Primordial Black Hole formation during the QCD phase transition

Ilia Musco

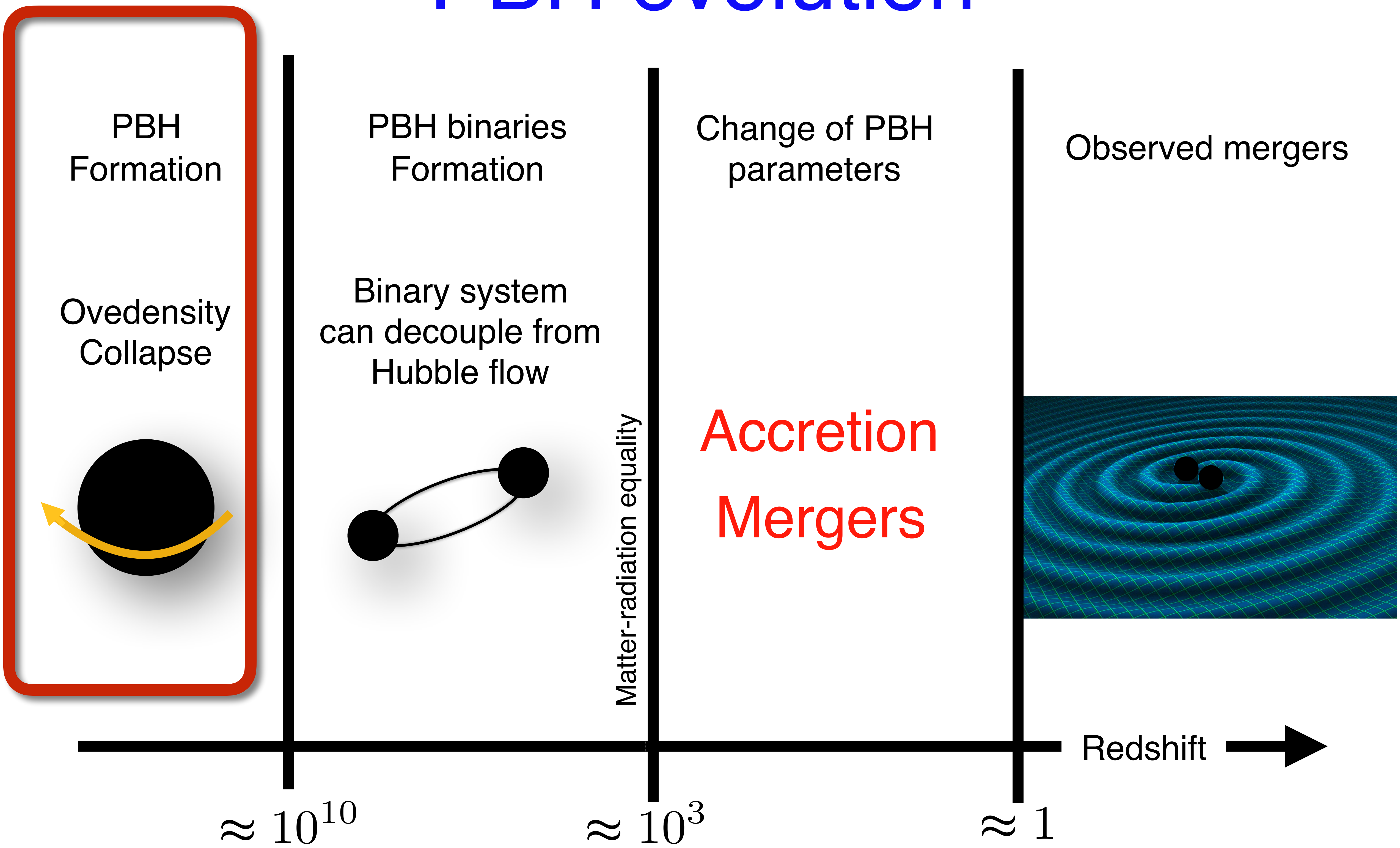
INFN Fellini Fellow - *SAPIENZA* University of Rome
Visitor - Queen Mary University of London (June - November 2023)

Cosmology in Miramare 2023 (Trieste)

1 September 2023

HORIZON 2020 G.A. n. 754496

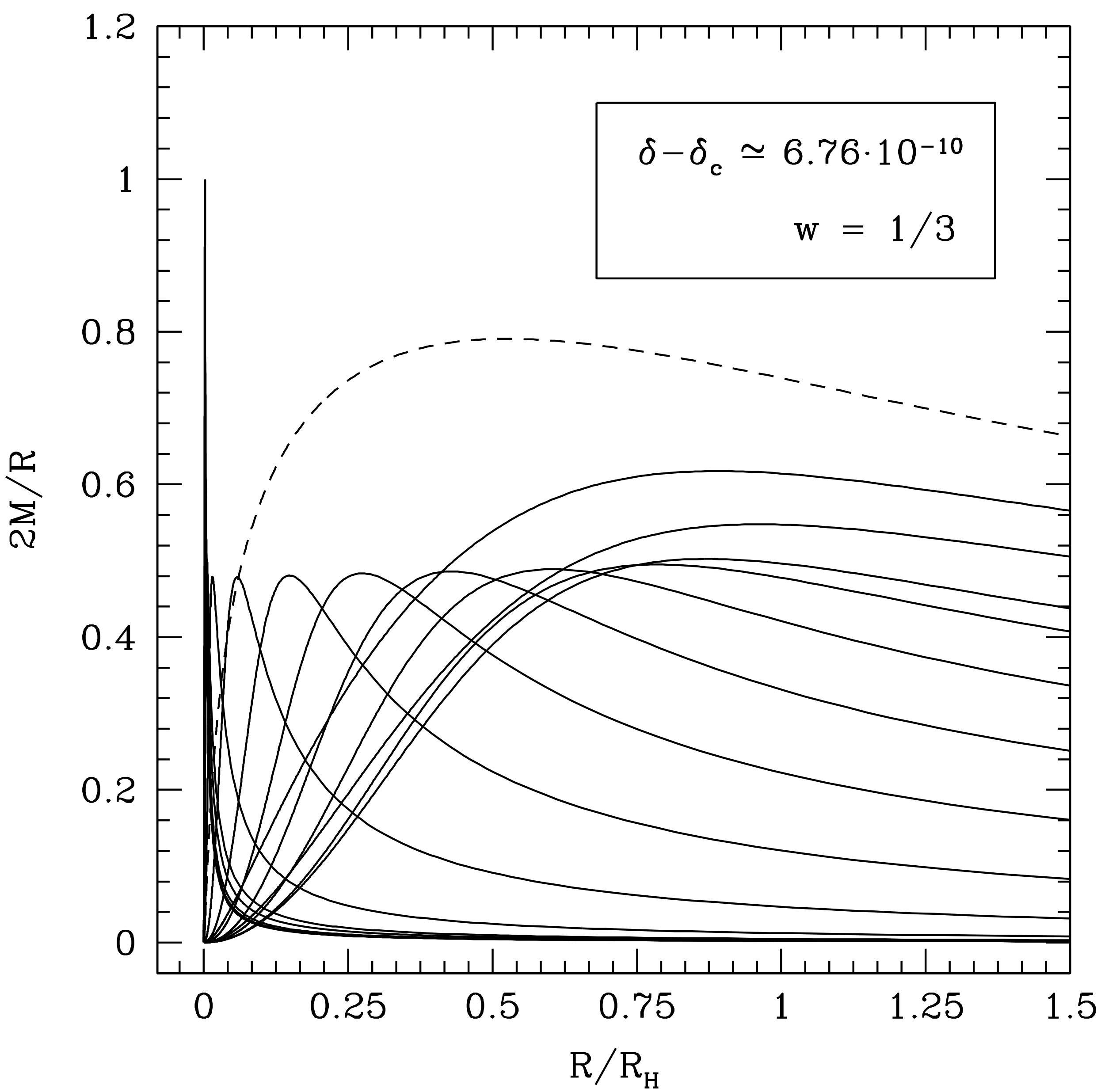
PBH evolution

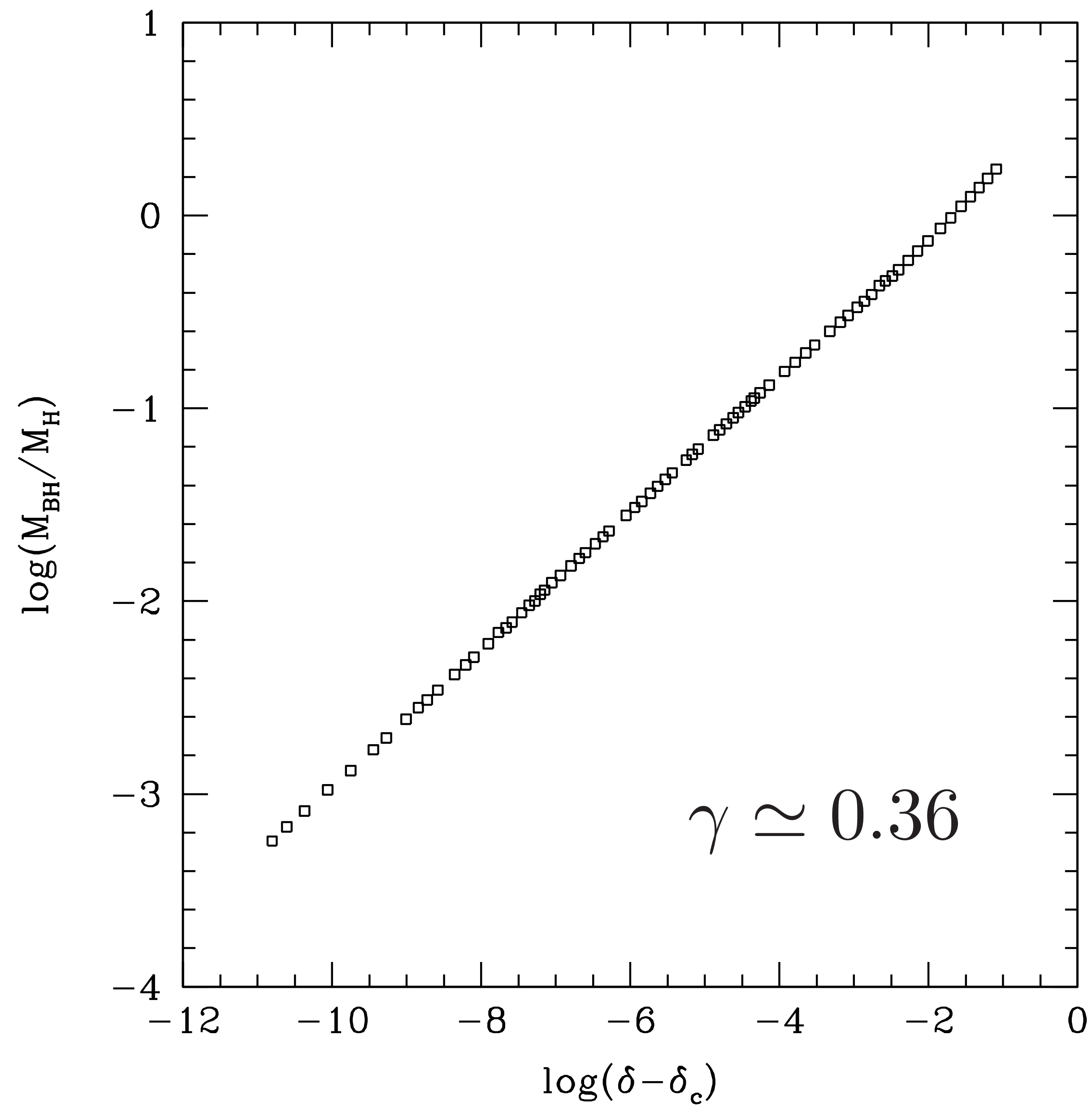


Numerical Results: PBH formation, mass distribution

$$R(r,t) = 2M(r,t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$





IM, J. Miller, Rezzolla, Polnarev - CQG (2005 - 2013)

$\mathcal{K}, \delta_c$ — shape dependent

Initial conditions: curvature profile

- The asymptotic metric ($t \to 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9}\left[\nabla^2\zeta(r) + \frac{1}{2}\left(\nabla\zeta(r)\right)^2\right] e^{-2\zeta(r)}$$

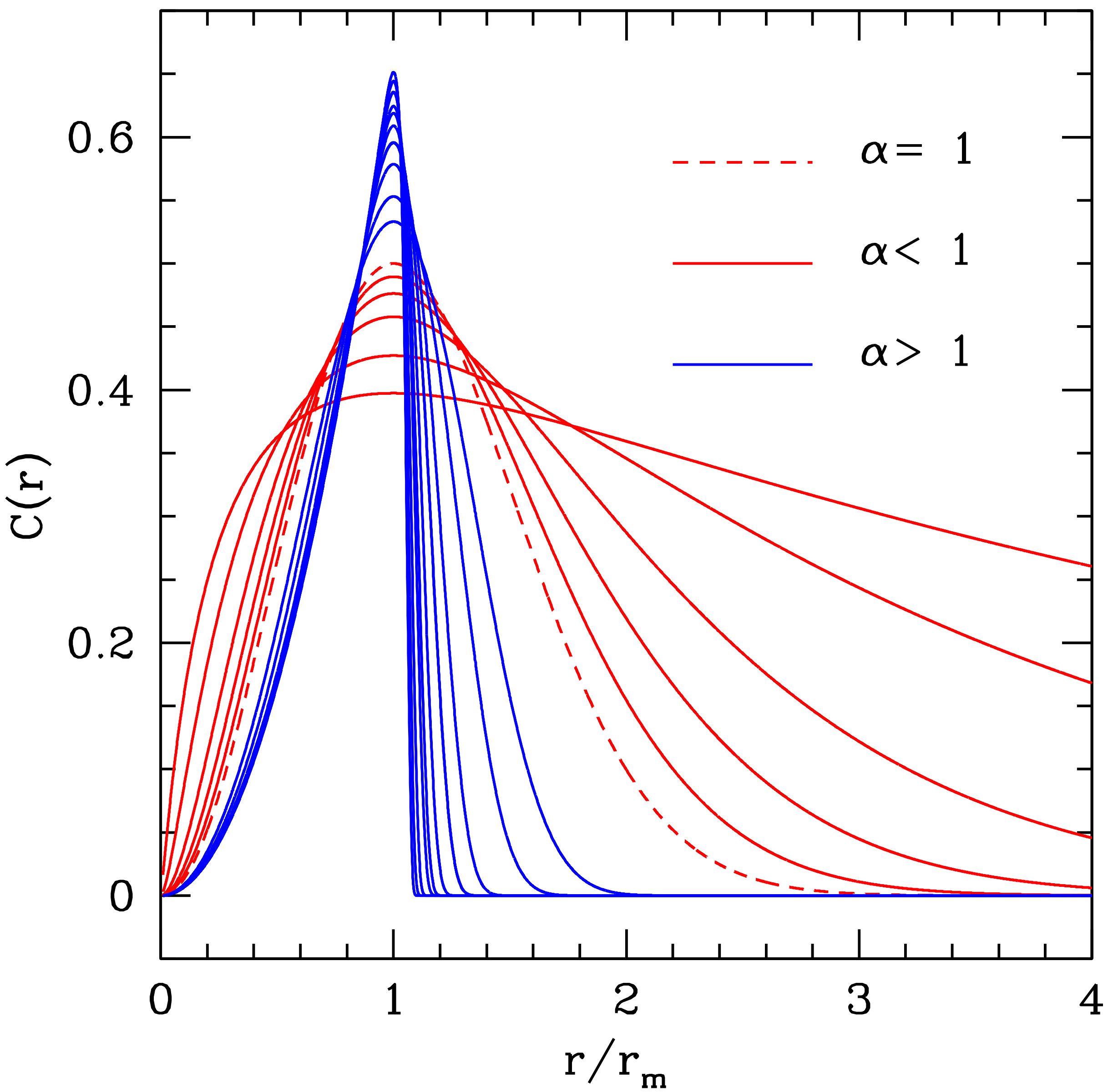
- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = -\frac{4}{3}\tilde{r}\zeta'(r)\left[1 + \frac{1}{2}\tilde{r}\zeta'(r)\right] \quad \Rightarrow \quad \delta = \delta_G\left[1 - \frac{3}{8}\delta_G\right]$$

Shape parameter

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\boxed{\delta(r_m, t_H) = 3\frac{\delta\rho}{\rho_b}(r_m, t_H)}$$



$$\tilde{r} = r e^{\zeta(r)}$$

$$\alpha \equiv -\frac{\mathcal{C}''(\tilde{r}_m)\tilde{r}_m^2}{4\mathcal{C}(\tilde{r}_m)} = \frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)(1 - \Phi_m)}$$

$$\boxed{0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}}$$

I. Musco - PRD (2019)

PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_*\hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_*\hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2\hat{r}_m^2 - 1)\frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos(k\hat{r}_m)\right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1 + F(\alpha)\right]\alpha = -\frac{1}{2}\left[1 + \hat{r}_m \frac{\int dk k^4 \cos(k\hat{r}_m) P_\zeta(k,\eta)}{\int dk k^3 \sin(k\hat{r}_m) P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5}e^{-1/\alpha}\frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha},\frac{1}{\alpha}\right)}}\,.$$

4. **The threshold δ_c:** compute the threshold as function of α, fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

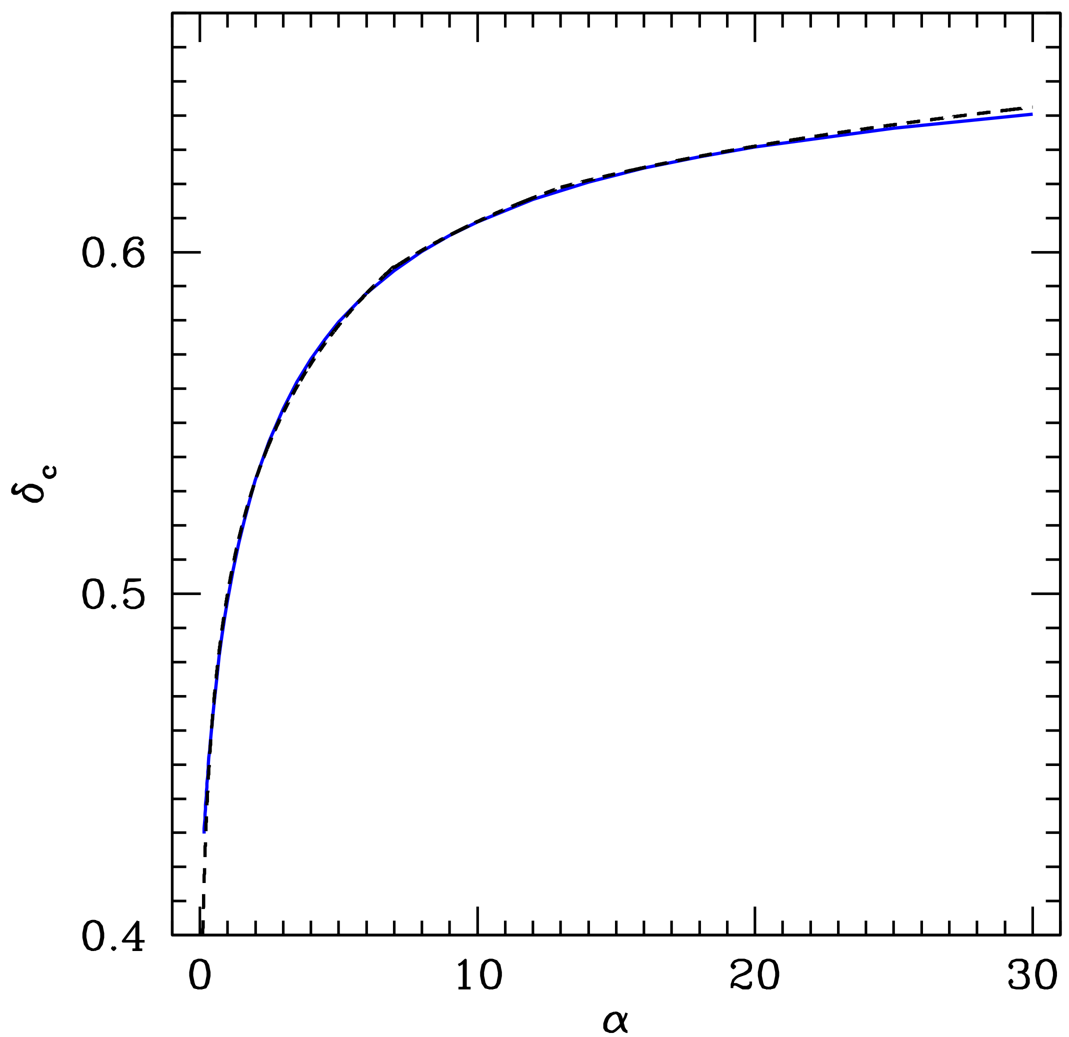
$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

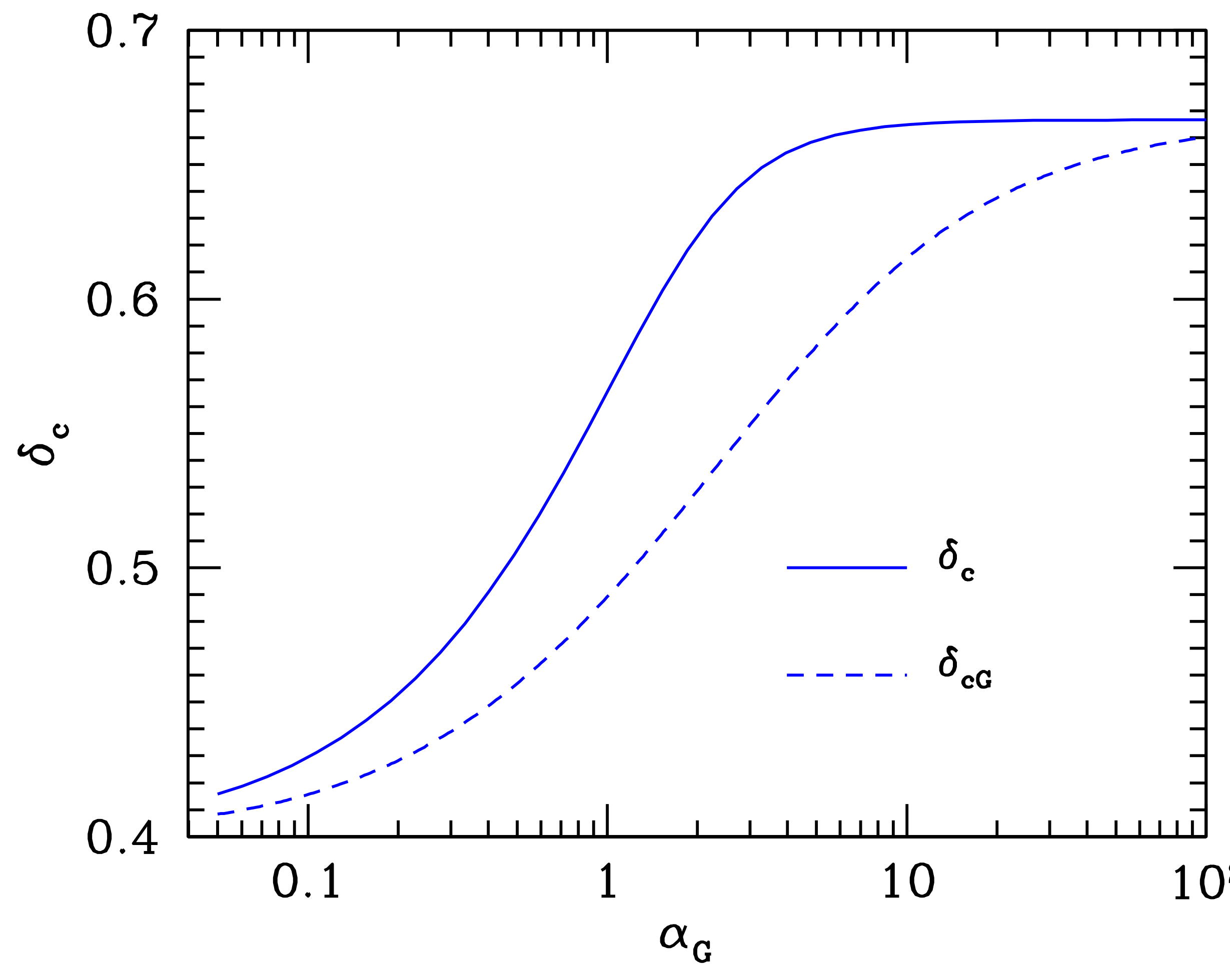
PBH threshold

- *IM, De Luca, Franciolini, Riotto -* PRD (2021)

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

$$\delta_m = \frac{4}{3}\Phi_m\left(1 - \frac{1}{2}\Phi_m\right) = \delta_G\left(1 - \frac{3}{8}\delta_G\right)$$





PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

- **PDF of δ follows a Gaussian distribution:** $$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (k r_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- **Narrow peak:** $$\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$$

- **Broad peak:** $$\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3 \times 10^{-3}$$

- **Non linear effects:** $$\delta = \delta_G \left[1 - \frac{3}{8}\delta_G\right] \;\Rightarrow\; 1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1 - \sqrt{1 - \frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$$

S.*Young, IM, C.Byrnes* JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

Equation of State of the Early Universe

The easy Universe goes through 3 main transitions before matter-radiation equality:

- Electroweak phase-transition
- **QCD phase- transition** (crossover)

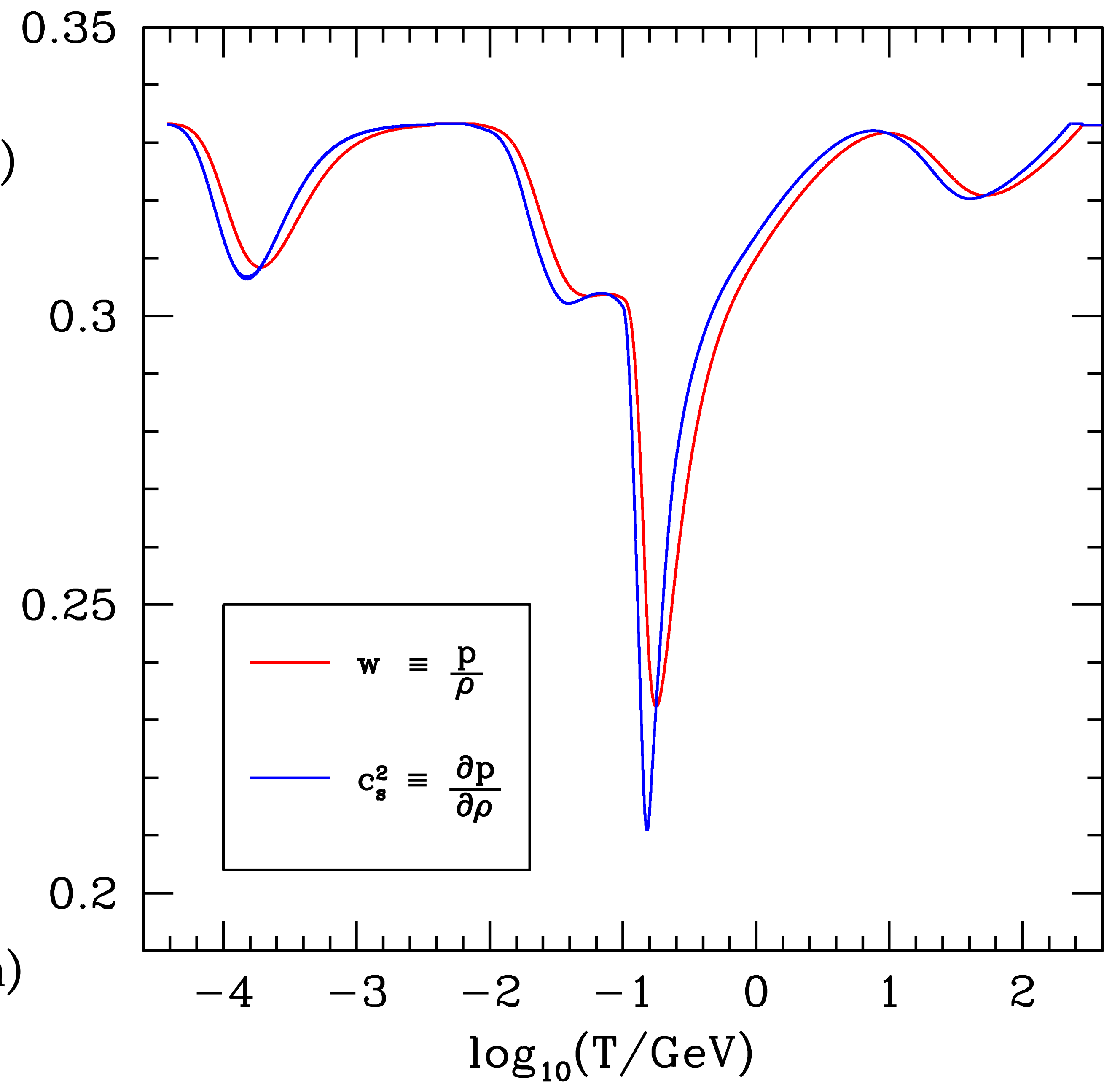
$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4 \qquad s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4 h_{\rm eff}(T)}{3 g_{\rm eff}(T)} - 1$$

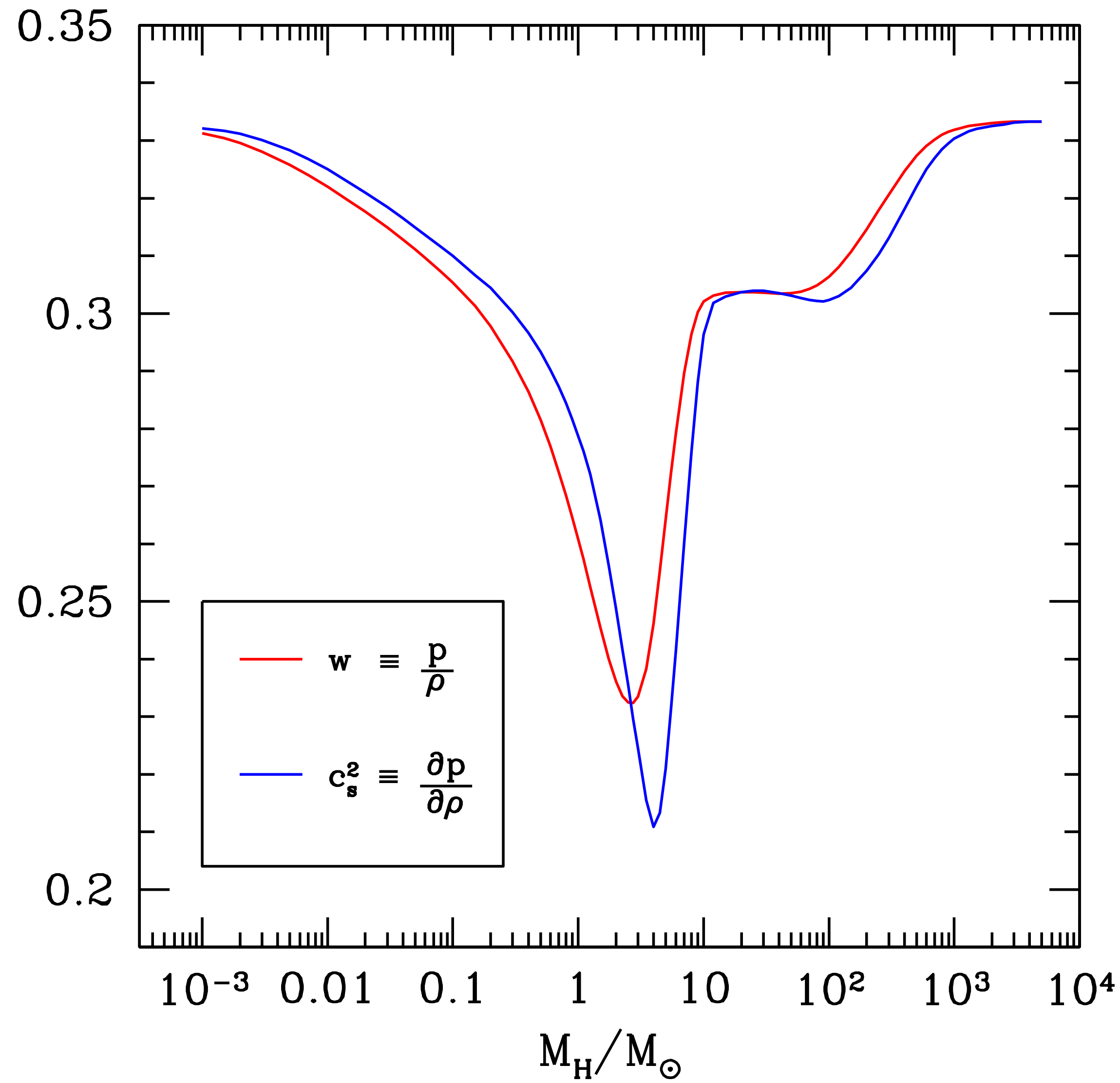
$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + T h'_{\rm eff})}{3(4 g_{\rm eff} + T g'_{\rm eff})} - 1$$

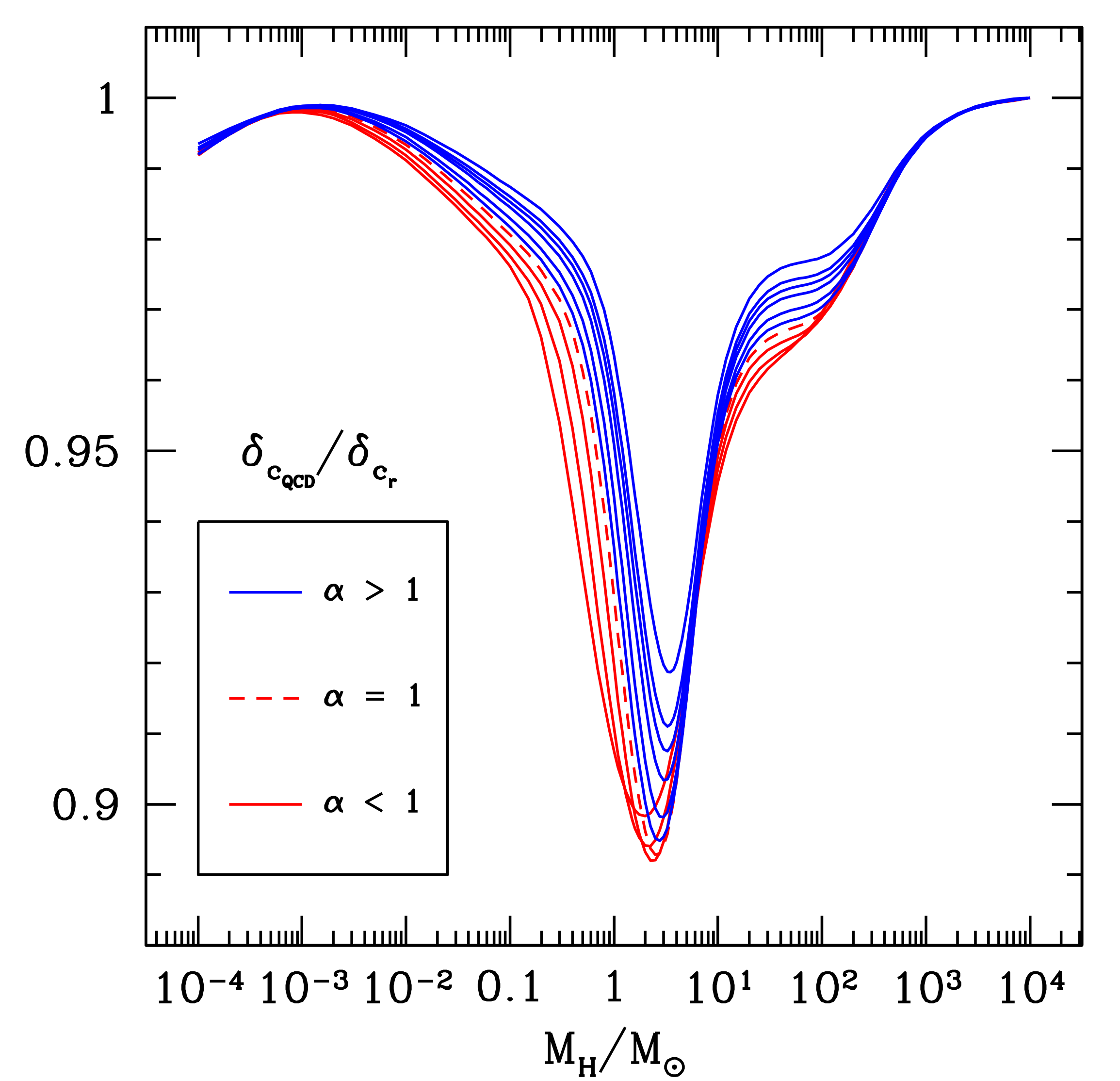
- Nucelosynthesis (e+e- annihilation)



PBH Threshold during the QCD

IM, K. Jedamzik, Sam Young - arXiv:2303.07980





Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of ***w(T)*.**

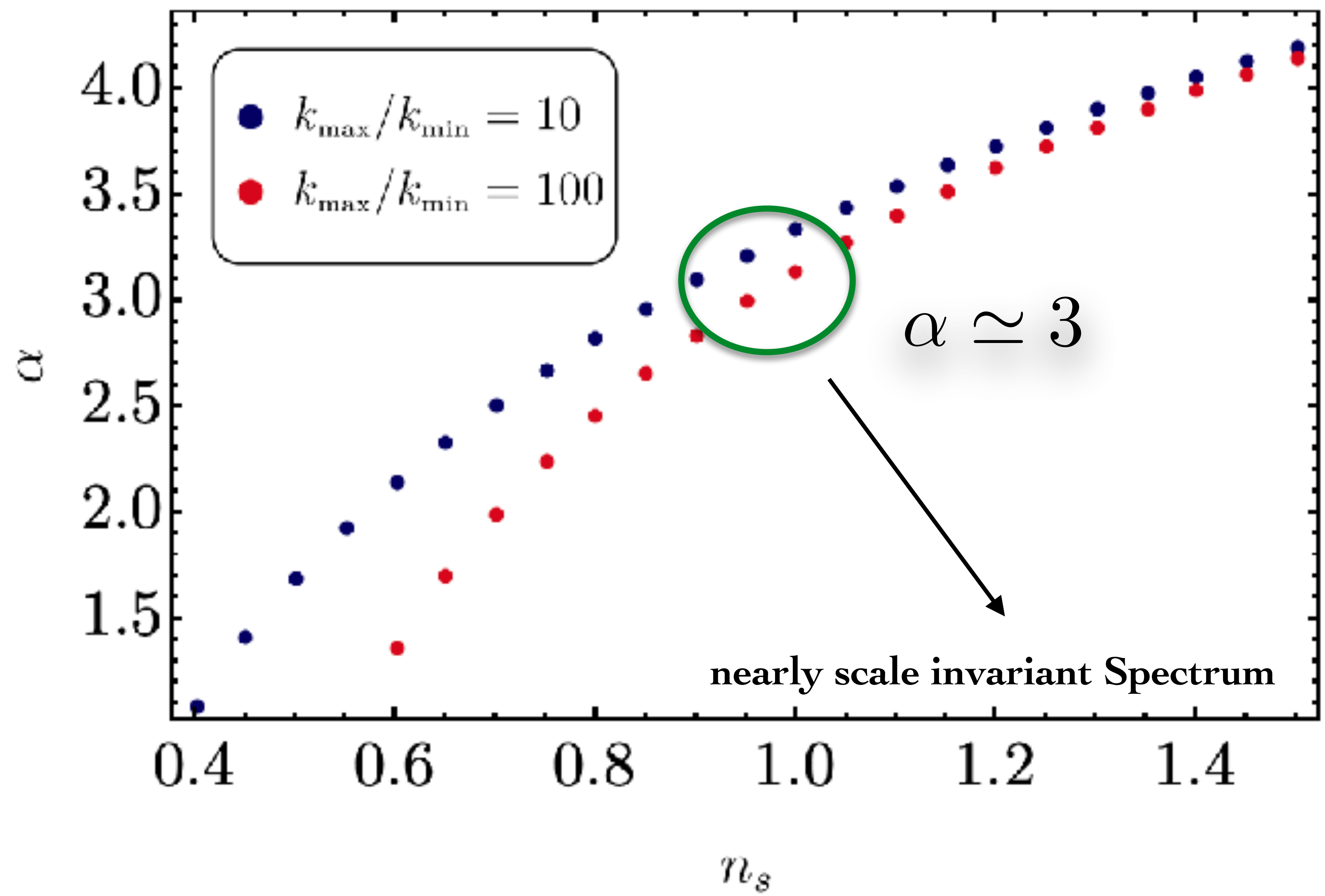
Significant enhancement of PBH formation around the solar mass scale: abundance increased of about O(3) with respect radiation!

Scale invariant Power Spectrum

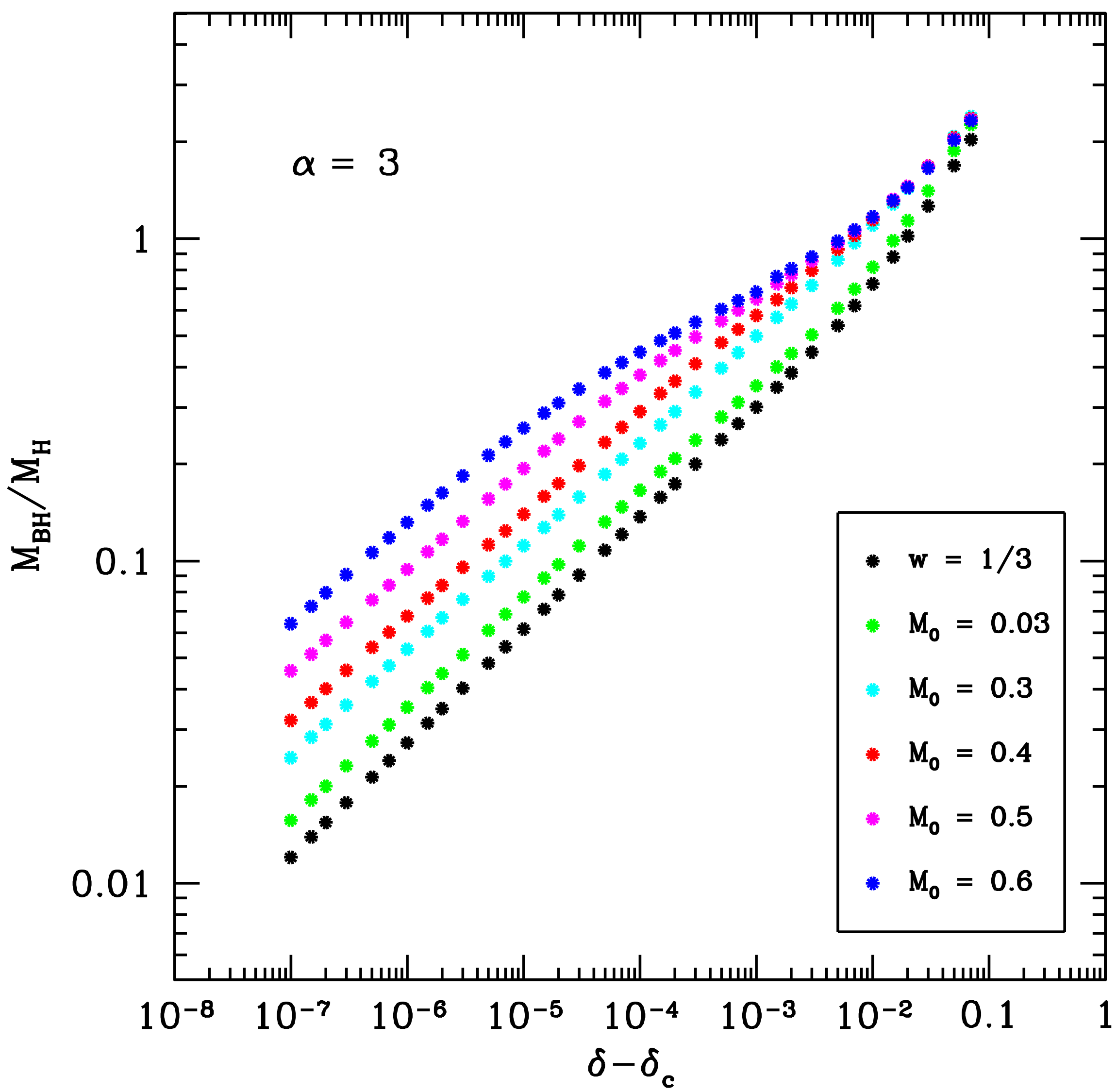
$$P_\zeta(k) = A\,(k/k_{\rm min})^{n_s-1}\,\Theta(k-k_{\rm min})\Theta(k_{\rm max}-k)$$

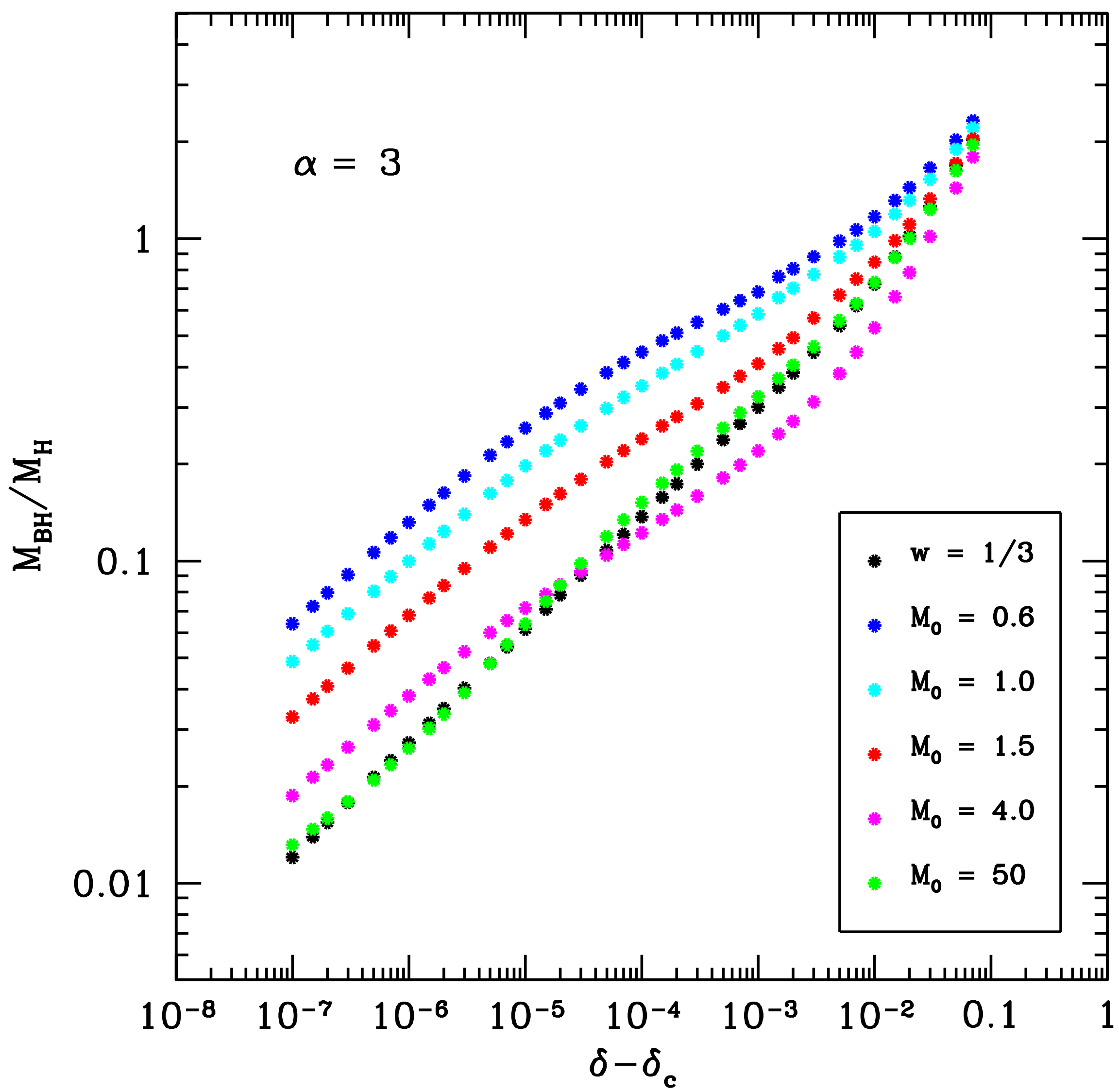
n_s — spectrum tilt

$k_{\rm max}/k_{\rm min}$ — cut-off scale



PBH scaling law during the QCD





$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

IM, K. Jedamzik, Sam Young - arXiv:2303.07980

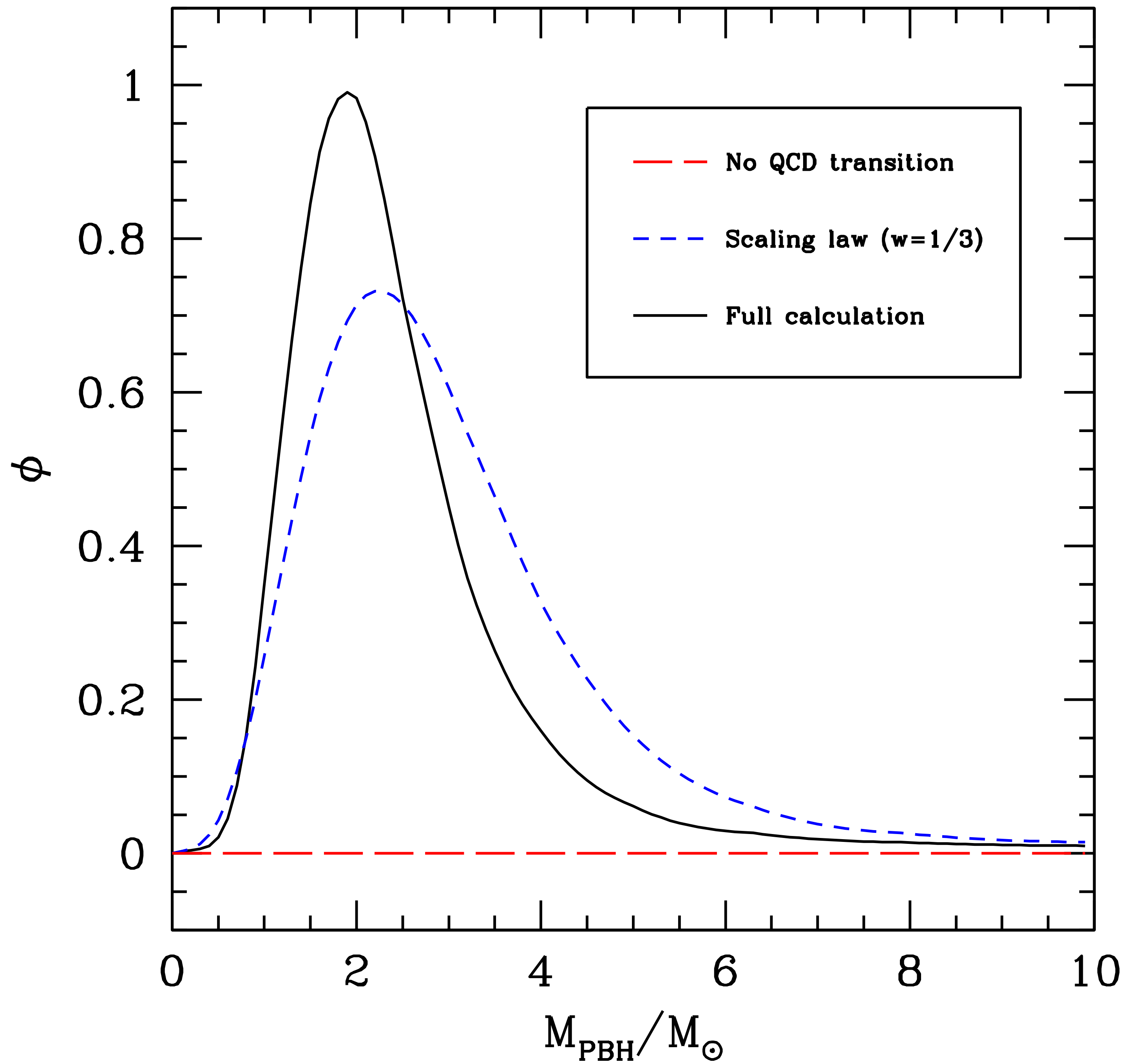
PBH mass function during the QCD

Mass Function ψ(mPBH): fraction of PBHs with mass in the infinitesimal interval of M_{PBH}

$$\phi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}M_{\rm PBH}}$$

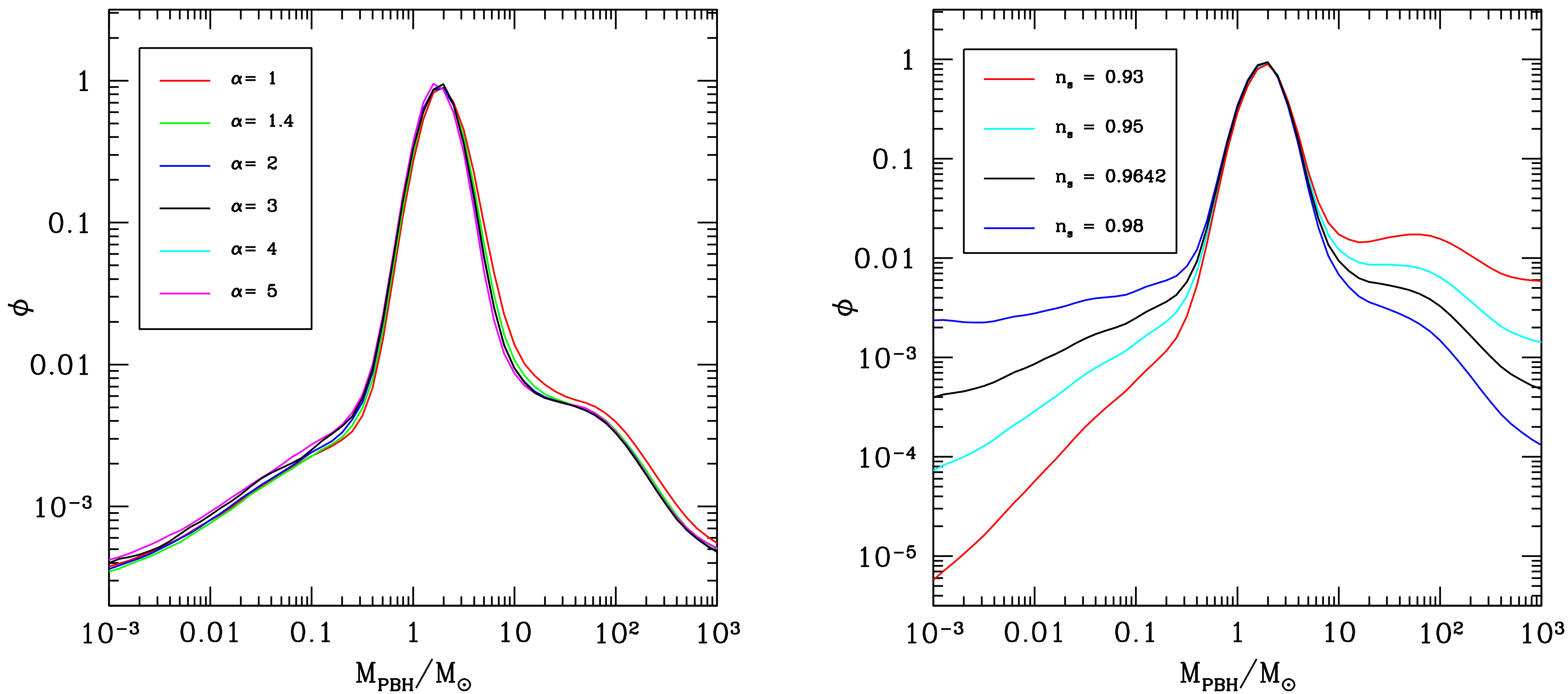
$$\int {\rm d}m_{\rm PBH}\ \phi(m_{\rm PBH}) = 1$$

- The main effect is given by the modification of the threshold.
- The modified scaling law gives a pile up of PBHs on smaller masses.



IM, K. Jedamzik, Sam Young - arXiv:2303.07980

PBH mass function during the QCD: shape/tilt dependence



- Given the PBH abundance, the shape does not play a significant role on the mass function (attractor solution)!
- The tilt of the power spectrum does not affect the peak of the mass function.

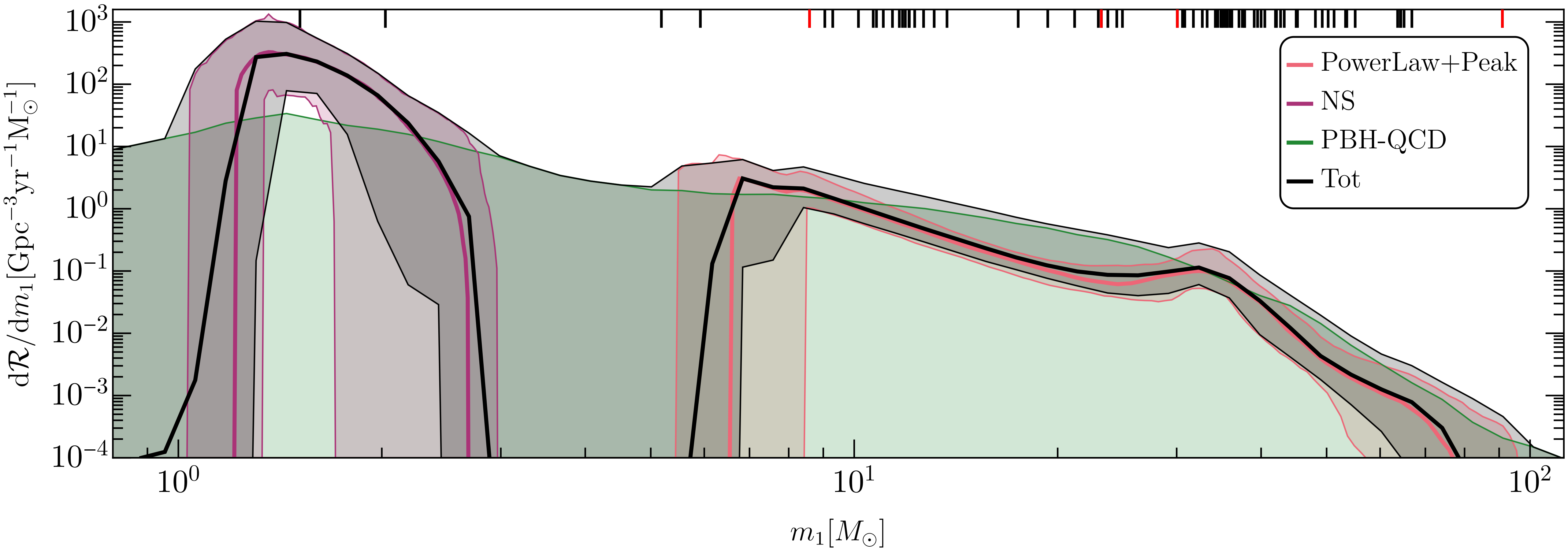
IM, K. Jedamzik, Sam Young - arXiv:2303.07980

GWs from PBH mergers

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

- Making **Bayesian inference analysis** we found that a <u>sub-population of PBHs is compatible with the LVK catalog</u>.

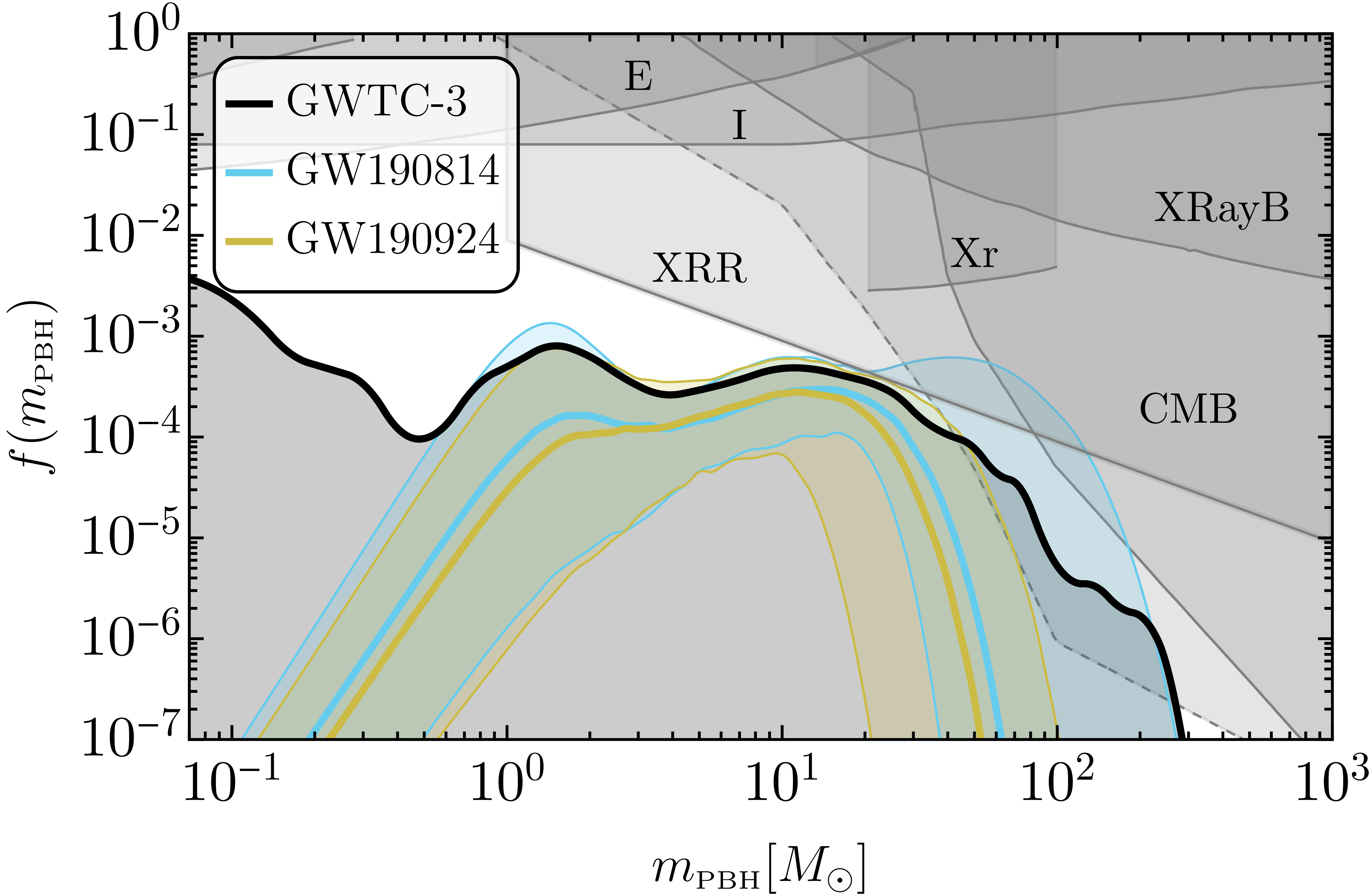
- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).



GW event	PBH prob. [%]	$m_1[M_\odot]$	$m_2[M_\odot]$
GW151012	1.2	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$
GW190412	25.4	$30.1^{+4.7}_{-5.1}$	$8.3^{+1.6}_{-0.9}$
GW190512_180714	1.6	$23.3^{+5.3}_{-5.8}$	$12.6^{+3.6}_{-2.5}$
GW190519_153544	1.5	$66.0^{+10.7}_{-12.0}$	$40.5^{+11.0}_{-11.1}$
GW190521	7.2	$95.3^{+28.7}_{-18.9}$	$69.0^{+22.7}_{-23.1}$
GW190602_175927	2.7	$69.1^{+15.7}_{-13.0}$	$47.8^{+14.3}_{-17.4}$
GW190701_203306	1.4	$53.9^{+11.8}_{-8.0}$	$40.8^{+8.7}_{-12.0}$
GW190706_222641	1.3	$67.0^{+14.6}_{-16.2}$	$38.2^{+14.6}_{-13.3}$
GW190828_065509	2.8	$24.1^{+7.0}_{-7.2}$	$10.2^{+3.6}_{-2.1}$
GW190924_021846	40.3	$8.9^{+7.0}_{-2.0}$	$5.0^{+1.4}_{-1.9}$
GW191109_010717	2.9	65^{+11}_{-11}	47^{+15}_{-13}
GW191129_134029	1.2	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$
GW190425	2.8	$2.0^{+0.6}_{-0.3}$	$1.4^{+0.3}_{-0.3}$
GW190426_152155	1.2	$5.7^{+3.9}_{-2.3}$	$1.5^{+0.8}_{-0.5}$
GW190814	29.1	$23.2^{+1.1}_{-1.0}$	$2.59^{+0.08}_{-0.09}$
GW190917_114630	3.0	$9.3^{+3.4}_{-4.4}$	$2.1^{+1.5}_{-0.5}$
GW200105_162426	3.6	$8.9^{+1.2}_{-1.5}$	$1.9^{+0.3}_{-0.2}$
GW200115_042309	1.2	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$

PBH - DM constraints

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)



Conclusions

- The **non linear threshold for PBH** and **the mass function** could be fully computed from the **shape of the power spectrum of cosmological perturbations**, making relativistic numerical simulations.

- A softening of the equation of state **(QCD) significantly enhances the formation of PBHs**, with a **mass distribution peaked between 1 and 2 solar masses (**the range of heavy NSs and light BHs).

- This could gives **a sub-population of BH mergers compatible with the LVK catalog**, explaining mass gap events as **GW190814**.

- Our analysis predicts a **constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%)**, compatible with the current observational constraints.

- **A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range** (USR inflation models).

Workshop (Rome 11-13 December): Future Perspectives on PBHs

- 2.5 days in the Botanic Garden of Trastevere (Rome downtown)
- 9 Invited talks, 5 contributed talks for young researches, 3 moderated discussions.
- Special Event 12 December: "**The Nature of Time and Mach Principle**"

https://agenda.infn.it/event/35854/

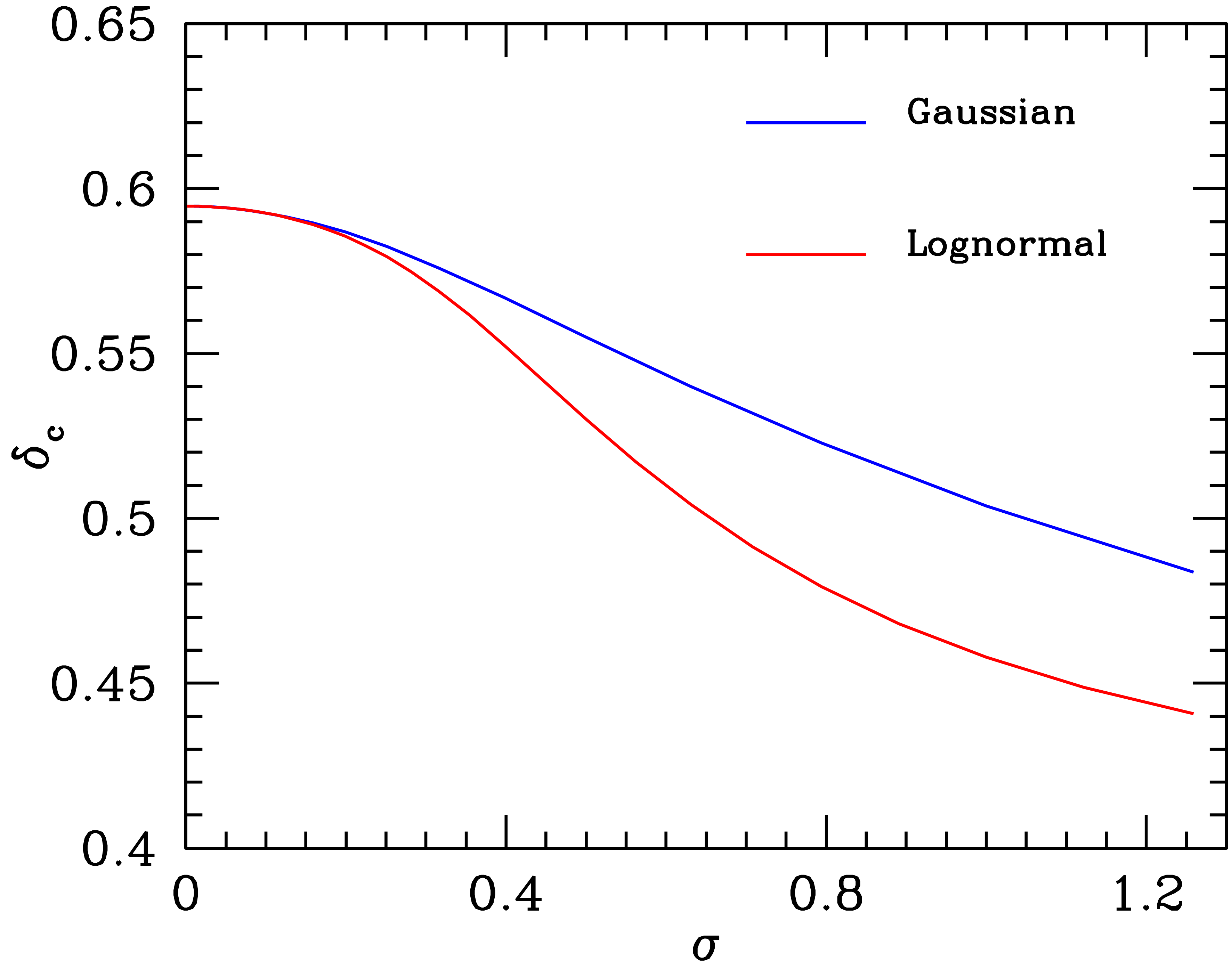
Istituto Nazionale di Fisica Nucleare

SAPIENZA UNIVERSITÀ DI ROMA

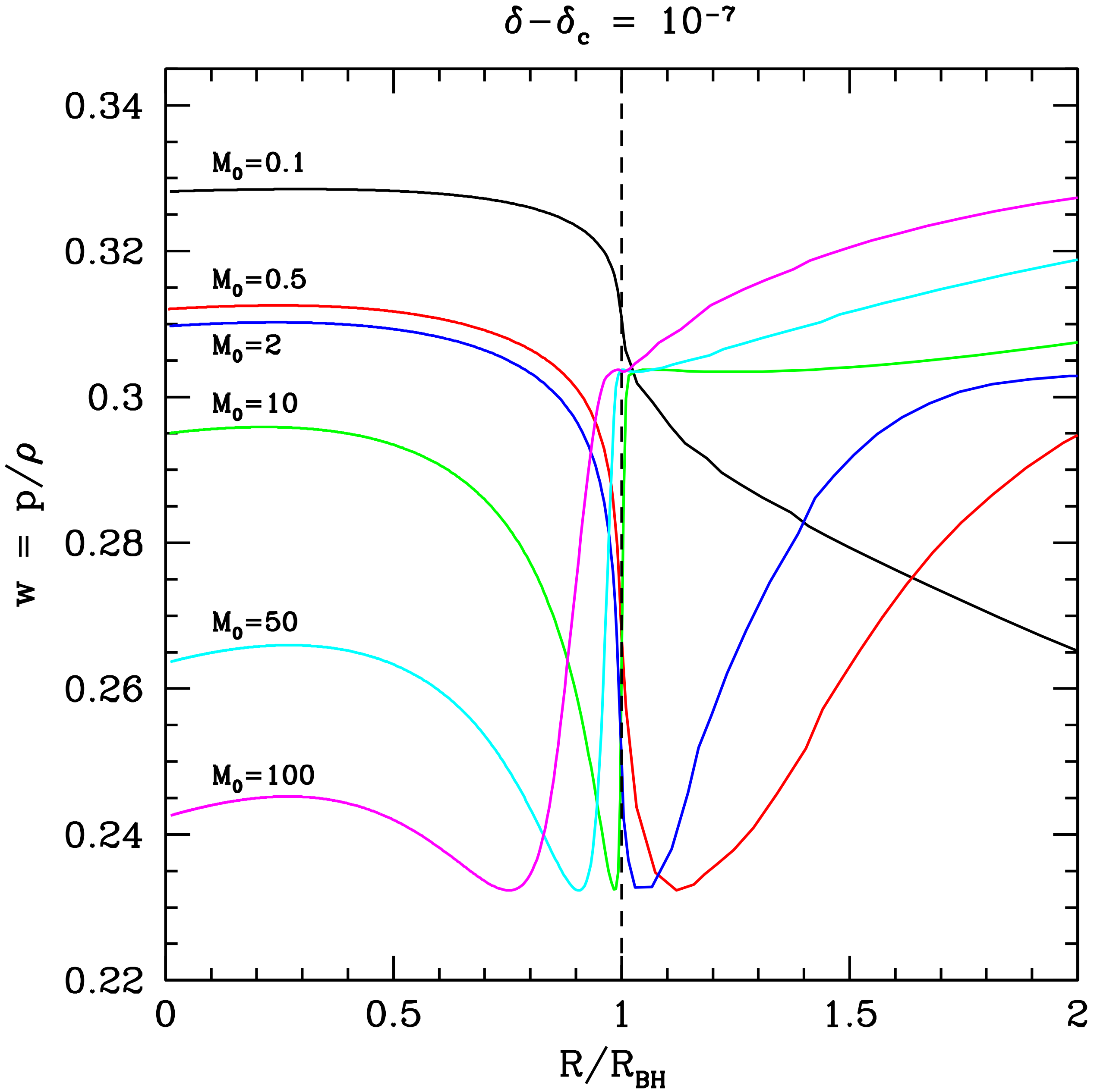
Power Spectrum:

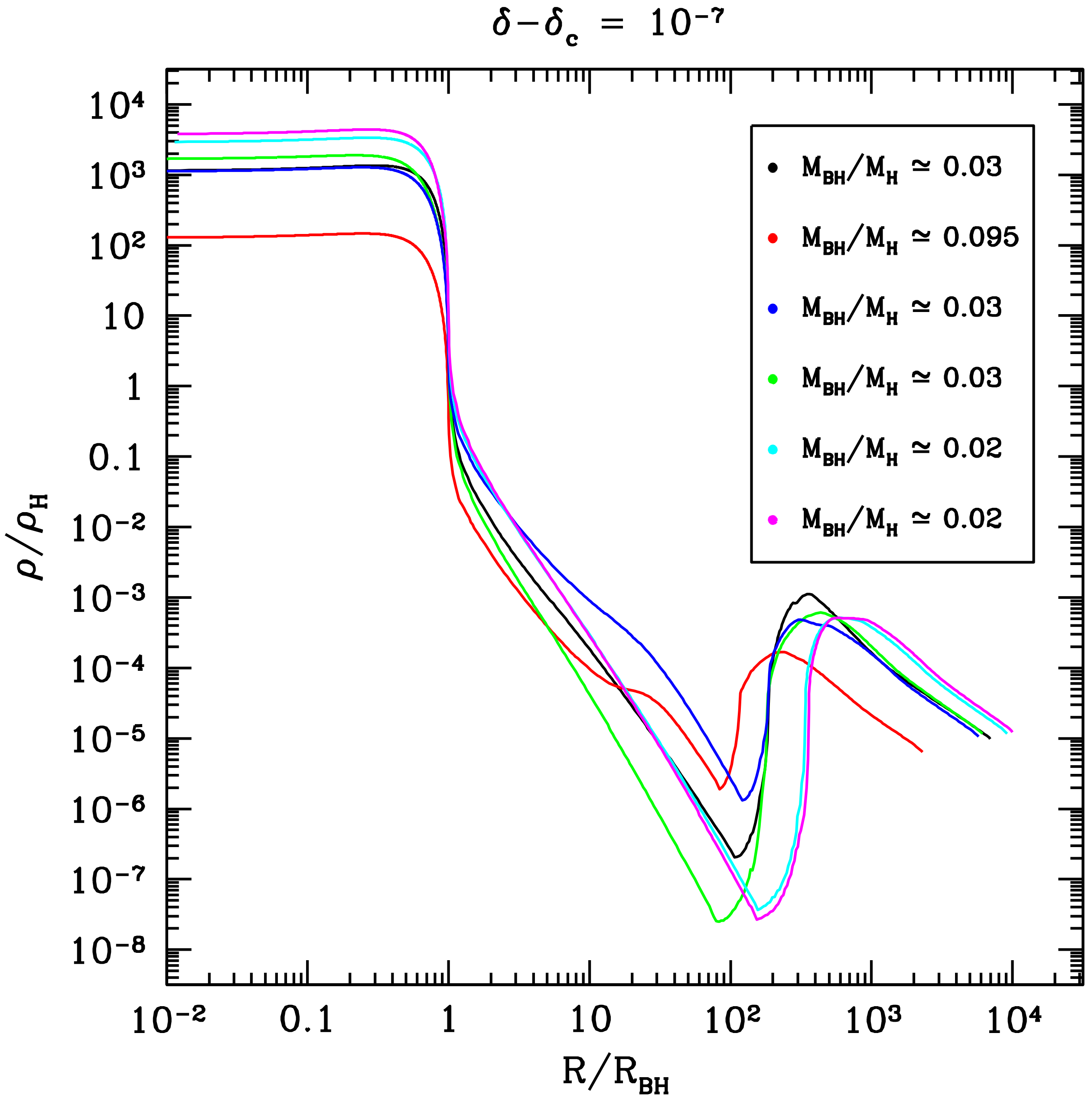
Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$

Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$



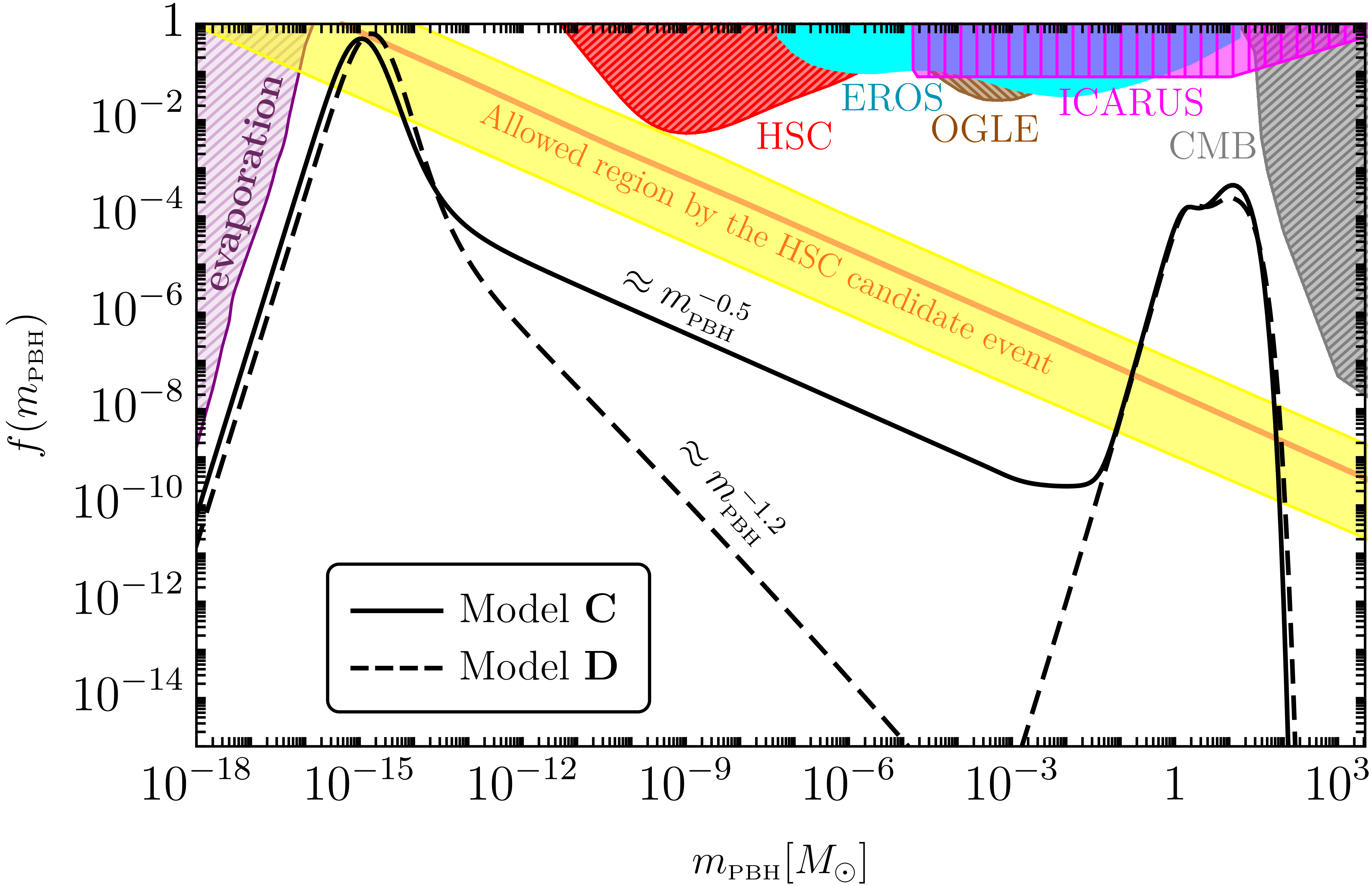
PBH formation during the QCD





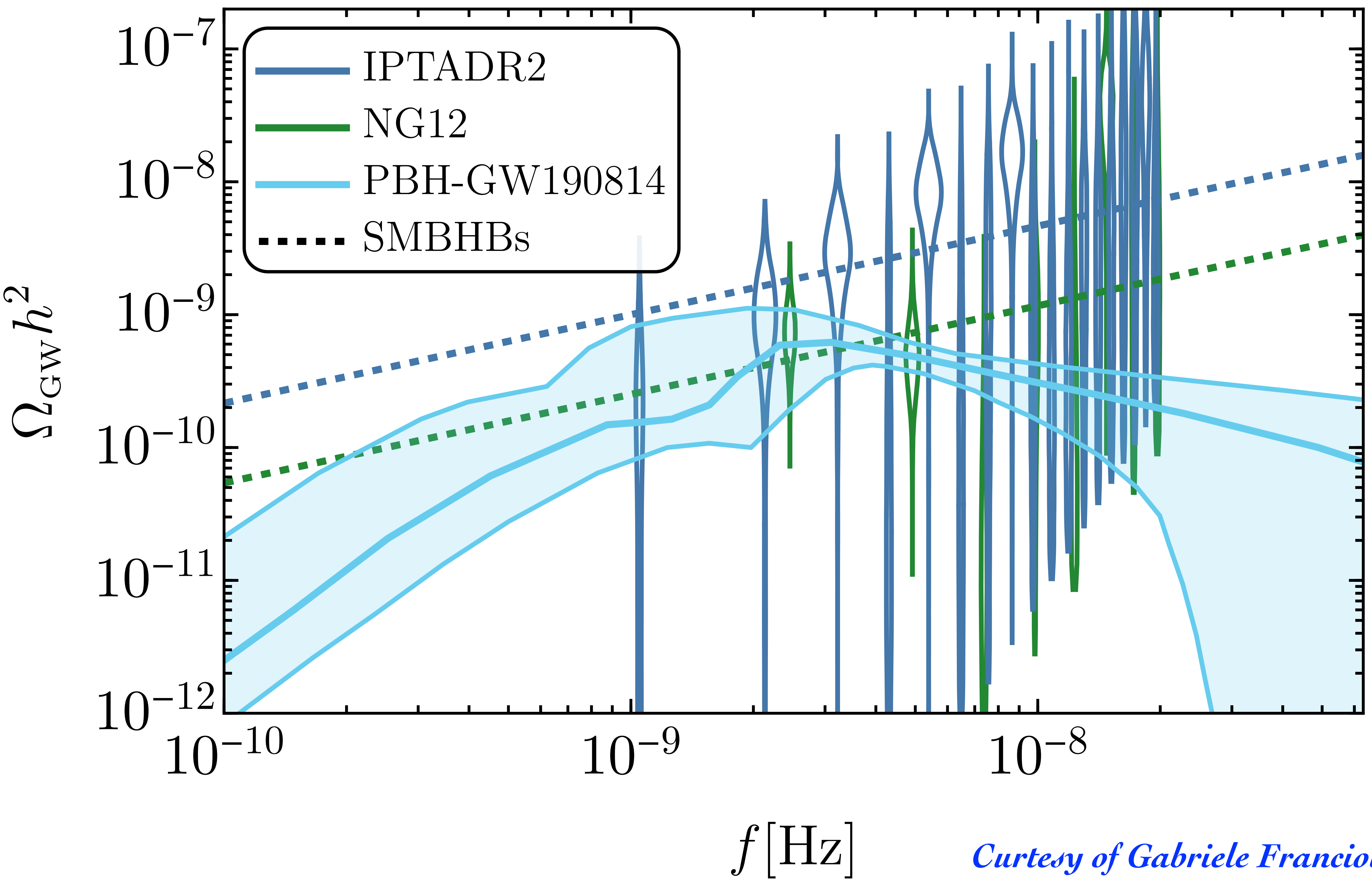
IM, K. Jedamzik, Sam Young - arXiv:2303.07980

PBHs and Dark Matter (asteroidal mass)



G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

PBH - DM constraints (PTA / NANOGrav)



Curtesy of Gabriele Franciolini

Relativistic Hydrodynamics in spherical symmetry

$$\mathrm{d}s^2 = -A^2(t,r)\mathrm{d}t^2 + B^2(t,r)\mathrm{d}r^2 + R^2(t,r)\mathrm{d}\Omega^2$$ - comoving (cosmic time) slicing

$$U \equiv D_t R \equiv \frac{1}{A}\frac{\partial R}{\partial t}\bigg|_r \qquad \Gamma \equiv D_r R \equiv \frac{1}{B}\frac{\partial R}{\partial r}\bigg|_t$$

$$D_t U = -\frac{\Gamma}{\rho + p_\mathrm{r}}\left[D_r p_\mathrm{r} + \frac{2\Gamma}{R}\left(p_\mathrm{r} - p_\mathrm{t}\right)\right] - \frac{M}{R^2} - 4\pi R p_\mathrm{r}$$

$$\frac{D_t \rho_0}{\rho_0} = -\frac{1}{R^2\Gamma} D_r\left(R^2 U\right)$$

$$\frac{D_t \rho}{\rho + p_\mathrm{r}} = \frac{D_t \rho_0}{\rho_0} + \frac{2U}{R}\frac{p_\mathrm{r} - p_\mathrm{t}}{\rho + p_\mathrm{r}}$$

$$\frac{D_r A}{A} = -\frac{1}{\rho + p_\mathrm{r}}\left[D_r p_\mathrm{r} + \frac{2\Gamma}{R}\left(p_\mathrm{r} - p_\mathrm{t}\right)\right]$$

$$D_r M = 4\pi R^2 \Gamma \rho$$

$$D_t M = -4\pi R^2 U p_\mathrm{r}$$

$$D_t \Gamma = -\frac{U}{\rho + p_\mathrm{r}}\left[D_r p_\mathrm{r} + \frac{2\Gamma}{R}\left(p_\mathrm{r} - p_\mathrm{t}\right)\right]$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

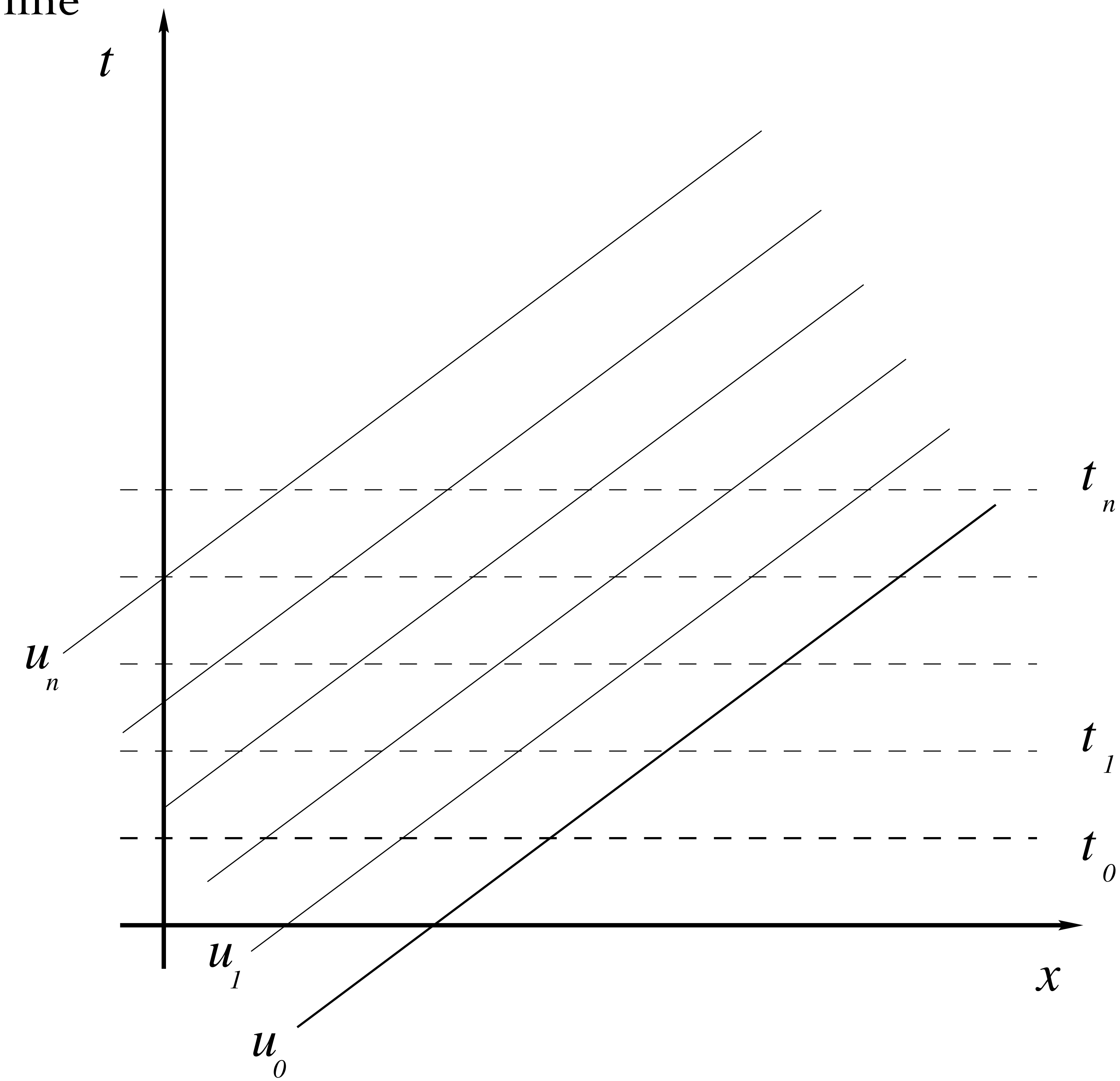
Numerical technique

The diagram show two different time foliation (space-time slicing):

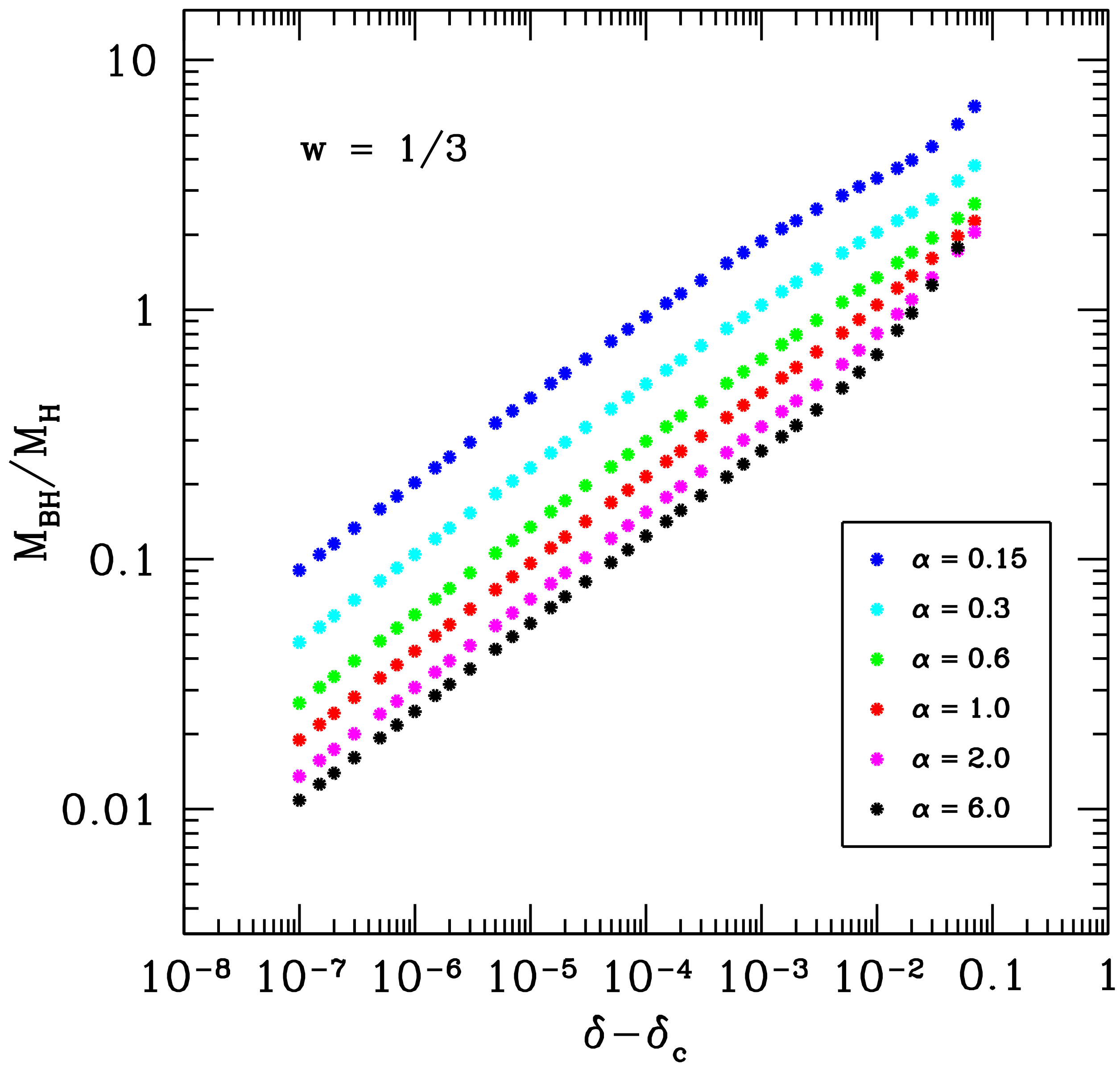
1. comoving (cosmic time) - solid line
2. null (observer time) - dashed line

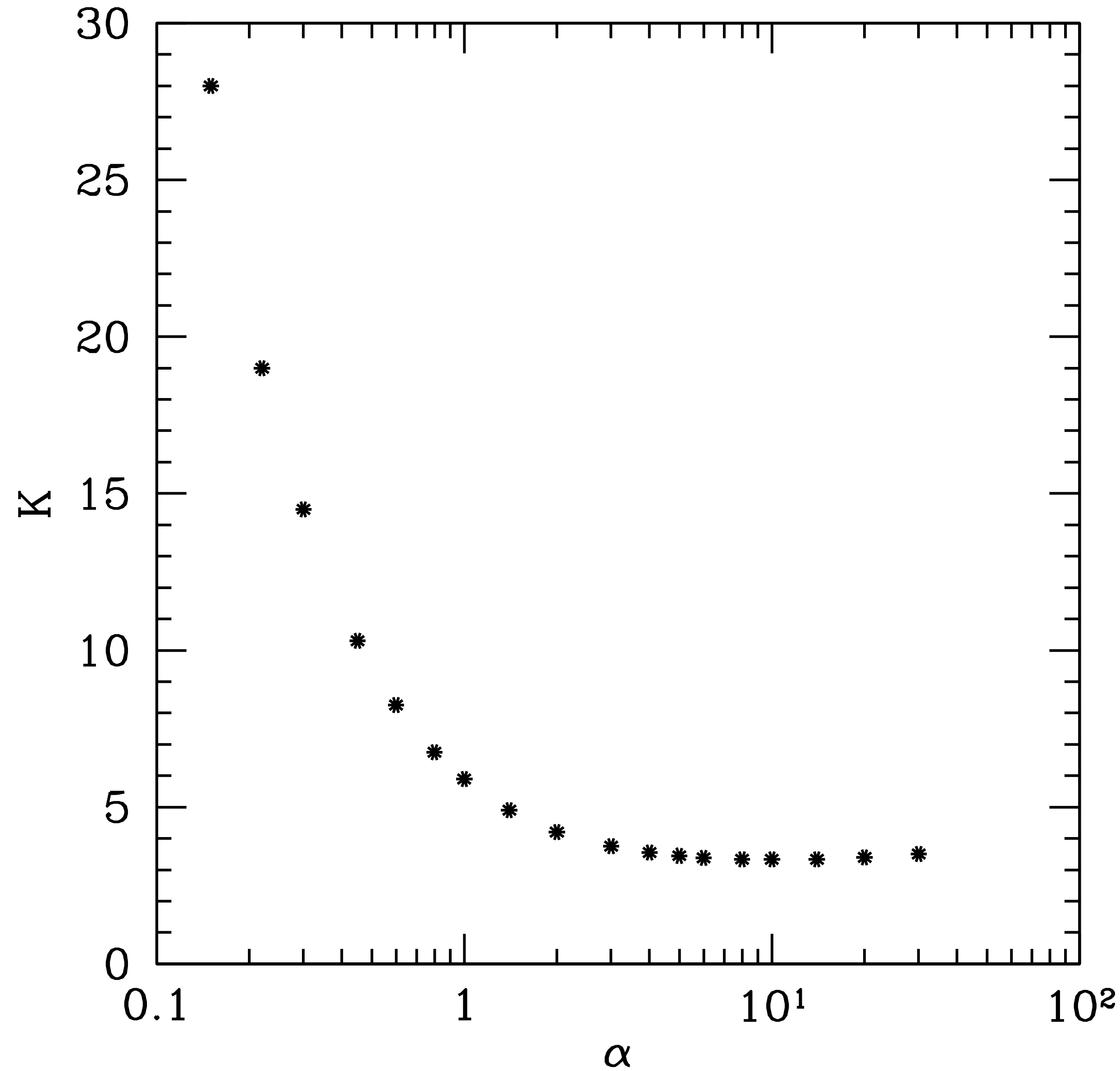
The initial conditions are specified on the cosmic time slicing. To follow the full evolution, and avoid the central singularity, one can transform the calculation into the observer time slicing, where the formation of the apparent horizon formation is infinitely redshifted. This allow to fully compute the final mass of the BH.

The code is using AMR.

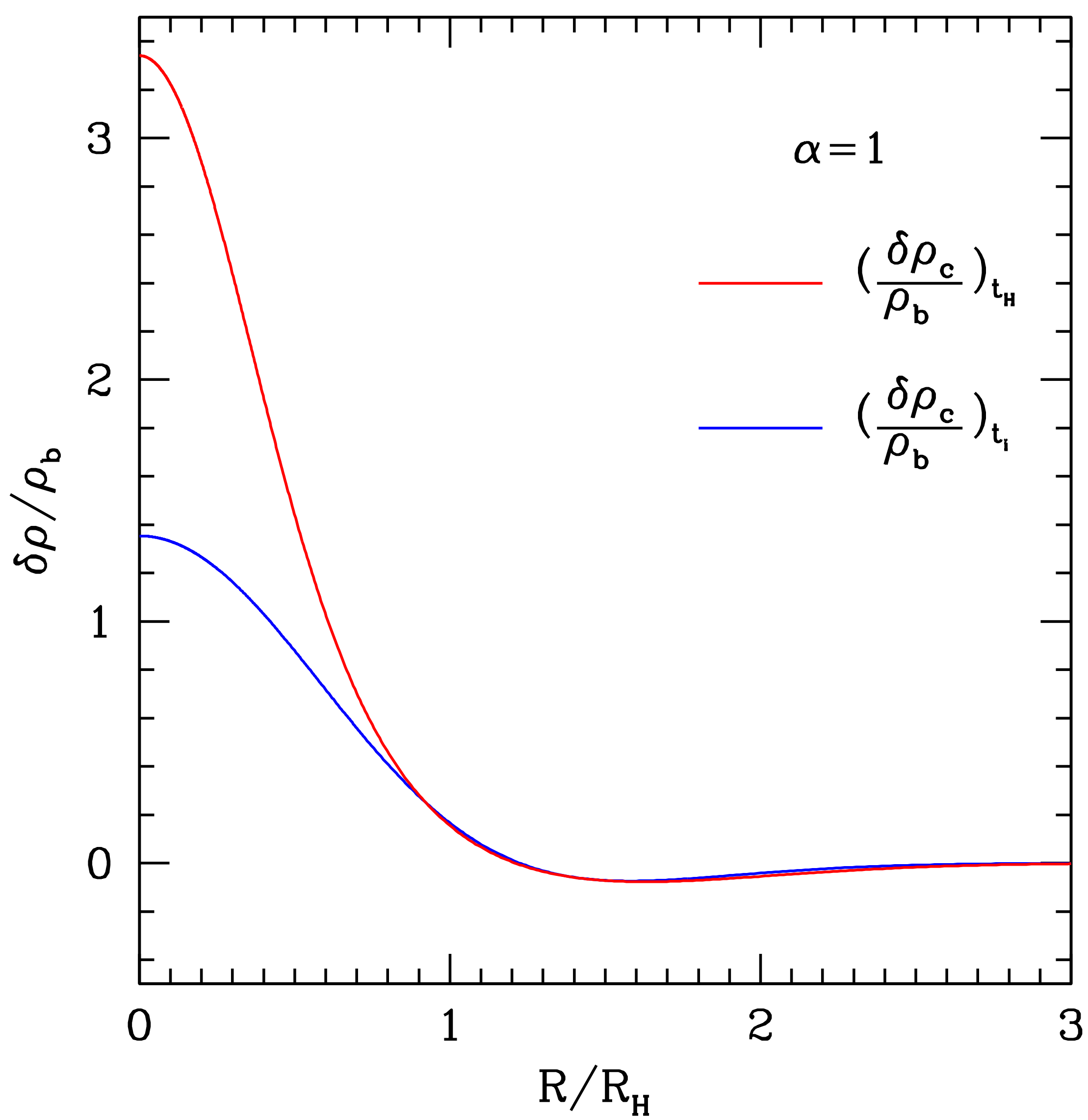


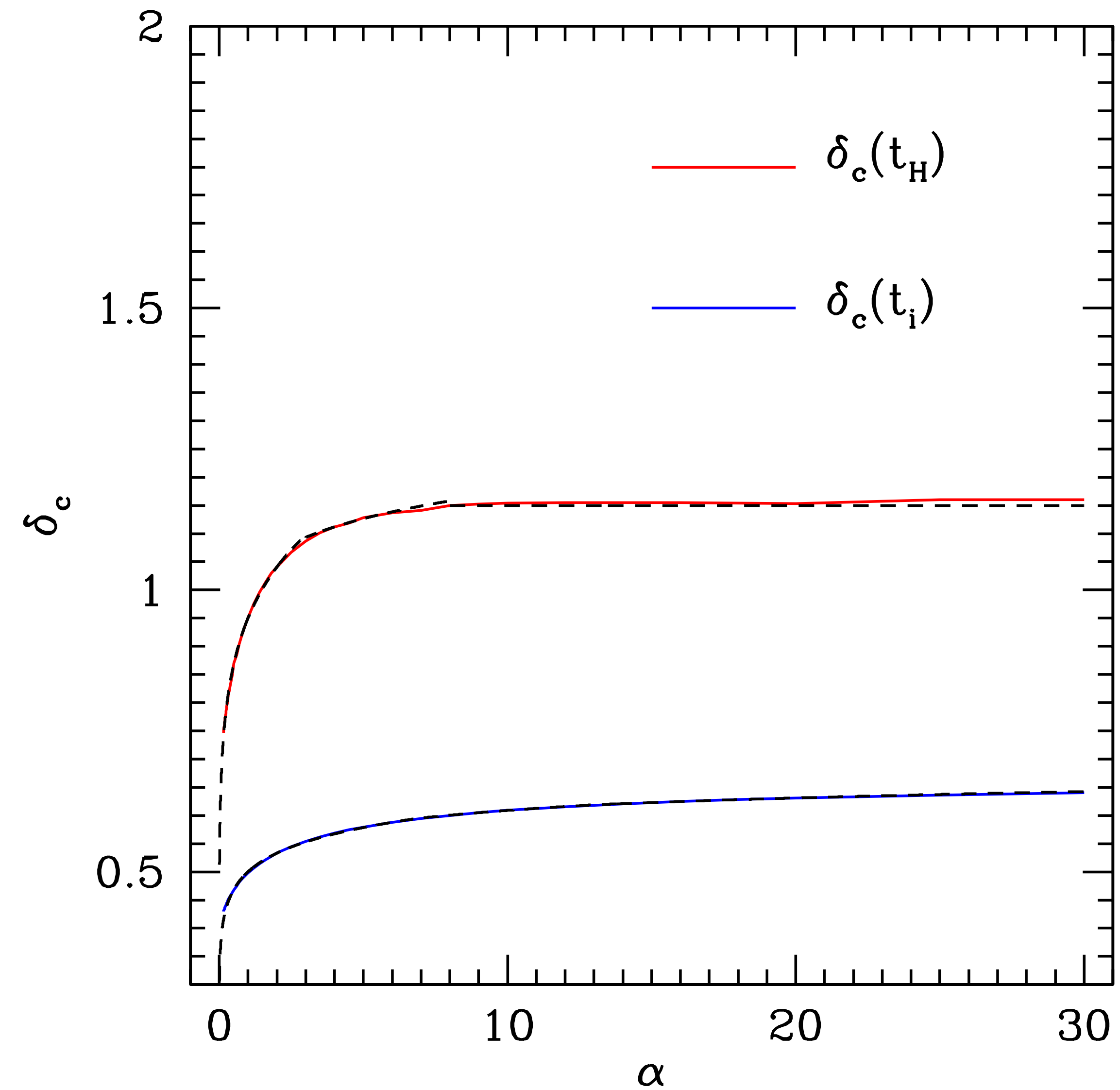
Scaling law (critical collapse)





Non linear horizon crossing

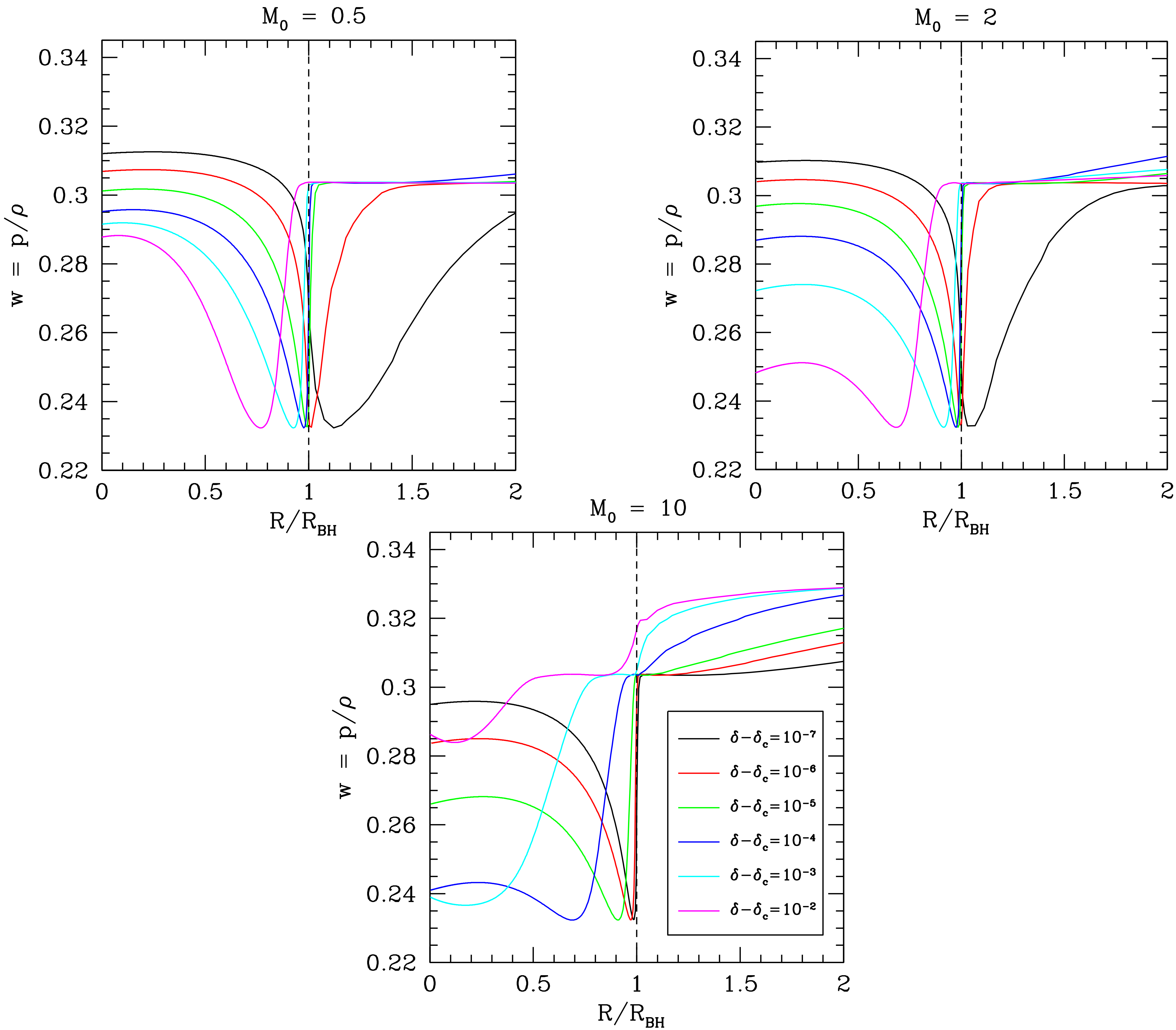




$$\delta_c(t_i) \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

$$\delta_c(t_H) \simeq \begin{cases} \alpha^{0.125} - 0.05 & 0.1 \lesssim \alpha \lesssim 3 \\ \alpha^{0.06} + 0.025 & 3 \lesssim \alpha \lesssim 8 \\ 1.15 & \alpha \gtrsim 8 \end{cases}$$

$$0.4 \leq \delta_c(t_i) < 0.6 \qquad 0.7 \lesssim \delta_c(t_H) \lesssim 1.15$$



M0 = 0.5
M0 = 2
M0 = 10
w = p/ρ
R/R_BH
0.22
0.24
0.26
0.28
0.3
0.32
0.34
0
0.5
1
1.5
2
δ−δc=10⁻⁷
δ−δc=10⁻⁶
δ−δc=10⁻⁵
δ−δc=10⁻⁴
δ−δc=10⁻³
δ−δc=10⁻²

Comparison with EBC

Escriva, Bagui, Clesse - arXiv:2209.06196

- Our (AMR) numerical scheme gives a larger variation of the threshold (about 25%).
- Scaling law was not considered: broader mass function profile.
- Press-Schechter or Peak Theory? Only perk theory allow to have a proper commutation of the abundance, and not overestimate PBHs