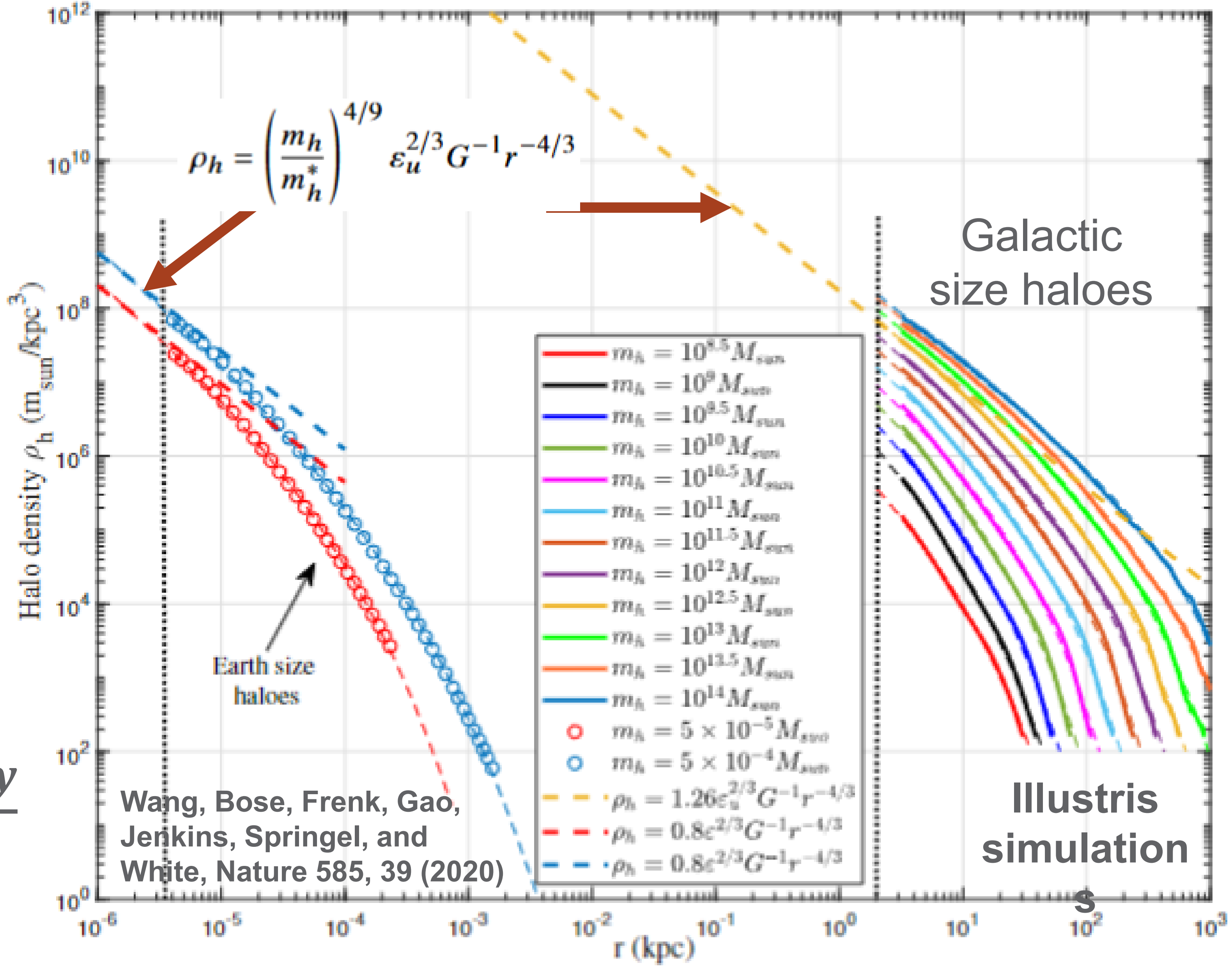
Dark constant: $$-\varepsilon_u \propto \frac{v_X^3}{l_X} = \frac{G^4(m_{X_c}^9)}{512\hbar^5} \approx 10^{-7} m^2/s^3,$$

$$\rho_h = \left(\frac{m_h}{m_h^*}\right)^{4/9} \varepsilon_u^{2/3} G^{-1} r^{-4/3}$$

$$\varepsilon_u: \frac{Energy}{time}$$

This halo density seems to be good from Earth-size to galactic haloes (across 10^{20} orders)

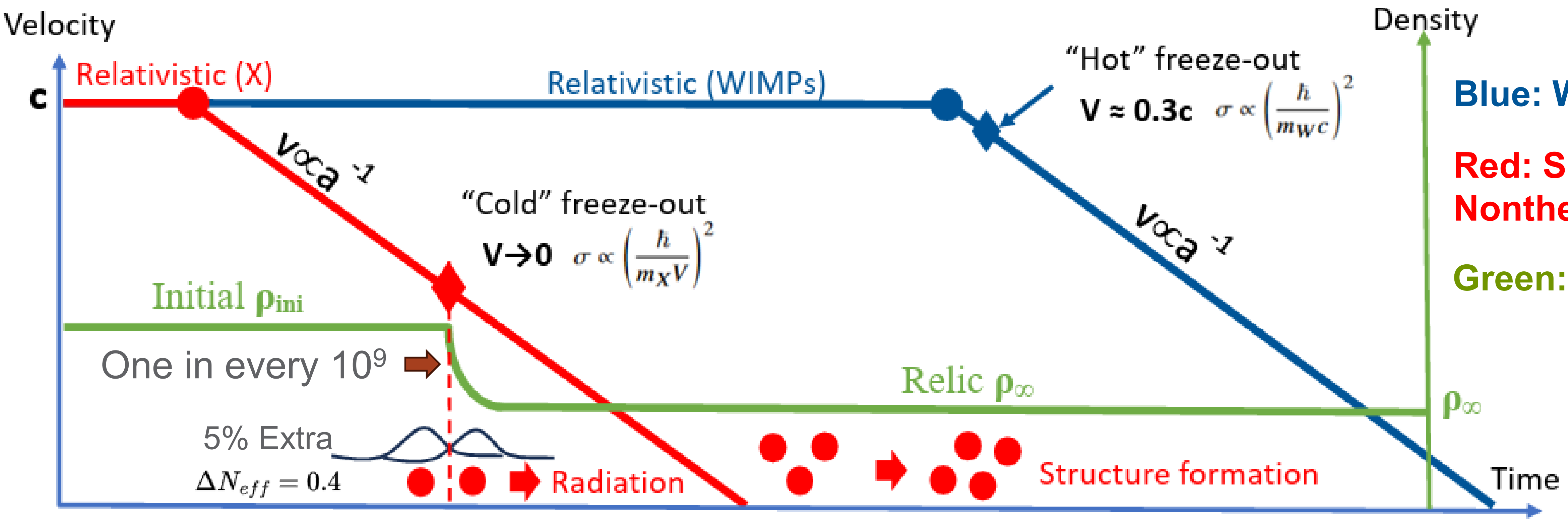
Why?



Wang, Bose, Frenk, Gao, Jenkins, Springel, and White, Nature 585, 39 (2020)

Earth Size To Galactic Halo Density Profiles

Take Away Message



Blue: WIMPs

Red: Superheavy Nonthermal X particles

Green: X particle density

Properties of X particles:

- Superheavy mass 10^{12} GeV
- Cross-section around 10^{-22} m^3/s
- Nonthermal and particle nature
- Fermions following the Pauli Exclusion
- Gravity only

Evolution of X particles:

- Become nonrelativistic early
- "Cold" freeze-out at 10^{-6}s (X miracle)
- Annihilation/decay into extra radiation
- Extremely early extra radiation for the Hubble tension
- Annihilation/decay into UHECRs