Thermal QCD axion production from the early universe

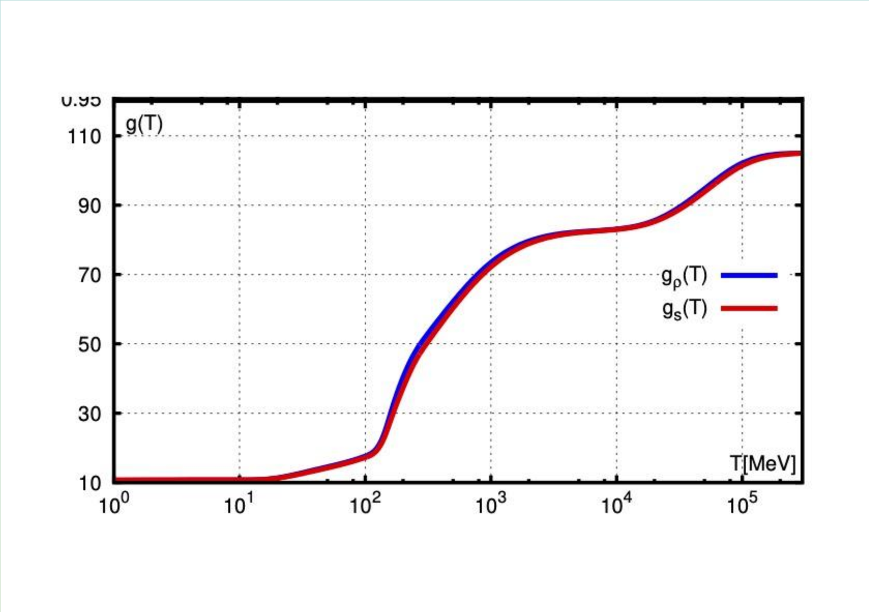
Alessio Notari

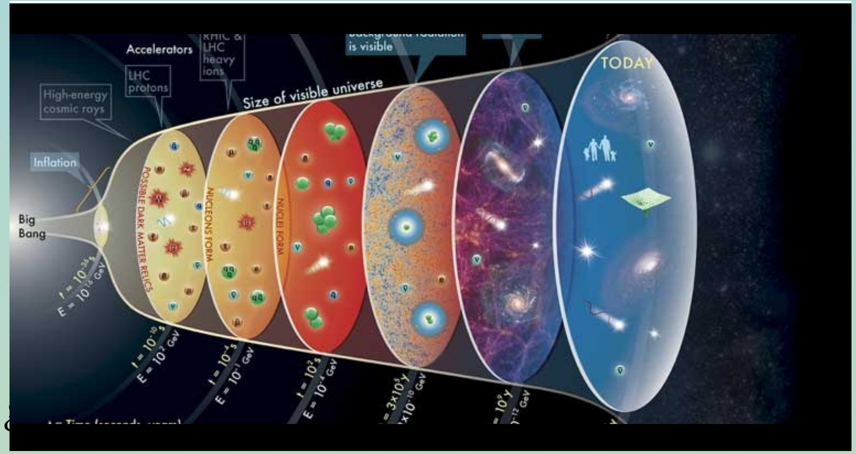
(Universitat de Barcelona,
On leave at : Galileo Galilei Institute, Firenze, Italy)

Phys.Rev.Lett. 131 (2023) 1, 011004, with F. Rompineve and G.Villadoro

Relic light particles in Cosmology

- Primordial plasma, g_* degrees of freedom and temperature T





Total plasma energy density: $\rho_{\text{TOT}} \propto g_* T^4$

$$g_* \equiv \sum_{i=\text{RELATIVISTIC BOSONS}} g_i + \frac{7}{8} \sum_{i=\text{RELATIVISTIC FERMIONS}} g_i$$

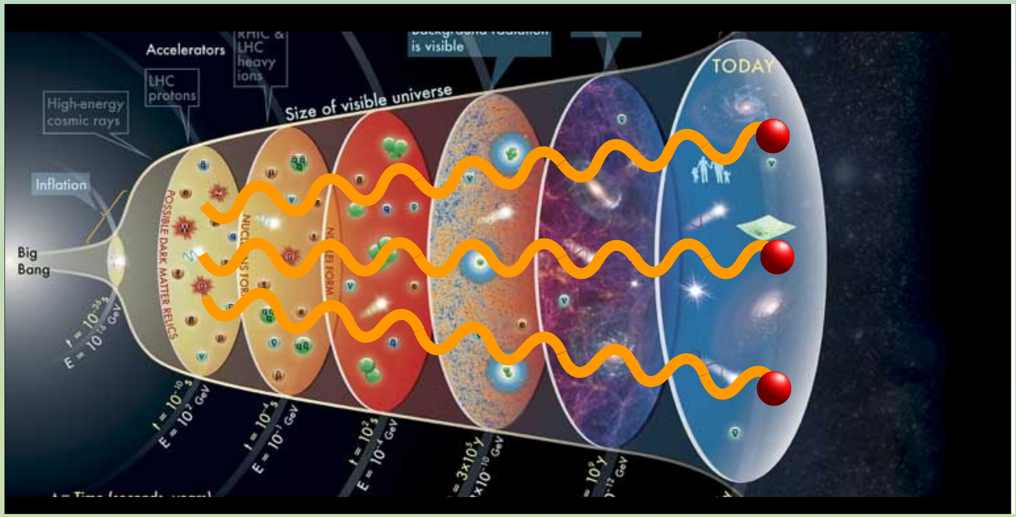
- Conservation of entropy: $g_*^{1/3} T \propto 1/a$
- When a species becomes non-relativistic (e.g. $e^+ - e^-$ at $T \ll m_e$) g_* decreases
  T slightly “increases” (photons get slightly “heated”)

Relic light particles in Cosmology

- Light particles with small interaction ("thermalization rate" Γ), (e.g. neutrinos, axions)

- Compare with Hubble rate H ⇨ Decoupling

- If Particle Decouples below some Temperature $T_{\rm DEC}$, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\rm DEC}/a$



Relic light particles in Cosmology

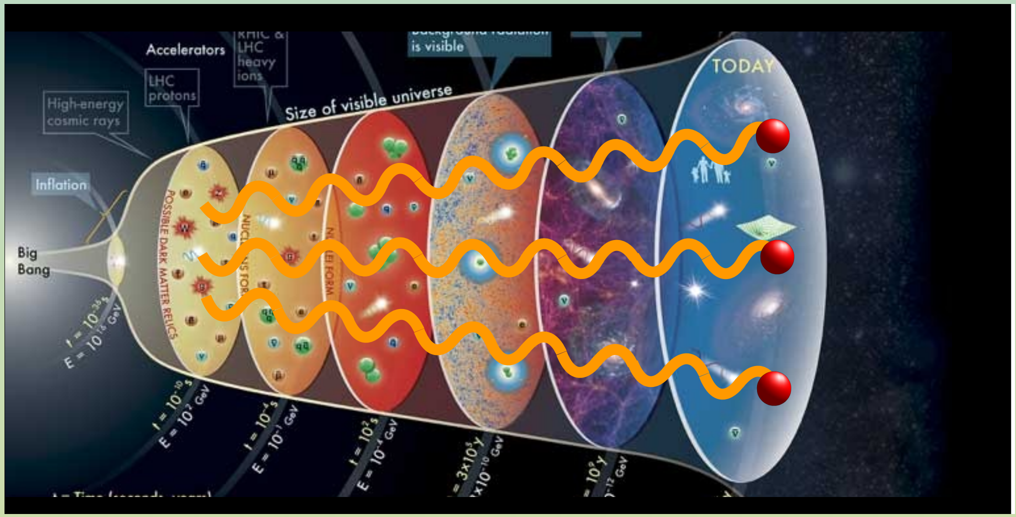
- Light particles with small interaction ("thermalization rate" Γ), (e.g. neutrinos, axions)

- Compare with Hubble rate ($H \equiv \dot{a}/a$)

- If Particle Decouples below some Temperature $T_{\rm DEC}$, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\rm DEC}/a$

- Compared to photons it does
  not get heated after decoupling

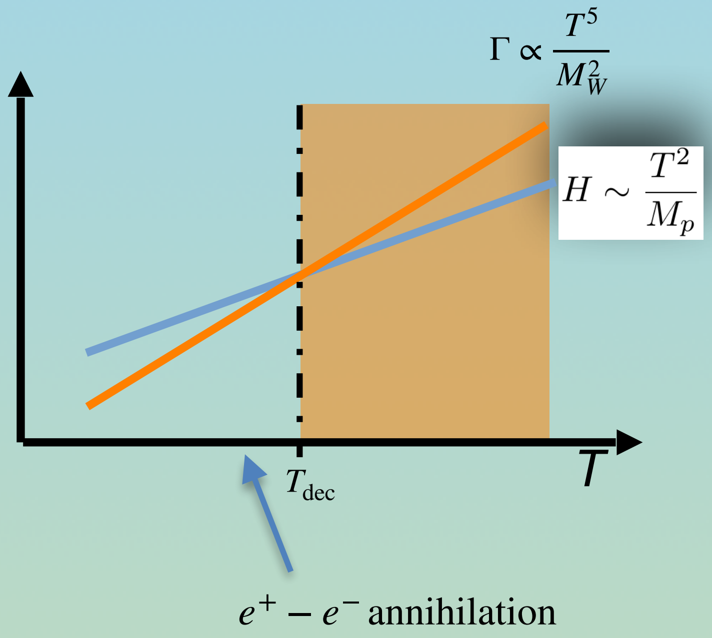
$$\rho_P/\rho_\gamma \propto T_P^4/T^4 \propto 1/g_{*DEC}^{4/3}$$

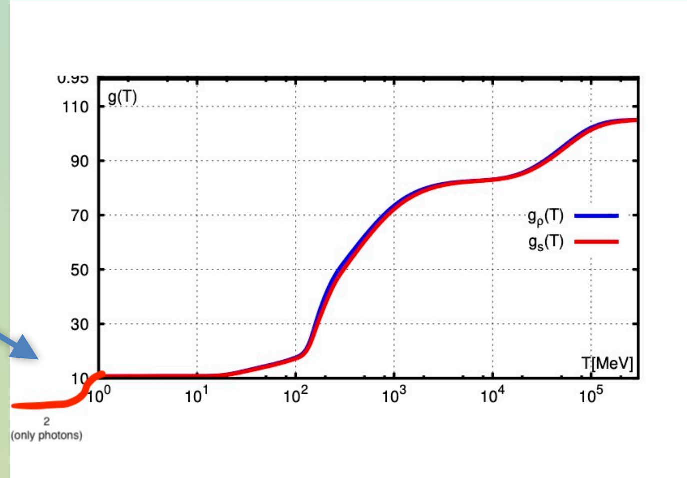


Example: Relic Neutrinos

- Neutrinos decouple at $T \approx MeV$

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{1}{g_{*,DEC}^{4/3}} = \left(\frac{4}{11}\right)^{4/3}, \qquad T_\nu \approx 0.7\, T_\gamma \approx 1.96\,\mathrm{K}$$

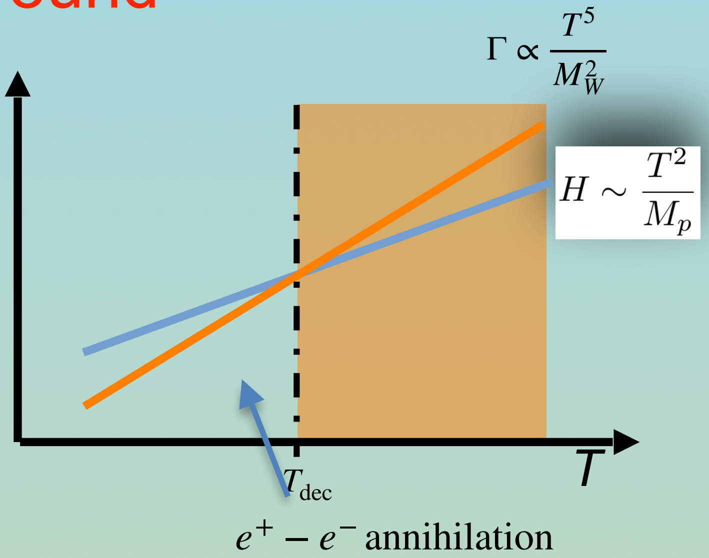




Example: Cosmic Neutrino Background

- Neutrinos decouple at $T \approx MeV$

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{1}{g_{*,DEC}^{4/3}} = \left(\frac{4}{11}\right)^{4/3}, \qquad T_\nu \approx 0.7\, T_\gamma \approx 1.96\,\mathrm{K}$$



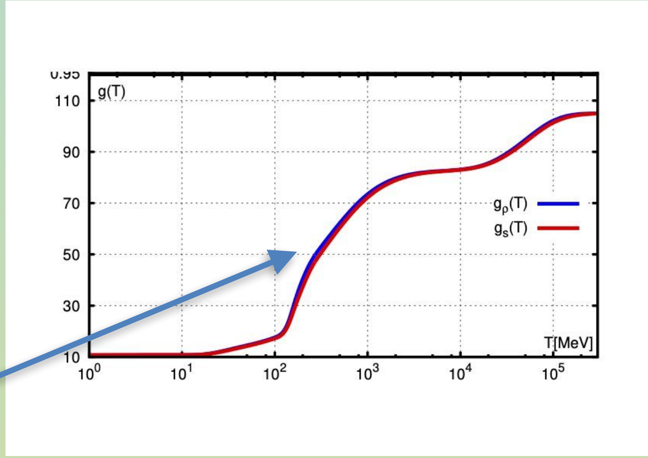
- Any light particle (axions,…) can do the same.

- Traditional parameterization as "extra neutrinos species":

$$\Delta N_{\rm eff} \equiv \left(\frac{8}{7}\right)\left(\frac{11}{4}\right)^{4/3} \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB}$$

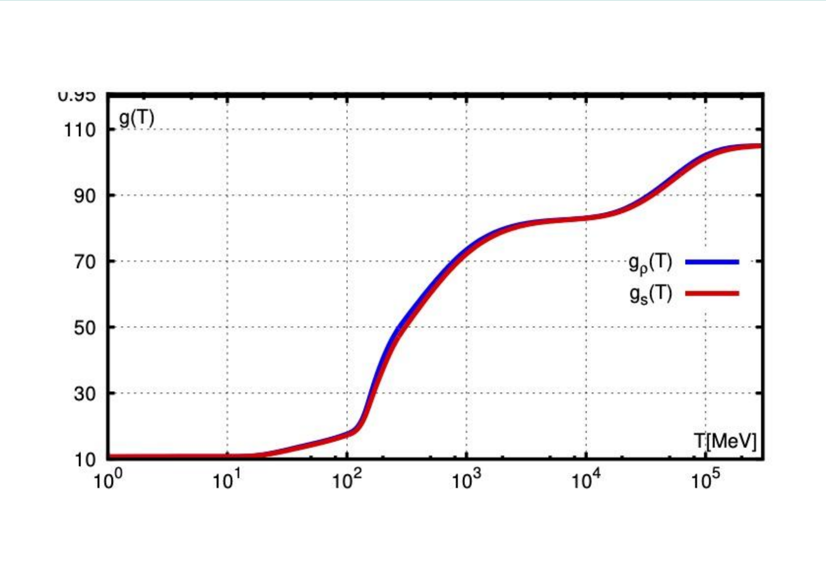
- Relic abundance suppressed as:

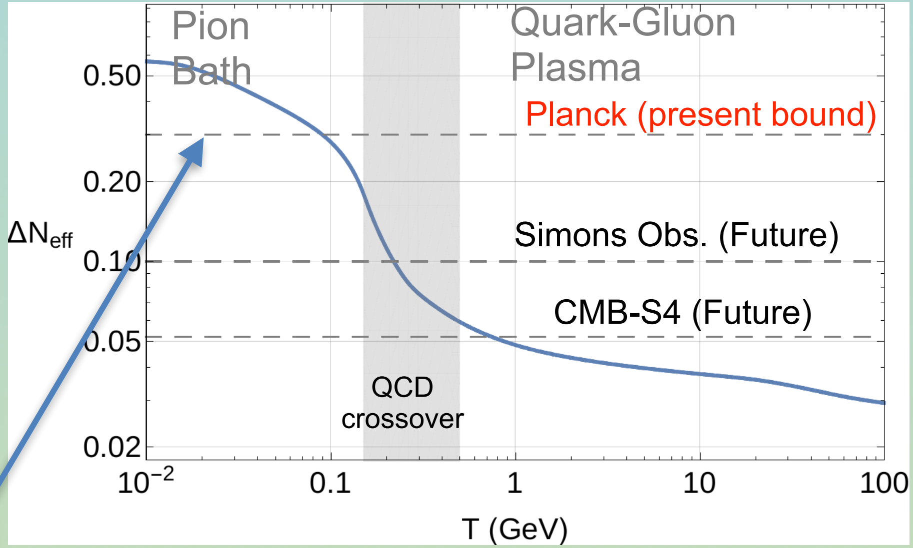
$$\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$$



Example: Relic Scalars

- Relic abundance $\Delta N_{\text{eff}} \propto \frac{\rho_P}{\rho_\gamma}|_{\text{CMB}} \propto \frac{1}{g_{*,DEC}^{4/3}}$

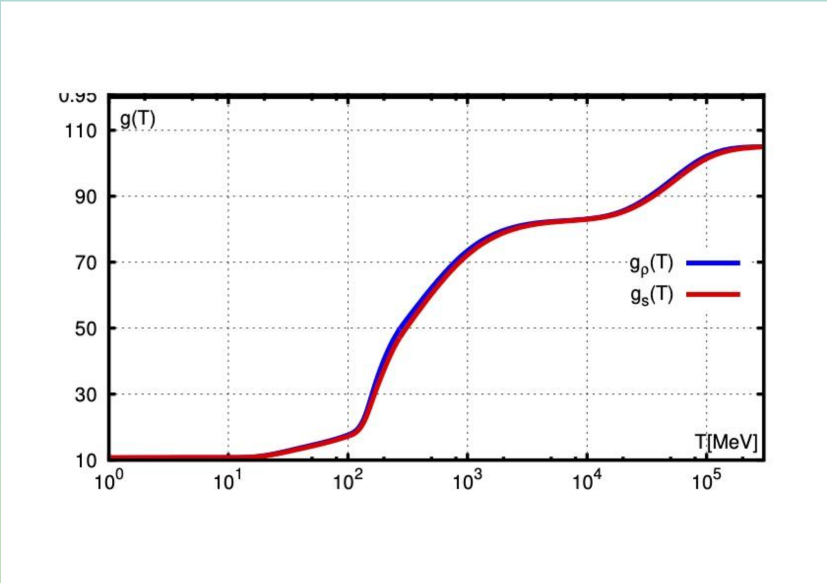


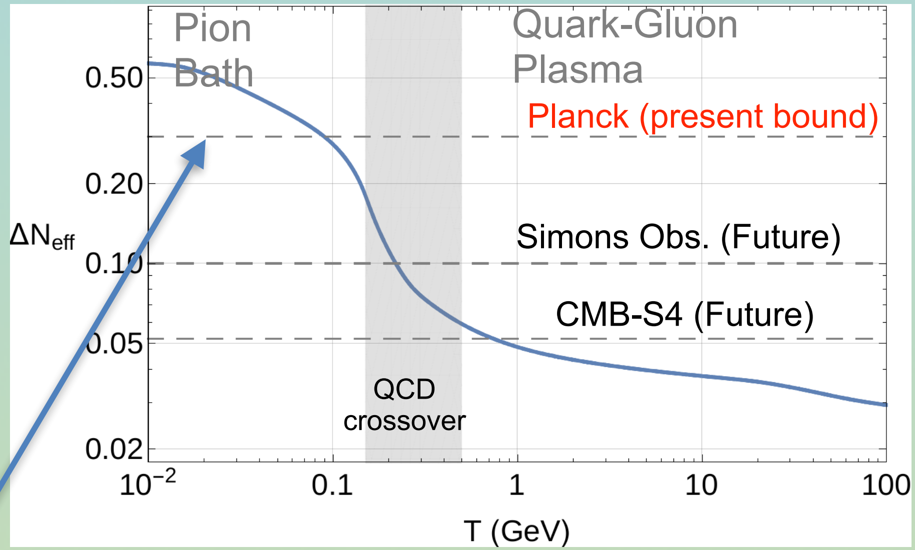


- Main effect: Extra "radiation" at CMB time ($T \approx 0.1 eV$) ⇨ ΔN_{eff} affects CMB spectra

Example: Relic Scalars

- Relic abundance $\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$





- Main effect: Extra "radiation" at CMB time ($T \approx 0.1eV$) ⇨ $\Delta N_{\rm eff}$ affects CMB spectra
- If massive ($m \lesssim 0.1\text{eV}$) becomes non-relativistic after CMB time ⇨ adds to Dark Matter and affects its fluctuations (more constrained)

The (Minimal) QCD Axion

$$\mathscr{L}_{\rm SM} \supset \theta_{\rm strong} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation in QCD is tiny $(\bar{\theta}_{\rm strong} \ll 1)$?

The (Minimal) QCD Axion

$$\mathscr{L}_{\mathrm{SM}} \supset \theta_{\mathrm{strong}} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation in QCD is tiny ($\bar{\theta}_{\mathrm{strong}} \ll 1$)?
- QCD Axion solution: promote θ_{strong} to a dynamical field $\rightarrow \dfrac{a}{f_a}$
- *Axion* potential minimized at $a = \bar{\theta}_{\mathrm{strong}} = 0$ (CP conserving)

The (Minimal) QCD Axion

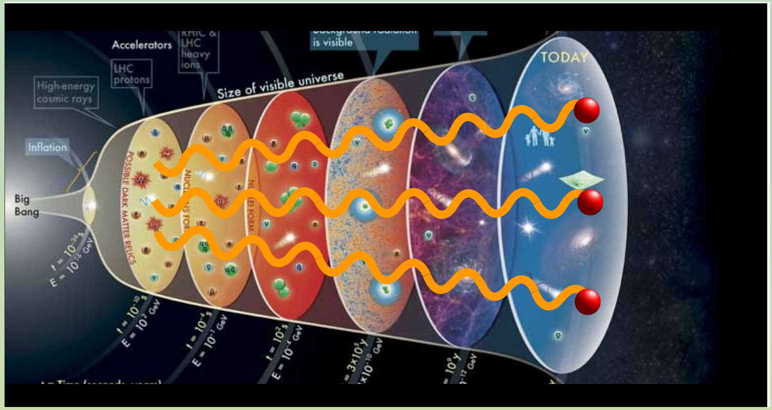
$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- Dynamical explanation of $\theta_{\rm strong} \ll 1$
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2/f_a \approx 0.57eV\left(\frac{10^7 GeV}{f_a}\right)$

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- Dynamical explanation of $\theta_{\text{strong}} \ll 1$
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2/f_a \approx 0.57eV\left(\frac{10^7 GeV}{f_a}\right)$
- Two populations of cosmological relic axions:
  - "Cold axions" candidate for Dark matter (or part of it), not covered in this talk.
  - "Thermal axions": relativistic at production, May become non-relativistic later ⇒ small part of dark matter (like relic neutrinos)



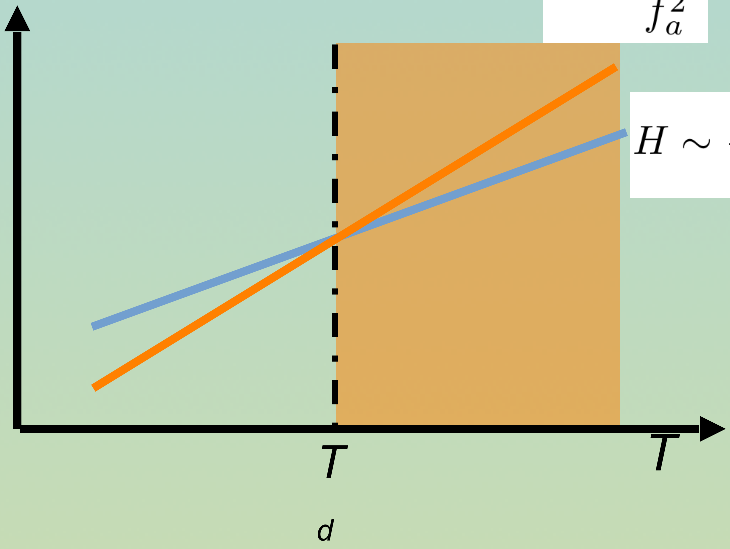
The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

Axion-gluon scatterings ($T \gtrsim T_{QCD}$)

$$\Gamma \sim \frac{T^3}{f_a^2}$$

$$H \sim \frac{T^2}{M_p}$$



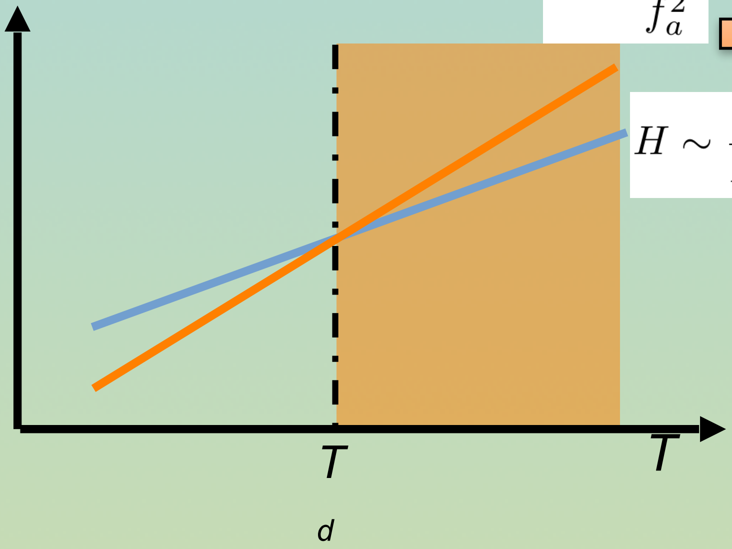
The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

Axion-gluon scatterings ($T \gtrsim T_{QCD}$)

$$\Gamma \sim \frac{T^3}{f_a^2}$$

$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \mathrm{GeV}}\right)^2$$



$$H \sim \frac{T^2}{M_p}$$

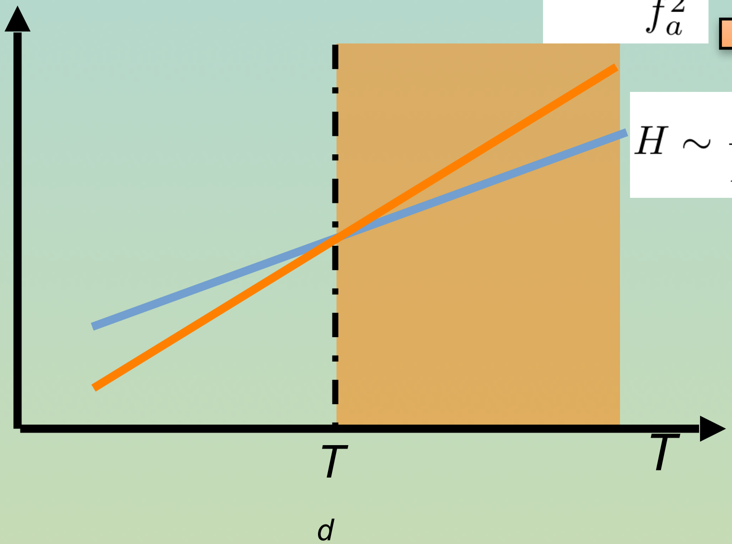
Cosmic Axion Background

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

Axion-gluon scatterings ($T \gtrsim T_{QCD}$)

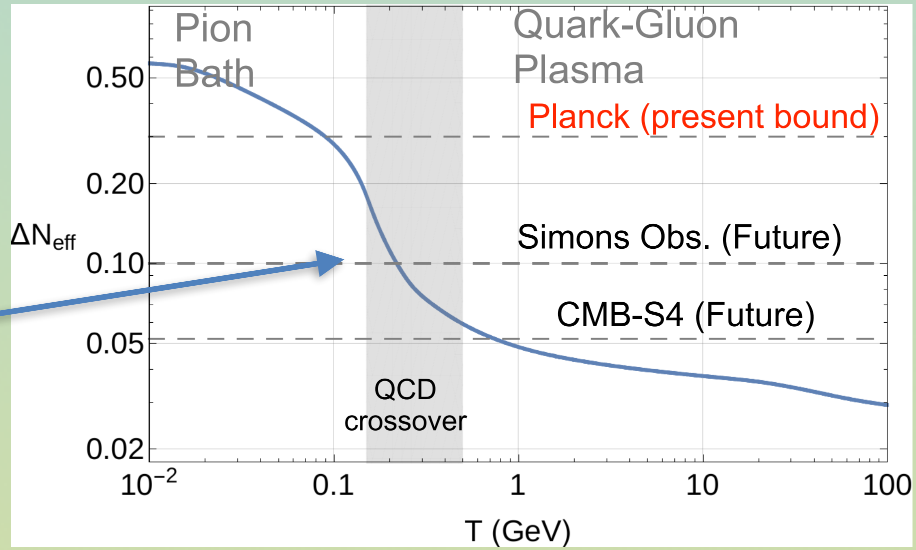
$$\Gamma \sim \frac{T^3}{f_a^2}$$

$$H \sim \frac{T^2}{M_p}$$



$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \mathrm{GeV}}\right)^2$$

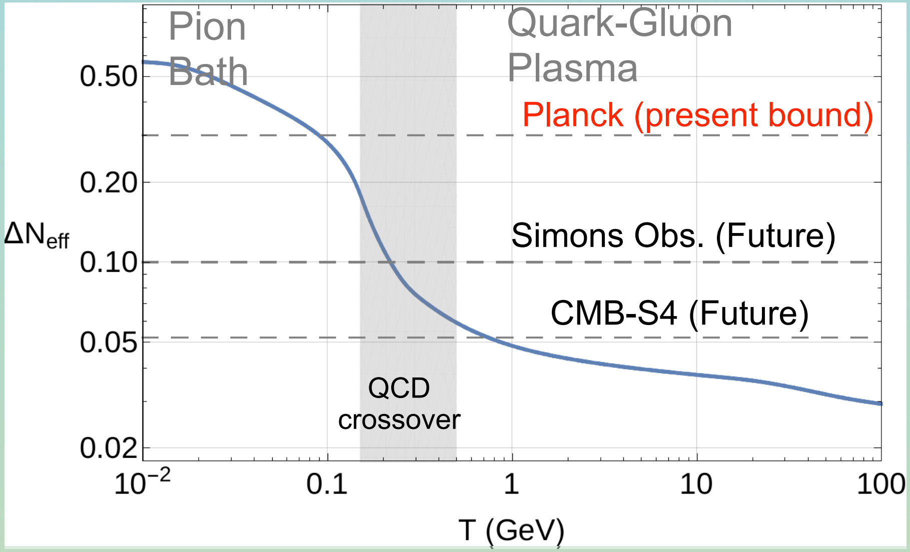
$$\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$$



Axion ΔN_{eff} has a long history:

Arias-Aragon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, AN, Piazza, Raffelt, Rompineve, Rota, Salvio, Sakharov, Silk, Slosar, Steffen, Strumia, Wallisch, Wong, Yun, Zsembinszki, Xue, …

$\pi\pi \leftrightarrow a\pi$ $\qquad$ $gg \leftrightarrow ga$



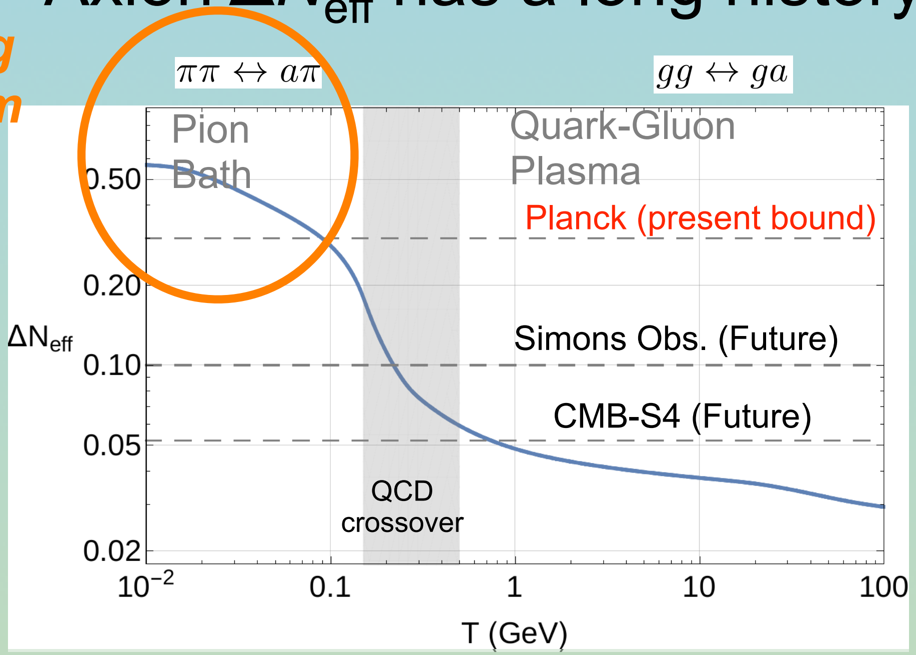
"Standard" treatments:

1. Instantaneous decoupling ($\Gamma = H$)
2. Single Boltzmann Eq.for abundance Y.

$$\frac{dY}{d\log x} = (Y^{\text{eq}} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right) \qquad (x \equiv m/T)$$

Axion ΔN_{eff} has a long history:

Our work: Improving present bounds from pion scatterings



Arias-Aragon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, AN, Piazza, Raffelt, Rompineve, Rota, Salvio, Sakharov, Silk, Slosar, Steffen, Strumia, Wallisch, Wong, Yun, Zsembinszki, Xue, …

"Standard" treatments:

1. Instantaneous decoupling ($\Gamma = H$)
2. Single Boltzmann Eq.for abundance Y.

$$\frac{dY}{d\log x} = (Y^{\mathrm{eq}} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right) \qquad (x \equiv m/T)$$

Momentum-dependent Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{<} = e^{-\frac{E}{T}}\Gamma^{>}$$

(Detailed balance, plasma particles in equilibrium)

Momentum-dependent Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{<} = e^{-\frac{E}{T}}\Gamma^{>}$$

(Detailed balance, plasma particles in equilibrium)

Perturbatively, due to scatterings with pions:

$$\Gamma^{<} = \frac{1}{2E}\int \left(\prod_{i=1}^{3}\frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\rm eq} f_2^{\rm eq}(1 + f_3^{\rm eq})(2\pi)^4\delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2_{2\,\leftrightarrow\,2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO chiral perturbation theory rate (Chang Choi '93)

NLO chiral perturbation theory rate (Chang Choi '93)

(Di Luzio, Martinelli, Piazza '21)

$\rightarrow$ breaks down at $T \gtrsim 60$ MeV !

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d} \frac{f_\pi}{2f_a}$$

1. The Thermalization Rate Γ

$\pi\pi \leftrightarrow a\pi$

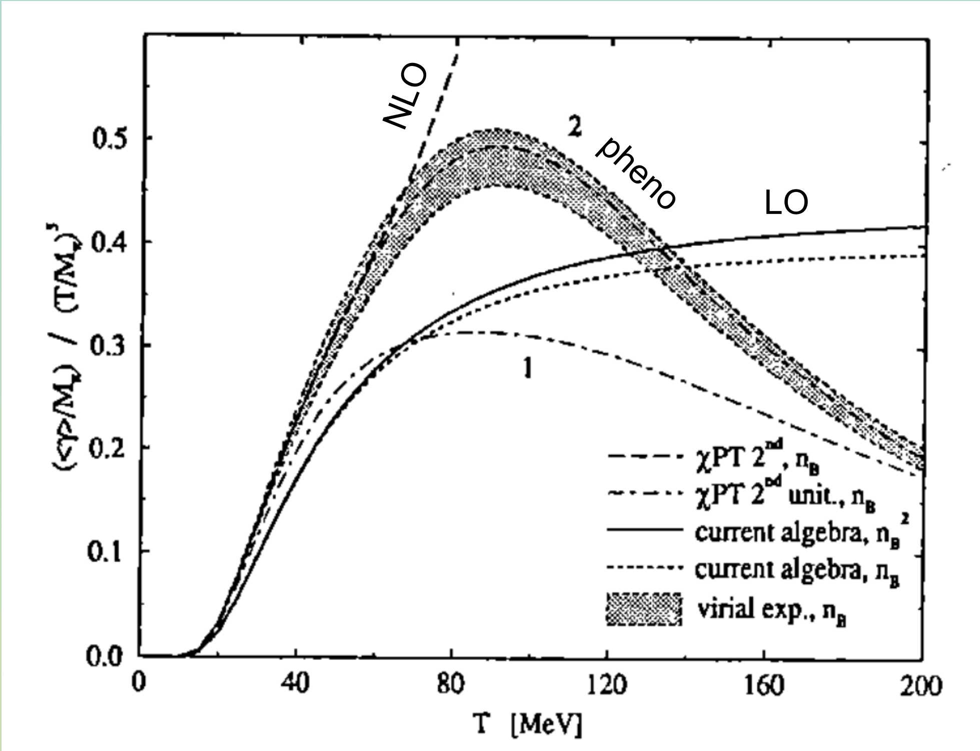
LO chiral perturbation theory rate (Chang Choi '93)

NLO chiral perturbation theory rate (Chang Choi '93)

(Di Luzio, Martinelli, Piazza '21)

→ breaks down at $T \gtrsim 60$ MeV !

$\pi\pi \leftrightarrow \pi\pi$



Schenk '94

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\rm phys} + \sin(\theta_{a\pi})\,a_{\rm phys} \simeq \pi^0_{\rm phys} + \theta_{a\pi} a_{\rm phys}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}$$

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right)q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\mathrm{phys}} + \sin(\theta_{a\pi})\,a_{\mathrm{phys}} \simeq \pi^0_{\mathrm{phys}} + \theta_{a\pi}a_{\mathrm{phys}}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}$$

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\mathrm{phys}} + \sin(\theta_{a\pi})\,a_{\mathrm{phys}} \simeq \pi^0_{\mathrm{phys}} + \theta_{a\pi} a_{\mathrm{phys}}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}$$

@ all orders in χPT

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

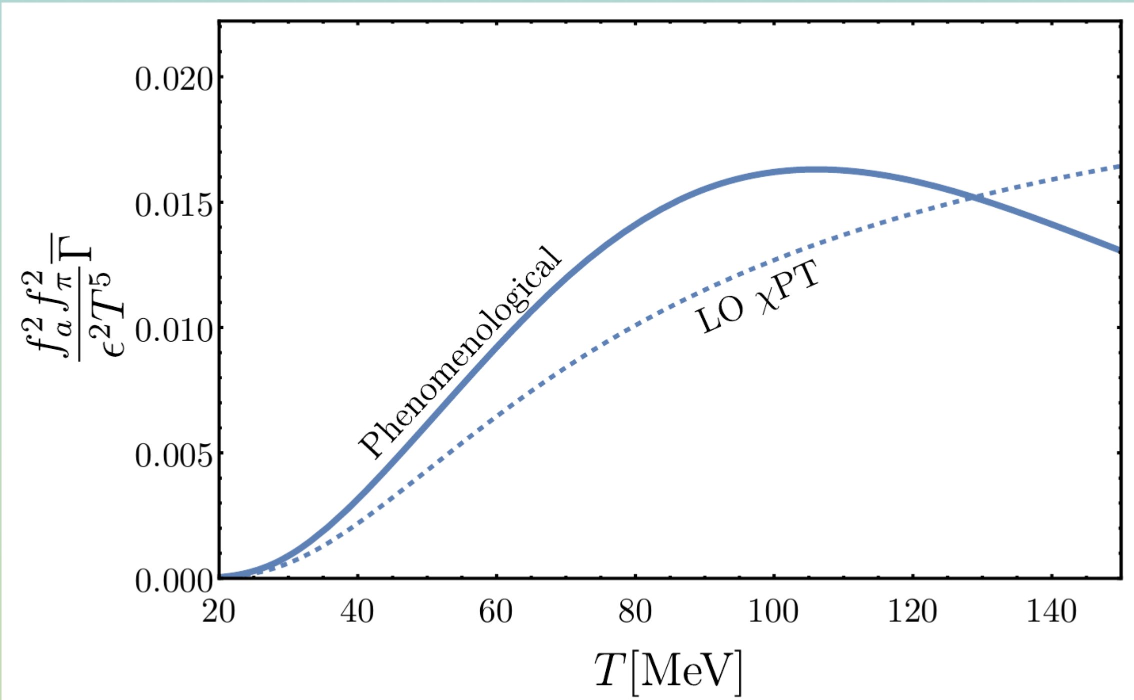
e.g. @ LO

$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta_{a\pi}^2\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

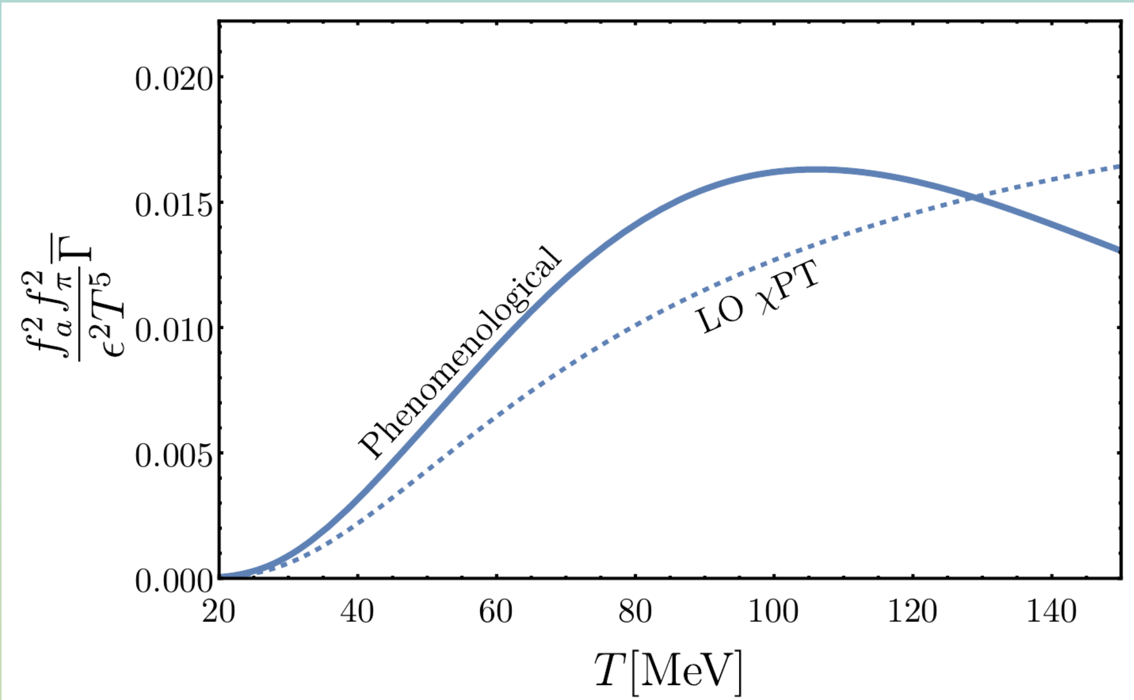
$$|\mathcal{M}^{\mathrm{LO}}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

$\lesssim 10\%$

1. The Axion Thermalization Rate Γ (from pions): our result

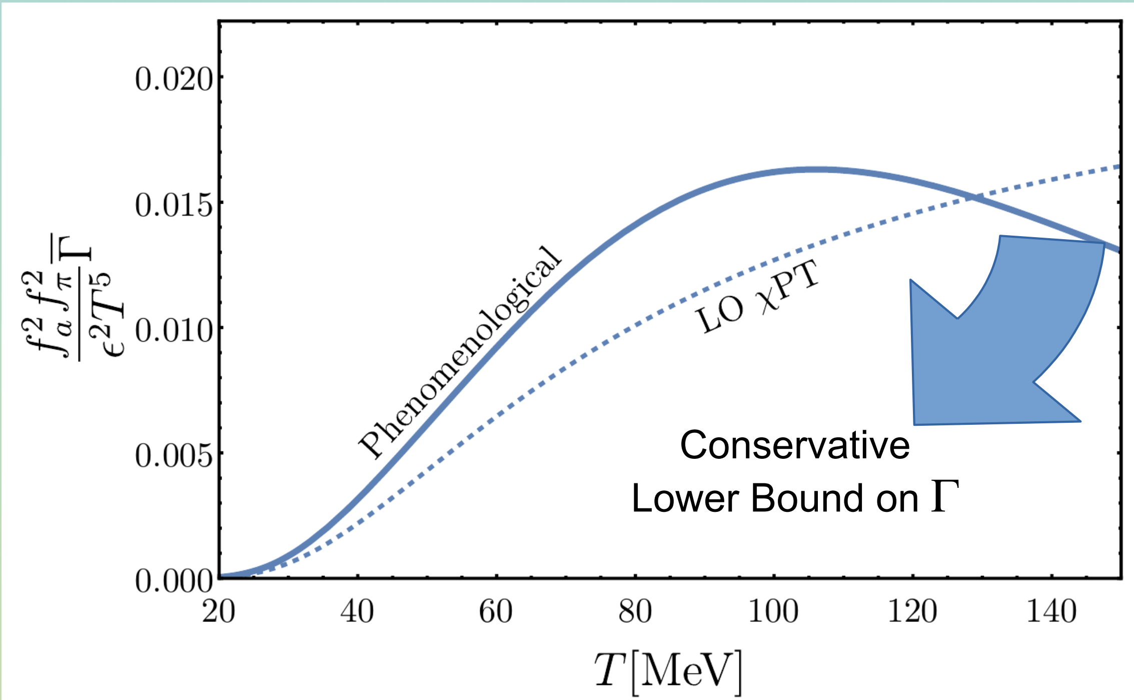


1. The Axion Thermalization Rate Γ (from pions): our result



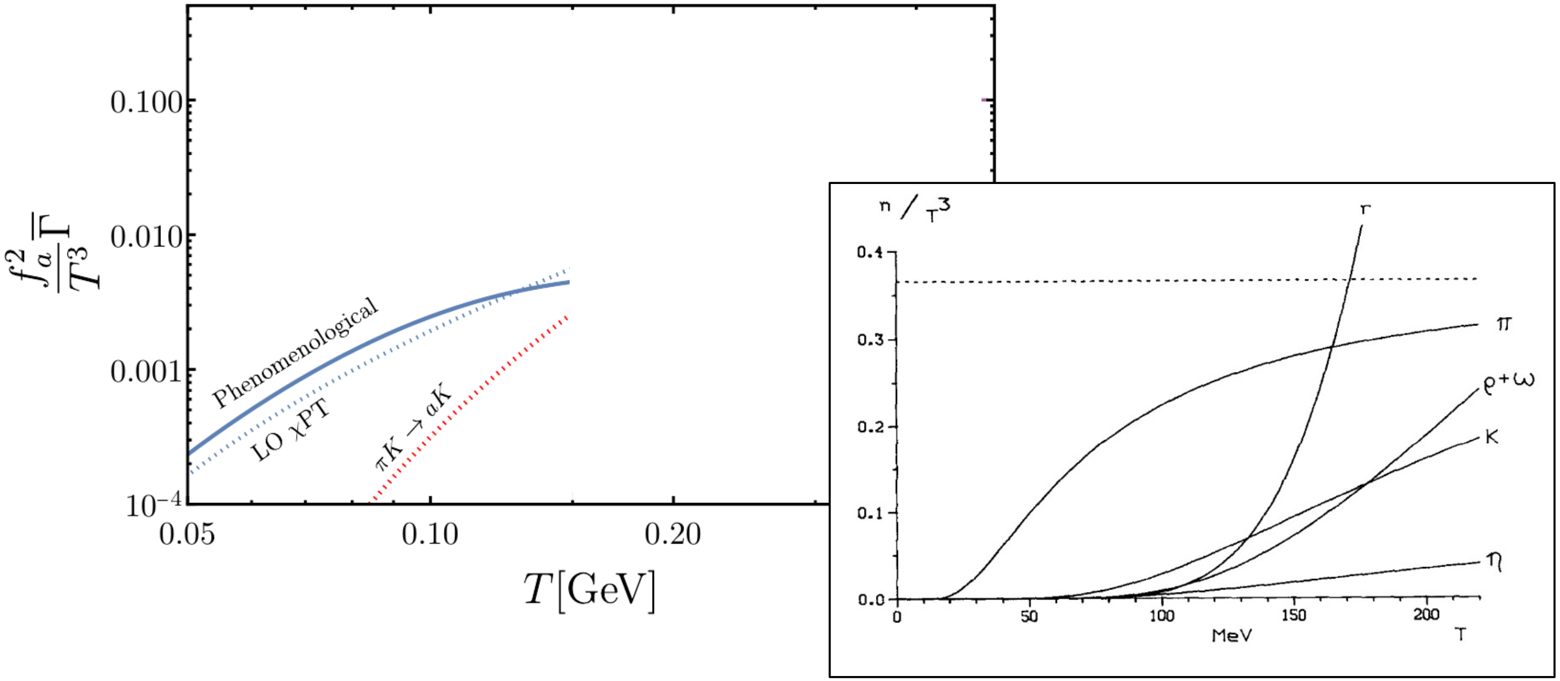
In reasonable agreement with: Di Luzio, Camalich, Martinelli, Oller, Piazza '22 (using NLO+unitarization)

1. The Axion Thermalization Rate Γ (from pions): our result



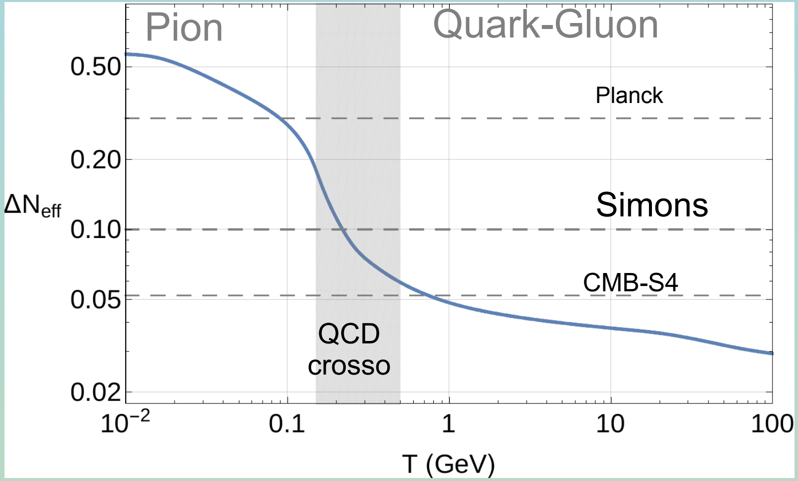
In reasonable agreement with:
Di Luzio, Camalich, Martinelli, Oller, Piazza '22
(using NLO+unitarization)

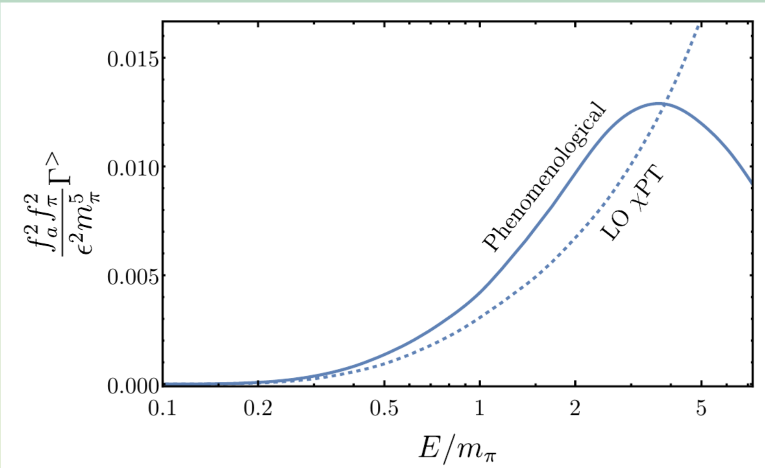
1. The Thermalization Rate Γ (Possible other channels: Kaons,…)



Gerber Leutwyler '89

2. Momentum Dependence





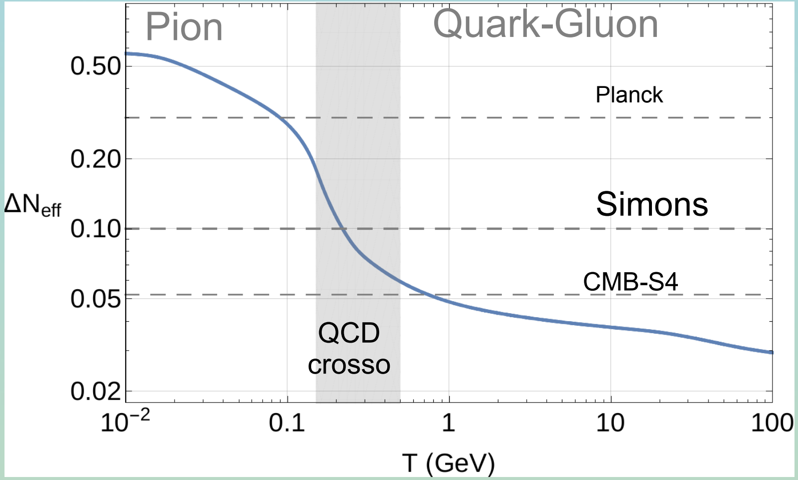
Boltzmann Eq.

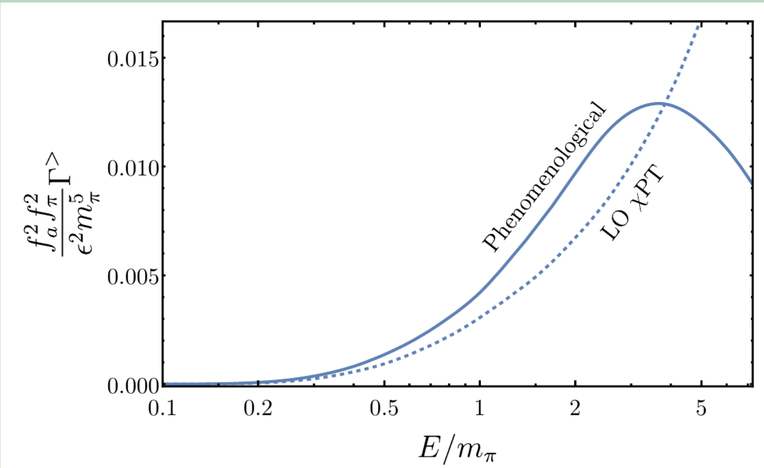
$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

High momenta k decouple later than low k

They see a lower g_* ⇨ More abundant

2. Momentum Dependence





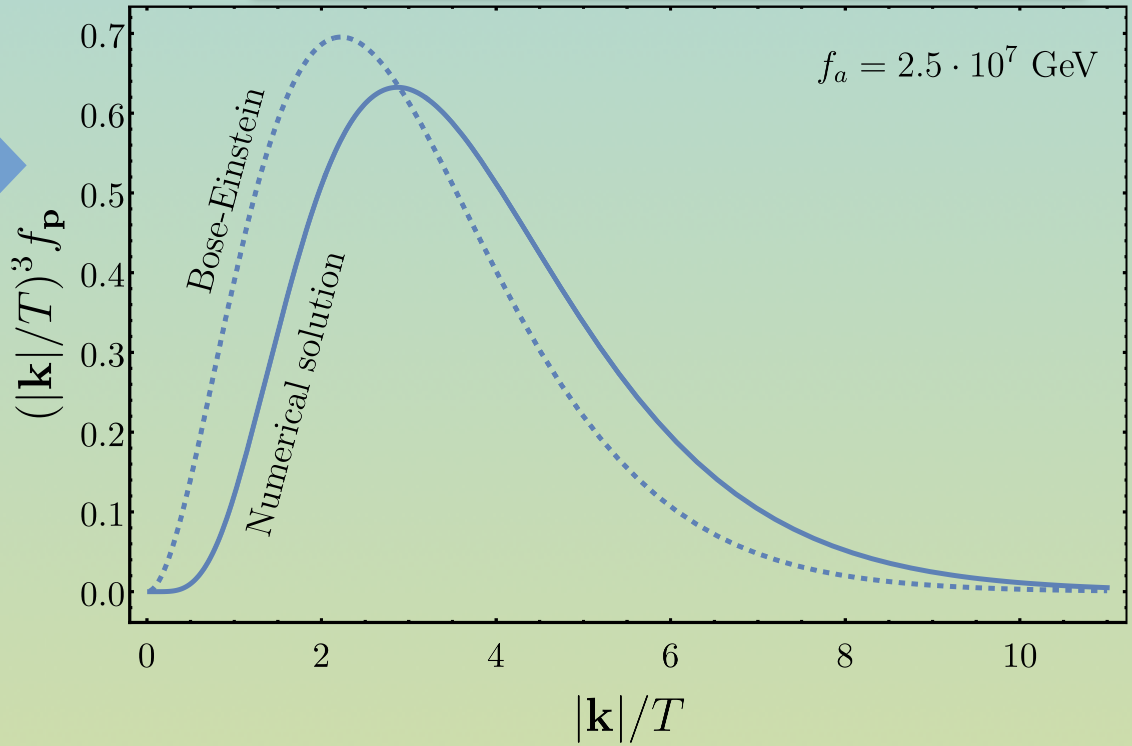
Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

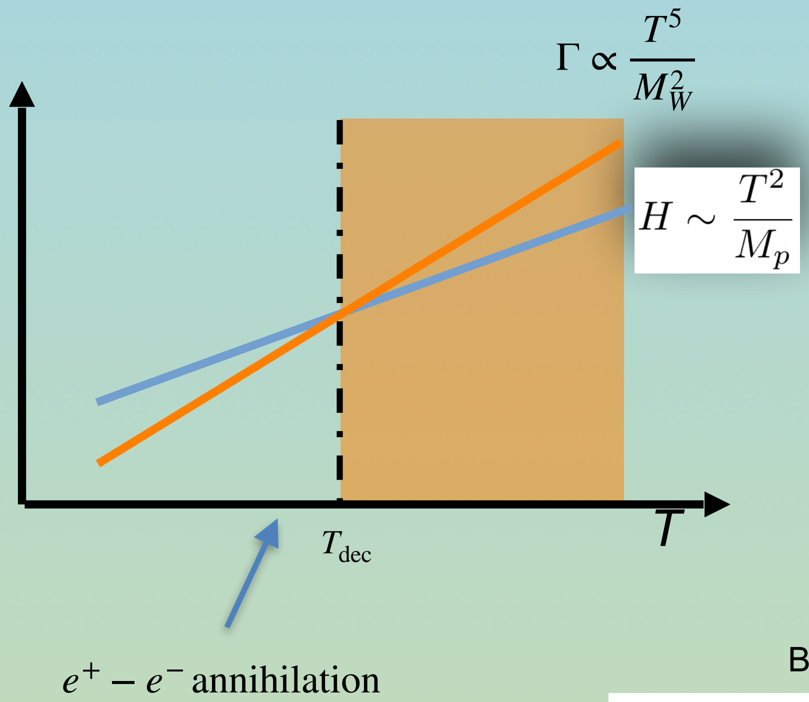
High momenta k decouple later than low k

They see a lower g_* → More abundant

~ 40% enhanced total abundance



2. Momentum Dependence: Neutrinos



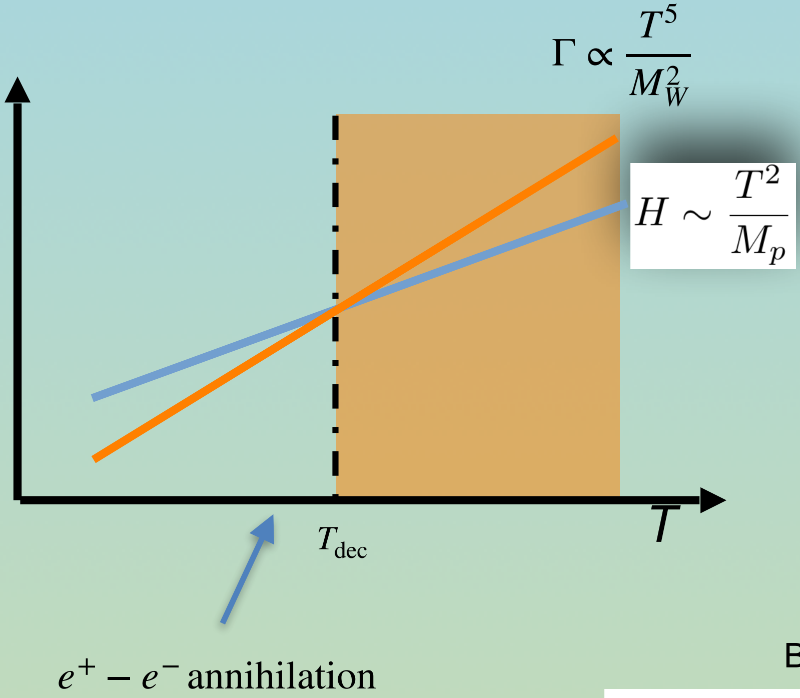
High momenta k decouple later than low k

They see a lower g_* → More abundant

Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

2. Momentum Dependence: Neutrinos



High momenta k decouple later than low k

They see a lower g_* ⇒ More abundant

Boltzmann Eq.

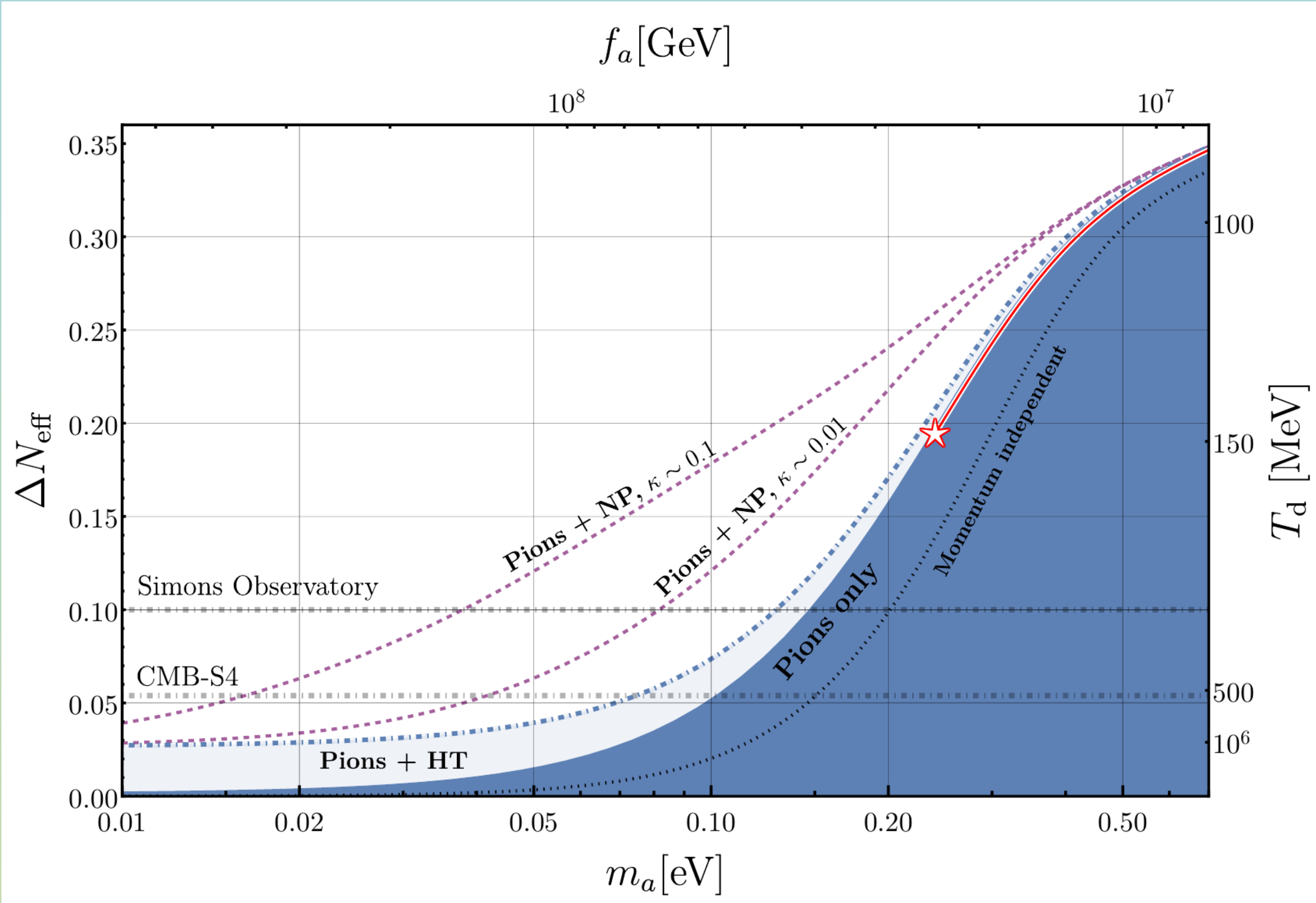
$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

But m_e and $T_{\rm dec}$ are more separated ⇒

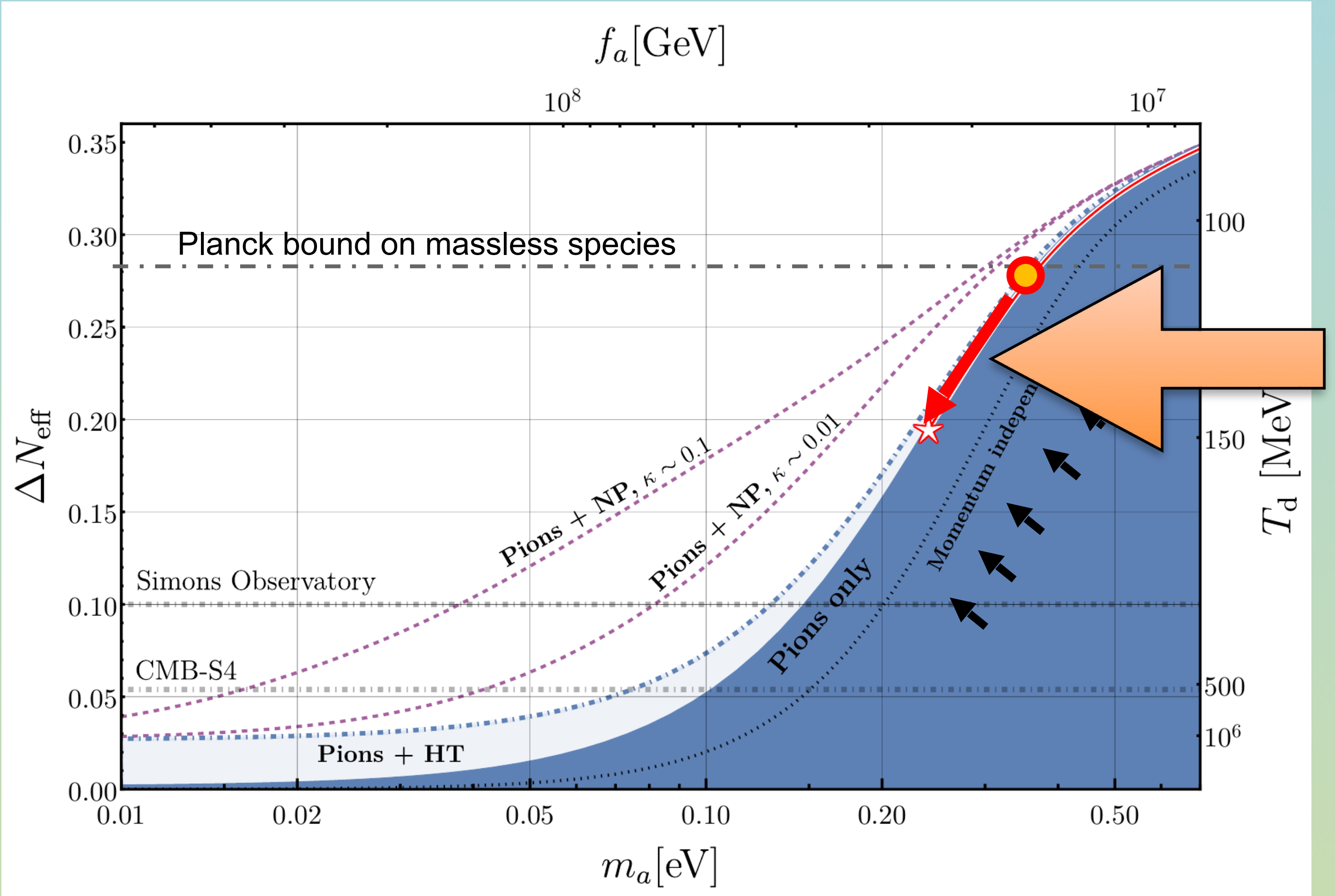
Only ~ 1% enhancement

$N_{\rm eff} \approx 3.044$

Present bound+Future Reach

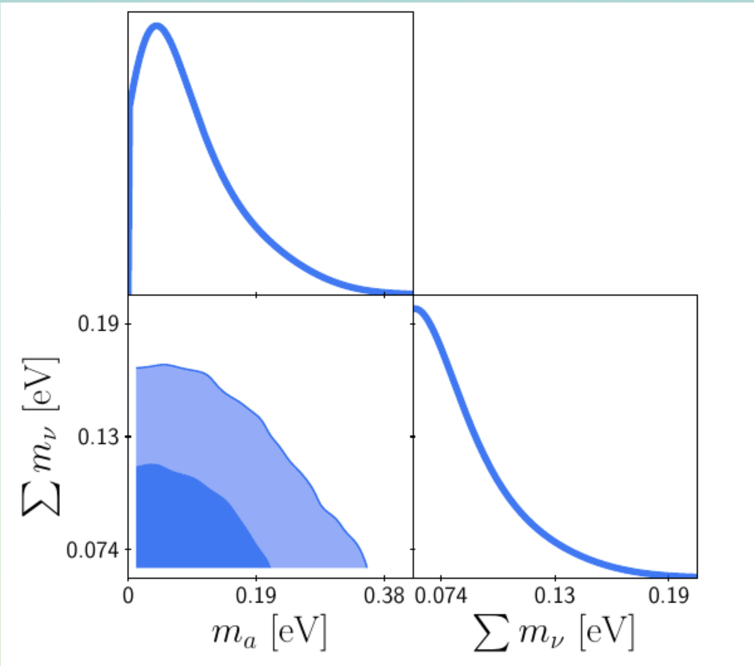


Present bound+Future Reach



Effect of mass

3. Combined cosmological Fit
(Λ_{CDM} + massive neutrinos + *axions*)



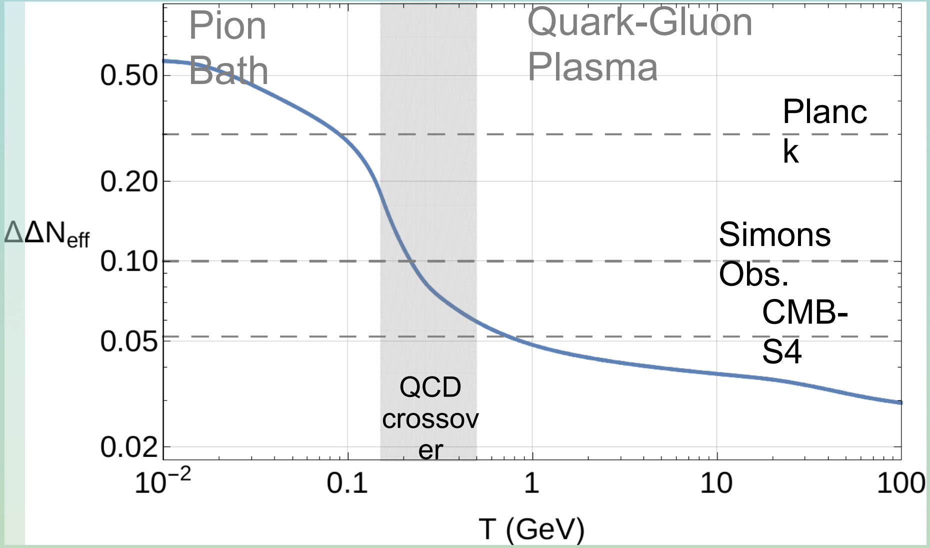
$$m_a \leq 0.24 \text{ eV}$$

$$f_a \geqslant 2.4 \cdot 10^7 \text{ GeV}$$

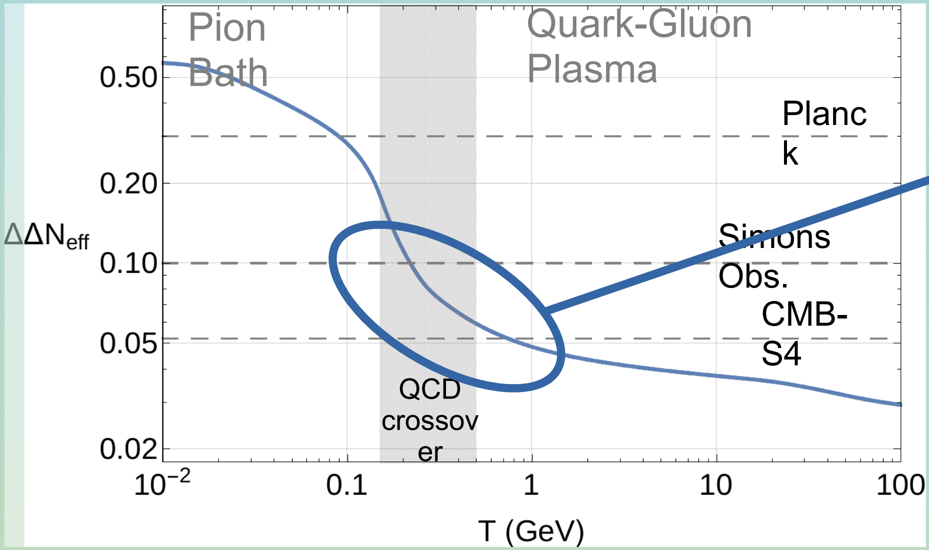
$$\Leftrightarrow$$

$$\Delta N_{\text{eff}} \lesssim 0.19$$

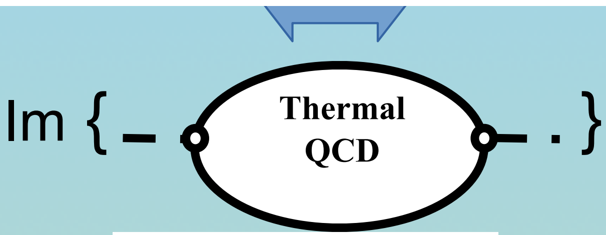
Future Reach



Future Reach



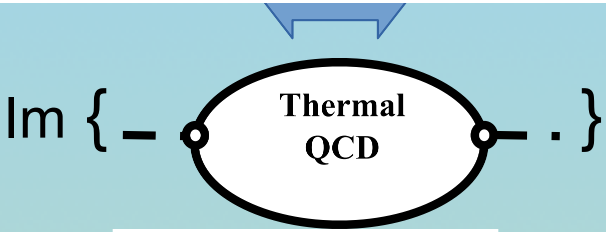
Non-Perturbative



$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

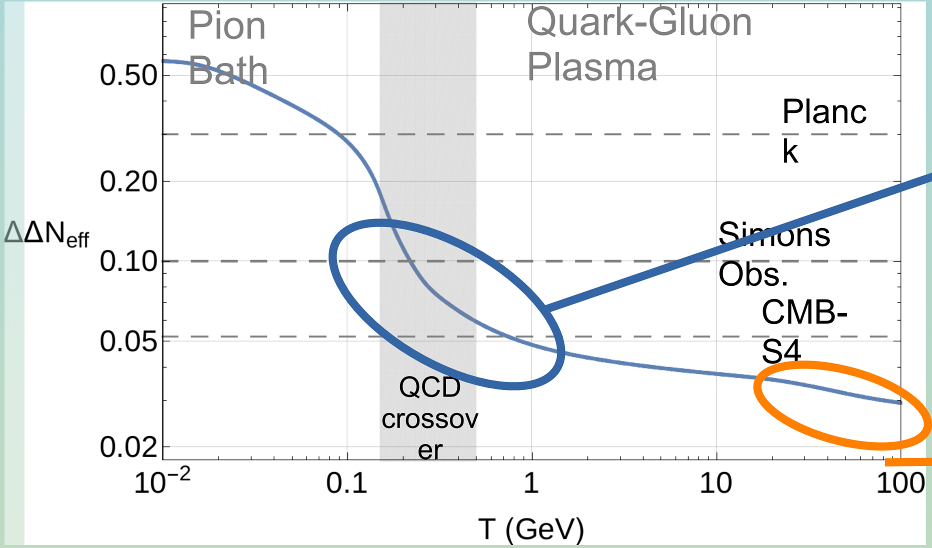
$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

Future Reach



$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$



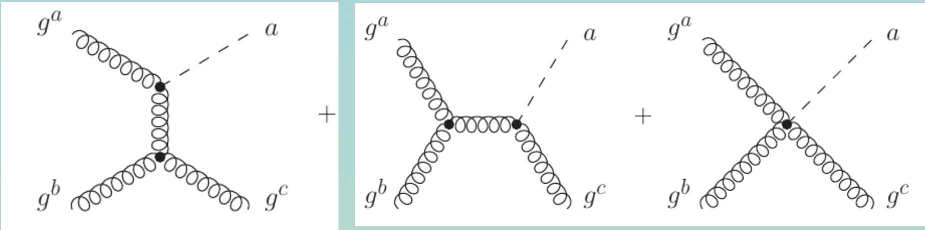
Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

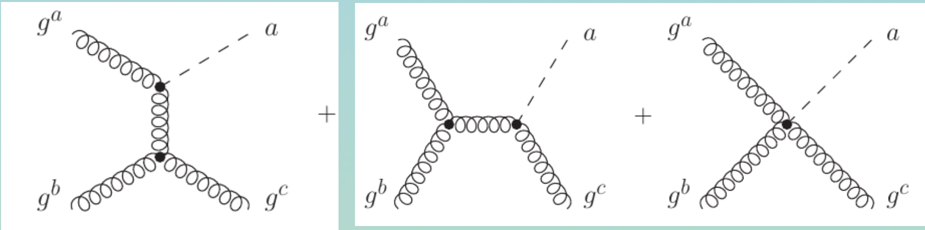
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

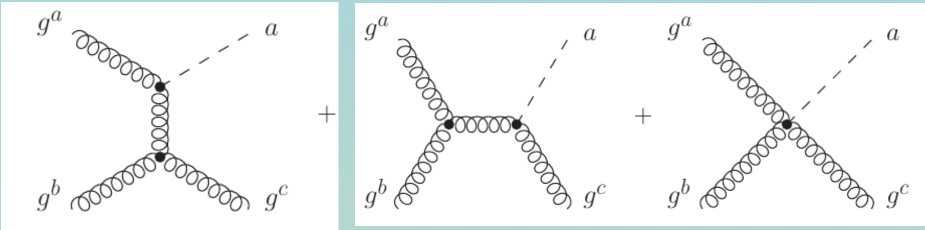
Masso, Rota, Zsembinszki '02
Graf, Steffen '10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

IR divergent

High Temperatures Regime



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

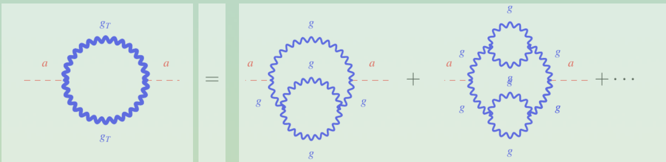
Masso, Rota, Zsembinszki '02
Graf, Steffen '10

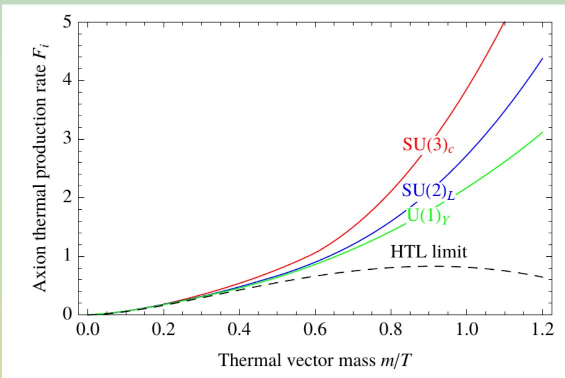
$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

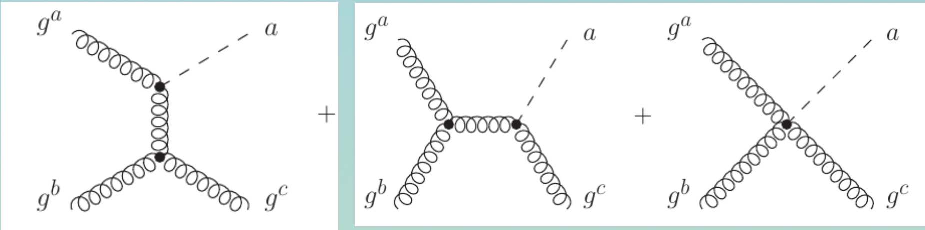
IR divergent

Unphysical negative F_3 cured by
Salvio, Strumia, Xue '13





High Temperatures Regime



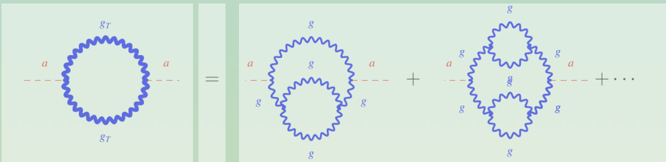
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

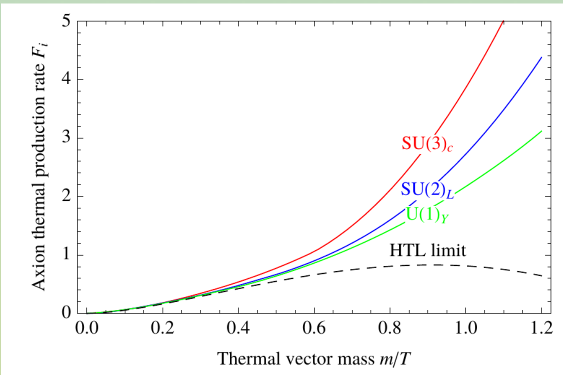
Masso, Rota, Zsembinszki '02
Graf, Steffen '10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

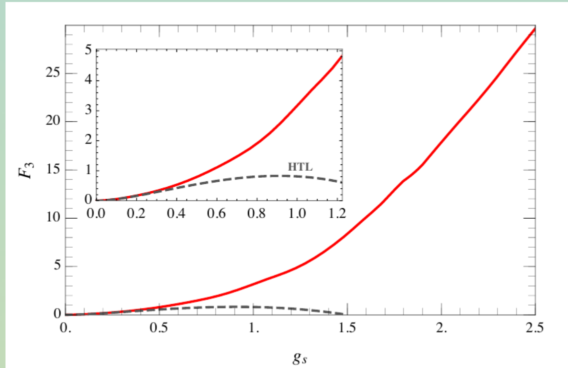
for $g_s \ll 1$ IR divergent

Unphysical negative F_3 cured by
Salvio, Strumia, Xue '13



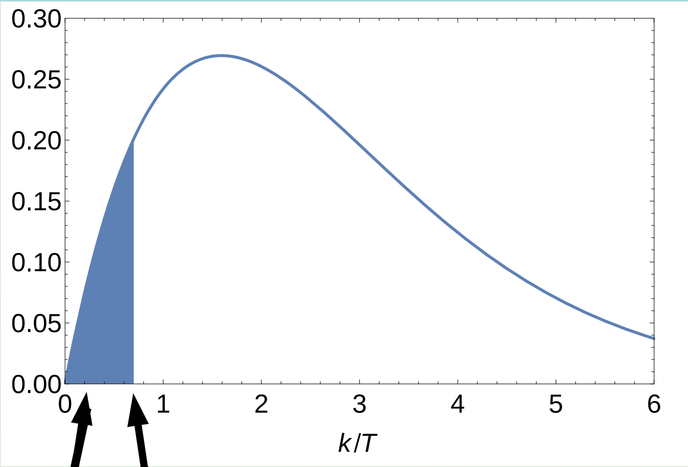


Recently D'Eramo, Hajkarim, Yun ('21):
extrapolated F_3 from Salvio et al. to $g_s > 1$
(Beyond regime of validity?)



*Matching gluon to pions through QCD crossover?
Pion-axion: suppressed by $\theta_{a\pi} \propto \dfrac{m_u - m_d}{m_u + m_d}$, gluon is not
Pion rates not monotonic with T
Rates could have sudden jumps, as g_* does

High Temperatures Regime



$k \sim m_e \sim g_s T$

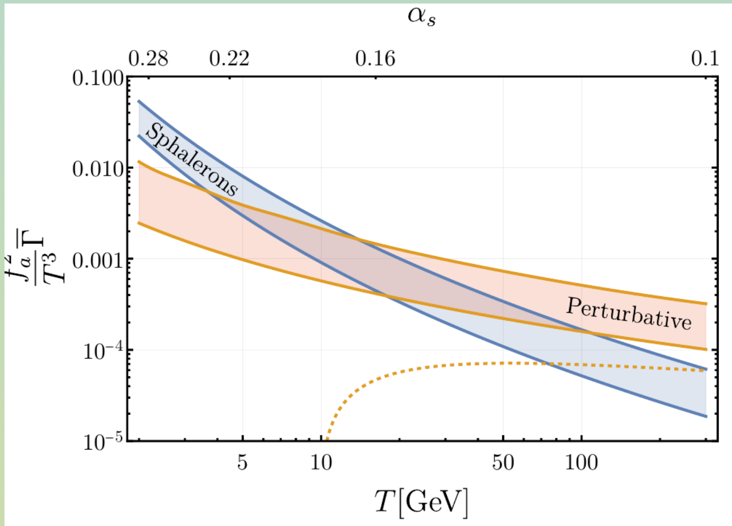
$k \sim m_m \sim g_s^2\, T$

$\# \sim 1 / g_s^2$

@ g_s << 1 :

large occupation numbers → dominated by semi-classical

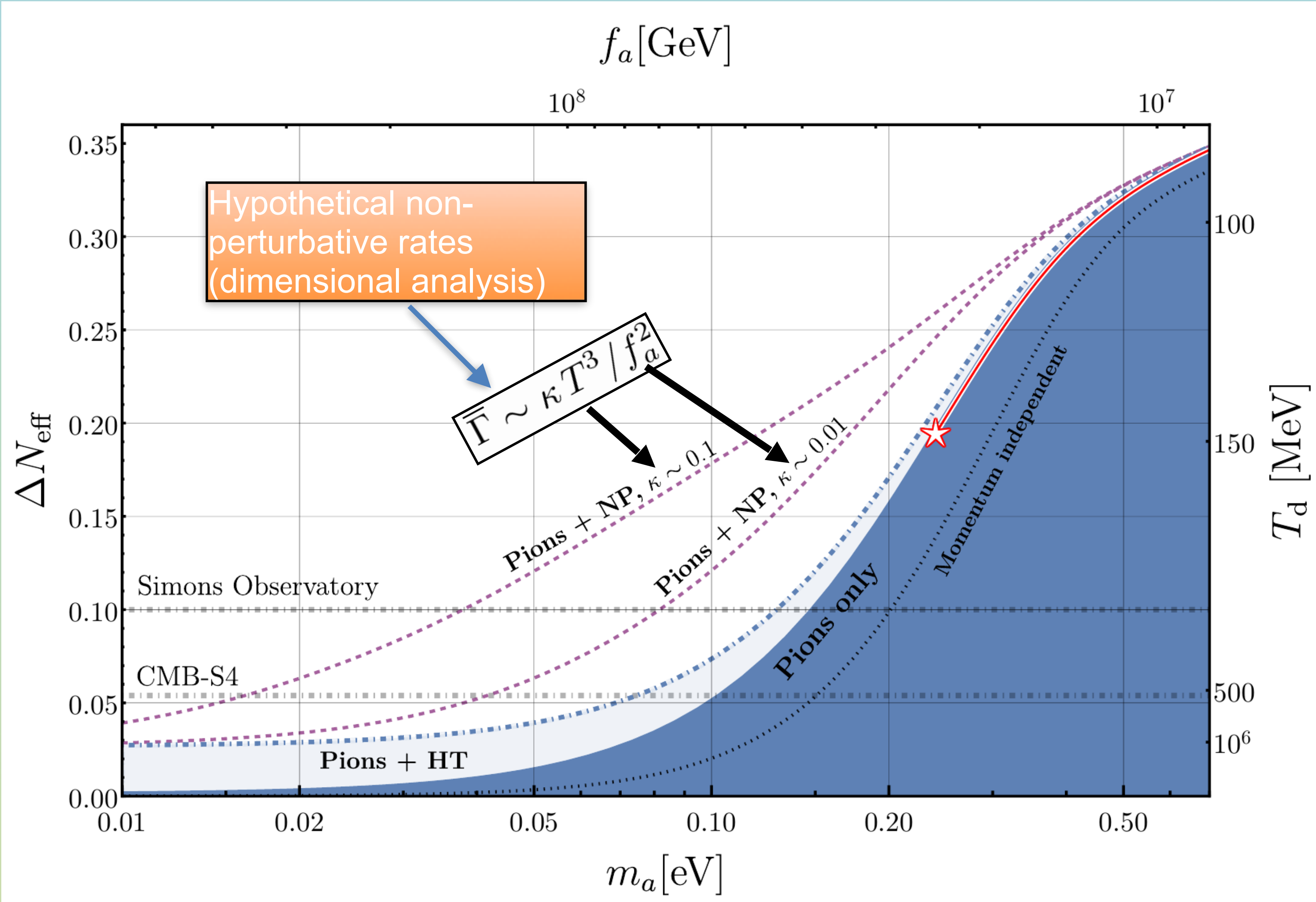
[non-linear YM equations - dissipation from strong sphalerons]



$$\Gamma_{\text{sphal}} \simeq \frac{(N_c \alpha_s)^5 T^3}{f_a^2}$$

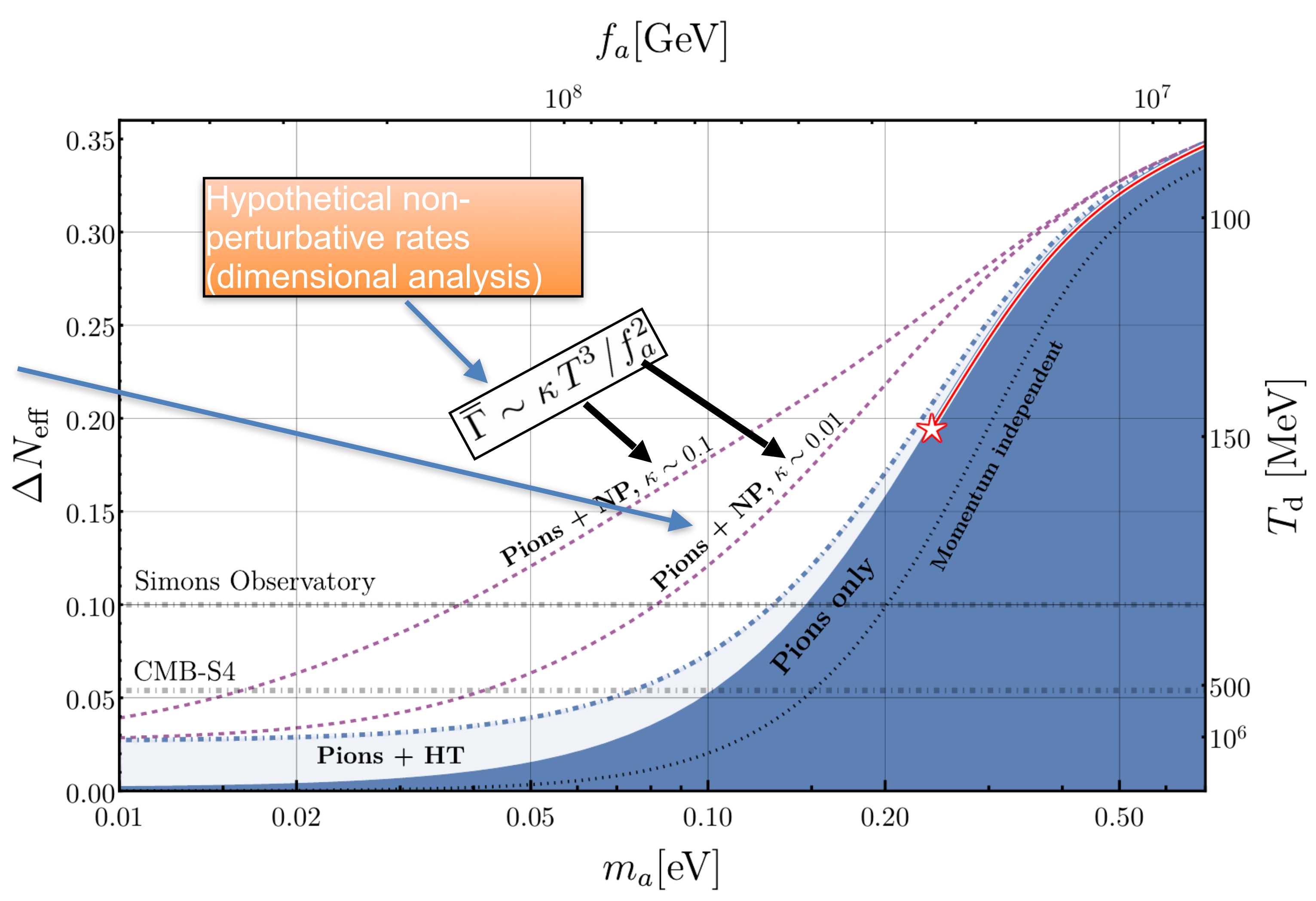
(Adapted from: Moore, Tassler '10)

Future Reach ($T_{\rm DEC} \gtrsim T_c$ region)



Future Reach ($T_{\rm DEC} \gtrsim T_c$ region)

Consistent with very recent Lattice QCD simulation (Bonanno et al. e-Print: 2308.01287) on $\Gamma_{\rm sphal}$



Conclusions:

- More reliable pion-axion rates and upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like QCD axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale

Conclusions:

- More reliable pion-axion rates and upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like QCD axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Doubts on Reliability of perturbative rates at $T \gg T_c$?
- Non-perturbative rates at $T \sim T_c$ crucial for upcoming CMB experiments $\Delta N_{\rm eff} \sim 0.1$

(*If axion couples directly to SM quarks and leptons: more production channels)

Conclusions:

- More reliable pion-axion rates and upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like QCD axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Doubts on Reliability of perturbative rates at $T \gg T_c$?
- Non-perturbative rates at $T \sim T_c$ crucial for upcoming CMB experiments $\Delta N_{\rm eff} \sim 0.1$

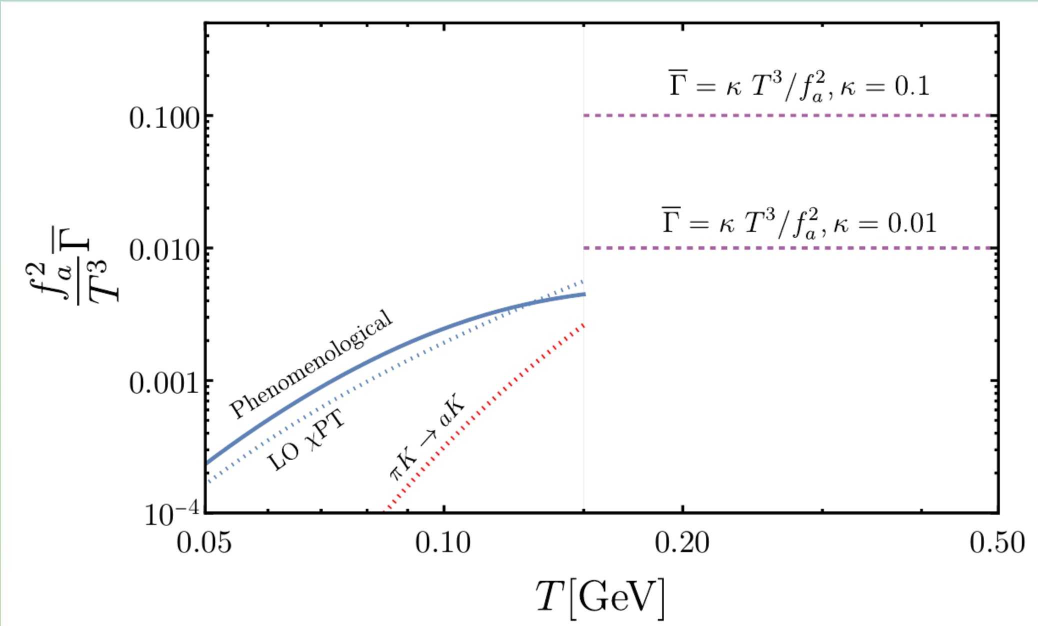
(*If axion couples directly to SM quarks and leptons: more production channels)

Thank you!

Back Up

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c \alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: Compute Γ_k for $T > T_c$

Existing Attempts (at k=0) e.g.

Moore, Tassler '10 : Classical SU(N) simulations

Kotov '18 ,

Altenkort et al. '20,

Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\rm sphal} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0},0) q(\vec{x},\tau)\rangle = -\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \, \rho(\omega) \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!