# Nonlinear equilibria and phase space transport in burning plasmas


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# Transport processes in burning plasmas

- Predicting the dynamics of a burning plasma on long time scales, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g., ITER;
- the crucial role of energetic particles Zonca et al. 2015; Chen and Zonca 2016, must be properly described;

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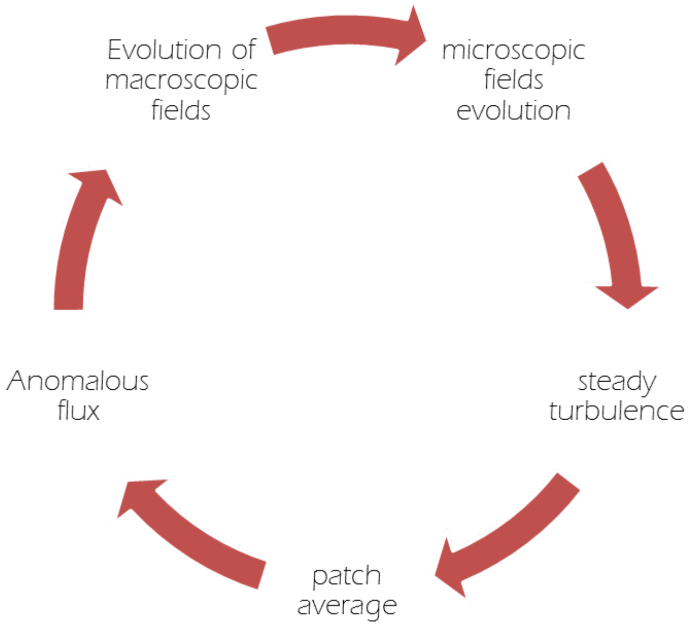
- the crucial role of energetic particles Zonca et al. 2015; Chen and Zonca 2016, must be properly described;
- a first-principle-based, self-consistent approach is crucial;
- extending gyrokinetic simulations to these time scales is a challenging task from the computational resource point of view, i.e., $\sim 10^{24}$grid points;
- simplifying assumptions based on physics understanding and first principles must be introduced;

# Transport processes in burning plasmas

- Transport equations define the evolution of radial profiles

$$\langle \partial_t n \rangle_\psi = - \langle \boldsymbol{\nabla} \cdot (n\boldsymbol{V}) \rangle_\psi \, ;$$

- consistent with a slowly evolving ($\omega^{-1}\partial_t \, \log p_0 \sim \delta^2$) equilibrium distribution function;



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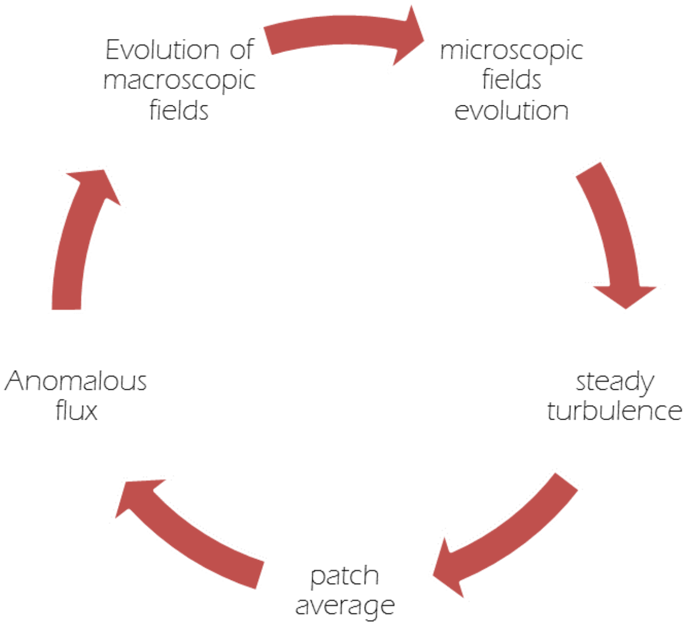
- consistent with a slowly evolving ($\omega^{-1}\partial_t \, \log p_0 \sim \delta^2$) equilibrium distribution function;
- Implicit separation of scales between equilibrium and fluctuations;
- Local Maxwellian is assumed;




Need for generalization!!!

# Purpose statement

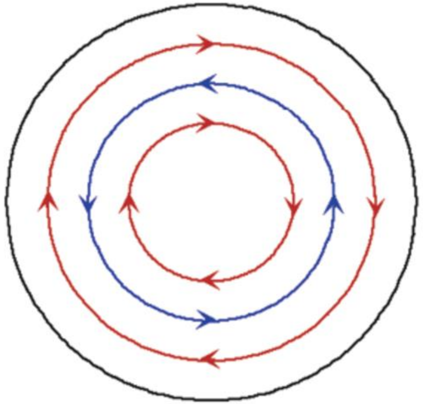
We aim at 1. providing the general expressions describing EPs (plasma) dynamics on long time scales (transport) and 2. introducing a framework to solve these equations within different levels of reduced dynamics.

By means of this approach it will be possible to:

- define the concept of nonlinear equilibrium;
- describe the physics of burning plasmas where alpha particles will play a key role in transport studies by interacting with thermal components;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the Phase space zonal structures theory, see Chen and Zonca 2016.

# Phase Space Zonal Structures

- Coupling between fluctuating fields can generate zonal flows and zonal fields;
- Crucial elements for regulating turbulent fluxes, e.g., by scattering instability turbulence to shorter radial wavelength stable domain…



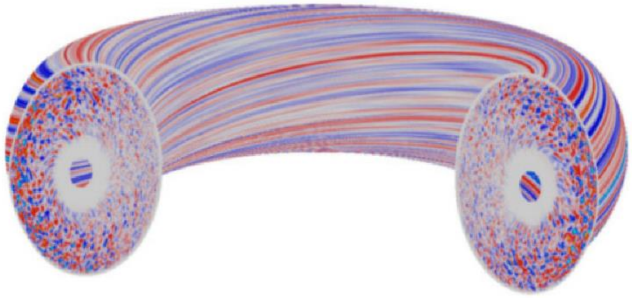


Figure: Courtesy of Y. Xiao et al., PoP 2015.

# Phase Space Zonal Structures

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- analogously, zonal structures in the density and temperature profiles are unaffected by rapid collision-less dissipation;
- collision-less undamped fluctuations in the phase space are called phase space zonal structures Chen and Zonca 2016, Falessi and Zonca 2019;

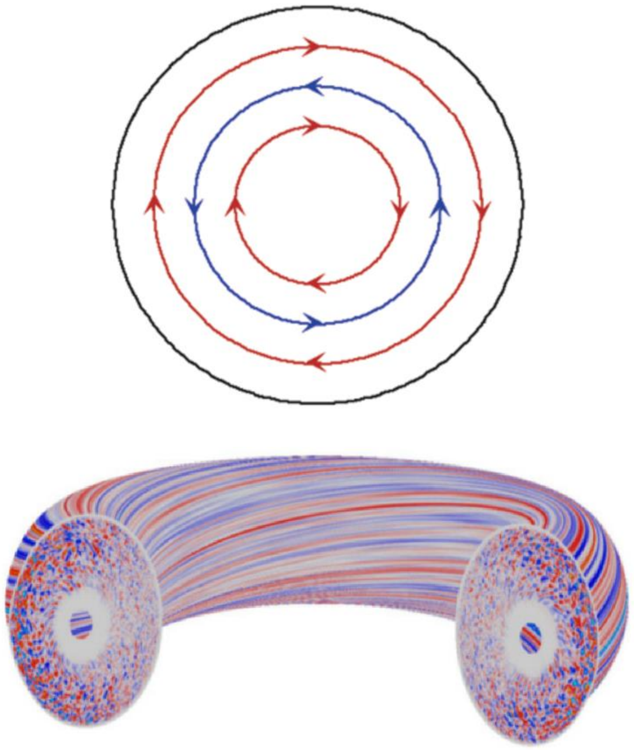

Figure: Courtesy of Y. Xiao et al., PoP 2015.

# Phase Space Zonal Structures equation

- We want to explicit identify the part of the toroidally symmetric distribution function unaffected by rapid collisionless dissipation that is the PSZS Falessi et al 2023 sub.;
- we start by writing the nonlinear Gyrokinetic equation:

$$\partial_t(DF) + \nabla \cdot \left(D\dot{\mathbf{X}}F\right) + \partial_{\mathcal{E}}\left(D\delta\dot{\mathcal{E}}F\right) = 0$$

$D$ is the Jacobian of the velocity space and $\dot{\mathbf{X}} = \dot{\mathbf{X}}_0 + \delta\dot{\mathbf{X}}$ is the gyrocenter velocity due to magnetic equilibrium and to fluctuating fields;

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- we decompose $F$ as $F_0 + \delta F$ where $F_0$ is the steady state solution of the lowest order gyrokinetic equation, i.e.:

$$F_0(\psi, \theta, \mathcal{E}, \mu) = e^{-iQ_z} F_{B0}(\psi, \theta, \mathcal{E}, \mu) = F_{B0}\left( \psi - \frac{F v_\parallel}{\Omega}, \mathcal{E}, \mu \right) = F_{B0}\left( \bar{\psi}(\psi, \theta, \mathcal{E}, \mu), \mathcal{E}, \mu \right)$$

where the drift/banana center shift operator $e^{iQ_z}$ with $Q_z \equiv F(v_\parallel/\Omega)k_z/(d\psi/dr)$

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where the drift/banana center shift operator $e^{iQ_z}$ with $Q_z \equiv F(v_\parallel/\Omega)k_z/(d\psi/dr)$

- we can calculate $D(\delta\dot{\mathbf{X}}\cdot\nabla + \delta\dot{\mathcal{E}}\partial_{\mathcal{E}})F_{B0}(\bar{\psi}(\psi,\theta,\mathcal{E},\mu),\mathcal{E},\mu)$...

# Phase Space Zonal Structures equation

- … and re-write the toroidally symmetric component of the Gyrokinetic equation:

$$D\left(\partial_t + \dot{\mathbf{X}}_0 \cdot \boldsymbol{\nabla}\right)\left(F_z - \frac{e}{m}\langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}}|_{\bar{\psi}} F_0 + \frac{F}{B_0}\frac{\partial F_0}{\partial \bar{\psi}}\langle \delta A_{\parallel g}\rangle_z\right) + D\frac{e}{m}\frac{\partial}{\partial \mathcal{E}}|_{\bar{\psi}} F_0 \partial_t \langle \delta L_g \rangle_z$$

$$+\frac{1}{J}\frac{\partial}{\partial \theta}\left(JD\delta\dot{\theta}\delta F\right) + \frac{1}{J}\frac{\partial}{\partial \psi}\left(JD\delta\dot{\psi}\delta F\right) + \frac{\partial}{\partial \mathcal{E}}\left(JD\delta\dot{\mathcal{E}}\delta F\right) = 0$$

where $\langle \delta L_g \rangle_z = J_0\left(\delta\phi_z - \frac{v_\parallel}{c}\delta A_{\parallel z}\right) + 2\frac{m}{\lambda e}\mu J_1 \delta B_{\parallel z}$ and $J$ is the Jacobian of the curvilinear coordinate system;

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$$G_z = F_z - \frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}}F_0 + \frac{F}{B_0}\langle\delta A_{\parallel g}\rangle_z\frac{\partial}{\partial\bar{\psi}}F_0$$

- We now apply $e^{iQ_z}$ and write the governing equation for $G_B = e^{iQ_z}G_z$

# Phase Space Zonal Structures equation

- … we obtain the following equation:

$$\partial_t(JDG_B) + \partial_\theta G_B$$
$$= e^{iQ_z}\left[-\frac{e}{m}\frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}}F_0\partial_t\big(JD\langle\delta L_g\rangle_z\big) - \frac{1}{J}\frac{\partial}{\partial\theta}\big(JD\delta\dot{\theta}\delta F\big) - \frac{1}{J}\frac{\partial}{\partial\psi}\big(JD\delta\dot{\psi}\delta F\big) - \frac{\partial}{\partial\mathcal{E}}\big(JD\delta\dot{\mathcal{E}}\delta F\big)\right]$$

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It can be shown that $e^{iQ_z}$, up to the required order, commute with partial derivatives;

- finally, integrating over $\theta$ and introducing the bounce/transit average $\overline{[\dots]} = \tau_b^{-1}\oint \frac{d\ell}{v_\parallel}[\dots]$, we obtain:

$$\partial_t\overline{G}_B = -\overline{e^{iQ_z}\frac{e}{m}\frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}}F_0\partial_t\langle\delta L_g\rangle_z} - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left(\tau_b\overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right) - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left(\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right)$$

# Particle transport

- We can re-write the low frequency (transport) component of $\delta f_z$ in term of only $\delta \overline{G}_B$;
- taking the time derivative of the surface averaged velocity integral we obtain:

$$\partial_t \left\langle \langle \delta f_z \rangle_v \right\rangle_\psi = \frac{e}{m} \left\langle \left[ 1 - \overline{\left(e^{-iQ_z} J_0\right)}\, \overline{\left(e^{iQ_z} J_0\right)} \right] \frac{\partial F_0}{\partial \mathcal{E}} \partial_t \delta \phi_z \right\rangle_v +$$

$$\frac{1}{V'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{\left(e^{-iQ_z} J_0\right)} \overline{\left[ c e^{iQ_z} R^2 \nabla \phi \cdot \nabla \langle \delta L_g \rangle \delta G \right]} \right\rangle_v \right\rangle_\psi$$

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- This equation describes the radial oscillations on any length-scale of the density profile in the absence of collisions and assuming GK ordering Falessi and Zonca 2019;
- mesoscales are spontaneously created by the turbulence;

# Equilibrium renormalization

- This naturally leads to define a reference distribution function $F_0$ at each instant of time. Going back to the previous derivation and rewriting terms such as:

$$\delta\dot{\mathbf{X}}\big|_z \cdot \nabla \delta F_z = \delta\dot{\mathbf{X}}\big|_z \cdot \nabla e^{-iQ_z}\left(\delta \bar{F}_{Bz} + \delta \tilde{F}_{Bz}\right)$$

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- $e^{-iQ_z}\delta\bar{F}_{Bz}$ analogously to $F_0$, is a function of the constants of motion;
- we obtain a newly defined $G_z$:

$$G_z = F_z - \frac{e}{m}\langle\delta L_g\rangle_z \frac{\partial}{\partial\mathcal{E}}\Big|_{\bar{\psi}} F_{0*} + \frac{F}{B_0}\langle\delta A_{\parallel g}\rangle_z \frac{\partial}{\partial\bar{\psi}} F_{0*}$$

- Where $F_{0*} = F_0 + e^{-iQ_z}\delta\bar{F}_{Bz}$ is the renormalized $F_0$;
- Phase Space Zonal Structures are the macro-/meso- scopic component of $F_{0*}$

# Orbit averaging

- Particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- Phase Space Zonal Structures equation is connected with the macro-/meso- scopic component, i.e. $[\dots]_S$, unperturbed orbit averaged distribution function (Falessi and Zonca 2019);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\,[F_, F_b] + \mathcal{S}\right)}_{z\,S}$$

where $\overline{(\dots)} = \tau_b^{-1}\oint d\theta/\dot{\theta}\,(\dots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\dots)\big(\bar{\psi},\theta\big)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\tilde{\psi}(\theta)$;

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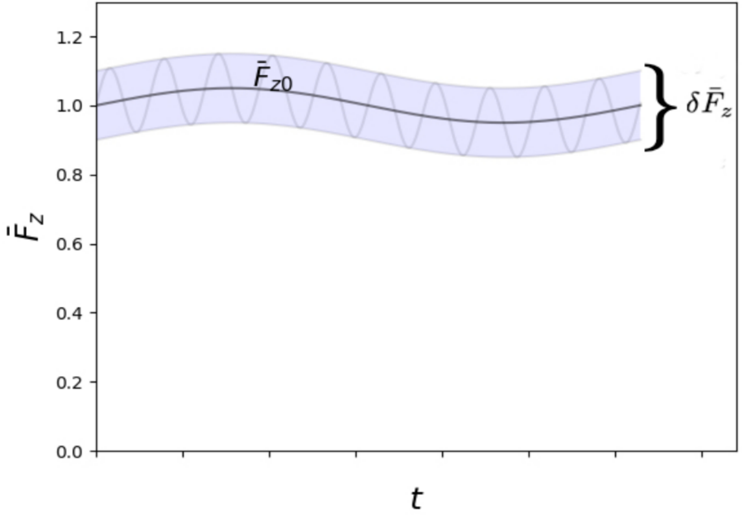
- equivalent to bounce/transit averaging a quantity shifted with the $e^{iQ_z}$ operator;
- This expression describe transport processes in the phase space due to fluctuations, collisions and sources.

# Neighboring nonlinear equilibria

- Having defined Phase Space Zonal Structures, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F_z}$;

$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



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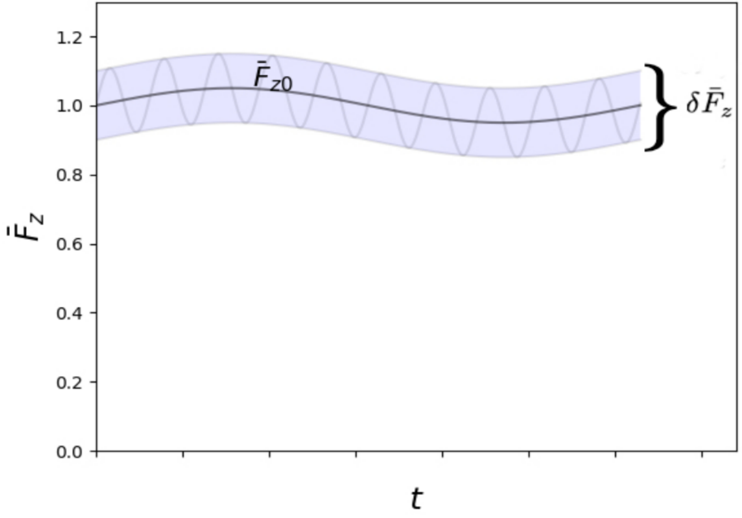
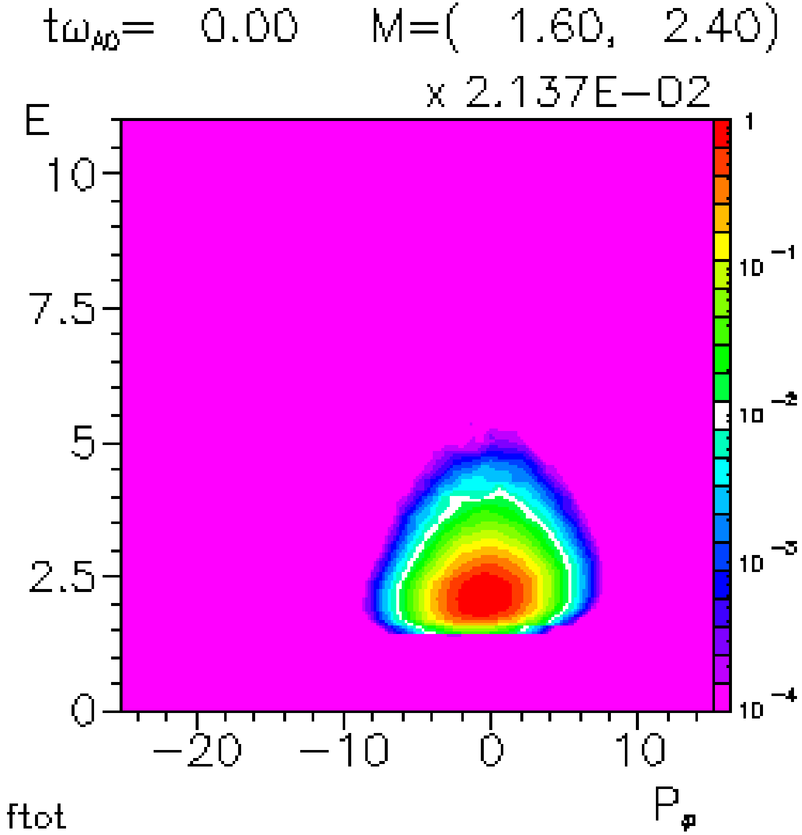
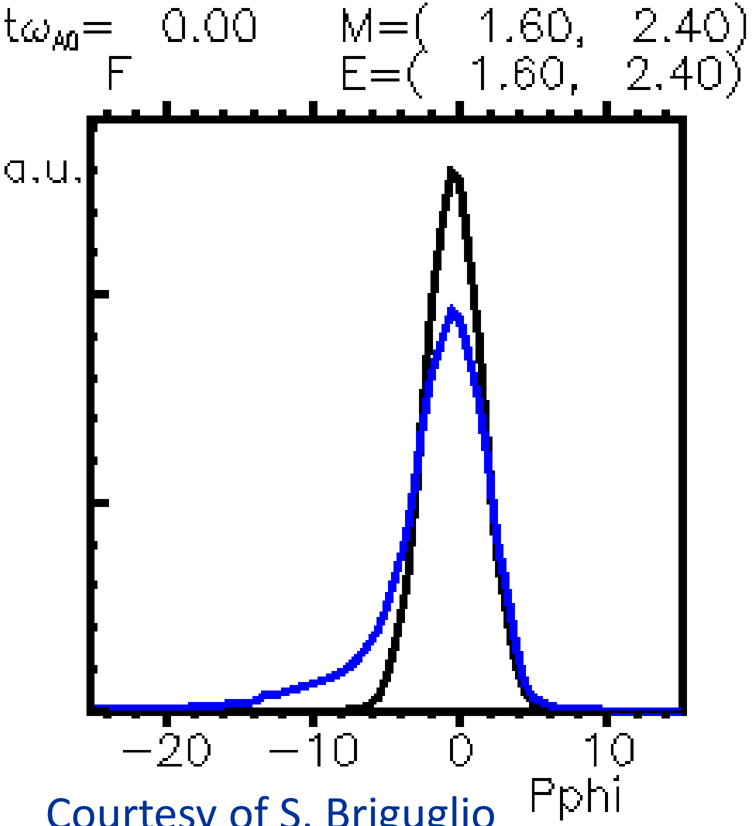
- micro-scales are accounted by $\delta\overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal fields, form a zonal state.

$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



# PSZS diagnostic in HMGC

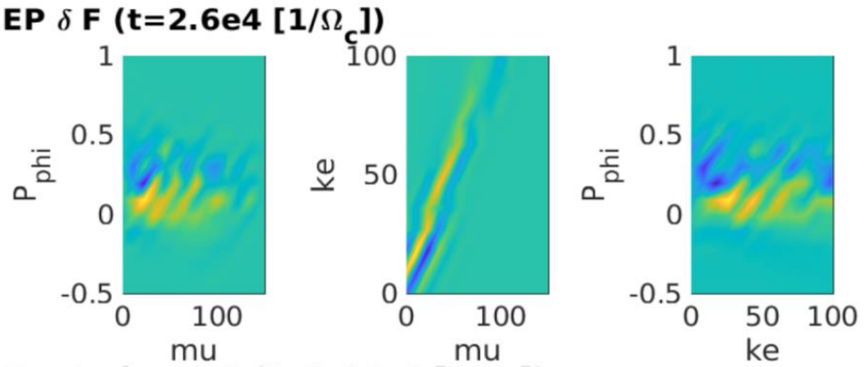


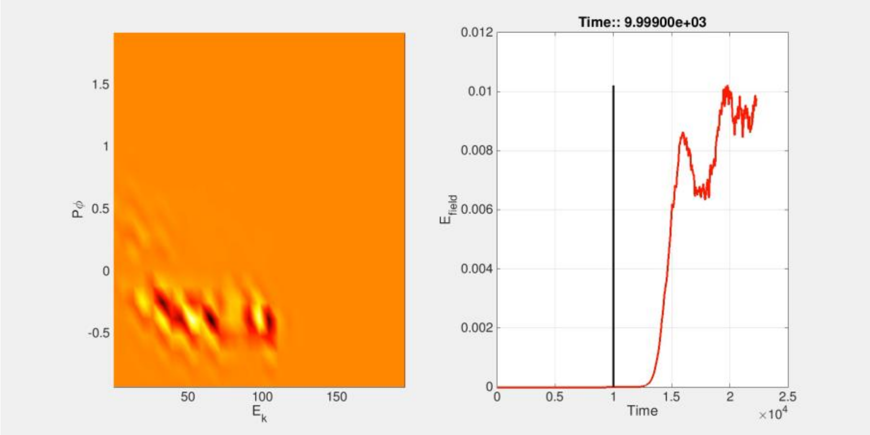




Courtesy of S. Briguglio

# Phase Space Zonal Structures diagnostic in ORB5

- an ORB5 diagnostic for PSZS has been developed, i.e., see Bottino et al (2022);
- PSZS can accumulate over time…
- A restart of ORB5 from PSZS data is the next step, see the contribution by A. Bottino @ EPS 2023;
- Illustration of ORB5 application to frequency chirping modes (Invited Talk by X. Wang @ EPS 2023)



# Reduced transport models

- PSZS transport is studied by means of a hierarchy of verified and validated reduced models within an EUROfusion Enabling Research Project with P. Lauber as P.I.;

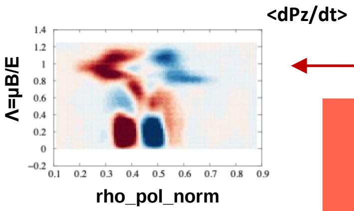
- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;

- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, Zonca et al 2021;

- the second and final level of simplification is the quasilinear model

# ATEP: kick model limit




ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

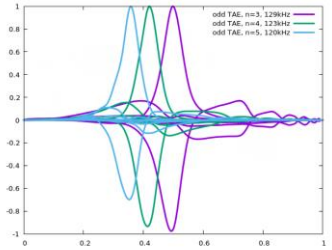
$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right] + \mathcal{S}\right)}_{z\,S}$$

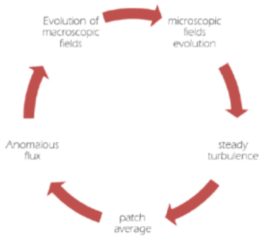
**calculate PSZS fluxes with prescribed amplitude**

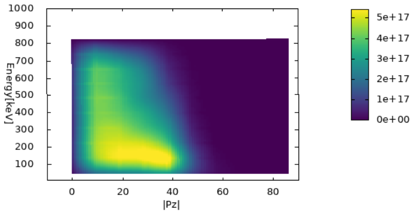
**advance $F_{EP}$ and return updated $F_{EP}$ into IDS, or its moments**

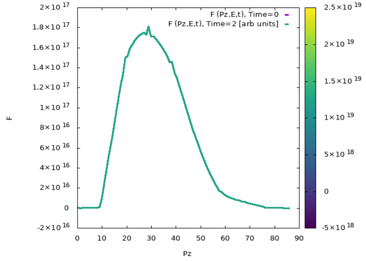
**calculate linear mode spectrum**

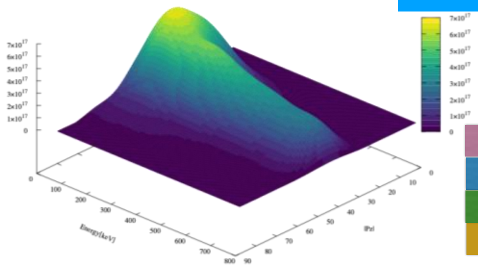
**DAEPS and LIGKA Are interchangeable Thanks to IMAS**




core_transport

equilibrium

mhd_linear

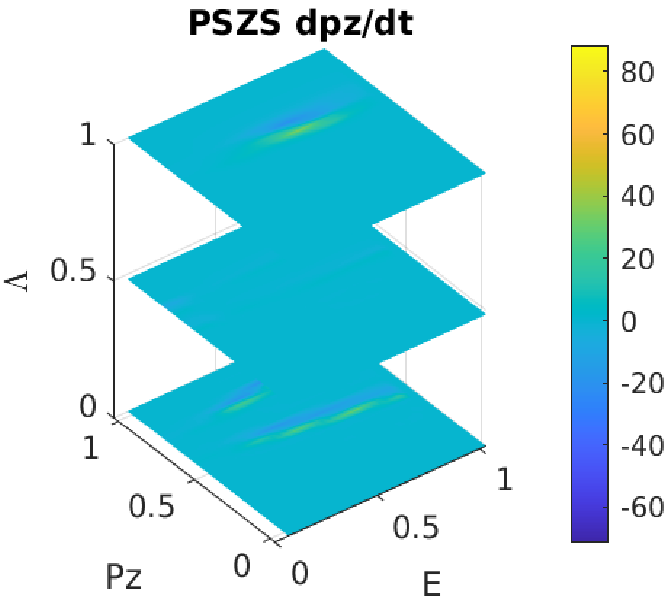
distributions

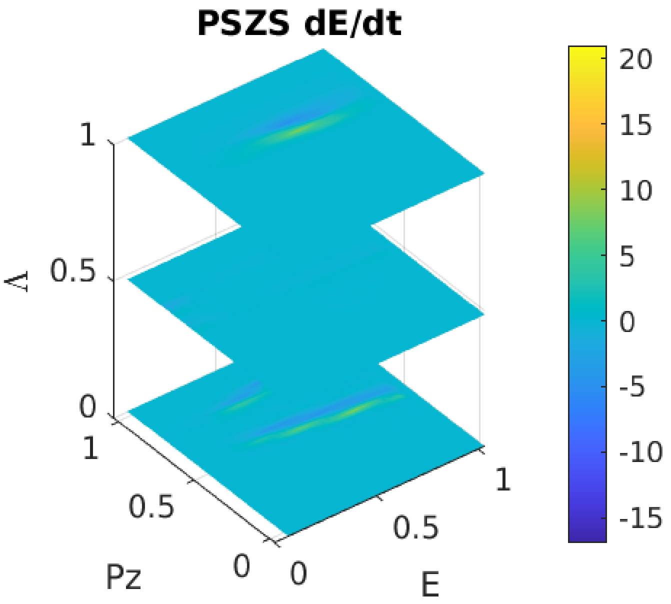
P. Lauber invited @ this conference

time

# Phase Space fluxes

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$

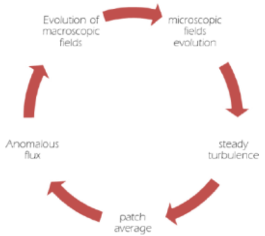



G. Meng et al. poster @ this conference

# Summary & Conclusions

- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;

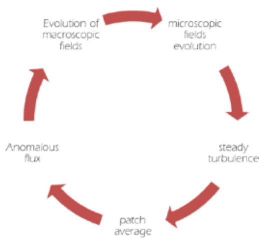


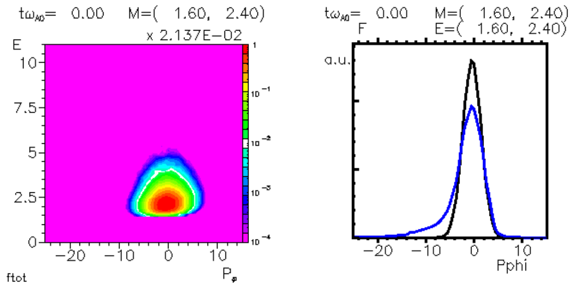
# Summary & Conclusions

- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;

- we have shown how to describe meso-scales and non-Maxwellian distribution functions using appropriate phase transport equations;

- we have introduced the concept of zonal state to describe the evolution of the plasma between neighboring nonlinear equilibria;



# Summary & Conclusions

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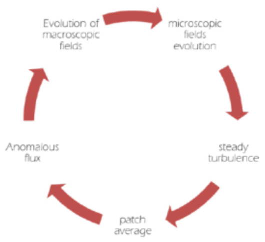
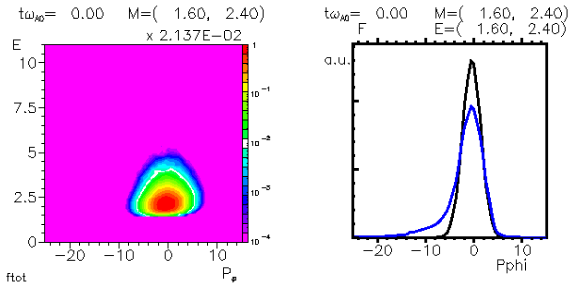
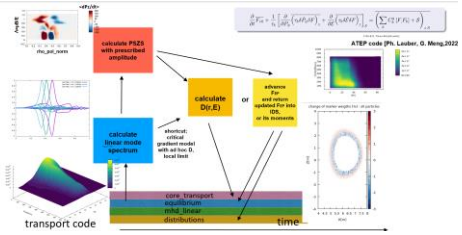
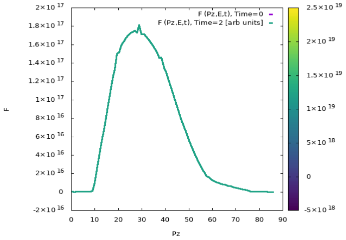
- we have introduced a framework which allow to study EP phase space transport over long time scales.



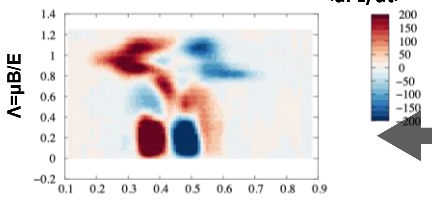



# ATEP: Quasilinear model

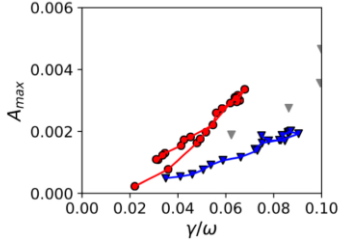
ATEP code: solve transport equation for PSZS with sources and collisions Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right] + \mathcal{S}\right)}_{z\,S}$$



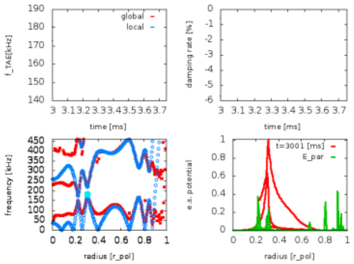

calculate PSZS

saturation rule: HAGIS or ORB5




use NL code/model for intensity closure

EP WF (LIGKA)

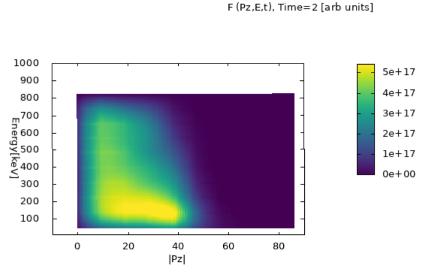



calculate linear mode spectrum

calculate D(r,E)

or

advance $F_{EP}$ and return updated distribution IDS, or its moments




t

t+1

transport code

core_transport
equilibrium
mhd_linear
distributions

time

# Collisional fluxes

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{C^g_{z0}} + [\bar{S}]_S$$

- expanding the collision operator we obtain:

$$\bar{C}^g_{z0} = \overline{C^g_z\left[\overline{F}_{z0}, \overline{F}_{z0}\right]} + \left[\overline{C^g_z\left[\overline{F}_{z0}, \delta F_z\right]} + \overline{C^g_z[\delta F, \delta F]}\right]_S$$

- the second term, in the presence of a Maxwellian reference state, describes neoclassical transport;
- corrections to neoclassical transport are given by the first and the third terms;
- PSZS evolution with collisions and sources have been studied in G. Meng et al. 2023

# Zonal fields

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations; this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium …

- Following Cary & Brizard 2009 , we write every moment of the zonal distribution function:

$$\mathbf{J}_z = e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D(T^{-1}\mathbf{v})\delta(\mathbf{X}+\boldsymbol{\rho}-\mathbf{r})\left[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0-\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\right]$$
$$+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]$$

- we can substitute $F_z$ for the Phase Space Zonal Structures $\bar{F}_{z0}=\left[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}\right]_S$;

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- we can substitute $F_z$ for the Phase Space Zonal Structures $\bar{F}_{z0}=\left[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}\right]_S$;

- we obtain, from a multipole expansion, an equilibrium consistent with a CGL pressure tensor:

$$\sigma\frac{\mathbf{J}_z\times\mathbf{B}}{c}=\nabla P_\parallel+(\sigma-1)\nabla\left(\frac{B^2}{8\pi}\right)+\frac{B^2}{4\pi}\nabla_\perp\sigma \qquad \Delta^*\psi+\nabla\ln\sigma\cdot\nabla\psi=-\frac{4\pi R^2}{\sigma}-\frac{1}{\sigma^2}\frac{\partial G}{\partial\psi}$$

# Zonal fields: macro- /meso- scopic component

- the current $\mathbf{J}_z$ and the pressure tensor $\mathbf{P}$ satisfy the following force balance equation:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1)\nabla\left(\frac{B^2}{8\pi}\right) + \frac{B^2}{4\pi}\nabla_{\perp}\sigma$$

where $\sigma = 1 + \frac{4\pi}{B^2}(P_{\perp} - P_{\parallel})$;

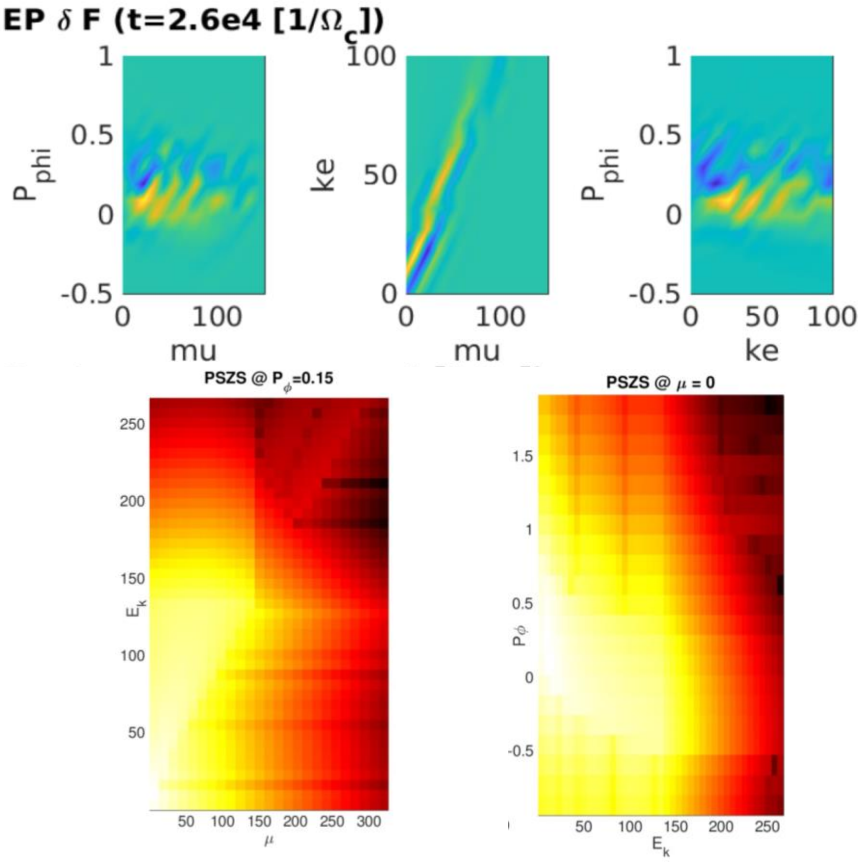
- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

$$\Delta^*\psi + \nabla\ln\sigma \cdot \nabla\psi = -\frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2}\frac{\partial G}{\partial\psi}$$

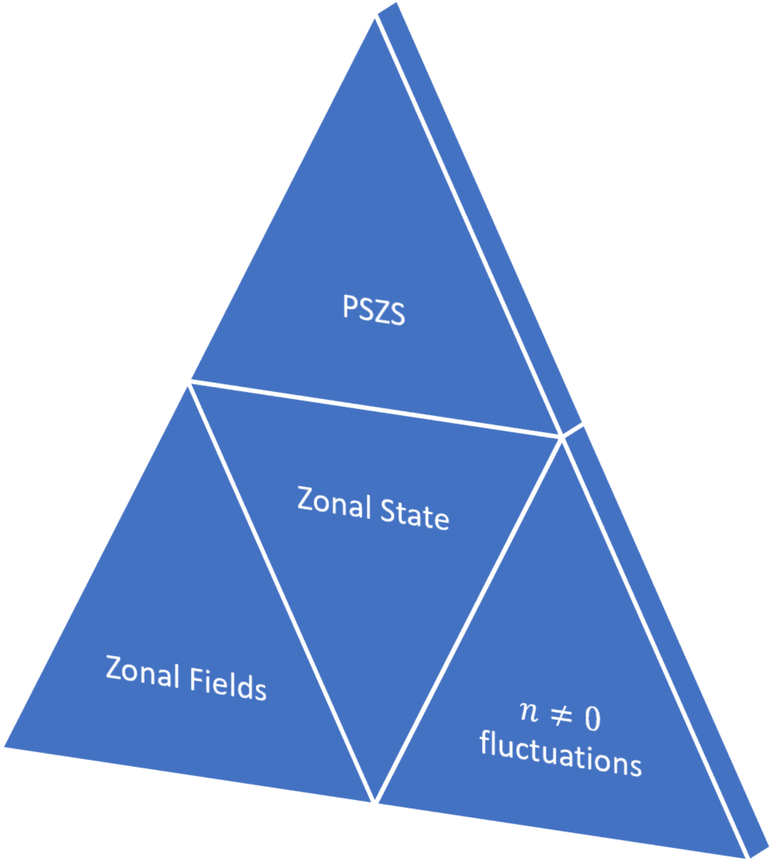
- where $G \equiv \sigma F^2/2$ is a flux function;

# Phase Space Zonal Structures diagnostic in ORB5

- an ORB5 diagnostic for PSZS has been developed, i.e., see Bottino et al (2022);
- PSZS can accumulate over time…
- A restart of ORB5 from PSZS data is the next step, see the contribution by A. Bottino @ EPS 2023;
- Illustration of ORB5 application to frequency chirping modes (Invited Talk by X. Wang @ EPS 2023)



# Zonal state



# Reduced transport models

## DAEPS (Y. Li et al 2020)

- Ballooning decomposition for fluctuations;
- Based on fish-bone like dispersion relation;
- Mode structure decomposition, separation of radial envelope and parallel mode structure;
- Calculate nonlinear fluxes by the DSM model or a saturation rule.

## LIGKA-HAGIS (Lauber et al 2007)

- Fourier decomposition for fluctuations;
- Solve linear gyrokinetic equation;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use IMAS-coupled EP stability WF (HAGIS/LIGKA) to calculate Phase Space Zonal Structures fluxes.

# Transport processes in burning plasmas

- EPs are generally non Maxwellian, and transport processes take place in the phase space!
- Separation of scales is questionable…
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;

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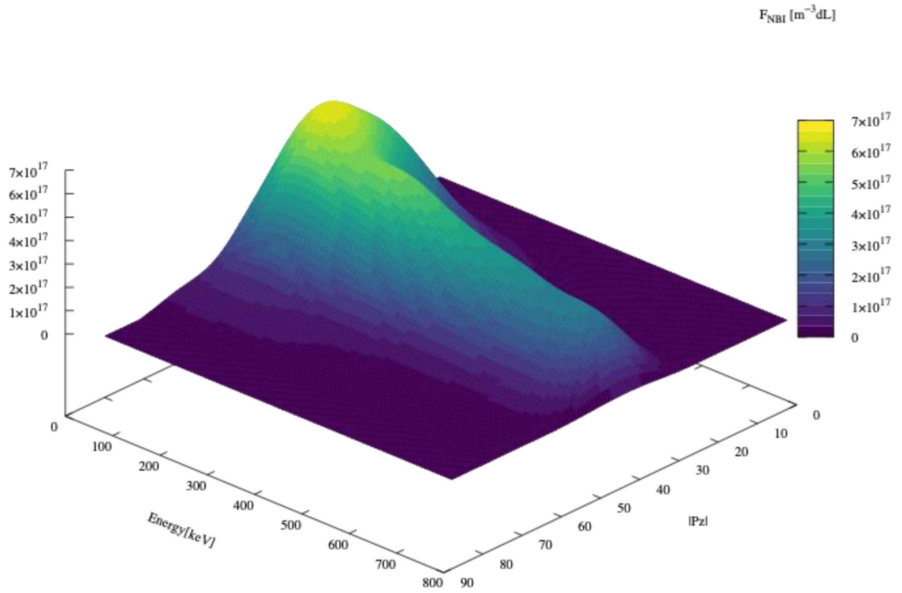
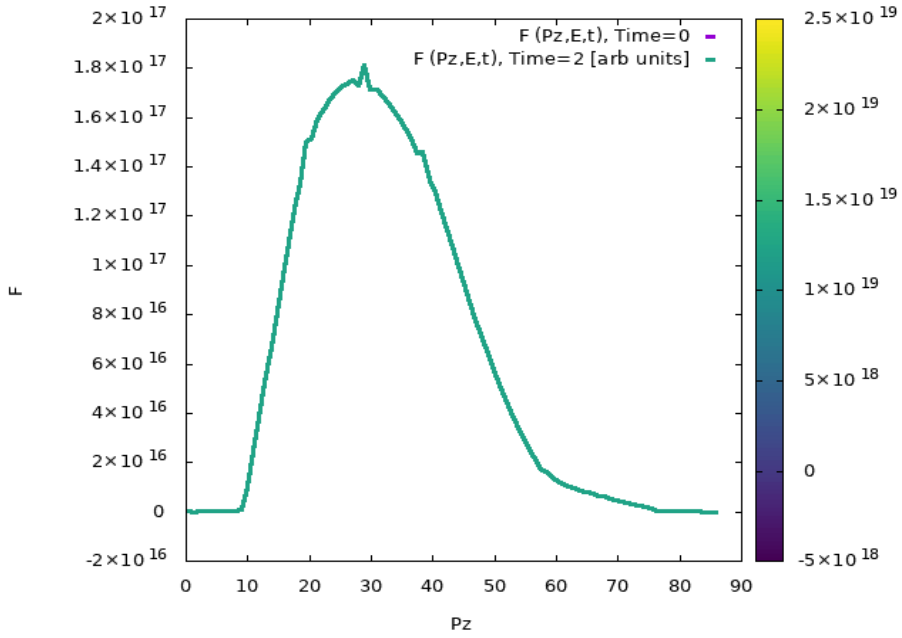
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;
- interaction with thermal plasma over long timescales can modify bulk transport processes;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the theory of Phase space zonal structures (PSZS), see Chen and Zonca 2016.

# PSZS evolution

**ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022**

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{z\,S}$$

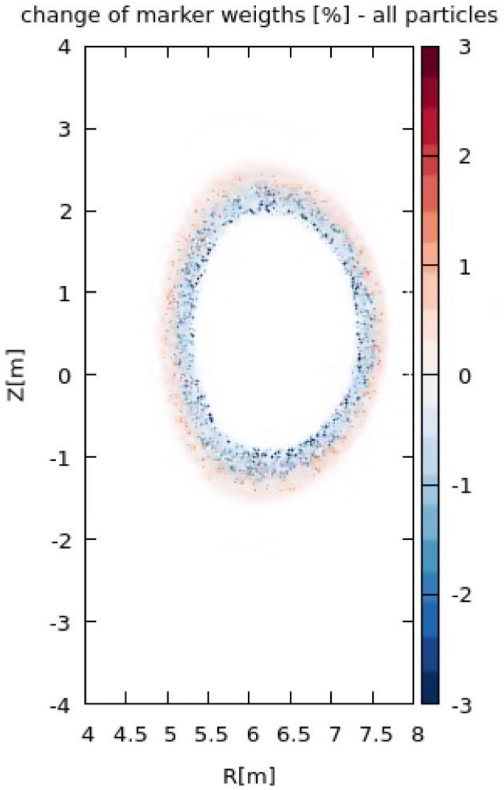


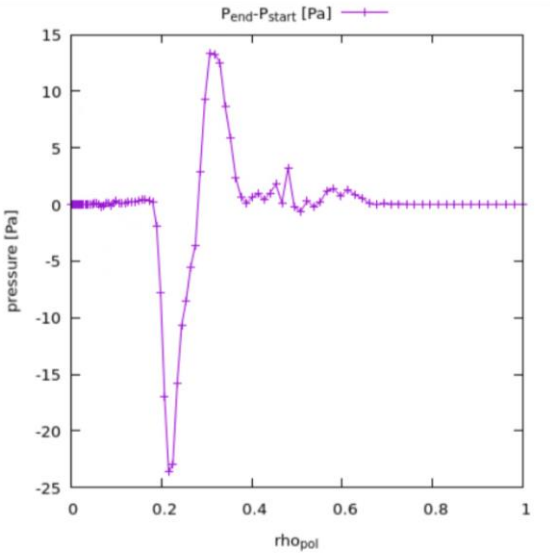




**ITER plasma from H&CD WF by M. Schneider**

# PSZS moments

- Mapping to 1d profile of PSZS;
- The CGL equilibrium can be readily constructed Falessi et al 2023 sub;
- transport is zonal by construction;
- Next step is the calculation of the corresponding magnetic equilibrium;




**ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022**