MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Stellarator optimisation: a brief review

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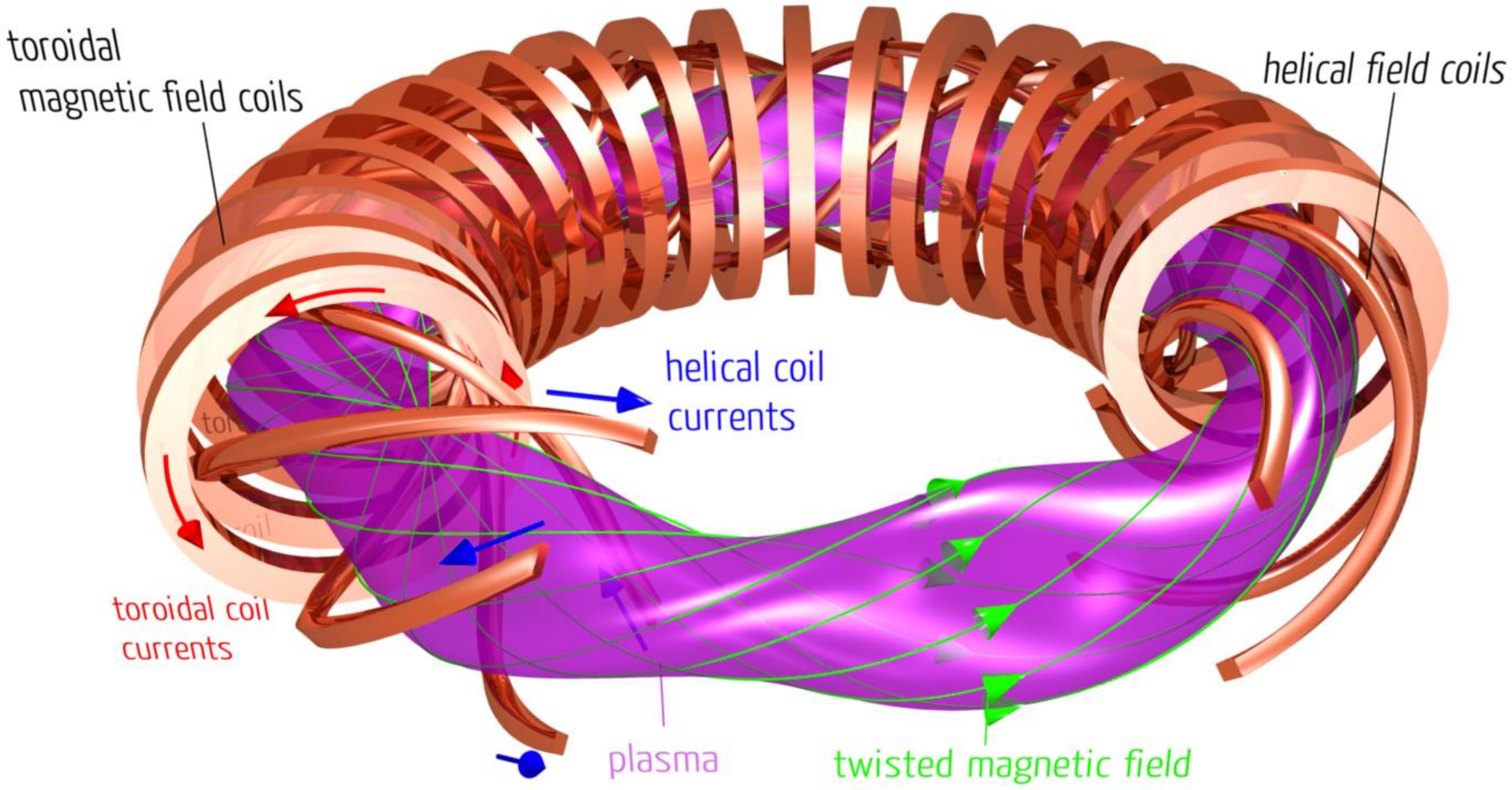
EUROfusion

Outline

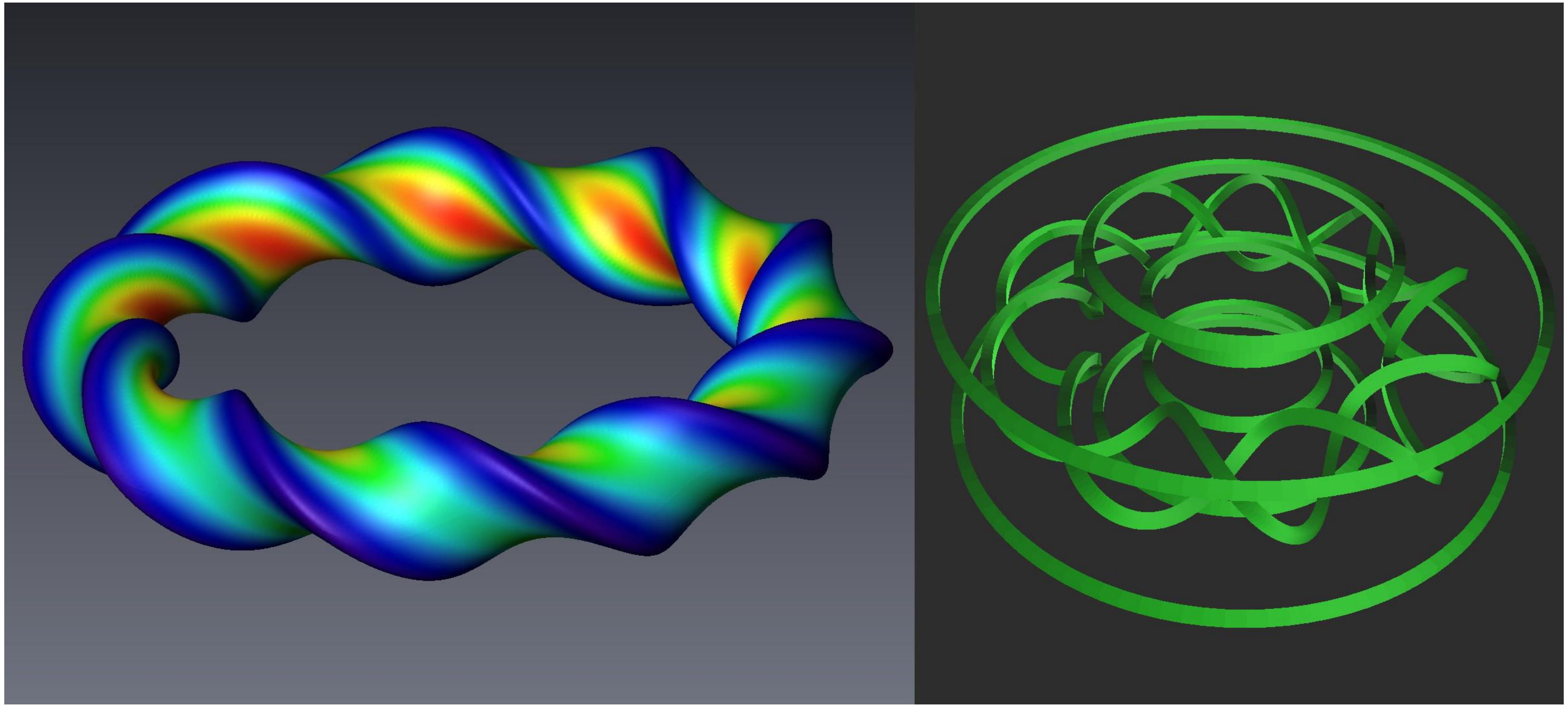
- Stellarators
- Optimisation and Wendelstein 7-X
- Optimisation of future stellarators
  - Microinstabilities and turbulence
  - Coils

Stellarators

The classical stellarator

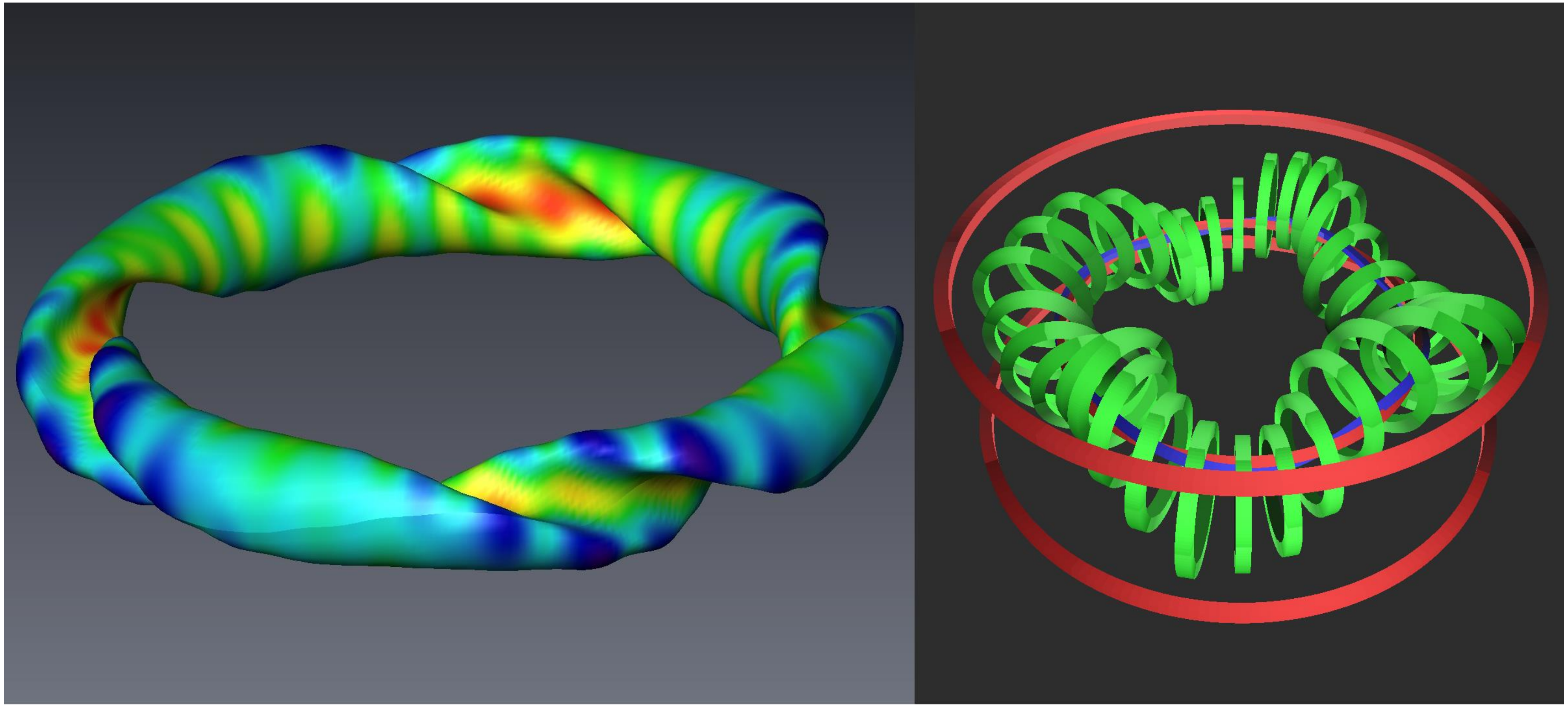


The torsatron/heliotron



Large helical device (LHD), National Institute for Fusion Science, Japan

The heliac



TJ-II, Madrid

Advantages and disadvantages

- Stellarators have many advatages over tokamaks:
  - No need for current drive
  - Smaller coils possible
  - Higher MHD beta limit
  - No disruptions
    - no runaway electrons
    - no Greenwald density limit:
      - higher-density operation possible
      - fewer alpha particles

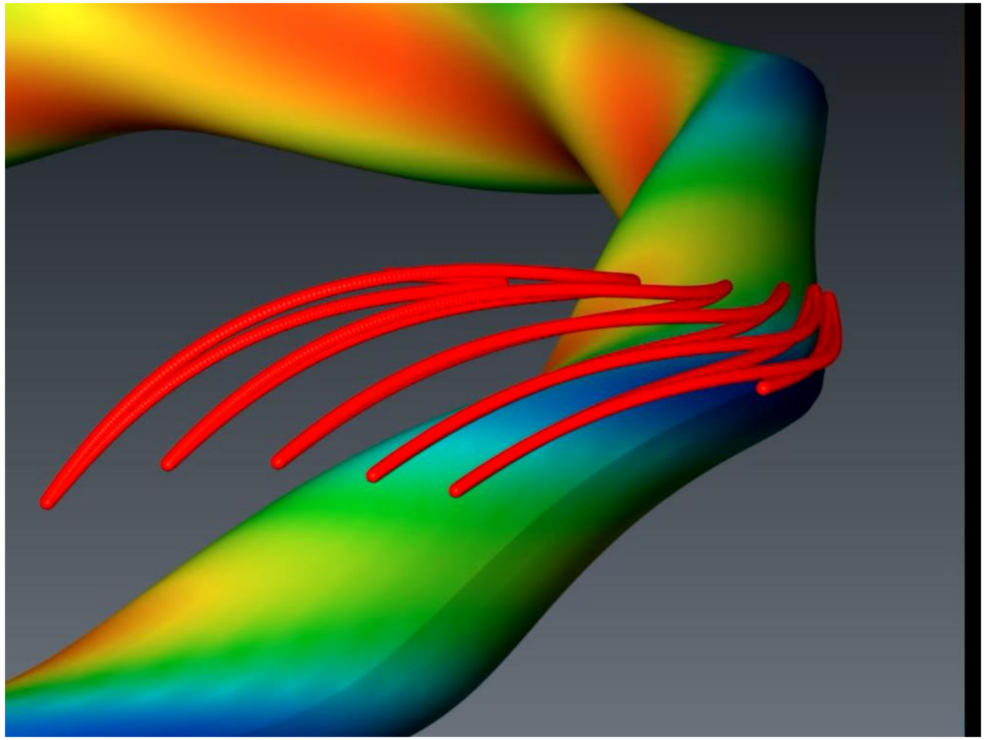
$$\boxed{\text{slowing-down time} \sim \frac{T_e^{3/2}}{n_e} = \frac{p_e^{3/2}}{n_e^{5/2}}}$$

- … and disadvatages:
  - More complicated geometry
  - Larger neoclassical transport
  - Less tested

Particle orbits in a classical stellarator

Collisionless orbits:

- Circulating particles well confined.
- Most trapped ones drift out.



With collisions:

- Large neoclassical energy transport.
- For electrons, typically diffusion coefficient

$$\chi_{\mathrm{nc}} \propto \epsilon_{\mathrm{eff}}^{3/2} \frac{T^{7/2}}{nB^2R^2}$$

$n = \text{density}$

$T = \text{temperature}$

$R = \text{radius}$

At high enough temperature, larger than turbulent transport

$$\chi_{\mathrm{gB}} \sim \frac{T_e^{3/2}}{RB^2}$$

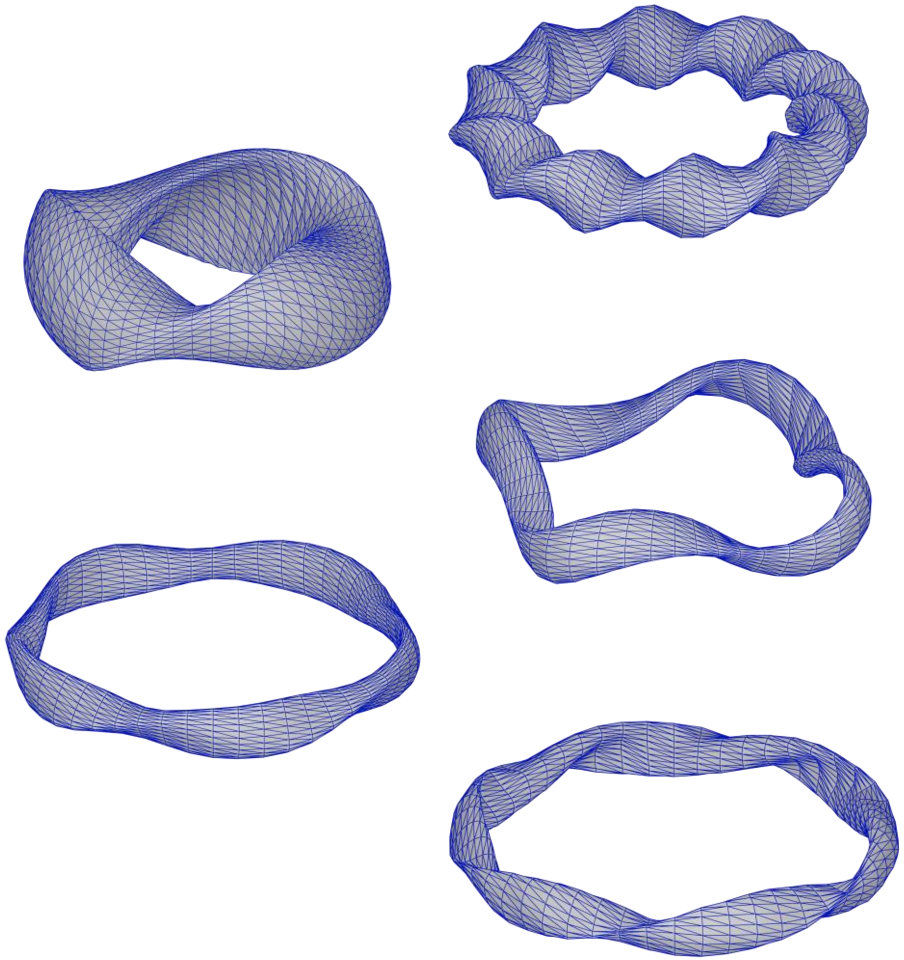
Optimisation and Wendelstein 7-X

Optimisation

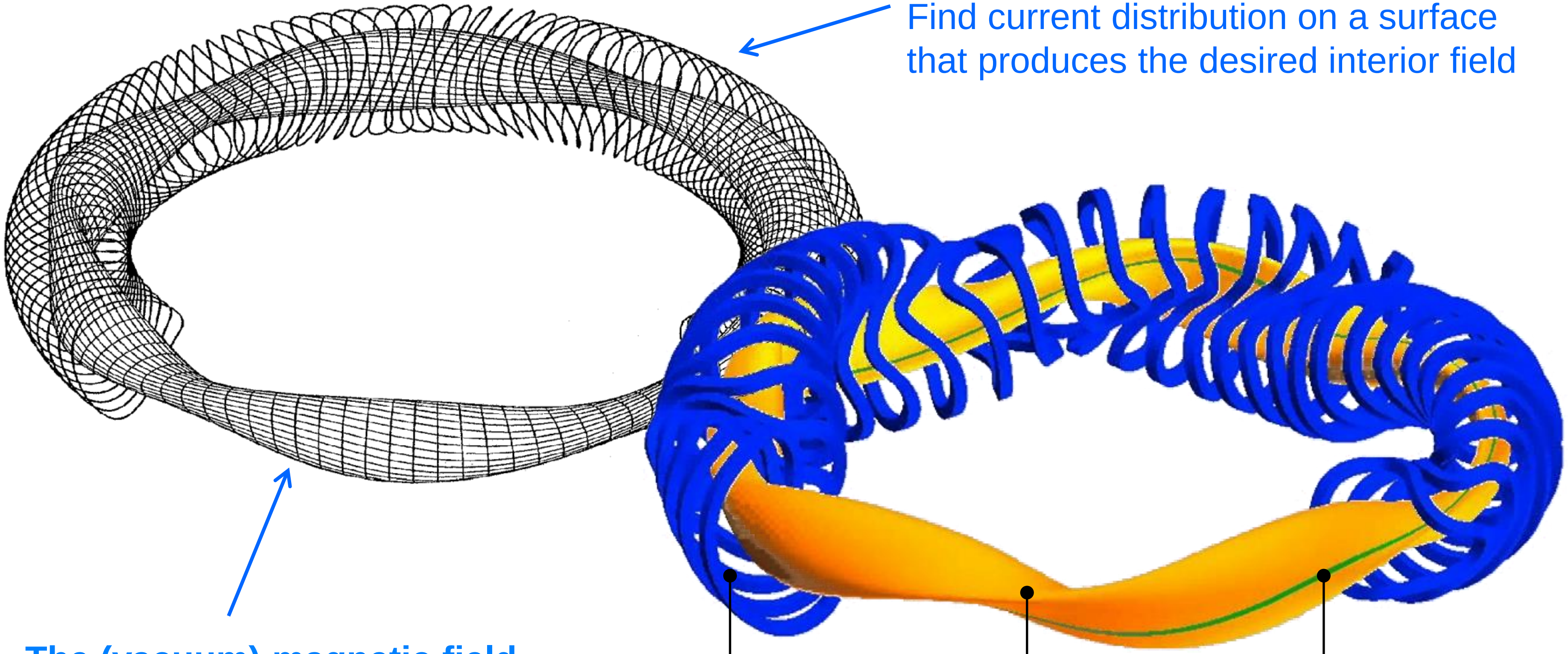
- Any toroidal MHD equilibrium with nested flux surfaces is determined uniquely by
  - the shape of the boundary, and
  - the radial pressure and current profiles.

- Number of degrees of freedom defining the shape of the boundary:
  - Tokamak: about 5
  - Stellarator: about 50

- How can this freedom be used to maximise performance?
  - Plasma equilibrium and stability
  - Thermal and fast-ion confinement
  - Coil complexity
  - …



Optimisation at β = 0



Find current distribution on a surface that produces the desired interior field

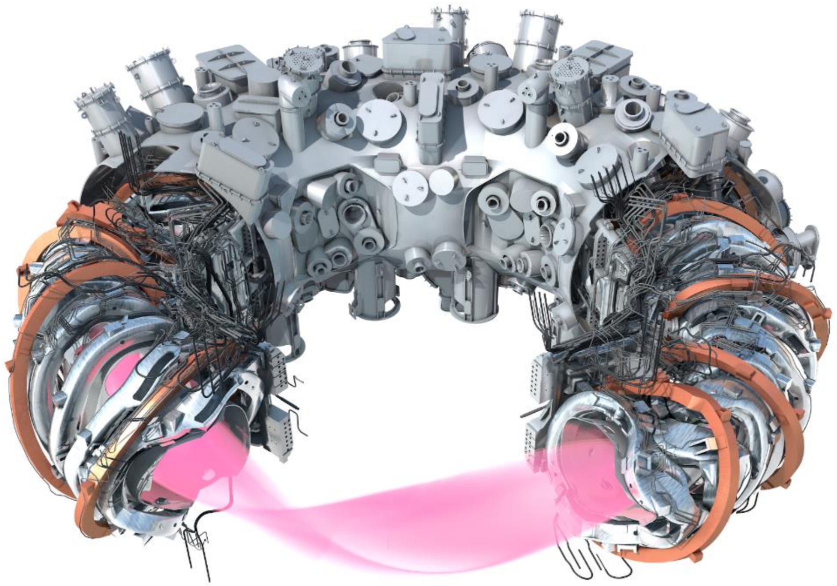
The (vacuum) magnetic field is defined by the shape of the boundary

$$\mathbf{B} = \nabla V$$

$$\nabla^2 V = 0$$

$$\mathbf{B} \cdot \hat{\mathbf{n}} = 0$$

Wendelstein 7-X



Technical parameters:

Major radius: 5.5 m

Plasma radius: 0.55 m

Plasma volume: 30 m^3

70 superconducting coils

Magnetic field (on axis): 2.5 T

Magnetic field energy: 620 MJ

W7-X was optimised for

Robust magnetic equilibrium:

good flux surfaces

small plasma current

- **pressure-driven currents minimised**

small Shafranov shift

Small neoclassical transport

experimentally demonstrated

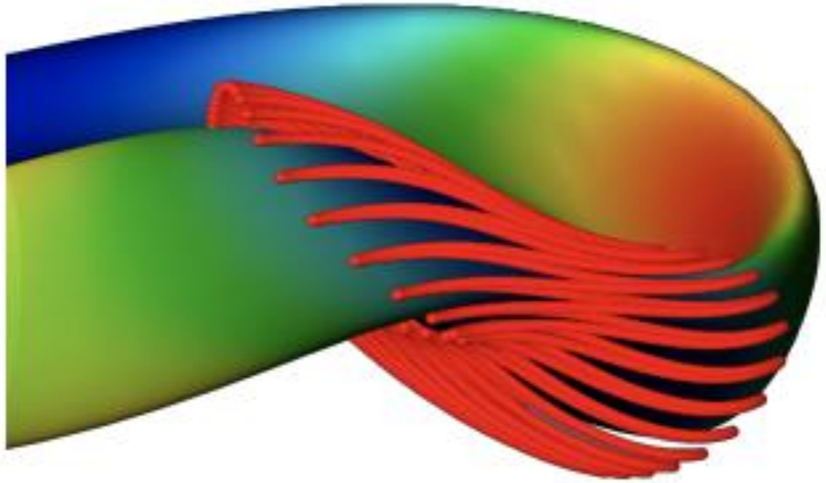
High threshold for ideal MHD pressure-driven instabilities: stable up to $\langle\beta\rangle \simeq 5\%$

Good fast-ion confinement at high β

Particle orbits in Wendelstein 7-X

Collisionless orbits:

- Circulating particles well confined
- Most trapped ones drift also well confined at sufficiently high β.
    - Drift poloidally rather than vertically.



With collisions:

- Much smaller neoclassial transport than in classical stellarators.

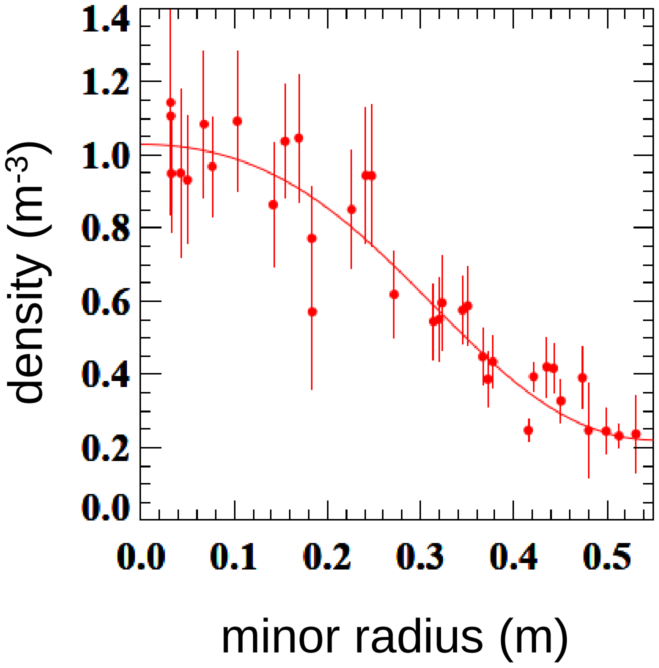
$$\chi_e^{\text{nc}} \sim \boxed{\epsilon_{\text{eff}}^{3/2}} \frac{T_e^{7/2}}{nR^2B^2}$$

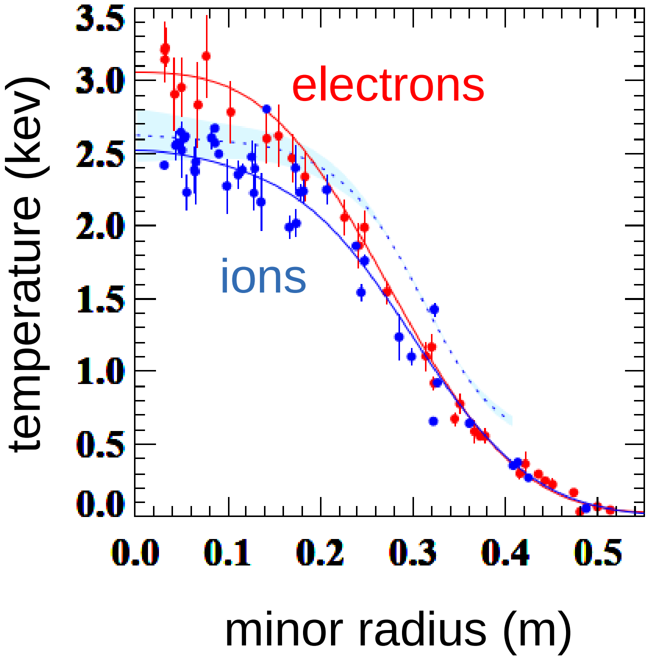
Record performance

Highest fusion triple product for a high-temperature stellarator plasma was transiently achieved in pellet discharges.

Pellets injected in rapid succession cause the density profile to steepen.

Stored energy, T_e and T_i all rise substantially after end of pellet injection. P_{ECRH} = 4.5 MW.



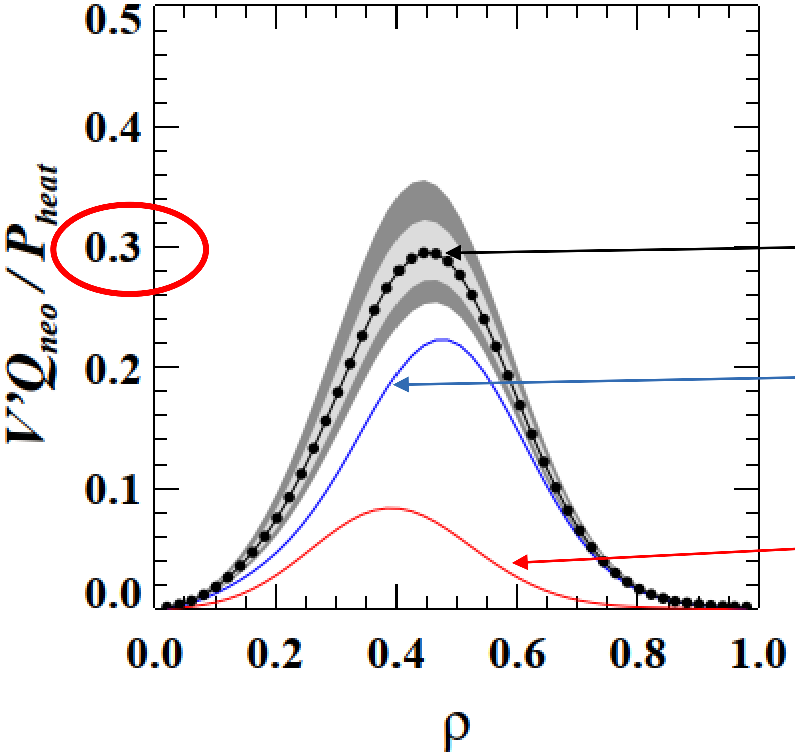


Neoclassical energy fluxes

Normalised (to heating power) energy flux as function of normalised minor radius:

Most of the losses are due to turbulence.

Gray zones correspond to density and temperature variations of 5% and 10%.



total neoclassical energy flux

neoclassical ion energy flux

neoclassical electron energy flux

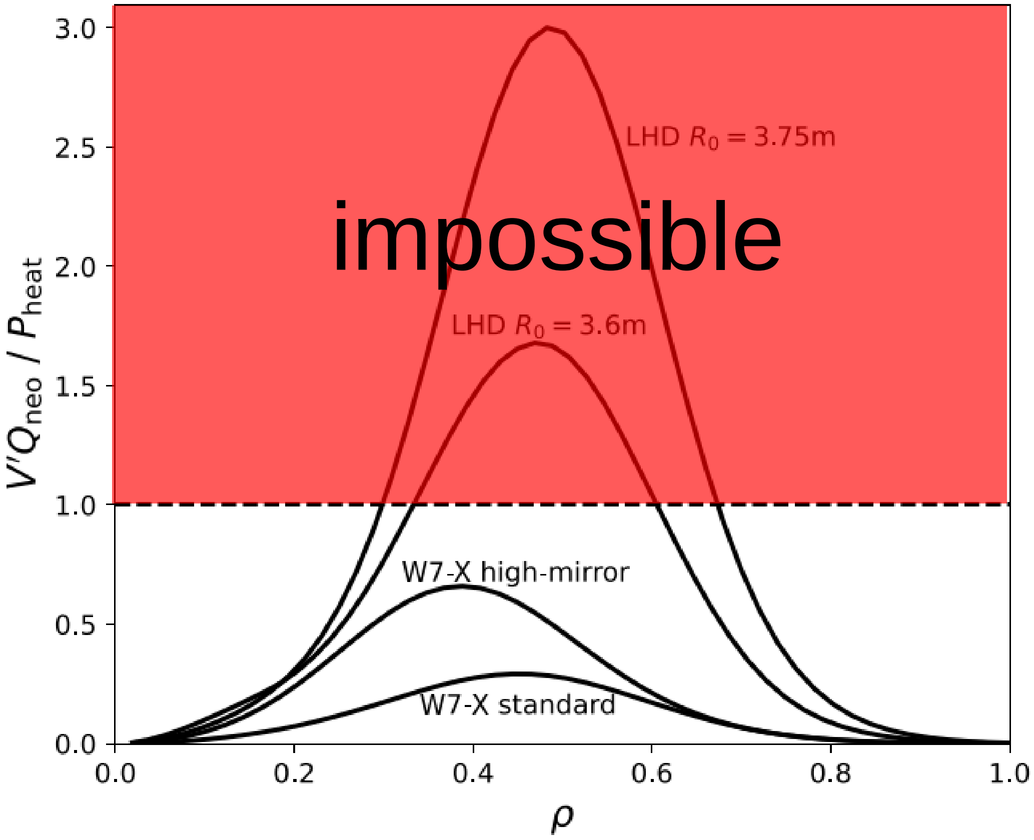
Other magnetic configurations

This plasma was created in the standard magnetic configuration of W7-X.

- We can, however, calculate neoclassical energy fluxes that would have arisen from the same density and temperature profiles in other magnetic configurations.
- The energy flux would have exceeded the heating power in less optimised stellarators.

Neoclassical optimisation of W7-X works.

Energy flux vs radius



Beidler et al, Nature (2021)

Optimisation of future stellarators

Opportunities for improvement

W7-X works very well, but could (and should!) be improved in a few ways:

1. **Fast-ion confinement**
2. **Coils**
3. **Turbulence**

Fast-ion confinement

- Untrapped (circulating) particle orbits are automatically well confined in any stellarator.
- For tokamak-like confinement of trapped particles, omnigenity is necessary:

$$\int (\mathbf{v}_d \cdot \nabla r)\, \frac{dl}{v_{||}} = 0$$

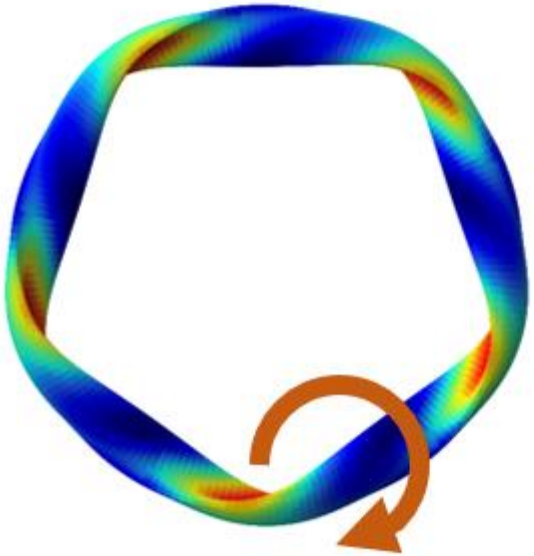
- Requires all level curves of

$$B(r,\theta,\varphi) = |\mathbf{B}(r,\theta,\varphi)|$$

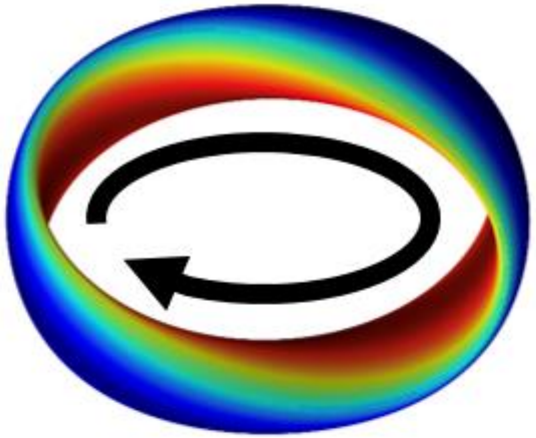
on each flux surface to have the same topology.

- 3 possbilities: These curves must close
    - toroidally (as in tokamaks),
    - helically
    - poloidally

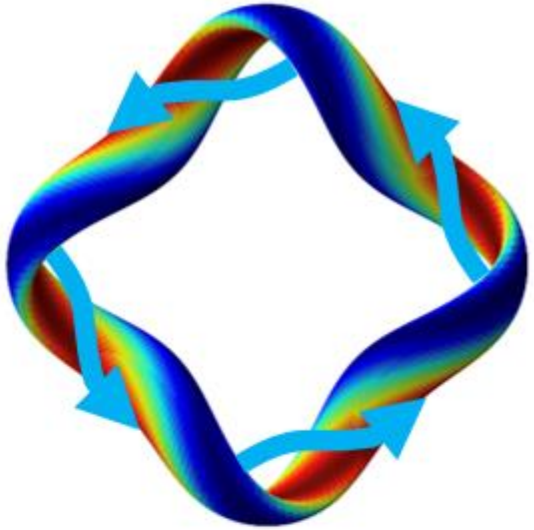
QA, QI, QH



Quasi-isodynamic
(QI)



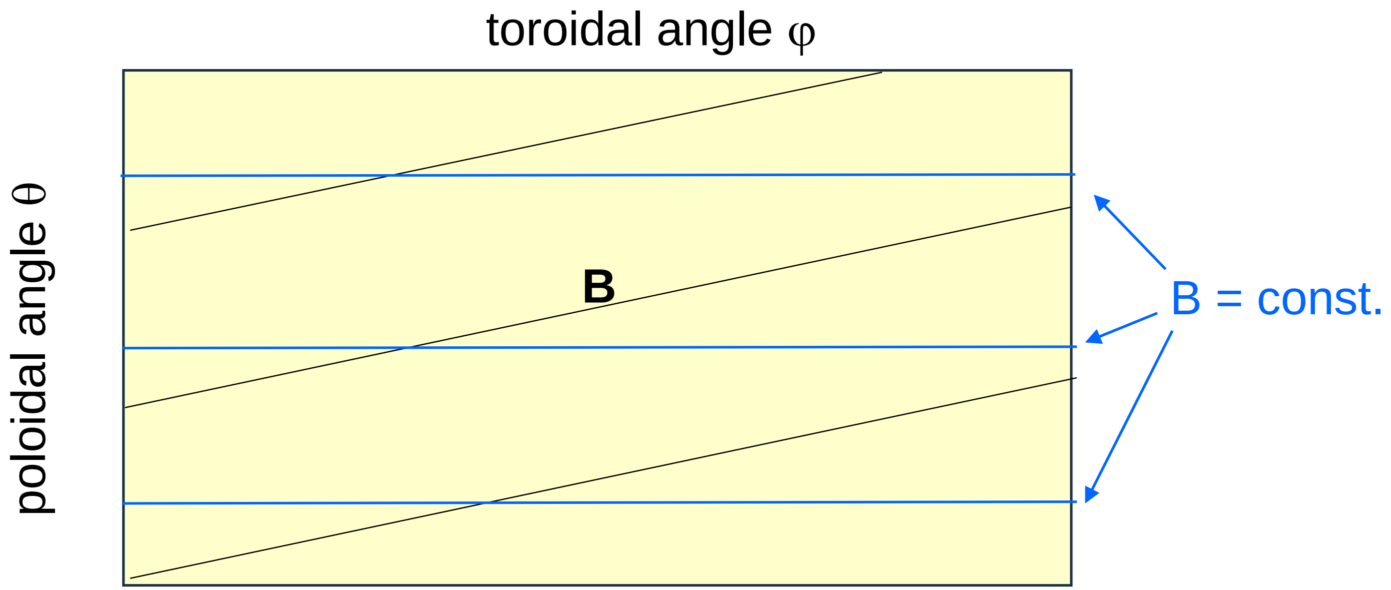
Quasi-axisymmetric
(QA)



Quasi-helically
symmetric (QH)

Quasisymmetry

Field lines are straight in magnetic coordinates

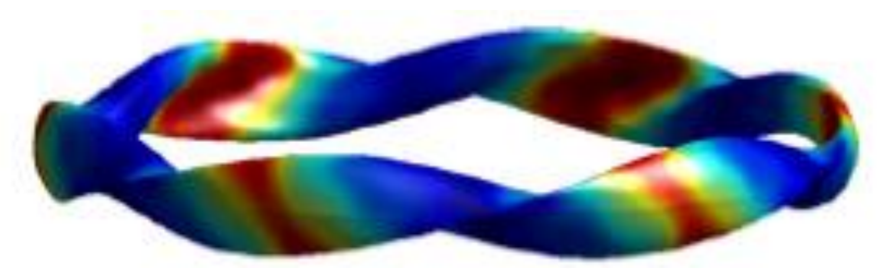


If all contours of constant B are also straight, the field is quasi-symmetric

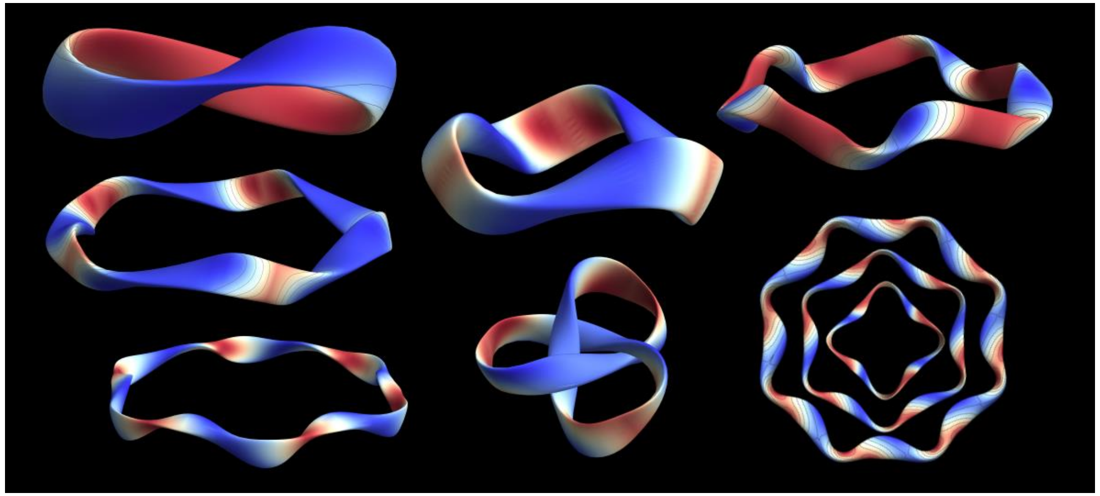
- All field lines are then equivalent, just like in a tokamak.
- Similar neoclassical properties, including bootstrap current.
- Can be achieved to extremely high accuracy (Landreman & Paul, PRL 2022).
- W7-X is <u>**not**</u> quasisymmetric.

Quasi-isodynamic stellarators

- Quasi-isodynamic (QI) stellarators are those that are omnigenous and have poloidally closed B-contours.
- Wendelstein 7-X is, very approximately, of this type.
- Perfectly QI stellarators have no bootstrap current.
  - makes B-field relatively robust



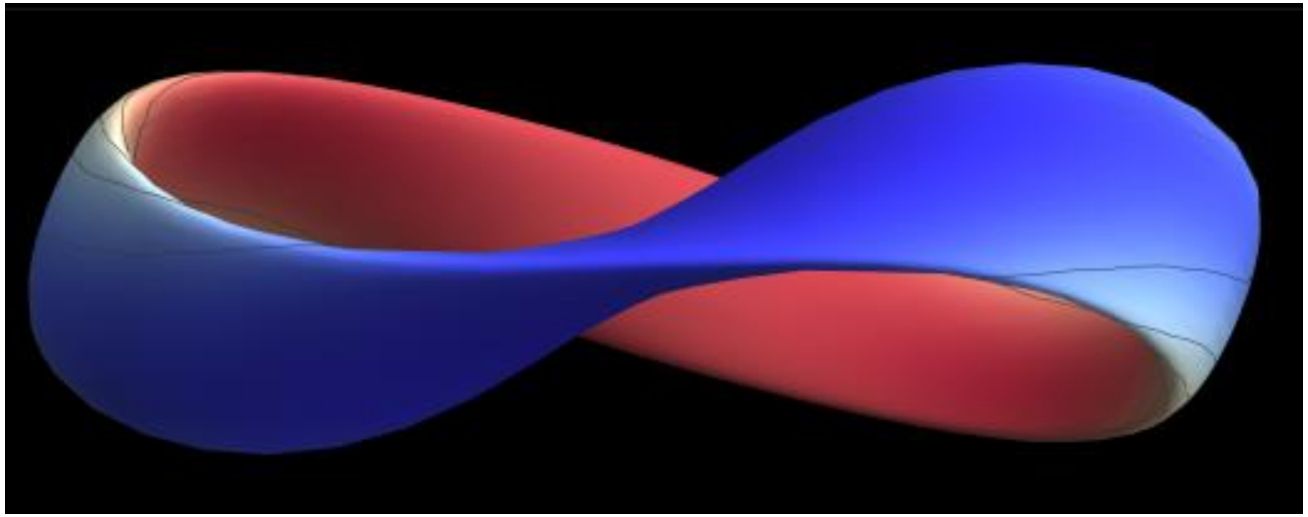
Subbotin et al, Nucl. Fusion (2006)



Plunk, unpublished 2023

Möbius stellarator

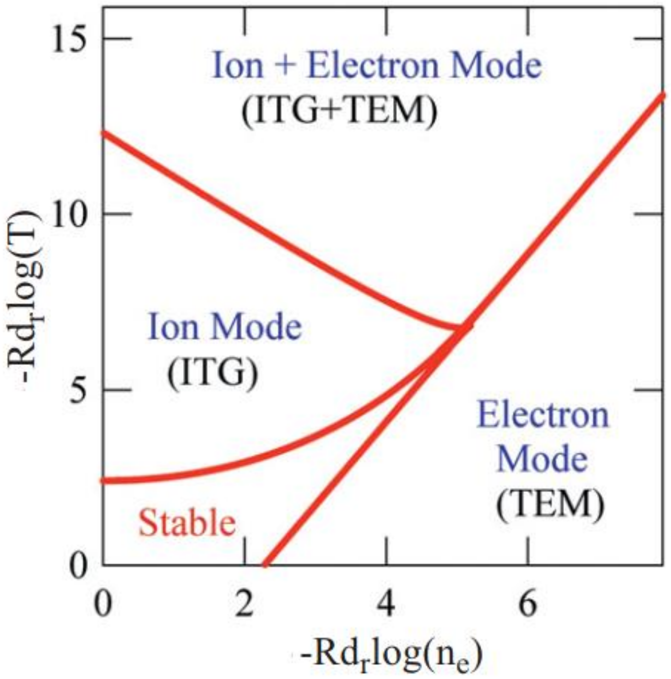
- QI stellarators with small number of field periods and small aspect ratio possible.
- Example: $N = 2, A = 5, \iota = 0.4, \epsilon_{\text{eff}} = 0.006$



Microinstabilities

Turbulence is electrostatic ($\delta\mathbf{B}=0$) at low β, and is usually believed to be caused mainly by two types of micro-instabilities:

- Ion-temperature-gradient (ITG) modes
  - stabilised by density gradient
- Trapped-electron modes (TEMs, driven by density gradient or electron temperature gradient)
  - Caused by trapped electrons in regions of ”bad curvature”



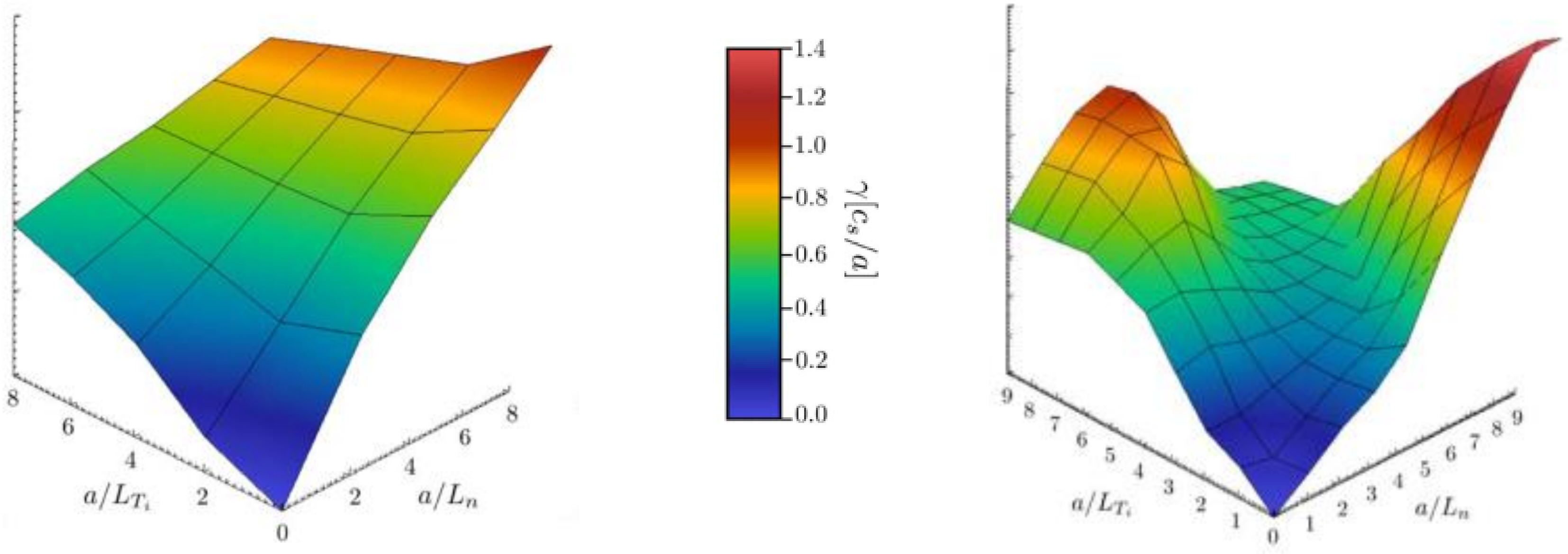
Garbet et al, PPCF 2004

How do these instabilities depend on magnetic geometry?

Stability diagram

Growth rate vs density and temperature gradient

GENE stellarator flux-tube runs with kinetic electrons, no collisions or electromagnetic fluctuations

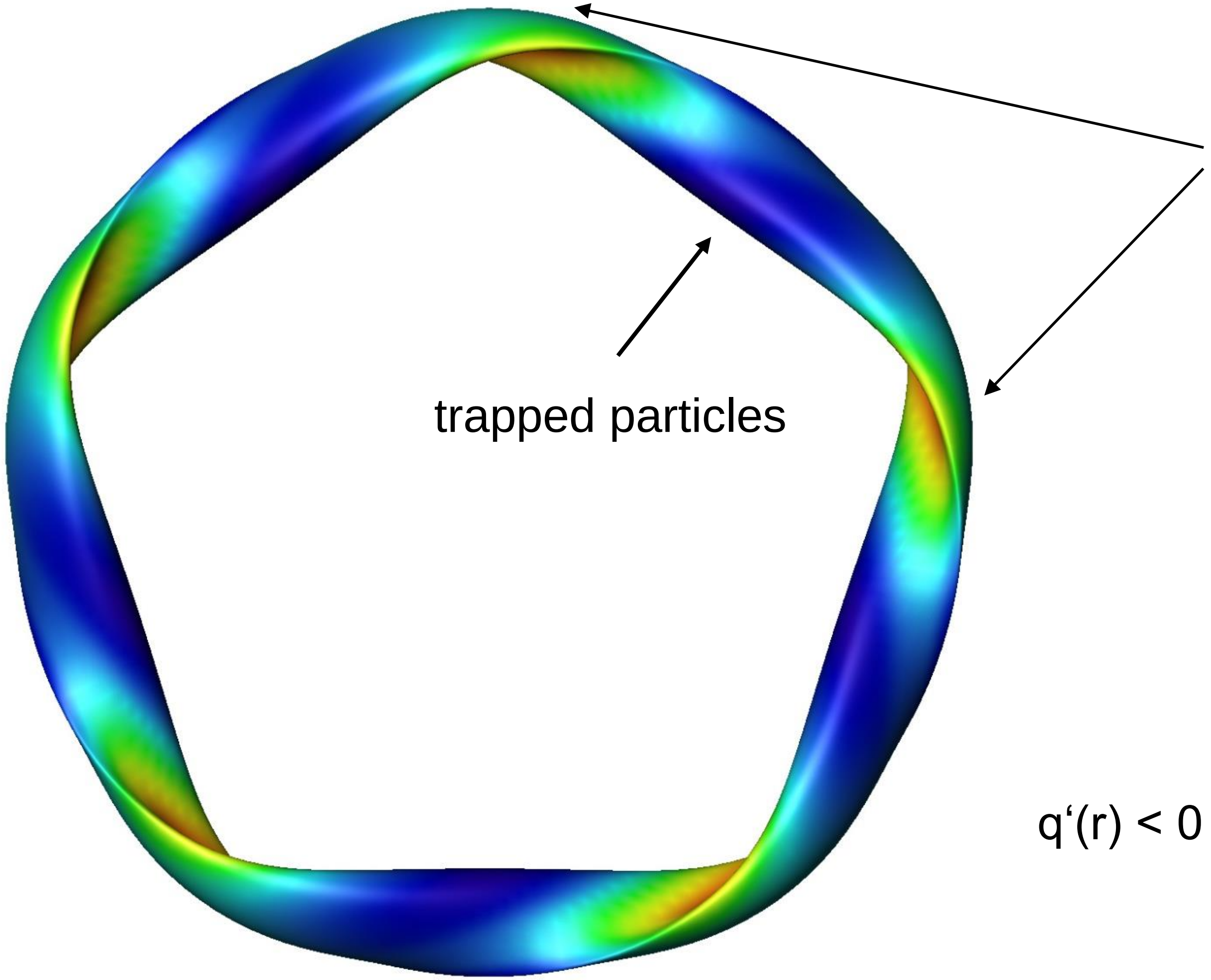


Tokamak

W7-X standard configuration

Alcuson et al, PPCF 2020

Wendelstein 7-X from above


bad curvature

trapped particles

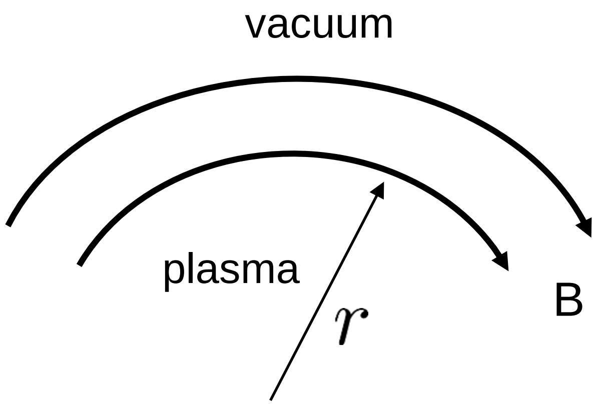
q‘(r) < 0 everywhere

Good and bad curvature

- Electrons move quickly along field lines.
  - bounce frequency >> turbulence frequency
- Adiabatic invariant of trapped partices is conserved:

$$J = \oint m v_{\|} dl$$

depends on minor radius r and energy E.



- Conservation of J implies for convex field lines

$$\Delta J = \frac{\partial J}{\partial r}\Delta r + \frac{\partial J}{\partial E}\Delta E = 0 \qquad \Rightarrow \qquad \frac{\Delta E}{\Delta r} = -\frac{\partial J/\partial r}{\partial J/\partial E} < 0 \quad \text{if} \quad \frac{\partial J}{\partial r} > 0$$

- It is thus energetically favourable to move particles outward.
  - vice versa if $\frac{\partial J}{\partial r} < 0$

Ion-temperature-gradient driven instabilities

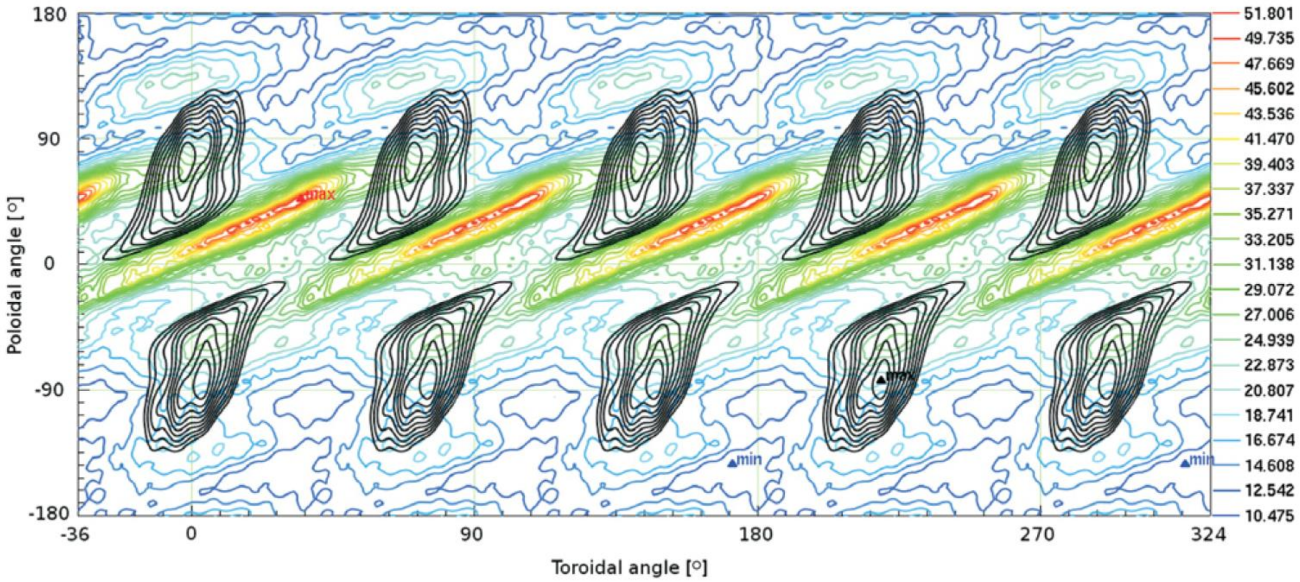
Roberg-Clark et al 2023

Ion-temperature gradient (ITG) instabilities difficult to eliminate.

- Slab branch exists in straight field.
- Curvature-driven branch more virulent.
  - Caused by ITG in regions of bad curvature.
  - Can try to reduce the ITG in such regions.

$$\text{Increase } |\nabla\alpha|^2, \text{ where } \mathbf{B} = \nabla\psi \times \nabla\alpha$$

  - Reduce connection length along B between regions of good and bad curvature, and add localised magnetic shear.



W7-X:

turbulence amplitude: colour

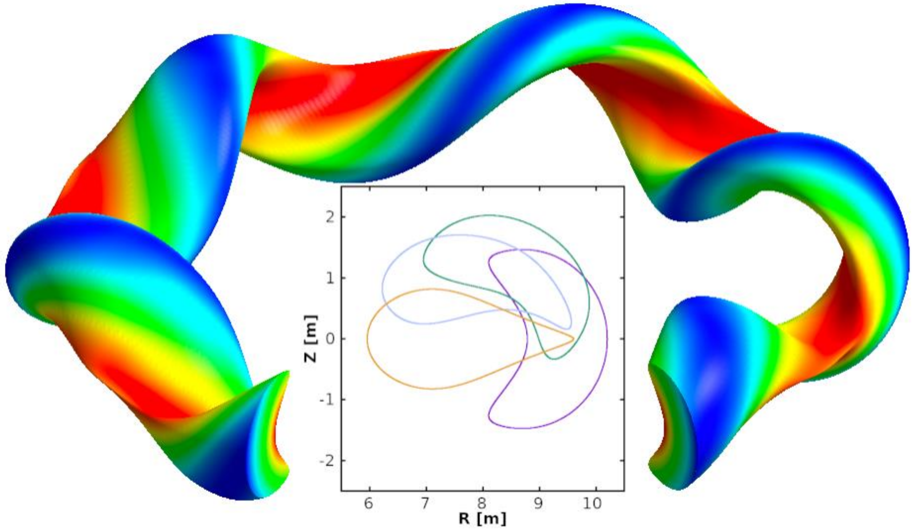
$|\nabla\alpha|^2$: black

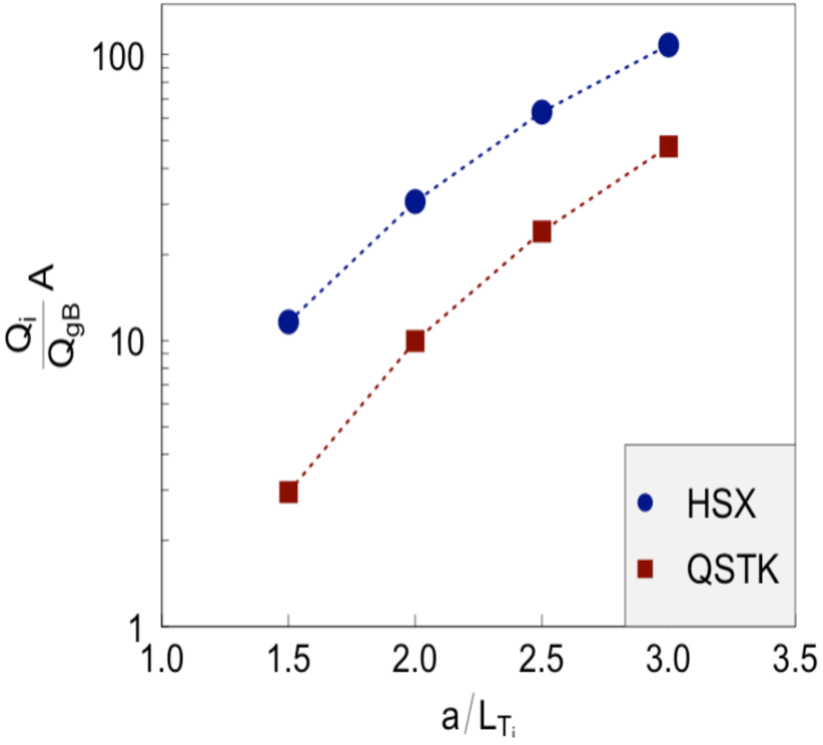
Helander et al. PPCF 2012

A stellarator with ITG turbulence reduction

QSTK: a 6-periodic QH stellarator:

- Better fast-ion confinement than W7-X
- Lower thermal neoclassical transport than W7-X
- MHD stable
- ITG turbulence pushed away from worst-curvature region
  - Lower turbulent transport



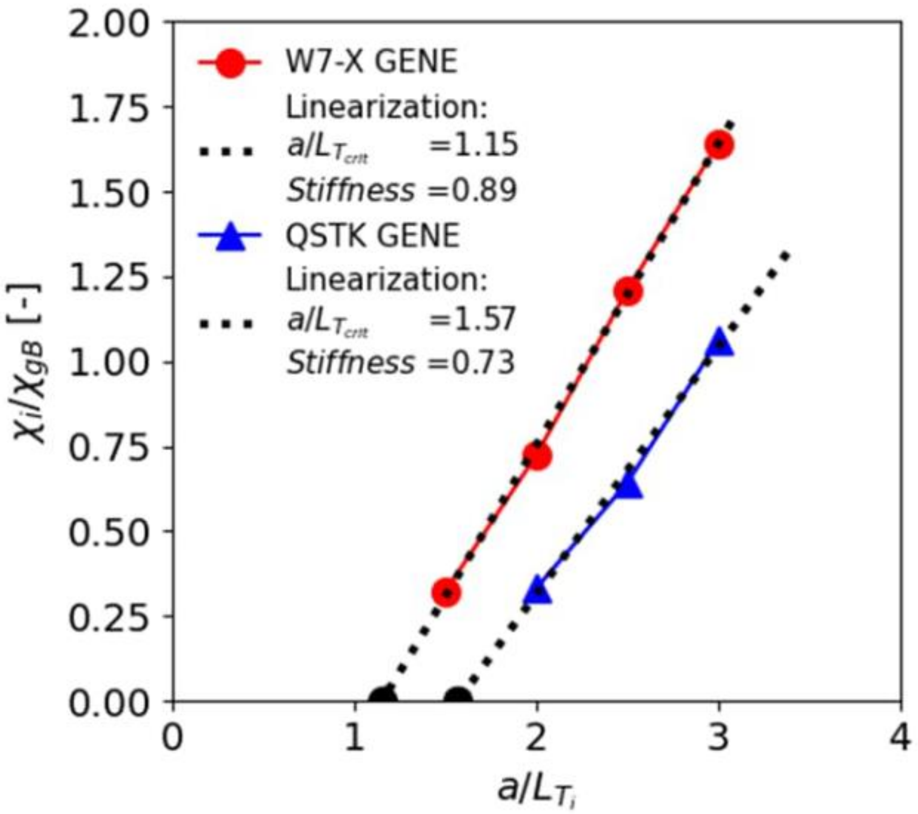


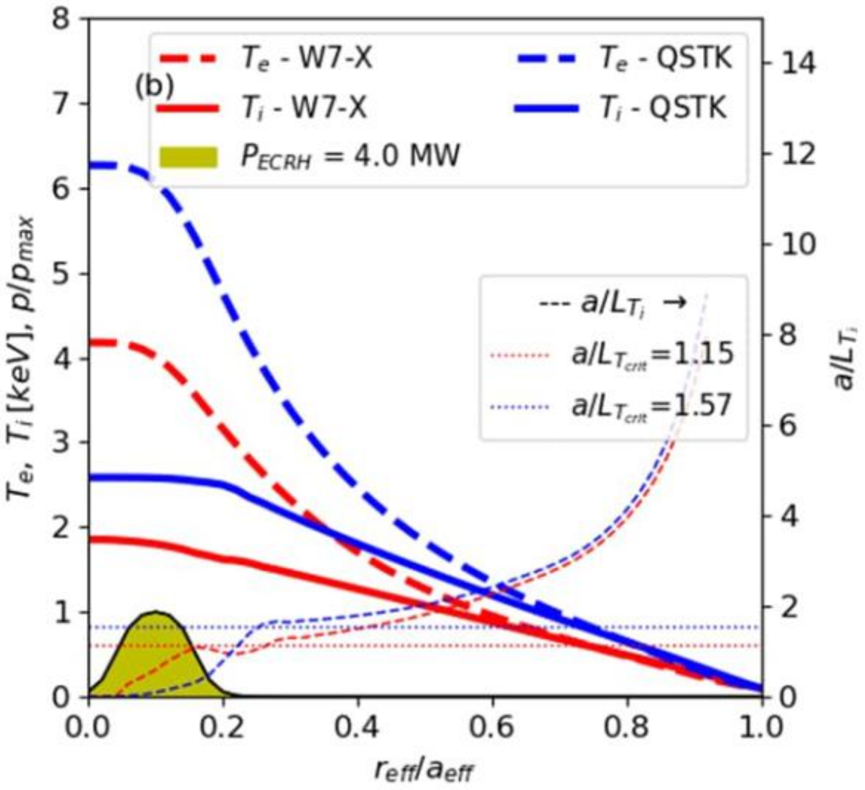
Energy flux vs temperature gradient in gyrokinetic simulations with adiabatic electrons.

Roberg-Clark et al. PRR 2023

Plasma performance of QSTK

Thought-experiment of QSTK scaled to same magnetic field, minor radius, and ECRH power as W7-X:



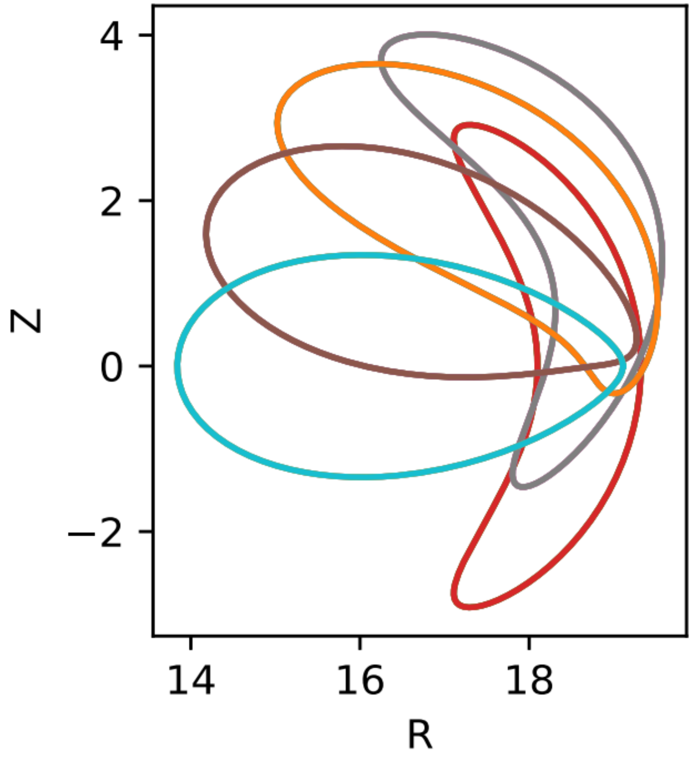


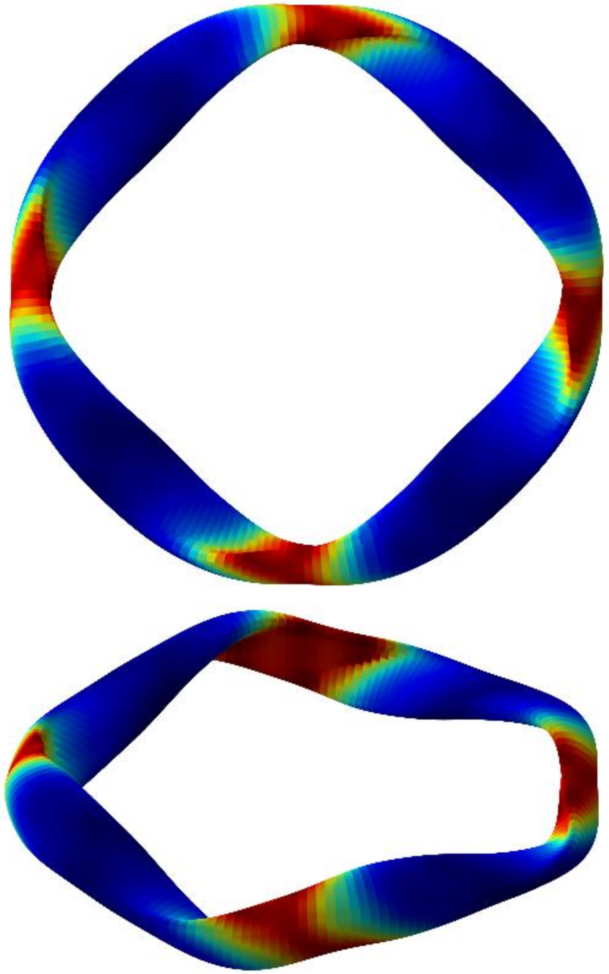
Beurskens et al., EPS 2023

Novel quasi-isodynamic stellarators

Very attractive QI stellarators becoming available, combining

- Good MHD stability



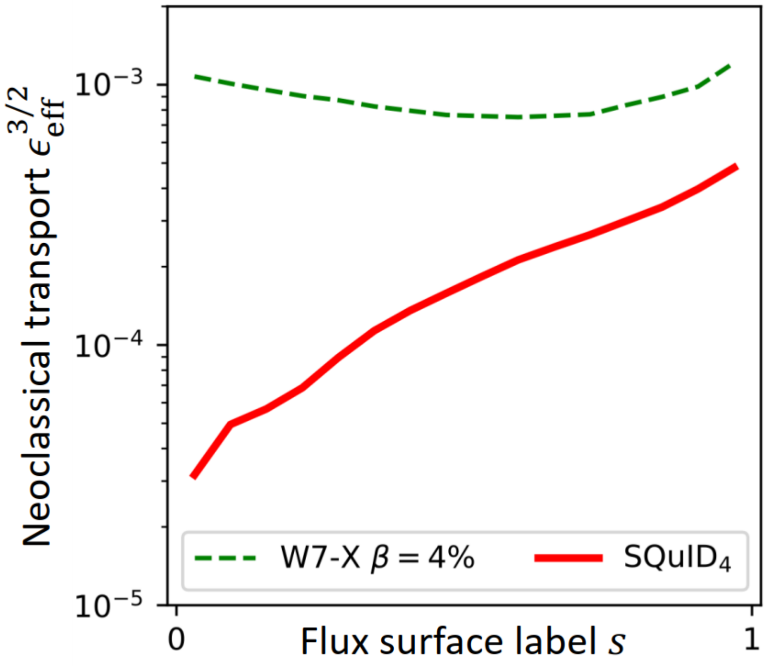


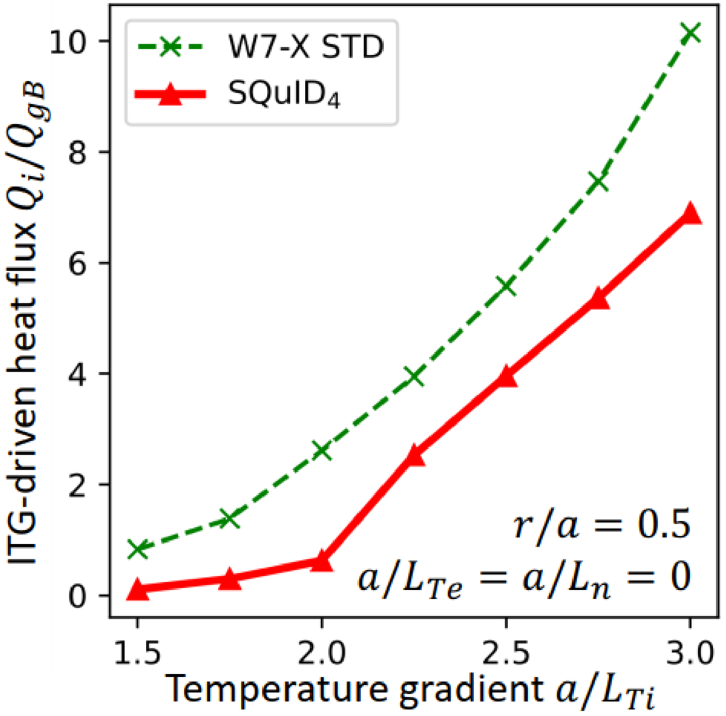
Goodman, this conference; see also next talk by Sanchez

Novel quasi-isodynamic stellarators

Very attractive QI stellarators becoming available, combining

- Good MHD stability
- Very small neoclassical transport and bootstrap current
- Reduced ITG and TEM turbulence

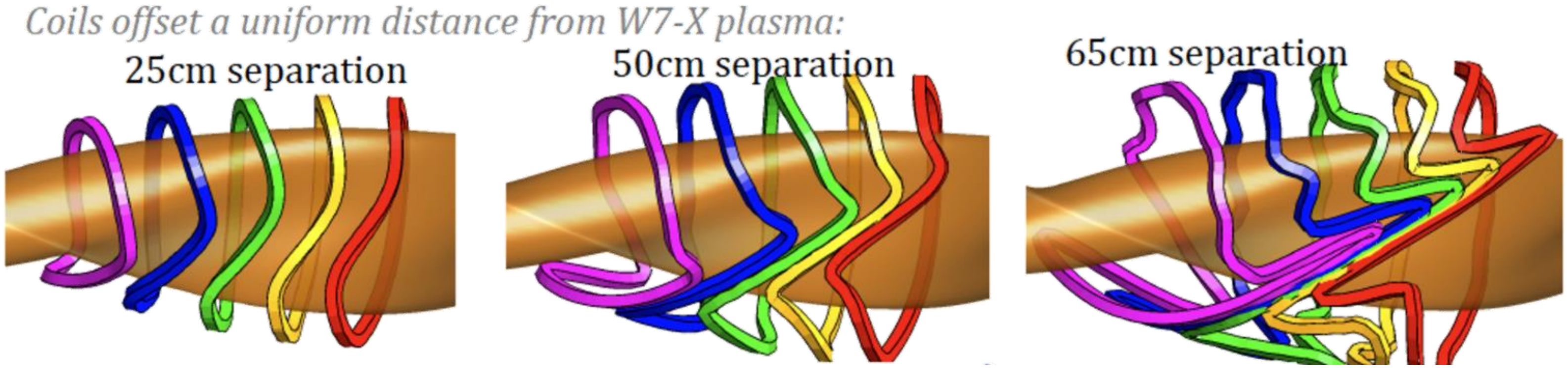




Goodman, this conference

Coils

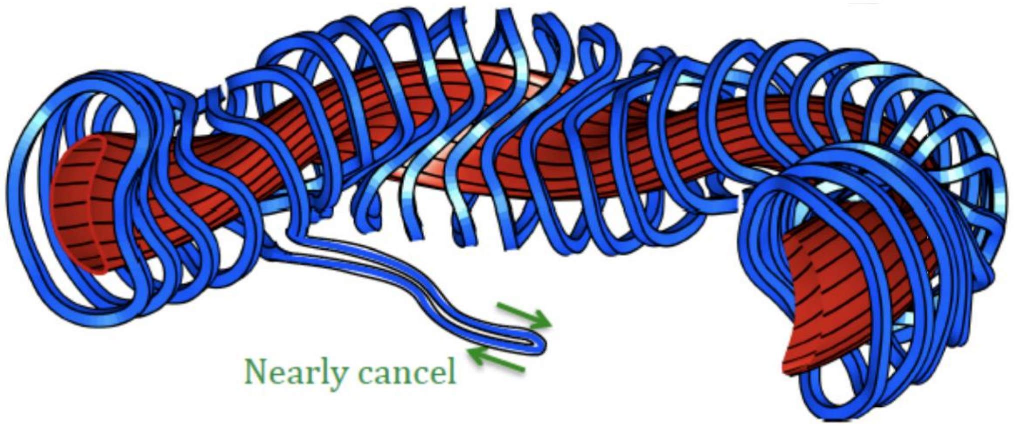
The coils should be situated some distance from the plasma. Can be difficult!



Landreman, 2019

A doubly ill-posed problem!

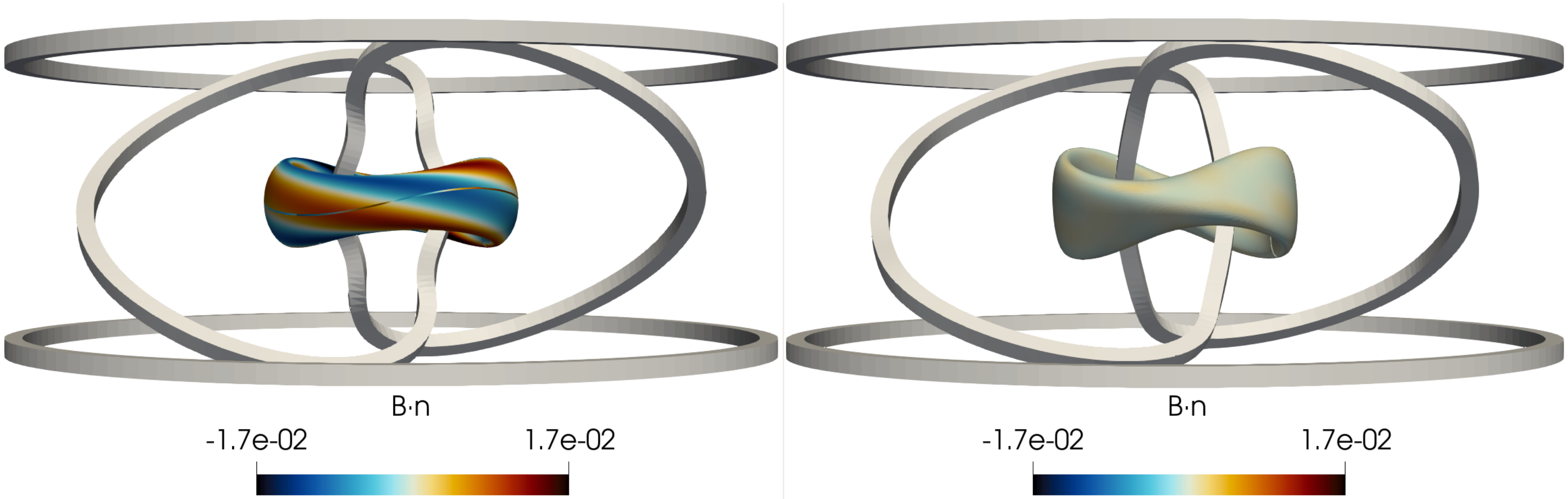
- In general, impossible to create exactly the desired field.
- Many different coils will create the same field.
  - Only therefore is it solvable in practice!



Simultaneous optimisation of plasma & coils

New development: combined optimisation of plasma & coils. (Henneberg et al. 2021, Jorge et al. 2023)

- Traditionally difficult because of unavailability of fast, robust, free-boundary equilibria.
- Now becoming possible.
- Example of a quasisymmetric stellarator with only 4 coils:



Optimisation of plasma first, coils later

Simulataneous optimisation of plasma & coils

Jorge 2023

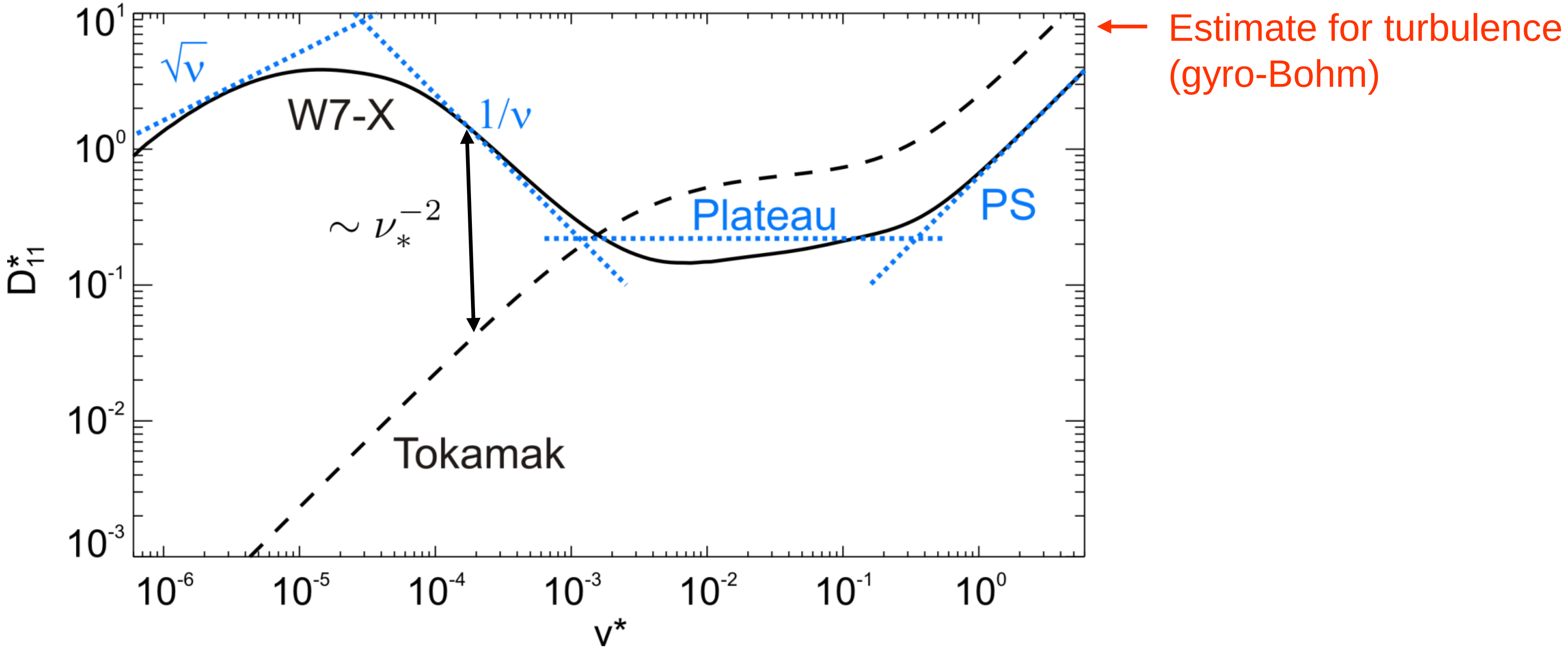
Summary

- In the past few years, stellarator optimisation has emerged as a vibrant new field.
  - Negligible neoclassical transport losses
  - Alpha-particle confinement as good as in tokamkas. Or better!
  - Possibility of controlling turbulence?
  - Simpler coils than in the past

- New kinds of stellarators possible.

Additional material

Neoclassical transport



Normalised “monoenergetic“ diffusion coefficient vs collisionality

$$\nu^* = \frac{\text{plasma radius}}{\text{mean free path}}$$

Neoclassical energy flux

The neoclassical energy flux can be calculated with some confidence.

depends on the radial electric field (particularly for the ions), but this can also be calculated.

fluxes of particles and energy

$$\begin{bmatrix} \Gamma^{\alpha}_{neo} \\ Q^{\alpha}_{neo}/T^{\alpha} \end{bmatrix} = -n^{\alpha} L^{\alpha}_{11} \left\{ \begin{bmatrix} 1 \\ \delta^{\alpha}_{21} \end{bmatrix} \left(\frac{1}{n^{\alpha}} \frac{\mathrm{d}n^{\alpha}}{\mathrm{d}r} - \frac{q^{\alpha} E_r}{T^{\alpha}} \right) + \begin{bmatrix} \delta^{\alpha}_{12} \\ \delta^{\alpha}_{22} \end{bmatrix} \frac{1}{T^{\alpha}} \frac{\mathrm{d}T^{\alpha}}{\mathrm{d}r} \right\}$$

Ambipolarity, $\Gamma^{i}_{neo} = \Gamma^{e}_{neo}$, implies

$$\frac{eE_r}{T^{\alpha}} = \frac{T^e T^i}{T^{\alpha} \left(L^e_{11} T^i + L^i_{11} T^e \right)} \left\{ \left(L^i_{11} - L^e_{11} \right) \frac{1}{n} \frac{\mathrm{d}n}{\mathrm{d}r} + L^i_{11} \delta^i_{12} \frac{1}{T^i} \frac{\mathrm{d}T^i}{\mathrm{d}r} - L^e_{11} \delta^e_{12} \frac{1}{T^e} \frac{\mathrm{d}T^e}{\mathrm{d}r} \right\} .$$

Works even in the presence of gyrokinetic turbulence, because regardless of E_r

$$\Gamma^{i}_{turb} = \Gamma^{e}_{turb}$$

E_r is neclassical, even if most of the transport is turbulent.

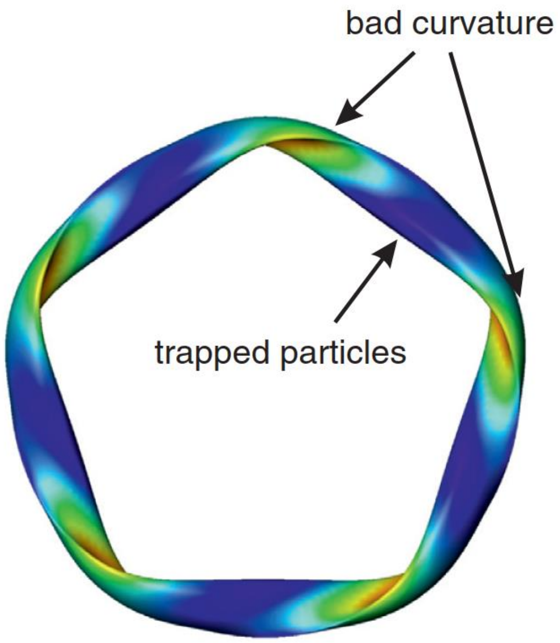
Usually agrees with experimental measurements.

Trapped-electron modes

Trapped-electron modes

Caused by electron trapping in bad-curvature regions

Overlap minimised in W7-X



Density-gradient-driven TEM instability requires

$$\overline{\omega}_{de}\omega_{*e} > 0$$

where

$$\overline{\omega}_{de} = \text{electron precession frequency} \propto -\frac{dJ}{dr}$$

$$\omega_{*e} = \text{electron diamagnetic frequency} \propto \frac{dn}{dr}$$

implying

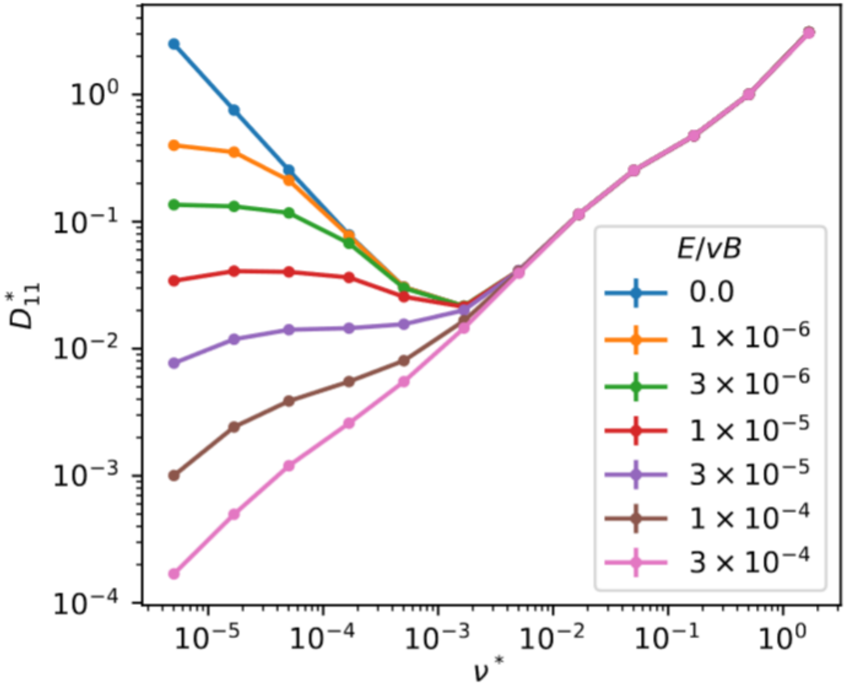
$$\frac{dJ}{\partial r}\frac{dn}{dr} < 0$$

Rosenbluth, PoF 1968
Proll et al, PRL 2012
Helander et al, PoP 2013

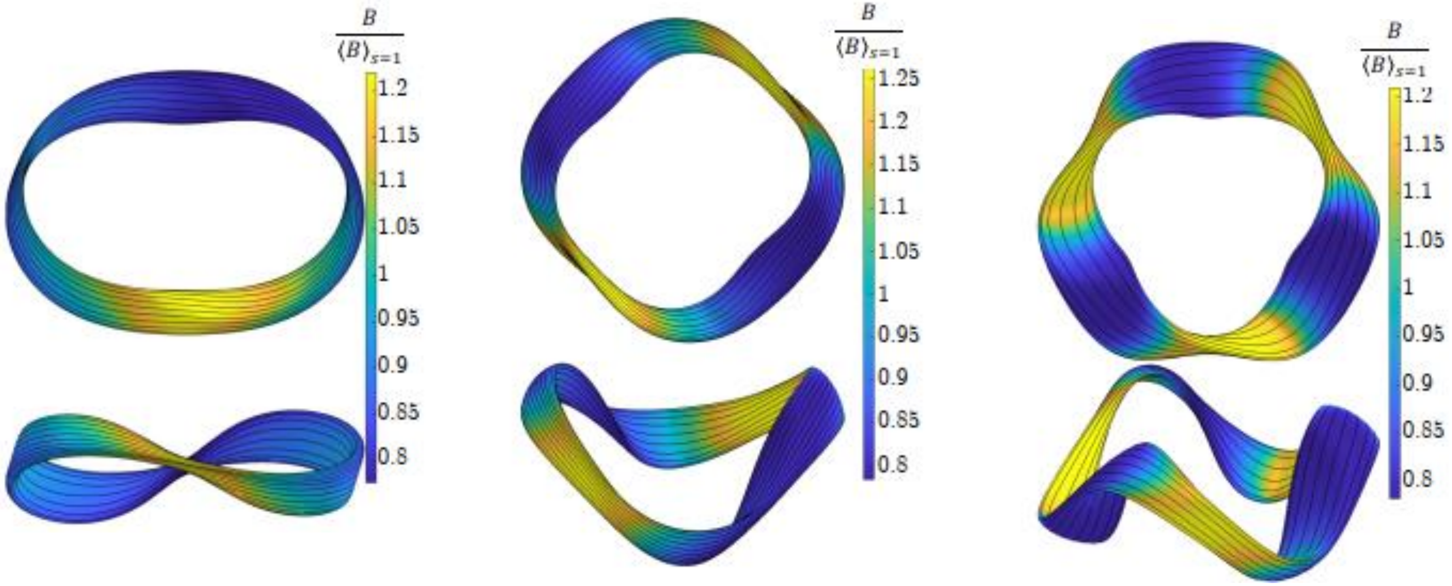
Novel quasi-isodynamic stellarators

- Virtually exactly QI fields achievable $(\epsilon_{\text{eff}} < 10^{-3})$
  - Very small alpha-particle losses and bootstrap current
  - Unusual neoclassical properties_
    - Small E_r eliminates $1/\nu$-regime
    - No plateau regime
    - Impurity expulsion possible

Neoclassical diffusivity vs collisionality



Goodman et al, JPP 2023



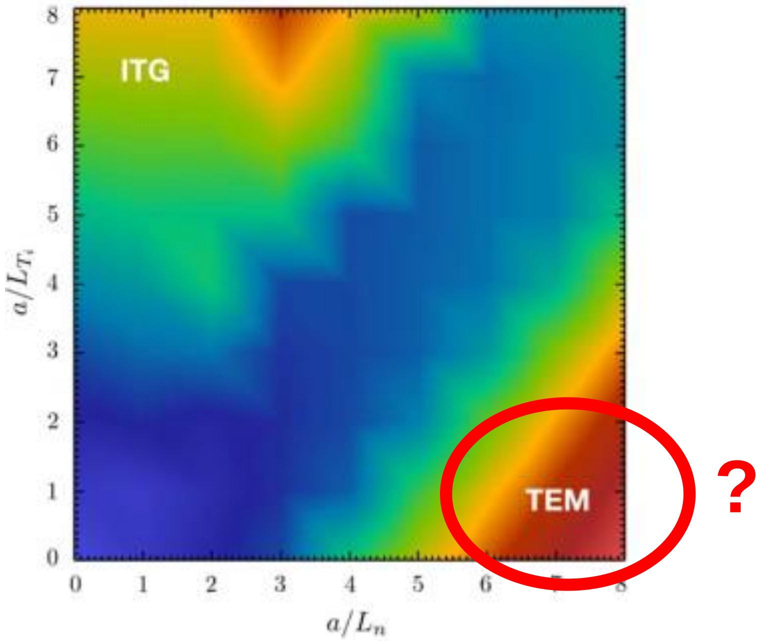
Microinstabilities in W7-X

- Ion-temperature-gradient-driven activity comparable to that in tokamaks.
  - Limits confinement and causes T_i clamping.

- Density-gradient-driven modes much more benign.

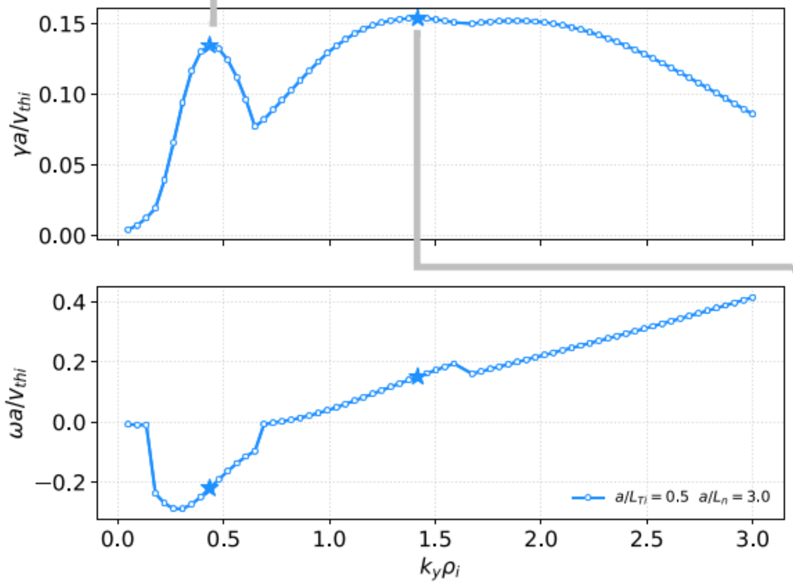
  - Analytical predictions (2012-2017)
    - Conventional trapped-electron-modes (TEMs) stable.
    - Instead: the so-called “universal mode” and “ion-driven TEMs”.

  - Are these expectations borne out by simulations?



Density-driven microinstabilities in W7-X

- Linear, flux-tube, electrostatic simulations in the high-mirror (KJM) magnetic configuration with kinetic electrons
- Finite ∇T_i and ∇n, $\nabla T_e = 0$



Growth rate and frequency vs wavenumber

Long wavelengths: the “universal mode“

Short wavelengths: ion-driven trapped-electron modes

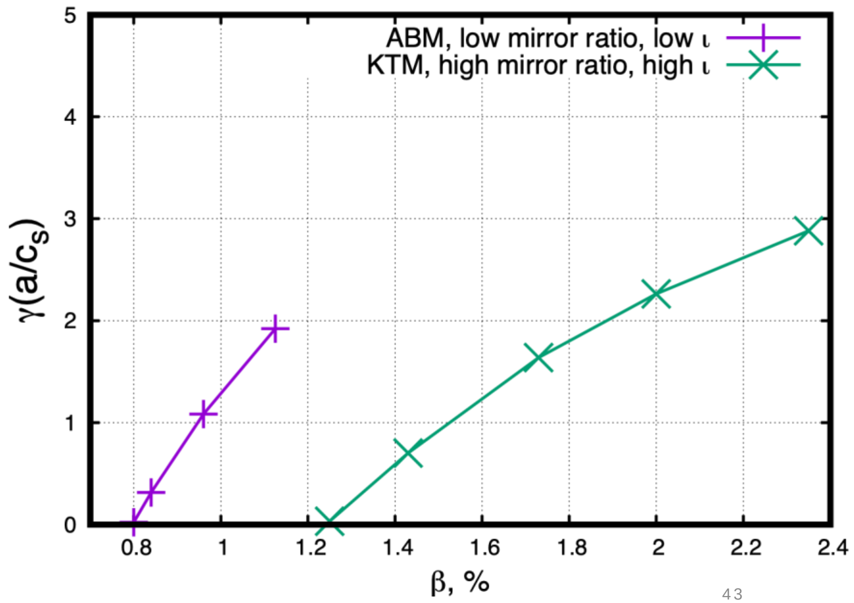
- Both types of modes destabilised by grad n and stabilised by grad T_i
- Investigation of global eigenmodes underway.

Electromagnetic microinstabilities

- How are microinstabilies affected by finite plasma β?

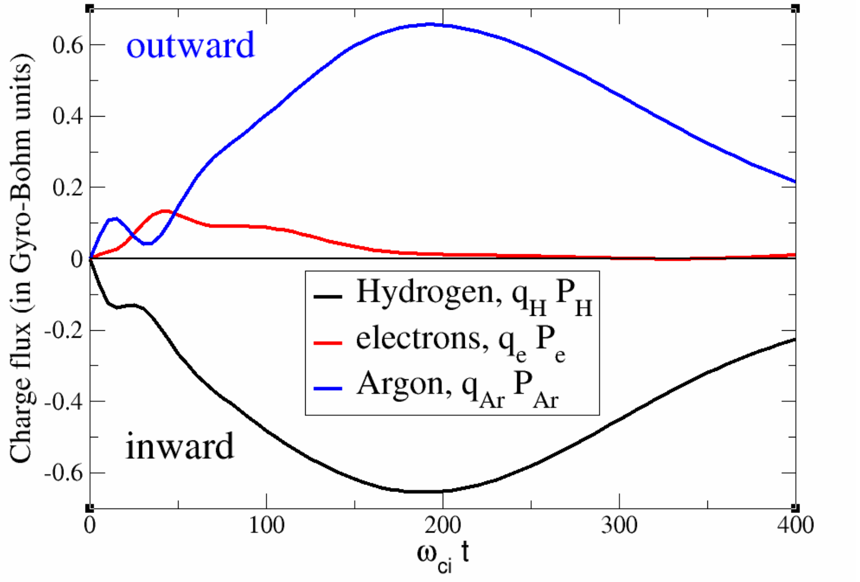
- How do electromagnetic instabilities (kinetic ballooning modes) depend on magnetic geometry?

  - Can we find configurations in W7-X where they are particularly stable?
    - High mirror ratio and high rotational transform help.
    - Configuration (KTM) identified and selected for experiments.



Impurity transport

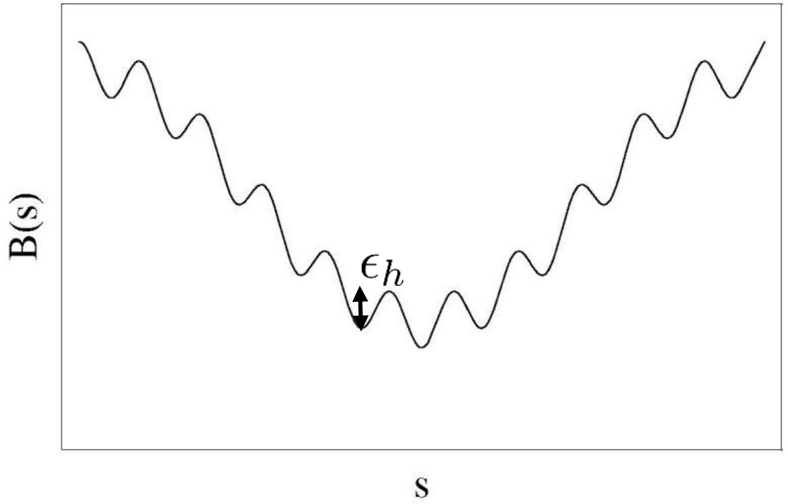
- Global simulations of electromagnetic turbulence with impurities in W7-X.
  - Argon goes out, hydrogen goes in.



- Further developments of the EUTERPE code:
  - GPU version
  - Equilibrium magnetic island capability
  - First steps towards a C++ version for compatibility with future HPC hierarchical architectures
  - Terms describing neoclassical physics added

Neoclassical transport

Random walk due to bad orbits:



- Step length

$$\Delta r \sim v_d \Delta t$$

- Time between steps

$$\Delta t \sim \epsilon_h / \nu$$

- Diffusion coefficient

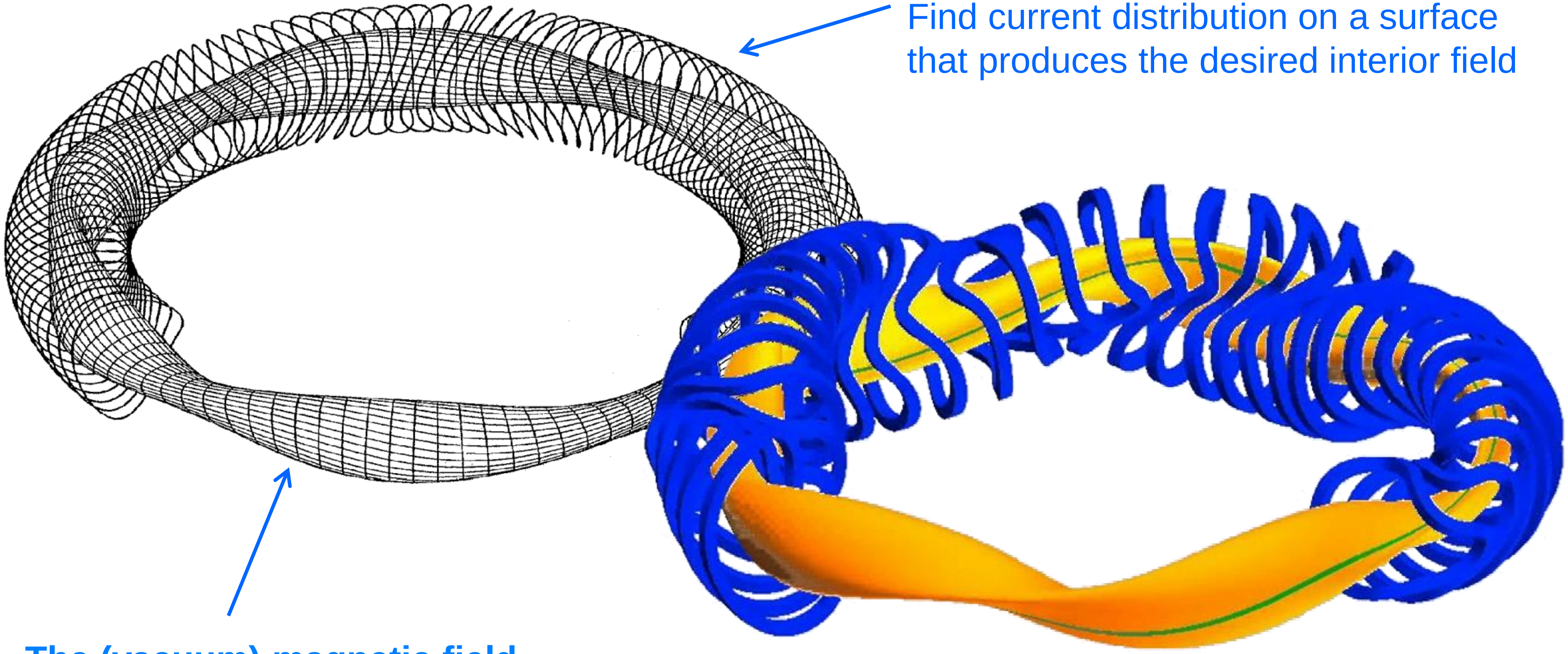
$$D_{1/\nu} \sim \epsilon_h^{1/2} \frac{\Delta r^2}{\Delta t} \sim \frac{\epsilon_h^{3/2} v_d^2}{\nu}$$

- Large at high temperatures, since

$$D_{1/\nu} \propto \frac{m^{1/2} T^{7/2}}{n B^2 R^2}$$

Coils

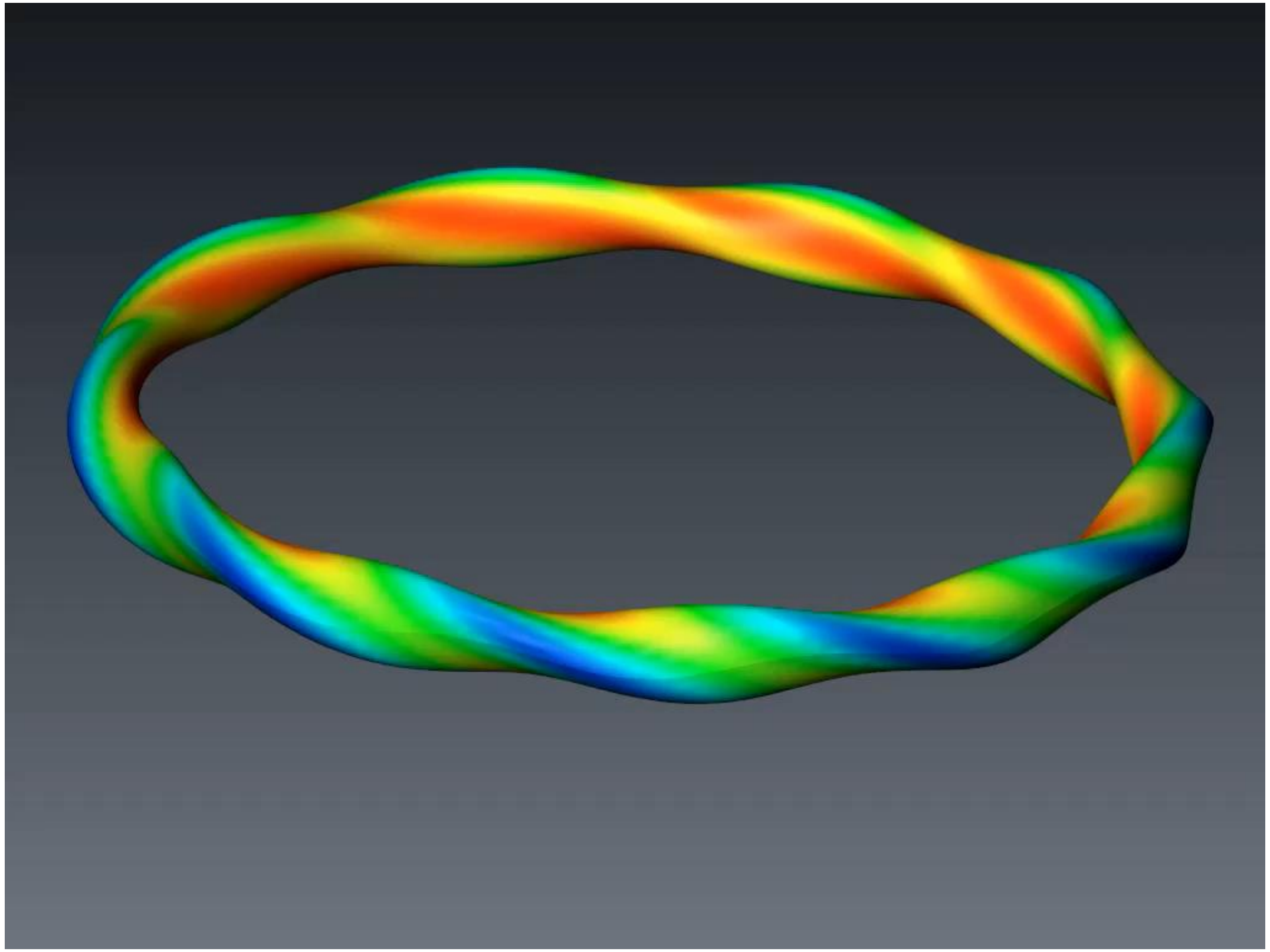
The second stage of (conventional) stellarator optimisation consists in finding coils that produce the desired magnetic field.



Find current distribution on a surface that produces the desired interior field

The (vacuum) magnetic field is defined by the shape of the boundary

Particle orbits in a classical stellarator



With collisions: so-called “neoclassical“ heat conductivity.

For electrons, typically

$$D_{1/\nu} \propto \frac{T^{7/2}}{nB^2R^2}$$

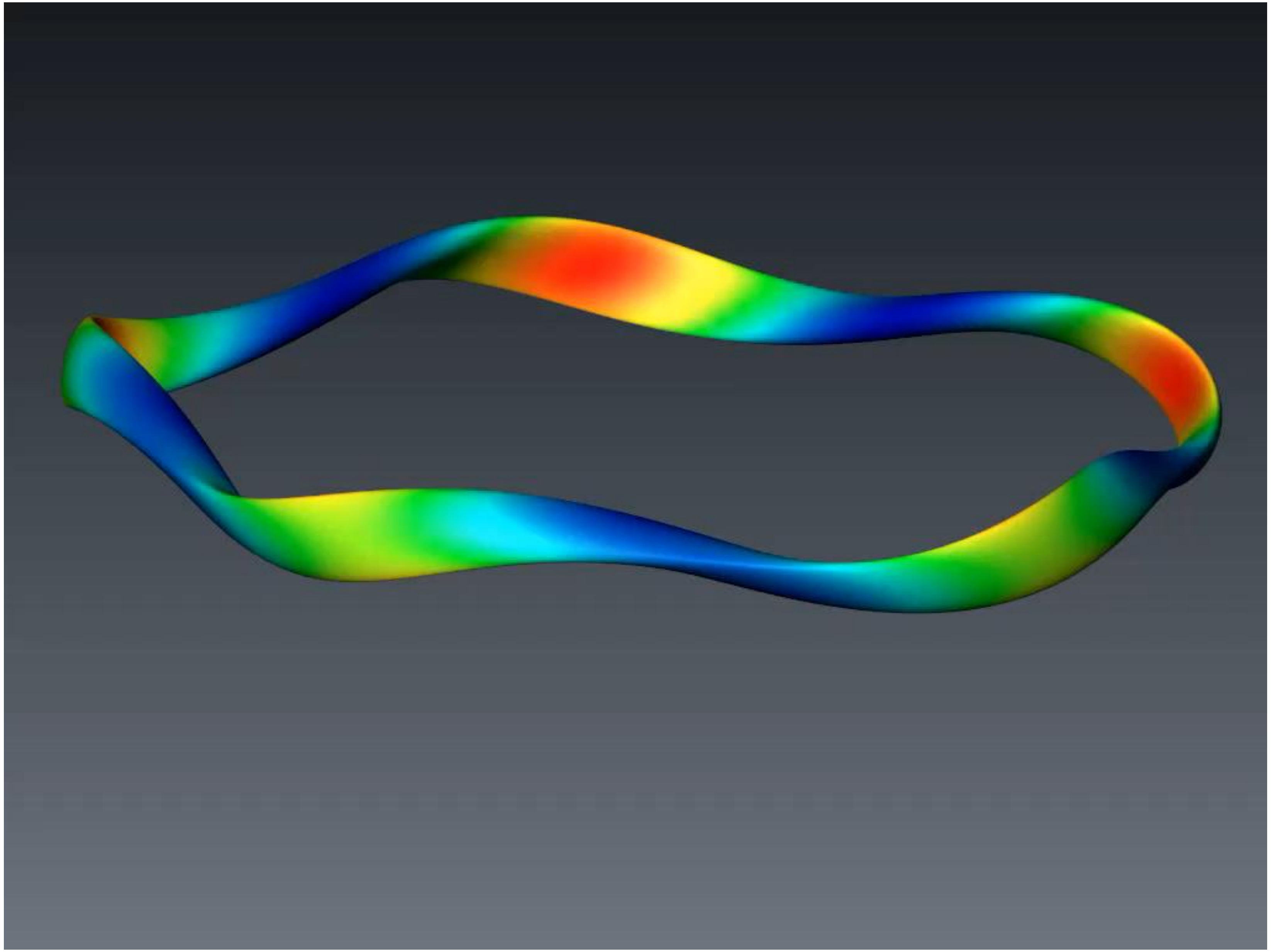
$n = \text{density}$

$T = \text{temperature}$

$R = \text{radius}$

$B = \text{field strength}$

Particle orbits in Wendelstein 7-X



Energy loss channels

Energy is lost from the plasma through

- Radiation
- Collisional “neoclassical“ transport
- Turbulent transport

In tokamaks, turbulence nearly always dominates.

In stellarators, neoclassical losses can be substantial at high temperature.

- Heat diffusivities scale as

$$\chi_e^{\mathrm{nc}} \sim \boxed{\epsilon_{\mathrm{eff}}^{3/2}} \frac{T_e^{7/2}}{nR^2B^2}, \qquad \chi_{\mathrm{gB}} \sim \frac{T_e^{3/2}}{RB^2}$$