Cosmic chronometers to calibrate the ladders and measure the curvature of the Universe. A model-independent study

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Tensions in Cosmology

Cosmological and astrophysical data have recently been found to be in some tension (2σ or larger) with the **standard ΛCDM model,** as specified by the *Planck* 2018 parameter values

Among others, there are

- *Hubble tension* (≈5σ) -> H_0
- *Growth tension* (2-3σ) -> σ_8 , $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$
- *CMB anisotropy anomalies* (2-3σ)

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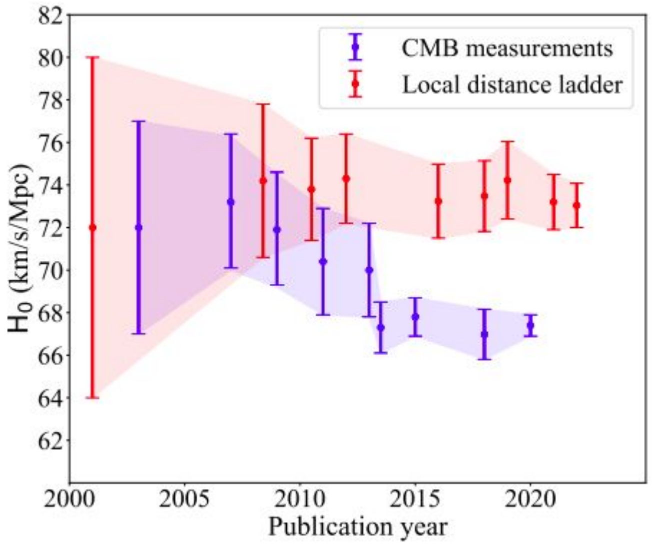

Perivolaropoulos+ 2021

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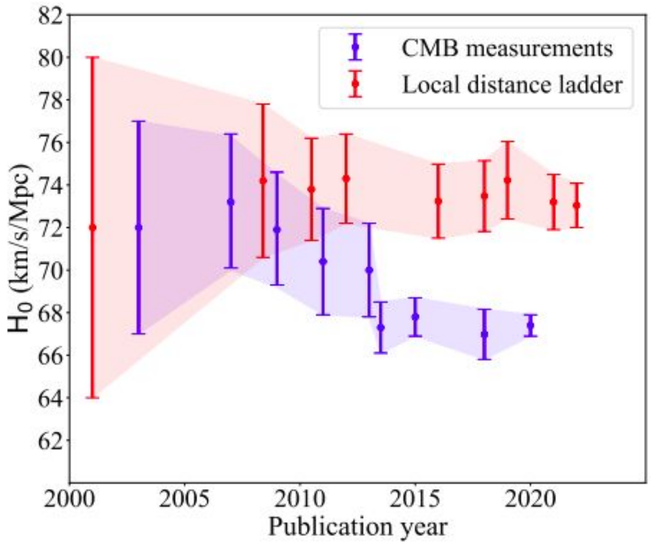

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Systematics in the data ?

New physics beyond flat ΛCDM ?

The cosmic distance ladder

Local distance ladder

model-independent estimates of H_0 in the local Hubble flow (z<0.15) through the calibration of the **absolute magnitude**, M, of standard candles such as Supernovae of type Ia (SNIa) at z<0.02

$$M = m(z) - 25 - 5\log\left(\frac{D_L(z)}{1\ Mpc}\right)$$

Inverse distance ladder

model-dependent estimates of H_0 : the standard ruler r_d, i.e. the sound horizon at the baryon-drag epoch, is used to calibrate cosmic distances and so the expansion rate $\boldsymbol{H(z)}$

$$\theta_* = \frac{\pi}{l_*} = \frac{r_d(z_*)}{D_A(z_*)} = \frac{\int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}}{\int_0^{z_*} \frac{dz}{H(z)}}$$

The **Hubble tension** can also be recast in a tension in the **calibrators of the local** and **inverse distance ladders**

A model-independent path to address the Hubble tension

We need alternative ways of calibrating the ladders

➔ Using low-redshift observations that are free from the main drivers of the current tensions: Cosmic Chronometers (CCH), SNIa and Baryon Acoustic Oscillations (BAO)

➔ Performing model-independent consistency tests of low-z data sets

➔ Tool: Gaussian Processes -> reconstruction of cosmological functions in an agnostic way

Low-*z* data sets for model-independent analysis

Cosmic chronometers (CCH)

32 direct measurements of *H(z)* using massive and passively-evolving galaxies in z = [0.07, 1.965]

⟶ rely only on the Cosmological Principle (CP) and General Relativity

Supernovae of type Ia (SNIa)

Pantheon+ compilation: 1701 light curves of 1550 spectroscopically confirmed SNIa in z = [0.001, 2.26] coming from 18 different surveys

⟶ We use only SNIa free from the local distance ladder calibration: 1624 data points

Baryon Acoustic Oscillations (BAO)

11 data points in z = [0.12, 1.48] from galaxy surveys (e.g., WiggleZ, BOSS, DES Y3, 6dFGS+SDSS), given in terms of angular BAO and radial BAO

Gaussian Processes (GPs)

How do we reconstruct the trend of our cosmological quantities in a model-independent way?

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Gaussian Processes: a bayesian tool designed to perform data-driven reconstruction of smooth trends from Gaussian distributed data

$$f(x) \sim GP(\mu(x), D[K(x,\tilde{x}), C])$$

Advantages:

- No need for a model – agnostic estimates of cosmological quantities
- Performs reconstruction of functions where we don't have data points
- Few assumptions: Choice of a kernel function

GaPP package by Seikel+ 2012

A model-independent path to address the Hubble tension

Test some of the **basic assumptions behind ΛCDM**, which are usually taken for granted:

1) the constancy of the SNIa absolute magnitude and **2)** the homogeneity property of the universe

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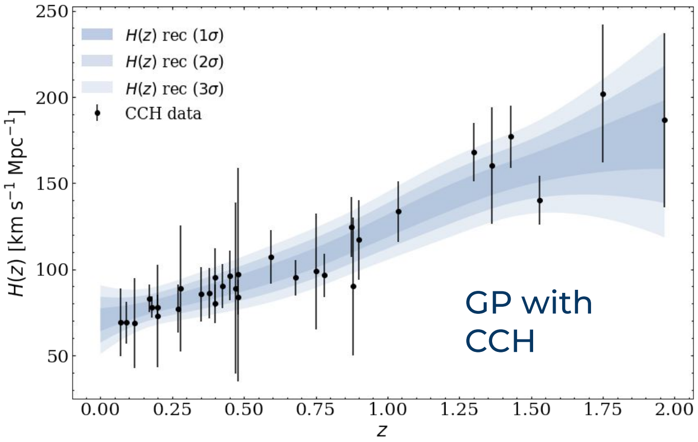
Test some of the **basic assumptions behind ΛCDM**, which are usually taken for granted:

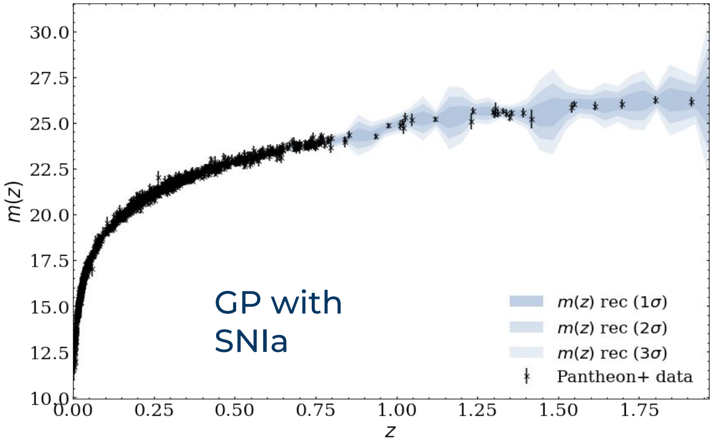
1) the constancy of the SNIa absolute magnitude and **2)** the homogeneity property of the universe

$$\Omega_k = 0$$

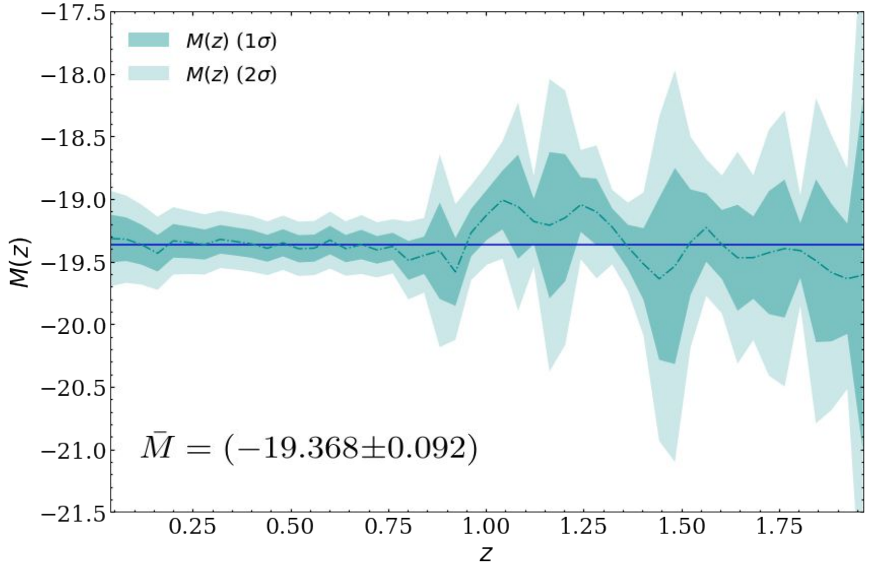
$$D_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}$$

$$M(z) = m(z) - 25 - 5\log\left(\frac{D_L(z)}{1\ Mpc}\right)$$





1) Testing the constancy of *M*



with the CCH and Pantheon+ data sets there is no significant statistical preference for the evolution of *M(z)*

2) Testing the Cosmological Principle

$\Omega_k(z)$ → $D_A(z) = \frac{D_L(z)}{(1+z)^2}$

1 $D_L(z) = 10^{m(z)-M(z)-25}$

2 $D_L(z) = \frac{c(1+z)}{\sqrt{\Omega_k H_0^2}} \sinh\left(\sqrt{\Omega_k H_0^2} \int_0^z \frac{dz'}{H(z')}\right)$

2) Testing the Cosmological Principle

$$\Omega_k(z) \longrightarrow D_A(z) = \frac{D_L(z)}{(1+z)^2}$$

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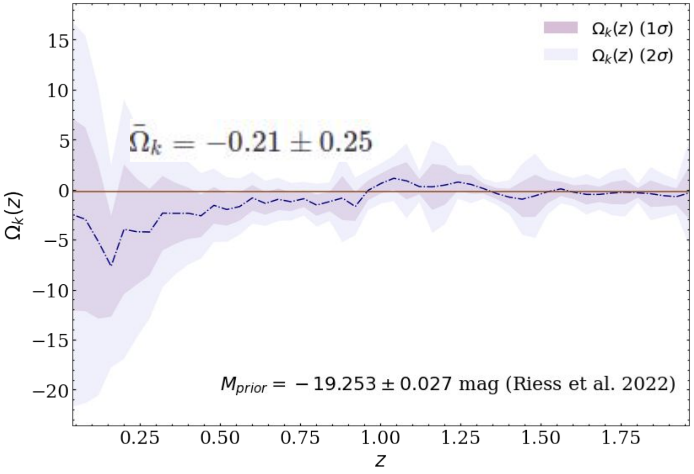

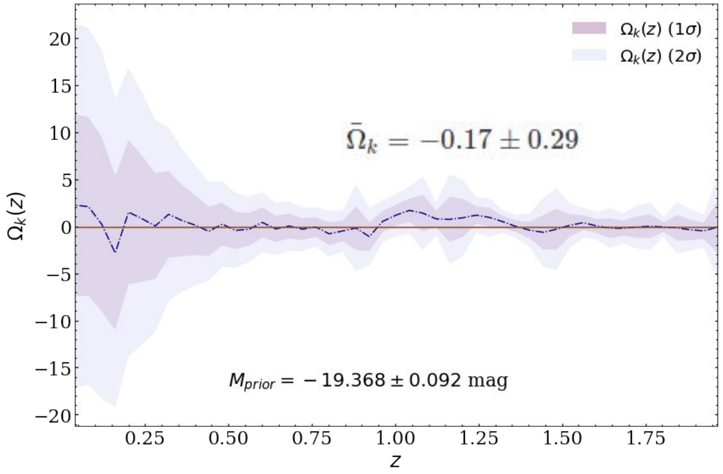

→ no hint for a violation of the CP from the CCH and Pantheon+ data sets

Consistency test for the BAO data

Angular and radial data: $r_d H(z)$, $D_V(z)/r_d$, $D_M(z)/r_d$

$$\Omega_k = 0$$

$$D_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}$$

⟸ GP with CCH

$$\begin{cases} D_A(z) = D_L(z)/(1+z)^2 \\ D_M(z) = (1+z)D_A(z) \\ \dfrac{D_V(z)}{r_d} = \dfrac{1}{r_d}\left[D_M^2(z)\,\dfrac{cz}{H(z)}\right]^{\frac{1}{3}} \end{cases}$$

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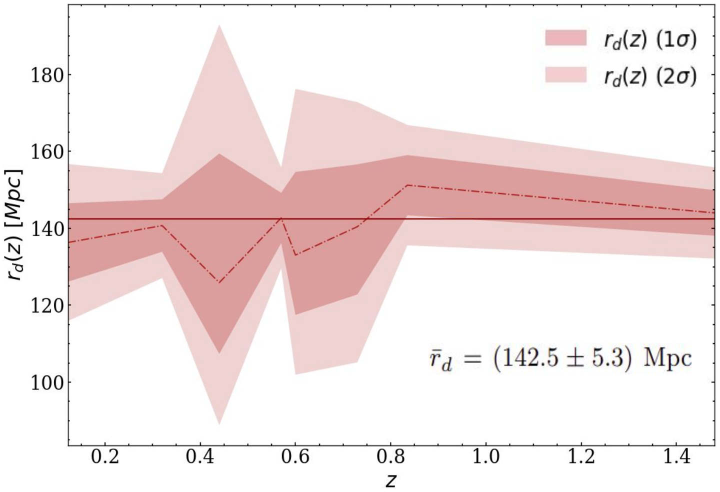
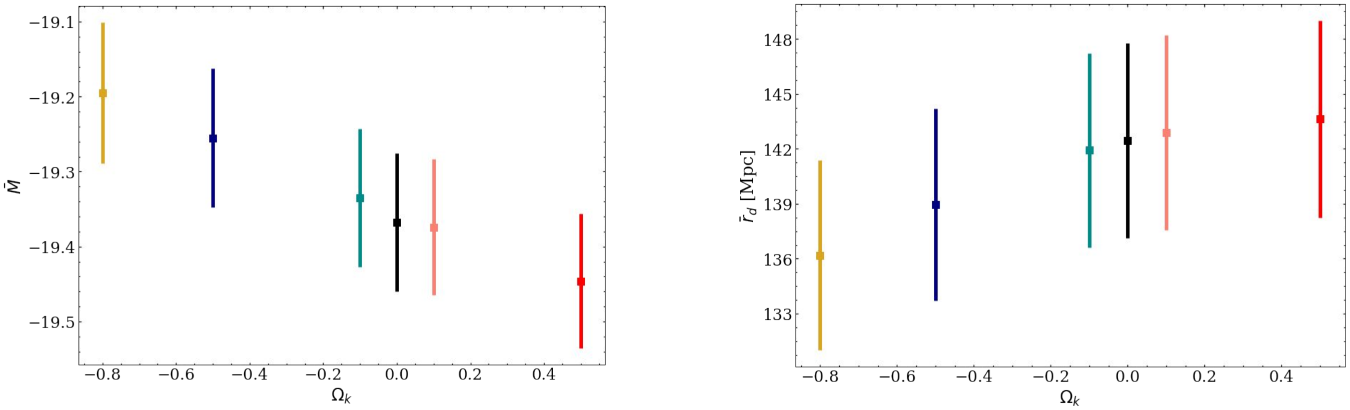


⟹ the values of r_d at different redshifts are statistically consistent with each other

Prior dependence

All the central values that we find in these analyses depend on the subjective choice of the priors:

$$D_L(z) = \frac{c(1+z)}{\sqrt{\Omega_k H_0^2}} \sinh\left(\sqrt{\Omega_k H_0^2} \int_0^z \frac{dz'}{H(z')}\right)$$



A more robust analysis has to be carried out in a multi-dimensional parameter space to jointly constrain M, Ω_k, r_d

Joint analysis

We combine the CCH, SNIa and BAO data sets to obtain joint constraints in the parameter space (M, Ω_k, r_d) through a grid-search method based on a χ^2 statistics

$$\chi^2_{\mu,i} = \sum_{k,l=1}^{1624} [m(z_k) - m_{rec,\mu,i}(z_k)] C^{-1}_{kl} [m(z_l) - m_{rec,\mu,i}(z_l)]$$ (CCH+SNIa)

$$\chi^2_{\mu,i} = \sum_{n,j=1}^{11} [x(z_n) - x_{rec,\mu,i}(z_n)] C^{-1}_{nj} [x(z_j) - x_{rec,\mu,i}(z_j)]$$ (CCH+BAO)

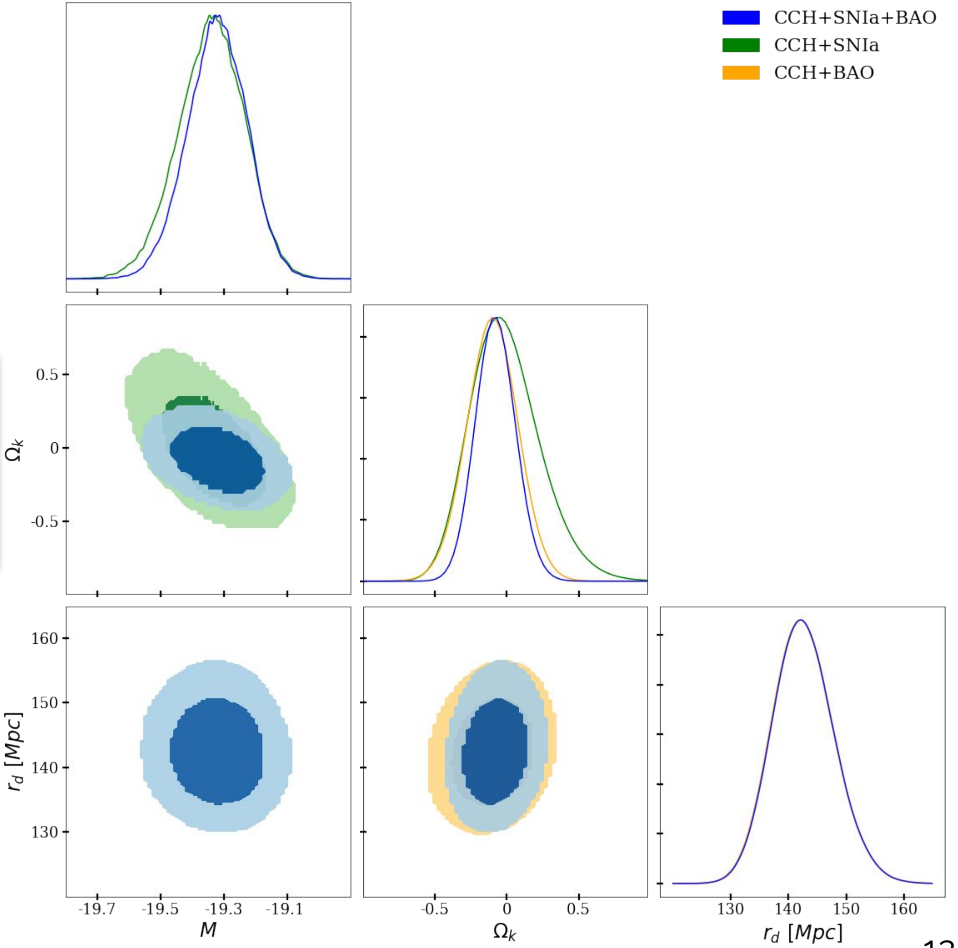
By combining the 2D analyses we get: $\chi^2(M, \Omega_k, r_d) = \chi^2(M, \Omega_k) + \chi^2(\Omega_k, r_d)$ (CCH+SNIa+BAO)

and we associate a weight to each grid point: $w_\mu \propto B_\mu \exp(-\bar{\chi}^2_\mu/2) \underbrace{\sum_{i=1}^{N} \exp(-[\chi^2_{\mu,i} - \bar{\chi}^2_\mu]/2)}_{\equiv f_\mu}$

$$\longrightarrow \quad \chi^2_{\mu,eff} = \bar{\chi}^2_\mu - 2\ln(B_\mu f_\mu)$$

Joint analysis - Results

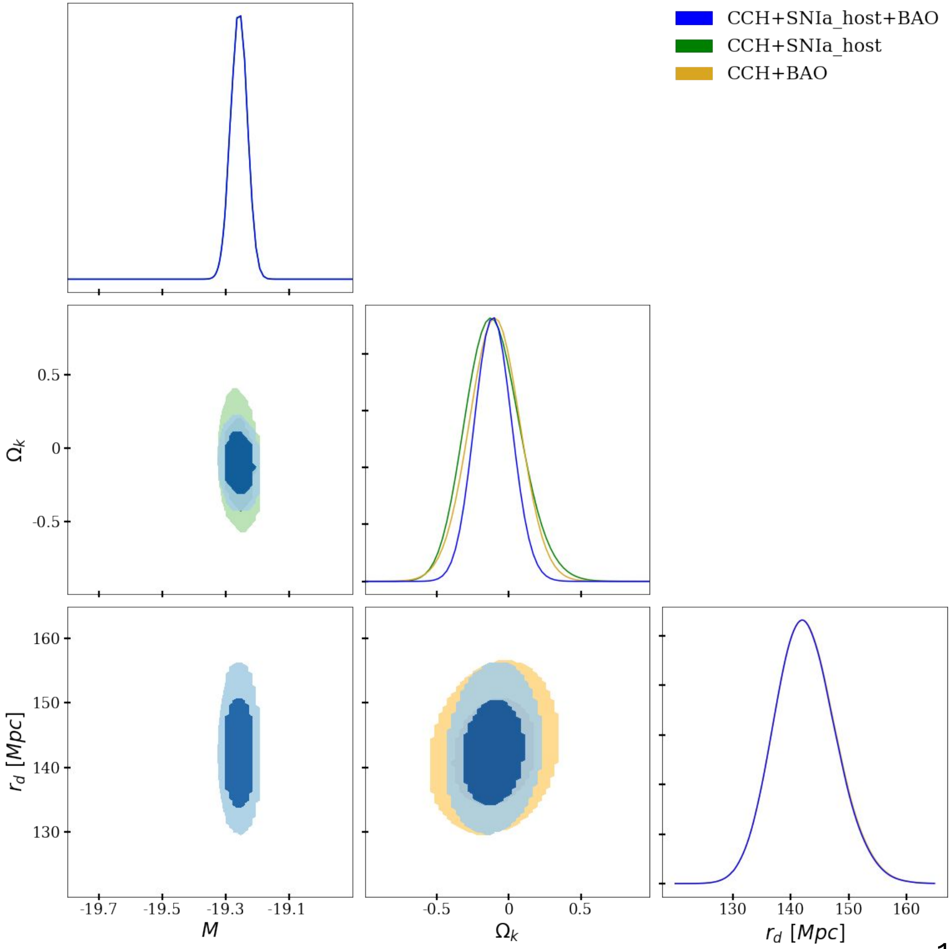
	CCH+SNIa	CCH+BAO	CCH+SNIa+BAO
M [mag]	$-19.344^{+0.116}_{-0.090}$		$-19.314^{+0.086}_{-0.108}$
Ω_k	$-0.07^{+0.27}_{-0.21}$	-0.10 ± 0.18	$-0.07^{+0.12}_{-0.15}$
r_d [Mpc]		$141.9^{+5.6}_{-4.9}$	142.3 ± 5.3



Joint analysis - Results

Including the SNIa located in the Cepheid host galaxies employed by **SH0ES** to calibrate the SNIa in the second rung of the cosmic distance ladder has a little impact on the results:

	CCH+SNIa_host+BAO
M [mag]	$-19.252^{+0.024}_{-0.036}$
Ω_k	$-0.10^{+0.12}_{-0.15}$
r_d [Mpc]	$141.9^{+5.6}_{-4.9}$



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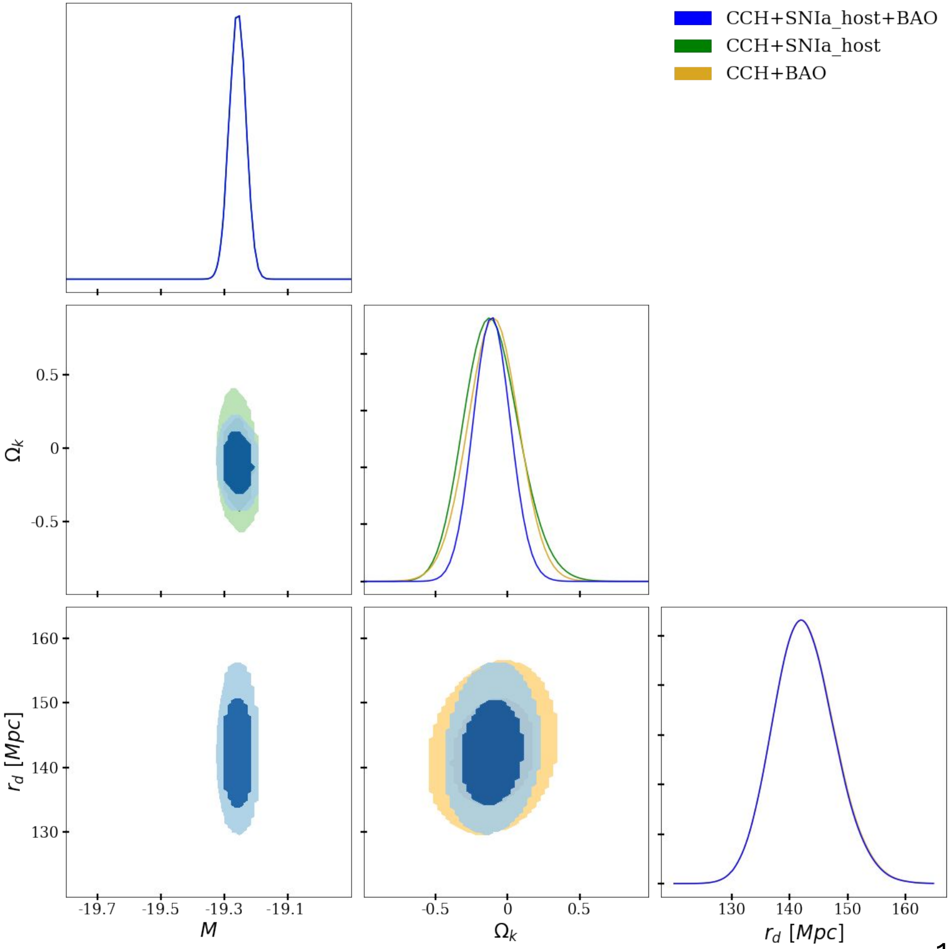
$$M^{R22} = (-19.253 \pm 0.027)\ \text{mag}$$

(Riess+, 2022)



H_0 measurement

We make use of the cosmographical expansion

$$D_L(z) = \frac{cz}{H_0}\left[1 + \frac{z}{2}(1 - q_0)\right] + \mathcal{O}(z^3)$$

Employing as a prior our CCH+SNIa+BAO constraint on M and the apparent magnitudes of the SNIa in the Hubble flow ($0.023 < z < 0.15$)

$$M(z) = m(z) - 25 - 5\log\left(\frac{D_L(z)}{1\,Mpc}\right)$$

We find: $H_0 = (71.5 \pm 3.1)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

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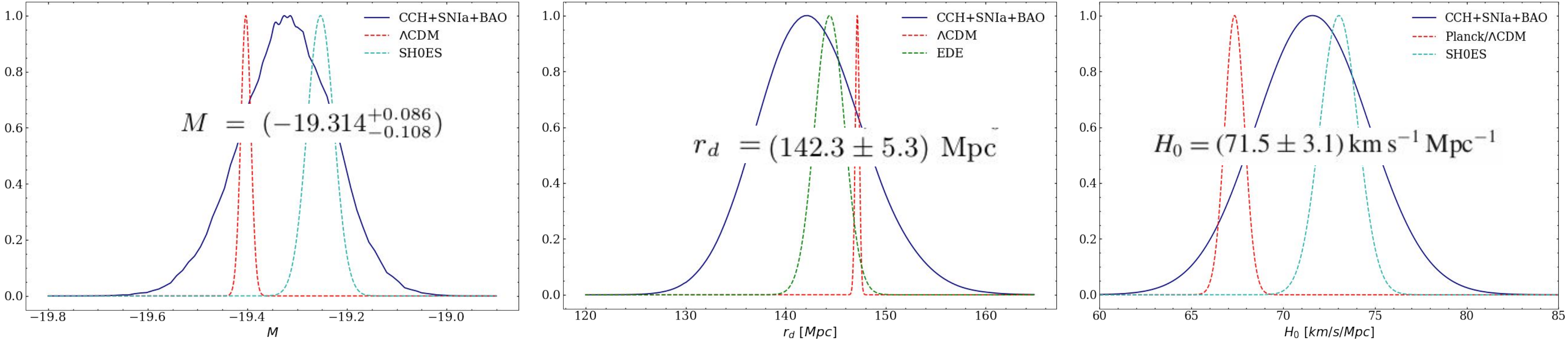
Main results & Conclusions

- ★ The CCH and Pantheon+ data sets do not point to a time evolution of the SNIa intrinsic luminosity nor a breaking of the homogeneity of the universe at large scales. Both, M and Ω_k are compatible at 68% C.L. with a constant.

- ★ Under the precision of the CCH, we find that the BAO data are all compatible with a common value of r_d .

- ★ Motivated by these results, we have jointly constrained with CCH, SNIa and BAO the constant values of Ω_k, M and r_d by applying a model-independent method which is also independent from the first rungs of the cosmic distance ladder employed by SH0ES and the CMB data from Planck, i.e. from the main data sets involved in the Hubble tension.

Main results & Conclusions



★ Nevertheless, the uncertainties we find are still too large to arbitrate the tension yet.

★ The proposed method will find interesting applications in the future with upcoming data (e.g., SNIa from the Vera C. Rubin Observatory's LSST and BAO from Euclid and DESI)

★ It will serve both as a discriminator of models beyond ΛCDM and an independent means of testing the calibration of the cosmic distance ladders.

Thank you for your attention!

Backup slides

About Gaussian Processes

$$\bar{\boldsymbol{f}}^* = \boldsymbol{\mu}^* + K(\boldsymbol{X}^*, \boldsymbol{X})[K(\boldsymbol{X}, \boldsymbol{X}) + C]^{-1}(\boldsymbol{y} - \boldsymbol{\mu})$$

$$cov(\boldsymbol{f}^*) = K(\boldsymbol{X}^*, \boldsymbol{X}^*) - K(\boldsymbol{X}^*, \boldsymbol{X})[K(\boldsymbol{X}, \boldsymbol{X}) + C]^{-1}K(\boldsymbol{X}, \boldsymbol{X}^*)$$

Kernel functions: →

Hyperparameters:

- l the characteristic length scale of significant changes in $f(x)$
- σ_f the variance, amplitude of significant changes in $f(x)$

$$K(x, \tilde{x}) = \sigma_f^2 exp\left[-\frac{(x-\tilde{x})^2}{2l^2}\right]$$

$$K(x, \tilde{x}) = \sigma_f^2 exp\left[\left(-\frac{\sqrt{3}|x-\tilde{x}|}{l}\right)\left(1 + \frac{\sqrt{3}|x-\tilde{x}|}{l}\right)\right]$$ Matérn 32 covariance function

$$\ln \mathcal{L} = \ln p(\boldsymbol{y}|\boldsymbol{X}, \sigma_f, l) = -\frac{1}{2}(\boldsymbol{y} - \boldsymbol{\mu})^T[K(\boldsymbol{X}, \boldsymbol{X}) + C]^{-1}(\boldsymbol{y} - \boldsymbol{\mu}) - \frac{1}{2}\ln|K(\boldsymbol{X}, \boldsymbol{X}) + C| - \frac{n}{2} ln2\pi$$

Backup slides

Weighted mean

$$\mathcal{L}(M) = \mathcal{N} \exp\left[-\frac{1}{2}\sum_{i,j=1}^{n_p}(M-\bar{M}_i)(M-\bar{M}_j)(C^{-1})_{ij}\right]$$

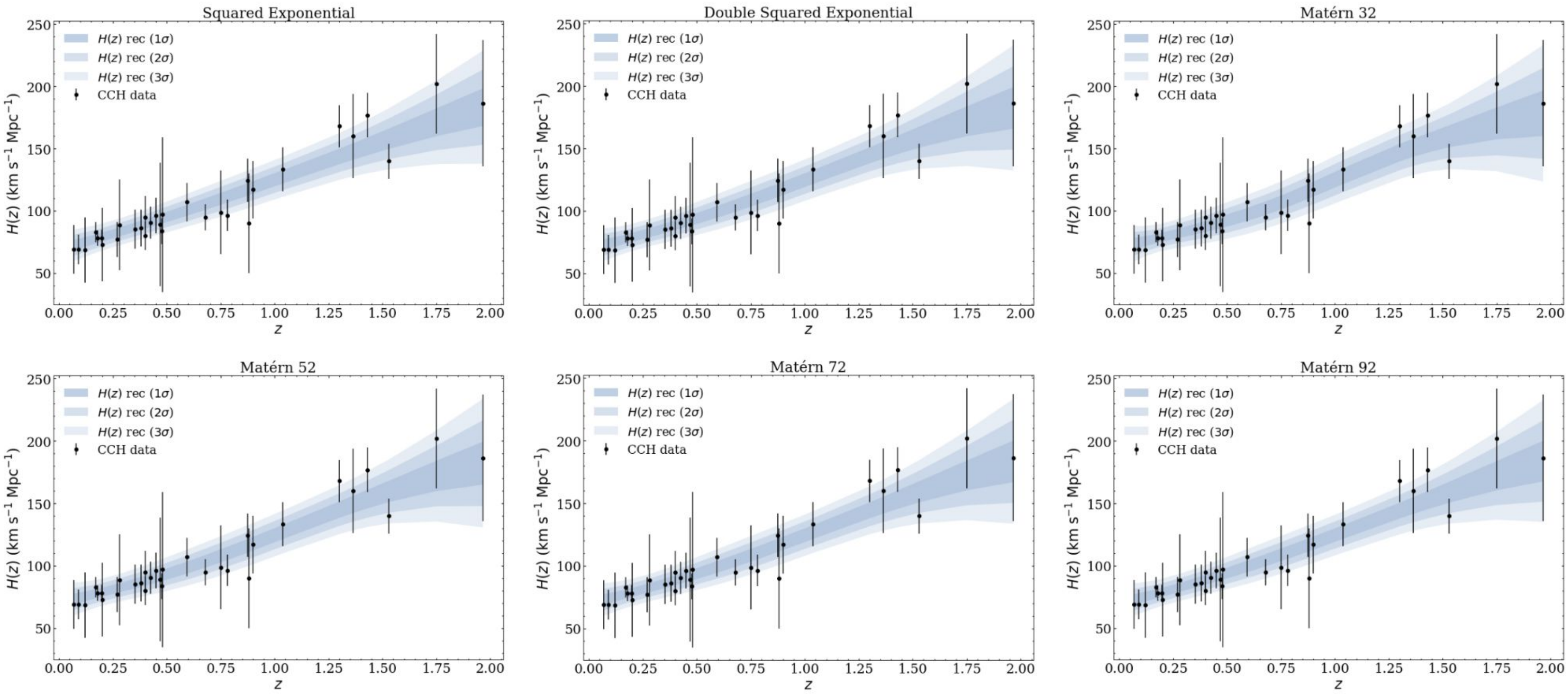
$$\mathcal{L}(M) = \tilde{\mathcal{N}} \exp\left[-\frac{1}{2}\left(\sum_{i,j=1}^{n_p} A_{ij}\right)\left(M - \frac{\sum_{i,j=1}^{n_p}\bar{M}_i A_{ij}}{\sum_{i,j=1}^{n_p} A_{ij}}\right)^2\right]$$

$$A_{ij} \equiv (C^{-1})_{ij} \qquad C_{ij} = \frac{1}{N_{real}}\sum_{\mu=1}^{N_{real}}(M_{\mu,i}-\bar{M}_i)(M_{\mu,j}-\bar{M}_j)$$

$$\bar{M} = \frac{\sum_{i,j=1}^{n_p}\bar{M}_i A_{ij}}{\sum_{i,j=1}^{n_p} A_{ij}} \quad ; \quad \sigma^2 = \frac{1}{\sum_{i,j=1}^{n_p} A_{ij}}$$

Backup slides

GP Kernels performance test on the reconstruction of the Hubble function with CCH

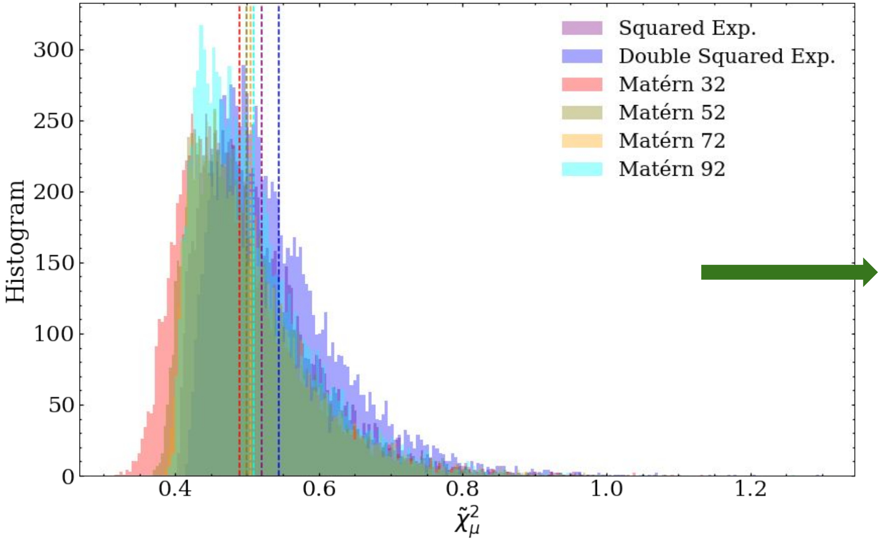


Backup slides

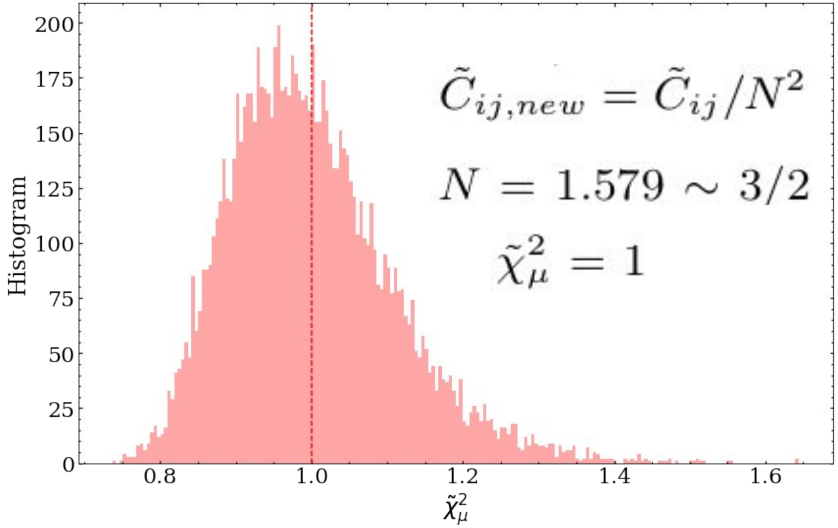
$$\chi^2_\mu = \sum_{i,j=1}^{32} [H(z_i) - H_{rec,\mu}(z_i)] \tilde{C}^{-1}_{ij} [H(z_j) - H_{rec,\mu}(z_j)]$$

$$P_{\tilde{\chi}^2_{K_i} < \tilde{\chi}^2_{K_j}} = \frac{1}{1 + P_j/P_i}$$

Kernels	P_{SE}/P_j
SE vs DSE	1.42
SE vs M32	**0.62**
SE vs M52	0.72
SE vs M72	0.82
SE vs M92	0.83



Considering smaller uncertainties in CCH data



Backup slides

Results

$$M = (-19.326^{+0.050}_{-0.068})$$

$$\Omega_k = 0.10^{+0.12}_{-0.15}$$

$$r_d = (142.6^{+3.9}_{-3.5})\ \mathrm{Mpc}$$