Covariant formulation of MOND from hyperconical metrics and observational constraints

$\mathscr{H}^4 := S^3_{\mathbb{R}_+}$

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Abstract

Modified Newtonian Dynamics (MOND) can partially explain the excess of rotation of galaxies, or the equivalent mass discrepancy-acceleration, without the requirement of dark matter halos. This work proposes a modification of GR based on the distorted stereographic projection of *hyperconical universes*, which leads to MOND effects at galactic scales. To describe the mass discrepancy-acceleration relation, a hypothesis on the centrifugal acceleration was assumed, which would show a small time-like contribution at large-scale dynamics due to the metric used. As a limit case, a covariant formulation compatible with MOND is obtained, and mass discrepancy-acceleration is satisfactorily modelled for a reference set of 123 galaxies collected from the SPARC dataset.

Introduction

Excess rotation in galaxies suggests a discrepancy between visible and total matter required to explain observations. However, local ratio between observed and expected speed (***v***) is predictable by the observed distribution of visible matter (M_b) in each galaxy, avoiding or reducing the need of dark matter. The empirical law is commonly known as mass-luminosity relation or baryonic Tully-Fisher relation (BTFR) that is $M_b \propto v^4$. Milgrom (1983) proposed modified Newtonian dynamics (MOND) as a possible alternative to the cold dark mass hypothesis. Milgrom's law is expressed in terms of the external force ***F*** and the acceleration ***a*** experienced by the objects, and a constant $a_0 \approx 1.2 \cdot 10^{-10}$ m/s^2,

$$F = m\,a \cdot \mu\left(\frac{a}{a_0}\right) \Rightarrow \begin{cases} \mu = 1 & \text{for } a >> a_0 \Rightarrow F = ma \Rightarrow v \approx \sqrt{\frac{GM}{r}} \quad \text{Keplerian rotation curve} \\ \mu = a/a_0 & \text{for } a << a_0 \Rightarrow F = m\frac{a^2}{a_0} \Rightarrow \frac{GMm}{r^2} = m\frac{\left(\frac{v^2}{r}\right)^2}{a_0} \Longrightarrow v^4 = GMa_0 \quad \text{BTFR law} \end{cases}$$

At cosmological scales, modified gravities also lead to phenomena similar to the produced by dark energy and dark matter. For instance, the *hyperconical model* [1–4] predicts: (1) ΛCDM-compatible (apparent) acceleration; (2) apparent dark energy and dark matter about $(\Omega_\Lambda, \Omega_M) \approx (0.70, 0.30)$; and (3) a Hubble tension between Planck- and SH0ES-based data (**Fig. 1**). Therefore, this work aims to explore ability of the same model (*hyperconical universe*) to contain relativistic MOND behavior and to explain the mass discrepancy observed in galaxies.

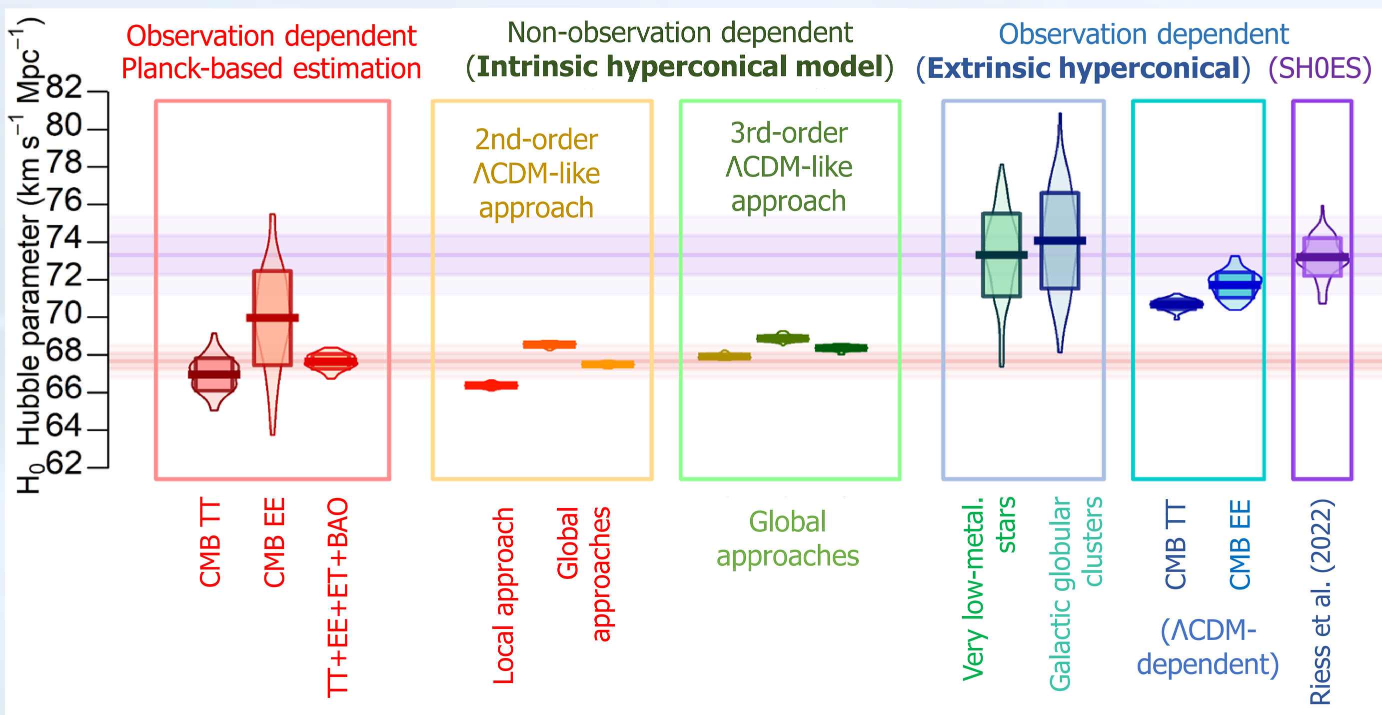

Figure 1. Comparison between observed (red and violet panels) and simulated (orange, green and blue panels) values of the Hubble tension, according to intrinsic and extrinsic geometry [4].

Relativistic formulation of MOND

The used method is a three-step procedure based on the hyperconical universe [1–4] (i.e. a linearly expanding manifold embedded in 5-Minkowskian space). Hyperconical metrics are locally flat and approach to the Friedmann–Lemaître–Robertson–Walker metric except by shift and lapse terms that lead to a radial inhomogeneity (or apparent acceleration if distorted projection is applied). That is:

$$ds^2 \approx dt^2\left(1 - kr'^2\right) - \frac{t^2}{t_0^2}\left(\frac{dr'^2}{1-kr'^2} + r'^2 d\Sigma^2\right) - \frac{2r't}{t_0^2}\frac{dr'dt}{\sqrt{1-kr'^2}}$$

Negligible for r'/t << 1

(1) Perturbation of vacuum energy density to obtain massive objects (Schwarzschild-like metric):

$$ds^2_{\mathscr{H}^4} \approx \left(1 - \frac{r^2}{t^2} - \frac{2GM}{r}\right)dt^2 - \frac{t^2}{t_0^2}\left[\left(1 - \frac{r^2}{t^2} - \frac{2GM}{r}\right)^{-1} dr'^2 + r'^2 d\Sigma^2\right]$$

(2) Perturbation of coordinates from the distorted stereographic projection (first-order approach):

$$\lambda_u \approx \frac{1}{1 - \frac{\gamma}{\gamma_0}} \Rightarrow \lambda_u^\alpha \approx 1 + \frac{\alpha\gamma}{\gamma_0} \approx 1 + \frac{\alpha r'}{\gamma_0 t_0 c} \Rightarrow \begin{cases} \hat{r}' = \lambda_u^\alpha r' \approx \left(1 + \frac{\alpha r'}{\gamma_0 t_0 c}\right) r' \\ \hat{t} = \lambda_u t \approx \left(1 + \frac{r'}{\gamma_0 t_0 c}\right) t \end{cases}$$

Projected angle:

$$\gamma_0(\gamma_M) :\approx \frac{\gamma_M}{\cos\gamma_M}$$

(3) Geodesic equations and centrifugal acceleration with an extra orthogonal contribution due to the curvature radius t of the universe:

$$\frac{d^2x^\mu}{d\tau^2} \approx \frac{1}{2}\eta^{\mu\nu}\frac{\partial}{\partial x^\nu}\hat{h}_{tt}\left(\frac{cdt}{d\tau}\right)^2 \Rightarrow \frac{d^2\hat{s}}{d\tau^2} \approx \left(\frac{1}{2}\frac{\partial}{\partial x^0}\hat{h}_{tt}\, e_t - \frac{1}{2}\frac{\partial}{\partial x^i}\hat{h}_{tt}\, e_i\right)\left(\frac{cdt}{d\tau}\right)^2$$

$$\frac{d^2\hat{s}}{c^2dt^2} \approx \frac{1}{2}\frac{\partial\hat{h}_{tt}}{c\partial t}e_t - \frac{1}{2}\frac{\partial\hat{h}_{tt}}{\partial r}\frac{\partial r}{\partial x^i}e_i \approx -\left(\frac{r}{\gamma_0 c^2t^2} + \frac{\alpha GM}{\gamma_0 t^2c^4}\right)e_t - \left(\frac{GM}{c^2r^2} + \frac{1}{\gamma_0 ct}\right)\frac{x^i}{r}e_i$$

$$\hat{a} := \frac{d^2\hat{s}}{c^2dt^2} = -\omega_t^2 ct\, e_t \sinh\theta - \omega_r^2 r\, e_{\vec{r}}\cosh\theta = -\left(\frac{1}{ct}e_te^t + \frac{1}{r}e_{\vec{r}}e^{\vec{r}}\right)\frac{v^2}{c^2}e_s =: S^{-1}\frac{v^2}{c^2}e_s$$

$$\frac{v^2}{c^2}e_s = -\left(cte_te^t + x^ie_ie^i\right)\frac{d^2s}{c^2dt^2} \approx ct\left(\frac{\alpha GM}{\gamma_0 t^2c^4} + \frac{r}{\gamma_0 c^2t^2}\right)e_t - \left(\frac{GM}{c^2r^2} + \frac{1}{\gamma_0 ct}\right)\frac{x^ix_i}{r}e_i$$

This corresponds to a time-like component in the total centrifugal force, whose squared modulus is proportional to ***v⁴***, and the term $r/(\gamma_0 ct)$ of the distorted projection contributed to it. Thus, the distorted stereographic projection contributes to deep MOND behavior with the acceleration c/t divided into the projected angle γ_0.

$$\frac{v^4}{c^4} = -\left\|\frac{v^2}{c^2}e_s\right\|^2 \approx \left(\frac{GM}{rc^2}\right)^2 + \frac{2GM}{\gamma_0 tc^3}$$

$$\begin{cases} v(M;\gamma_0) \approx \sqrt[4]{\left(\frac{GM}{r_M}\right)^2 + \frac{2GMc}{\gamma_0 t}} \xrightarrow{a << a_0} \text{BTFR} \\ \frac{v^4}{v_K^4} = 1 + \frac{r}{v_K^2}\frac{2c}{\gamma_0 t} \Longrightarrow \left(\frac{v}{v_K}\right)^2 = \sqrt{1 + \frac{1}{|a_N|}\frac{2c}{\gamma_0 t}} \end{cases}$$

$\frac{2c}{\gamma_0 t} = a_0$

Mass-discrepancy acceleration relation (MDAR)

Observational constraints

Using SPARC data of 123 late-type galaxies, the proposed fictitious acceleration explains mass discrepancy (M/M_b) slightly better (~1%) than the BTFR empirical fitting (RMAE = 0.115 and R^2 = 0.91 versus RMAE = 0.108 and R^2 = 0.92, respectively) (**Fig. 2**) and also better for individual galaxy rotation curves (RMAE = 0.080 and R^2 = 0.953 versus RMAE = 0.070 and R^2 = 0.957, respectively), even considering a unique parameter. Furthermore, the new model explains the transition between the Newtonian and cosmological scales in a natural way [5].

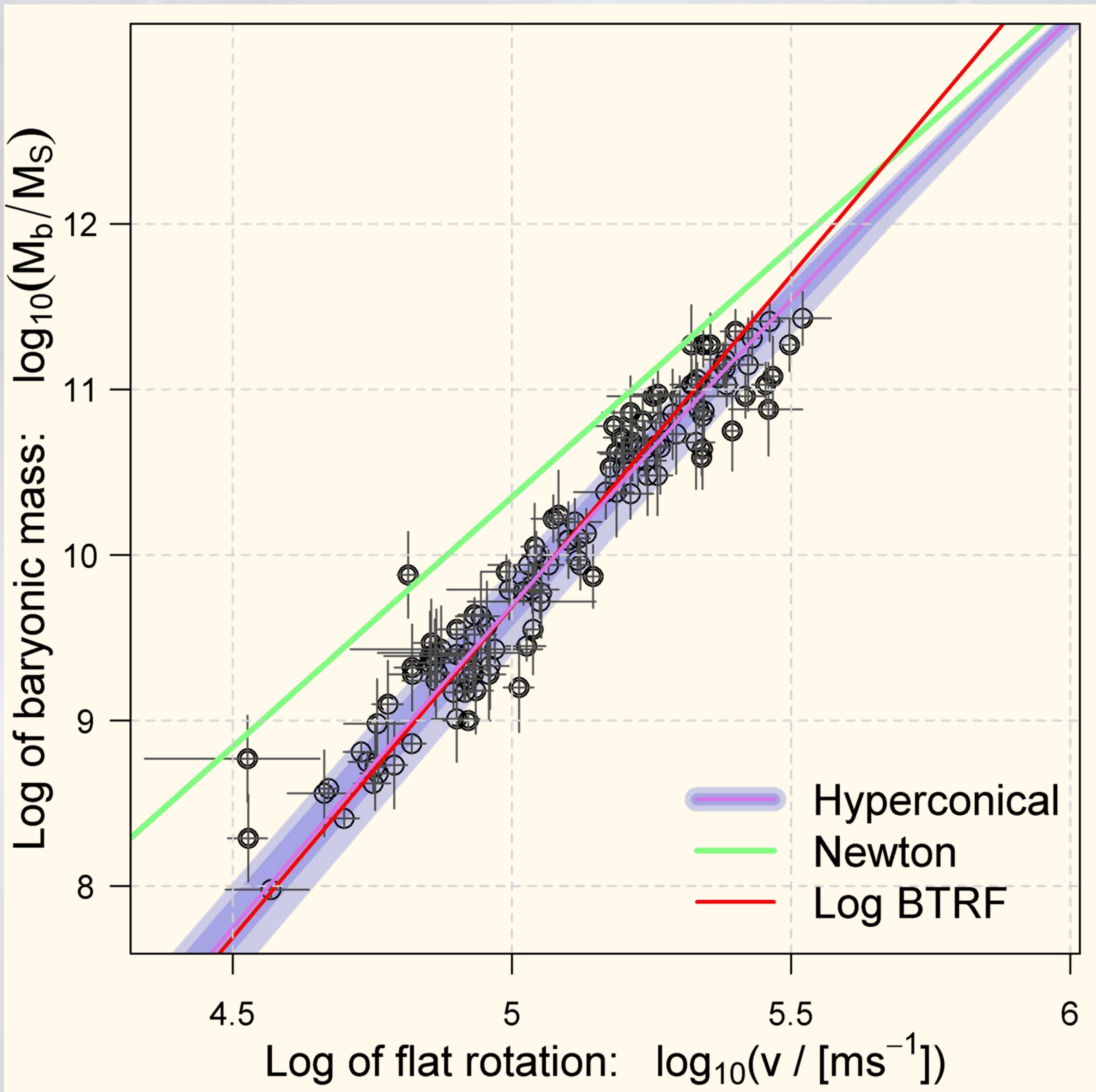

Figure 2. MDAR/BTFR modeling. Fitting of baryonic Tully–Fisher relation for 123 galaxies with 'flat' velocity (v) and baryonic mass (M_b), expressed in terms of solar mass (M_S). The Newton line represents the Keplerian orbits. The best fit is found for γ_M/π = 0.460 ± 0.002, which corresponds to $a_0 = 2c/t \cdot \cos(\gamma_M)/\gamma_M$ = 1.20±0.07·10^{-10}ms^{-2}, and the interquartile of $\gamma_M/\pi \in$ (0.436, 0.470), that is $a_0 \in$ (0.9, 20) ·10^{-10} ms^{-2}.

Conclusions

1. Dark energy and dark matter can be interpreted as two **geometrical consequences** of the same distorted stereographic projection.
2. The **first-order perturbation approach** in that projection is adequate to formulate a relativistic MOND and then to obtain rotation curves.
3. Two essential ingredients are: (1) the **mass of perturbations** with respect to the vacuum energy and (2) a **time-direction** centrifugal acceleration.
4. Lagrangian density of GR does not need to change but the **background Ricci curvature scalar** needs to be subtracted since it is equal to the vacuum energy density (i.e. the background of the mass perturbations).
5. Finally, the proposed fictitious acceleration explains the mass discrepancy (M/M_b) slightly better (~1%) than the BTFR empirical fitting

References

[1]. Monjo, R. (2017). Phys. Rev. D, 96, 103505, doi:10.1103/PhysRevD.96.103505, arXiv:1710.09697
[2]. Monjo, R. (2018). Phys. Rev. D, 98, 043508, doi:10.1103/PhysRevD.98.043508, arXiv:1808.09793
[3]. Monjo, R. and Campoamor-Stursberg R. (2020). Class. Quant. Grav. 37, 205015, doi: 10.1088/1361-6382/abada
[4]. Monjo, R. and Campoamor-Stursberg R. (2023). Class. Quant. Grav. doi: 10.1088/1361-6382/aceacc
[5]. Monjo, R. (2023). Class. Quant. Grav. CQG-110747 (under review).

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