Dissipative Inflation via Scalar Production

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based on 2305.07696, with **Paolo Creminelli**, **Soubhik Kumar** and **Luca Santoni**

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Introduction

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They don't have to thermalize!, e.g. axion coupled to $U(1)$: $\phi F \tilde{F}$ by Anber and Sorbo '09.

Natural inflation, strong backreaction

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\,.$$

One of the photon polarizations **grow exponentially** due to instability

$$\frac{\mathrm{d}^2 A_\pm}{\mathrm{d}\tau^2} + \left(k^2 \pm 2k\frac{\xi}{\tau}\right)A_\pm = 0\,, \qquad \xi = \frac{\alpha\dot{\phi}_0}{2fH}\,.$$

Instability starts at $|k\tau| \simeq 2\xi$ and continues up to **superhorizon scales**. Total amount of enhancement is $A \sim e^{\pi\xi}$. Inflaton equation of motion

$$\phi'' + 2aH\phi' - \nabla^2\phi + a^2V' = a^2\frac{\alpha}{f}\vec{E}.\vec{B}\,.$$

Quntum fluctuations in the $\vec{E}.\vec{B}$ term sources primordial perturbations.

Difficulties: Large power spectrum, non-locality of the response, resonant instability.

Anber and Sorbo '09, '12.
Domcke et al '20, Caravano et al '22, Peloso and Sorbo '22.

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Complex Scalar field

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Remark: EFT with dissipation

Single field (clock) models

Single clock with dissipation

Multiple field models

Our model is an example of **Effective field theory of inflation with dissipation**, Nacir, Porto, Senatore and Zaldarriaga '11.

Outline

1. The Model
2. Linear Perturbations
3. Non-Gaussianity

The Model

Inflaton + Additional Degrees Of Freedom (ADOF)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2|\chi|^2 \right.$$
$$\left. - i\frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \boxed{\frac{1}{2}m^2(\chi^2 + \chi^{*2})}\right].$$

- For $m = 0$ the action is $U(1)$ invariant. One can remove the current coupling by $\chi \to e^{-i\phi/f}\chi$, which changes $M^2 \to M^2 + (\partial\phi)^2/f^2$.
- We consider $M^2(X)$ and $m^2(X)$. With hindsight, M^2 is defined with the unconventional sign.
- The only shift-symmetry breaking term is the potential $V(\phi)$.

Bodas, Kumar, and Sundrum '20

Dynamics of ADOF

Equation of motion for χ will be

$$\Box\chi + \frac{2i}{f}\nabla^\mu\phi\nabla_\mu\chi + \left(M^2 + i\frac{\Box\phi}{f}\right)\chi - m^2\chi^* = 0\,.$$

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We have $\phi = \phi_0$ with $\boxed{\rho \equiv \dot{\phi}_0/f}$, also define $\chi = (\sigma_1 + i\sigma_2)/\sqrt{2}a^{3/2}$

$$\ddot{\sigma}_1 - \frac{\vec{\nabla}^2\sigma_1}{a^2} - \left(M^2 - m^2\right)\sigma_1 - 2\rho\dot{\sigma}_2 = 0\,,$$
$$\ddot{\sigma}_2 - \frac{\vec{\nabla}^2\sigma_2}{a^2} - \left(M^2 + m^2\right)\sigma_2 + 2\rho\dot{\sigma}_1 = 0\,.$$

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$$\ddot{\sigma}_2 - \frac{\vec{\nabla}^2\sigma_2}{a^2} - \left(M^2 + m^2\right)\sigma_2 + 2\rho\dot{\sigma}_1 = 0\,.$$

Neglecting expansion one can find the natural modes of the system assuming, in Fourier space, $\sigma \sim e^{-i\omega t}$ and obtains

$$\omega_\pm^2 = \left(\sqrt{k^2 + \rho^2 - M^2 + \frac{m^4}{4\rho^2}} \pm \rho\right)^2 - \frac{m^4}{4\rho^2}\,,$$

Dynamics of ADOF (cont.)

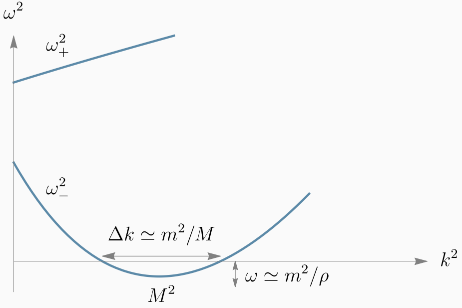


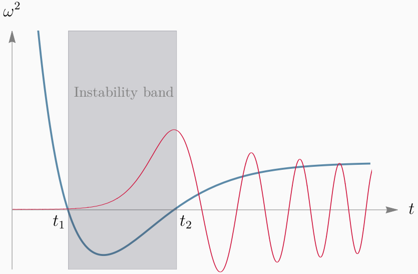
Figure 1: Dispersion relation

- Complex scalar field, two modes.
- ω_- has a minimum located at $k = M$ controlled by the value of m.
- For $m = 0$, $U(1)-$invariant case, the band closes.
- The location of the band will be $M^2 - m^2 < k^2 < M^2 + m^2$.
- Very large and very small scales are healthy if

$$m \ll M \lesssim \rho$$

ADOF in expanding universe

Including expansion, momenta gets redshifted $k \to k/a$. Therefore, the instability is regulated by the limited amount of time spent in the band controlled by H.



- Length of the band

$$H\Delta t \sim \frac{m^2}{M^2} \ll 1\,.$$

- Total growth

$$\pi\xi \equiv \int_{t_1}^{t_2} \mathrm{d}t\,|\omega_-| \sim \frac{m^4}{H\rho M^2}\,.$$

- Exponential enhancement of the fields $\chi \sim e^{\pi\xi}$.

Demanding $\boxed{H \ll m \ll M \lesssim \rho}$ we get $\xi = \mathcal{O}(1)$.

Canonical quantization of ADOF

Quantization of χ field

$$\sigma_i(t,\vec{x}) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{3/2}}\, \mathrm{e}^{i\vec{k}\cdot\vec{x}} \left[(F_k(t))_{ij}\hat{a}_j(\vec{k}) + (F_k^*(t))_{ij}\hat{a}_j^\dagger(-\vec{k})\right] .$$

The matrix F plays the role of mode functions. It has to be a matrix since the two fields are strongly coupled by presence of $\rho\dot{\sigma}$ term. Mode functions satisfy

$$\ddot{F}_k + \begin{pmatrix} 0 & -2\rho \\ 2\rho & 0 \end{pmatrix} \cdot \dot{F}_k + \begin{pmatrix} \frac{k^2}{a^2} - M^2 + m^2 & 0 \\ 0 & \frac{k^2}{a^2} - M^2 - m^2 \end{pmatrix} \cdot F_k = 0\,.$$

Bunch–Davies initial condition implies

$$F_k(t \to -\infty) \to \frac{\mathrm{e}^{-ik\tau}}{\sqrt{2k/a}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

WKB solution

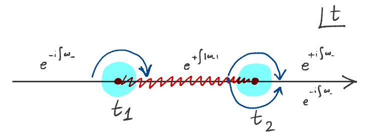
Focusing on each column

$$\vec{F}_{\text{column}} = \vec{Q}(t) \exp\left(-i \int \mathrm{d}t\ \omega(t)\right),$$

with $D(\omega).\vec{Q} = 0$. For Nontrivial solutions $\det D(\omega_\pm) = 0$. In addition, $\vec{Q}$ is the null vector of $D(\omega)$. Normalization is fixed by looking at NLO WKB

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\vec{Q}_\pm^\dagger \begin{pmatrix} \omega_\pm & -i\rho \\ i\rho & \omega_\pm \end{pmatrix} \vec{Q}_\pm\right] = 0\,.$$

General solution is addition of $F_\pm$ and $F_\pm^*$. WKB is valid if $\frac{\dot{\omega}}{\omega^2} \ll 1$, therefore it breaks down at $\omega^2(t_{1,2}) = 0$. Need to do matching at $t_{1,2}$:



Weinberg 1961, Dufaux, et al '06, Landau QM

WKB solution (cont.)

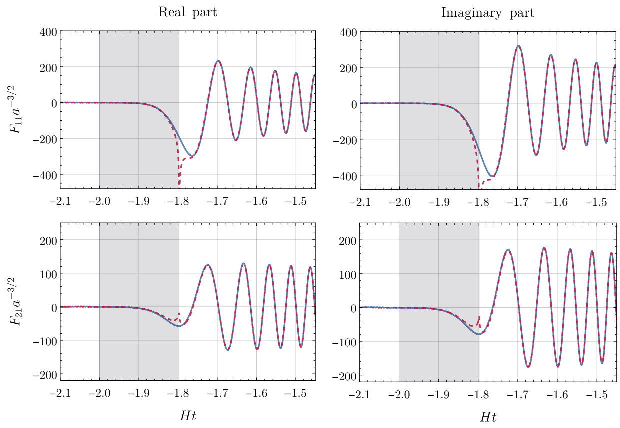


Figure 2: Comparison of numeric (solid) and analytic (dashed) solution. Gray region is the instability band.

Inflaton dynamics

Equation of motion for the inflaton is

$$\nabla_\mu \left[\left(1 + \frac{(M_X^2 - 2\rho^2)}{\rho^2 f^2} |\chi|^2 - \frac{m_X^2}{2\rho^2 f^2} (\chi^2 + \chi^{*2}) \right) \nabla^\mu \phi \right] - V'(\phi) + \frac{i m^2}{f} (\chi^2 - \chi^{*2}) = 0 \, .$$

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Define $\mathcal{O}\equiv -i(\chi^2-\chi^{*2})$, neglect $\ddot{\phi}_0$, $\dot{H}$ at the background level

$$3H\dot{\phi}_0+V'+\frac{m^2}{f}\langle\mathcal{O}\rangle\simeq 0\,.$$

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$$3H\dot{\phi}_0+V'+\frac{m^2}{f}\langle\mathcal{O}\rangle\simeq 0\,.$$

- Backreaction could be large since $\langle\mathcal{O}\rangle\simeq\frac{m^2}{2\pi^2}e^{2\pi\xi}$.
- For moderate values of f ($\gg M$) we get $2\pi\xi\sim\log fV'/m^4$
- For $\dot{H}/H^2\ll 1$ we require $V\gg$ kinetic of ϕ and χ and therefore, $3M_{\rm Pl}^2H^2\approx V$.

- We can neglect the other terms in the equation

$$\frac{m_X^2 \left\langle \chi^2 + \chi^{*2} \right\rangle}{(M_X^2 - 2\rho^2) \left\langle |\chi|^2 \right\rangle} \simeq \frac{m^4}{\rho^4} \ll 1 \,, \qquad \frac{\frac{H\dot{\phi}_0}{f^2} \left\langle |\chi|^2 \right\rangle}{\frac{im^2}{f} \left\langle \chi^2 - \chi^{*2} \right\rangle} \simeq \frac{H\rho^3}{m^4} \simeq \frac{1}{8\xi} \lesssim 1 \,.$$

- The sign of the backreaction term is correct

$$\dot{\phi}_0 > 0 \implies -i \left\langle \chi^2 - \chi^{*2} \right\rangle > 0 \,.$$

- Require an attractor solution: $\frac{\mathrm{d}\xi}{\mathrm{d}\dot{\phi}_0} > 0$. We have seen that

$$\xi \simeq \frac{m(\dot{\phi}_0)^4}{8H\left(\frac{\dot{\phi}_0}{f}\right) M(\dot{\phi}_0)^2} \,.$$

- Without $M^2(X)$ and $m^2(X)$ tends to move away from the desired solution. Sign of M^2 can be a consequence of inflating background.

Linear Perturbations

General remarks

Much easier to perturb the equations of motion. Parametrize deviations $\phi = \phi_0 + \delta\phi$ and $\mathcal{O} = \langle\bar{\mathcal{O}}\rangle + \delta\mathcal{O}$ and assume decoupling limit.

It is single-clock inflation and the main observable is $\zeta = -H\delta\phi/\dot{\phi}_0$.

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For any operator $\mathcal{O}$, deviations from $\langle\bar{\mathcal{O}}\rangle$ can be decomposed into intrinsic **noise** and induced **response** fluctuations

$$\delta\mathcal{O} = \delta\mathcal{O}_S + \delta\mathcal{O}_R\,.$$

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$$\delta\mathcal{O} = \delta\mathcal{O}_S + \delta\mathcal{O}_R\,.$$

By suitable assumptions it is enough to focus on $\boxed{\mathcal{O} = -i(\chi^2 - \chi^{*2})}$ in the equation of motion

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\vec{\nabla}^2\delta\phi}{a^2} + V''\delta\phi = -\frac{m^2}{f}(\delta\mathcal{O}_S + \delta\mathcal{O}_R)\,,$$

while other operators like $|\chi|^2$, $\chi^2 + \chi^{*2}$ etc. could be neglected.

Response and Locality

At leading order, response is the change in $\langle \mathcal{O} \rangle$ as a result of perturbation $\delta\phi$, i.e. $\delta\mathcal{O}_R = \langle \mathcal{O} \rangle_{\phi} - \langle \mathcal{O} \rangle_{\phi_0}$.

- **Hierarchy of scales** variation of $\delta\phi$ is much slower/longer than χ, WKB solution can be extended to include $\delta\phi$.
- **Local operator** certain class of operators that $\langle \mathcal{O} \rangle$ is dominated by modes around the instability band.

The response in this case is **local**

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$$\delta\mathcal{O}_R \simeq \frac{\partial \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0} \delta\dot{\phi} \,.$$

The equation will become

$$\delta\ddot{\phi} + (3H + \gamma)\delta\dot{\phi} - \frac{\vec{\nabla}^2 \delta\phi}{a^2} + V'' \delta\phi = -\frac{m^2}{f} \delta\mathcal{O}_S \,,$$

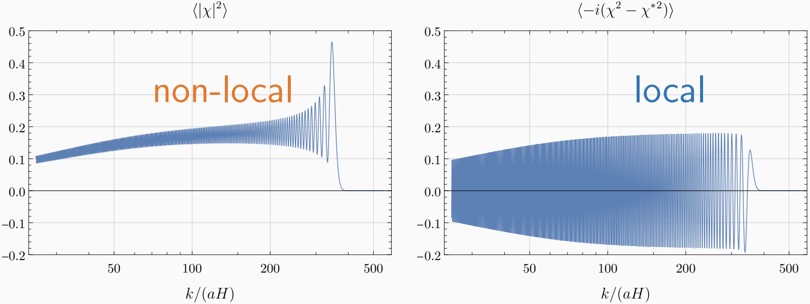
with $\gamma/H \sim \xi^2 e^{2\pi\xi} M^2/f^2 \gg 1$.

Local vs Non-Local

For a generic operator of the form $\mathcal{O} = \frac{1}{a^3} A_{ij}\sigma_i\sigma_j$ we have

$$\langle \mathcal{O} \rangle = \frac{1}{a^3} \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \,\mathrm{Tr}\Big(A^T F_k F_k^\dagger\Big)\,.$$

For a homogeneous perturbation, each mode is mostly sensitive to the value $\dot{\phi}$ at the moment of instability.



We get rid of non-locality for $\xi \gtrsim 1$, fine tuning, etc.

Statistics of the Noise

Noise is quantum mechanical fluctuation $\delta\mathcal{O}_S = \mathcal{O} - \langle\mathcal{O}\rangle$. Eventually we are interested in correlation functions

$$\left\langle \delta\mathcal{O}_S(t,\vec{k})\delta\mathcal{O}_S(t',\vec{k}')\right\rangle' = \int \frac{2\,\mathrm{d}^3\vec{p}}{(2\pi)^3 a^3 a'^3}\,\mathrm{Tr}\,F_q^\dagger(t)AF_p(t)F_p^\dagger(t')AF_q(t')\,,$$

in which $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $q = |\vec{k} - \vec{p}|$.

The integrand is dominated by the instability band. We are interested in long distance correlations $k \ll p \sim q$. This is delta function in real space.

In addition, the correlation decrease for large temporal separations, $t - t' \gg m^{-1}$, due to oscillations after the instability band.

$$\boxed{\left\langle \delta\mathcal{O}_S(t,\vec{k})\delta\mathcal{O}_S(t',\vec{k}')\right\rangle \simeq (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{\delta(t-t')}{a^3}\nu_{\mathcal{O}}}$$

with $\nu_{\mathcal{O}} = M\,e^{4\pi\xi}/4\pi^2 m$.

$$\delta\ddot{\phi} + (3H + \gamma)\delta\dot{\phi} - \frac{\vec{\nabla}^2\delta\phi}{a^2} + V''\delta\phi = -\frac{m^2}{f}\delta\mathcal{O}_S$$

Linear perturbations

$$\ddot{\delta\phi} + (3H+\gamma)\dot{\delta\phi} + \left(\frac{k^2}{a^2} + V''\right)\delta\phi = -\frac{m^2}{f}\delta\mathcal{O}_S\,.$$

The generic solution is a linear combination of homogeneous and the sourced part. In the limit that $\gamma \gtrsim H$, vacuum fluctuations becomes exponentially suppressed. Therefore, the main source for fluctuations come from the noise

$$\delta\phi(\tau,\vec{k}) = -\frac{m^2}{f}\int \mathrm{d}\tau' a'^2 G_k(\tau,\tau')\delta\mathcal{O}_S(\tau',\vec{k})\,.$$

Eventually power spectrum can be written

$$\left\langle \zeta_{\vec{k}}\zeta_{\vec{k}'}\right\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H^2m^4}{\rho^2 f^4}\nu_{\mathcal{O}}\int \mathrm{d}\tau' G_k(0,\tau')^2\,.$$

The amplitude

$$\boxed{\Delta_s^2 \simeq \frac{1}{32\xi^2}\left(\frac{\gamma}{\pi H}\right)^{3/2}\frac{MH^4}{m^5}} \sim 10^{-9}$$

Non-Gaussianity

Beyond linear order

Genuine test of the model is provided by the non-Gaussian features of perturbations. We need to expand the e.o.m beyond linear order.

Two types of non-Gaussianities:

- Non-Gaussian **statistics** of the noise term $\delta\mathcal{O}_S$. It can be shown
- Non-linear **dynamics** of the system, i.e. quadratic terms in the e.o.m. The relevant contribution is the non-linear response: $\delta\mathcal{O}_R$ up to quadratic order.

Non-Gaussian Noise

Similar to the two point function we get

$$\langle \delta\mathcal{O}_S(1)\delta\mathcal{O}_S(2)\delta\mathcal{O}_S(3)\rangle \simeq (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\delta(\tau_1-\tau_2)\delta(\tau_1-\tau_3)\, H^8\tau_1^8\nu_{\mathcal{O}^3}\,,$$

with $\nu_{\mathcal{O}^3} \simeq \mathrm{e}^{6\pi\xi}\,/\pi^2 m^2$. The three point function of the inflaton will be

$$\langle \delta\phi_{\vec{k}_1}\delta\phi_{\vec{k}_2}\delta\phi_{\vec{k}_3}\rangle = -\left(\frac{m^2}{f}\right)^3 \int \Big(\mathrm{d}\tau_i\, a_i^2 G_{k_i}(0,\tau_i)\Big)^3 \langle \delta\mathcal{O}_S(1)\delta\mathcal{O}_S(2)\delta\mathcal{O}_S(3)\rangle$$

which leads to

$$f_{\mathrm{NL}}^{\mathsf{eq}} = \frac{5}{18}\,\frac{\int \mathrm{d}y\, y^2\tilde{G}(0,y)^3}{\left(\int \mathrm{d}y\,\tilde{G}(0,y)^2\right)^2}\,\frac{\nu_{\mathcal{O}^3}H^2}{\frac{H}{\rho f}\frac{m^2}{f}\nu_{\mathcal{O}}^2} \simeq \boxed{\frac{40\pi}{9}\xi\frac{m^2}{M^2}}\,.$$

Non-linear Response

We expect that local approximation remains valid up to higher orders.

In the Gaussian approximation, two parameters that can change influenced by $\delta\phi$: **mean** and **variance**

$$\delta\mathcal{O}_R \simeq \boxed{\frac{\partial\langle\mathcal{O}\rangle}{\partial\dot{\phi}_0}\left(\dot{\delta\phi} - \frac{(\partial_i\delta\phi)^2}{2\dot{\phi}_0 a^2}\right) + \frac{1}{2}\frac{\partial^2\langle\mathcal{O}\rangle}{\partial\dot{\phi}_0^2}\dot{\delta\phi}^2} + \boxed{\frac{1}{2\nu_{\mathcal{O}}}\frac{\partial\nu_{\mathcal{O}}}{\partial\dot{\phi}_0}\dot{\delta\phi}\delta\mathcal{O}_S} + \dots ,$$

The first two terms: $\delta\langle\mathcal{O}\rangle(\sqrt{\partial_\mu\phi\partial^\mu\phi})$, the last term is the change in $\langle\delta\mathcal{O}^2\rangle$.

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Therefore, one would obtain

$$\ddot{\delta\phi} + (3H+\gamma)\dot{\delta\phi} - \frac{\vec{\nabla}^2\delta\phi}{a^2} + V''\delta\phi = \frac{\gamma}{2\rho f}\left[\frac{(\vec{\nabla}\delta\phi)^2}{a^2} - 2\pi\xi\dot{\delta\phi}^2\right] - \frac{m^2}{f}\left(1 + 2\pi\xi\frac{\dot{\delta\phi}}{\rho f}\right)\delta\mathcal{O}_S\,.$$

Bispectrum

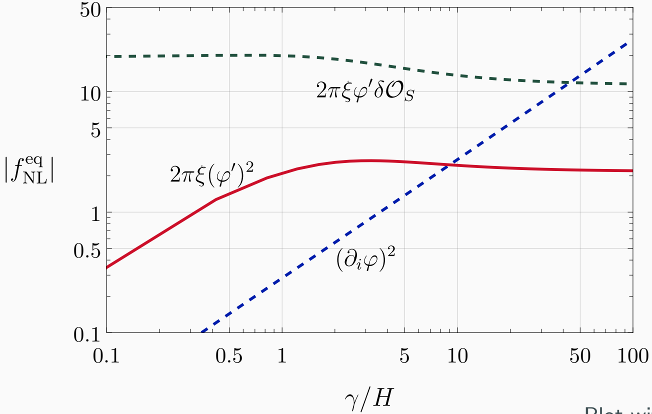
$$\delta\phi^{\rm NLO}(\tau,\vec{k}) = -\int {\rm d}\tilde{\tau}\, G_k(\tau,\tilde{\tau}) \int \frac{{\rm d}^3\vec{p}}{(2\pi)^{3/2}} \left[\frac{\gamma}{2\rho f}\left(\vec{p}.\vec{q}\,\delta\phi_p\delta\phi_q + 2\pi\xi\delta\phi'_p\delta\phi'_q\right) + 2\pi\xi\tilde{a}^2\frac{m^2}{\rho f^2}\delta\phi'_q\delta\mathcal{O}_S(\tilde{\tau},p)\right],$$

with $\vec{q} = \vec{k} - \vec{p}$ and $\delta\phi$ the is linear order solution, i.e. $\delta\phi \sim \int G\,\delta\mathcal{O}_S$.
The 3-point function of curvature perturbation is given by

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle_{\rm NL} = -\left(\frac{H}{\rho f}\right)^3 \left[\left\langle \delta\phi^{\rm NLO}_{\vec{k}_1}\delta\phi_{\vec{k}_2}\delta\phi_{\vec{k}_3}\right\rangle + \vec{k}_1 \leftrightarrow \vec{k}_2 + \vec{k}_1 \leftrightarrow \vec{k}_3\right]$$
$$\equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(k_1,k_2,k_3)\,.$$

We parametrize the 3-point function with the magnitude at equilateral triangle

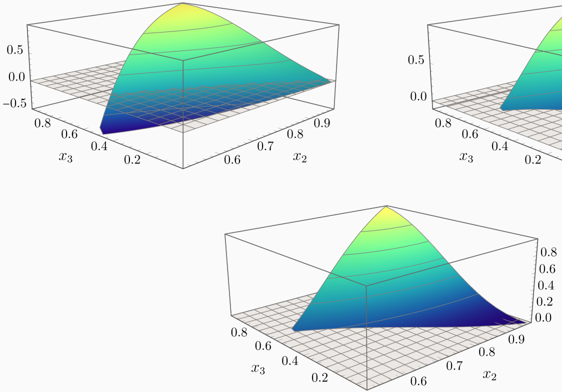
$$B(k,k,k) = \frac{1}{k^6}\frac{18}{5} f_{\mathrm{NL}} (2\pi^2 \Delta_s^2)^2 \,.$$



Plot with $\xi = 2$.

$f_{\rm NL}$ (cont.)

- The coefficient of $(\vec{\nabla}\delta\phi)^2$ is fixed by nonlinear realization of Lorentz symmetry and $f_{\rm NL}^{\rm eq} \simeq -\gamma/4H$. Same sign as the reduced speed of sound contribution.
- In the limit of small friction the only remaining term is $\delta\dot{\phi}\delta\mathcal{O}_S$ with $f_{\rm NL}^{\rm eq} \simeq -5.7\xi$.

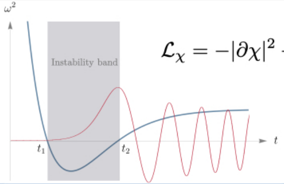


Shapes corresponding to (from left to right) terms $(\nabla\delta\phi)^2$, $\xi\delta\dot{\phi}^2$ and $\xi\delta\dot{\phi}\delta\mathcal{O}_S$.

Shapes (cont.)

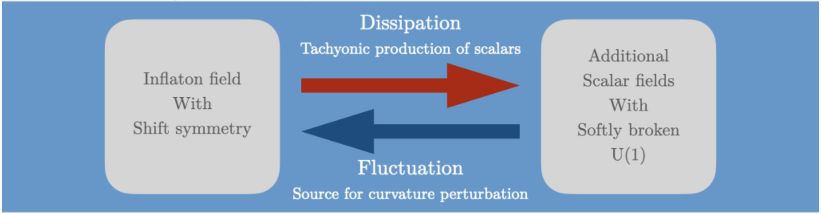
- The peak is at the equilateral configuration.
- Squeezed limit vanishes since the model is single clock.
- Partial enhancement in the collinear configuration.

Summary



$$\mathcal{L}_\chi = -|\partial\chi|^2 + M^2|\chi|^2 - i\frac{\partial_\mu\phi}{f}(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi) - \frac{1}{2}m^2(\chi^2 + \chi^{*2})$$

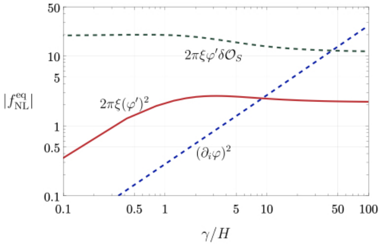
Tachyonic instability is triggered by coupling to the inflaton which causes particle production due to U(1) breaking.



Curvature perturbation is sourced by the stochastic fluctuation of the additional scalars. Power spectrum is is the evolution of the noise power

$$\Delta^2 \sim \frac{H^4 M}{m^5}$$

Non-linear evolution of the Gaussian noise is the souse of non-Gaussianities. The shape is equilateral with amplitude shown in the figure.



Future directions

- Gravitational Waves
- Thermalization
- Fermions (Adshead, et. al. 18)

Thank you