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# A Lagrangian formulation of interacting dark sectors

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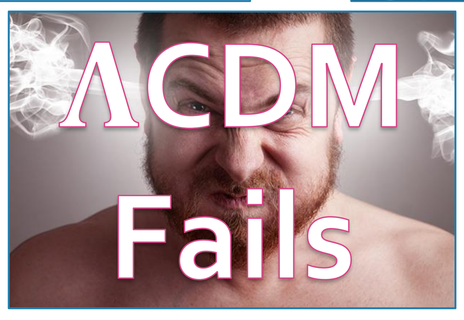
## Introduction

**ΛCDM Model**

| Indirect Measurement | Direct Measurement |
|---|---|
| $H_0 = 66 - 68$ km $s^{-1}$ Mpc$^{-1}$ | $H_0 = 72 - 74\ km\ s^{-1}\ Mpc^{-1}$ |

Alternatives: **Early Dark Energy**




Alternatives: **Extra Dark Radiation**

Alternatives: **Interacting DM-DE**

$\dot{\rho}_M + 3H(\rho_M + P_M) = Q$; $\dot{\rho}_{DE} + 3H(\rho_{DE} + P_{DE}) = -Q$.

$Q = \xi\, \rho_{DE}\, H$ --- Valentino et. al

Planck 2018, $H_0 = 72.8^{+3.0}_{-1.5}$ km $s^{-1}$ Mpc$^{-1}$ with $\xi = -0.54^{+0.12}_{-0.28}$

DM — Energy flow → DE

## Problem with Interaction

- Lack of generalization of $Q$.
- Lack of physical motivation of Hubble parameter $H$ appearing in the interaction.
- Instability in the dark sector at the perturbation level.

## Origin of Interaction

- **The Lagrangian of the interacting DM-DE can be written as**

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_M(\psi) + \mathcal{L}_{DE}(\phi) + \mathcal{L}_{int}(\psi, \phi)$$

- **Variation yields:**

$$3\,H^2 = \rho_M + \rho_{DE} + \rho_{int}$$
$$2\,\dot{H} + 3\,H^2 = -P_M + P_{DE} + P_{int}$$
$$\dot{\rho}_M + 3\,H\,\rho_M = Q_M$$
$$\dot{\rho}_{DE} + 3\,H\,(P_{DE} + \rho_{DE}) = Q_{DE}$$

- **The constraint relation:**

$$\dot{\rho}_{int} + 3\,H\,(\rho_{int} + P_{int}) + Q_M + Q_{DE} = 0.$$

## Equivalence of two picture

| Former approach | Lagrangian approach |
|---|---|
| $\bar{\rho}_M + \bar{\rho}_{DE}$ | $\rho_M + \rho_{int} + \rho_{DE}$ |
| $\bar{P}_M + \bar{P}_{DE}$ | $P_M + P_{DE} + P_{int}$ |
| **Linear Transformation** | |
| $\bar{\rho}_M = \alpha\,\rho_M + \beta\,\rho_{DE} + \gamma\,\rho_{int}$<br>$\bar{\rho}_{DE} = (1-\alpha)\rho_M + (1-\beta)\rho_{DE} + (1-\gamma)\rho_{int}$ | |
| $\dot{\bar{\rho}}_M + 3\,H\,(\bar{\rho}_M + \bar{P}) = (\alpha - \gamma)Q_M + (\beta - \gamma)Q_{DE}$<br>$\dot{\bar{\rho}}_{DE} + 3\,H\,(\bar{\rho}_{DE} + \bar{P}_{DE}) = -(\alpha - \gamma)Q_M - (\beta - \gamma)Q_{DE}$ | |
| $Q \equiv (\alpha - \gamma)Q_M + (\beta - \gamma)Q_{DE}$ | |

- Lagrangian construction is equivalent to the former.
- It modifies the Friedmann equations and generalizes the interaction.

## Action of an Interacting DM-DE

**Fluid Action**

$$S = \int \sqrt{-g}\,\frac{R}{2\,\kappa^2} - \sqrt{-g}\,\rho(n,s) + J^\mu(\varphi_{,\mu} + s\,\theta_\mu + \beta_A \alpha^A_\mu)$$
$$-\sqrt{-g}\mathcal{L}(X,\phi) - \sqrt{-g}\,\alpha_c f_c(\rho,\phi,X) + \alpha_d f_d(\rho,\phi,X)\,J^\mu\phi_\mu$$

k-essence Field — Kinetic Interaction — Derivative Interaction

$\mathcal{L} = -\frac{\delta^2}{\kappa^2\phi^2}(-X + X^2)$ & $X = \frac{1}{2}\dot{\phi}^2$

$J^\mu = \sqrt{-g}\,n\,u^\mu,$

- **The covariant derivative of stress tensor becomes,**

$$\nabla_\mu T^{\mu 0}_\phi = Q^0 = \alpha_c f_{c,\phi}\dot{\phi} + \alpha_c f_{c,X}\,\dot{\phi}\ddot{\phi} = -\nabla_\mu T^{\mu 0}_M$$

- **For the Derivative interaction**

$$\nabla_\mu T^{\mu 0}_\phi = Q^0 = n^2 \alpha_d 3\,H\,\dot{\phi}\,f_{d,n} = -\nabla_\mu T^{\mu 0}_M$$

## Dynamical Stability analysis

$$y = \frac{\kappa^2 V}{3H^2}, \Omega_M = \frac{\kappa^2\rho}{3H^2}, \Omega_\phi = \frac{\kappa^2\rho_\phi}{3H^2}, z = \frac{\kappa^2 f}{3\,H^2}, x = \dot{\phi}$$

**Stability:** Eigenvalues of $J_{ij} = (E_1, E_2, E_3)$
Stable: (-,-,-). Unstable: (+,+,+) Saddle: (-,+,+).

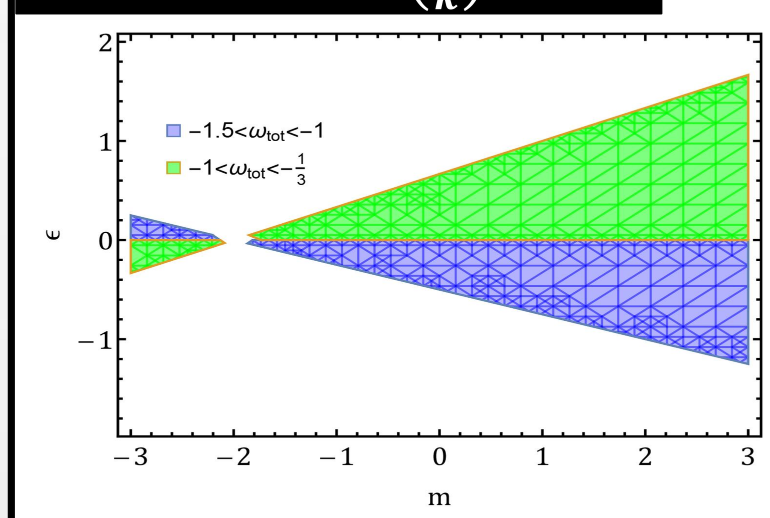
**Autonomous Equations**
$x_i{}' = h_i(x,y,z)$

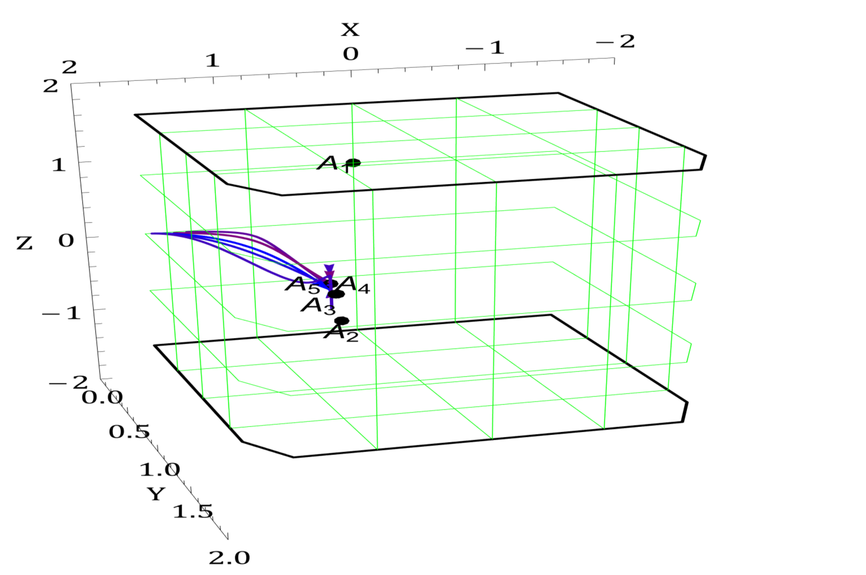
**Critical points:** $x_i' = 0$

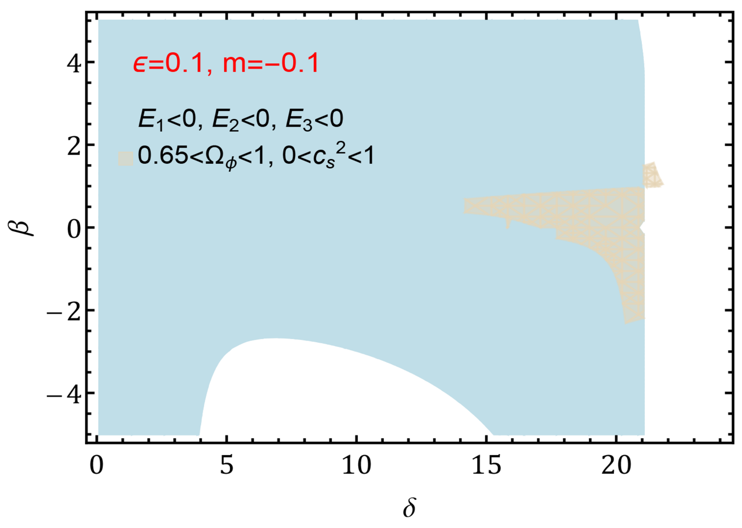
**Jacobian Matrix**
$J_{ij} = \left(\frac{dx_i{}'}{dx_j}\right)_{3\times 3}$

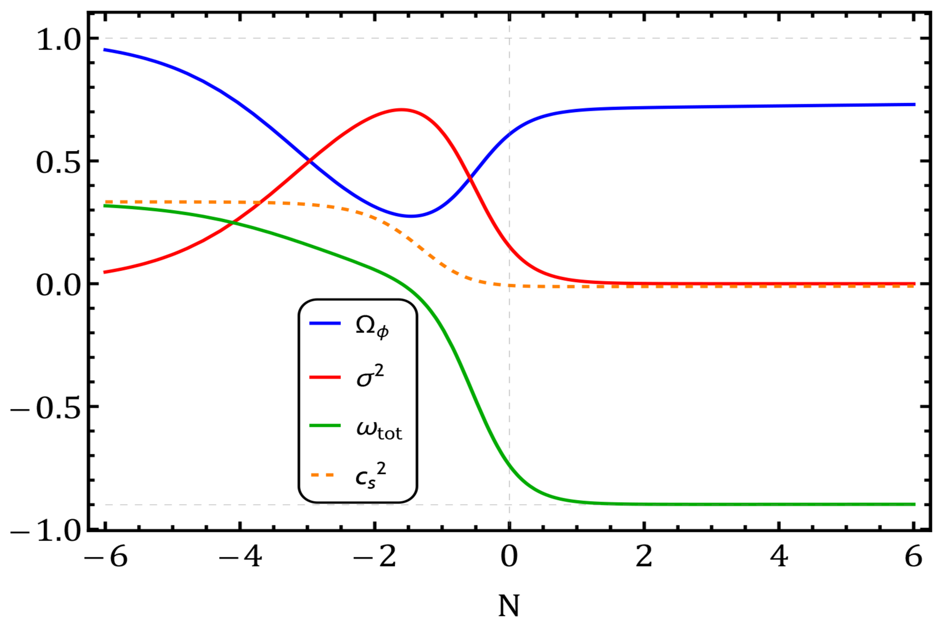
**Kinetic Model**

$$f_c = M^{4-4\epsilon}\,\rho^\epsilon\left(\frac{\phi}{\kappa}\right)^m e^{\beta\,X}$$




## Conclusion & References

- The coupling introduced at the Lagrangian level **generalizes the interaction**, making it easy to **identify instabilities**.
- Constructed two types of coupling: (i) one in which the interaction is a function of both field and fluid parameter, and using dynamical analysis with an **exponential interaction form**, the model demonstrates **stable accelerating solution** during the late-time phase; and (ii) another in which **field velocity is coupled with particle flux** density, resulting in ***H* in the continuity equation.**

[1] A. Pourtsidou, C. Skordis, and E. J. Copeland. "Models of dark matter coupled to dark energy". In: *Phys. Rev. D* 88.8 (2013), p. 083505.

[2] Anirban Chatterjee, Saddam Hussain, and Kaushik Bhattacharya. "Dynamical stability of the k-essence field interacting nonminimally with a perfect fluid". In: Phys. Rev. D 104.10 (2021), p. 103505.

[3] Kaushik Bhattacharya, Anirban Chatterjee, and Saddam Hussain. "Dynamical stability in presence of non-minimal derivative dependent coupling of k-essence field with a relativistic fluid". In: Eur. Phys. J. C 83.6 (2023), p. 488.

[4] Nicola Tamanini. "Phenomenological models of dark energy interacting with dark matter". In: Phys. Rev. D 92.4 (2015), p. 043524

[5] Christian G. Boehmer, Nicola Tamanini, and MatthewWright. "Interacting quintessence from a variational approach Part I: algebraic couplings". In: Phys. Rev. D 91.12 (2015), p. 123002.

[6] Christian G. Boehmer, Nicola Tamanini, and MatthewWright. "Interacting quintessence from a variational approach Part II: derivative couplings". In: Phys. Rev. D 91.12 (2015), p. 123003