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An efficient conservative solver for plasma transport equations in tokamaks

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Integrated modelling for magnetically confined tokamak plasmas is an indispensable tool in interpreting and in guiding the tokamak experiments. To evolve the plasma profiles (current, densities, temperatures), integrated modelling schemes solve a system of stiff diffusion-advection transport equations, constructed using a set of physical models for equilibrium, transport and sources. It is crucial for the solver of the transport equations to be accurate, numerically stable, conservative and fast converging to ensure robustness and to reduce the number of calls to numerically expensive physical models.
In this work we adapt the conservative Interpolated Differential Operation (IDO) scheme [1] to solve the Generalized Form of Transport Equations [2] (backwards-Euler time advance)

$$\frac{aY-bY_M}{\tau}+\frac{1}{c}\frac{\partial}{\partial x}\left(-d\frac{\partial Y}{\partial x}+eY\right)=f-gY, \tag{1}$$

which is used to formulate transport equations in ETS [2] for linearized iterations in the non-linear convergence loop. Here, $Y(x), Y_M(x)$ are the predicted profile on the present and previous time-steps respectively, τ is the time-step, and $a(x), b(x), .., g(x)$ are the transport coefficients obtained from physical models. To approximate $Y(x)$ we use 4th-order polynomials centred at the inner spatial grid points $\{x_i\}_{i=2}^{N-1}$ with grid size N. The coefficients of i-th polynomial are determined from $\{Y_{i-1}, Y_i, Y_{i+1}, Y^x_{i-1/2}, Y^x_{i+1/2}\}$, with the profile values $Y_i := Y(x_i)$ and the cell-integrated values $Y^x_{i+1/2} := \int_{x_i}^{x_{i+1}} Y(x)\mathrm{d}x$. The spatial derivatives $\frac{\partial Y}{\partial x}, \frac{\partial^2 Y}{\partial x^2}$ are available via the approximating polynomials. A system of discretized linear equations is formed from: eq. (1) discretized at $\{x_i\}_{i=2}^{N-1}$; cell-integrals of eq. (1) at intervals $\{[x_i, x_{i+1}]\}_{i=1}^{N-1}$; and the boundary conditions. The free variables are $\{Y_{i+1}, Y^x_{i+1/2}\}_{i=1}^{N-1}$. We evaluate integrals of the form $\int_{x_i}^{x_{i+1}} a(x)Y(x)\mathrm{d}x$ using Gauss-Legendre quadrature, where the off-grid values of $Y(x), a(x)$ are obtained from the approximating polynomials and a cubic splines representation respectively.
The proposed scheme has 4th order of convergence in space, which enables significant reduction of the spatial grid size while retaining high accuracy, thus minimizing calls to expensive physical models [3]. The proposed scheme is conservative by design, since it directly solves cell-integrated version of eq. (1). Additionally, as compared to IDO-based implementations [3], the scheme proposed here does not require the second-order derivative of diffusion and the first-order derivative of the source term, thus avoiding issues related to instabilities when taking numerical derivatives of transport coefficients computed by physical codes. The non-linear convergence using under-relaxation is similar to that reported in [3].

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Authors: Dr LUDVIG-OSIPOV, Andrei (Chalmers University of Technology); Dr YADYKIN, Dmytro (Chalmers University of Technology); Prof. STRAND, Pär (Chalmers University of Technology)

Presenter: Dr LUDVIG-OSIPOV, Andrei (Chalmers University of Technology)

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