Nonlocal transport theory in relativistic plasmas

K. Bendib-Kalache and A. Bendib
Laboratoire Electronique Quantique, Faculty of Physics, USTHB, Algiers, Algeria

Linear semicollisional transport coefficients in relativistic plasmas are calculated from the Vlasov-Landau equation. Non local expressions of electron heat flux and of transfer of momentum from ions to electrons in collisions were derived for arbitrary relativistic regime defined by the relativistic parameter , $z = m_e c^2/T$ where m_e is the electron mass, *T* is the electron temperature (in energy units) and *c* is the speed of light. New transport coefficients valid for arbitrary relativistic regime and collisionality were derived which can be used as reliable closure relations in fluid equations. The asymptotic collisional and collisionless regime for the thermal conductivity, frictional force and thermal force reported in Refs. 1-3 were recovered.

KINETIC MODEL

•Relativistic Vlasov-Landau equation

$$\frac{\partial F}{\partial t} + \vec{v}\cdot\frac{\partial F}{\partial \vec{r}} - e\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\frac{\partial F}{\partial \vec{p}} = C_{ee}(F,F) + C_{ei}(F)$$

Approximations:

✓Gradients along the x-axis

✓ the plasma is split into a global equilibrium and a perturbed state

•Equilibrium state defined by the temperature T_0 , the density n_0 and the Maxwell-Boltzmann-Jüttner (MBJ) electron distribution function (EDF)

$f_{MBJ} = \frac{n_0 z}{4\pi m^3 c^3} exp(-z\gamma)$ where $\gamma = \sqrt{1 + (p/m_e c)^2}$ and $\vec{p} = \gamma m \vec{v}$

•Perturbed state defined by the EDF δf the fluid velocity δV the density δn and the electric field δE

• the perturbed Vlasov-Landau equation

$$\frac{\partial \delta f}{\partial t} + c^2\frac{p_x}{\varepsilon}\frac{\partial \delta f}{\partial x} - e\delta E_x\frac{\partial f_{MBJ}}{\partial p_x} - \frac{\varepsilon}{c^2}\frac{\partial f_{MBJ}}{\partial p_x}\frac{\partial \delta V_x}{\partial t} - p_x\frac{\partial f_{MBJ}}{\partial p_x}\frac{\partial \delta V_x}{\partial x} = C_{ee}(\delta f) + C_{ei}(\delta f)$$

where $\varepsilon = \gamma m_e c^2$, $C_{ee}(f) = -\nu_{ee}\left(f - f_{MBJ}\right)$ $C_{ei}(f) = \nu_{ei}(p)\frac{\partial}{\partial p_i}\left(p_i p_j - p^2\delta_{ij}\right)\frac{\partial f}{\partial p_j}$

Resolution :

✓spatial Fourier transform $(x \leftrightarrow k)$ and Laplace transform $(t \leftrightarrow s)$

in the limit of the stationary approximation, $(s \to 0)$

✓Expansion in the Legendre polynomial basis: $\delta f = \sum_{l=0}^{\infty} \delta f_l(p) P_l(\mu)$

where $P_l(\mu)$ is the l^{th}-order Legendre polynomial,and $\mu = \frac{p_x}{p}$

✓Lorentz approximation: $C_{ee} \ll C_{ei}$

It results an infinite set of equations

$$\lim_{s\to 0} s\delta f_0 - \delta f_{MBJ}(t=0) + \frac{c^2}{\sqrt{3}}\frac{p}{\varepsilon}ik\delta f_1 + \frac{1}{3}\frac{p^2}{m\varepsilon}zf_{MBJ}ik\delta V_x = -\nu_{ee}\left(\delta f_0 - \delta f_{MBJ}\right)$$

$$\lim_{s\to 0} s\delta f_1 + c^2\frac{p}{\varepsilon}ik\left(\frac{1}{\sqrt{3}}\delta f_0 + \frac{2}{\sqrt{15}}\delta f_2\right) + \frac{1}{\sqrt{3}}\frac{e}{m}\frac{p}{\varepsilon}zf_{MBJ}\delta E_x = -2\nu_{ei}(p)\,\delta f_1$$

$$\lim_{s\to 0} s\delta f_2 + c^2\frac{p}{\varepsilon}ik\left(\frac{2}{\sqrt{15}}\delta f_1 + \frac{3}{\sqrt{35}}\delta f_3\right) + \frac{2}{\sqrt{45}}\frac{p^2}{m\varepsilon}zf_{MBJ}ik\delta V_x = -6\nu_{ei}(p)\,\delta f_2$$

$$\lim_{s\to 0} s\delta f_l + c^2\frac{p}{\varepsilon}\frac{\partial}{\partial x}\left(\frac{l}{\sqrt{4l^2-1}}\delta f_{l-1} + \frac{l+1}{\sqrt{2l+1}\sqrt{2l+3}}\delta f_{l+1}\right) = -l(l+1)\nu_{ei}(p)\,\delta f_l \quad \text{for } l \geq 3$$

Resolution with the used of the continued fraction : $F_l = \frac{1}{\nu_{ei} + k^2c^2\frac{(l+1)^2}{4(l+1)^2-1}\frac{\gamma^2-1}{\gamma^2}F_{l+1}}$

The solution reads:

$$\delta f_0 = \delta f_0(\delta n)\frac{\delta n}{n_0} + \delta f_0(\delta T)\frac{\delta T}{T_0} + \delta f_0(\delta V)\frac{\delta V}{c} + \delta f_0(\delta U)\frac{\delta U}{c}$$

where δU is the velocity between ions /electrons

$$\delta f_1 = \delta f_1(\delta n)\frac{\delta n}{n_0} + \delta f_1(\delta T)\frac{\delta T}{T_0} + \delta f_1(\delta V)\frac{\delta V}{c} + \delta f_1(\delta U)\frac{\delta U}{c}$$

where the components depend on integrals of the form: $\int_1^{\infty} F_0F_1(\gamma^2-1)^n\gamma^m\, exp(-z\gamma)d\gamma$

•Heat flux δq and the transfer of momentum from ions to electrons δR_{ei}

$$\delta q = \frac{4\pi}{3}m_e^4c^6\int_1^{\infty}\gamma(\gamma^2-1)\delta f_1 d\gamma \quad \text{and} \quad R_{ei} = -\frac{1}{3}\frac{Z_I n_e m_e e^4 ln\Lambda}{\varepsilon_0^2}\int_1^{\infty}\gamma\delta f_1 d\gamma$$

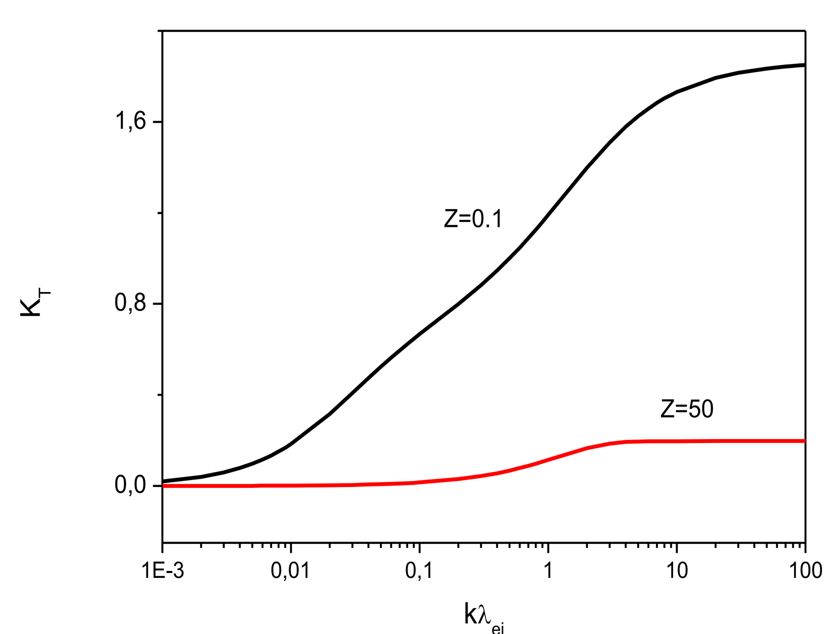
Using the Braginskii notation, they are written follows

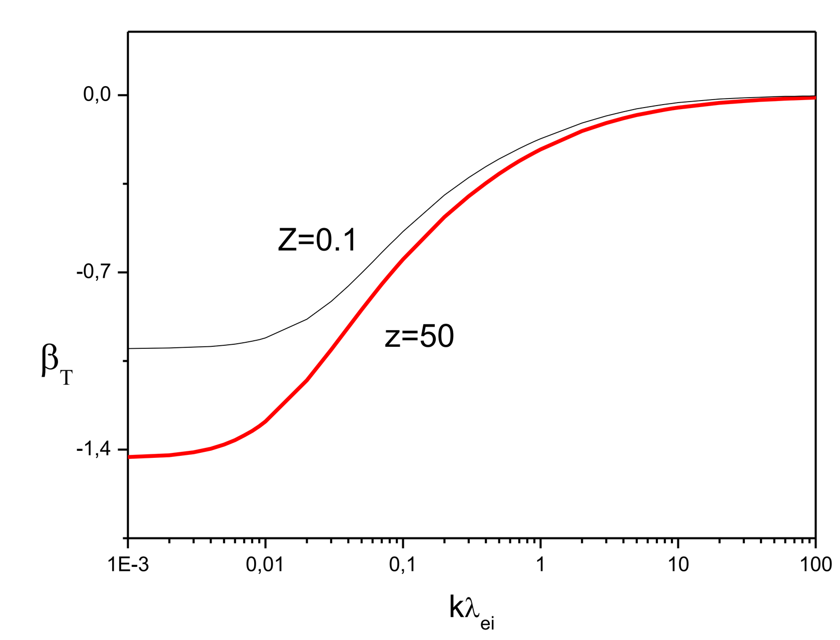
$$\delta q = -K_T n_e c\frac{ik}{k}\delta T + \alpha_u n_e T\delta u + \alpha_V n_e \delta V$$

$$\delta R_{ei} = \beta_T n_e ik\delta T + \beta_u\frac{c}{\lambda_{ei}}n_e m\delta u$$

where λ_{ei} the electon-ion mean free path

•Numerical results





Discussion:

✓The thermal conductivity K_T strictly decreases from the collisional limit to the collisionless one

✓For increasing relativistic effects, the thermal conductivity increases

✓We recover the Onsager symetry $\beta_T = \alpha_u$ in the whole collisionalty regime.

✓We found that α_u strictly decreases from the collisional value to zero in the collisionless limit

✓The coefficient α_V is purely non local coefficient. It strictly increases from zero to -0.4 in the collisionless limit

CONCLUSION

In present and future ICFexperiments:

❑ temperture T ranges from 5keV $(z = 100)$ up to 15 keV $(z = 34)$.This regime is moderately relativistic. Thus the relativistic effects should be accounted for

❑All the dispersion relations of stable (plasma waves, ion acoustic waves…) and instable modes instabilities (Weibel and parametric instabilities…) have to be calculated in the whole collisionality regime. In Ref. 4, we found that the growth rate of the Weibel modes is maximum in the intermediate collisionality $(k\lambda_{ei}{\sim}1)$.

REFERENCES

[1] S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, NewYork, 1965),Vol. 1,p. 205.
[2] K. Bendib-Kalache, A. Bendib, and K. Mohammed El Hadj, Phys. Rev. E **82**, 056401 (2010).
[3] B. Touil, A. Bendib and K. Bendib-Kalache Phys. Plasmas **24**, 022111 (2017).
[4] Y. Medjkoune, K. Bendib-Kalache and A. Bendib, Phys. Plasmas **25**, 012116 (2018).