Minkowski Functionals as a tool to study Non-Gaussianity and anisotropy: new extensions to CMB polarization and beyond

Javier **Carrón Duque**

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In collaboration with:
Alessandro **Carones**
Domenico **Marinucci**
Marina **Migliaccio**
Nicola **Vittorio**

31st August 2023
Cosmology 2023 in Miramare

ACCORDO ATTUATIVO N. 2021-43-HH.0

dell'Accordo Quadro ASI/INFN n. 2021-8-Q.0

Codice Unico di Progetto (CUP) F85F21006430005

PER

"Realizzazione di attività tecniche e scientifiche presso lo Space Science Data Center - SSDC"

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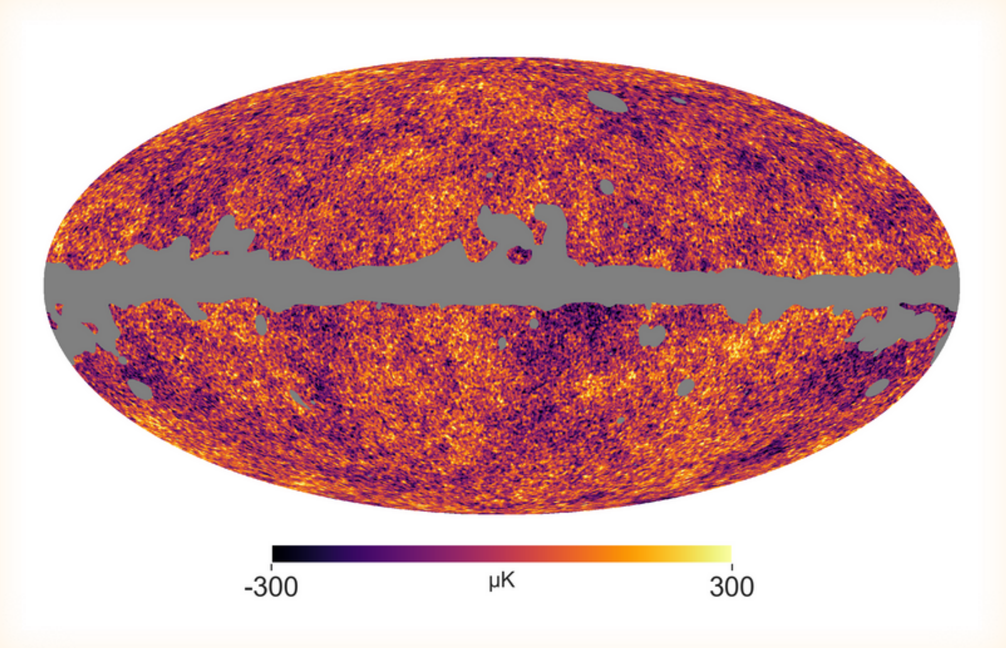
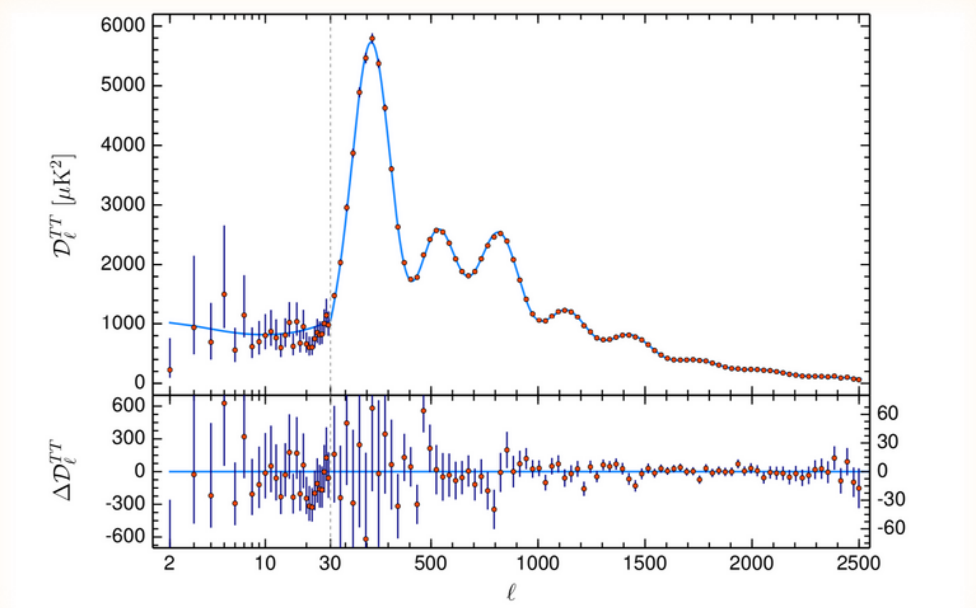
Outline

- Introduction
- Minkowski Functionals on CMB polarization
- Applications of Minkowski Functionals
- Software
- Conclusions

Gaussian fields are easy to describe

- Gaussian → Physical process fully described by 2pt correlation function





Source: Planck Collaboration

- Blind to non—Gaussianity and anisotropy
- Other tools: 3/4pt correlation functions, extrema statistics, Betti numbers, persistent homology, field-level inference, Machine Learning, Minkowski Functionals...

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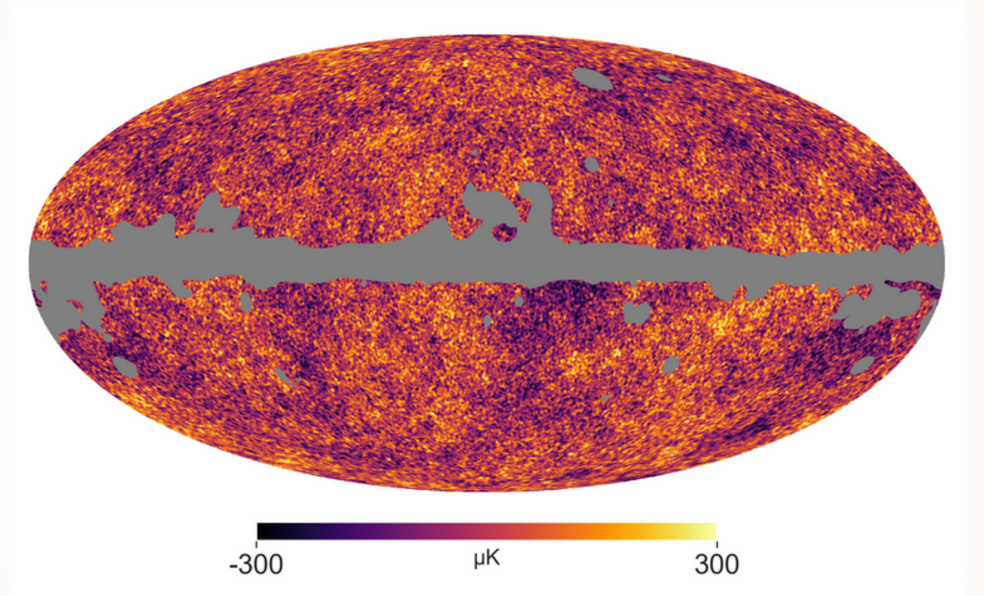

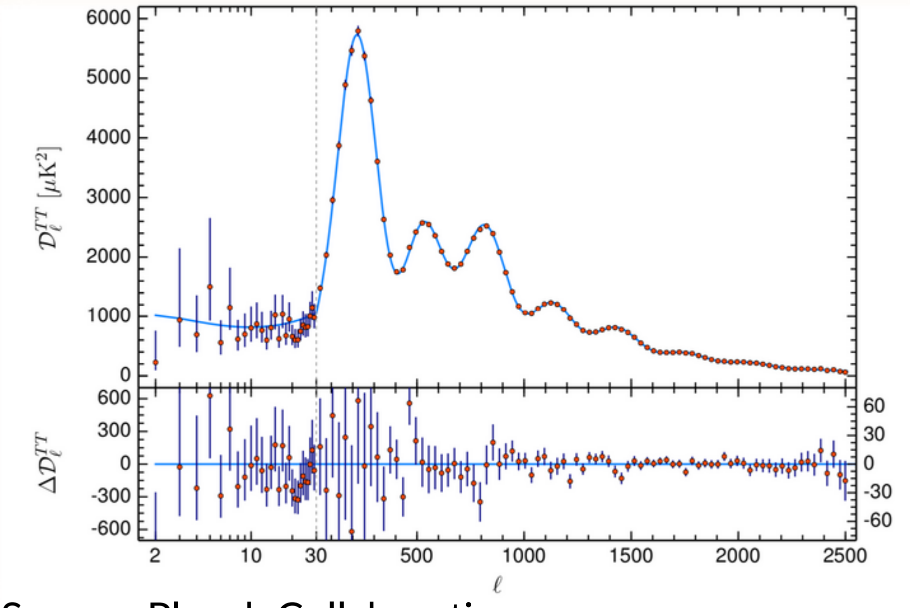

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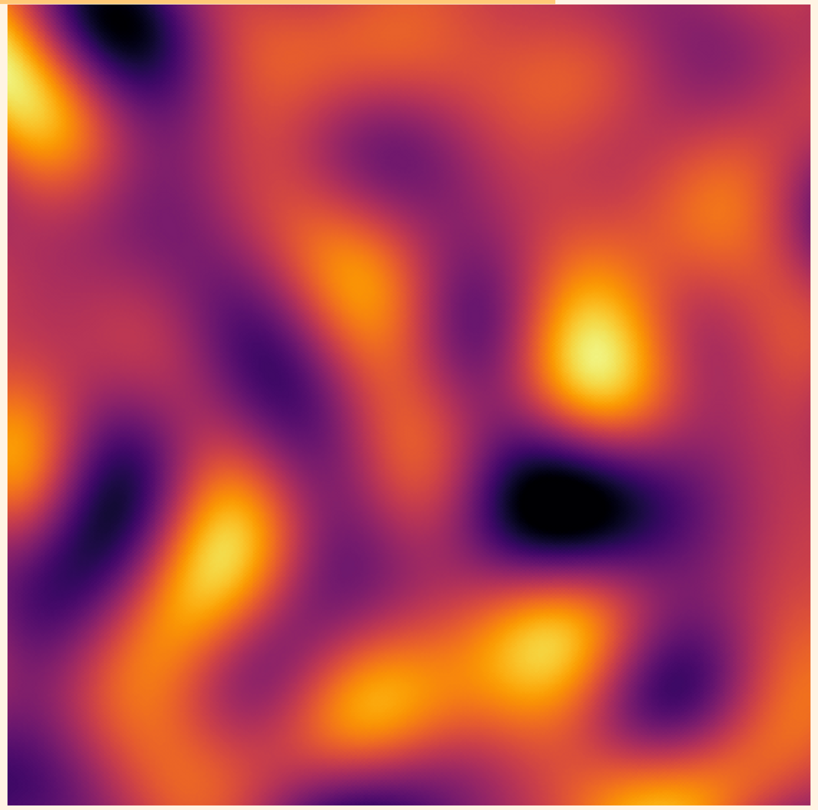
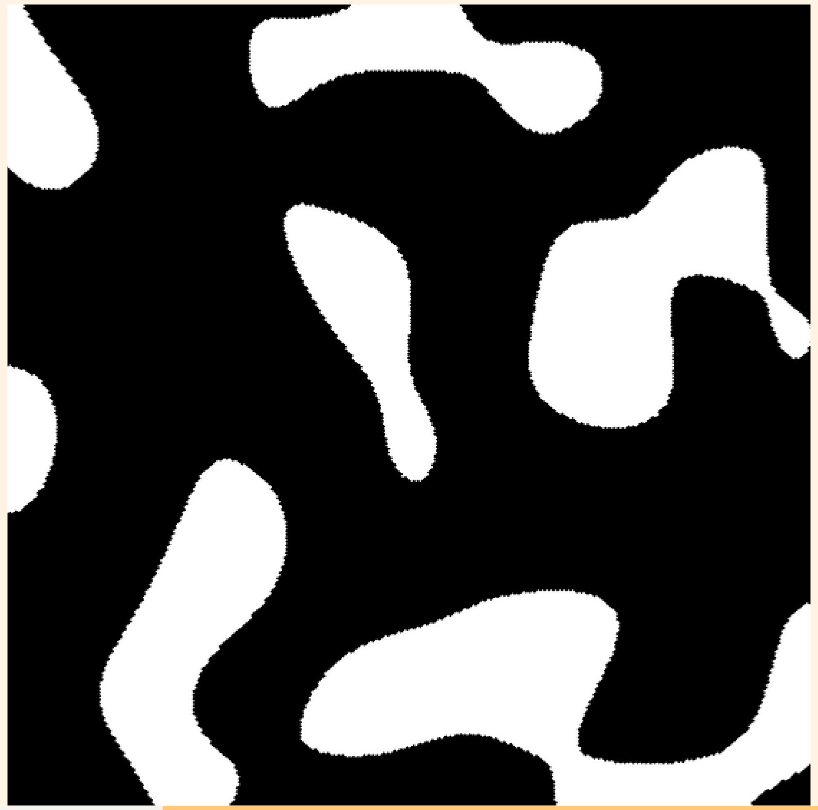
- Other tools: 3/4pt correlation functions, extrema statistics, Betti numbers, persistent homology, field-level inference, Machine Learning, Minkowski Functionals...
- f_{NL} is very important, but not the only way

Minkowski Functionals are higher order statistics

- We consider a field (*e.g.*, T or δ)
- Let u be a threshold (*e.g.*, 2σ)
- We define the **excursion set** $A(u)$ as the regions of the field above u
- Minkowski Functionals (MFs) are:
  - V_0 : area of $A(u)$
  - V_1 : boundary length of $A(u)$
  - V_2 : Euler—Poincaré characteristic of $A(u)$ (#regions – #holes)





MFs are accurately predicted

- For isotropic Gaussian fields, the expectation is known and variance is small
- The three factors decouple:

$$\mathbb{E}\left[V_j(A_u)\right] \approx \rho_j(u)\; V_0(\mathbb{S}^2)\; \mu^{j/2}$$

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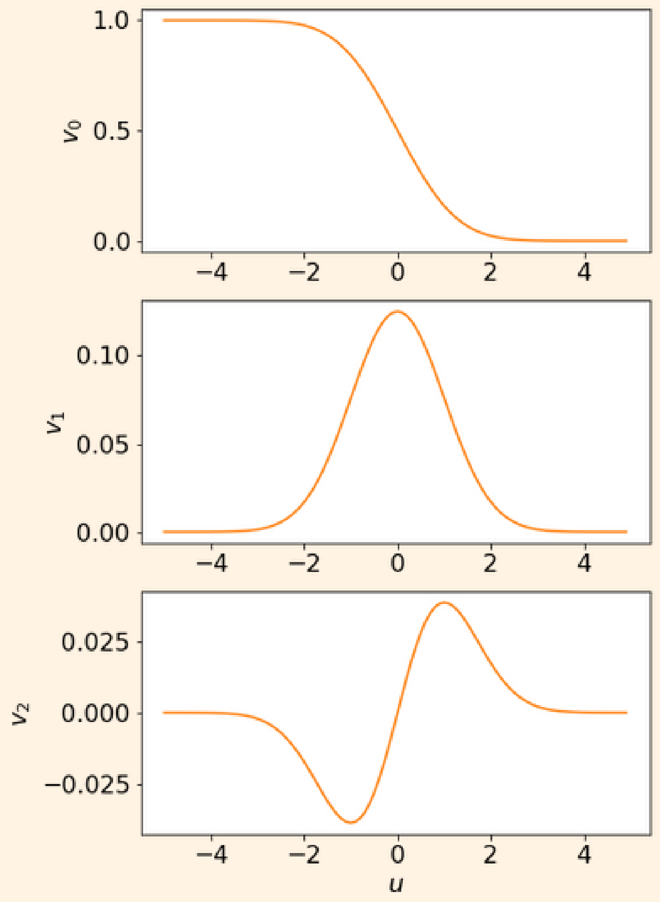
Threshold — Ambient manifold — Property of the map (correlation length)



$$\frac{\mathbb{E}\left[V_0(A_u)\right]}{4\pi} = 1 - \Phi(u)$$

$$\frac{\mathbb{E}\left[V_1(A_u)\right]}{4\pi} = \frac{1}{8}\exp(-\frac{u^2}{2})\mu^{1/2}$$

$$\frac{\mathbb{E}\left[V_2(A_u)\right]}{4\pi} = \frac{\mu}{\sqrt{(2\pi)^3}}\exp(-\frac{u^2}{2})u$$

MFs have plenty of applications

- Any deviation is due to non—Gaussianity and/or anisotropy
- Early Universe (*e.g.*, T): test for primordial non—Gaussianity
  - Planck 2018 VII (isotropy & statistics)

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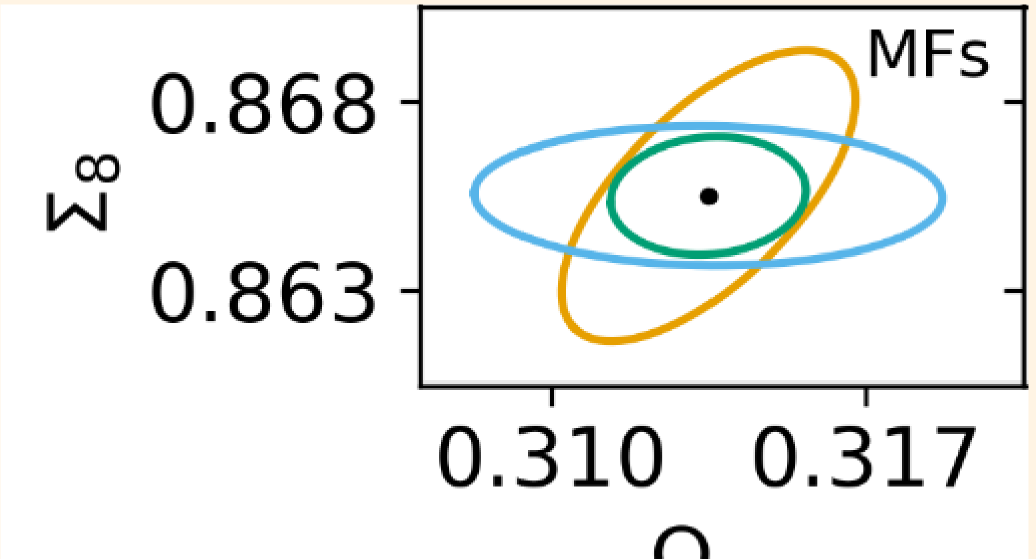

Euclid Preparation XXIX

- Any deviation is due to non−Gaussianity and/or anisotropy
- Early Universe (*e.g.*, T): test for primordial non−Gaussianity
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- Late Universe (*e.g.*, κ): extract more cosmological information
  - Euclid Preparation XXIX (2023), Grewal+ (2022), Zürcher+ (2022), ...

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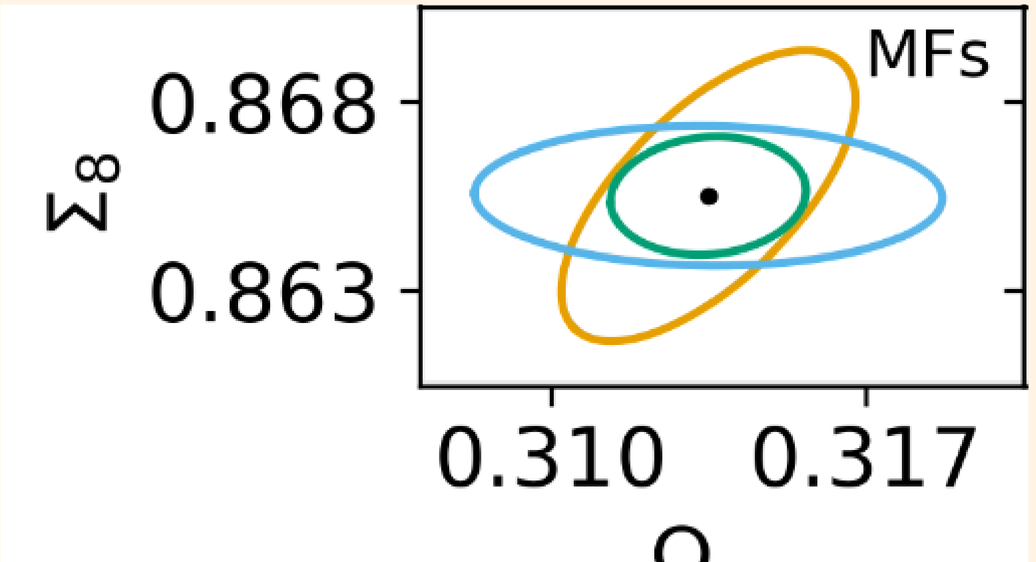

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- Foregrounds (*e.g.*, Galactic):
  - Martire+ (2023), Krachmalnicoff+ (2020), ...

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- Foregrounds (*e.g.*, Galactic):
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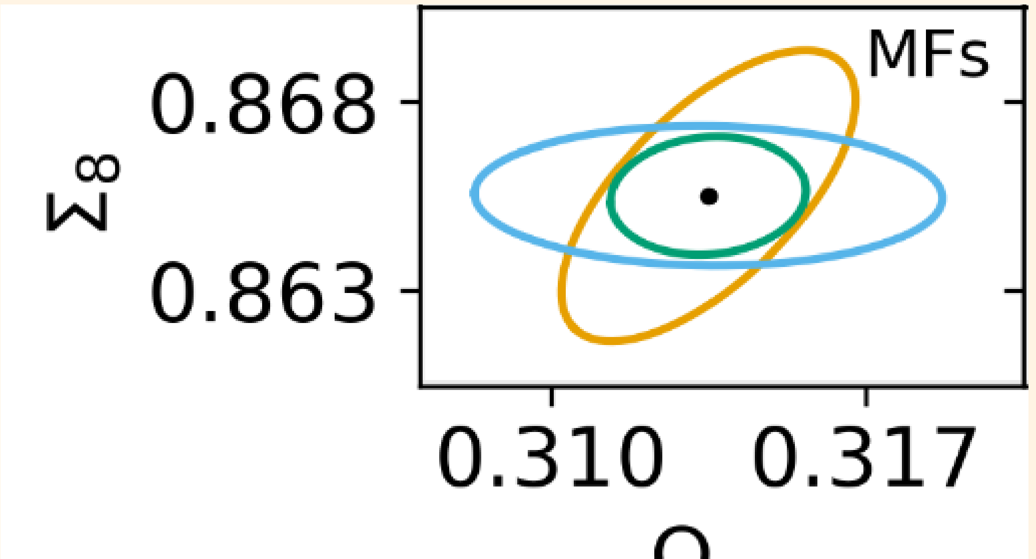
- Large Scale Structure (*e.g.*, galaxy distribution):
  - Liu+ (2023), Appleby+ (2022), Spina (2021), ...



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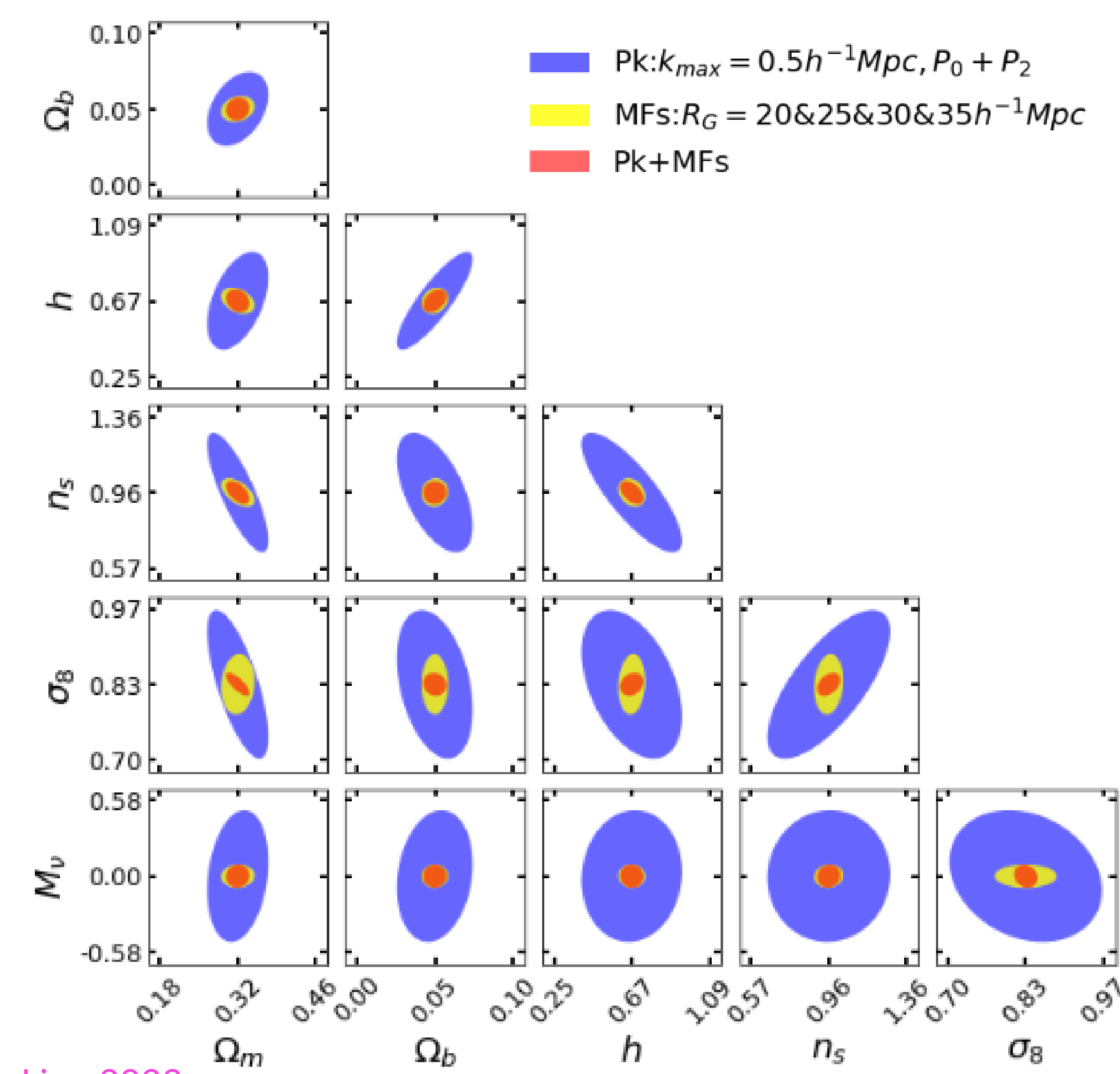


Liu+ 2023

We extend MFs to modulus of polarization P^2

Minkowski Functionals of CMB polarisation intensity with Pynkowski: theory and application to Planck data

Alessandro Carones,[1,2★] Javier Carrón Duque,[1,2] Domenico Marinucci,[3] Marina Migliaccio,[1,2] Nicola Vittorio[1,2]

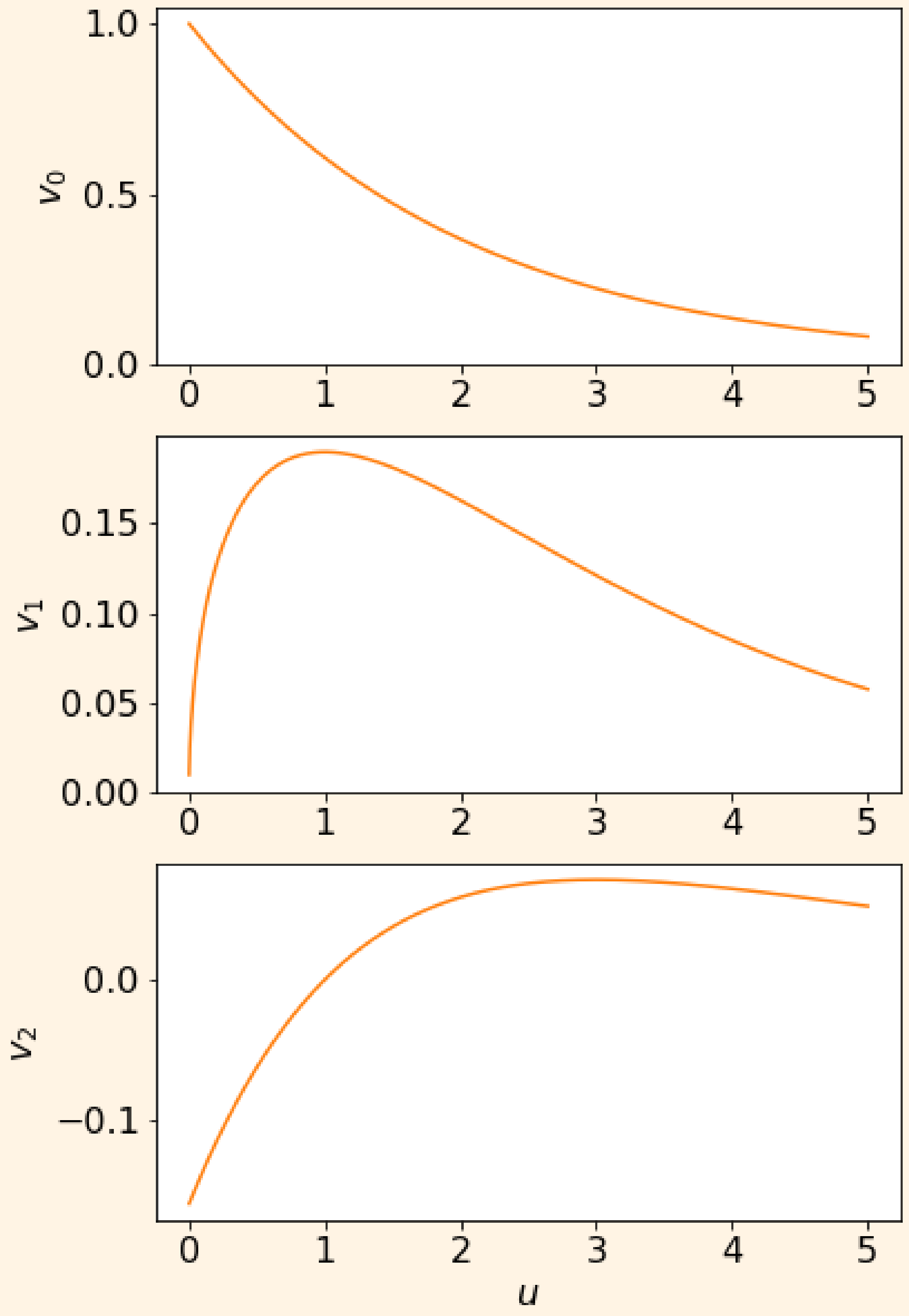
arXiv: 2211.07562

- We generalize the theoretical formula for $P^2 = Q^2 + U^2$

$$\frac{\mathbb{E}\left[V_0(A_u)\right]}{4\pi} = \exp(-u/2)$$

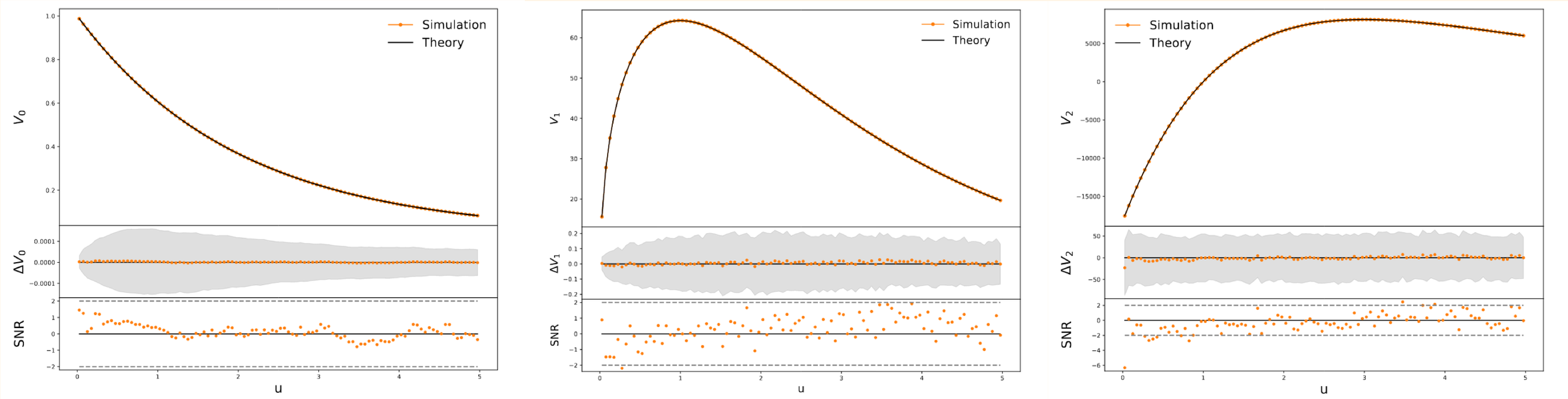
$$\frac{\mathbb{E}\left[V_1(A_u)\right]}{4\pi} = \frac{\sqrt{2\pi}}{8}\sqrt{\mu u}\exp(-\frac{u}{2})$$

$$\frac{\mathbb{E}\left[V_2(A_u)\right]}{4\pi} = \mu\frac{(u-1)\exp(-u/2)}{2\pi}$$



Simulations are compatible with the theory (P^2)

- Excellent compatibility between theory and isotropic Gaussian simulations

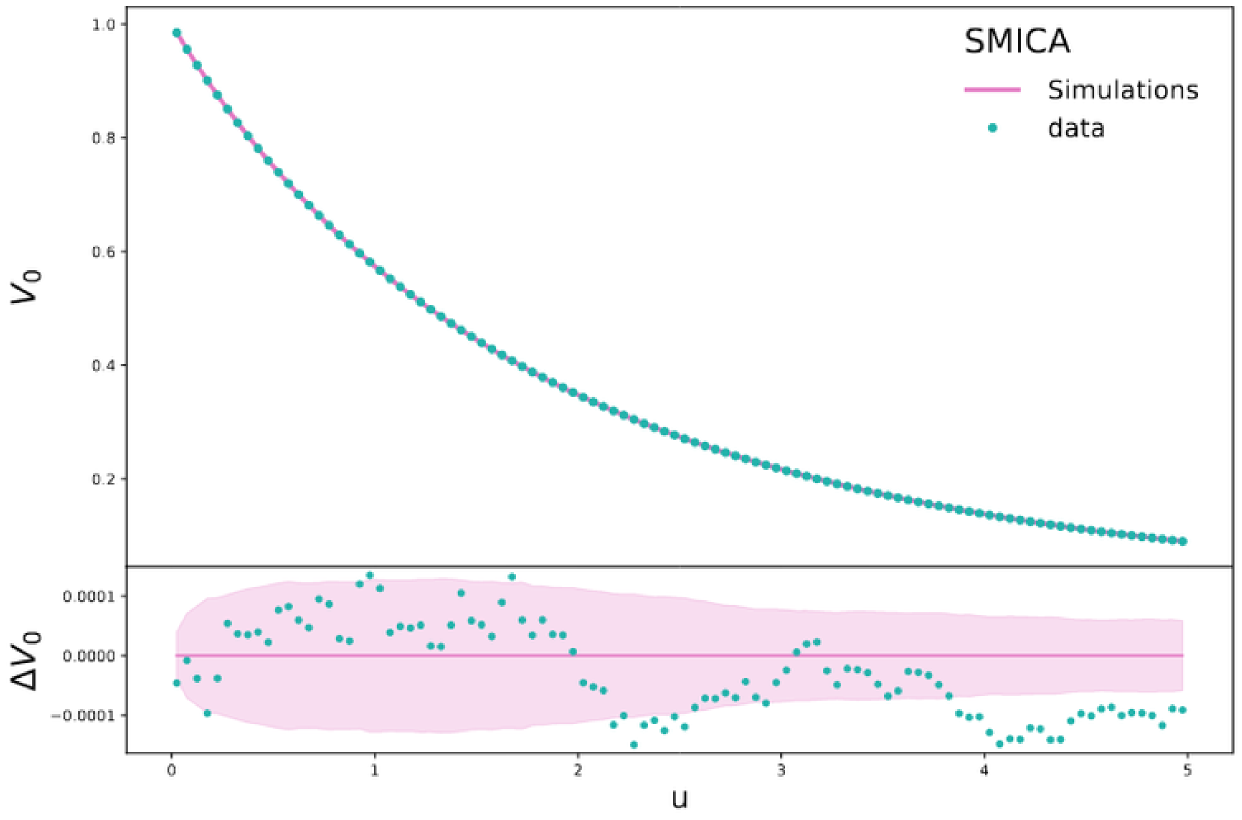


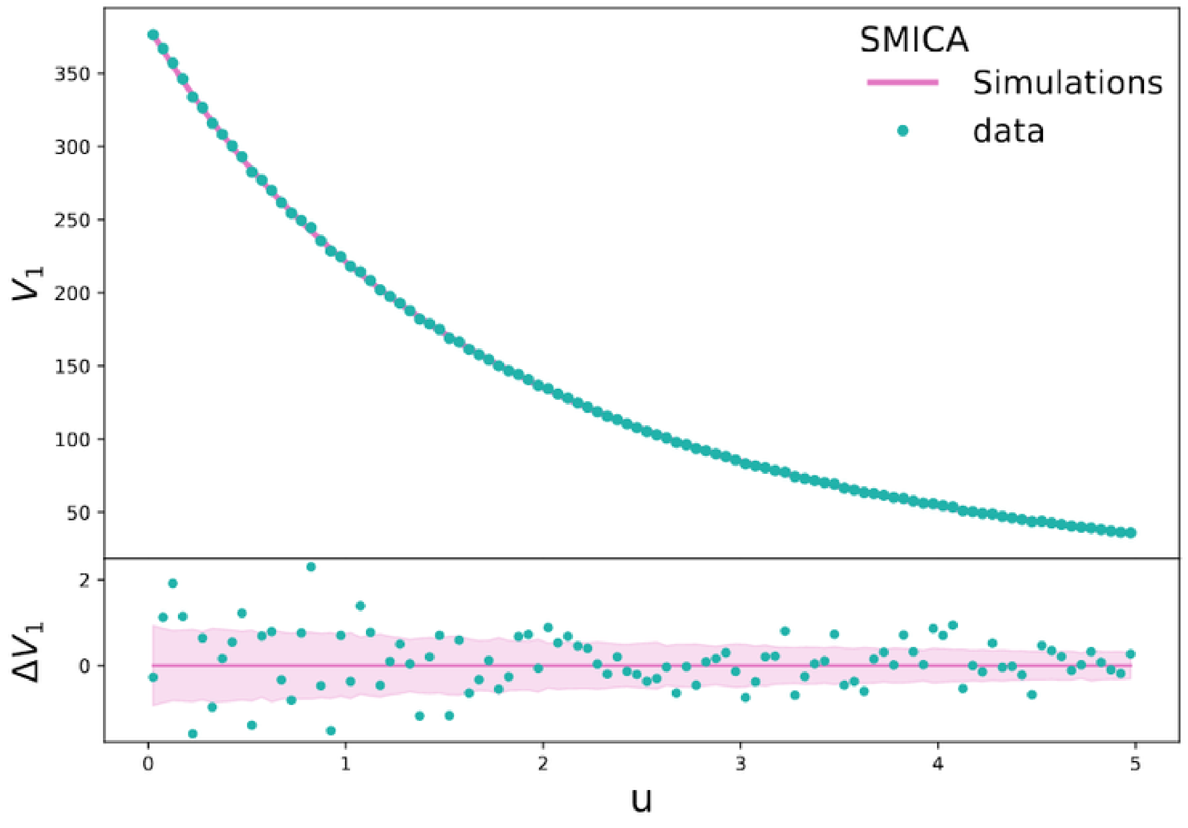
- Shown:
  - MF
  - Residual in simulations (1σ)
  - mean / σ(mean)

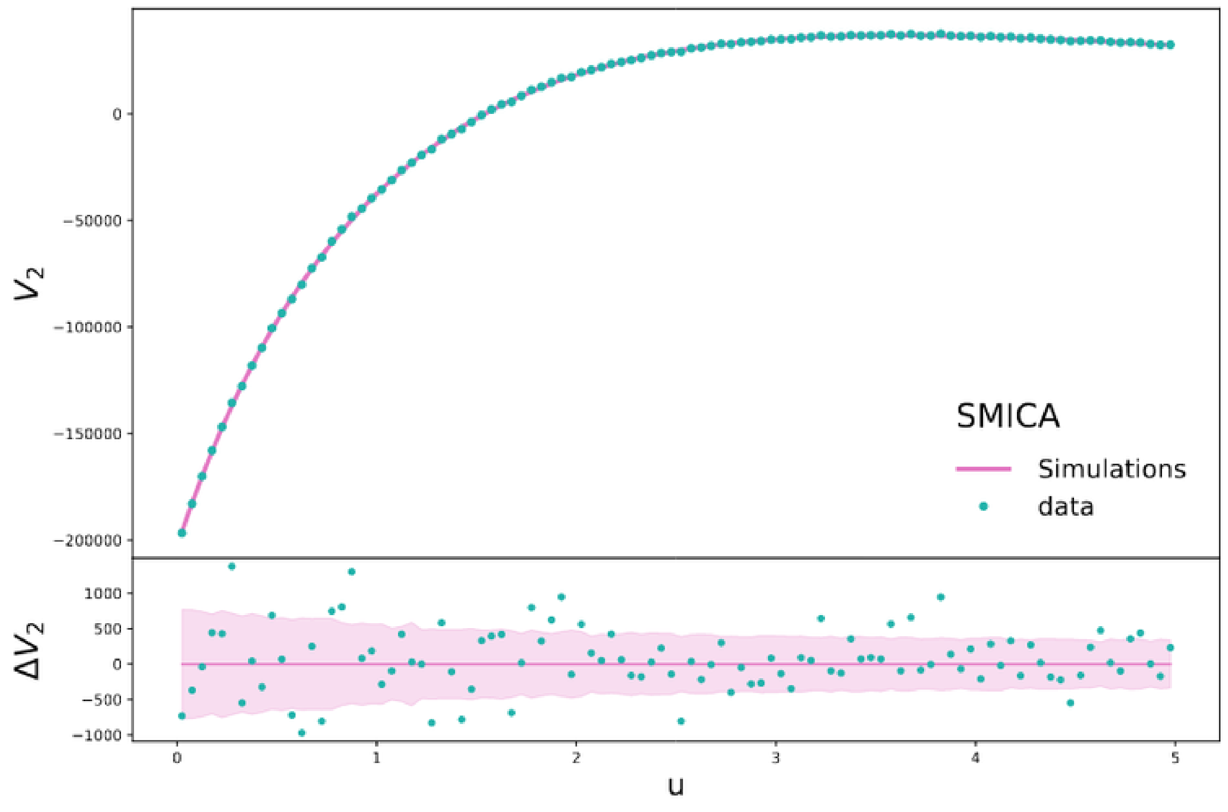
Planck is compatible with realistic simulations (P^2)

- Realistic simulations with anisotropic noise (observational strategy)
- No significant deviation (SMICA & SEVEM)

		χ^2	p_{exc} (%)
V_0	SMICA	1.012	44.7
	SEVEM	0.993	47.0
V_1	SMICA	1.010	47.7
	SEVEM	1.144	17.0
V_2	SMICA	0.812	86.7
	SEVEM	1.084	30.7







There is more information in the polarization field

- Polarization is a spin−2 complex field
- Information is lost in any scalar projection (P, E, B, Q, U, ...)

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- Polarization is a spin−2 complex field
- Information is lost in any scalar projection (*P*, *E*, *B*, *Q*, *U*, ...)
- We analyse the full polarization information using

$$f(\phi,\theta,\psi) = Q(\phi,\theta)\cos(2\psi) - U(\phi,\theta)\sin(2\psi)$$

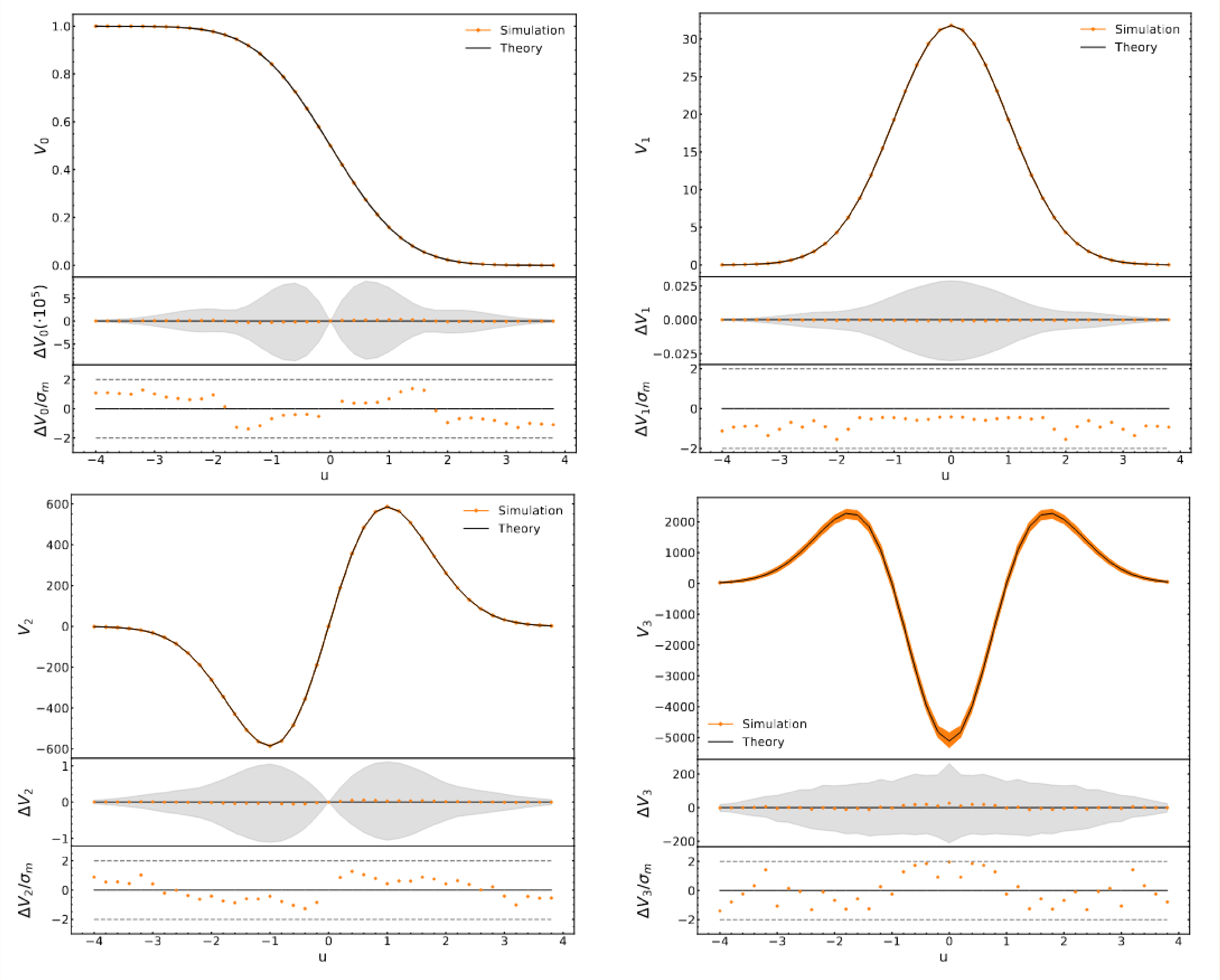
- This is defined in *SO(3)*, a 3D manifold

Minkowski Functionals in $SO(3)$ for the spin–2 CMB polarisation field

J. Carrón Duque,[a,b,1] A. Carones,[a,b] D. Marinucci,[c] M. Migliaccio,[a,b] and N. Vittorio[a,b]

arXiv: 2301.13191

Simulations are compatible with the theory (*f*)



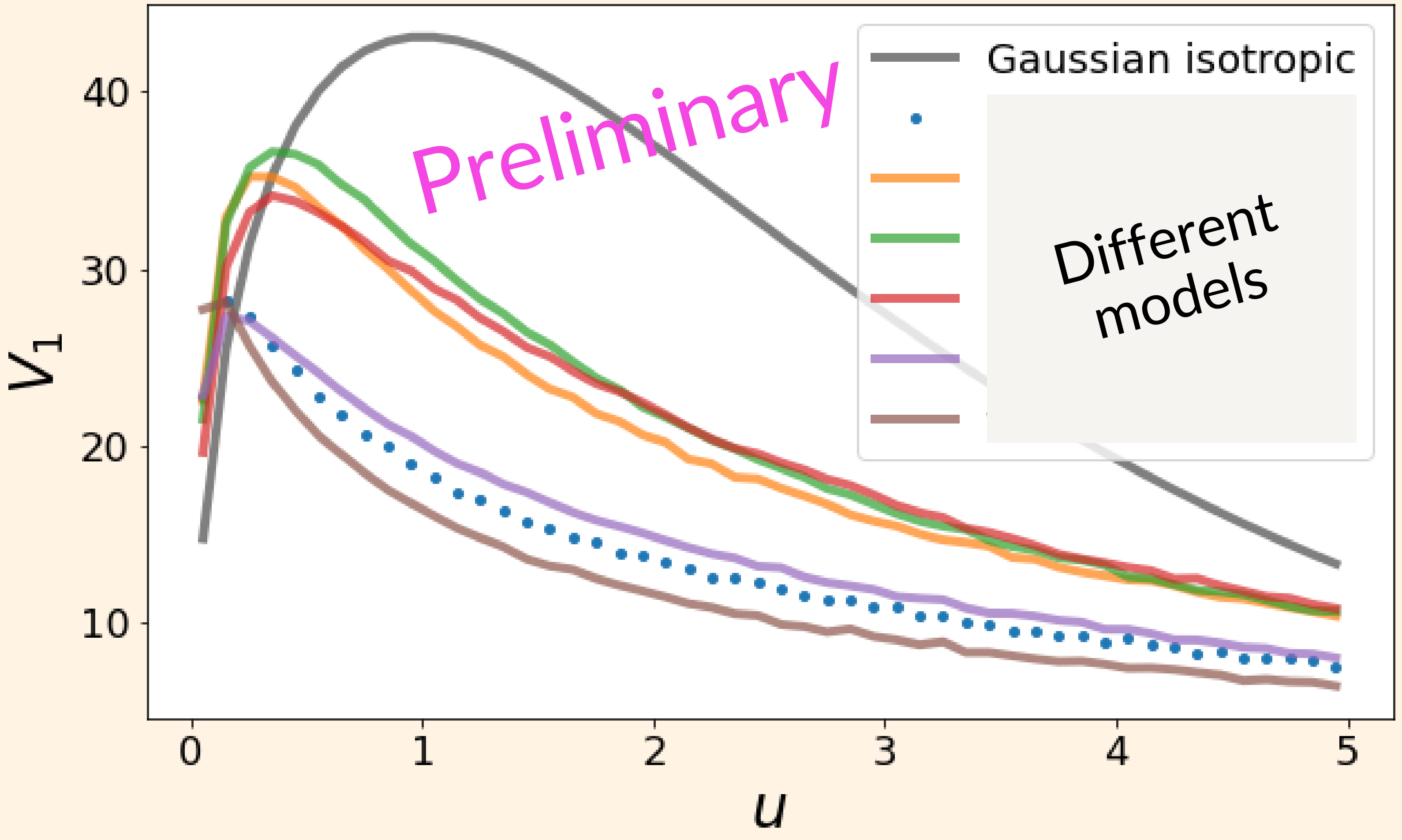
- 3D space → 4 MFs
- Shown:
  - MF
  - Residual in simulations (1σ)
  - mean / σ(mean)
- No significant deviation is found

We analyse Galactic dust models

(w/ Giuseppe **Puglisi**)

- Non−Gaussian foregrounds are important to test component separation methods
- Simulating non−Gaussian foregrounds is not trivial:
  - Modulation
  - Very different approaches from PySM
  - Generative Neural Networks

    (forse: Krachmalnicoff&Puglisi, 2020)
- We are assessing different models



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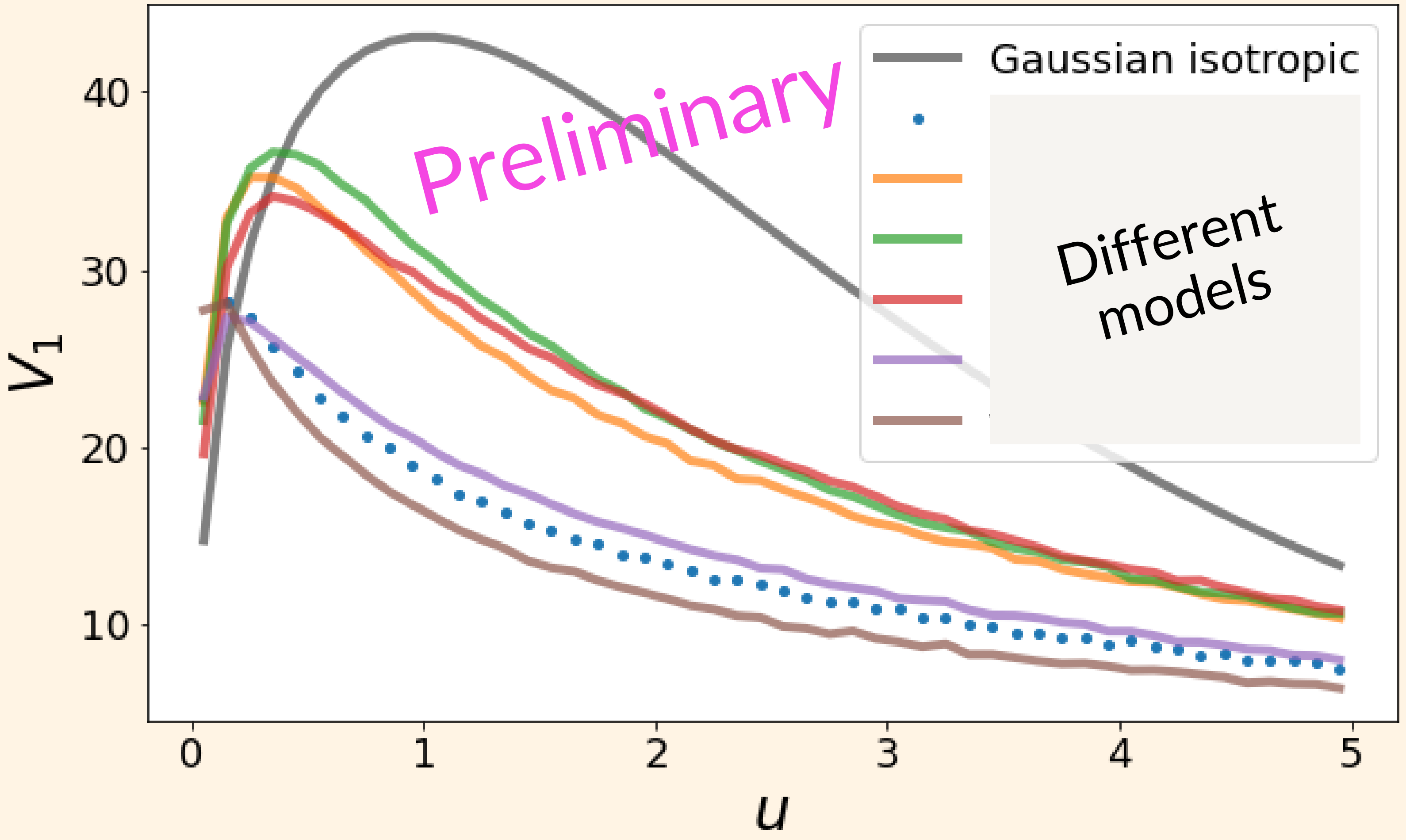
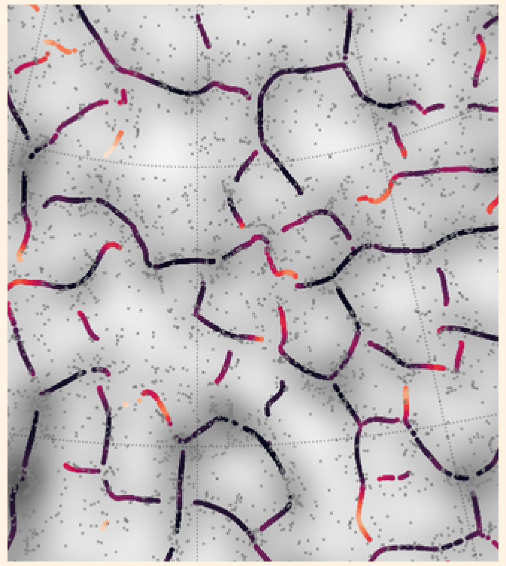


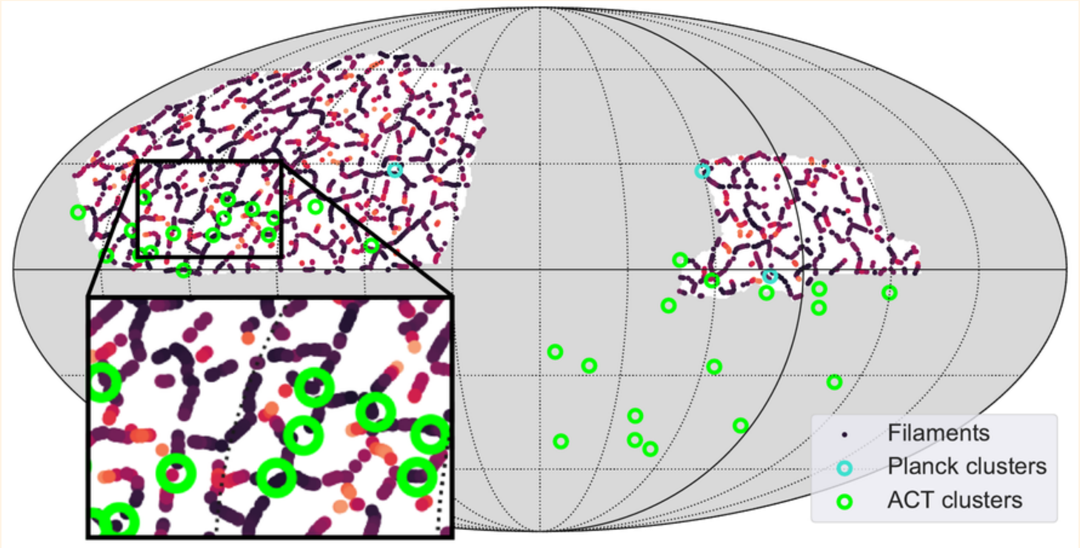
We produced a Cosmic Filaments catalogue

- Publicly available: www.javiercarron.com/catalogue
- *0.05 < z < 2.2*
- Promising results in different areas

A novel cosmic filament catalogue from SDSS data*

Javier Carrón Duque[1,2], Marina Migliaccio[1,2], Domenico Marinucci[3], and Nicola Vittorio[1,2]





MFs can be applied to the 3D density field

- The LSS is NOT Gaussian: lots of information in its non−Gaussianities

Primordial non−Gaussianities

- Consequence of Inflation
- MFs are well suited for some models
- Blind or model dependent

Late Universe non−Gaussianities

- Consequence of Gravity and Baryonic effects
- Dominant, especially at small scales

- Can MFs distinguish both origins?
  - Can we include the effect of Gravity?
- Can MFs constrain cosmological parameters effectively?
  - Yes, at least with forward modelling
- How do they compare to other statistics?
  - Theoretical models, degeneracies, systematics, ...

MFs can have many other applications

- We are exploring, among others:
  - Galactic dust polarised emission
  - Morphology of LSS
  - Forecast for future missions
  - CMB power asymmetry
  - **+ new ideas?**

We develop Pynkowski as a Python package

- Pynkowski is fully documented and modular
- Theory module: theoretical prediction of different kinds of fields (Gaussian, χ^2, f, ...)
- Data module: different kinds of data structures (np arrays, healpix maps, ...)
- Stats module: different higher-order statistics (MFs, maxima/minima distribution, ...)
- All modules are easy to expand

Now available!

https://github.com/javicarron/pynkowski

```
$ pip install pynkowski
```

Pynkowski is easy to use

https://github.com/javicarron/pynkowski

```
$ pip install pynkowski
```

```python
import numpy as np
import healpy as hp

import pynkowski as mf    # For Minkowski Functionals

# Define the thesholds for the excursion sets
us = np.linspace(-5., 5., 100)

# Load the CMB map and angular power spectrum
my_map = ...
my_cls = hp.anafast(my_map)    # or load from file

# Compute the Minkowski Functionals on my map
data_map = mf.Healpix(my_map, normalise=True, mask=None)     # Default parameters

v0_data = mf.V0(data_map, us)
v1_data = mf.V1(data_map, us)
v2_data = mf.V2(data_map, us)

# Compute the Minkowski Functionals on a Gaussian random field with the same power spectrum
gaussian_field = mf.SphericalGaussian(my_cls, normalise=True, fsky=1.)       # Default parameters

v0_theory = mf.V0(gaussian_field, us)
v1_theory = mf.V1(gaussian_field, us)
v2_theory = mf.V2(gaussian_field, us)
```

Python

Takeaway points

- Minkowski Functionals are useful tools to study **non—Gaussianities** and isotropy, with many applications in both the Early and Late Universe
- We have expanded the formalism to **CMB polarization** in two ways: the polarization intensity P^2, and the full information in the spin map
- We have created **Pynkowski** to ease the application of MFs to the cosmological community

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Thank you!