



Nonlinear equilibria and phase space transport in burning plasmas

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- Predicting the dynamics of a **burning plasma** on **long time scales**, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g., **ITER**;
- the crucial role of **energetic particles** [Zonca et al. 2015](#); [Chen and Zonca 2016](#), must be properly described;



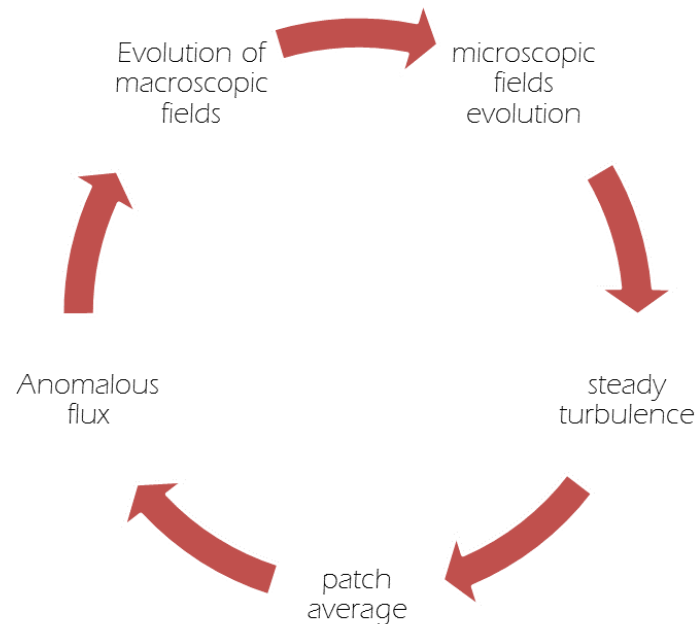
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- the crucial role of **energetic particles** Zonca et al. 2015; Chen and Zonca 2016, must be properly described;
- a **first-principle-based**, self-consistent approach is crucial;
- extending **gyrokinetic simulations** to these time scales is a challenging task from the computational resource point of view, i.e., $\sim 10^{24}$ grid points;
- **simplifying assumptions** based on physics understanding and **first principles** must be introduced;



- Transport equations define the evolution of **radial profiles**

$$\langle \partial_t n \rangle_\psi = - \langle \nabla \cdot (n \mathbf{V}) \rangle_\psi ;$$

- consistent with a **slowly evolving** ($\omega^{-1} \partial_t \log p_0 \sim \delta^2$) **equilibrium** distribution function;

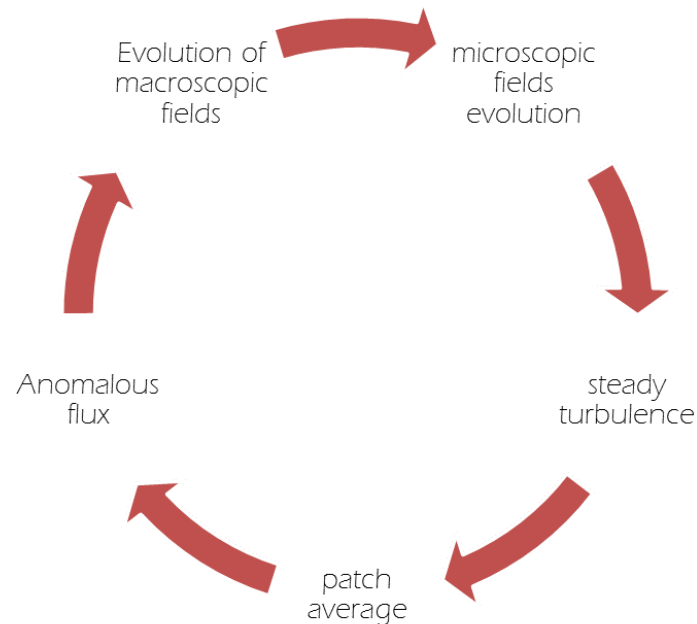




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- consistent with a **slowly evolving** ($\omega^{-1} \partial_t \log p_0 \sim \delta^2$) **equilibrium** distribution function;
- Implicit **separation of scales** between equilibrium and fluctuations;
- Local **Maxwellian** is assumed;



Need for generalization!!!



We aim at 1. providing the **general expressions** describing **EPs** (plasma) dynamics on long time scales (**transport**) and 2. introducing a framework to solve these equations within different levels of **reduced dynamics**.

By means of this approach it will be possible to:

- define the concept of **nonlinear equilibrium**;
- describe the physics of **burning plasmas** where alpha particles will play a **key role in transport studies** by interacting with thermal components;
- the derivation, see **Falessi 2017**; **Falessi and Zonca 2019**, is based on the **Phase space zonal structures** theory, see **Chen and Zonca 2016**.



- Coupling between fluctuating fields can generate **zonal flows** and **zonal fields**;
- **Crucial elements for regulating turbulent fluxes**, e.g., by scattering instability turbulence to shorter radial wavelength stable domain...

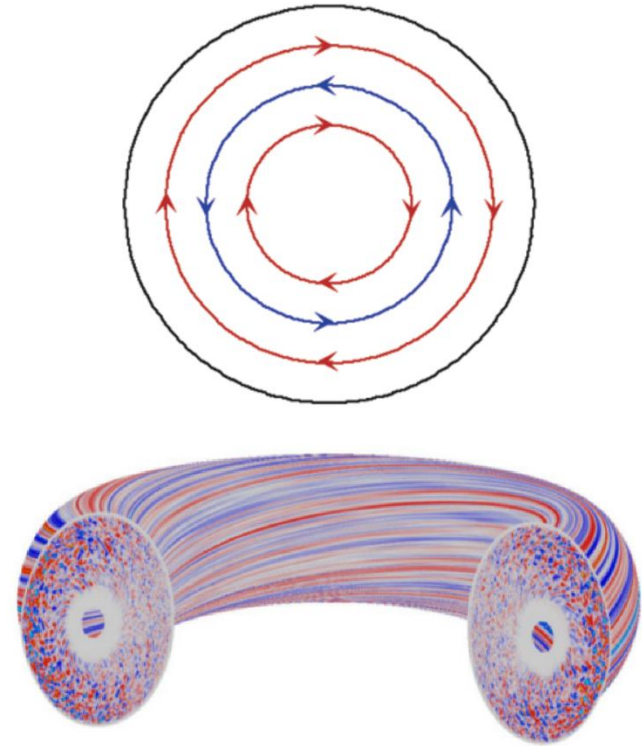


Figure: Courtesy of Y. Xiao et al., PoP 2015.



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- **Crucial elements for regulating turbulent fluxes**, e.g., by scattering instability turbulence to shorter radial wavelength stable domain...
- analogously, **zonal structures** in the **density and temperature profiles** are unaffected by rapid collision-less dissipation;
- collision-less undamped fluctuations in the phase space are called **phase space zonal structures** [Chen and Zonca 2016](#), [Falessi and Zonca 2019](#);

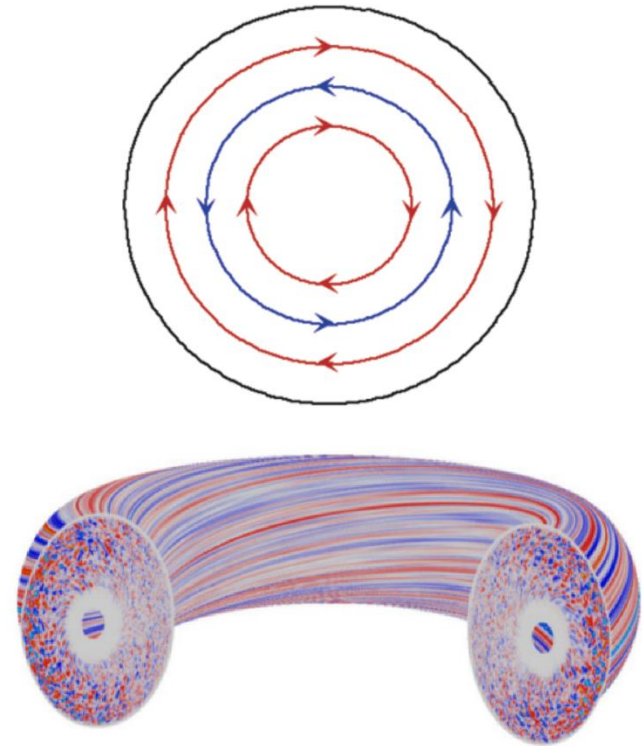


Figure: Courtesy of Y. Xiao et al., PoP 2015.



- We want to explicit identify the part of the toroidally symmetric distribution function **unaffected by rapid collisionless dissipation** that is the PSZS [Falessi et al 2023 sub.](#);
- we start by writing the nonlinear Gyrokinetic equation:

$$\partial_t(DF) + \nabla \cdot (D\dot{\mathbf{X}}F) + \partial_\varepsilon(D\delta\dot{\mathbf{E}}F) = 0$$

D is the Jacobian of the velocity space and $\dot{\mathbf{X}} = \dot{\mathbf{X}}_0 + \delta\dot{\mathbf{X}}$ is the gyrocenter velocity due to magnetic equilibrium and to fluctuating fields;

Phase Space Zonal Structures equation



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- we decompose F as $F_0 + \delta F$ where F_0 is the **steady state solution of the lowest order gyrokinetic equation**, i.e.:

$$F_0(\psi, \theta, \varepsilon, \mu) = e^{-iQ_z} F_{B0}(\psi, \theta, \varepsilon, \mu) = F_{B0} \left(\psi - \frac{F v_{\parallel}}{\Omega}, \varepsilon, \mu \right) = F_{B0} \left(\bar{\psi}(\psi, \theta, \varepsilon, \mu), \varepsilon, \mu \right)$$

where the **drift/banana center shift operator** e^{iQ_z} with $Q_z \equiv F(v_{\parallel}/\Omega)k_z / (d\psi / dr)$

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where the **drift/banana center shift operator** e^{iQ_z} with $Q_z \equiv F(v_{\parallel}/\Omega)k_z / (d\psi / dr)$

- we can calculate $D(\delta\dot{\mathbf{X}} \cdot \nabla + \delta\dot{\mathbf{E}}\partial_\varepsilon)F_{B0}(\bar{\psi}(\psi, \theta, \varepsilon, \mu), \varepsilon, \mu) \dots$



- ... and re-write the **toroidally symmetric** component of the **Gyrokinetic equation**:

$$D(\partial_t + \dot{\mathbf{X}}_0 \cdot \nabla) \left(F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 + \frac{F}{B_0} \frac{\partial F_0}{\partial \bar{\psi}} \langle \delta A_{\parallel g} \rangle_z \right) + D \frac{e}{m} \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 \partial_t \langle \delta L_g \rangle_z \\ + \frac{1}{J} \frac{\partial}{\partial \theta} (JD \delta \dot{\theta} \delta F) + \frac{1}{J} \frac{\partial}{\partial \psi} (JD \delta \dot{\psi} \delta F) + \frac{\partial}{\partial \mathcal{E}} (JD \delta \dot{\mathcal{E}} \delta F) = 0$$

where $\langle \delta L_g \rangle_z = J_0 \left(\delta \phi_z - \frac{v_{\parallel}}{c} \delta A_{\parallel z} \right) + 2 \frac{m}{\lambda e} \mu J_1 \delta B_{\parallel z}$ and J is the Jacobian of the curvilinear coordinate system;



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$$G_z = F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 + \frac{F}{B_0} \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} F_0$$

- We now apply e^{iQ_z} and write the governing equation for $G_B = e^{iQ_z} G_z$



- ... we obtain the following equation:

$$\begin{aligned} & \partial_t(JD G_B) + \partial_\theta G_B \\ & = e^{iQ_z} \left[-\frac{e}{m} \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 \partial_t (JD \langle \delta L_g \rangle_z) - \frac{1}{J} \frac{\partial}{\partial \theta} (JD \delta \dot{\theta} \delta F) - \frac{1}{J} \frac{\partial}{\partial \psi} (JD \delta \dot{\psi} \delta F) - \frac{\partial}{\partial \mathcal{E}} (JD \delta \dot{\mathcal{E}} \delta F) \right] \end{aligned}$$

It can be shown that e^{iQ_z} , up to the required order, commute with partial derivatives;



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It can be shown that e^{iQ_z} , up to the required order, commute with partial derivatives;

- finally, integrating over θ and introducing the **bounce/transit average** $\overline{[\dots]} = \tau_b^{-1} \oint \frac{d\ell}{v_\parallel} [\dots]$, we obtain:

$$\partial_t \overline{G_B} = -e^{iQ_z} \frac{e}{m} \frac{\partial}{\partial \mathcal{E}} |\bar{\psi} F_0 \partial_t \langle \delta L_g \rangle_z - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left(\tau_b \overline{e^{iQ_z} \delta \psi \delta F} \right) - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left(\tau_b \overline{e^{iQ_z} \delta \dot{\mathcal{E}} \delta F} \right)$$



- We can re-write the **low frequency (transport) component** of δf_z in term of only $\delta \bar{G}_B$;
- taking the time derivative of the surface averaged velocity integral we obtain:

$$\partial_t \langle \langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{(e^{-iQ_z J_0})} \overline{(e^{iQ_z J_0})} \right] \frac{\partial F_0}{\partial \varepsilon} \partial_t \delta \phi_z \right\rangle_v +$$
$$\frac{1}{v'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{(e^{-iQ_z J_0})} \left[c e^{iQ_z R^2} \nabla \phi \cdot \nabla \langle \delta L_g \rangle \delta G \right] \right\rangle_v \right\rangle_\psi$$



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- This equation describes the **radial oscillations on any length-scale** of the density profile in the absence of collisions and assuming GK ordering [Falessi and Zonca 2019](#);
- **mesoscales** are spontaneously created by the turbulence;



- This naturally leads to define a reference distribution function F_0 at each instant of time. Going back to the **previous derivation** and rewriting terms such as:

$$\delta\dot{\mathbf{X}}|_z \cdot \nabla \delta F_z = \delta\dot{\mathbf{X}}|_z \cdot \nabla e^{-iQ_z} (\delta\bar{F}_{Bz} + \delta\tilde{F}_{Bz})$$

- $e^{-iQ_z} \delta\bar{F}_{Bz}$ analogously to F_0 , is a function of the **constants of motion**;



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- we obtain a newly defined G_z :

$$G_z = F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_{0*} + \frac{F}{B_0} \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} F_{0*}$$

- Where $F_{0*} = F_0 + e^{-iQ_z} \delta\bar{F}_{Bz}$ is the renormalized F_0 ;
- **Phase Space Zonal Structures** are the macro-/meso- scopic component of F_{0*}



- Particle motion in the reference magnetic field is characterized by **three integrals of motion**, i.e. P_ϕ, μ, \mathcal{E} ;
- **Phase Space Zonal Structures** equation is connected with the **macro-/meso- scopic component**, i.e. $[\dots]_S$, unperturbed orbit averaged distribution function (Falessi and Zonca 2019);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \overline{(\tau_b \delta \dot{P}_\phi \delta F)}_z + \frac{\partial}{\partial \mathcal{E}} \overline{(\tau_b \delta \dot{\mathcal{E}} \delta F)}_z \right]_S = \overline{\left(\sum_b C_b^g [F, F_b] + \mathcal{S} \right)}_{zS}$$

where $\overline{(\dots)} = \tau_b^{-1} \oint d\theta / \dot{\theta} (\dots) = \oint d\theta / \dot{\theta} e^{iQ_z} (\dots) (\bar{\psi}, \theta)$, with $\tau_b = \oint d\theta / \dot{\theta}$ and $\psi = \bar{\psi} + \delta\tilde{\psi}(\theta)$;



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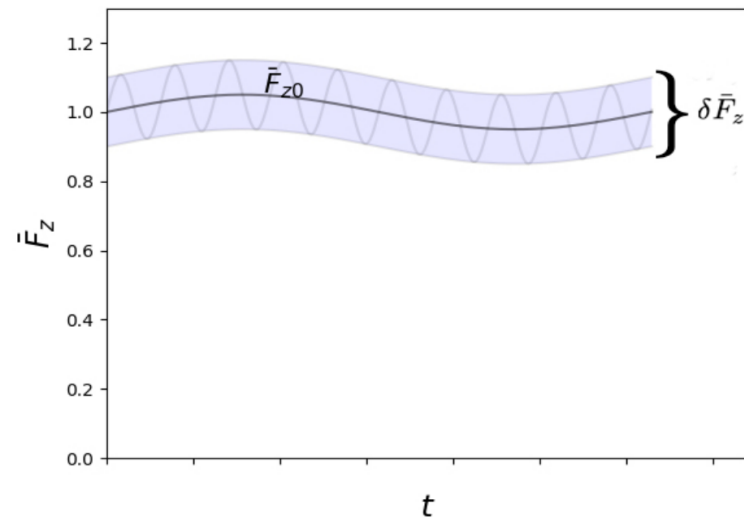
where $\overline{(\dots)} = \tau_b^{-1} \oint d\theta / \dot{\theta} (\dots) = \oint d\theta / \dot{\theta} e^{iQ_z} (\dots) (\bar{\psi}, \theta)$, with $\tau_b = \oint d\theta / \dot{\theta}$ and $\psi = \bar{\psi} + \delta\tilde{\psi}(\theta)$;

- equivalent to **bounce/transit averaging** a quantity shifted with the e^{iQ_z} operator;
- This expression describe **transport processes in the phase space** due to fluctuations, collisions and sources.



- Having defined **Phase Space Zonal Structures**, we can decompose the **toroidally symmetric distribution function**;
- $\overline{F_{z0}}$ describe **macro- & meso-scales**;
- **micro-scales** are accounted by $\delta\tilde{F}_z$;

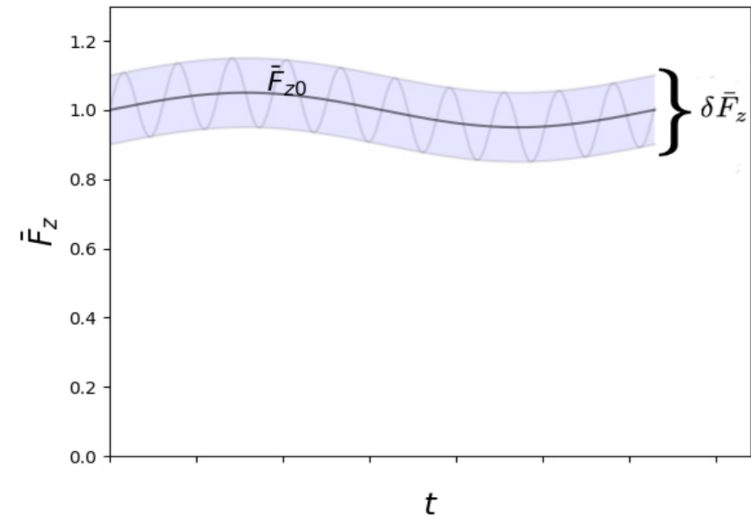
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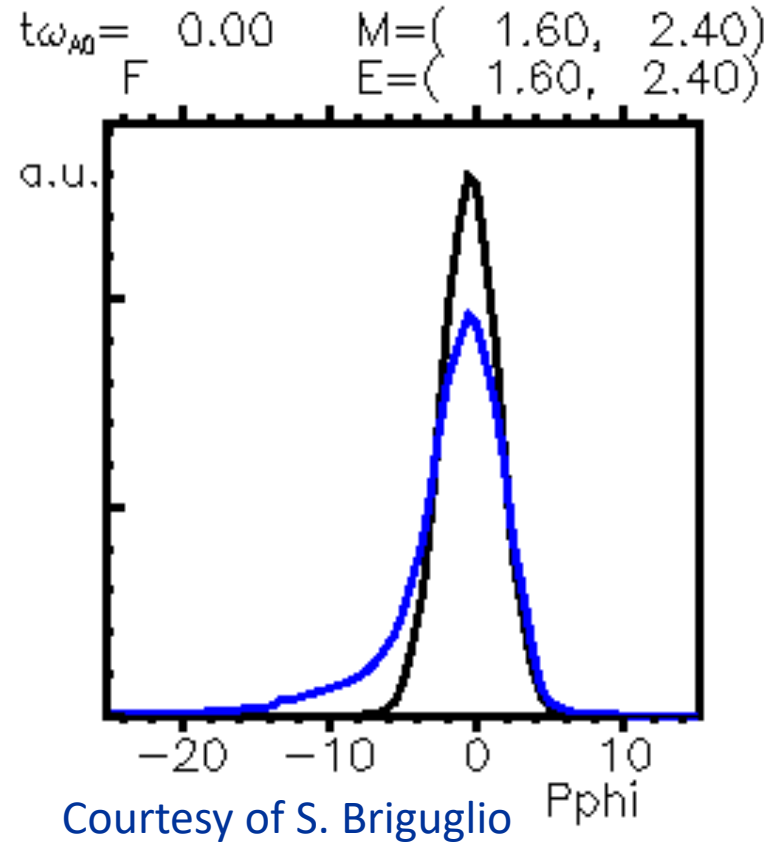
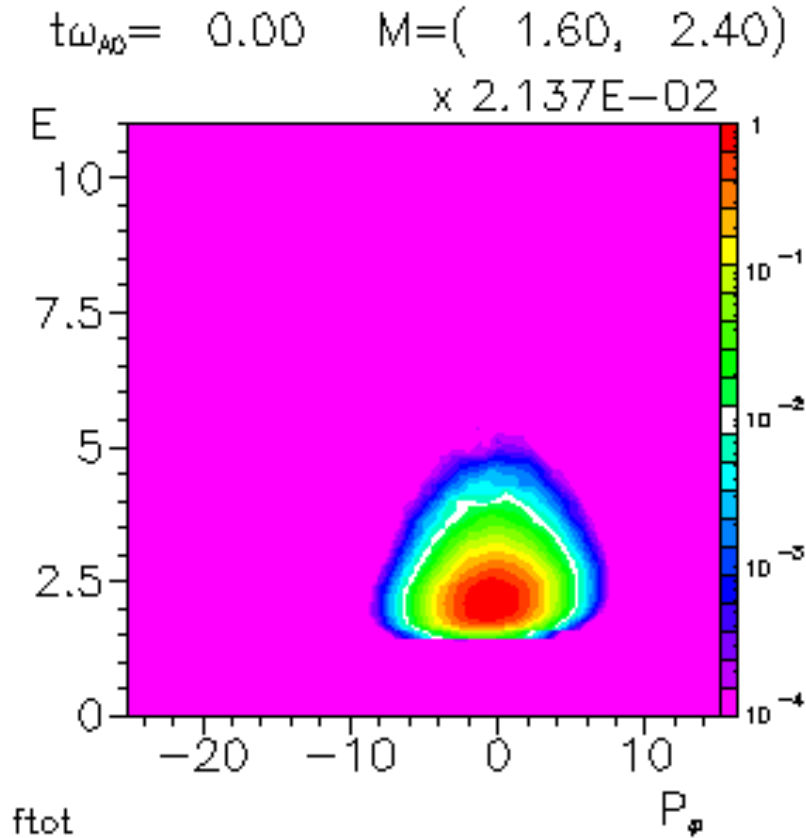


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- $\overline{F_{z0}}$ describe **macro- & meso-scales**;
- **micro-scales** are accounted by $\delta\overline{F_z}$;
- they describe system transitions between **neighboring nonlinear equilibria**, see [Chen and Zonca 2007](#); [Falessi and Zonca 2019](#);
- **nonlinear equilibria**, together with **zonal fields**, form a **zonal state**.

$$F_z = \overline{F_{z0}} + \delta\overline{F_z} + \delta\tilde{F}_z$$



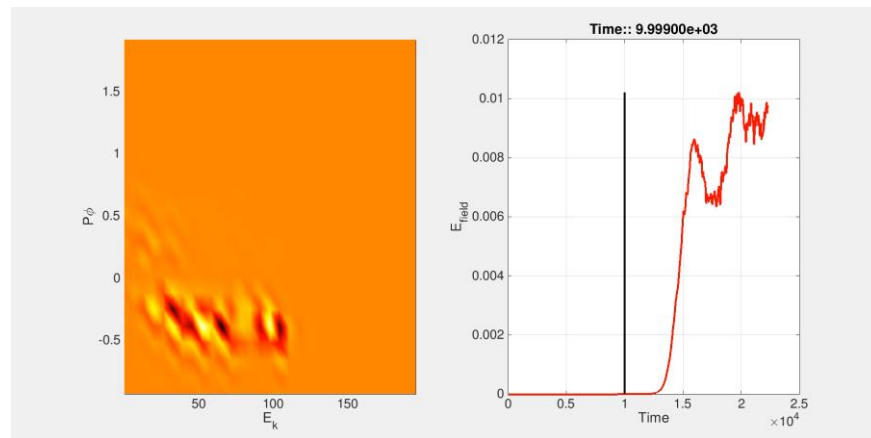
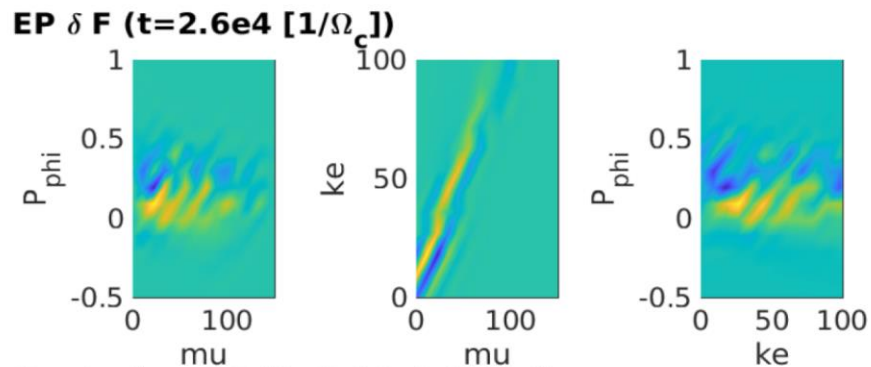
PSZS diagnostic in HMGC



Phase Space Zonal Structures diagnostic in ORB5



- an **ORB5** diagnostic for **PSZS** has been developed, i.e., see [Bottino et al \(2022\)](#);
- **PSZS** can **accumulate** over time...
- A restart of **ORB5** from **PSZS** data is the next step, see the contribution by [A. Bottino @ EPS 2023](#);
- Illustration of ORB5 application to frequency chirping modes ([Invited Talk by X. Wang @ EPS 2023](#))





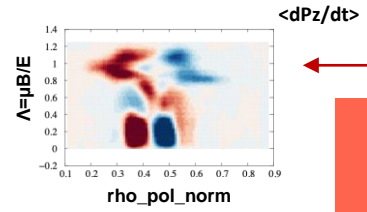
- PSZS transport is studied by means of a **hierarchy of verified and validated reduced models** within an **EUROfusion Enabling Research Project** with **P. Lauber** as P.I.;
- explicit expression of **EP fluxes in PSZS** equations have been calculated within the following hierarchy of simplifying assumptions:
 - the **zeroth level of simplification** consist in the gyrokinetics description of plasma dynamics;
 - the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, [Zonca et al 2021](#);
 - the second and final level of simplification is the **quasilinear model**

ATEP: kick model limit

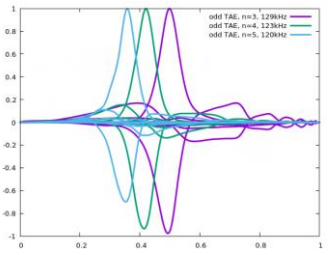


ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)

$$\frac{\partial}{\partial t} \overline{F}_{z=0} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \left(\tau_b \delta \dot{P}_\phi \delta F \right) \right]_z + \frac{\partial}{\partial \mathcal{E}} \left(\tau_b \delta \dot{\mathcal{E}} \delta F \right) \Big|_S = \left(\sum_b C_b^q [F, F_b] + S \right) \Big|_{z_S}$$



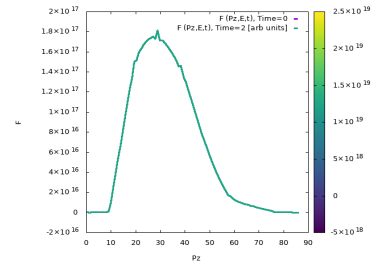
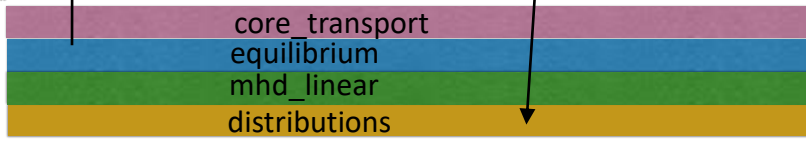
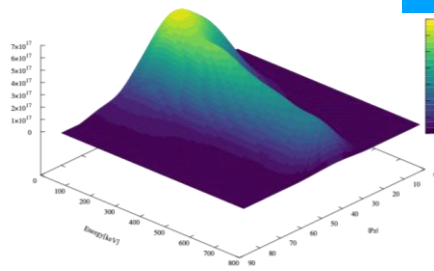
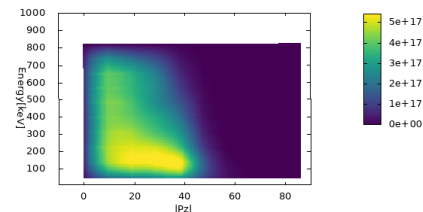
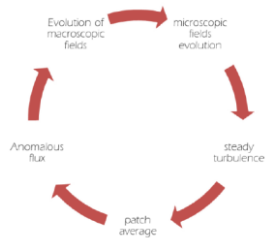
calculate PSZS fluxes with prescribed amplitude



calculate linear mode spectrum

DAEPS and LIGKA Are interchangeable Thanks to IMAS

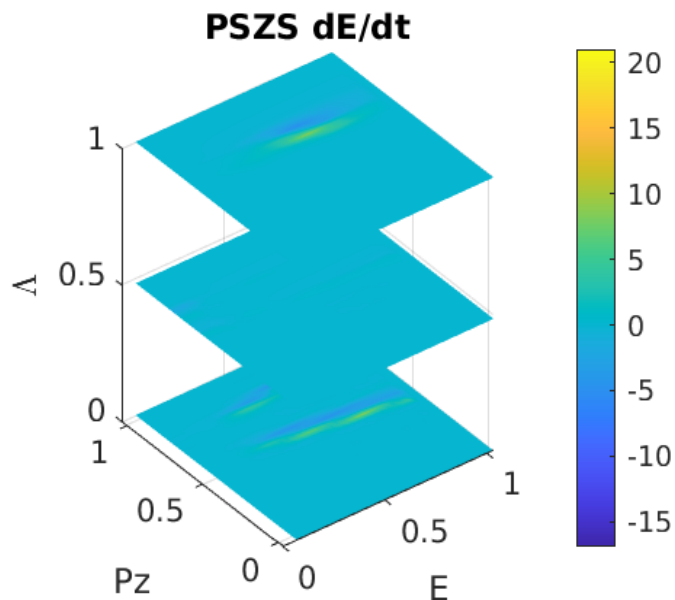
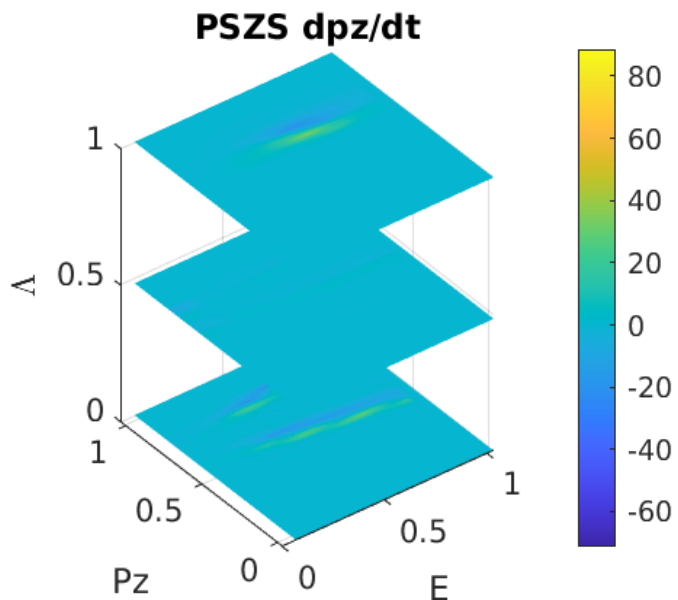
advance F_{EP} and return updated F_{EP} into IDS, or its moments



P. Lauber invited @ this conference time



$$\frac{\partial \overline{F_{z0}}}{\partial t} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \overline{(\tau_b \delta \dot{P}_\phi \delta F)}_z + \frac{\partial}{\partial \mathcal{E}} \overline{(\tau_b \delta \dot{\mathcal{E}} \delta F)}_z \right]_S = \overline{\left(\sum_b C_b^g [F, F_b] + S \right)}_{zS}$$

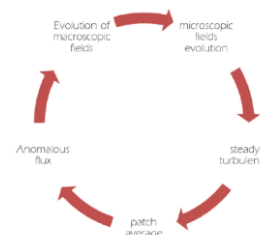


G. Meng et al. poster @ this conference

Summary & Conclusions



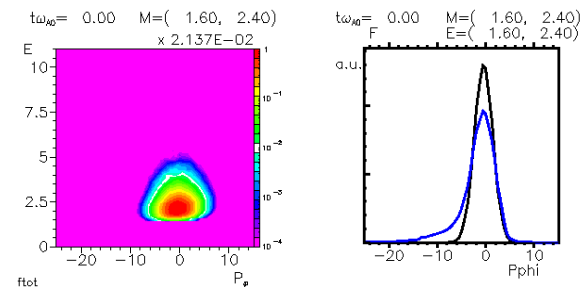
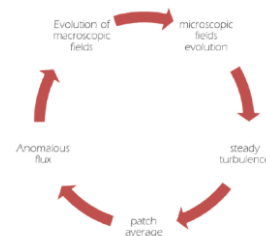
- Need of **predictive transport models** on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing **EPs**;



Summary & Conclusions



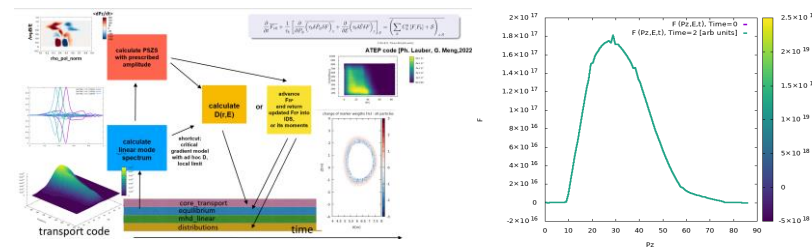
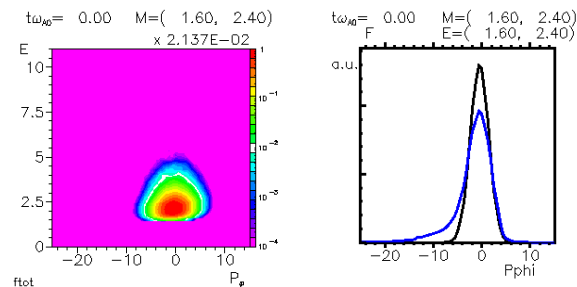
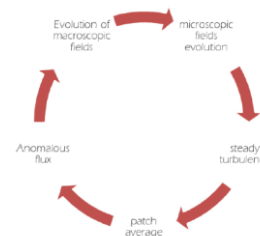
- Need of **predictive transport models** on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing **EPs**;
- we have shown how to describe **meso-scales** and **non-Maxwellian distribution functions** using appropriate phase transport equations;
- we have introduced the concept of **zonal state** to describe the evolution of the plasma between neighboring nonlinear equilibria;



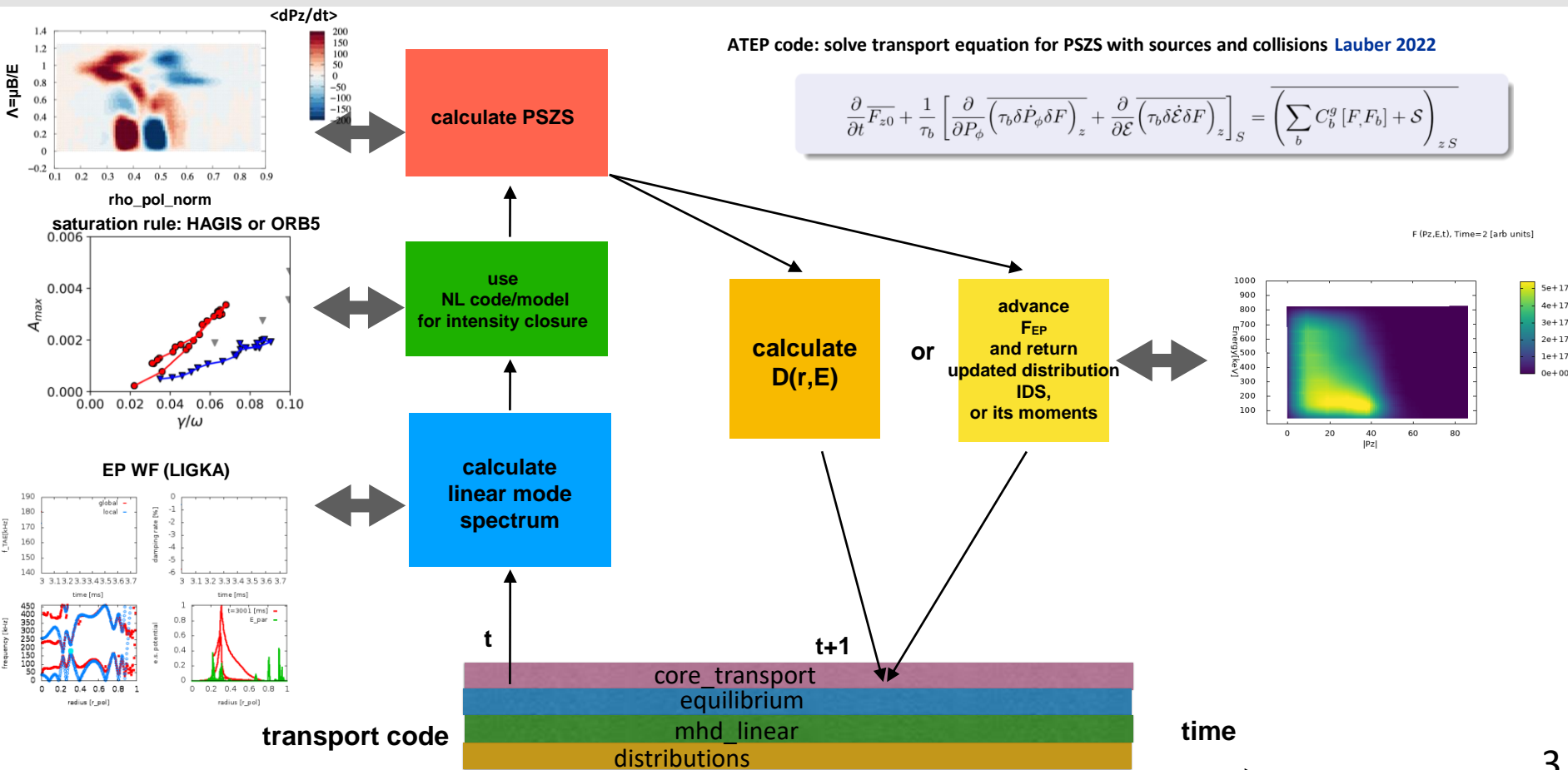
Summary & Conclusions



- Need of **predictive transport models** on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing **EPs**;
- we have shown how to describe **meso-scales** and **non-Maxwellian distribution functions** using appropriate phase transport equations;
- we have introduced the concept of **zonal state** to describe the evolution of the plasma between neighboring nonlinear equilibria;
- we have introduced a framework which allow to **study EP phase space transport** over long time scales.



ATEP: Quasilinear model





$$\frac{\partial}{\partial t} \overline{F_{z0}} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \overline{(\tau_b \delta \dot{P}_\phi \delta F)}_z + \frac{\partial}{\partial \mathcal{E}} \overline{(\tau_b \delta \dot{\mathcal{E}} \delta F)}_z \right]_S = \overline{C_{z0}^g} + [\bar{S}]_S$$

- expanding the **collision operator** we obtain:

$$\bar{C}_{z0}^g = \overline{C_z^g [\bar{F}_{z0}, \bar{F}_{z0}]} + \left[\overline{C_z^g [\bar{F}_{z0}, \delta F_z]} + \overline{C_z^g [\delta F, \delta F]} \right]_S$$

- the second term, in the presence of a Maxwellian reference state, describes **neoclassical transport**;
- **corrections to neoclassical transport** are given by the first and the third terms;
- PSZS evolution with collisions and sources have been studied in [G. Meng et al. 2023](#)



- We have shown how to calculate an **evolving renormalized distribution function** consistent with the finite level of fluctuations; this is connected to the **evolution of a macro-/meso- scopic corresponding CGL equilibrium** ...
- Following **Cary & Brizard 2009** , we write every moment of the zonal distribution function:

$$\begin{aligned} \mathbf{J}_z &= e \int d\mathcal{E} d\mu d\alpha d^3\mathbf{X} D(T^{-1}\mathbf{v}) \delta(\mathbf{X} + \boldsymbol{\rho} - \mathbf{r}) \left[F_0 + \delta F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} F_0 - \frac{e}{m} \frac{\partial}{\partial \mu} F_0 \langle \delta L_g \rangle_z \right] \\ &+ \frac{e^2}{m} \int d\mathcal{E} d\mu d\alpha d^3\mathbf{X} D \left[\frac{\partial}{\partial \mathcal{E}} F_0 \delta \phi_z + \frac{1}{B_0} \frac{\partial}{\partial \mu} F_0 \delta L_z \right] \end{aligned}$$

- we can **substitute F_z for the Phase Space Zonal Structures $\bar{F}_{z0} = [F_0 + e^{-iQ_z} \delta \bar{F}_{Bz}]_S$** ;



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- we can **substitute** F_z for the **Phase Space Zonal Structures** $\bar{F}_{z0} = [F_0 + e^{-iQ_z} \delta \bar{F}_{Bz}]_S$;
- we obtain, from a **multipole expansion**, an equilibrium consistent with a **CGL pressure tensor**:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1) \nabla \left(\frac{B^2}{8\pi} \right) + \frac{B^2}{4\pi} \nabla_{\perp} \sigma \quad \Delta^* \psi + \nabla \ln \sigma \cdot \nabla \psi = - \frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2} \frac{\partial G}{\partial \psi}$$



- the current \mathbf{J}_z and the pressure tensor \mathbf{P} satisfy the following force balance equation:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1) \nabla \left(\frac{B^2}{8\pi} \right) + \frac{B^2}{4\pi} \nabla_{\perp} \sigma$$

where $\sigma = 1 + \frac{4\pi}{B^2} (P_{\perp} - P_{\parallel})$;

- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

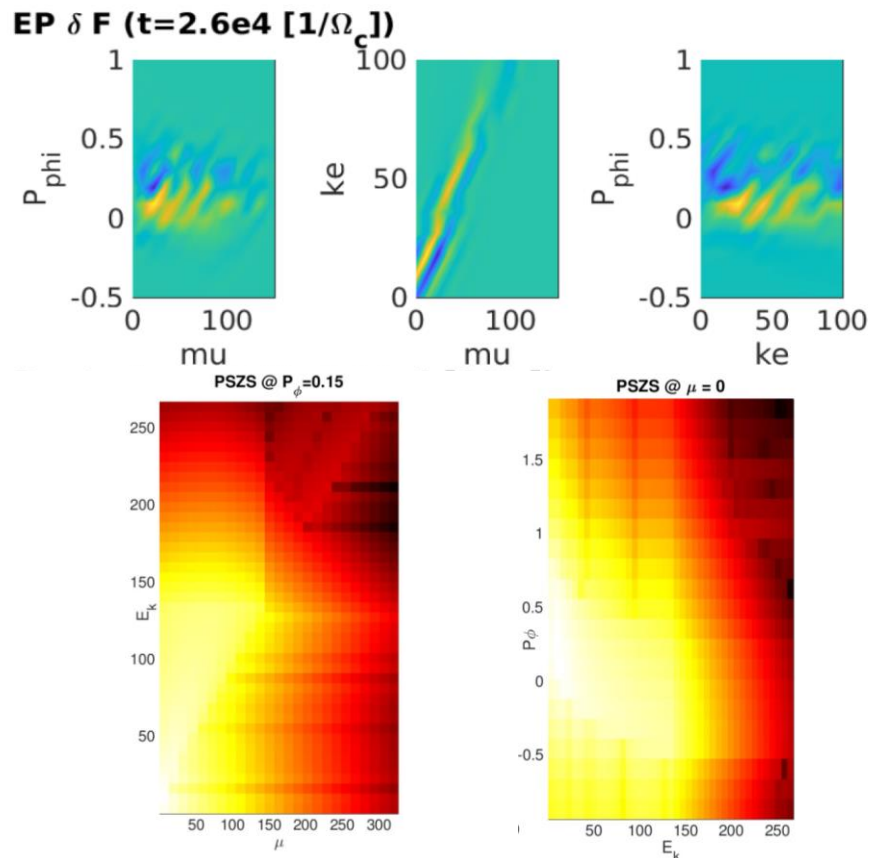
$$\Delta^* \psi + \nabla \ln \sigma \cdot \nabla \psi = - \frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2} \frac{\partial G}{\partial \psi}$$

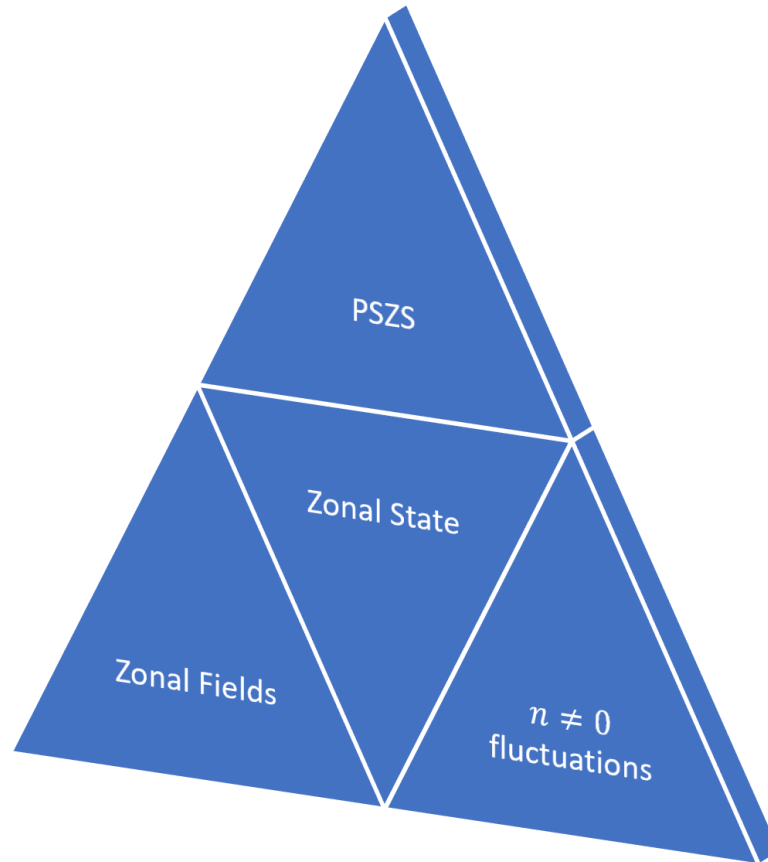
- where $G \equiv \sigma F^2 / 2$ is a flux function;

Phase Space Zonal Structures diagnostic in ORB5



- an **ORB5** diagnostic for **PSZS** has been developed, i.e., see [Bottino et al \(2022\)](#);
- **PSZS** can **accumulate** over time...
- A restart of **ORB5** from **PSZS** data is the next step, see the contribution by [A. Bottino @ EPS 2023](#);
- Illustration of ORB5 application to frequency chirping modes ([Invited Talk by X. Wang @ EPS 2023](#))







DAEPS (Y. Li et al 2020)

- **Ballooning decomposition** for fluctuations;
- Based on **fish-bone like dispersion relation**;
- Mode structure decomposition, **separation of radial envelope and parallel mode structure**;
- Calculate nonlinear fluxes by the **DSM model** or a saturation rule.

LIGKA-HAGIS (Lauber et al 2007)

- **Fourier decomposition** for fluctuations;
- Solve **linear gyrokinetic equation**;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use **IMAS-coupled EP stability WF (HAGIS/LIGKA)** to **calculate Phase Space Zonal Structures fluxes**.



- EPs are generally non Maxwellian, and transport processes take place in the phase space!
- Separation of scales is questionable...
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;

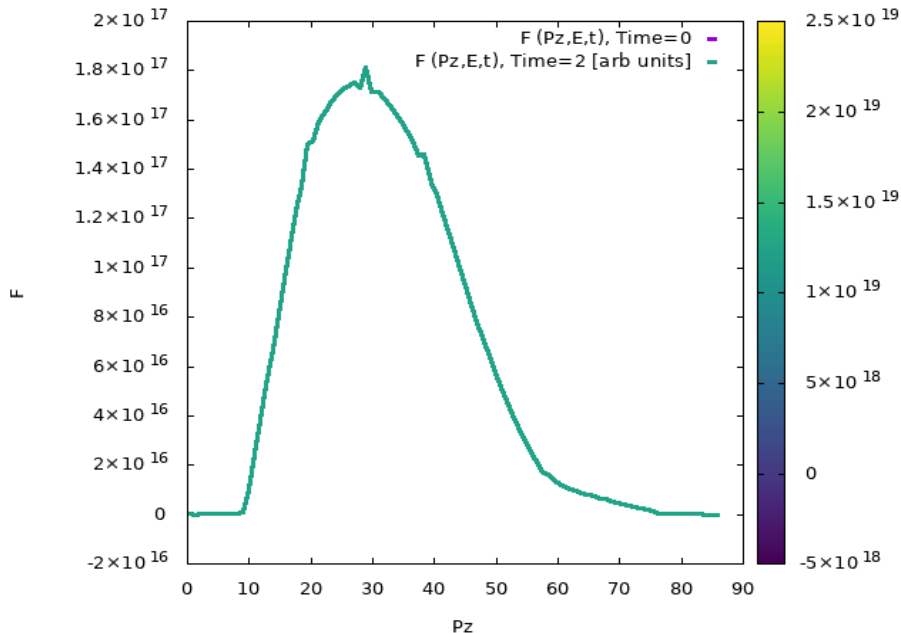
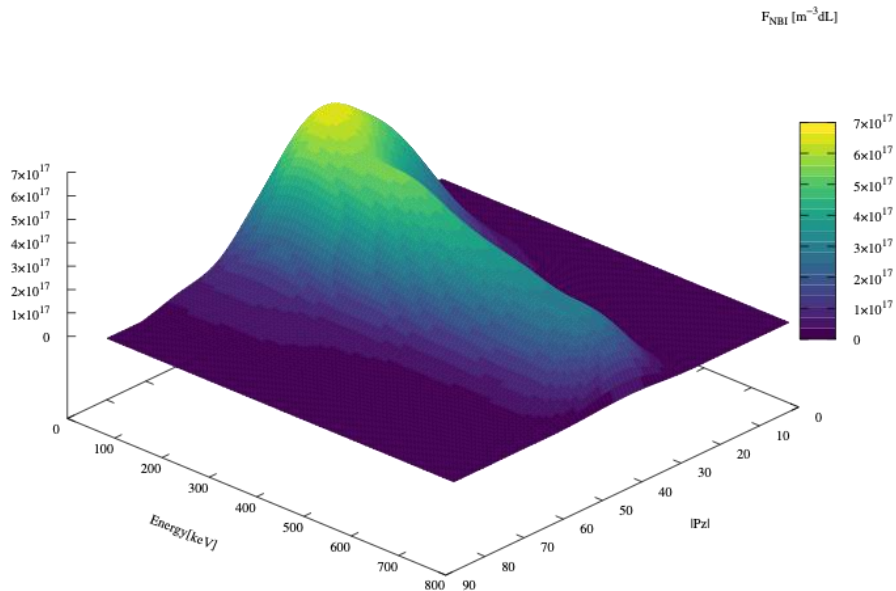


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- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;
- interaction with thermal plasma over long timescales can modify bulk transport processes;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the theory of Phase space zonal structures (PSZS), see Chen and Zonca 2016.



ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)

$$\frac{\partial \overline{F_{z0}}}{\partial t} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \left(\tau_b \delta \dot{P}_\phi \delta F \right)_z + \frac{\partial}{\partial \mathcal{E}} \left(\tau_b \delta \dot{\mathcal{E}} \delta F \right)_z \right]_S = \left(\sum_b C_b^g [F, F_b] + S \right)_{zS}$$

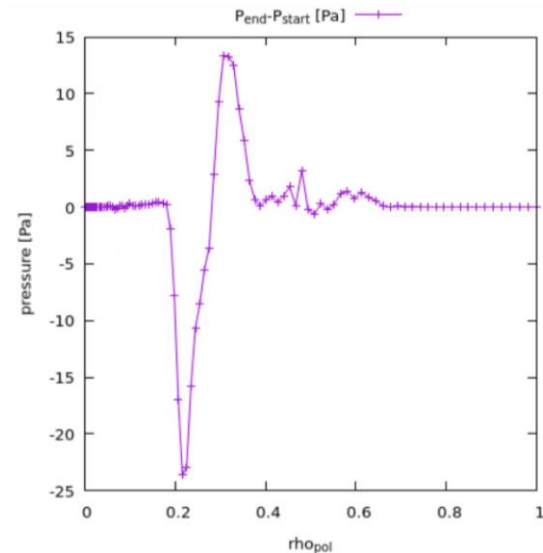
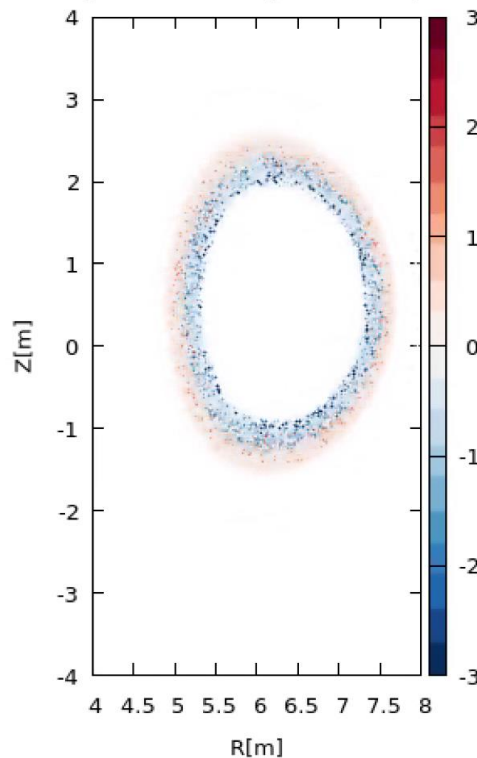


ITER plasma from H&CD WF by M. Schneider



- Mapping to 1d profile of **PSZS**;
- The **CGL equilibrium** can be readily constructed [Falessi et al 2023 sub](#);
- transport is **zonal** by construction;
- Next step is the calculation of the corresponding **magnetic equilibrium**;

change of marker weights [%] - all particles



ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)