MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Global fluid turbulence simulations in stellarators

Wendelstein 7-X

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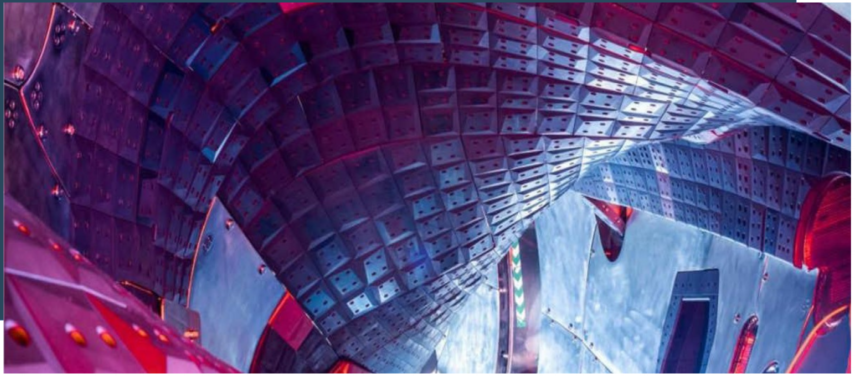

EUROfusion

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Outline

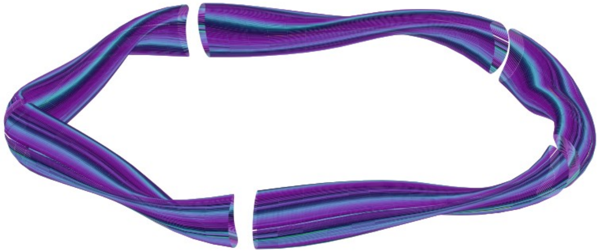
An introduction

- **Stellarator SOL physics**
- **BOUT++**
- **Flux Coordinate Independent (FCI) method**

Global fluid turbulence simulations

- **analytic stellarator**
  - equilibrium transport
  - fluctuation dynamics
- **Wendelstein 7-X**

Conclusions and future work



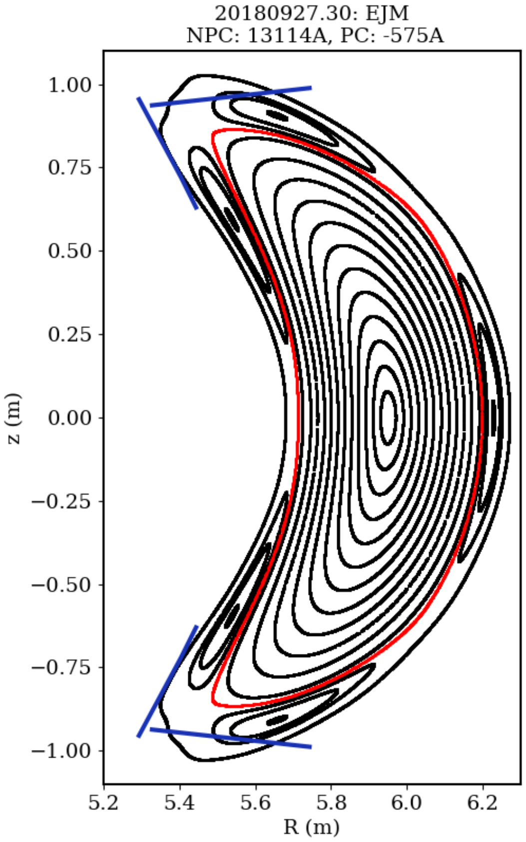
Background and motivation

The scrape-off-layer (SOL) of W7-X provides a novel environment for SOL physics

- **magnetic islands and chaotic field lines**
- **nonuniform curvature and abrupt changes in $L_{||}$ due to the island divertor**

The study of turbulence within the W7-X SOL is in its infancy **[1,2]**

While local simulations are useful, global edge turbulence simulations are becoming available **[3,4]**



Poincaré plot of W7-X indicating island edge topology **[courtesy C Killer]**

[1] Killer et al., *NF* (2019)
[2] Killer et al., *NF* (2021)
[3] Shanahan et al., *PPCF* (2019)
[4] Coelho et al., *NF* (2022)

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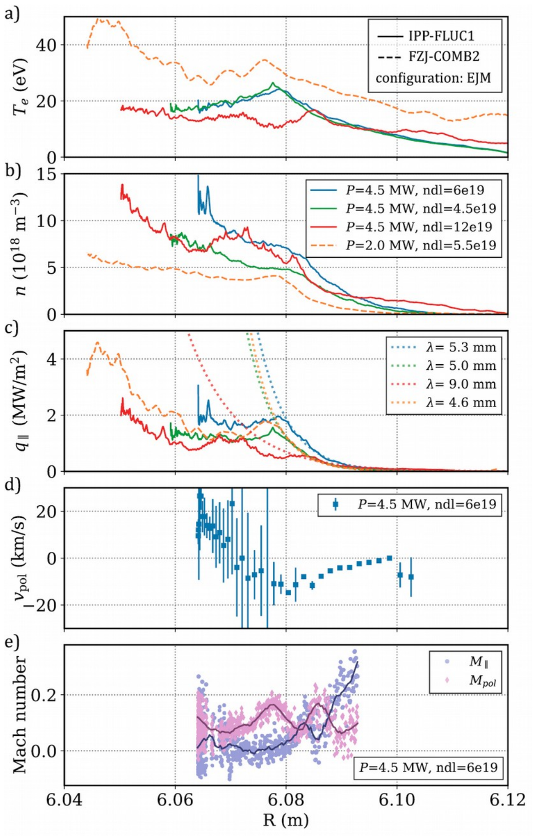


Measurements of plasma parameters in the W7-X SOL; Island influence can be seen in i.e. poloidal velocity flip. **[1]**

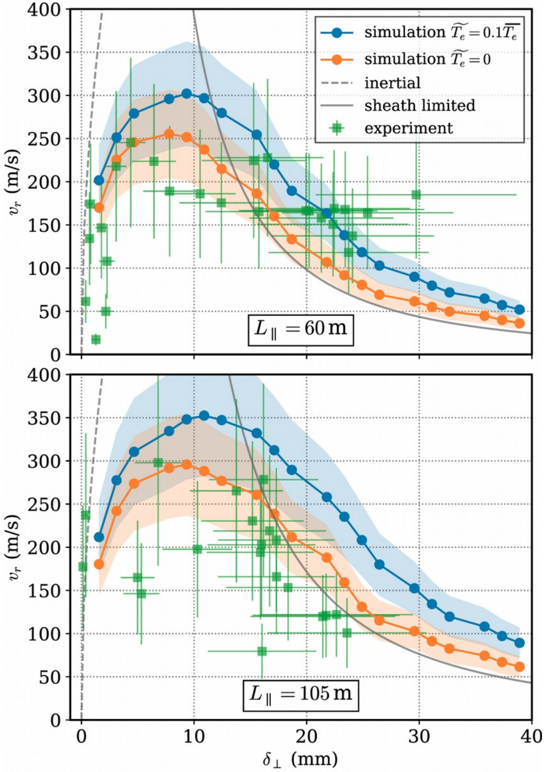
BOUT++

BOUT++ **[5]** is a framework for nonlinear plasma fluid simulations

- **No set model, geometry, or numerical methods**

Previous applications to stellarators:

- **Understand filament propagation due to:**
  - Nonuniform curvature
    - Globally **[3]** and locally **[6,7]**
  - Abrupt changes in connection length **[8]**
- **Comparing filament simulations to probe measurements [9]**



Comparison of probe measurements (green) and seeded blob simulations (orange/blue) **[9]**

[3] Shanahan et al., *PPCF* (2019)
[5] Dudson et al., *CPC* (2009)
[6] Shanahan et al., *JP;CS* (2018)
[7] Huslage et al., *PPCF Submitted* (2023)
[8] Shanahan & Huslage *JPP* (2020)
[9] Killer & Shanahan et al., *PPCF* (2020)

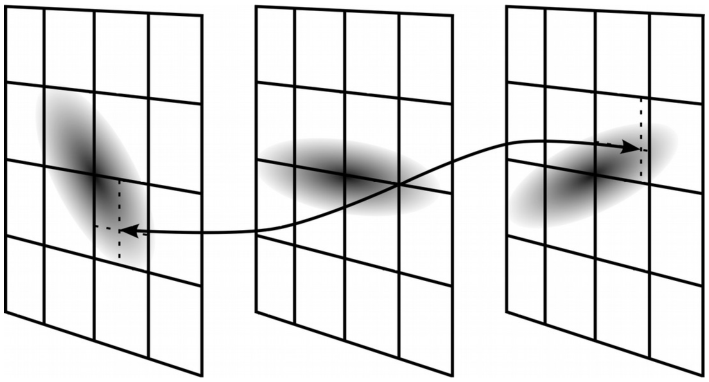
Stellarator stimulations in BOUT++ require lateral thinking

BOUT++ conventionally uses field-aligned coordinates (∇ψ, ||, ϕ) due to the flute-like nature of fluid turbulence

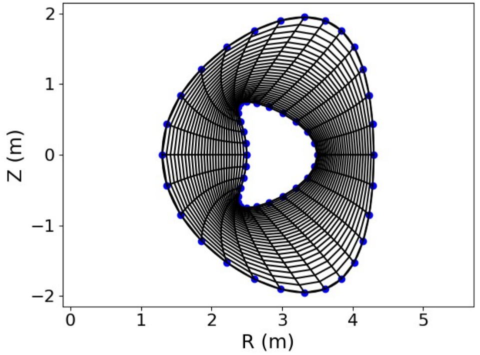
- **difficult in complicated topologies**

The Flux-Coordinate-Independent (FCI) method for parallel derivatives allows for complicated magnetic topologies

- **Interpolation of field lines onto poloidal planes – "locally-aligned" coordinates.**
    - BOUT++, GRILLIX, FELTOR, FENICIA…
- **3D metrics in BOUT++ allow for curvilinear grids**
    - Inner and outer boundaries can be aligned to flux surfaces or plasma facing components.



A schematic of the FCI method



A curvilinear FCI grid for BOUT++

Simulation geometry and initial conditions

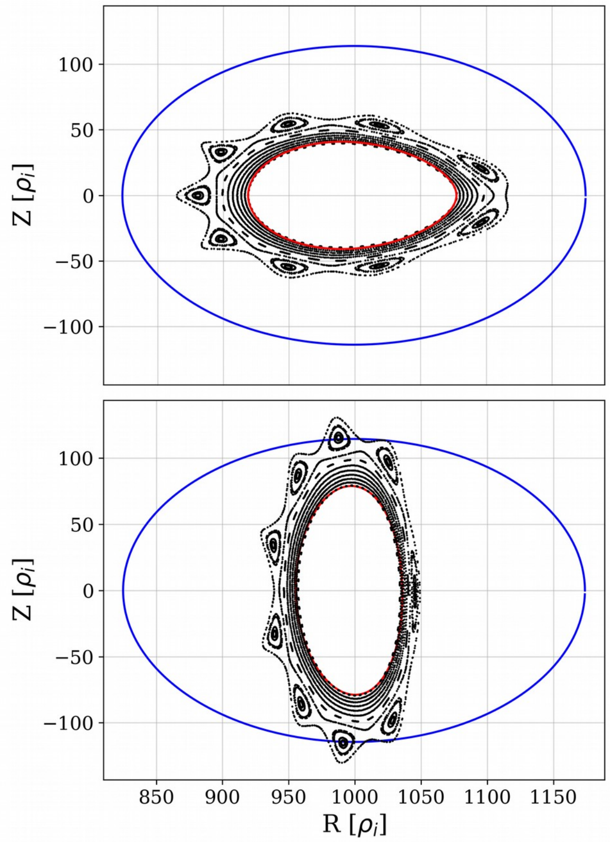
Geometry is the same as **[4]**, but without the core.

- **5-field period, m=9 island at the edge – generated from Dommaschk potential**
- **Islands intersect wall at discrete toroidal locations.**
- **Curvature follows 1/R (to lowest order)**

Isothermal simulations are given an initial perturbation, and settle down into a steady-state where the losses through the sheath balance the source

- **Source located at the inner flux surface**
- **Evolve full-field density, vorticity and parallel ion momentum**

[4] Coelho et al., *NF* (2022)



Simulation domain of an analytic stellarator with islands intersecting the boundary

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$$\frac{\partial n}{\partial t} = -\nabla \cdot \left(n\mathbf{V}_{E\times B} + n\mathbf{V}_{mag,e}\right) - \nabla_{||}\left(nv_{||e}\right) + S_n$$

$$\frac{\partial \omega}{\partial t} = \nabla \cdot \left[e\left(p_e + p_i\right)\nabla \times \frac{\mathbf{b}}{B}\right] + \nabla_{||}j_{||} - \nabla \cdot \left(\omega \mathbf{V}_{E\times B}\right)$$

$$\frac{\partial}{\partial t}\left(m_i n v_{||i}\right) = -\nabla \cdot \left[m_i n v_{||i}\left(\mathbf{V}_{E\times B} + \mathbf{b}v_{||i} + \mathbf{V}_{mag,i}\right)\right] - \partial_{||}p_e - \partial_{||}p_i$$

$$\omega = \nabla \cdot \left[\frac{e}{\Omega B}\left(n_0 \nabla_\perp \phi\right)\right]$$

$$J_{||} = en(v_{||,i} - v_{||,e}) = -\tfrac{1}{\nu}\partial_{||}\phi - \tfrac{1}{n_e}\partial_{||}p_e$$

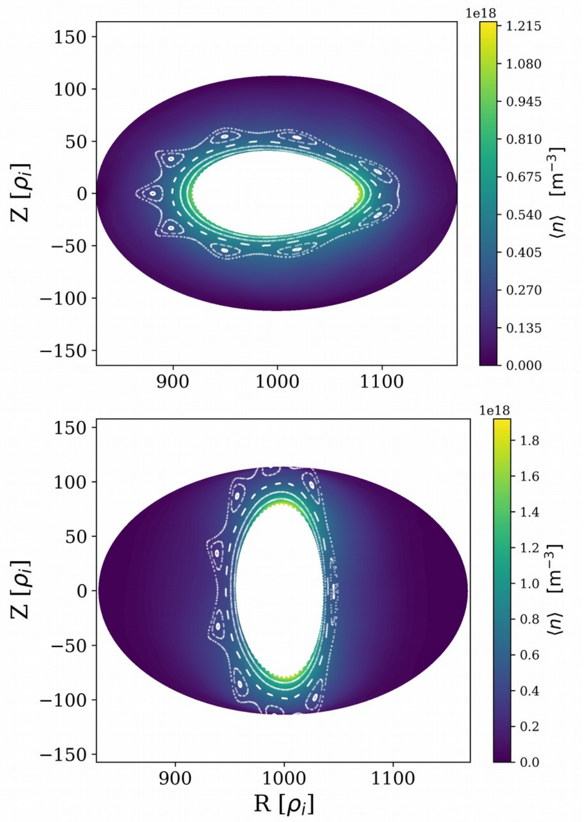
$$\mathbf{V}_{E\times B} = \frac{\mathbf{b}\times\nabla\phi}{B} \qquad \mathbf{V}_{mag,e} = -\frac{T_e}{e}\nabla\times\frac{\mathbf{b}}{B} \qquad \mathbf{V}_{mag,i} = \frac{T_i}{e}\nabla\times\frac{\mathbf{b}}{B}$$

[4] Coelho et al., *NF* (2022)

Equilibrium transport

The overall transport follows the curvature drive

- **Equilibrium density profile broader on the outboard side**
- **steady-state potential contours indicate transport following curvature drive via ExB motion**
- **mean-field radial flux primarily shows transport along curvature direction**
- **When intersecting the boundary, the sheath connection interupts flows toward the outboard side.**
  - sheath connection disrupts potential
- **radial correlation lengths range from 8ρ_i (outboard, vertical) to 50ρ_i (inboard, horizontal)**



Steady-state equilibrium density profiles

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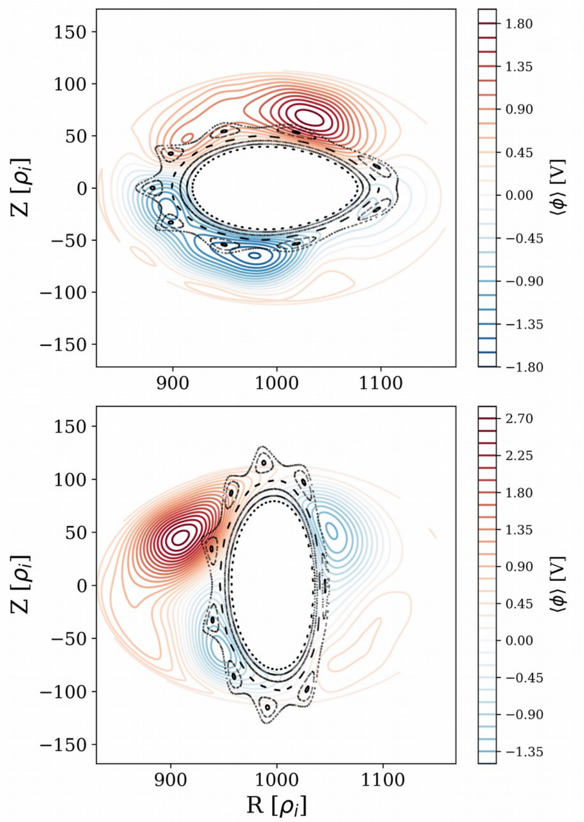

Steady-state equilibrium potential profiles

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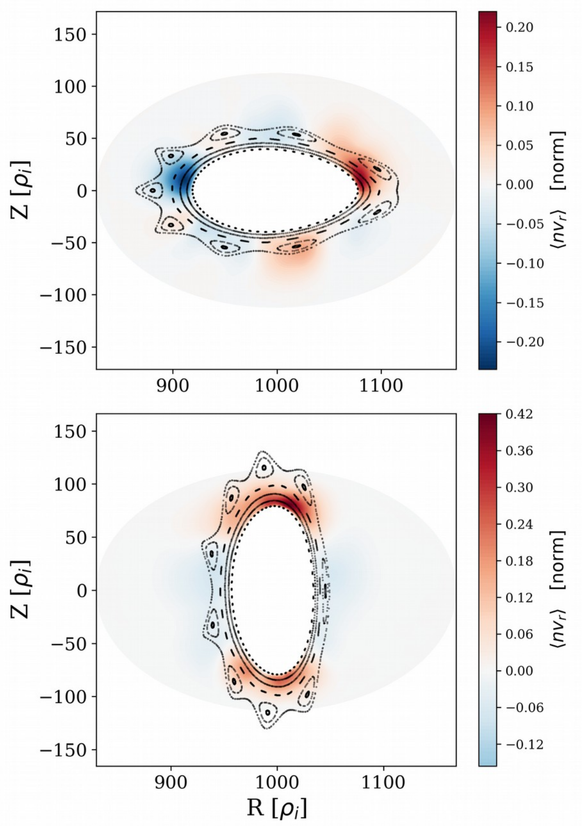


Steady-state equilibrium radial flux

Fluctuations in the SOL

Large fluctuations ($2<k_{perp}\rho_i<15$) are present throughout the SOL.

- **A slight increase in density fluctuation amplitude and extent seen on outboard midplane**

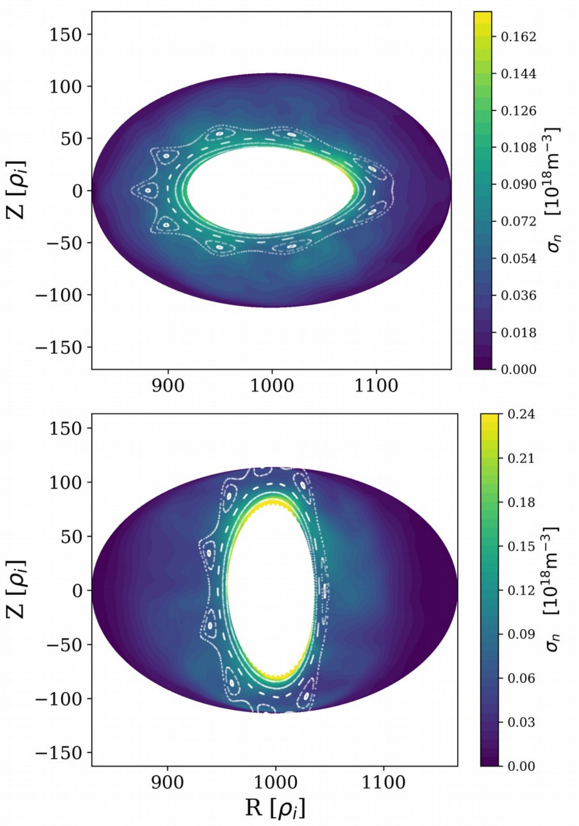
Inboard activity seen in the vertical cross section.

- **Also seen in [4]**
- **Correlated to island width?**

The radial flux of fluctuations does not follow curvature drive.

Overall transport is blob-like.

- **A positive skewness indicates positive perturbations.**



Standard deviation of the density signal

[4] Coelho et al., *NF* (2022)

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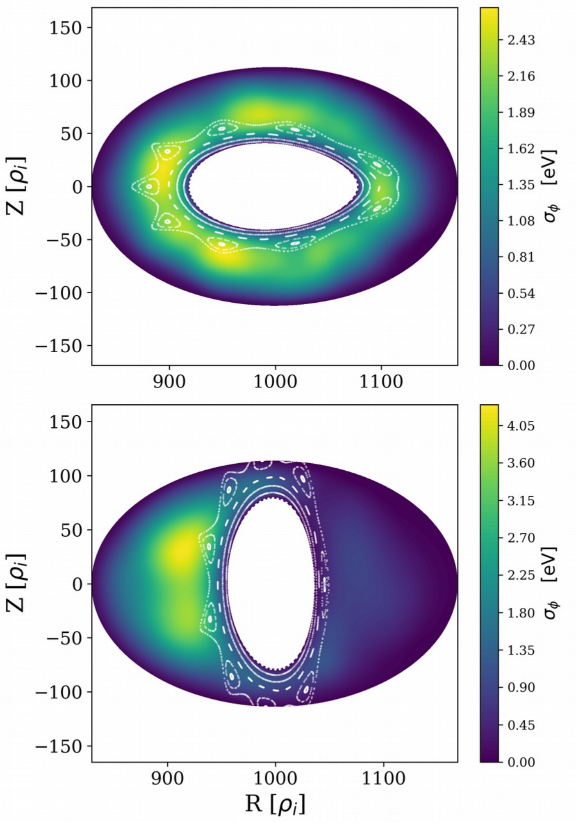

Standard deviation of the potential signal

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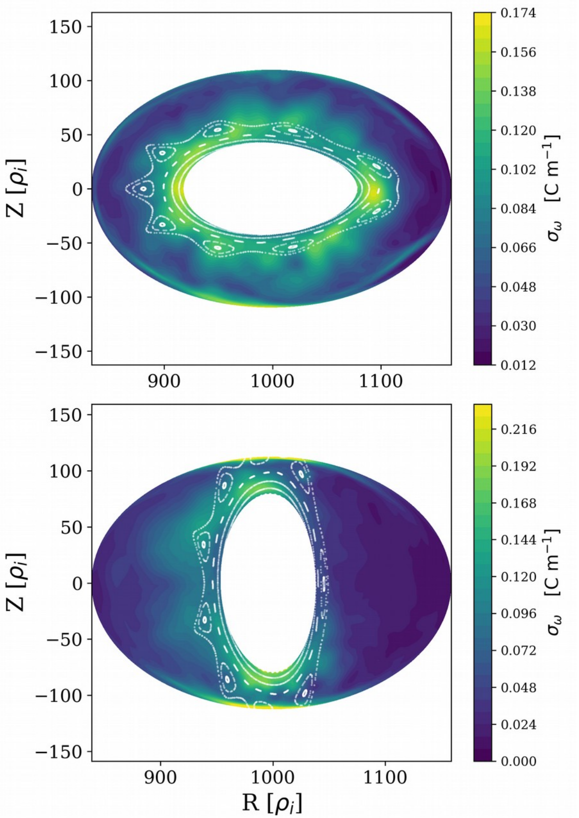

Standard deviation of the vorticity signal

[4] Coelho et al., *NF* (2022)

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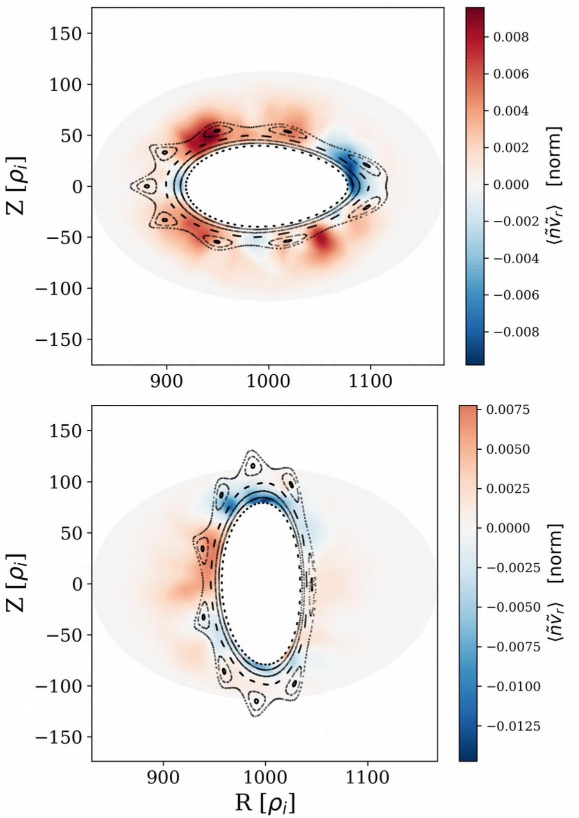

Radial flux of the fluctuations

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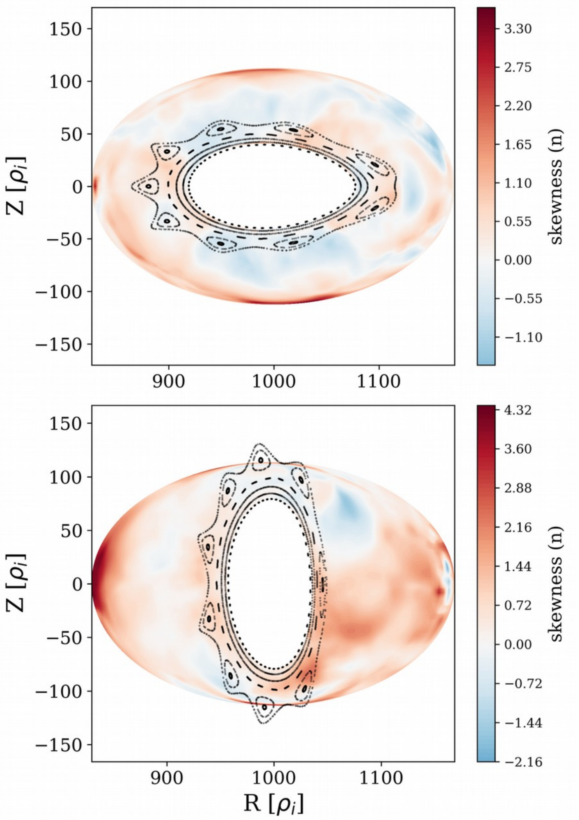


Skewness of the fluctuations

[4] Coelho et al., *NF* (2022)

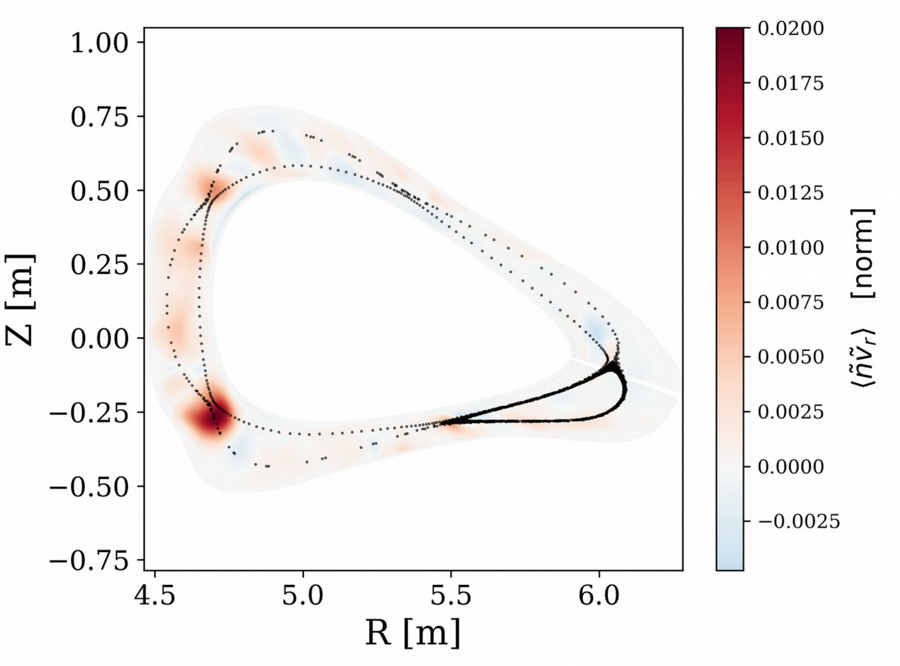
Preliminary results in Wendelstein 7-X

The first global turbulence simulations have been performed in Wendelstein 7-X.

- **Outer boundary includes the divertor geometry**
- **Full island geometry included**

Perturbation flux outward into the SOL is highest near islands.

- **perturbation flux follows curvature drive**



Radial flux of the fluctuations in the W7-X SOL.

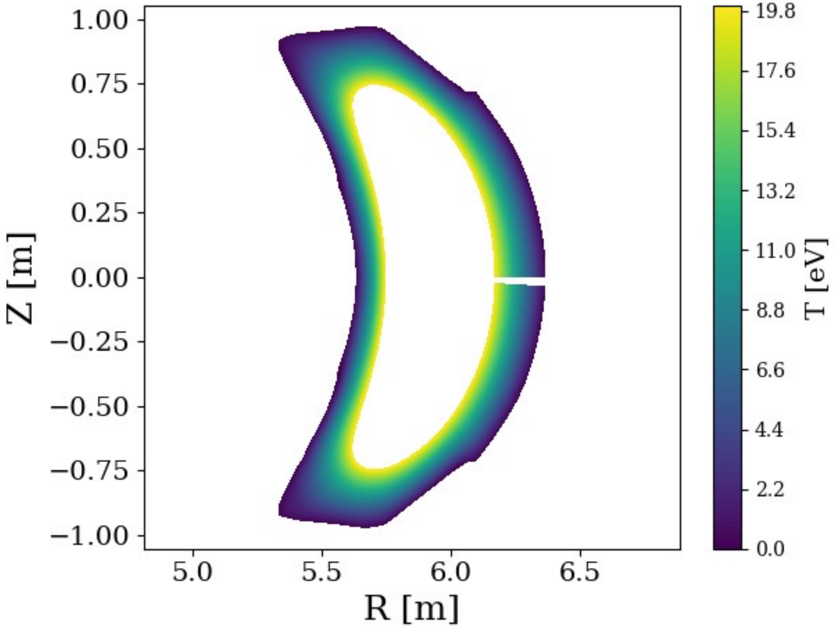
Other models for Wendelstein 7-X

Other models have been implemented for W7-X:

- **EMC3-lite**
- **Heat transport**

With others in development:

- **EMC3-like**
- **Hot-ion turbulence**
- **Hermes-3**
  - Multifluid, arbitrary number of ion and neutral species, fluid neutral models



Steady-state "T" profile in BOUT++ with the EMC3-lite model

Conclusions

BOUT++ is a flexible tool for understanding the physics at the edge and SOL of stellarators.

Simulations have been performed in an analytical stellarator with an island divertor.

- **Equilibrium transport follows curvature drive**
- **Large, blob-like fluctuations are present throughout the SOL, leading to transport away from the core.**
- **Potential influence of the island SOL on fluctuations**

Extensions to W7-X are underway.

- **isothermal turbulence simulations have been performed**
    - Further analysis and nonisothermal simulations soon to come

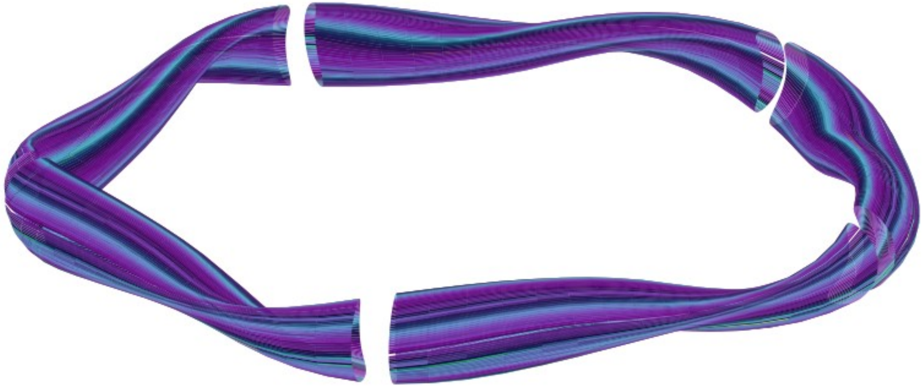
Hermes-3: Multifluid simulations

Hermes-3 [10] is a multifluid model which:

- **simulates 1D, 2D or 3D transport or turbulence**
- **can have an arbitrary number of ion and neutral species (determined at runtime)**
- **uses ADAS & AMJUEL, fluid neutral models**
- **Includes "relax_potential" option for steady-state potential (+drifts)**

Initial tests in W7-X geometry seem to work

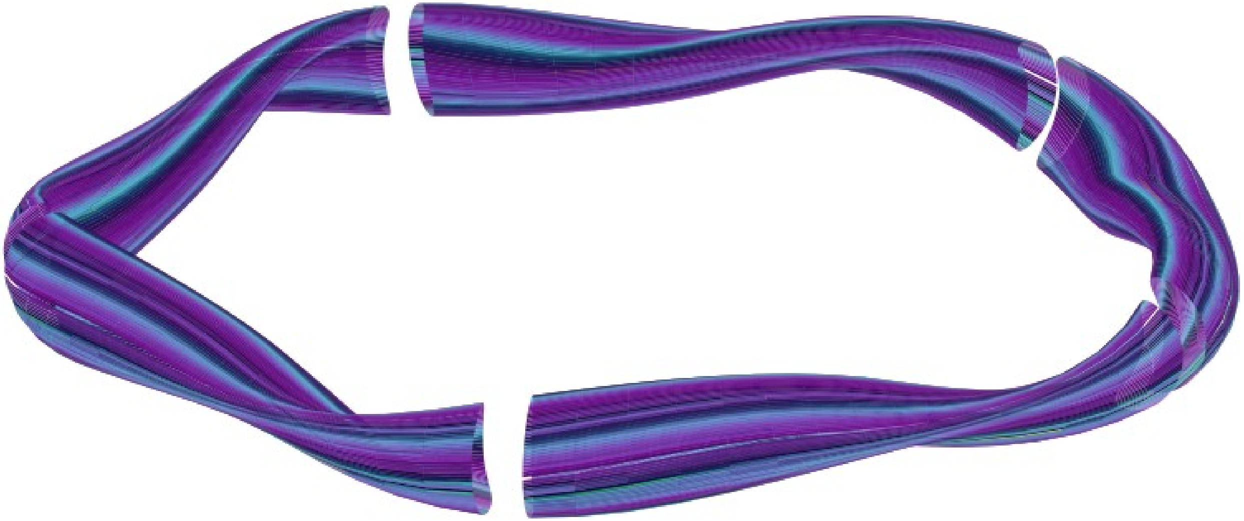
- **Transport, turbulence within closed field lines in the near-term**



Numerical test in W7-X geometry for Hermes-3

[10] B Dudson et al., *arXiv:2303.1213* (2023)

Backup slides



Field-aligned systems in complicated topologies

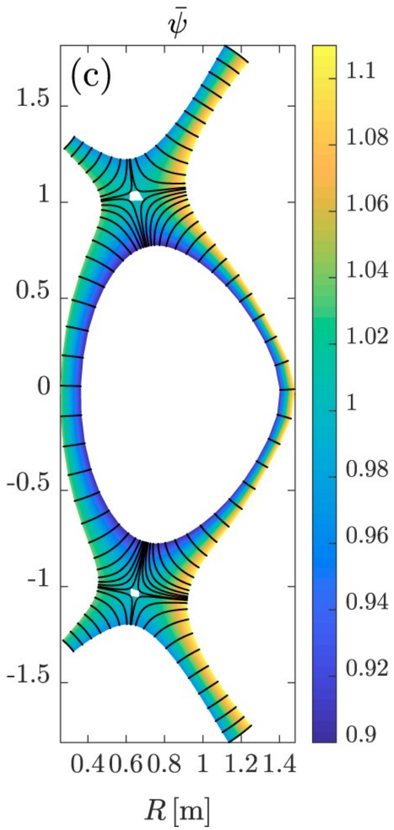
Field-aligned systems (∇ψ, ||, ϕ) are often preferred in fluid turbulence due to the flute-like nature of fluid turbulence.

At O- and X-points, however, this system fails.

- **The edge of W7-X exhibits an island topology (and stochastic region)**

We can use either:

- **an unstructured mesh (XGC, BoRis)**
- **or non-field-aligned system**
  - 3D Cartesian (GBS, BOUT++)
  - Locally-aligned (BSTING/BOUT++)



Comparison of probe measurements (green) and seeded blob simulations (orange/blue)
[F Riva et al., PPCF 61 095013 (2019)]

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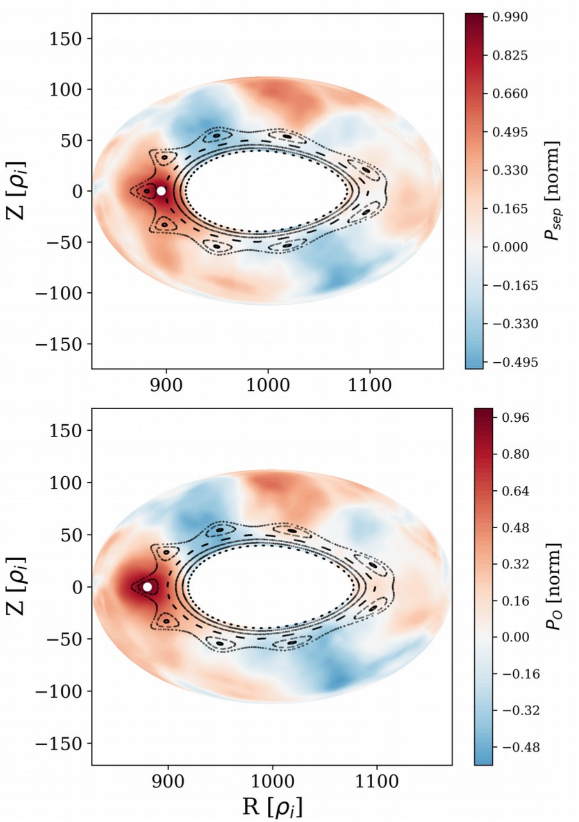
Correlations indicate the role of the separatrix for transport into the SOL

- **The separatrix is correlated to the outer SOL**
- **radial correlation lengths range from 8ρ$_i$ (outboard, vertical) to 50ρ$_i$ (inboard, horizontal)**



Pearson correlation coefficients with two reference points: one on the separatrix (top) and one in an O-point (bottom)

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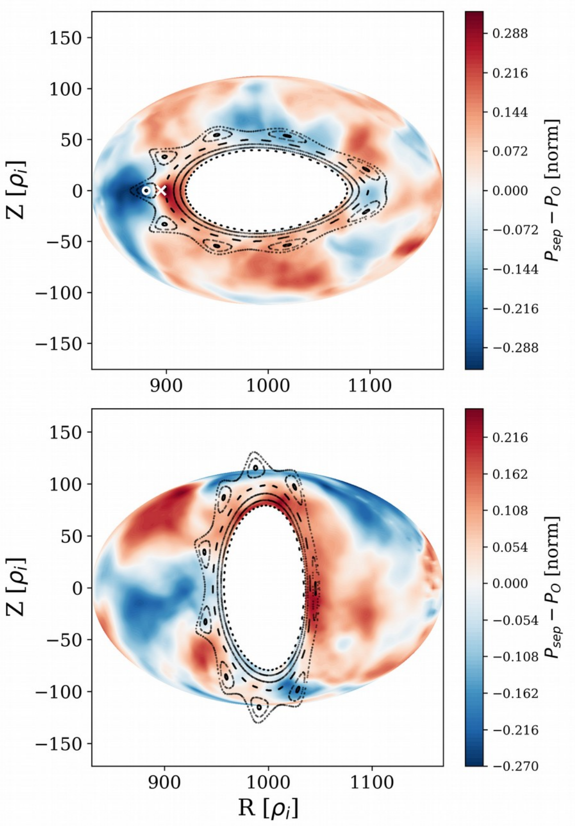
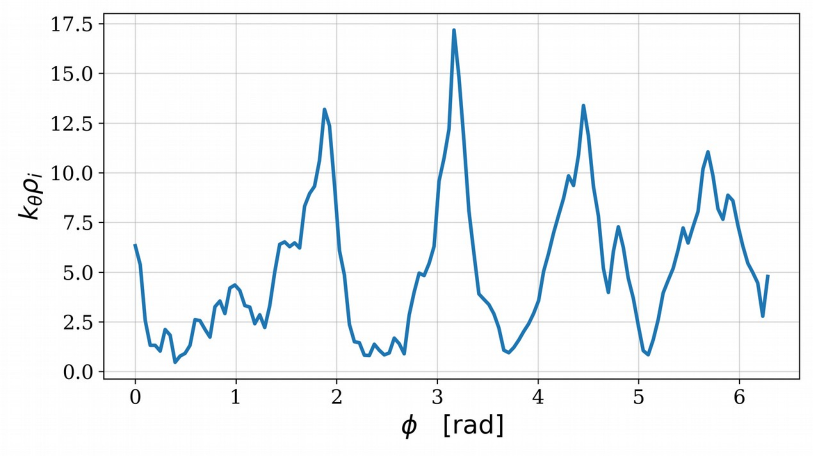


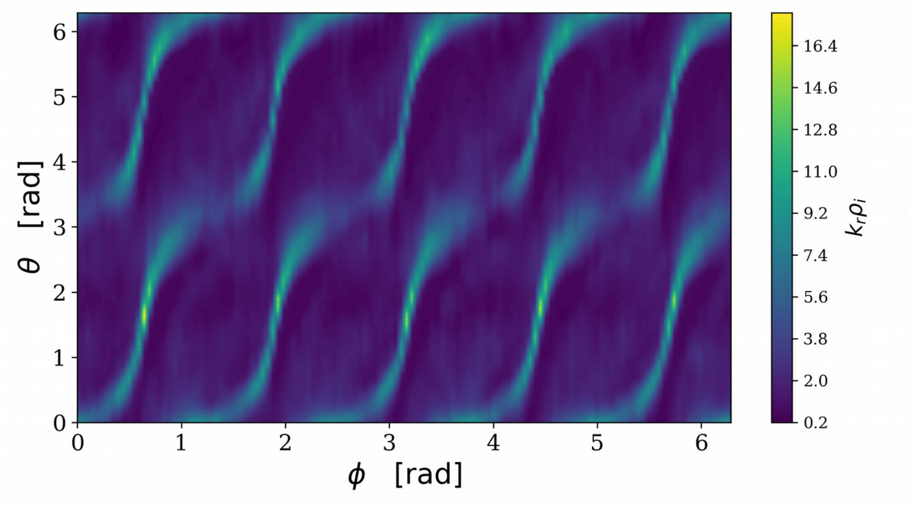
The difference between Pearson correlation coefficients with two reference points: one on the separatrix (**X**) and one in an O-point (**O**)

Fluctuation structure

Large-scale fluctuations are dominant

Some evidence of m=2, n=5 fluctuations





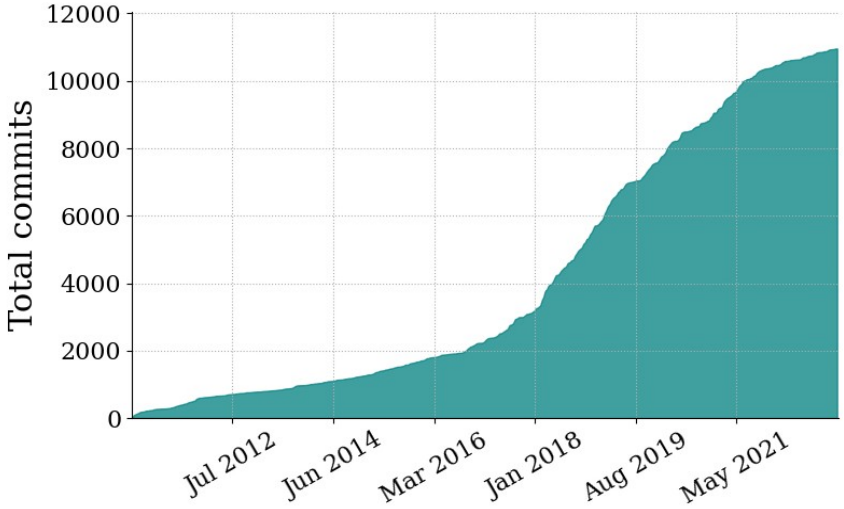
BOUT++

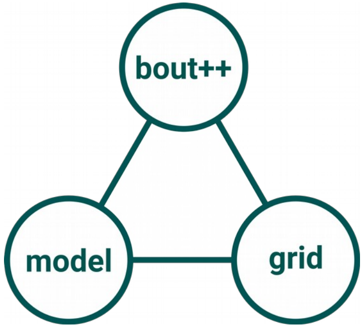
BOUT++ is a modular, open source framework for nonlinear fluid (turbulence) simulations

Simple syntax, allowing for several physics models with many applications:

- **Edge turbulence and transport, blobs, detachment, magnetic reconnection, nonlinear MHD, edge-localized modes (ELMs), chocolate bubbles …**

Active development; new features and bug fixes added regularly.





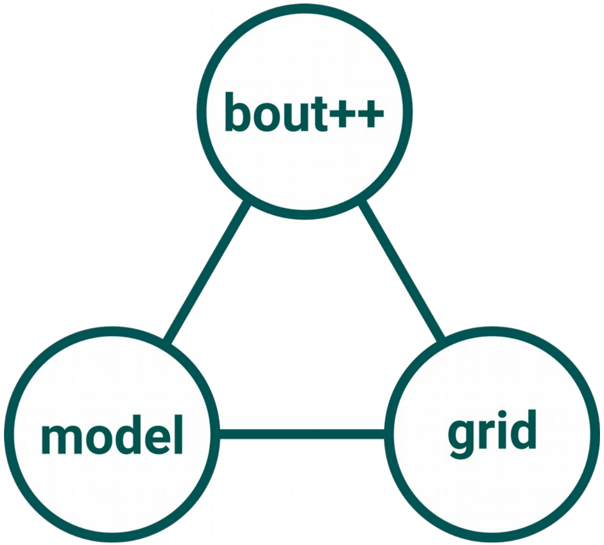
BOUT++ soups have three ingredients

BOUT++ is a framework, not a set model

Models are developed which use the methods within BOUT++

As such, new developments are either:

- **Improving BOUT++ internal methods**
- **Developing a new model**
- **Creating grids for required geometry**



Hermes-3

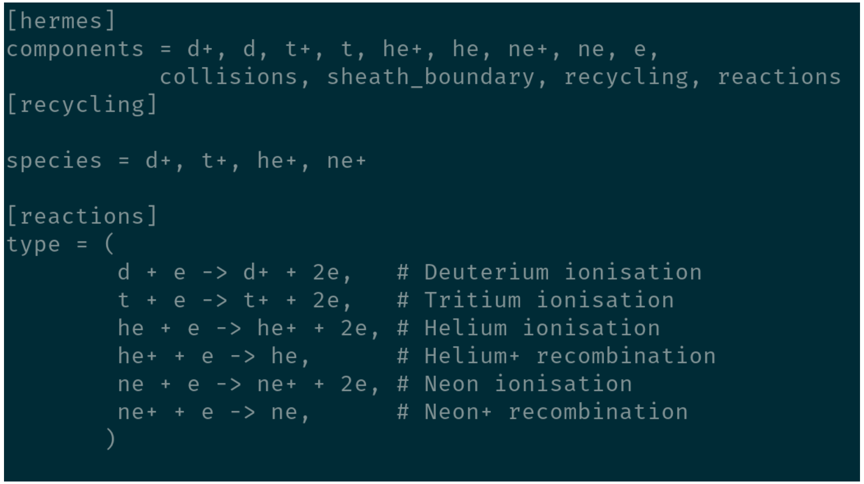
Hermes-3 is a new model using BOUT++ for edge applications **[B Dudson 2023 Submitted; https://arxiv.org/abs/2303.12131]**

- **Multifluid, 1D, 2D or 3D for transport or turbulence**
- **Arbitrary number of ion and neutral species (determined at runtime)**
- **Uses ADAS & AMJUEL, fluid neutral models**
- **"relax_potential" option for steady-state potential**

Actively developed, online manual.

No development needed for 1D/2D applications.

Even easier syntax in input files



```
[hermes]
components = d+, d, t+, t, he+, he, ne+, ne, e,
           collisions, sheath_boundary, recycling, reactions
[recycling]

species = d+, t+, he+, ne+

[reactions]
type = (
        d + e -> d+ + 2e,   # Deuterium ionisation
        t + e -> t+ + 2e,   # Tritium ionisation
        he + e -> he+ + 2e, # Helium ionisation
        he+ + e -> he,      # Helium+ recombination
        ne + e -> ne+ + 2e, # Neon ionisation
        ne+ + e -> ne,      # Neon+ recombination
       )
```

Example input from a Hermes-3 simulation with cross-field diffusion, collisions between species, sheath boundary conditions, and recycling

Hermes-3 (1D)

An example from **[Dudson et al., 2023, submitted]**

- **no-flow upstream, sheath boundary downstream**
- **Evolving all electron and ion species**
  - **Neon, Deuterium**
- **heat conduction, 100% recycling, ionization of neturals, charge exchange, feedback control of upstream density**
- **Thermal force included.**

Immediately applicable to W7-X, with the relevant parameters – no development needed.

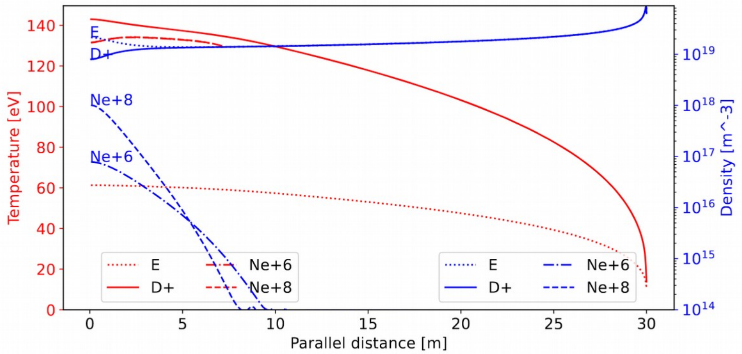


Figure 7: Steady state solution with 100% recycling, evolving all charge states of neon as separate fluids with their own densities, temperatures and flow velocities. A subset of species densities (blue lines) are shown on a logarithmic scale. Simulation inputs in `examples/1D-neon` of the Hermes-3 repository.

The stellarator two-point model in Hermes-3

Previous work by Feng et al. [1] created a stellarator-two point model.

- **Effects of cross-field transport introduced through the field line pitch – Θ – the ratio the radial distance to the parallel arc length in the SOL**
- **In tokamaks, Θ is about 2 orders of magnitude larger than in stellarators (0.1 vs. 0.001)**

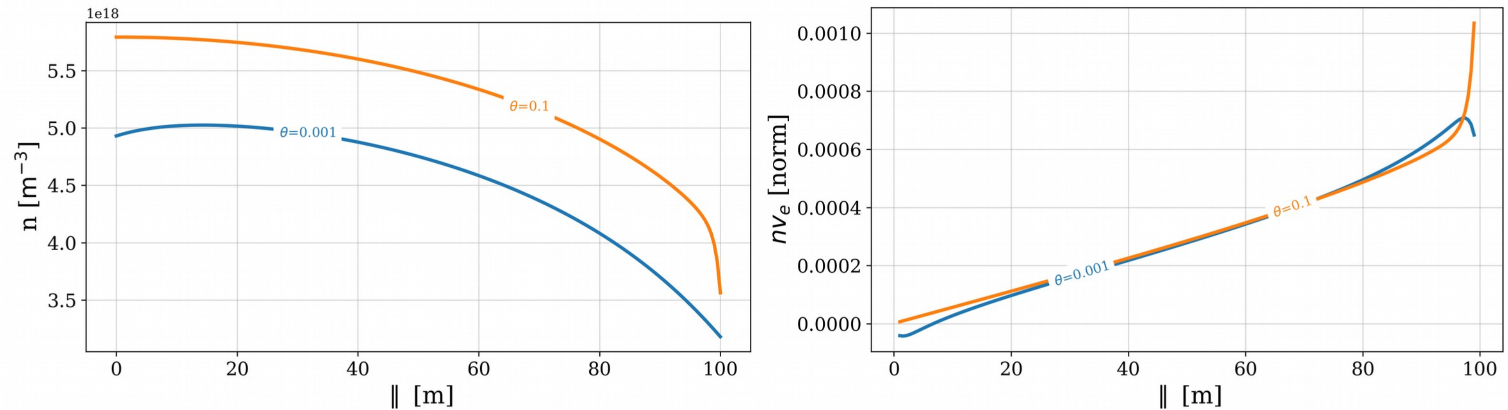
Rewriting the terms in [1] in the parallel direction.

- **D, χ, and ν are prescribed coefficients**

$$\frac{\partial N}{\partial t} = \ldots + \nabla \cdot \left(\mathbf{b} \frac{D}{\Theta} \partial_{||} N \right)$$

$$\frac{\partial P}{\partial t} = \ldots + \frac{2}{3} \nabla \cdot \left(\mathbf{b} \frac{\chi}{\Theta} N \partial_{||} T \right)$$

$$\frac{\partial}{\partial t} (NV) = \ldots + \nabla \cdot \left(\mathbf{b} \frac{\nu}{\Theta} \partial_{||} NV \right)$$



Initial simulations of the stellarator 2-pt model in Hermes-3, showing the difference between tokamak and stellarator transport in the SOL.

[1] Y Feng et al., *PPCF* 53 024009 (2011)

Higher-frequency fluctuations on the outboard side