



# Advanced transport models for energetic particles

20th European Fusion Theory Conference, 2.-5. October 2023, Padova, Italy

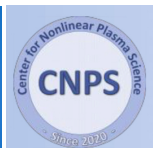
ATEP team:

Philipp Lauber (PI), Matteo Falessi (Co-PI), Alessandro Biancalani, Sergio Briguglio, Alessandro Cardinali, Nakia Carlevaro, Valeria Fusco, Thomas Hayward-Schneider, Florian Holderied, Axel Könies, Yang Li, Yueyan Li, Guo Meng, Alexander Milovanov, V.-Alin Popa, Stefan Possanner, Gregorio Vlad, Xin Wang, Markus Weiland, Alessandro Zocco, Fulvio Zonca

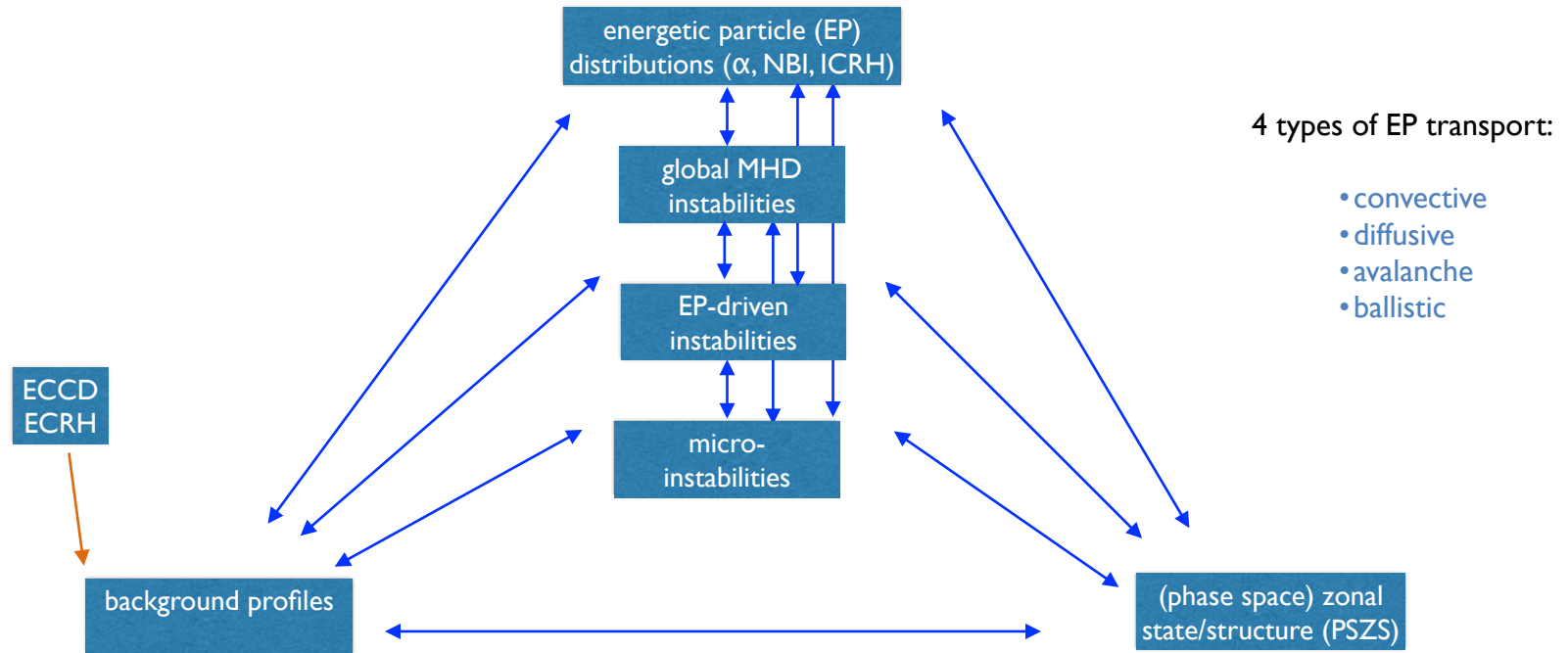
and A. Bottino, M. Schneider, S.D. Pinches, O. Hoenen, TSVV10 team, ASDEX Upgrade team



MAX-PLANCK-INSTITUT  
FÜR PLASMAPHYSIK



This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.



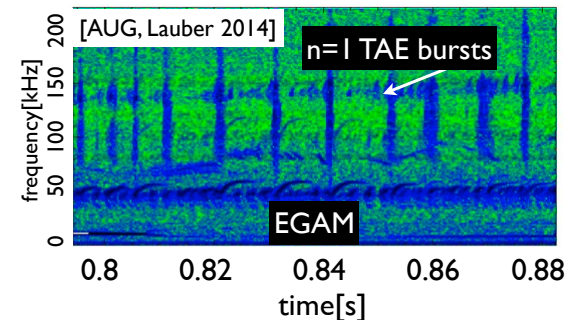
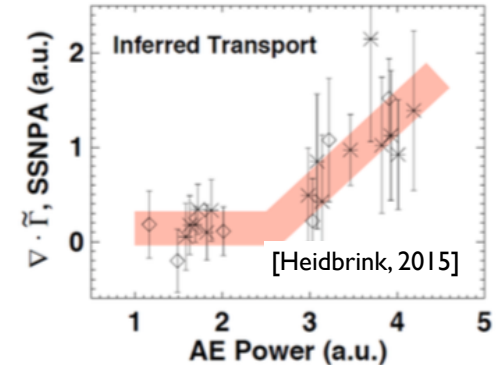
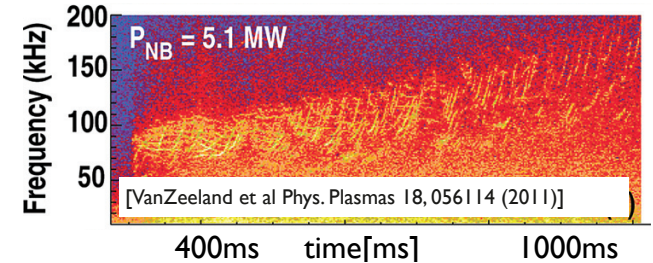
- final goal: predicting the self-organisation of a burning plasma
- challenge: complex interdependence on vastly different spatial and temporal scales

# EP transport: experiment



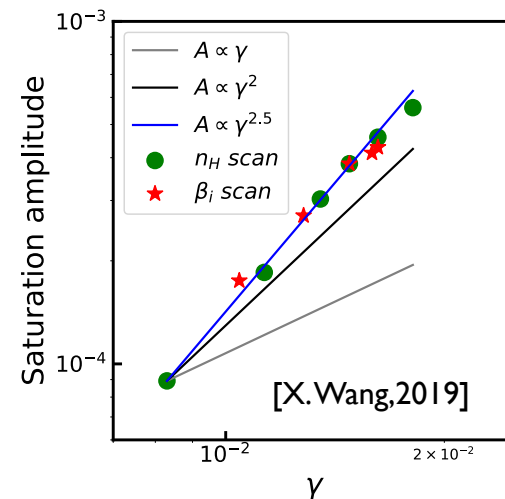
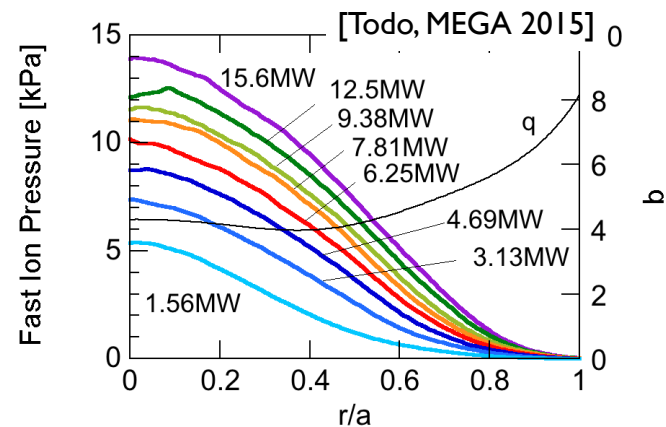
- for multiple overlapping Alfvén eigenmodes (AEs) resonances: stiff EP transport found at DIII-D [Collins, Heidbrink 2015-2018 ], as predicted by QL theory [Sagedeev&Galeev, Kaufman 1972, ...]; high q, large orbits, dominated by losses rather than redistribution
- in JET re-deposition of EPs (ICRH) was observed: core-localised TAEs redistribute EPs, redistributed EPs drive edge-TAE [Nabais et al, PPCF 2019]
- in ITER, both core and edge TAEs are weakly damped and can be driven non-linearly [Pinches, Lauber, Schneller 2014/2015, T Hayward 2019, ORB5]
- mode chirping and avalanches-type events found in many experiments [Kusama, Shinohara, J-T-60U 1999+]
- bursting, non-linear mode-mode couplings and EP transport (FIDA, INPA) measured in ASDEX Upgrade EP super-shots [Lauber 2014+], .i.e. further development of AUG NLED benchmarks case [Vlad 2020-2023, Vannini 2019, Rettino 2021-23]

for a comprehensive review please refer to dedicated review articles, e.g.  
[NF ITPA special issue 2006, update 2023/24, Heidbrink 2008, Breizman& Sharapov 2011, Lauber 2013, Chen&Zonca RMP 2015, Gorelenkov&Pinches Toi 2014, Todo 2019, ...]



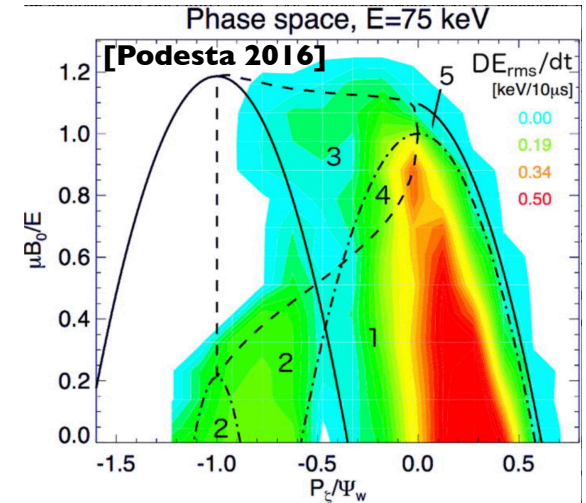
- MHD-hybrid simulations of DIII-D case: transport due to steady and increasingly intermittent EP fluxes for higher power [MEGA, Todo 2015]
- multi-phase MEGA simulations for TAE- avalanches at JT-60U [Bierwage 2016,17];
- at increased EP pressure, so-called energetic particle modes start to exist: simulations as pioneered by (X)HMGC and HYMAGYC teams [Briguglio PoP1998, Bierwage 2012-16]; many dedicated diagnostics developed for phase space analysis
- chirping AE/EPs: XHMGC simulations [X.Wang, S. Briguglio, 2021]: AE saturation level (and thus related EP transport) due to chirping modes is larger than standard quadratic scaling:  

$$A \sim \gamma^{2.5}$$
- global GENE and GTC simulations highlight interaction with micro-turbulence [Citrin, diSiena 2019-2023, Brochard 2021-23]
- global ORB5 simulations with increasing complexity start to capture experimentally relevant regimes [A. Biancalani, T Hayward-Schneider, A. Bottino, F.Vannini, B. Rettino 2013-2023] and compare in with MHD-hybrid results [Vlad 2020-23]
- **difficult to disentangle various non-linearities in comprehensive codes- verify results?**
- **transport-time scales?**
- **vast parameter regime - sensitivity scans ?**
- **how to reduce to reasonably fast models?**





- diffusion coefficients for impurity transport by background turbulence, no e.m. EP-driven modes [Angioni 2009, Püschel, etc]
- critical gradient model [R. Waltz, E. Bass, 2014 -2023]: use local AE stability threshold, add upshift of transport threshold using  $(ExB)_{\text{turb}}$  shearing rate; above threshold set  $D_{EP}$  to ad hoc values [e.g.  $10\text{m}^2/\text{s}$ ] to clamp EP's radial gradient to critical value
- kick model [M. Podesta, 2014-2022]: calculate probability density function of kick in  $P_z$  and  $E$  for given amplitude
- RBQ model, 1D, 2D [N. Gorelenkov 2015-2022]: use resonance broadening QL theory connected to NOVA-K to evolve mode amplitude consistently with evolution of  $F_{EP}$
- gyrofluid model [D Spong, 2019-2022], TAEFL code: fluid closures simplify problem, runs on longer time scales
- GENE-Tango model [A. di Siena, 2022-23]: relies on global kinetic GENE runs + power balance
- transport models as derived from general non-linear gyrokinetic theory [Chen, Zonca RMP 2015, Z. Qiu et al 2017-2023] using phase space zonal structure (PSZS) transport theory [M.-V. Falessi, F. Zonca, 2017-2023] [see talk M. Falessi at this conference](#)



**within Eurofusion Enabling research project ATEP: based on general theoretical framework, develop and implement hierarchy of (reduced) phase space zonal structure (PSZS) transport models**

# ingredients for reduced energetic particle (EP) transport models:



needed for scaling from TCV-AUG-JET, W7X... to JT-60SA-DTT-ITER-DEMO, in particular burning plasmas

## required models:

4. self-organisation - back reaction of EP transport on profiles and background transport

non-linear/quasi-linear global kinetic e.m.+ background transport

3. EP transport and losses

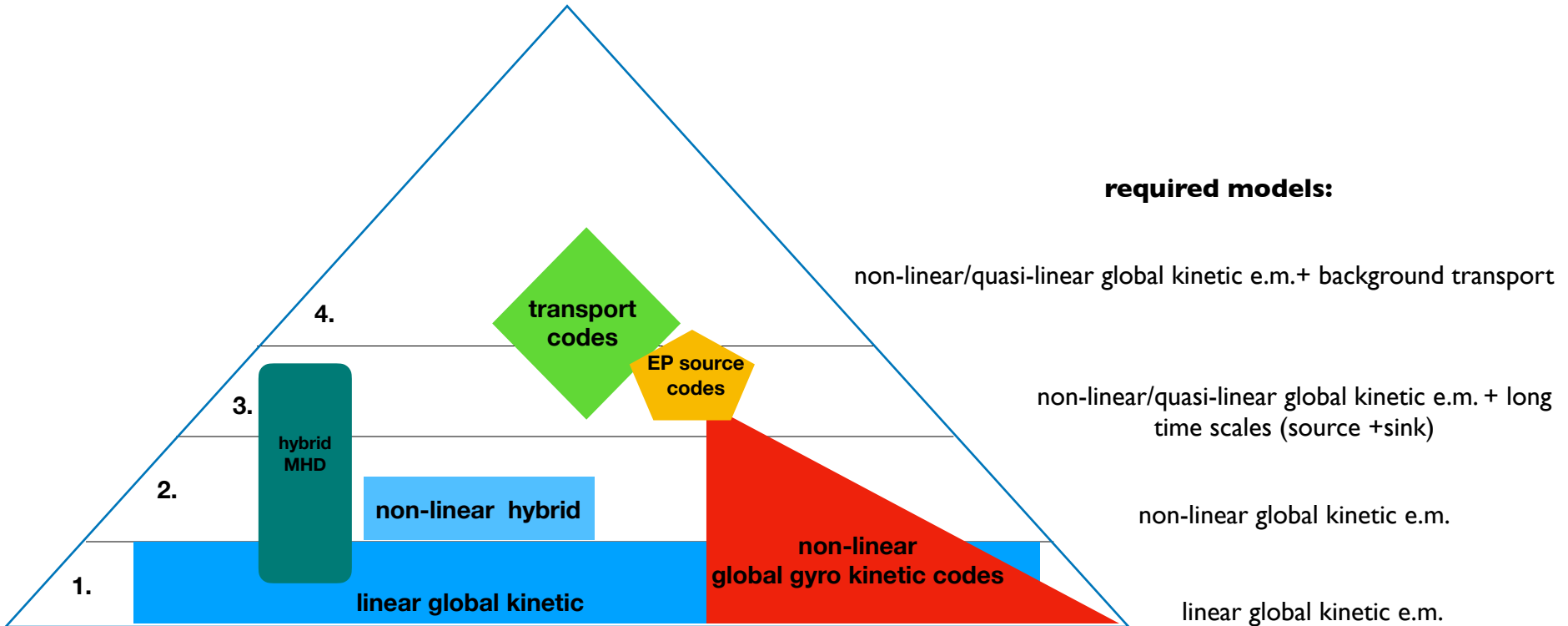
non-linear/quasi-linear global kinetic e.m. + long time scales (source +sink)

2. non-linear mode evolution, saturation mechanisms

non-linear global kinetic e.m.

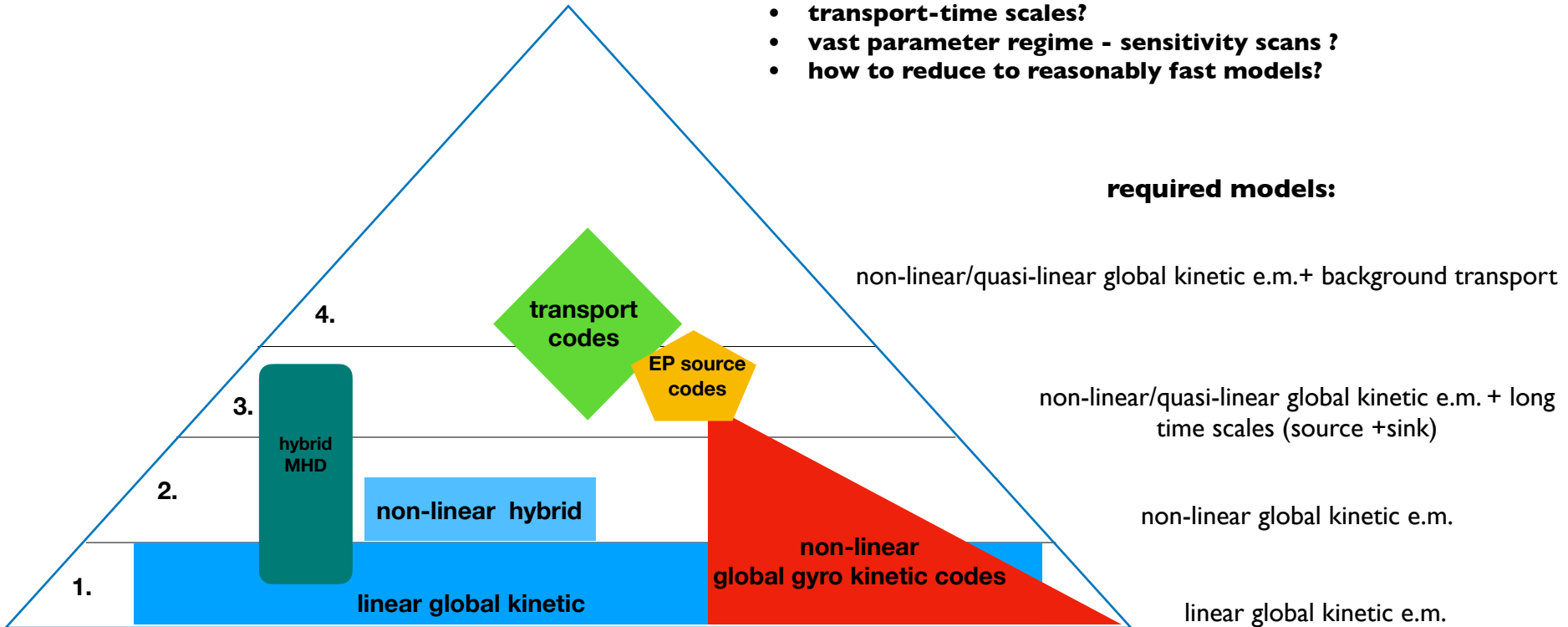
1. mode stability

linear global kinetic e.m.





- **difficult to disentangle various non-linearities in comprehensive codes- verify results?**
- **transport-time scales?**
- **vast parameter regime - sensitivity scans ?**
- **how to reduce to reasonably fast models?**



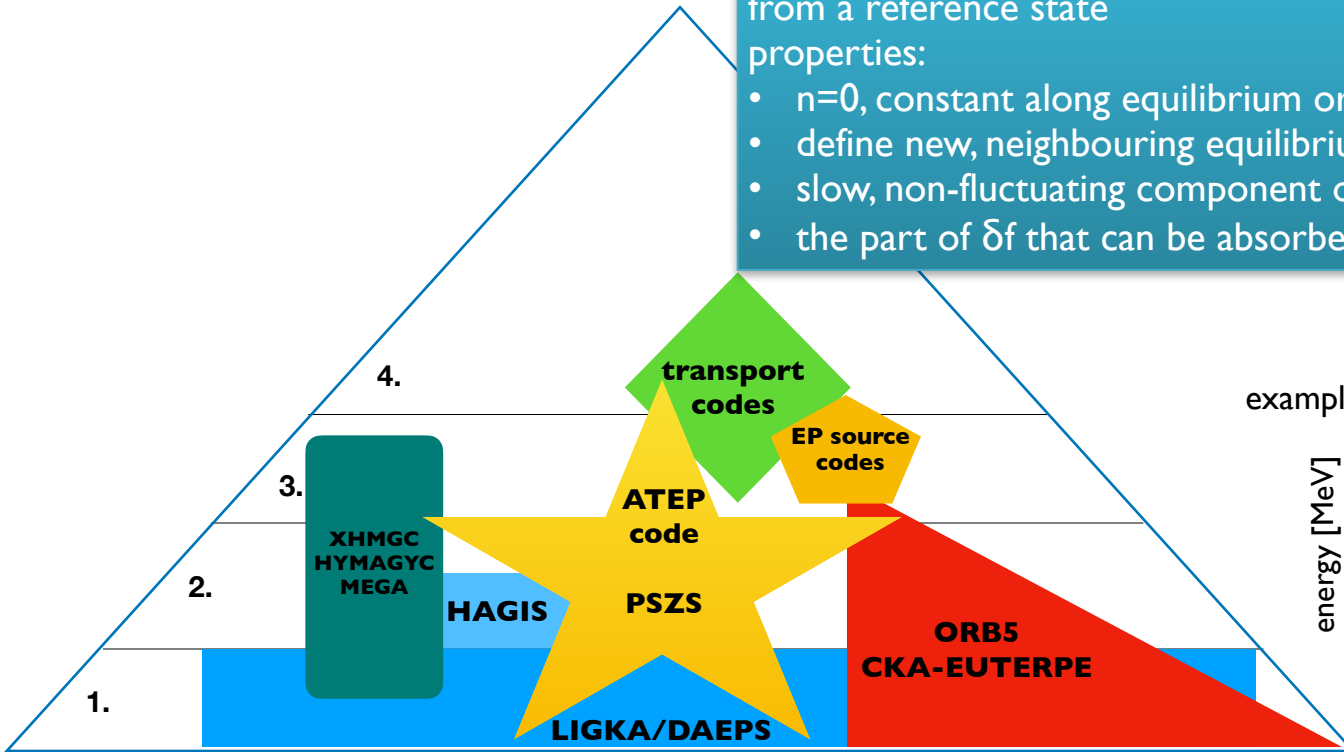




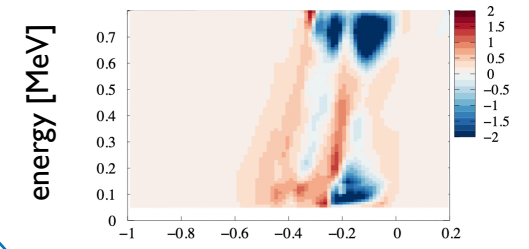
phase space zonal structures (PSZS) are collision-less undamped, long-lived nonlinear deviations of the plasma from a reference state

properties:

- $n=0$ , constant along equilibrium orbits
- define new, neighbouring equilibrium reference state
- slow, non-fluctuating component of  $F_{EP}$  evolution
- the part of  $\delta f$  that can be absorbed in new  $F_0$



example in CoM space representation:



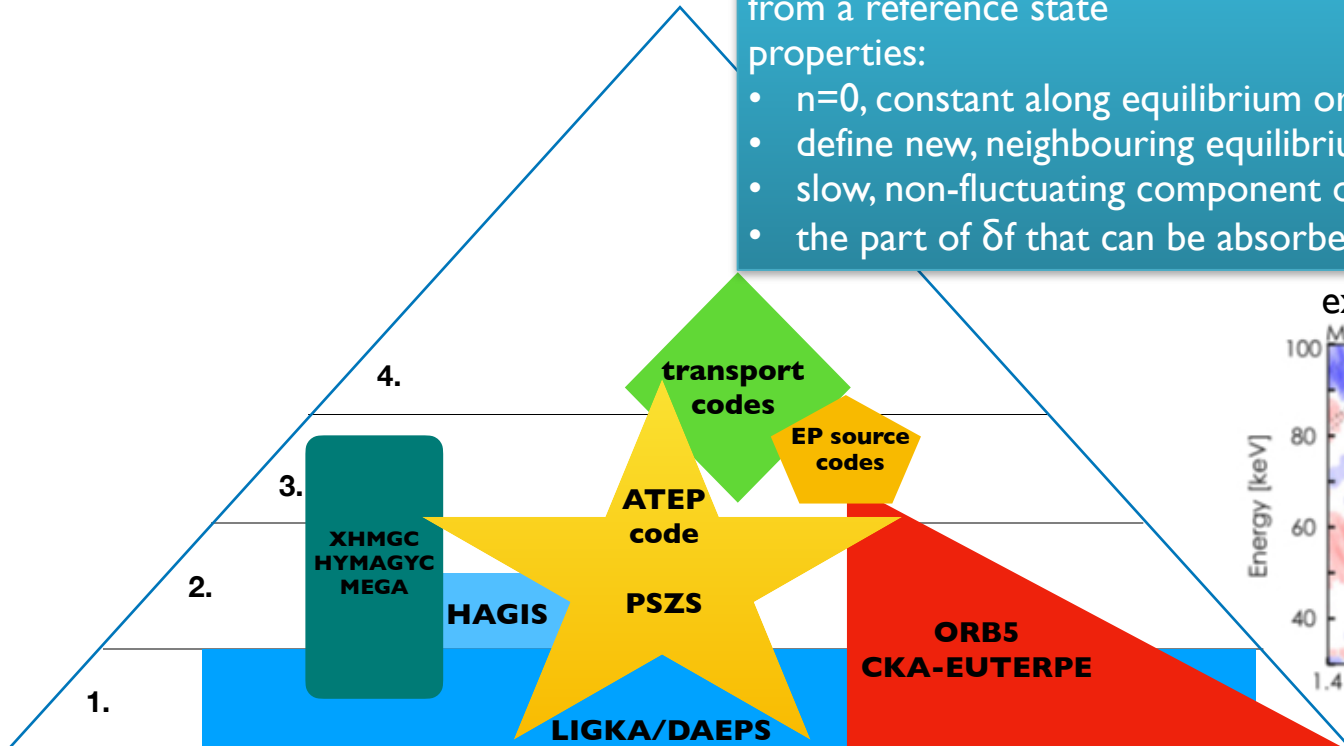
canonical toroidal momentum  $P_\phi$



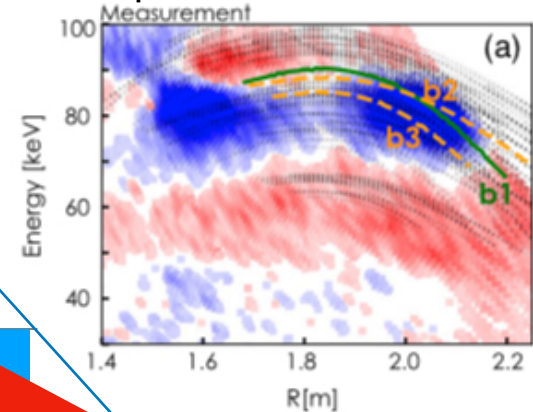
phase space zonal structures (PSZs) are collision-less undamped, long-lived nonlinear deviations of the plasma from a reference state

properties:

- $n=0$ , constant along equilibrium orbits
- define new, neighbouring equilibrium reference state
- slow, non-fluctuating component of  $F_{EP}$  evolution
- the part of  $\delta f$  that can be absorbed in new  $F_0$



experiment:



DIII-D, INPA [Du et al PRL 2021]  
 AUG, INPA J. Rueda [FEC 2023]

theory: explicit calculation of PSZs for specific cases (see talk M. Falessi)



- PSZS theory and overall implementation strategy
- general distribution functions in constants of motion space (CoM)
- linear mode spectrum: the Energetic Particle Stability Workflow (EP-WF)
- phase space transport coefficients
- evolve transport equation in kick model and quasi-linear (QL) limit
- back mapping to real space and non-linear equilibria
- verification and validation - common effort of ENR ATEP team



$$\frac{\partial \overline{F_{z0}}}{\partial t} + \frac{1}{\tau_b} \left[ \frac{\partial}{\partial P_\phi} \left( \overline{\tau_b \delta \dot{P}_\phi \delta F} \right)_z + \frac{\partial}{\partial \mathcal{E}} \left( \overline{\tau_b \delta \dot{\mathcal{E}} \delta F} \right)_z \right]_S = \left( \sum_b C_b^g [F, F_b] + \mathcal{S} \right)_{zS}$$

$\nabla_z \cdot \Gamma$

wave-induced phase space flux

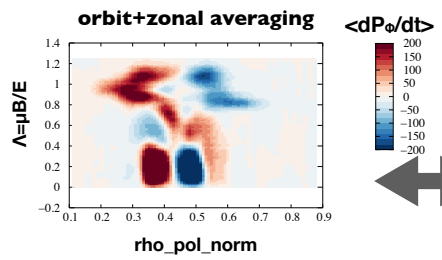
collisions + source

[M-V. Falessi, F. Zonca, 2017-2023]  
 continuity equation in phase space;  
 valid for single or multiple modes  
 in general valid for all regimes;  
 interactions with background fluctuations  
 can be consistently kept

- kick limit: fix perturbations amplitudes for calculating  $\langle dP_\phi/dt \rangle$  and evolve continuity equation in CoM space
- in the QL limit, assuming overlapping resonances, flux can be split into convective and diffusive component [L Chen, JGR 104, 1999]
- diffusion coefficients can be evaluated by determining  $D_{P_\phi P_\phi} = |dP_\phi/dt|^2 \tau_{ac}$ , similar for  $D_{EE}$ , and off diagonal terms (if present), resonant and non-resonant contributions can be separated
- in [L Chen, JGR 104, 1999] also the importance of  $E_{||}$  is discussed (KAW physics), leading to additional convective flux contributions (linear GK code LIGKA provides this information - see below)



# implementation: QL limit

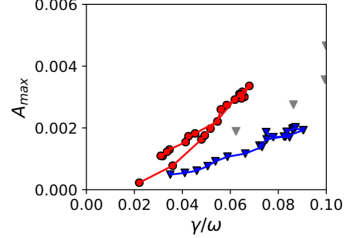


**calculate PSZS**

**PSZS transport theory [M. Falessi 2017-2022]**

$$\frac{\partial \overline{F_{z0}}}{\partial t} + \frac{1}{\tau_b} \left[ \frac{\partial}{\partial P_\phi} \left( \overline{\tau_b \delta \dot{P}_\phi \delta F} \right)_z + \frac{\partial}{\partial \mathcal{E}} \left( \overline{\tau_b \delta \dot{\mathcal{E}} \delta F} \right)_z \right]_S = \left( \sum_b C_b^g [F, F_b] + S \right)_{zS}$$

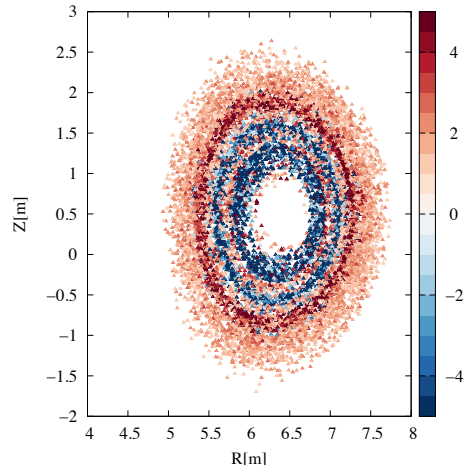
**saturation rule/energy conservation (HAGIS model)**



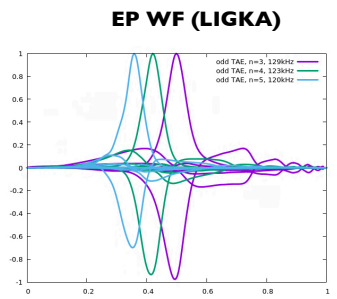
**use NL code/model for intensity closure**

**calculate  $D(r,E)$**

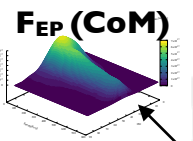
**advance  $F_{EP}$  and return updated distribution IDS, or its moments**



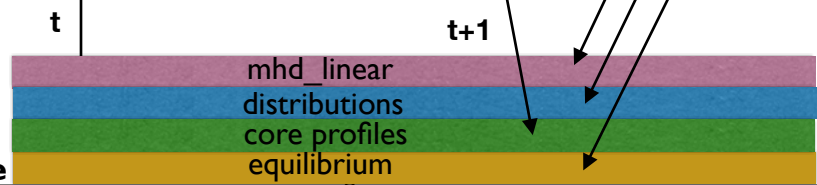
**newly developed ATEP/ ATEP-3D code [Ph. Lauber, G. Meng, 2022-23]**



**calculate linear mode spectrum**



**transport code**





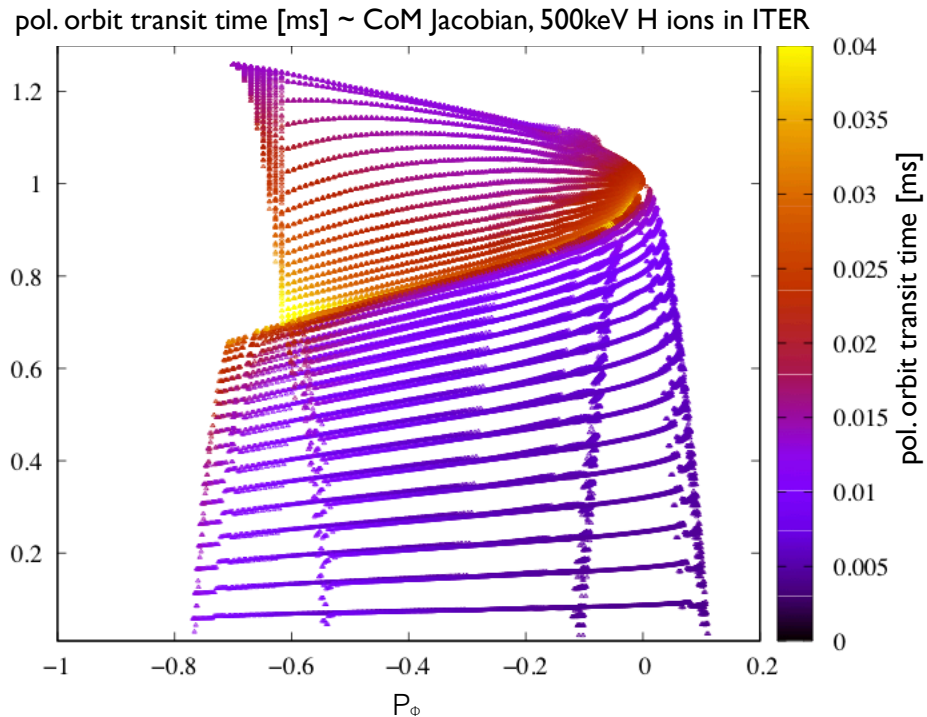
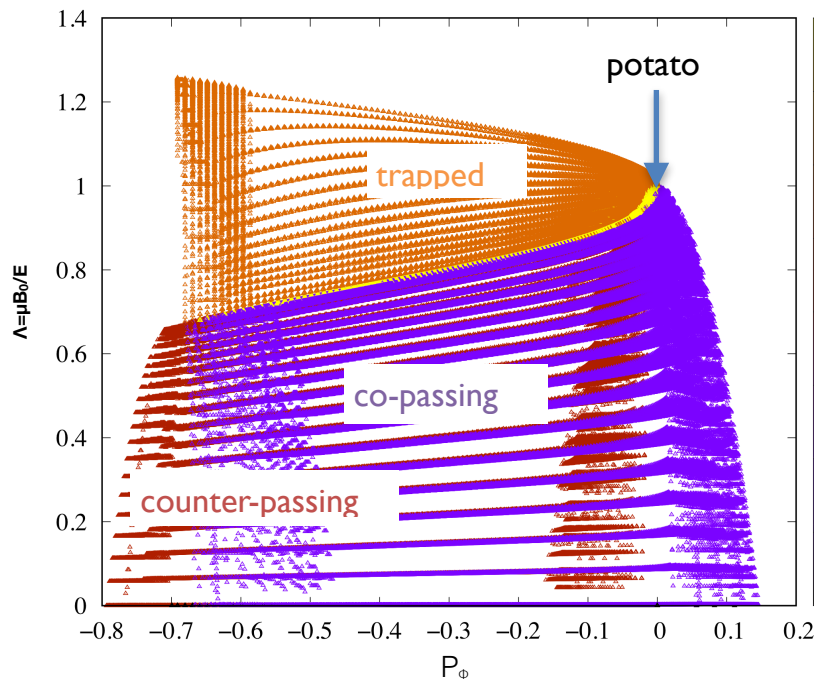
## **determining $F_{EP}$ in constants of motion space (CoM)**

# determining $F_{EP}$ in constants of motion space (CoM)



several recent papers [Bierwage 2022, G. Brochard, FEC 2023, Salewski 2020-21] using a similar procedure:

- establish orbit database to classify particles
- determine CoM Jacobian ( $P_\phi, E, \Lambda, \mu B_0/E, \sigma$ )
- set up grid in CoM space
- bin markers as given by neoclassical physics codes [NEMO/Spot, ASCOT, RABBIT, etc...], here ITER H-pre-fusion case 100015, I [M. Schneider, 2018]





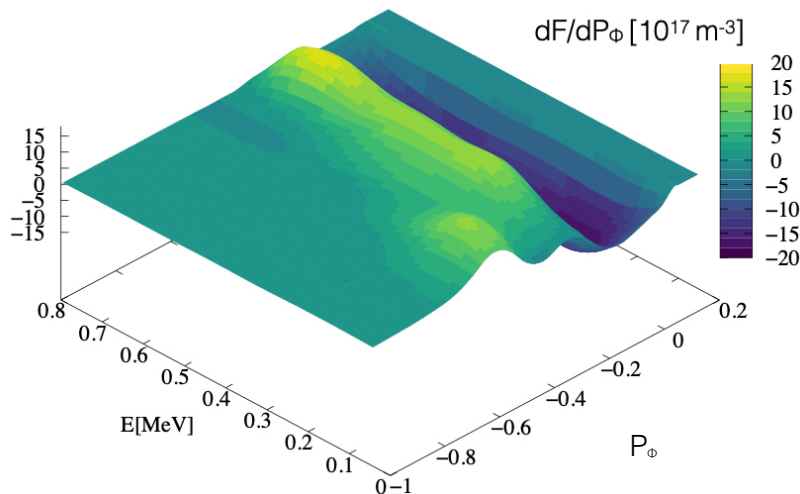
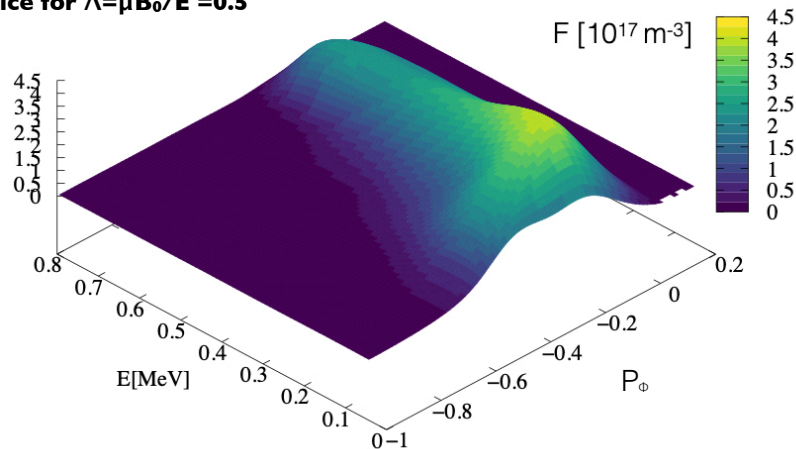
# determining $F_{EP}$ in constants of motion space (CoM)



several recent papers [Bierwage 2022, G. Brochard, FEC 2023] using a similar procedure:

- establish orbit database to classify particles
- determine CoM Jacobian
- set up grid in CoM space
- bin markers as given by neoclassical physics codes [NEMO/Spot, ASCOT, RABBIT, etc...], here ITER H-pre-fusion case 100015, I [M. Schneider, 2018]
- use 2D cubic splines in each sub-space to create fine sub-grids, then create 3D spline for  $F_{EP}$
- back-transform in other coordinate systems possible, if needed
- here, all calculations are using IMAS interfaces (equilibrium, transport code, orbit tracer (HAGIS [S.D. Pinches]), ATEP code)

slice for  $\Lambda = \mu B_0 / E = 0.5$





## **Calculating the mode spectrum**



LIGKA [Qin 1998, Lauber 2003, JPC 2007, Lauber PLREP 2013, Bierwage&Lauber 2017, Lauber JPCS 2018]

- gyrokinetic moment equation (GKM):

shear Alfvén law

<https://git.iter.org/projects/STAB/repos/ligka/>

$$\begin{aligned}
 & - \frac{\partial}{\partial t} \left[ \nabla \cdot \frac{1}{v_A^2} \nabla_{\perp} \phi \right] + \mathbf{B} \cdot \nabla \frac{\nabla \times (\nabla \times (\frac{\nabla \psi}{i\omega})_{\parallel} \mathbf{b})}{B} + (\mathbf{b} \times \nabla (\frac{\nabla \psi}{i\omega})_{\parallel} \mathbf{b}) \cdot \nabla \frac{\mu_0 j_{\parallel}}{B} \\
 & = - \sum_a \mu_0 \int d^2v e_a \{ \mathbf{v}_d \cdot \nabla J_0 f \}_a + \sum_a \left[ \mathbf{b} \times \nabla \left( \frac{\beta_{a\perp}}{2\Omega_a} \right) \right] \cdot \nabla \nabla_{\perp}^2 \phi \\
 & + \sum_a \frac{3w_{th,a}^2}{8v_A^2 \Omega_a^2} \nabla_{\perp}^4 \frac{\partial \phi}{\partial t} + \mathbf{B} \cdot \nabla \frac{1}{B} \sum_a \frac{\beta_a}{4} \nabla_{\perp}^2 (\frac{\nabla \psi}{i\omega})_{\parallel} \mathbf{b}
 \end{aligned}$$

'pressure' tensor - curvature drift coupling

reduced MHD as limit;  
various successful benchmarks  
with ORB5 [Hayward-Schneider,  
2021-23]

- quasi neutrality (QN):

$$0 = \sum_a e_a \int d^2v \{ J_0 f \}_a + \nabla_{\perp} \cdot \frac{m_i n_i \nabla_{\perp} \phi}{B^2}$$

- non-adiabatic response for perturbed distribution function:

resonances (circ/trapped):

$$\hat{h} = ie \sum_m \int_{-\infty}^t dt' e^{i[n(\varphi' - \varphi) - m(\theta' - \theta) - \omega(t' - t)]} e^{-im\theta}$$

propagator → resonance
→
{

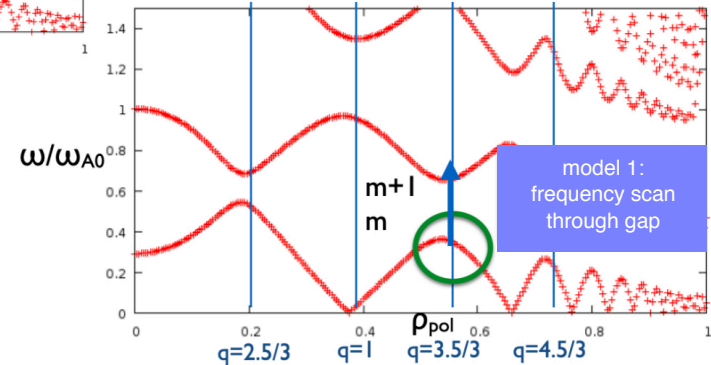
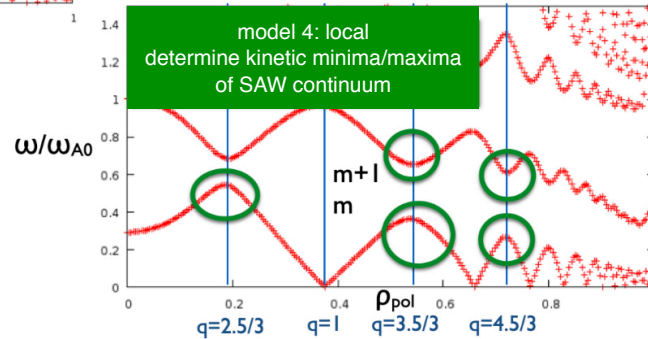
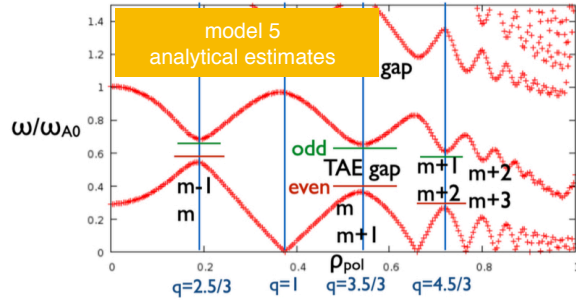
$$\left\{ \begin{aligned} \omega_{AE} - \omega_{prec} - (nq - m + k) \cdot \omega_t &= 0 \\ \omega_{AE} - \omega_{prec} - k \cdot \omega_b &= 0 \end{aligned} \right.$$

$$\frac{\partial F_0}{\partial E} [\omega - \hat{\omega}_*] J_0^2(k_{\perp} \varrho_i) \left[ \phi_m(r') - \left( 1 - \frac{\omega_d(r', \theta')}{\omega} \right) \psi_m(r') \right]$$

free energy

for all species, including electrons and energetic particles

# Linear mode spectrum: Energetic particle stability workflow (EP-WF)





training course & additional material: <https://indico.euro-fusion.org/event/2729/>

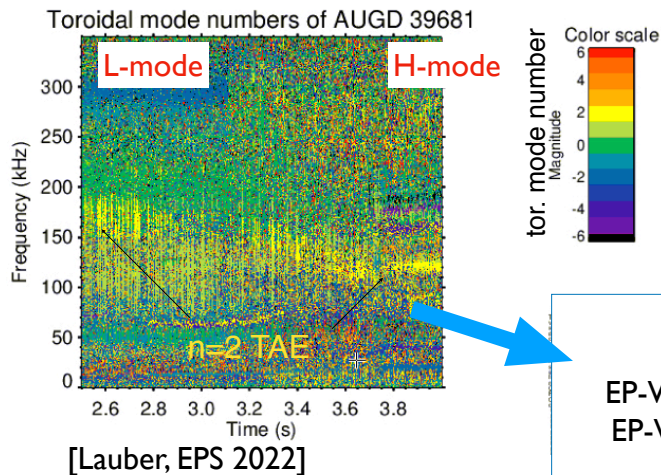
- fully IMAS compatible (python)
- git version control
- module installations available
- gui and non-gui versions
- batch job submission

The screenshot displays the EP Workflow GUI with several panels:

- WORKFLOW PARAMETERS:** Includes fields for user (public), machine (ITER), shot\_nr (130012), run\_in (2), machine\_out, test\_DB, run\_out (10), and itime (15-17.19). It also has buttons for 'Save Configuration', 'Save and Run', 'Save Configuration as', 'Load Configuration', and 'Restore Default'.
- ACTOR SELECTION:** A dropdown menu showing 'Ligka\_m5' selected.
- FURTHER SETTINGS:** Checkboxes for 'ligka\_541', 'ligka\_5412', 'pulse\_list', 'fast\_particles', and 'hd5'. Below are buttons for 'Save Configuration', 'Save and Run', 'Save Configuration as', 'Load Configuration', and 'Restore Default'.
- SCENARIO PARAMETERS (m):** A table of parameters with values set to 1, including r\_E, r\_H, r\_D, r\_T, r\_Be, r\_C, r\_Ne, r\_He4\_ash, r\_He4\_EP, T\_E, T\_H, T\_D, T\_T, T\_Be, T\_C, T\_Ne, T\_He4\_ash, and T\_He4\_EP.
- Bulk Ions:** Parameters for H (0.02), D (0.02), T (0.02), and Ar (0.02).
- Impurities:** Parameters for Be (0.02), Ne (0.02), He4 (0.02), C (0.02), Tu (0.02), and Ar (0.02).
- Fast Ions:** Parameters for H (0.001), D (0.001), and He4 (0.001).
- LIGKA PARAMETERS:** A list of parameters such as modus (5), min\_n\_tor (10), max\_n\_tor (10), min\_m (11), max\_m (11), sidebands (5), sidebands\_asy (2), mode\_type (1), even (0), cosp (1), start\_pos (1), force\_m (false), npsi\_out (256), kr\_read (0.050), q0 (0.050), rad\_start (0.050), rad\_end (1.050), and offset\_d (0.050).
- IDS Merge:** A panel with 'Inputs' and 'Settings' sections. Inputs include user\_in\_1, machine\_in\_1, shot\_in\_1, run\_in\_1, HD5\_1, user\_in\_2, machine\_in\_2, shot\_in\_2, run\_in\_2, HD5\_2, and 'Output' fields for machine\_out, shot\_out, run\_out, and HD5\_out. Settings include itime (15-17.19) and Equilibrium\_copy.

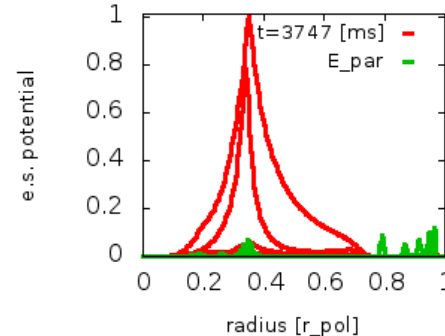
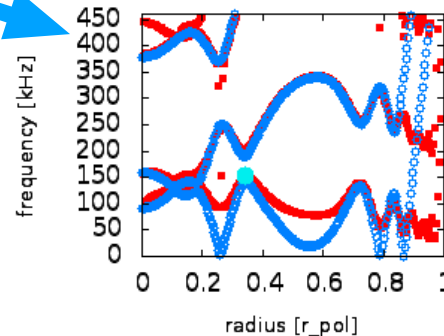
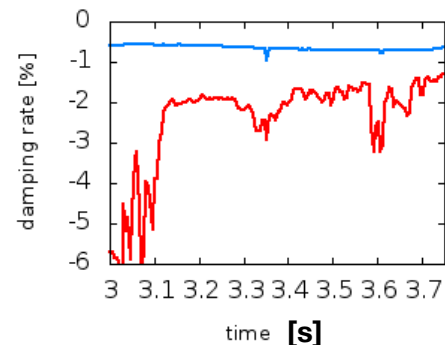
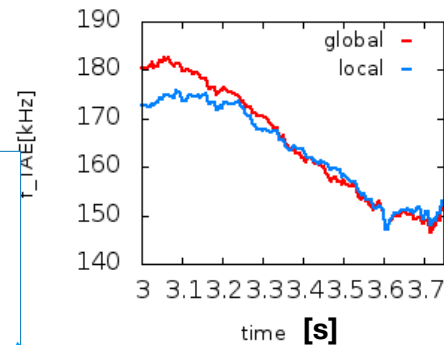


AUG EP 'supershot' scenarios: D NBI into D plasma  
(further development of NLED AUG benchmark case)

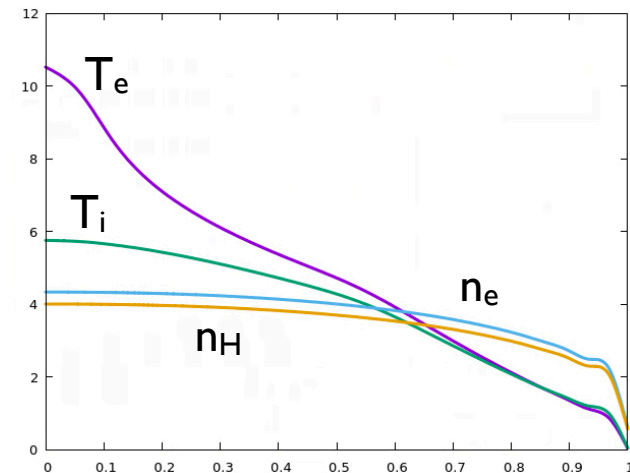


IDA +  
TRVIEW +  
EP-WF: LIGKA local +  
EP-WF: LIGKA global

- analyse L-mode, H-mode and transition phase using
- systematic uncertainty quantification feasible
- bursty and steady-state phases visible, in agreement with damping analysis and drive
- speed up WF using ML methods [V.-A. Popa; in preparation]

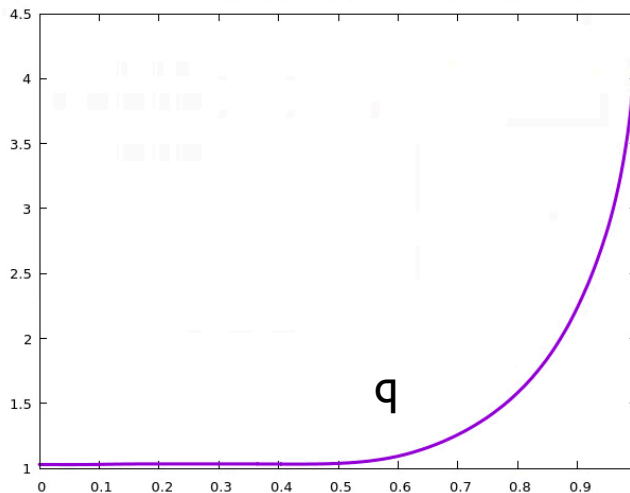


# ITER pre-fusion H scenario 100015, I [Metis, M. Schneider NF (2021)]



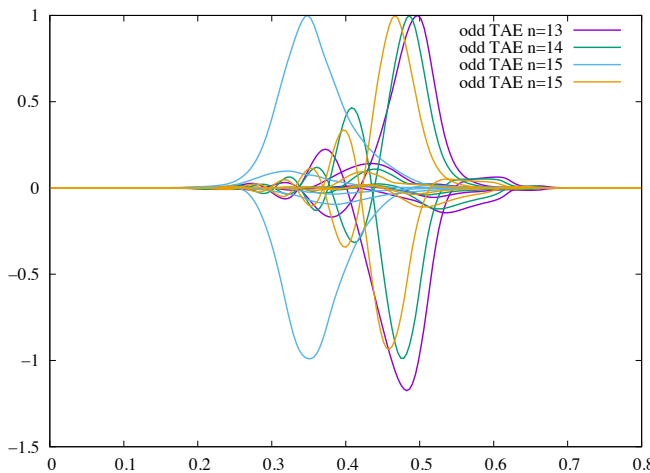
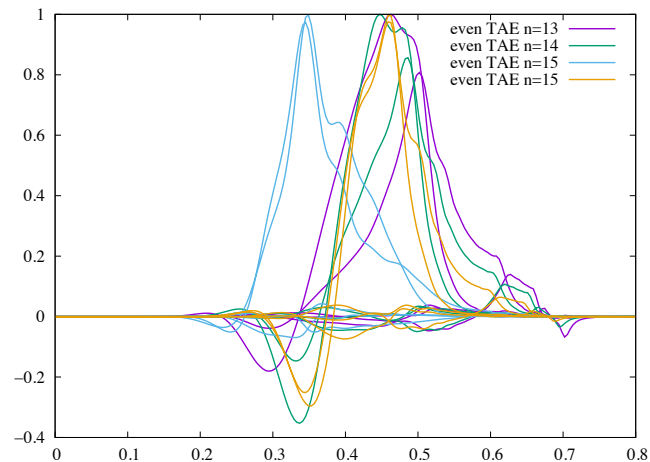
use set of mid-n TAEs  
as test case  
(as all data is consistently  
available in  
IMAS thx to M. Schneider)

$B = -5$  T  
 $I = -1.8$  MA



small damping rates:  
1-5%

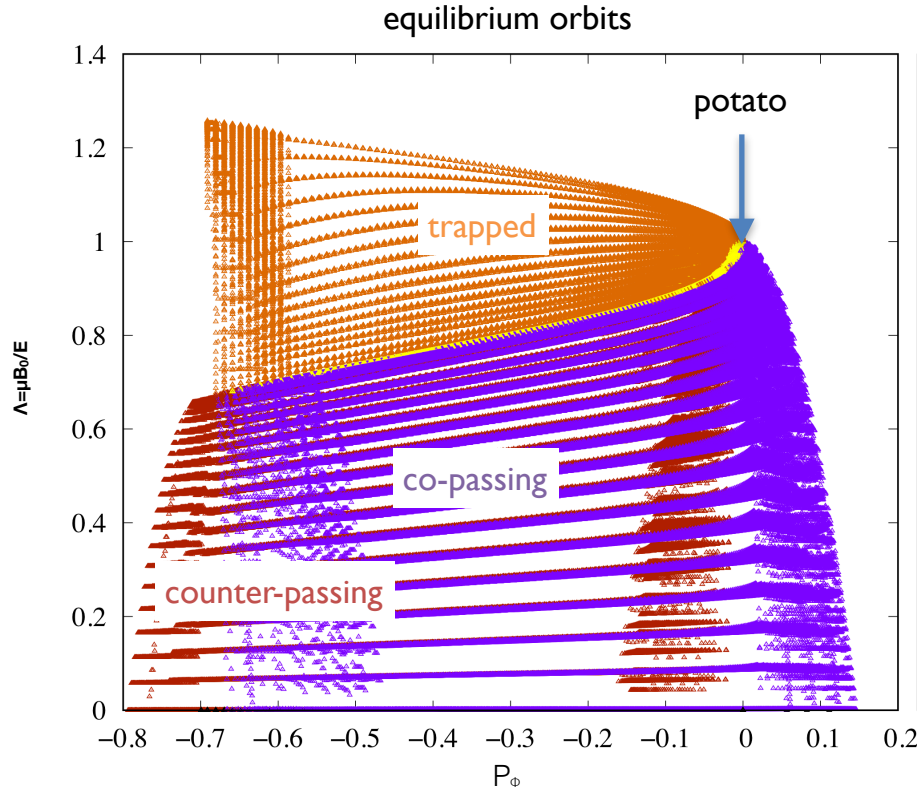
with H beam (870 keV) marginally  
unstable





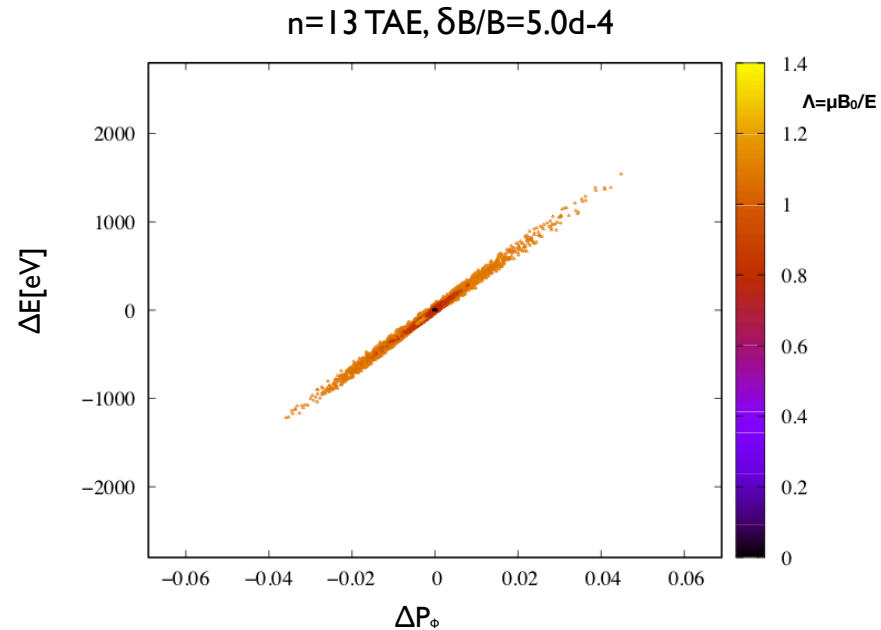
**determine phase space transport coefficients**





use wrapper ('finder') for HAGIS [S.D. Pinches]  
to efficiently set up marker space

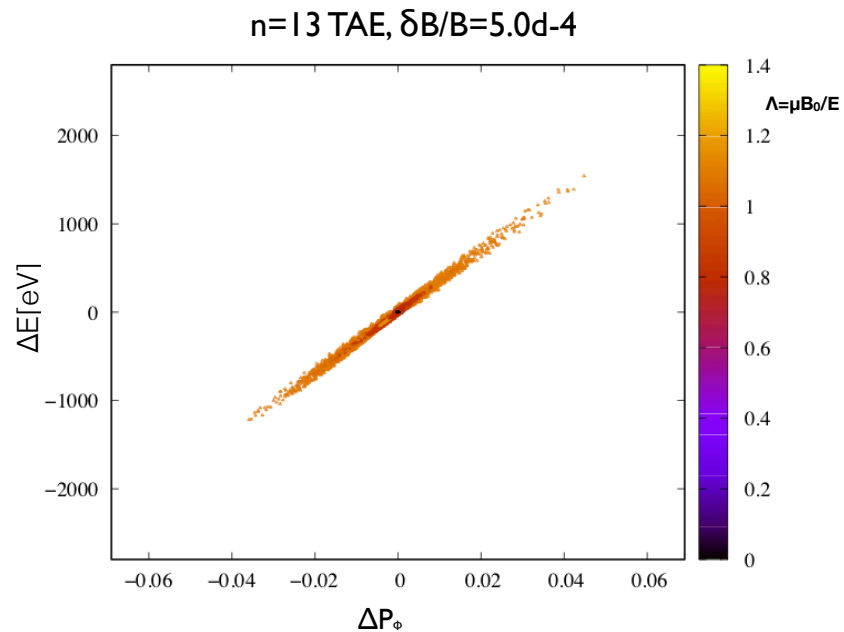
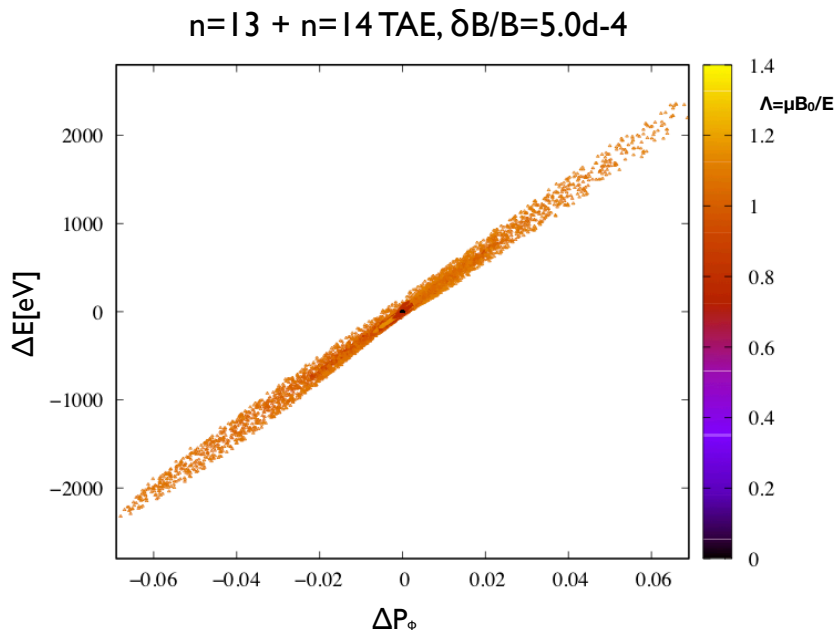
with perturbation:  $dP/dt$  and  $dE/dt$  of resonant particles  
in single wave are proportional to each other



off-diagonal elements play a role  
in multi-mode cases, or with  $E_{//}$



with perturbation:  $dP/dt$  and  $dE/dt$  of resonant particles in single wave are proportional to each other [Southwood, 1969]

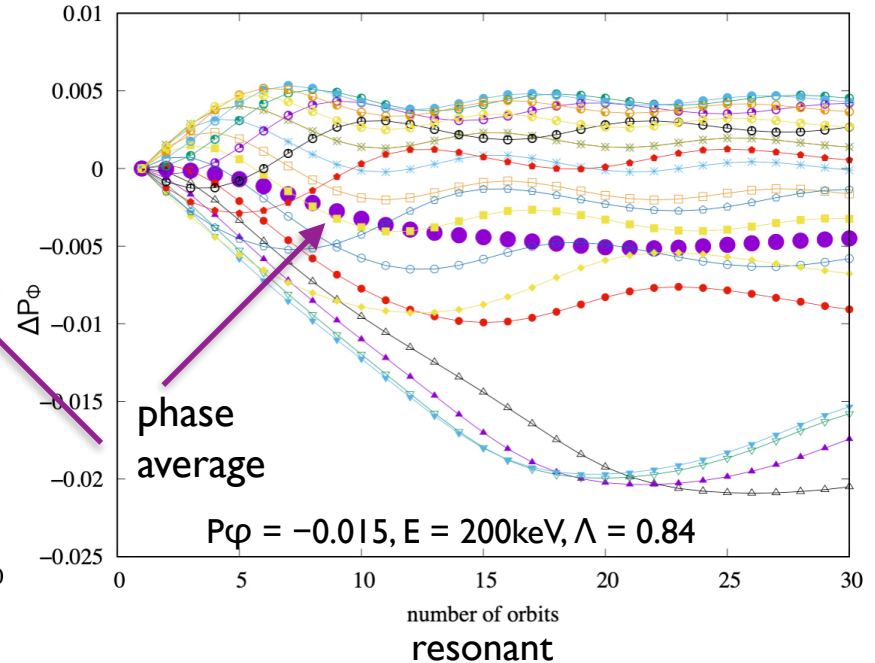
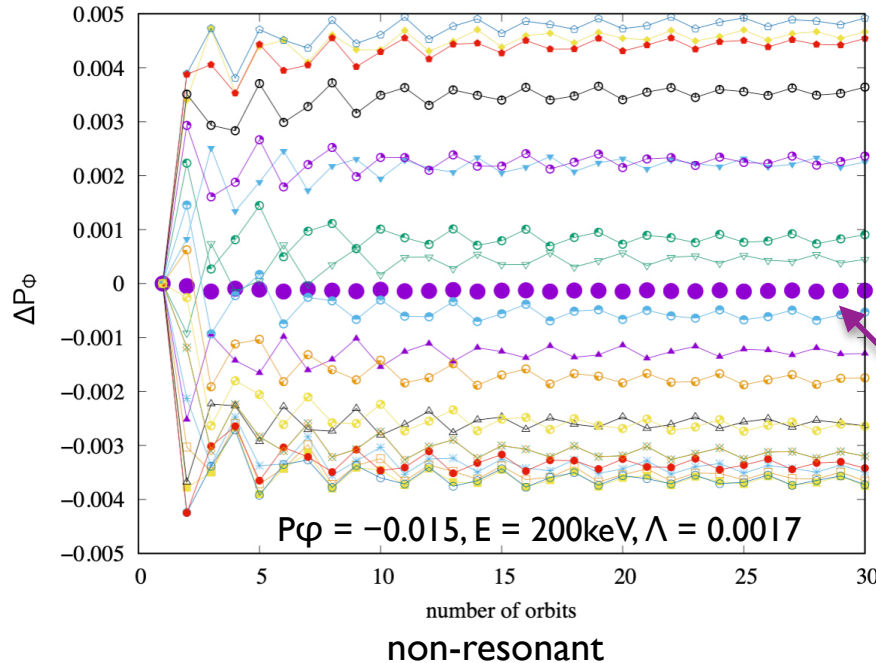


off-diagonal elements play a role in multi-mode cases, or with  $E_{//}$

# zonal and orbit averaging



start particles with different phase shifts with respect to wave: ( $2\pi/n$ , or random), follow typical 3-5 orbits to account for higher resonances, then average ( $n=13TAE$ ;  $\delta B/B = 5 \cdot 10^{-4}$ )



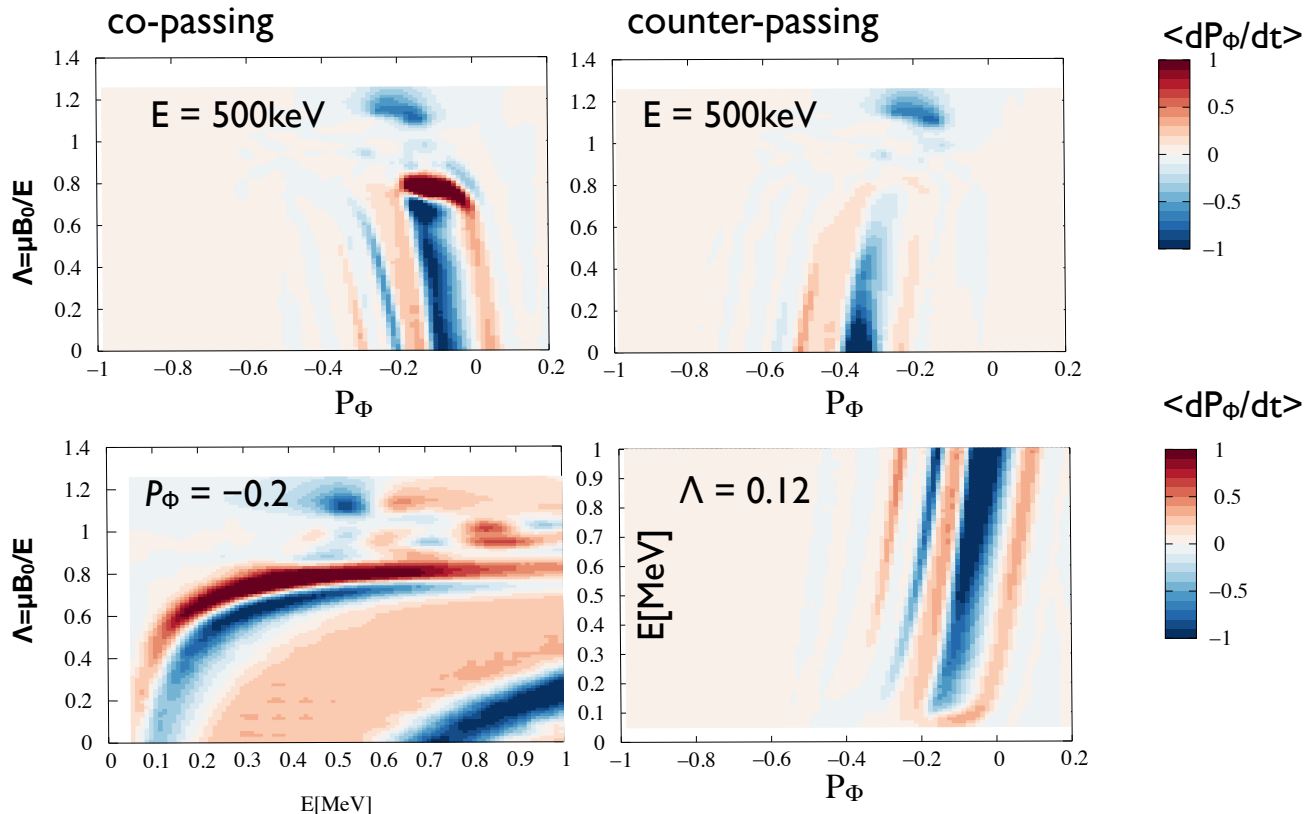
caveat: this procedure is reducing the full dynamics: valid in small-amplitude/QL/limit, transport time scales can be improved, relaxed if needed (ballistic transport cases);  
note also close relation to  $P_\phi$  grid resolution/Courant criterion; accounts for resonance broadening consistently

# zonal and orbit averaging: $\langle dP_\phi/dt \rangle$ in CoM representation

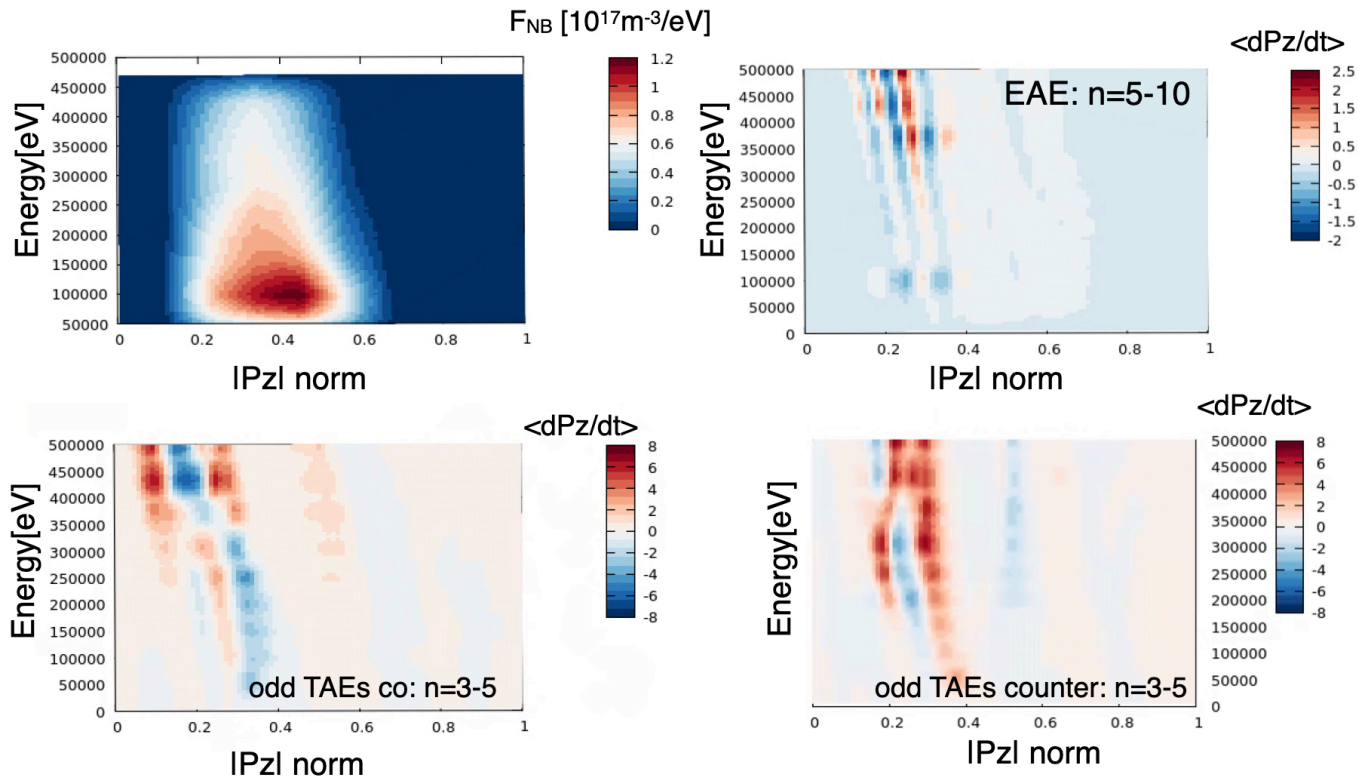


$$\delta B/B = 5 \cdot 10^{-6}$$

- typically follow 128x40x40x4 markers
- store in IDS (distributions)
- use multi-level spline interpolation [Lee 1997]
- use cartesian grid in CoM space (96x96x96)



# PSZS for EAEs and odd TAEs



all plots for  $\Lambda = \mu B_0 / E = 0.24$

resonances with both positive and negative gradients of  $F_{EP}$  possible



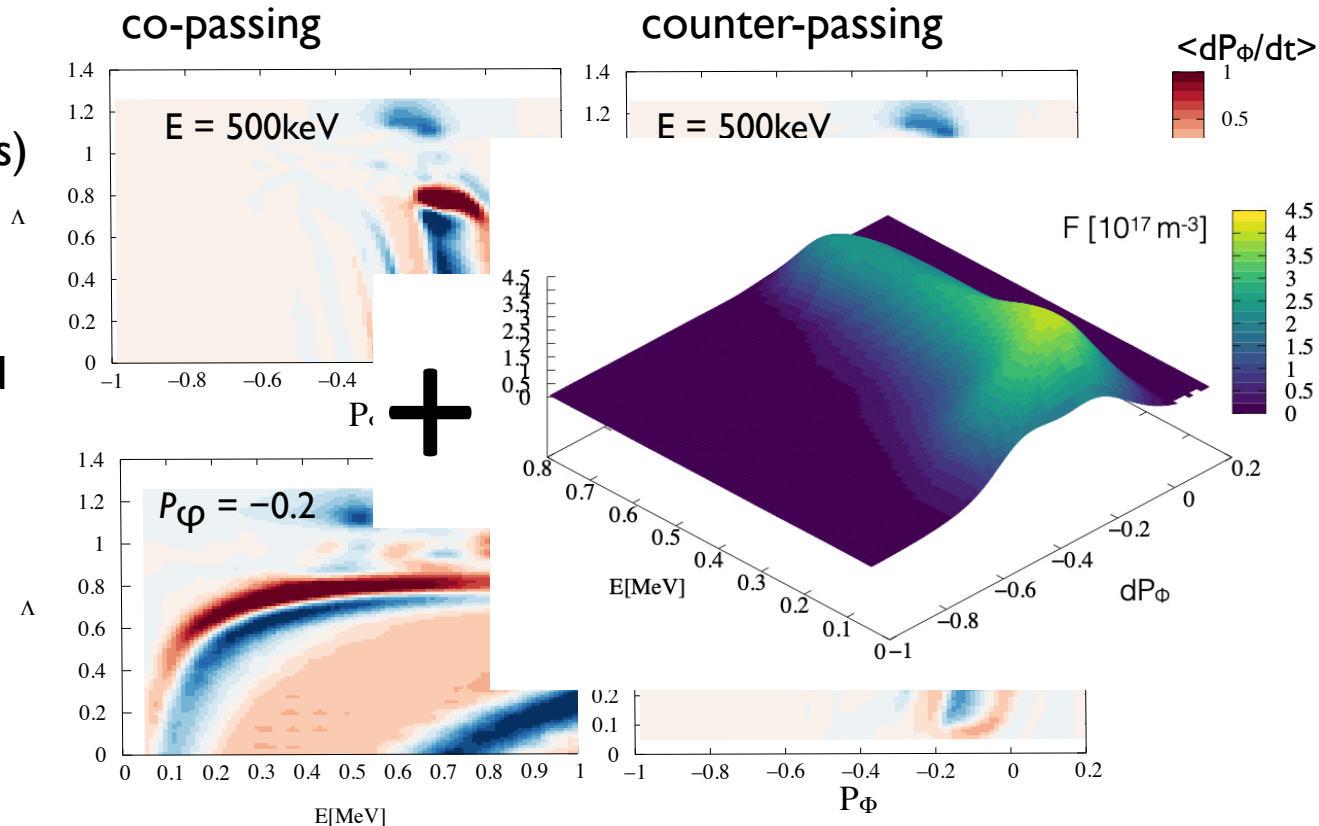
## **evolve transport equations in kick-model limit**

# zonal and orbit averaging: $\langle dP_\phi/dt \rangle$ in CoM representation



$$\delta B/B = 5 \cdot 10^{-6}$$

- typically follow 128x40x40x4 markers
- store in IDS (distributions)
- use multi-level spline interpolation [Lee 1997]
- use cartesian grid in CoM space (96x96x96)



# evolve continuity equation for $F_z$ in CoM space

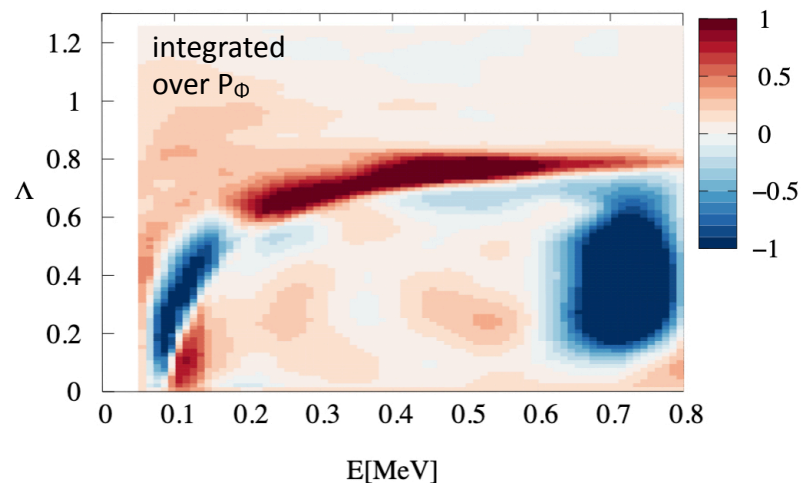
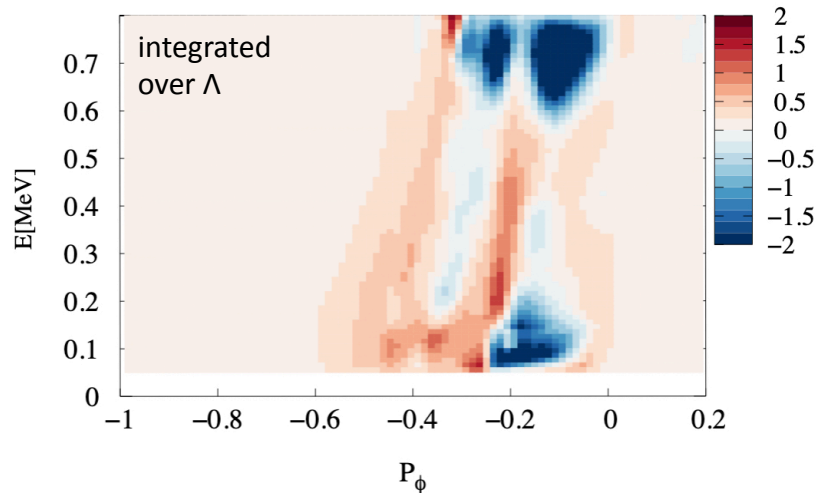


$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi} \left( \left\langle \frac{dP_\phi}{dt} \right\rangle F_z \right) - \frac{\partial}{\partial E} \left( \left\langle \frac{dE}{dt} \right\rangle F_z \right) \quad \mathbf{v}_{P_\phi, E} = \left( \left\langle \frac{dP_\phi}{dt} \right\rangle, \left\langle \frac{dE}{dt} \right\rangle \right)$$

advection equation, assuming  $\nabla \cdot \mathbf{v}_{P_\phi, E} = 0$  i.e. incompressible phase space flow is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

$$\delta F_{EP} = F_{EP}(t = 700\text{ms}) - F_{EP}(t = 0) [10^{16} \text{m}^{-3}]$$

with constant  $\delta B(t)/B = 10^{-5}$





# evolve continuity equation for $F_z$ in CoM space



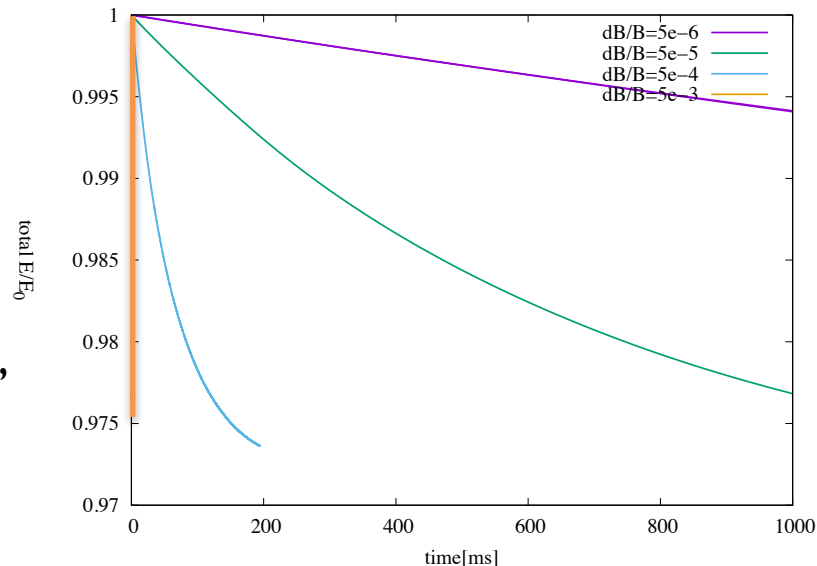
$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi} \left( \left\langle \frac{dP_\phi}{dt} \right\rangle F_z \right) - \frac{\partial}{\partial E} \left( \left\langle \frac{dE}{dt} \right\rangle F_z \right) \quad \mathbf{v}_{P_\phi, E} = \left( \left\langle \frac{dP_\phi}{dt} \right\rangle, \left\langle \frac{dE}{dt} \right\rangle \right)$$

advection equation, assuming  $\nabla \cdot \mathbf{v}_{P_\phi, E} = 0$ , i.e. incompressible phase space flow  
is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

phase space density is conserved  
add energy diagnostic:

$$\mathcal{E}(t) = \int dV_{P_\phi, E, \Lambda} E \cdot F_{EP}(t) / E_0$$

if perturbations are consistently chosen  
i.e. as unstable eigenfunctions of the equilibrium,  
energy stored in gradients of  $F_{EP}$  is depleted



# evolve continuity equation for $F_z$ in CoM space



$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi} \left( \left\langle \frac{dP_\phi}{dt} \right\rangle F_z \right) - \frac{\partial}{\partial E} \left( \left\langle \frac{dE}{dt} \right\rangle F_z \right) \quad \mathbf{v}_{P_\phi, E} = \left( \left\langle \frac{dP_\phi}{dt} \right\rangle, \left\langle \frac{dE}{dt} \right\rangle \right)$$

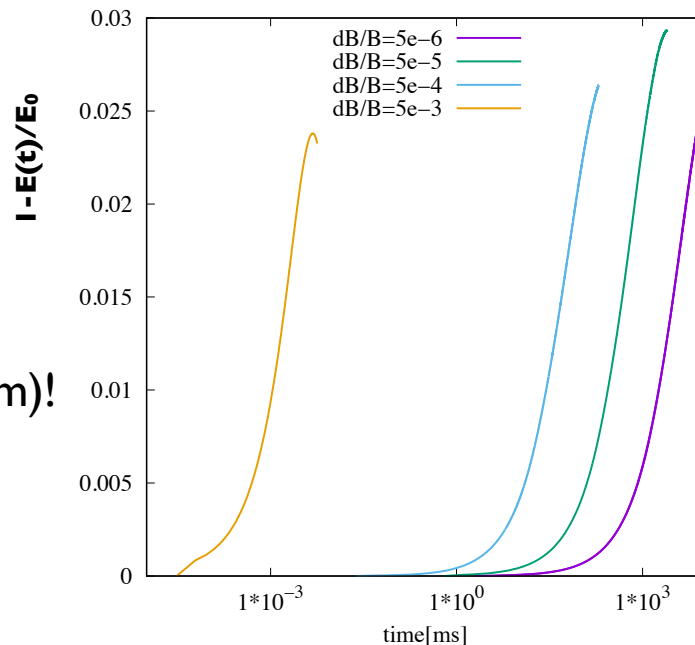
advection equation, assuming  $\nabla \cdot \mathbf{v}_{P_\phi, E} = 0$  i.e. incompressible phase space flow  
is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

phase space density is conserved  
add energy diagnostic:

$$\mathcal{E}(t) = \int dV_{P_\phi, E, \Lambda} E \cdot F_{EP}(t) / E_0$$

note that energy can also increase (forced driven system)!

find minimum in energy, defining the maximally relaxed state of  $F_{EP}$  in presence of a fixed perturbation



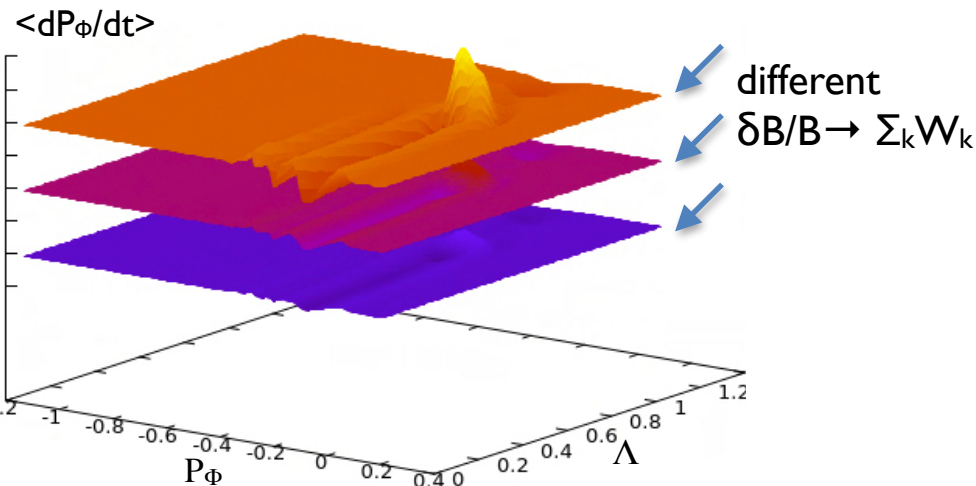


## **evolve transport equations in quasi-linear limit (QL)**

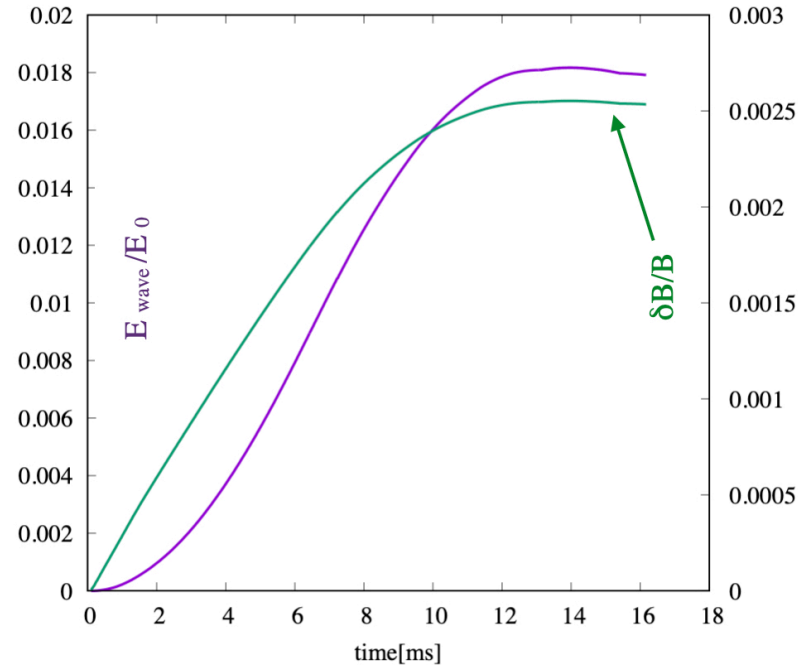
$$\frac{d}{dt} \left( \mathcal{E} + \sum_k W_k \right) = -2 \sum_k \gamma_{d,k} W_k$$

$$\mathcal{E}^\circ(t) = \int dv P_{\phi, E, \Lambda} E \cdot F_{EP}(t)$$

amplitude dependent  $\langle dP_\phi/dt \rangle$ ,  $\langle dE/dt \rangle$  needed!

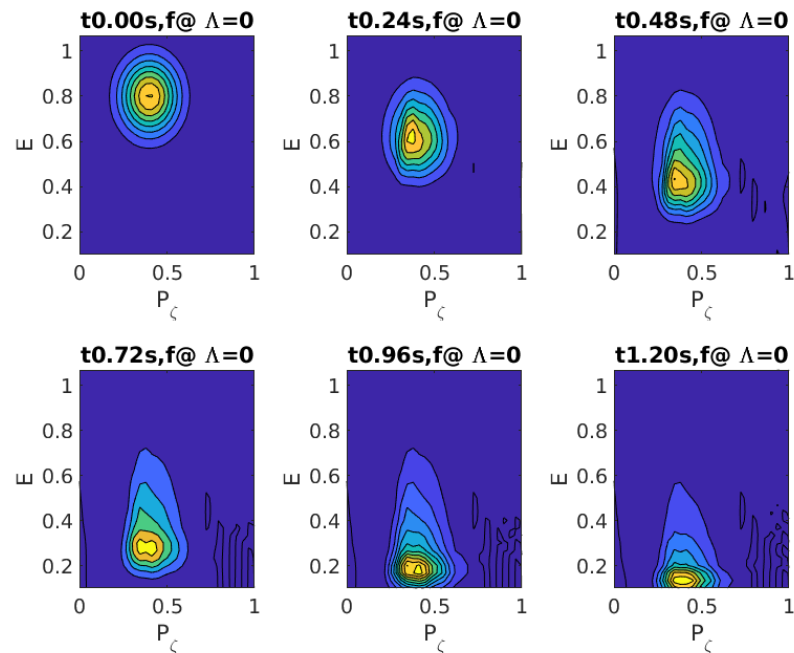
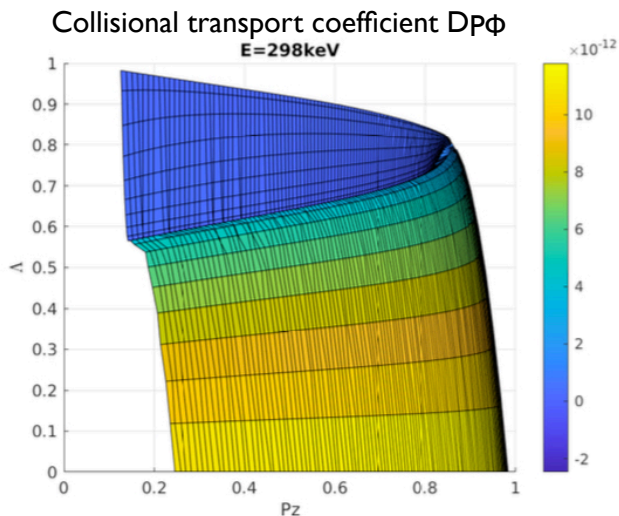


- run previously developed WF for calculating PSZS (FINDER/HAGIS) and store in different IDS occurrences
- import into ATEP code (typically 3-5 different amplitudes  $\delta B/B = 5 \cdot 10^{-6}, 5 \cdot 10^{-5}, 5 \cdot 10^{-4}, 5 \cdot 10^{-3}$ )
- interpolate in CoM space, then construct 4D object
- it includes resonance broadening and transitions from isolated to overlapping modes
- it is NOT yet self-consistent, i.e. ratio of mode amplitudes is fixed (radial envelope equation not solved)
- use E-conservation of PSZS transport equation to determine energy transfer to mode and change mode amplitude(s) accordingly

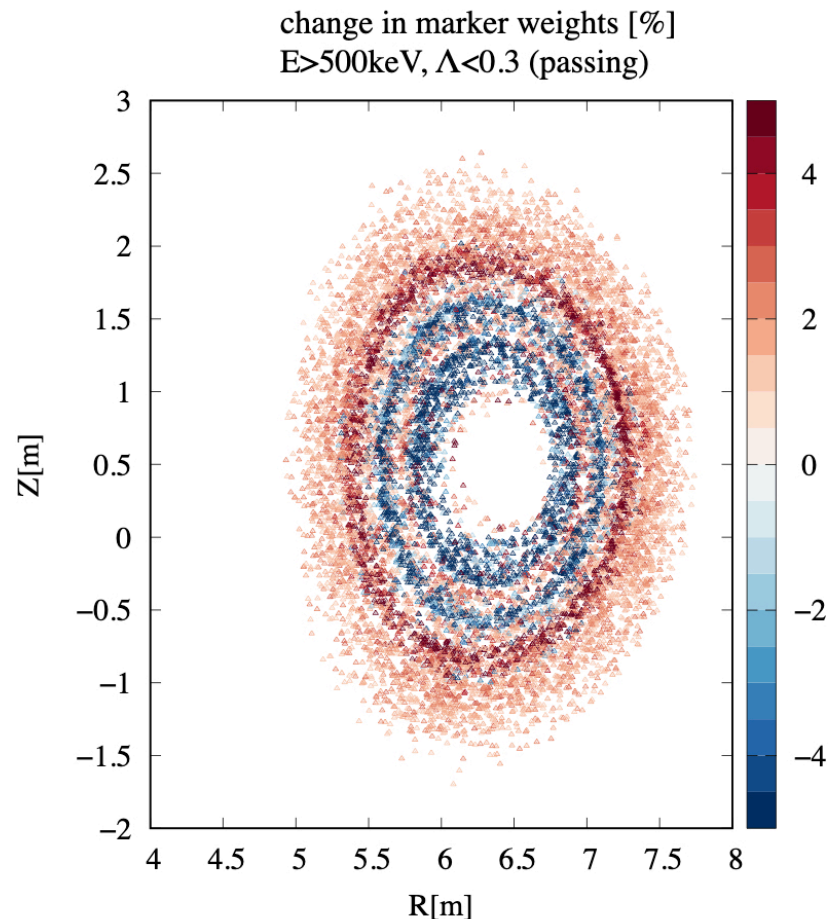


- energy conserving model - energy stored in  $F_{EP}$  gradients is converted into wave energy: non-linear hybrid model á la HAGIS (non-linear wave particle interaction Lagrangian)
- relative amplitudes of modes remain fixed, as given by linear growth rates ( $\gamma^2 \sim A$ )
- here, no damping was used yet; mode growth stops after energy of  $F_{EP}$  has been exhausted
- for steady state, mode decay has to be balanced by collisions

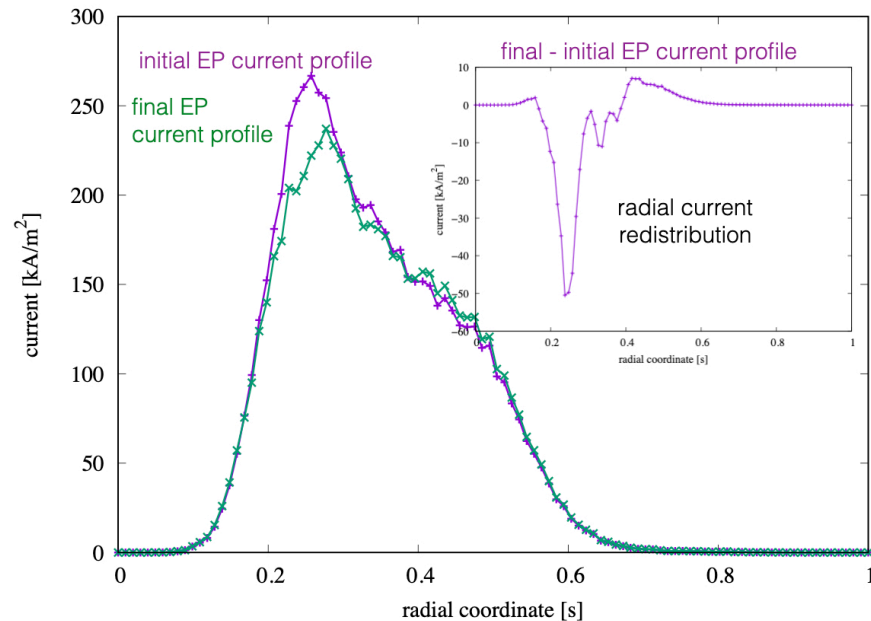
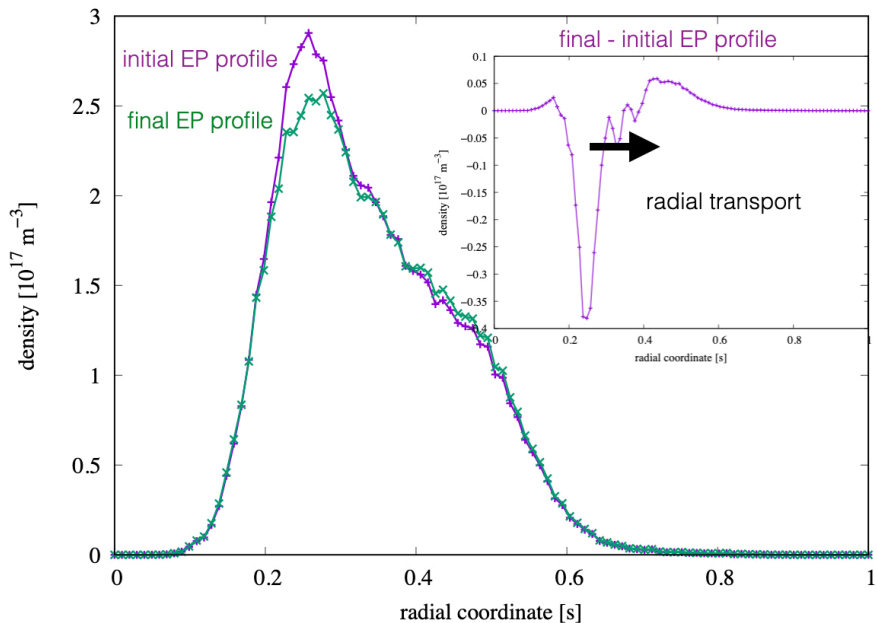
- collision operators are typically given in  $E, v//$  space (explicit pitch angle dependence)
- use framework above (IMAS based wrapper for HAGIS) with neoclassical HAGIS version [A. Bergmann, PoP 2001] to obtain orbit-averaged collision coefficients (linearised collision operator)
- use the same CoM grid as for PSZS part
- general 3D solver (implicit solver)
- details: poster [G. Meng, at this conference](#)



- use map created for setting up orbits quantities (see above) to assign new weights to markers as given by initial input from heating code or SD model
- only ‘weights’ in CoM are transported, not markers themselves
- transport is by construction ‘zonal’ - taking moments of evolved state allows us to define new non-linear equilibrium:



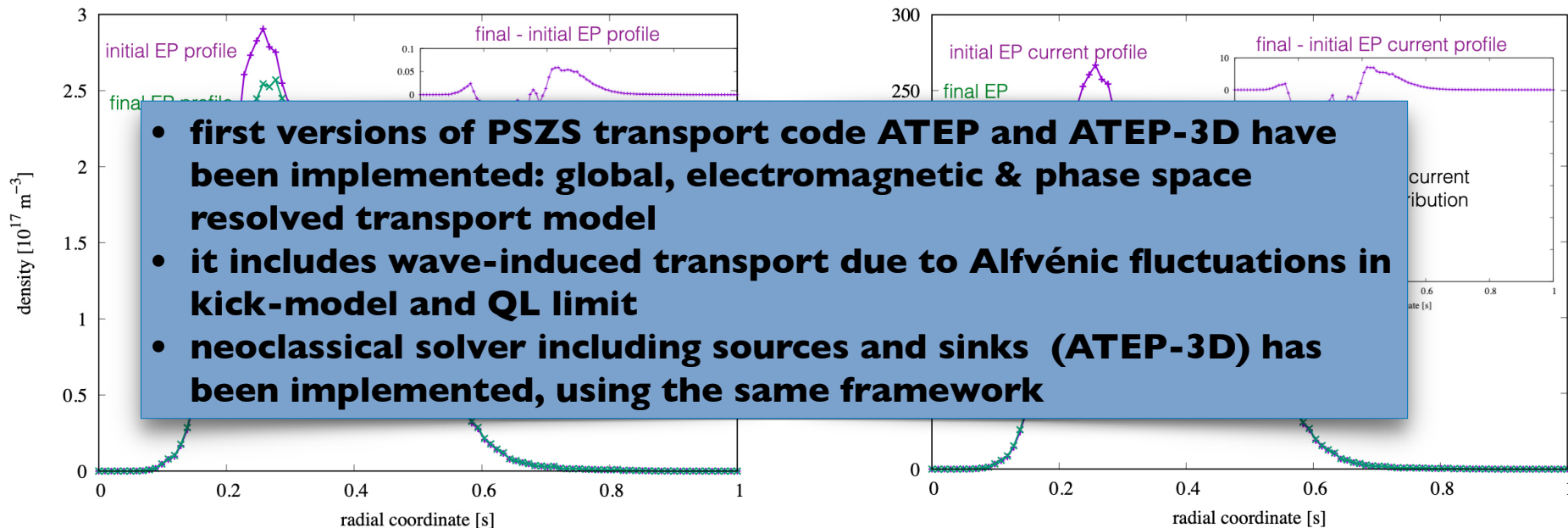
back-mapping and calculating moments given EP transport in physical units:



can be passed to transport/equilibrium code to calculate new consistent non-linear equilibrium



back-mapping and calculating moments given EP transport in physical units:



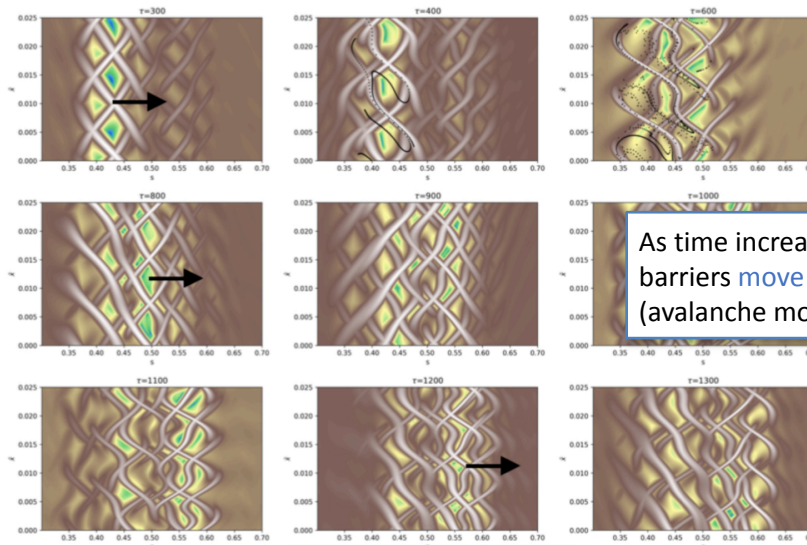
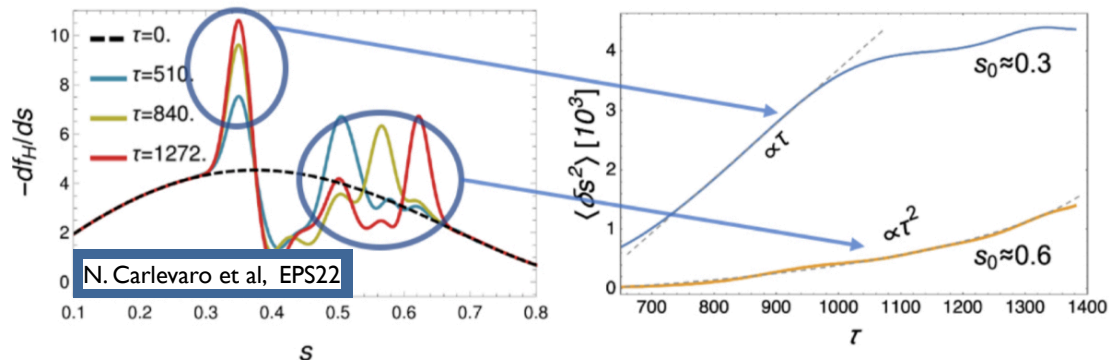
can be passed to transport/equilibrium code to calculate new consistent non-linear equilibrium



**verify, validate and evolve models - ENR ATEP team effort**

- benchmark with original HAGIS model
- benchmark with DAEPS code - calculates fluxes explicitly based on separation of radial and parallel mode structures
- started extension to 3D geometry [A. Zocco, 2023]
- benchmark with 1D beam-plasma system [N. Carlevaro, PPCF 2022]:
  - bump on tail model
  - partition phase space in slides of maximal power exchange
  - use LIGKA linear mode information
  - successful comparison with LIGKA-HAGIS model
- tracers dynamics studied with Lagrangian Coherent structures: relevant structures/barriers change during non-linear evolution: from inner to outer radial transport peak (see ITER case above):

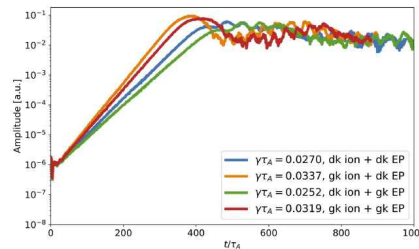
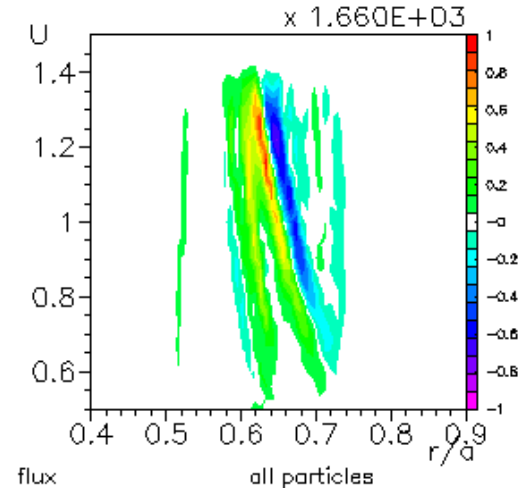
add tracers to system and determine diffusive ( $\tau$ ) vs. convective ( $\tau^2$ ) scaling:



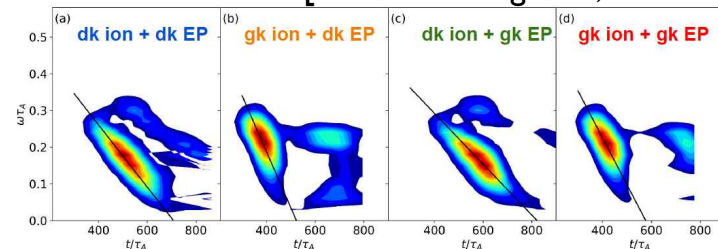
As time increases the more robust barriers move radial outwards (avalanche mode excitation)

- benchmark with XHMGC calculations, featuring advanced features for transport analysis: Hamiltonian mapping diagnostics & explicit flux ‘measurements’
- implemented also in HYMAGYC [G.Vlad,V. Fusco]
- benchmark with STRUPHY code: MHD-kinetic hybrid code based on new stringent mathematical formulation: structure preserving geometric finite elements + PIC  $\Rightarrow$  improved non-linear stability [F Holderried, S Possanner 2020-2023]
- compare with ORB5 PSZS diagnostics [A. Bottino Varenna 2022] (see talk *M. Falessi*)  
compare to various ORB5 results; e.g. use scaling for chirping modes  
ORB5 runs are available also in presence of turbulence
- analyse and plan new experiments based on AUG EP ‘Supershots’  
INPA measurements of phase space transport!  
[J. R. Rueda, FEC 2023]

$t\omega_{A0} = 696.00$  [X.Wang, S. Briguglio et al 2021]



[ORB5: X Wang et al, 2022-23]

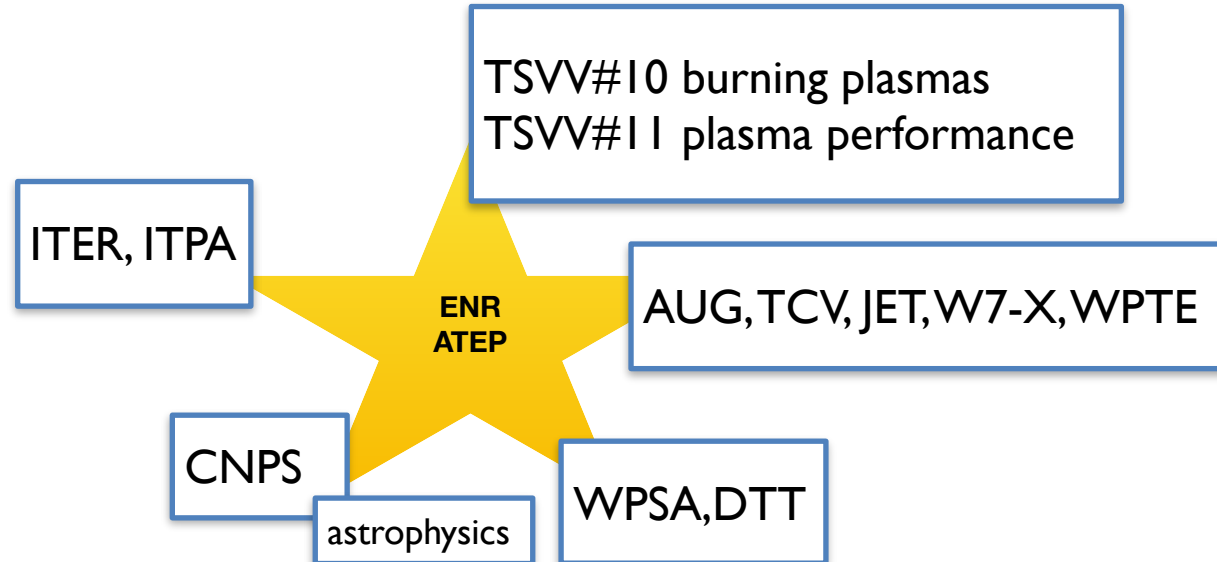




started enable new routes to EP transport analysis and prediction via:

- new theoretical framework
- new common concept of connecting non-linear code results to reduced models (PSZS)
- new common EP transport code developments
- newly implemented analysis methods
- new IMAS based infrastructure

established and growing connections to other groups and experiments

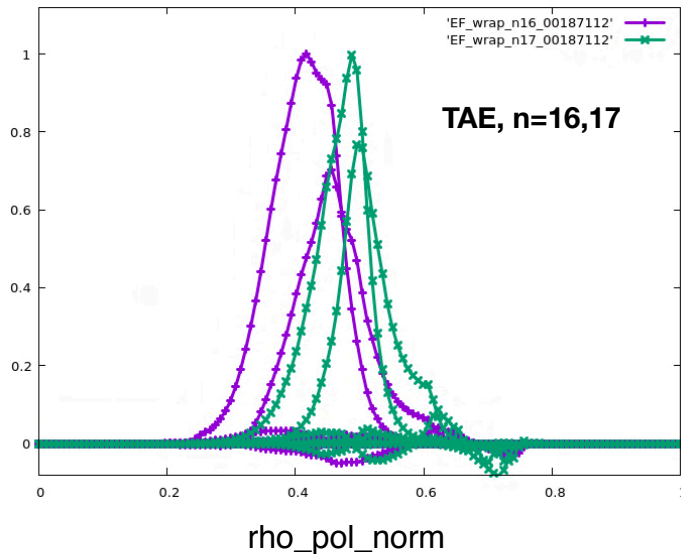




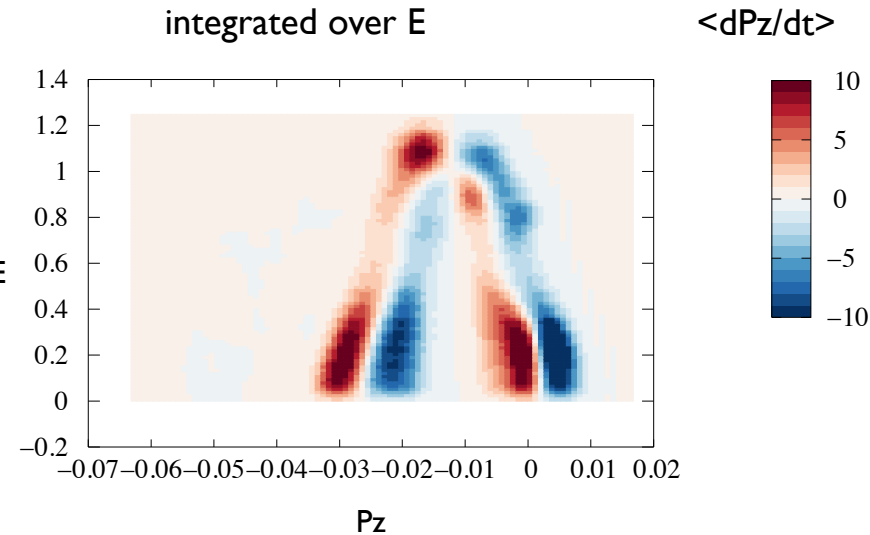
backup



- calculate  $\langle dP_z/dt \rangle$  ,  $\langle dE/dt \rangle$  for given fixed mode structures - here: scan amplitudes in 2 mode system



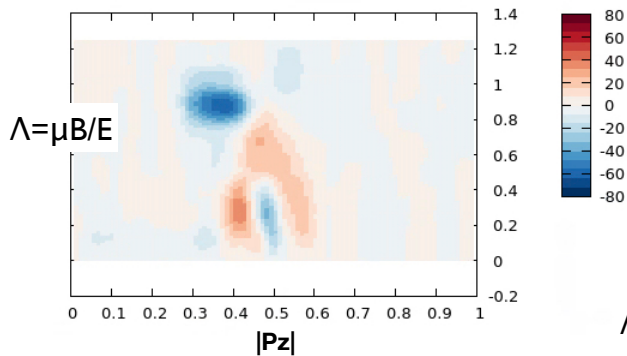
$\Lambda = \mu B/E$



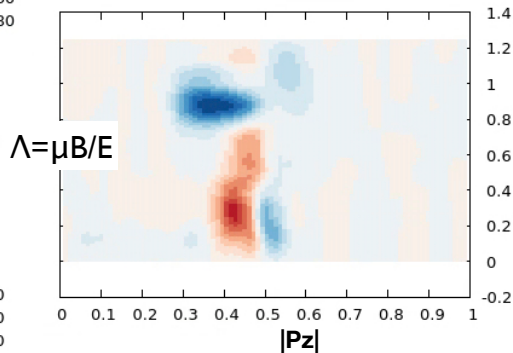
typical grid:  $(P_z, E, \Lambda)$  (128x40x40)



**n=16 single**

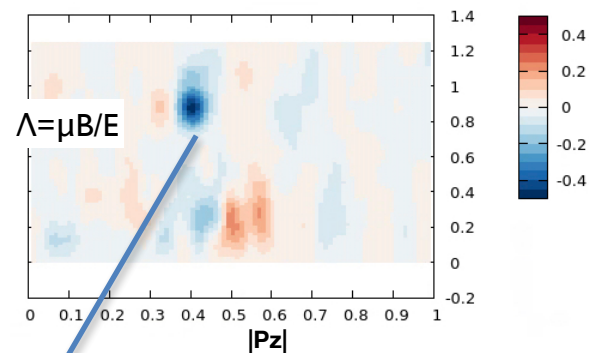


**n=16+17 both**

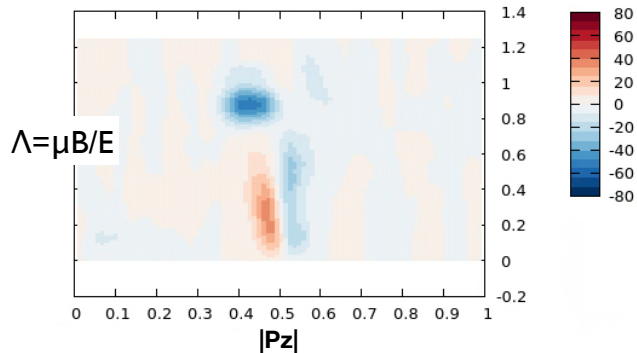


**relative difference**

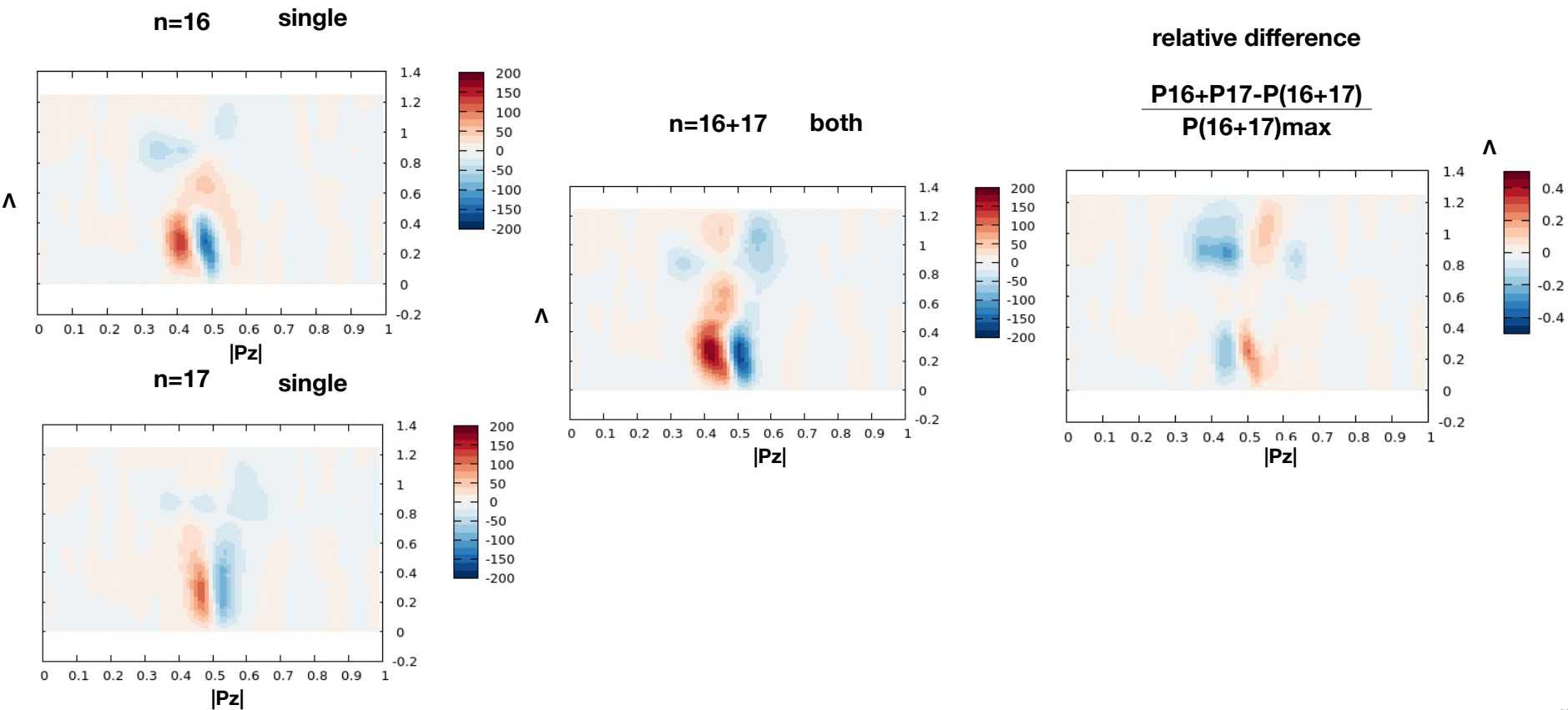
$$\frac{(P_{16+17}) - P_{(16+17)}}{P_{(16+17)max}}$$

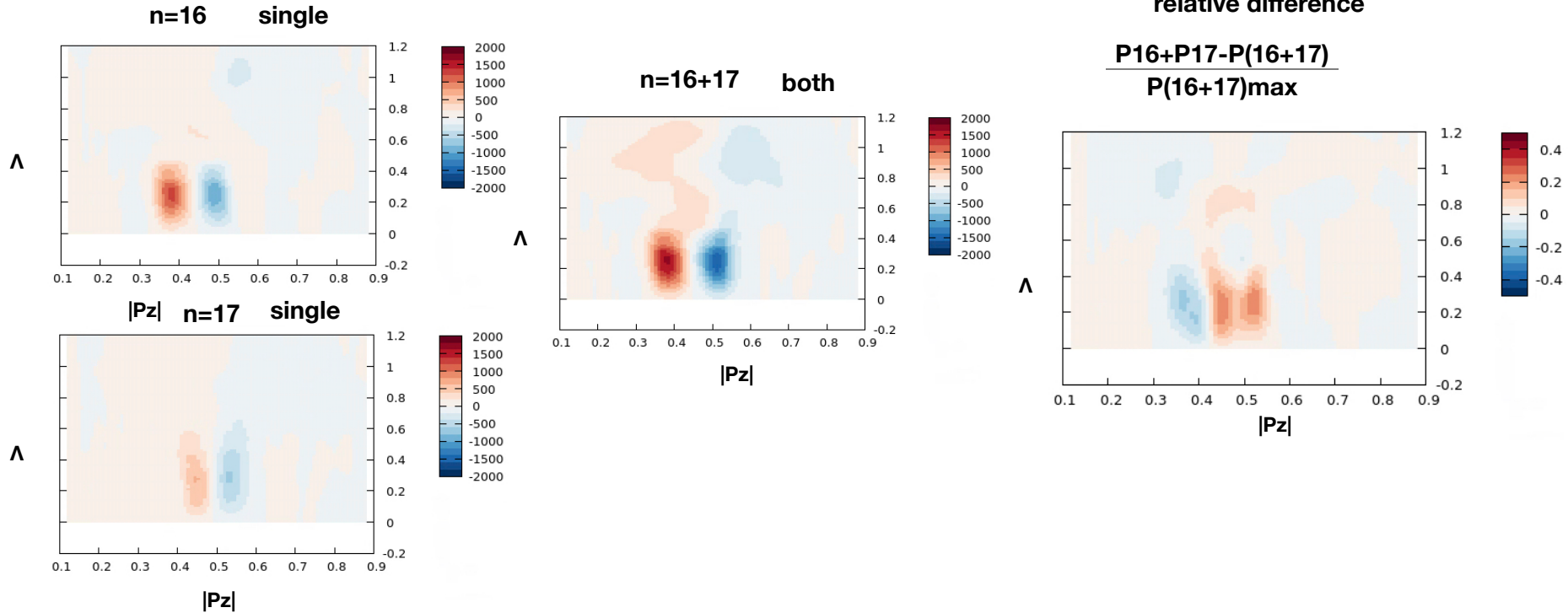


**n=17 single**





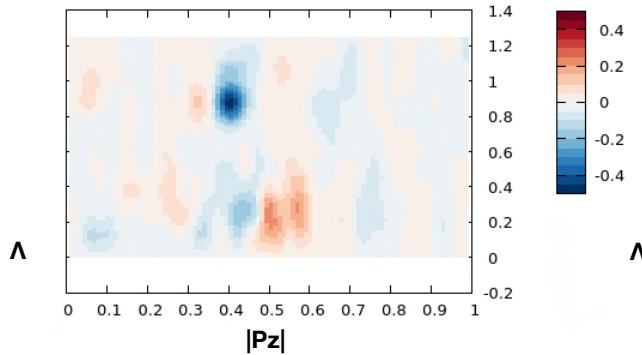




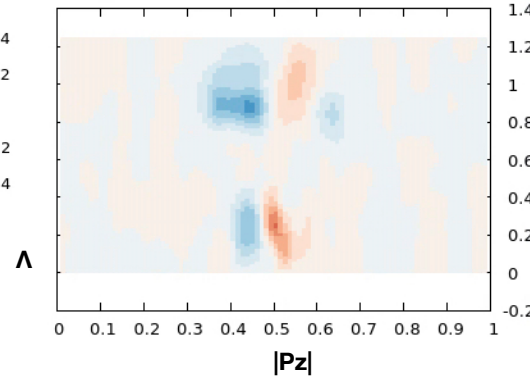


$$\text{relative difference} = \frac{P(16+P17)-P(16+17)}{P(16+17)\text{max}}$$

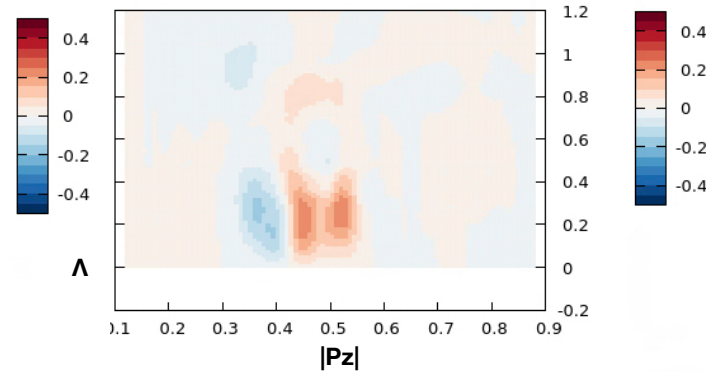
$\text{dB}/B=0.5 * 10^{-3}$



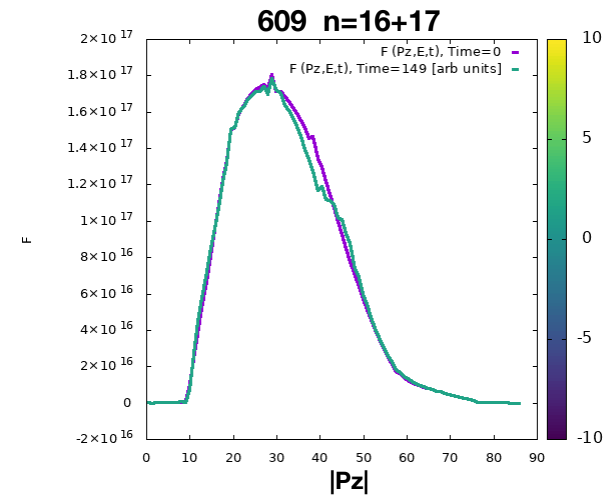
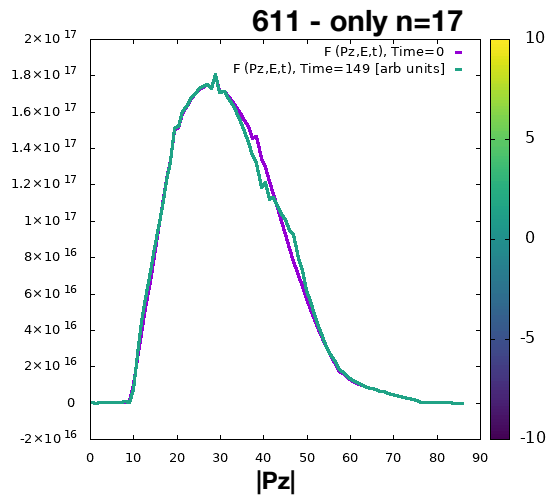
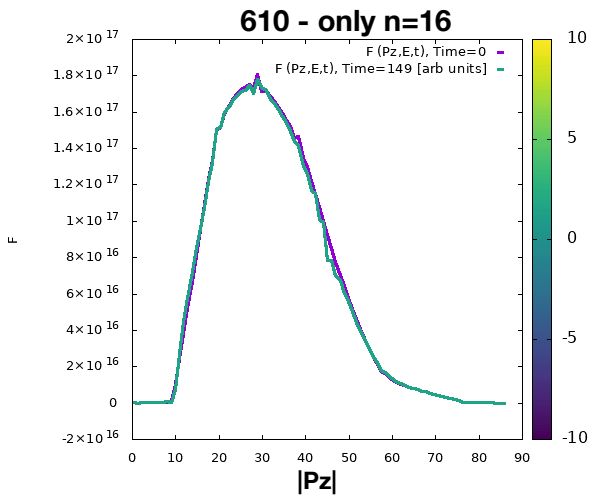
$\text{dB}/B=1 * 10^{-3}$



$\text{dB}/B=5 * 10^{-3}$

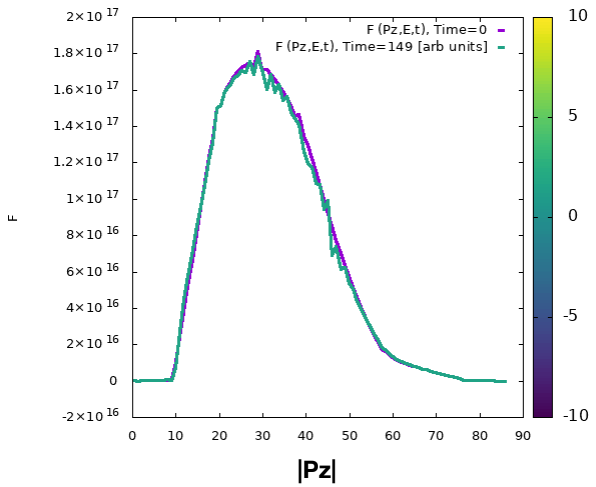


- multi-mode systems need careful treatment when going from isolated mode case to resonance-overlap (diffusive) regime:
- depending on amplitude, trapped and passing particles show different relative importance for causing resonance overlap (FOW vs resonance width) - **consistent treatment of resonance broadening**

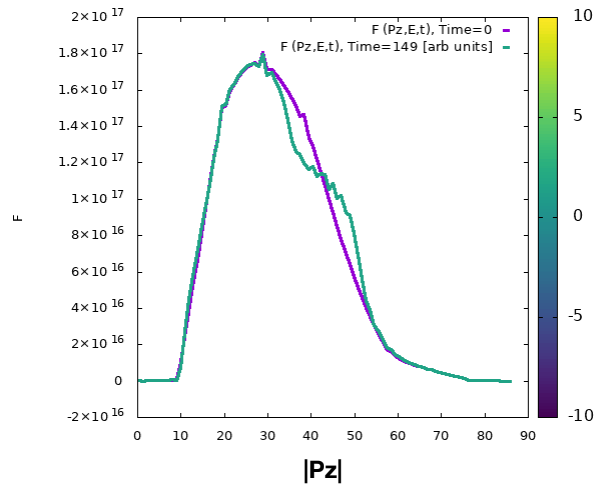




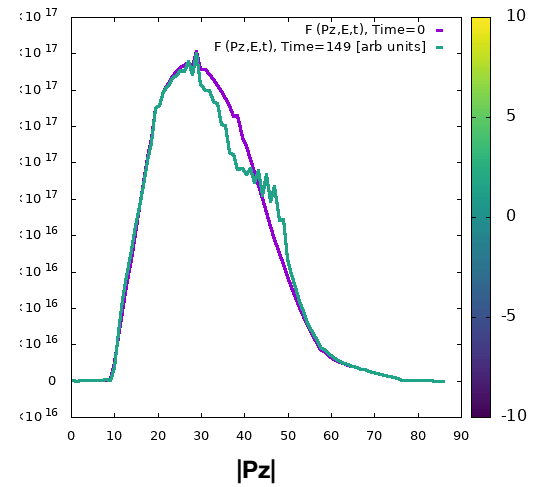
603 - only  $n=16$

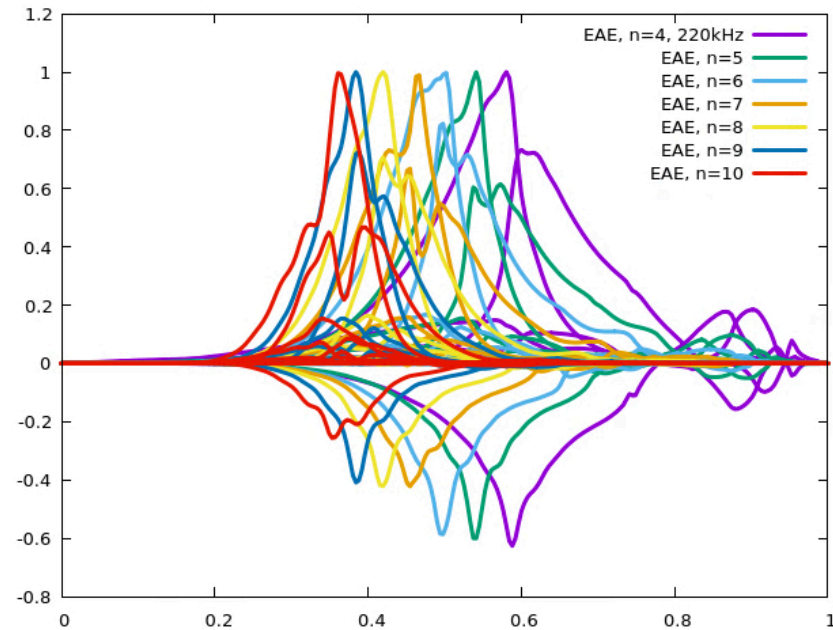
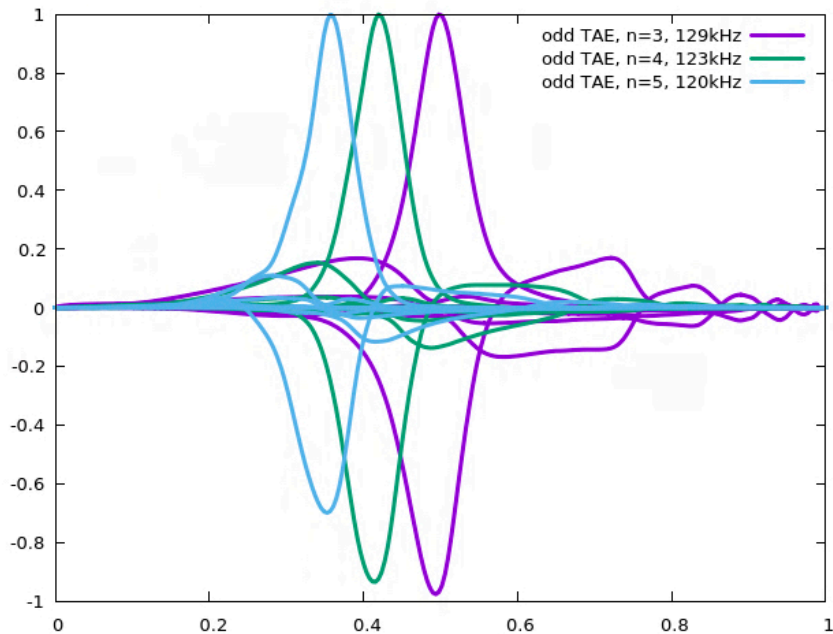


605 - only  $n=17$



604  $n=16+17$

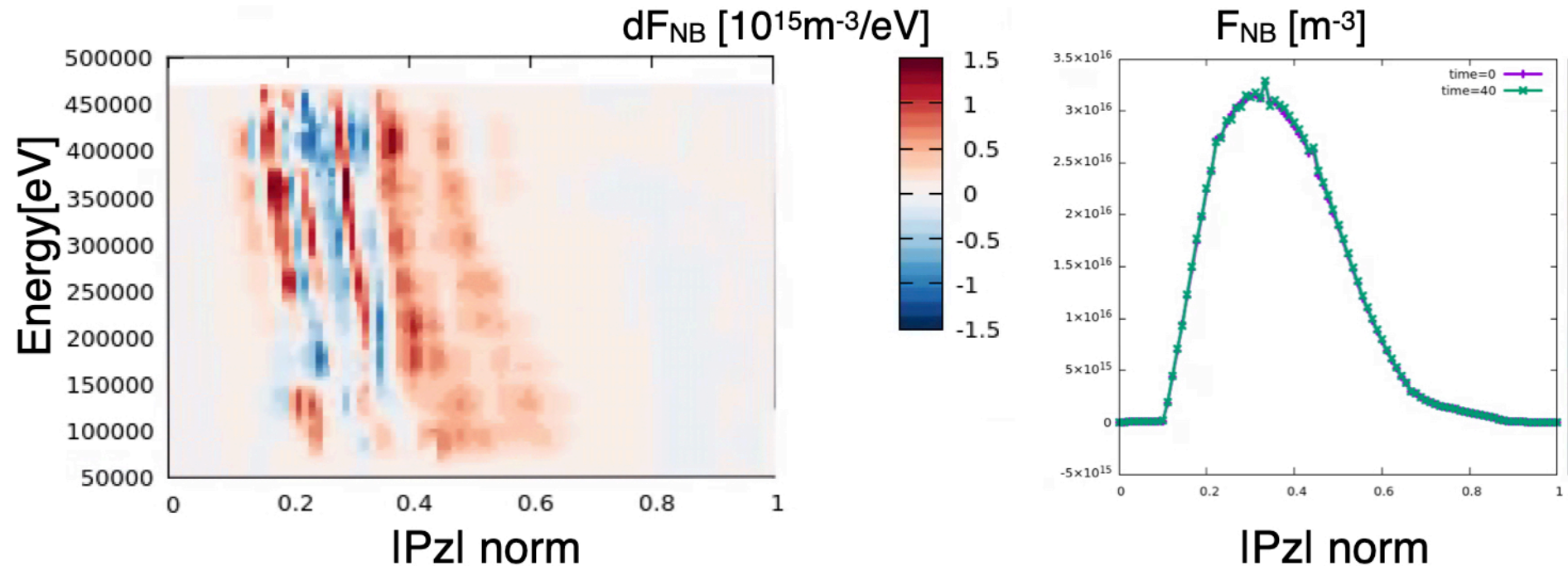




EP-WF has been adopted to cope with co- and counter propagating modes

# solving the PSZS equation (kick-model limit): EAEs

$\delta F(t) = F(t=40) - F(t=0)$  in CoM space ( $\Lambda = \mu B_0 / E = 0.24$ ) for the set of **co and counter propagating EAEs**

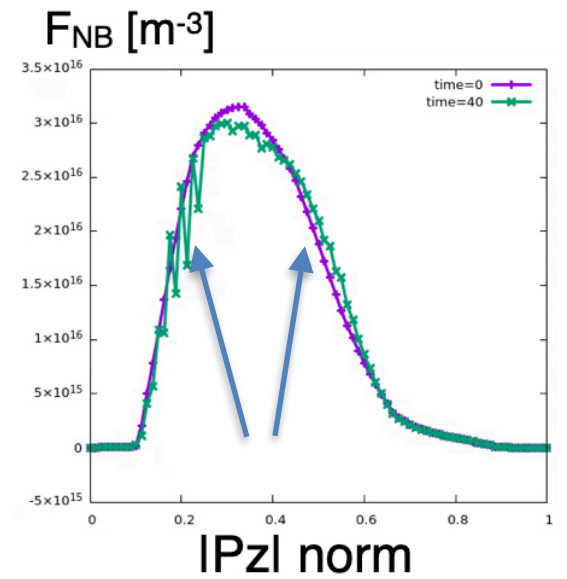
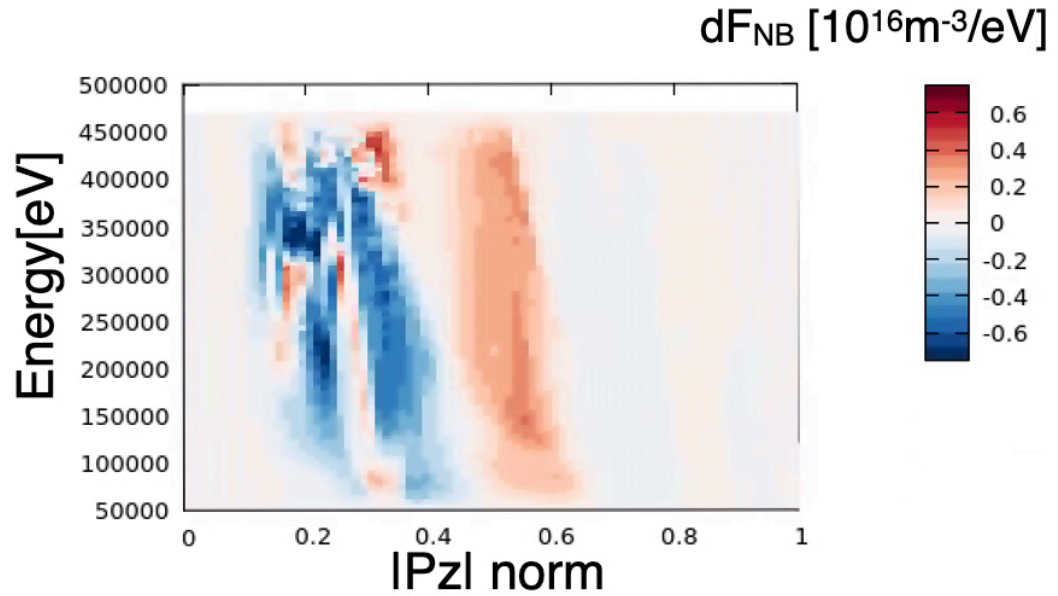


much smaller EP transport (4 times smaller) than odd TAEs (using the same saturation rule  $\gamma \sim A^2$ )  
 next: how do these modes affect the current deposition? mapping back and take moments...

# solving the PSZS equation (kick-model limit):TAEs

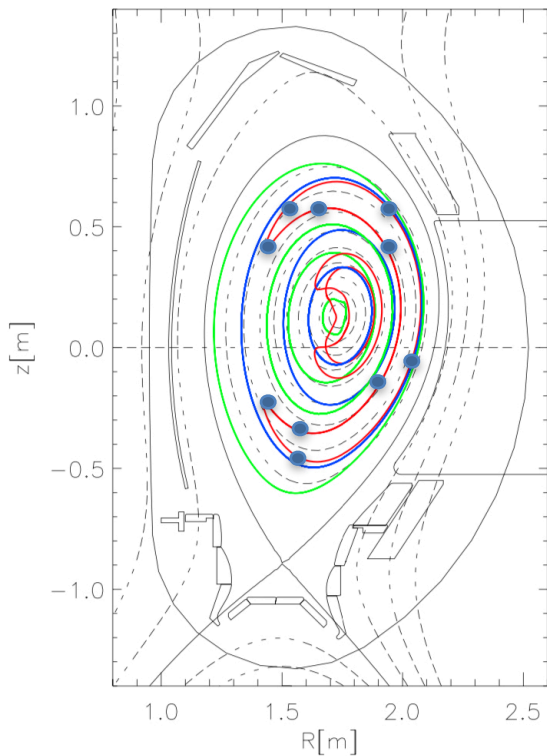


$\delta F(t)=F(t=40)-F(t=0)$  in COM space ( $\Lambda=\mu B_0/E=0.24$ ) for the set of odd co and counter propagating TAEs



both gradients are depleted

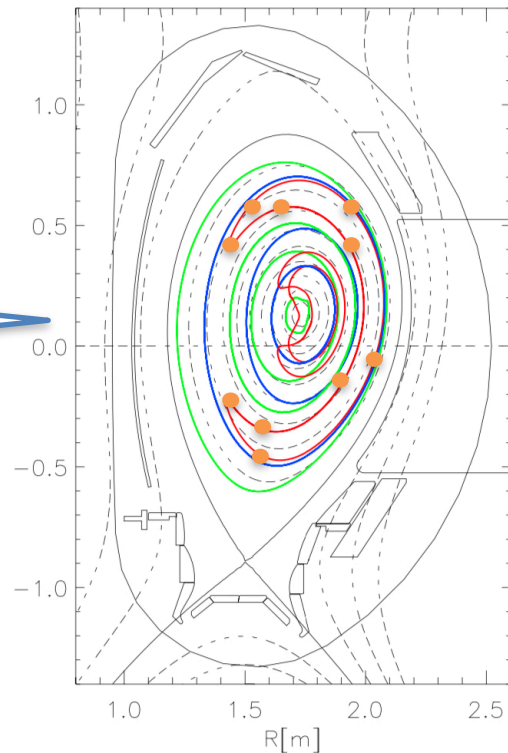




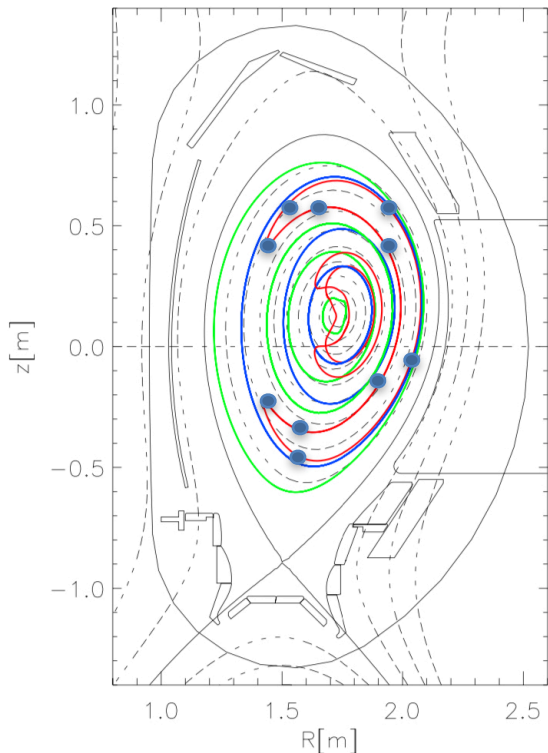
bin all markers on same orbit to  $(P_z, E, \Lambda)$ - grid and sum over weights to obtain density in COM space  $F(P_z, E, \Lambda)$

evolve PSZS  
transport equation  
i.e. update  $F_u(P_z, E, \Lambda)$

$$\frac{\partial F_{EP}}{\partial t} = \frac{\partial P_z}{\partial t} \frac{\partial F_{EP}}{\partial P_z} + \frac{\partial E}{\partial t} \frac{\partial F_{EP}}{\partial E}$$



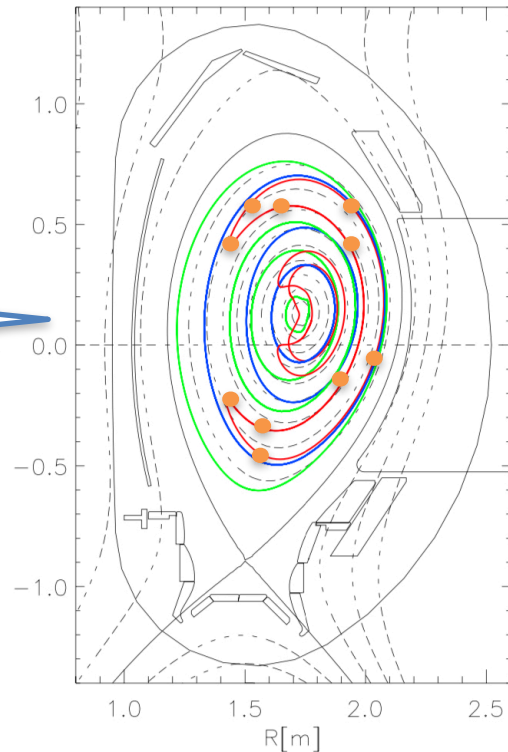
distribute new weight proportional to original weights, i.e. scale all marker weights of certain bin by  $F_u(P_z, E, \Lambda)/F(P_z, E, \Lambda)$



evolve PSZS  
transport equation  
i.e. update  $F_u(P_z, E, \Lambda)$

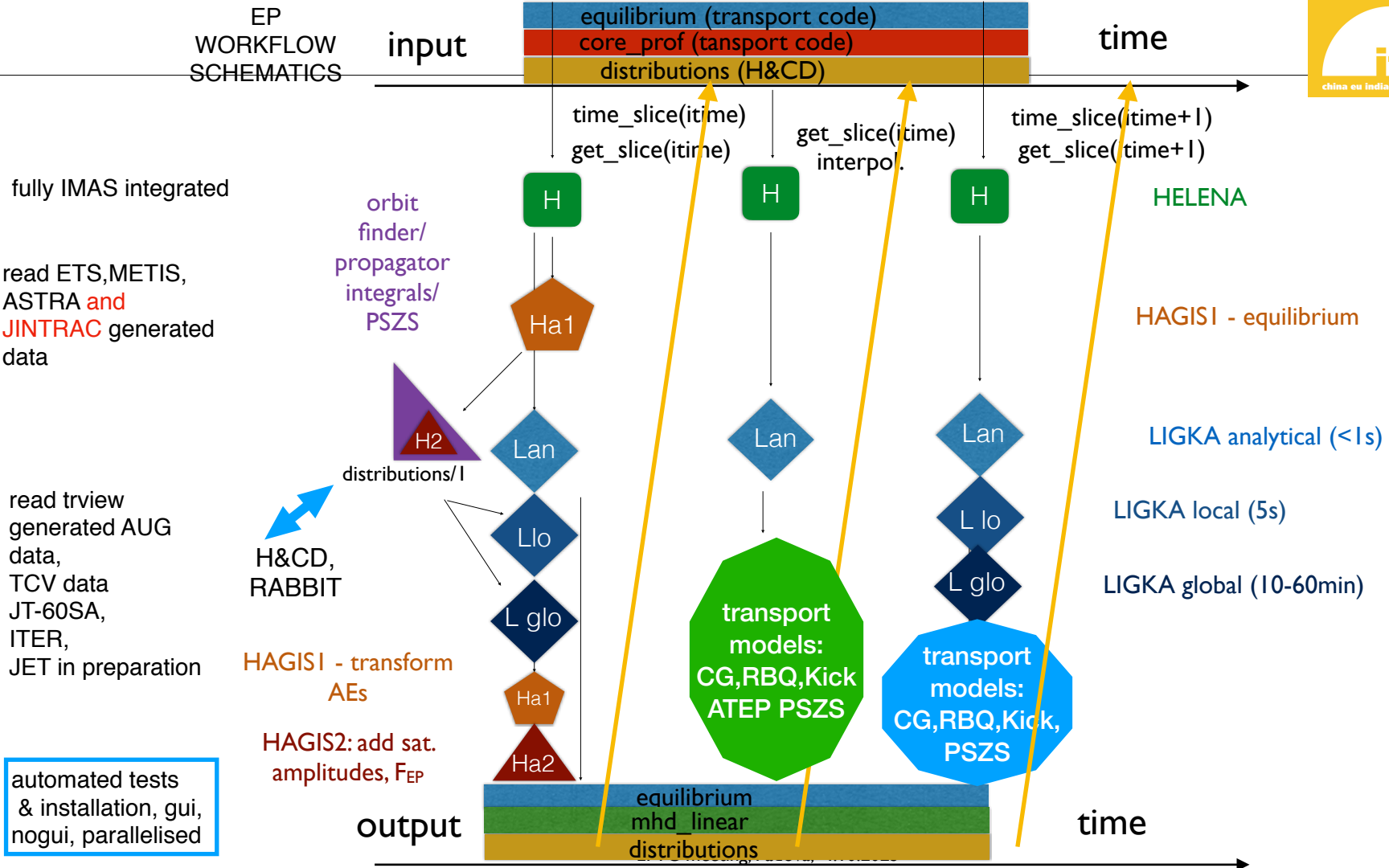


$$\frac{\partial F_{EP}}{\partial t} = \frac{\partial P_z}{\partial t} \frac{\partial F_{EP}}{\partial P_z} + \frac{\partial E}{\partial t} \frac{\partial F_{EP}}{\partial E}$$



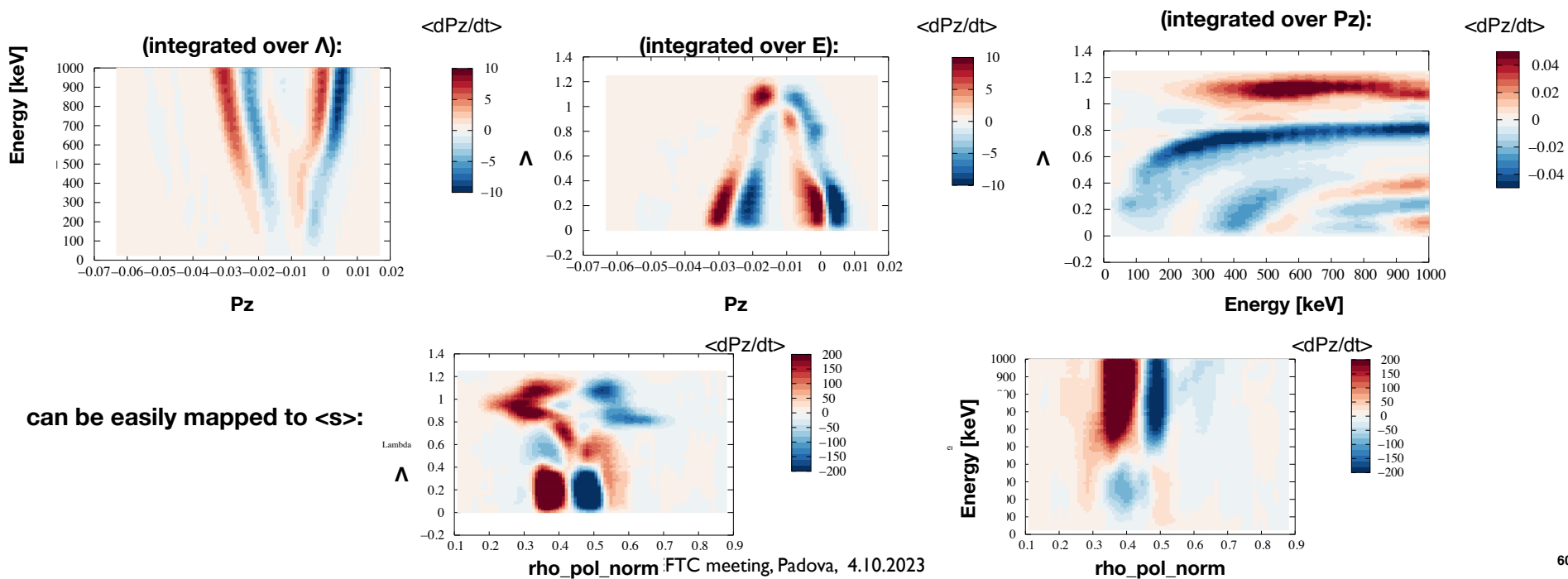
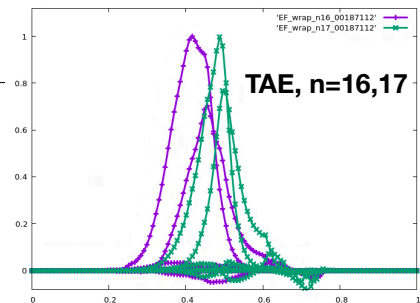
bin all markers on same orbit to  
( $P_z, E, \Lambda$ )- grid and sum over weights to  
obtain density in COM space  $F(P_z, E, \Lambda)$

then calculate moments and/or transport  
coefficients to be used in connected codes

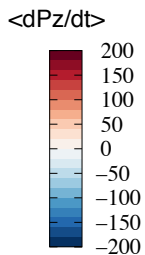
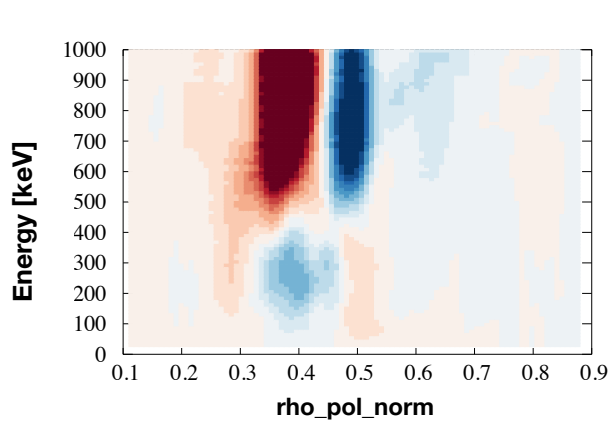


WP3.3-M1 Extend unperturbed orbit integration routines and averaging procedures in order to calculate phase space fluxes in HAGIS mid 2022 (fully)

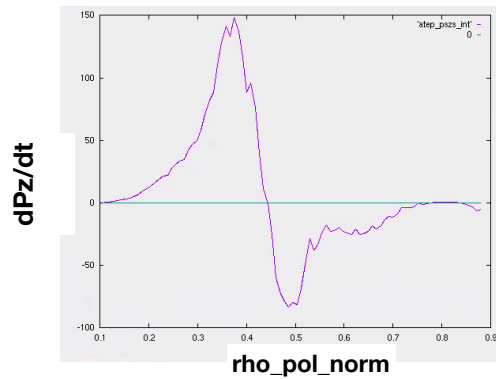
by zonal averaging of a representative particle ensemble, calculate  $\langle dP_z/dt \rangle$ , i.e. radial transport for given set of fixed mode structures at fixed amplitudes, write as IDS object in COM Pz,E, $\Lambda$  [Lauber DTT seminar, 5/2022, Bierwage et al, ID: 30554]



## calculation of diffusions coefficients: $D(s,E)$ and $D(s)$



E integration



to be done: transform from  $\langle dP_z \rangle^2 / \langle dt \rangle$  to  $D(s,E) = \langle ds \rangle^2 / \langle dt \rangle$

and feed back to transport code

# EP properties in various (planned) experiments



scaling of difference machines with respect to energetic particle (EP)  $\beta$

$\alpha$ -particle pressure (gradient) has to increase for DEMO: properties of EP confinement?

	$B_T$ [T]	$T_i(0)$ [keV]	$\beta(0)_{back}$ [%]	$\beta(0)_{\alpha,EP}$ [%]	$\beta_{\alpha}/\beta_{back}$	$R$ [m]	$R_0 \nabla \beta_{\alpha,EP}/\beta_{back}$
TFTR-DT	5.0	28	4.6	0.2	0.04	2.5	0.11
JET-DT	3.8	23	5.7	0.4	0.07	3.0	0.21
ITER-I5MA	5.3	25	4.8	0.9	0.19	5.2 (2)m	0.50
AUG, off axis	2.2	1.2	0.4	0.4	<1.0	1.65(0.5)m	<3.3
JT-60SA	2.25	6.0	3.8	<2.3	<0.6	2.96(1.2)m	1.48
DEMO PPPT, 2015	5.7	30	7.0	6.0	0.85	9.0 (3)m	2.55

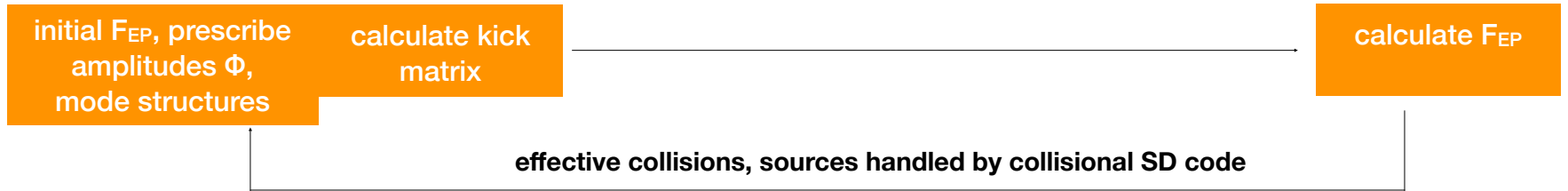
can  
overcome  
EPM  
threshold

known from JT-60U, AUG, spherical tokamaks: EP-driven mode dynamics and transport changes if EPM threshold is reached:

$$q^2 R_0 \nabla \beta_{EP} / \beta_i (T_i / T_{EP}) \gg \epsilon^{3/2} \quad [\text{Zonca et al NF, 45, 2005}]$$

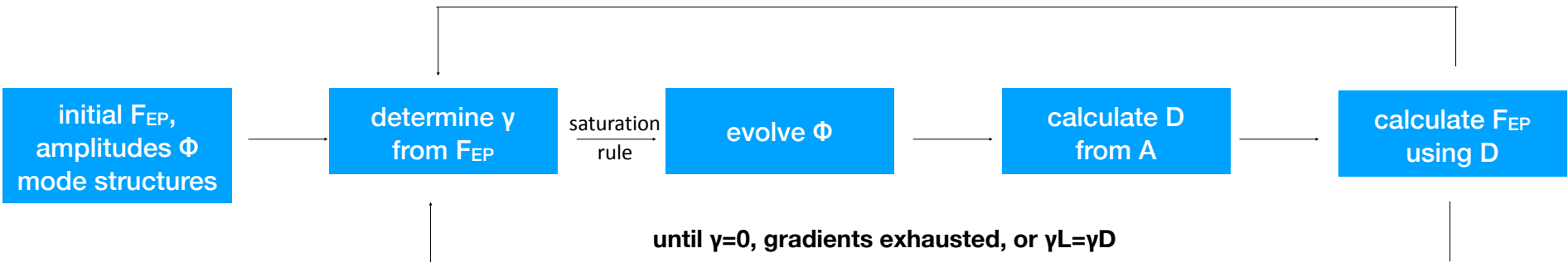


**kick model scheme [Podesta 2014]:**



**typical quasi-linear scheme [Sagdeev/Galeev 1969, Kaufman 1972, Gorelenkov 2018]:**

**add effective collisions, sources**



$$\gamma_n = 2\pi^2 \frac{e^2}{m} \frac{v_n}{|k_n|} \frac{\partial f(v_n)}{\partial v}$$

$$\frac{\partial}{\partial t} W_n = 2\gamma_n W_n$$

$$D(v) = \frac{2\pi e^2}{m^2} \sum_n |k_n \phi_{n0}|^2 \delta(\Omega_n)$$

$$W_n = \frac{|k_n \phi_{n0}|^2}{2\pi v_n}$$

**+self consistent resonance broadening**

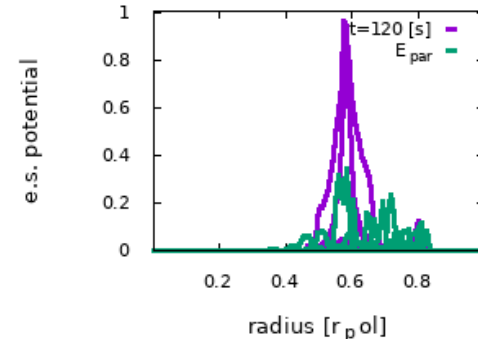
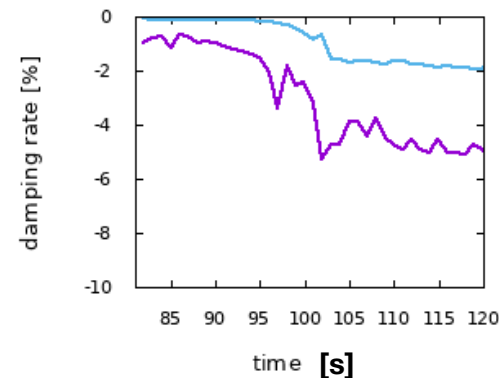
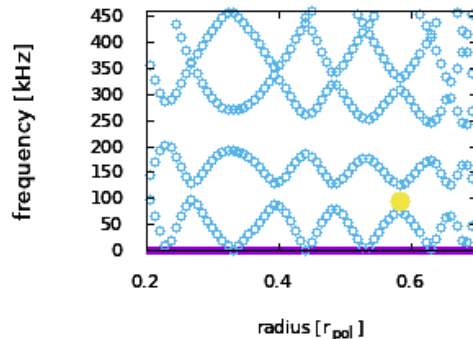
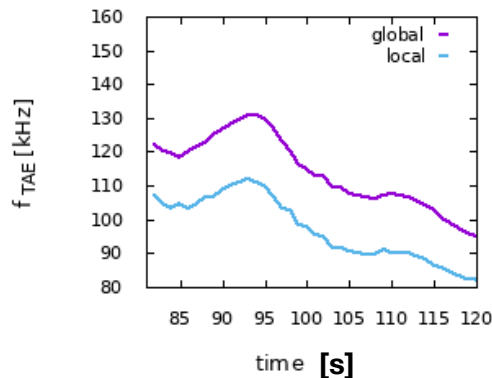
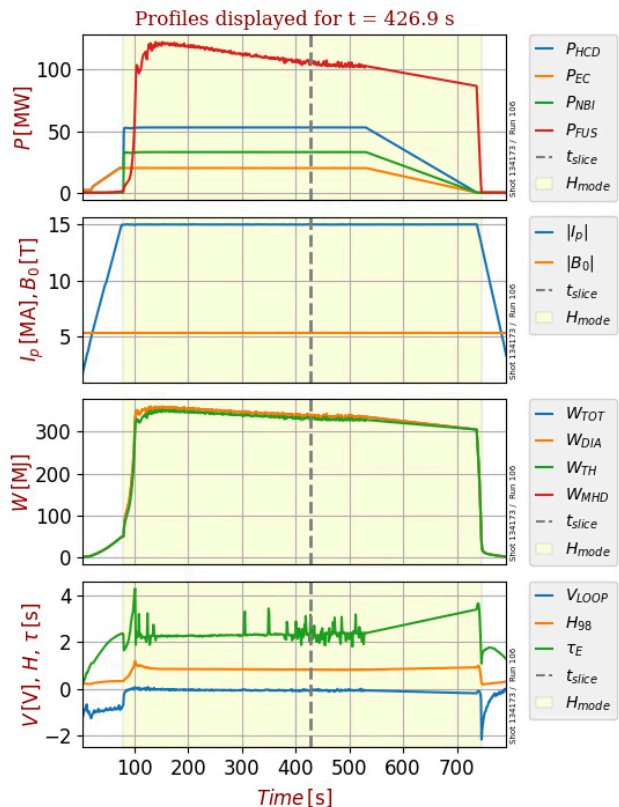
$$\frac{\partial f}{\partial t} = \hat{Q}f \equiv \frac{\partial}{\partial v} \left( D(v) \frac{\partial f}{\partial v} \right)$$

# Linear mode spectrum: Energetic particle stability workflow



[S.D. Pinches, plenary EPS 2022] #134173, 106 TAE n=18

[V.-A. Popa, NF accepted 2023]



identified end of power ramp-up phases as most critical time points for in-depth EP transport analyses



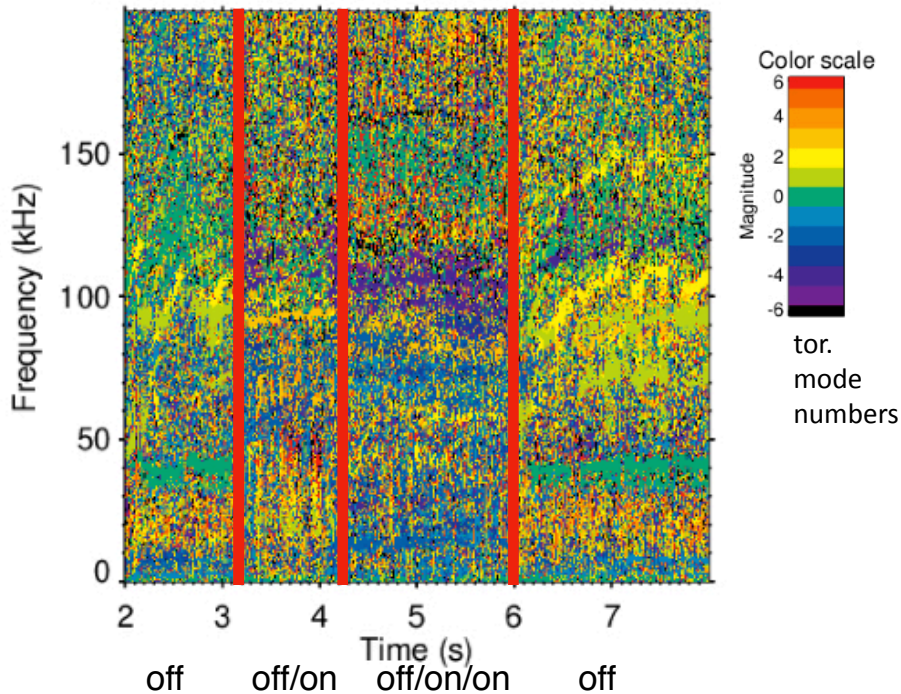
# Energetic particle stability workflow: validation at ASDEX Upgrade



for theory very important: isotope scans (e.g. finite orbit width effects)

- D beam in D plasma
- D beam in H plasma
- H beam in H- plasma
- D beams into He plasma

### Toroidal mode numbers of AUGD 41437



for some shots: FIDA data available [B. Geiger]

