Modulational instability of Geodesic-Acoustic-Mode packets

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ABSTRACT

Geodesic-acoustic-modes (GAMs) are oscillations of the zonal flows (ZFs) that are observable in toroidal fusion reactors and play an important (but fairly complex) part in the turbulence-flows interactions. In order to gain a deeper understanding of the involved dynamics it has recently been demonstrated that GAM packets can be described by a reduced model - a (focusing) nonlinear Schrödinger equation (NLSE) [Poli 21], which predicts susceptibility of GAM packets to the modulational instability (MI).

The necessary conditions for this instability are analyzed analytically and numerically using the NLSE model. The predictions of the NLSE are compared to gyrokinetic simulations. Here, an instability of the GAM packets with respect to modulations is observed, thus validating the NLSE approach. However, significant differences in the dynamics of the small scales are discerned between the NLSE and gyrokinetic simulations, most notably with respect to the damping of higher spectral components.

QUALITATIVE INTRODUCTION

- Zonal flow: axisymmetric perturbation of the $\vec{E}\times\vec{B}$-flow velocity.
- ZF velocity is not constant along the poloidal coordinate due to changing magnetic field strength

$\Rightarrow m = \pm 1$ flow divergence develops

Small flow divergence | Large flow divergence

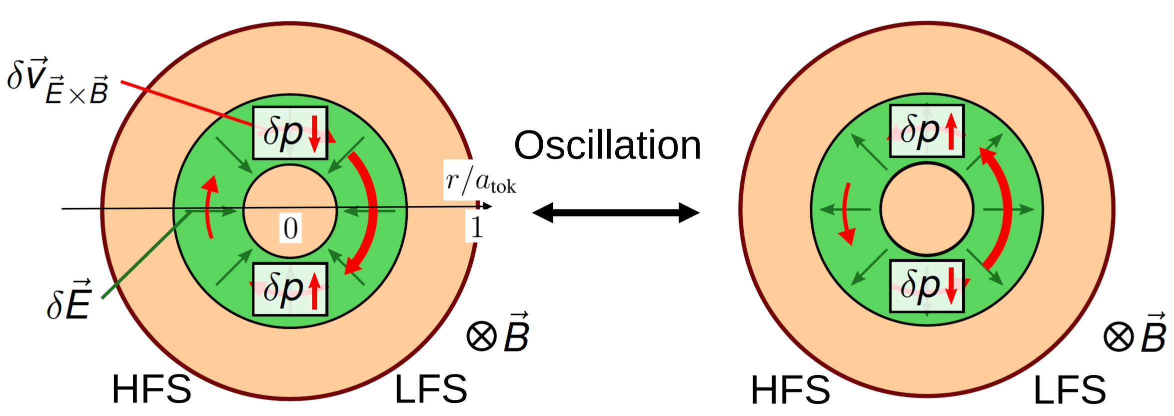
- can be compensated by parallel flows $v_\parallel$

$\Rightarrow$ **Result:** Stationary (zero-frequency) ZF

- cannot be compensated, $m=\pm 1$ pressure mode emerges
- Radial currents form and reverse the electric field

$\Rightarrow$ **Result:** Oscillation of the ZFs, i.e. the GAM

GAM OSCILLATION



Adapted from [Conway 22]

LINEAR DISPERSION RELATION

- GAM needs to be treated with gyrokinetics; one obtains the following (linear) result:

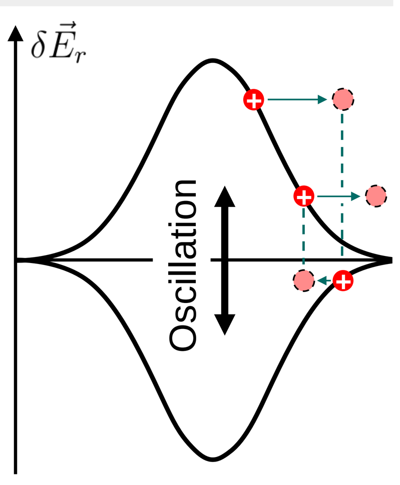
$$\omega = \mathcal{F} + \frac{1}{2}\mathcal{G}k^2, \qquad (1)$$

- $\mathcal{F}\propto v_{\rm Th,i}/R_0$ is the dispersionless GAM frequency (ASDEX Upgrade: 5-25 kHz [Conway et al., 08]).
- $\mathcal{G}\propto \mathcal{F}\rho_i^2 D(\tau_e)$ is the dispersive coefficient, which can be positive or negative depending on $\tau_e = T_e/T_i$.

NONLINEAR BEHAVIOUR

- GAM electric field affects plasma background (e.g. ponderomotive force, see right figure)
- Altered background changes in turn GAM dynamics

$\Rightarrow$ Nonlinear self-coupling of GAM radial electric field E_r, effect is proportional to $|E_r|^2$.



NONLINEAR SCHRÖDINGER EQUATION

The dynamics of the lin. GAM disp. relation, eq. (1), is contained in the linear Schrödinger equation:

$$\mathrm{i}\partial_t\psi = \mathcal{F}\psi - \frac{1}{2}\mathcal{G}\partial_r^2\psi \text{ , with } \mathfrak{Re}\{\psi\} = E_r. \qquad (2)$$

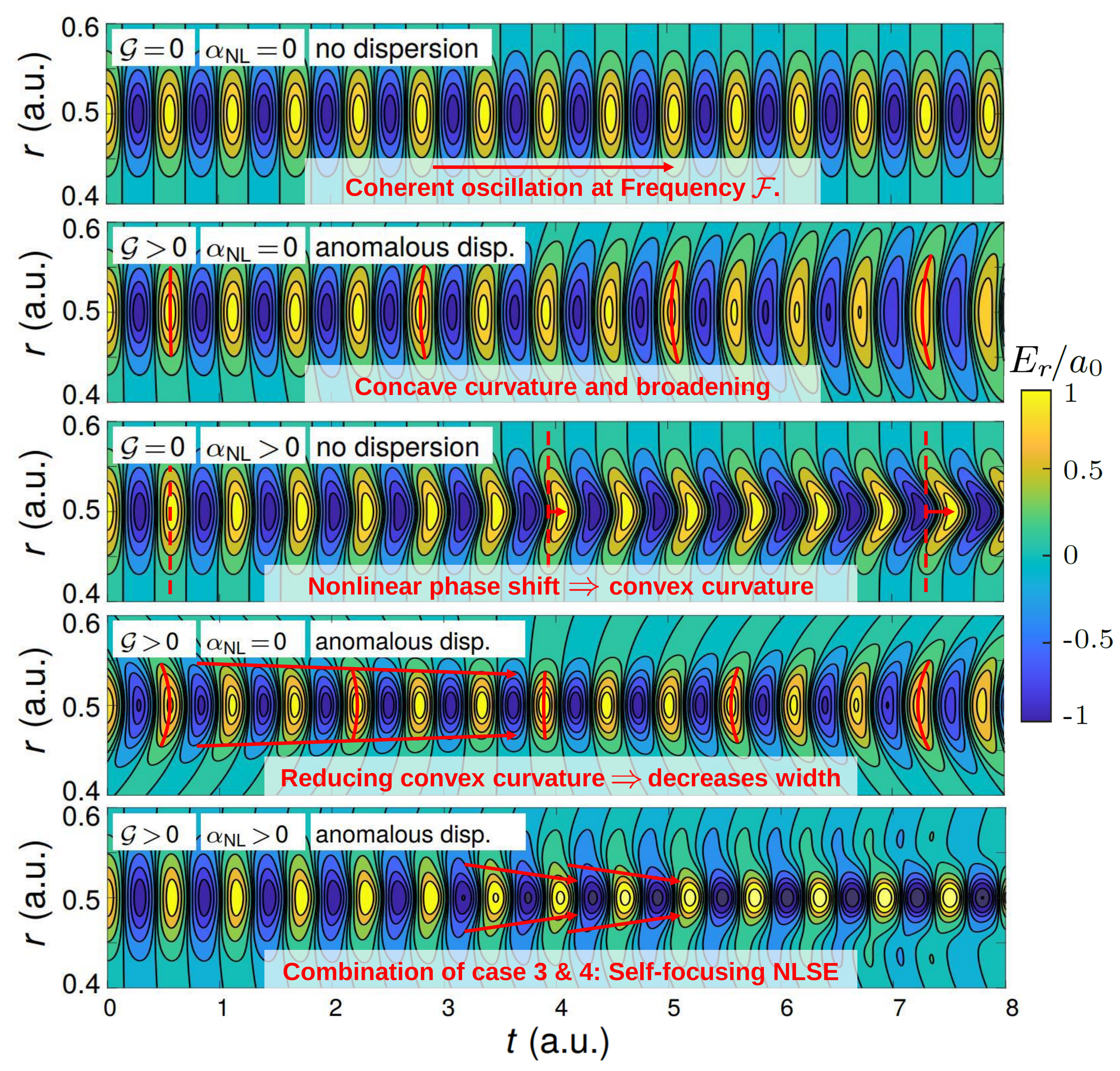
- To confirm this, compare the Fourier transform of eq. (2) with eq. (1).
- Expand eq. (2) that it includes the nonlinear effects from the previous section (term $\propto |E_r|^2 = |\psi|^2$)

$$\Rightarrow \mathrm{i}\partial_t\psi = \mathcal{F}\psi - \frac{1}{2}\mathcal{G}\partial_r^2\psi - \alpha_{\rm NL}|\psi|^2\psi,\ \mathfrak{Re}\{\psi\} = E_r. \text{ (NLSE)}$$

- Currently no analytical expression for nonlinear coefficient, simulations show that $\alpha_{\rm NL} > 0$ for GAMs.

(FOCUSING) NLSE DYNAMICS

We compare now the effect of the different coefficients $\mathcal{F}$, $\mathcal{G}$, and $\alpha_{\rm NL}$ in the NLSE on the dynamics of the GAM radial electric field $\delta\vec{E}_r$, which is initialized as a Gaussian profile:



The interaction of anomalous dispersion ($\mathcal{G}>0$) and the nonlinear phase-shift creates a self-focusing effect of maxima in the NLSE dynamics, which is the mechanism behind the modulational instability.

MODULATIONAL INSTABILITY

Textbook theory [Agrawal 13]: Wave envelope consisting of

- a constant background and
- a weak sinusoidal perturbation (i.e. a modulation)

is unstable under the following two conditions:

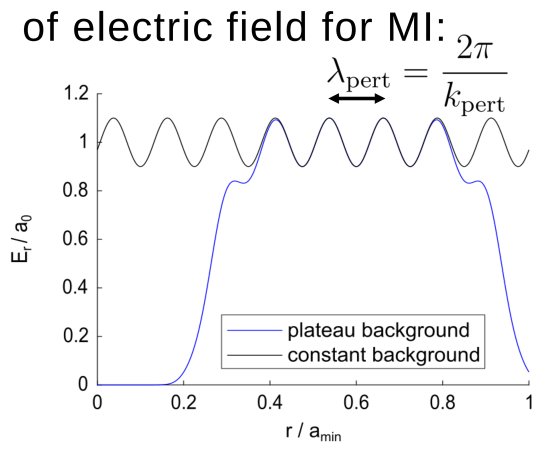
1. **The NLSE is self-focusing**

   See the previous section, $\mathcal{G}>0 \Leftrightarrow \tau_e < 5.45$.

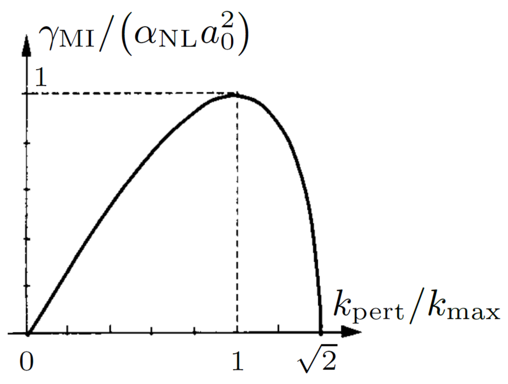
2. **The perturbation wavevector $k_{\rm pert}$ fulfills**

$$k_{\rm pert} < 2a_0\sqrt{\frac{\alpha_{\rm NL}}{\mathcal{G}}} = \sqrt{2}k_{\rm max}.$$

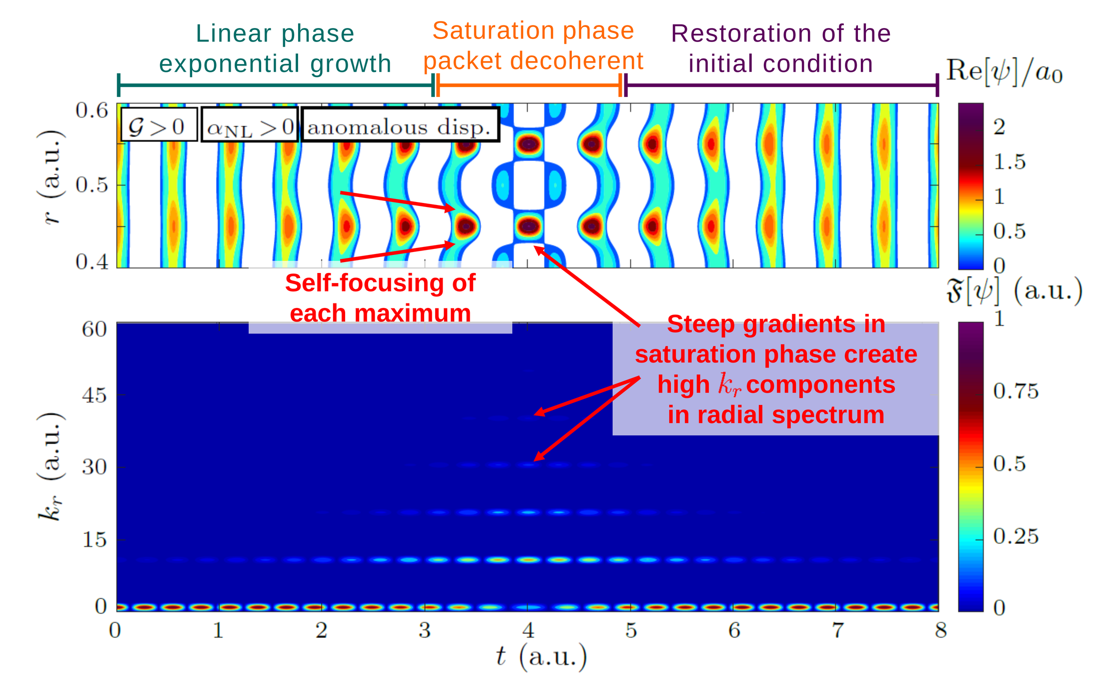
Idealized initial condition of electric field for MI: $\lambda_{\rm pert} = \frac{2\pi}{k_{\rm pert}}$



NLSE predicts the following MI growth rate $\gamma_{\rm MI}$:



The MI consists of 3 phases:

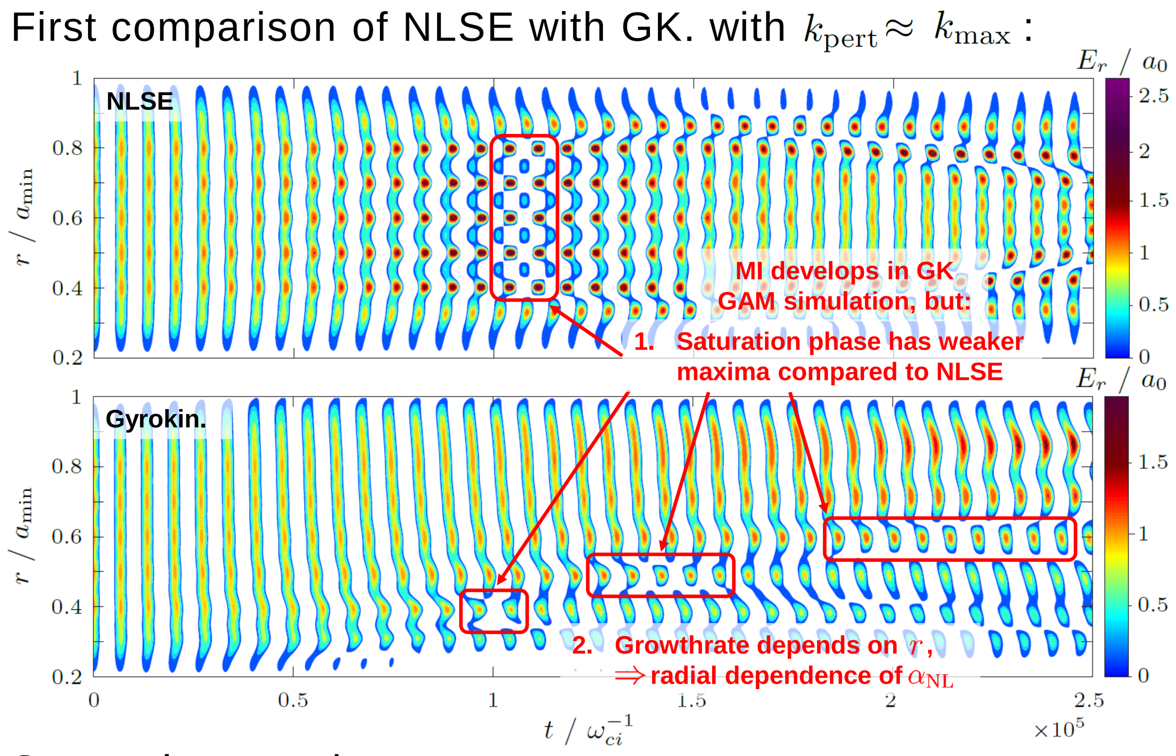


GYROKINETIC SIMULATIONS OF MOD. INSTABILITY IN GAMS

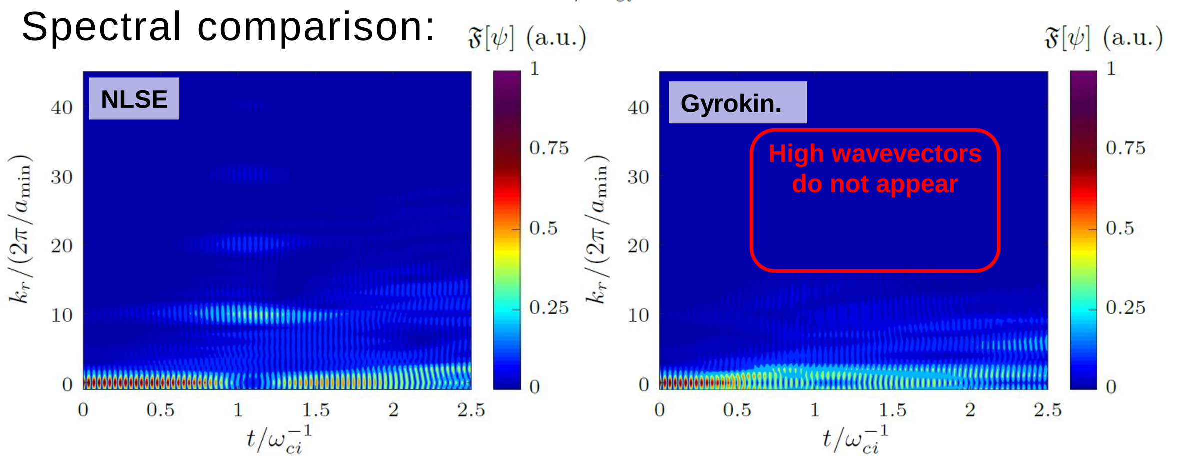
To fulfill the conditions of MI and negligible damping:

- $\tau_e = 3 < 5.45$, (anomalous disp., self focusing NLSE)
- $q_s = 15$, (negligible damping of the initial spectrum)
- $k_{\rm pert} \overset{!}{<} 14.6\,\frac{2\pi}{a_{\rm min}} = \sqrt{2}\cdot k_{\rm max} \approx \sqrt{2}\cdot 10.3\,\frac{2\pi}{a_{\rm min}}$ (for the given set of parameters)

First comparison of NLSE with GK. with $k_{\rm pert} \approx k_{\rm max}$:



Spectral comparison:



INCLUSION OF DAMPING

The results from the previous section, i.e.

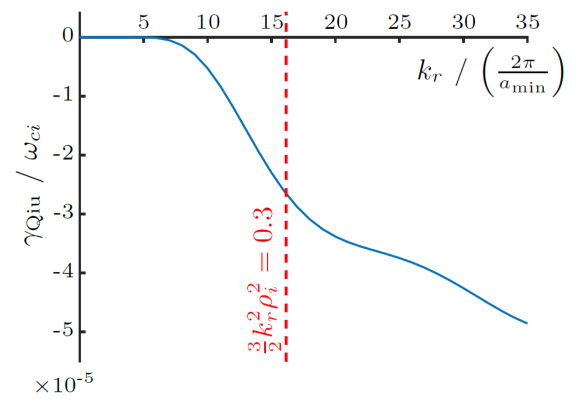
- the weak amplitude in the MI saturation phase and
- the missing high k_r components in the spectrum

indicate that a damping term should be included in the NLSE. Due to $1/q_s^2 \ll k_r^2\rho_i^2 \ll 1$ for the chosen parameters, the term derived in [Qiu 09] is adopted:

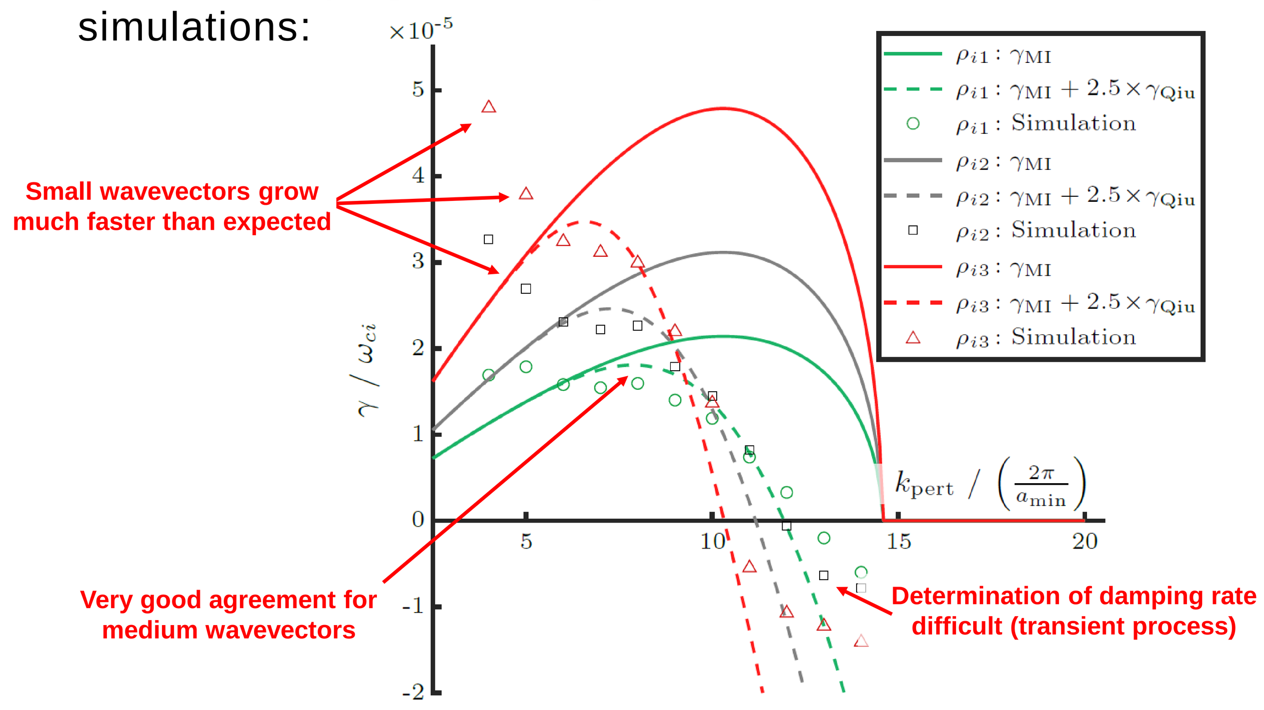
$$\gamma_{\rm Qiu} = -\frac{|\omega_b|}{\sqrt{2}b}\exp\left\{-\sigma\frac{\omega_b}{\omega_{\rm dt}}\right\}\left[1 + b\frac{v_{Ti}^2}{\omega_b^2R_0^2}\left(\frac{31}{16}+\frac{9}{4}\tau_e+\tau_e^2\right)\right.$$
$$-b\frac{v_{Ti}^4}{\omega_b^4R_0^4}\left(\frac{747}{32}+\frac{481}{32}\tau_e+\frac{35}{8}\tau_e^2+\frac{1}{2}\tau_e^3\right) - 2\frac{v_{Ti}^4}{\omega_b^4R_0^4q_s^2}\left(\frac{23}{8}+2\tau_e+\frac{1}{2}\tau_e^2\right)\Big]$$
$$\times\left\{1+\frac{1}{24}\omega_b\omega_{\rm dt}^2\left(-\sigma\frac{4}{\omega_{\rm dt}^3}+\frac{\omega_b}{\omega_{\rm dt}^4}\right)+\sigma\frac{\omega_{\rm dt}}{\omega_b}\tau_e+\left(\tau_e^2+\frac{5}{4}\tau_e+1\right)\frac{\omega_{\rm dt}^2}{\omega_b}-2b\right\}$$

Dependency of $\gamma_{\rm Qiu}$ on k_r:

- Mainly small scales (high k_r) are affected
- Red line marks limit of applicability



Compare theoretic prediction of analytic MI growth rate $\gamma_{\rm MI}$ and damping $\gamma_{\rm Qiu}$ with growth rates measured in GK simulations:



CONCLUSION

The NLSE model predictions for MI are confirmed, and MI is observed in GK GAM simulations.

The damping of nonlinearly generated high k_r components hampers the full development of the MI in GK simulations. Damping can be included in the NLSE.

Future work should take $\alpha_{\rm NL} = \alpha_{\rm NL}(r)$ into account.

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