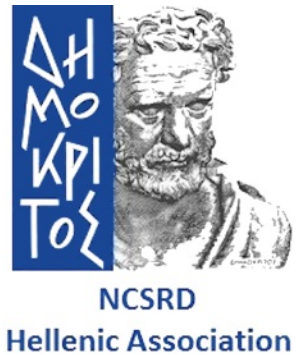


Breakdown time estimation for EC-assisted start-up in tokamaks

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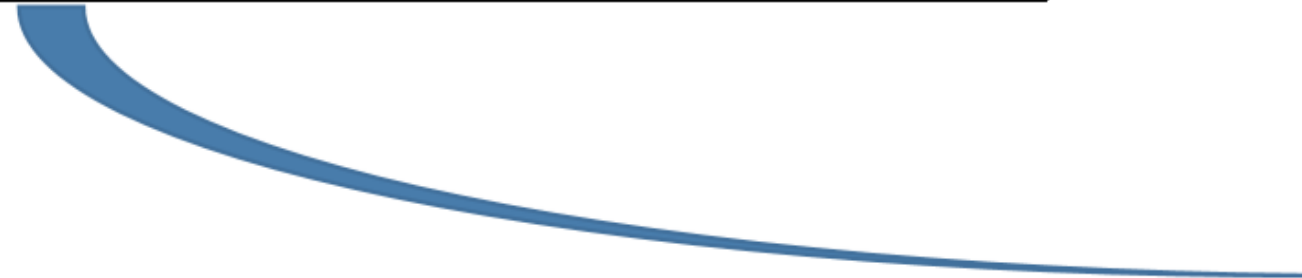
Introduction

A numerical tool modelling the excitation and evolution of electron avalanche ionization in the breakdown phase of start-up in tokamaks is presented. We estimate the energization efficiency of the nonlinear interaction of spatially localized Gaussian EC-field propagating in vacuum with an ensemble of seed electrons. This process is coupled with the acceleration of electrons due to the induced loop voltage along the vacuum vessel, as well as the impact ionization and elastic collision events that lead to the abrupt increase of electron density during the avalanche process. Special care is taken to incorporate the effect of the toroidal magnetic field in the collision statistics. A simple analytical auxiliary tool based on the dynamics of avalanche evolution is developed in order to estimate the breakdown time as a function of the RF-field parameters, the loop voltage and the prefill pressure of the neutral gas.

**Simulation of the breakdown phase
(RFAVION)**

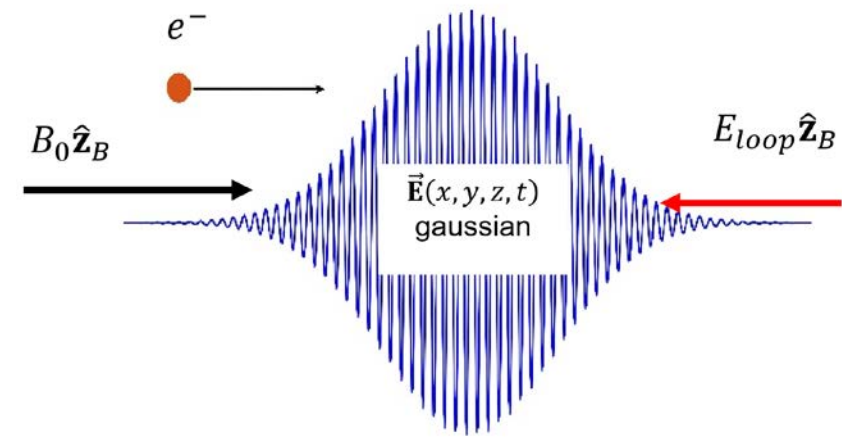


Nonlinear electron-field interaction
(“Energization” of low temperature electrons)





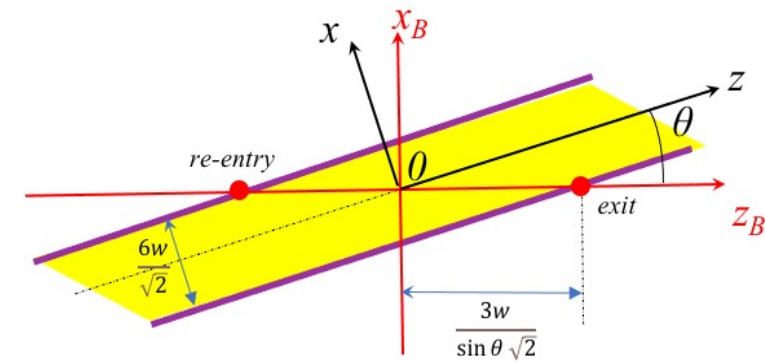
Conditional mapping
Electron acceleration - Electron collisions



$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0(x, y) e^{ik_z z - i\omega t} = -\frac{\partial \vec{\mathbf{a}}}{\partial t} - \nabla \phi$$

$$|\vec{\mathbf{E}}_0(x, y)|^2 \sim P_0 \cdot \exp\left(-\frac{(x^2 + y^2)}{w^2}\right)$$

Inclination ($\varphi_{\text{tor}} \equiv 90^\circ - \theta$)



- Electron – beam nonlinear interaction: Hamiltonian formulation (relativistic form)

$$h(\chi, \psi, \zeta, U_x, U_\psi, U_\zeta, \tau) = \gamma - \hat{\Phi} - \hat{\Phi}_L$$

$$\gamma^2 = 1 + \left| \vec{\mathbf{U}} + \vec{\mathbf{A}} + \vec{\mathbf{A}}_0 \right|^2 = 1 + |\vec{\mathbf{u}}|^2$$

$$\vec{\mathbf{A}} = \text{Re} \vec{\mathbf{A}}_\omega, \quad \vec{\mathbf{A}}_\omega = \frac{e \vec{\mathbf{A}}_\omega}{m_e c} = \hat{\mathbf{y}} \alpha e^{-\frac{\chi^2 + \psi^2}{2\hat{w}^2}} e^{i\kappa(\zeta - \tau) + i\varphi_0}$$

$$\hat{\Phi} = \text{Re} \hat{\Phi}_\omega, \quad \hat{\Phi}_\omega \equiv \frac{e \Phi_\omega}{m_e c^2} = i \alpha \frac{\psi}{\hat{w}^2 \kappa} e^{-\frac{\chi^2 + \psi^2}{2\hat{w}^2}} e^{i\kappa(\zeta - \tau) + i\varphi_0}$$

$$\vec{\mathbf{A}}_0 = (\chi \cos \theta + \zeta \sin \theta) \hat{\mathbf{y}} \quad \hat{\Phi}_L = \varepsilon_L (\zeta \cos \theta - \chi \sin \theta)$$

$$\alpha \equiv \frac{e E_0}{m_e c \omega} \quad \varepsilon_{\varepsilon L} = \frac{-e E_{\text{loop}}}{m_e c \omega}$$

(Small parameters)

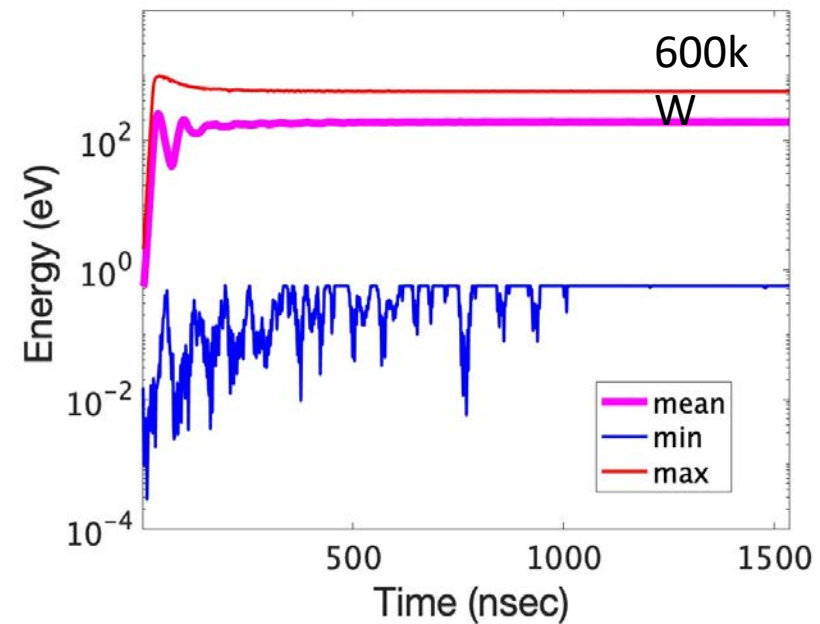
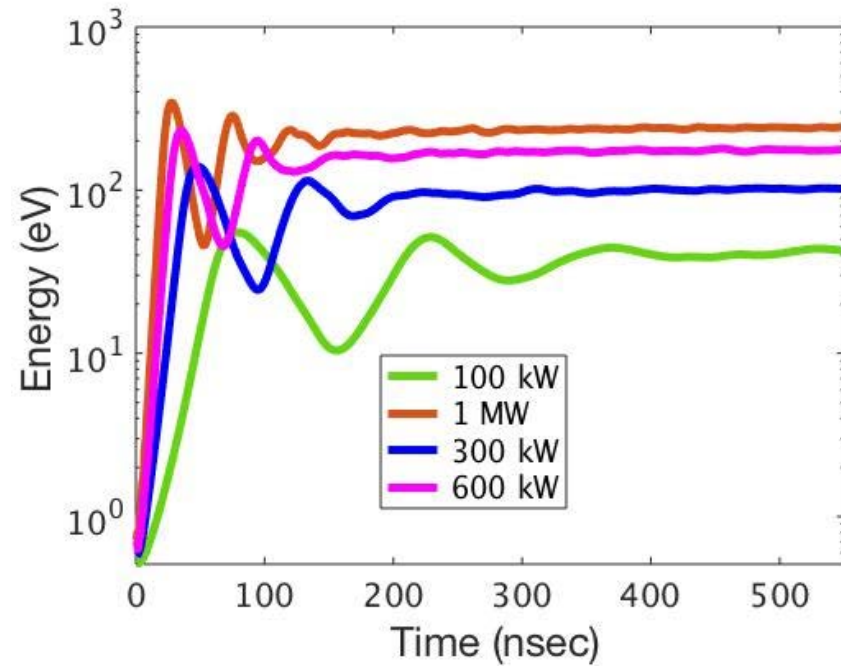
$$E_0 = \sqrt{\frac{2\eta P_0}{\pi w^2}} \quad \kappa \equiv \frac{\omega}{\omega_{ce}}$$

- Numerically integrate Hamilton – Jacobi equations for the canonical coordinates

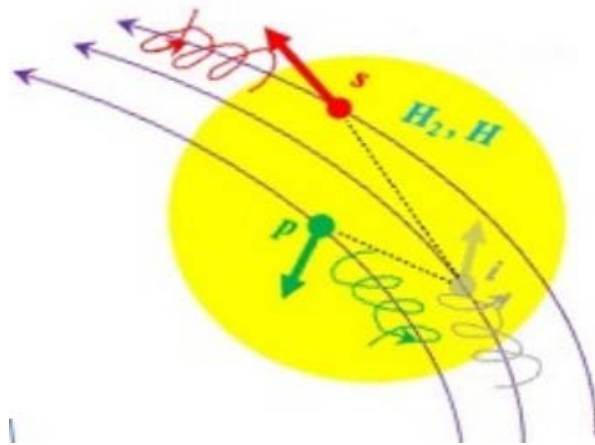
$$dq/d\tau = \partial h/\partial p, \quad dp/d\tau = -\partial h/\partial q$$

$$p = \{\chi, \psi, \zeta\} \quad q = \{U_\chi, U_\psi, U_\zeta\}$$

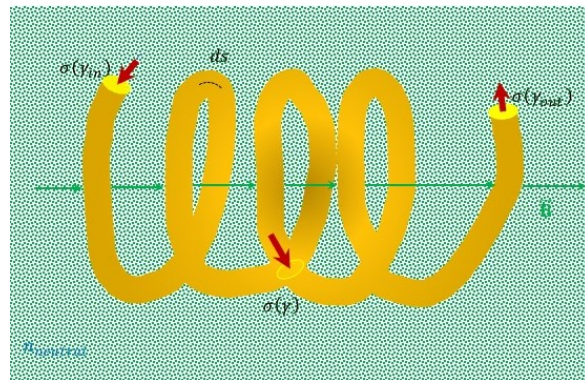
- Electron position/momenta after the interaction (trapping/passing/reflected orbits)



“energization” (collisionless heating) of seed electrons solely through their interaction with RF beam
($B_\phi = 1.475 T$, X2-mode, TCV major radius)



- Conditional Mapping (Hamiltonian formulation-analytical)
 - position/momenta** of electrons **exiting** EC-beam → **new position/momenta** electrons **re-entering** EC-beam
- Conditionality: monitoring impact ionization & elastic collision events:
 - mean free path evaluation (analytical)

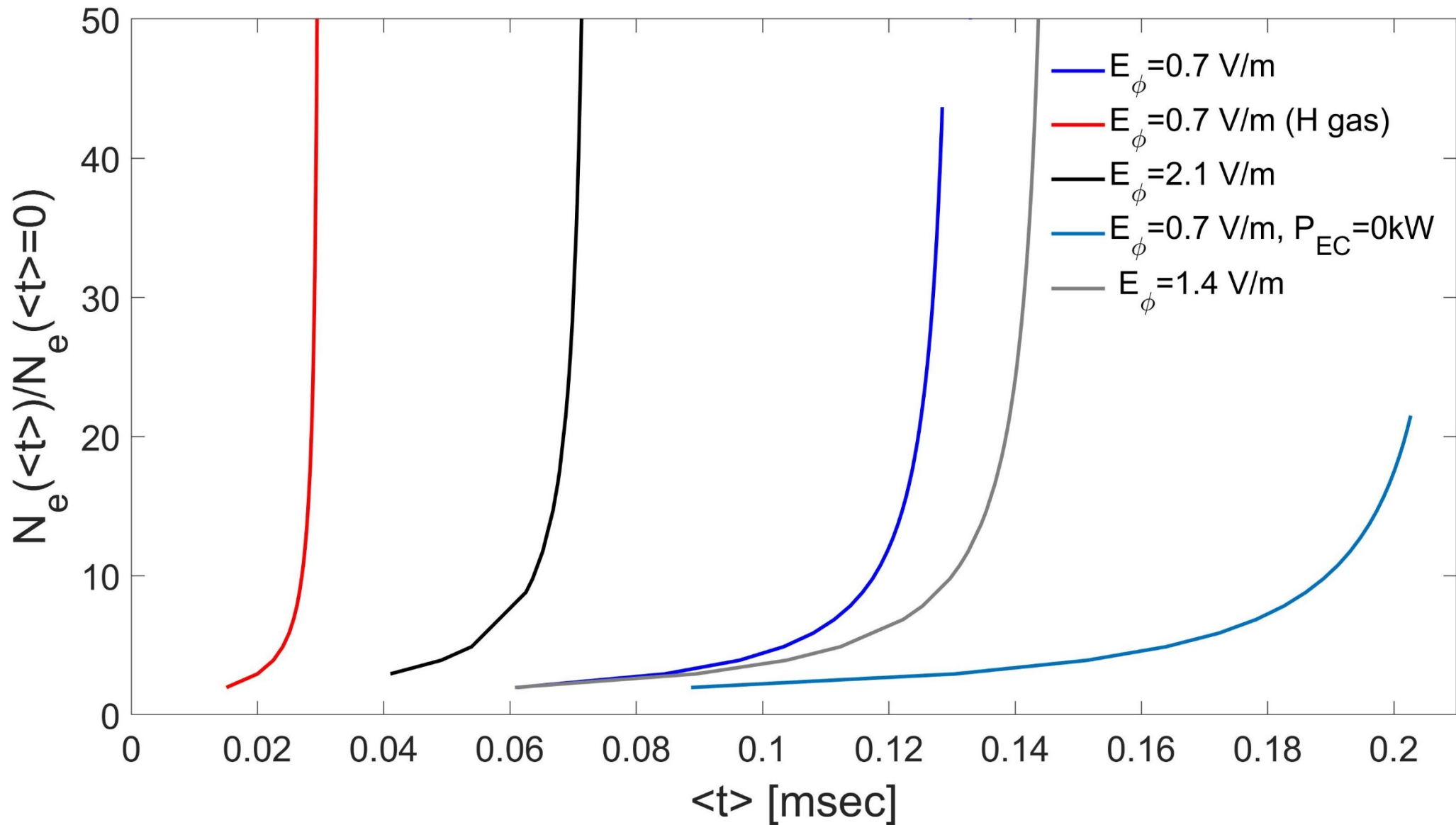


$$y([s_{in}, s_{out}]) = n_{neutral} \int_{s_{in}}^{s_{out}} \sigma(\gamma) ds$$

evaluated following
electron gyrating path

$$y([s_{in}, s_{out}]) = 1 \Rightarrow \lambda_{MFP} = |s_{out} - s_{in}|$$

- collision mechanics model



Simulated electron population growth (impact ionization and elastic collisions as well as losses considered
($B_\phi = 1.475$ T, X2-mode, initial RF power $P_{EC} = 600$ kW, connection length $L_c = 600$ m, prefill pressure $p = 20$ mPa, D neutral gas, TCV major radius)

Simplified analytical estimation of breakdown time (only impact ionization considered)

A simple approach of the avalanche through impact ionization as a “cataclysmic” event until a threshold density n_{th} (i.e. 5% of $n_{neutral}$) is reached, is to consider the rate of change of the normalized density as a singular function of time, dependent on the density itself, proportional to the energy dependent average cross section $\sigma(t)$, and to the neutral gas pressure p

$$\frac{dv(t)}{dt} = c_f(t)v(t), \quad v(t) = \frac{n(t)}{n_{th}} \quad c_f(t) = \frac{C_0 p \sigma(t)}{[1-v(t)]^\lambda}, \quad \lambda > 0,$$

The **impact ionization process is optimal** (has the maximum likelihood to produce multiple new electrons) when the electron energy is in the range of $[3U_{ion}, 6U_{ion}]$ (U_{ion} : ionization potential), where it can be approximated by:

$$\sigma(t) = \sigma_{max} \left\{ 1 - C_1 [e_{opt} - e(t)]^3 \right\}$$

Where C_1 , σ_{max} and e_{opt} : $2.4 \times 10^{-6} \text{ eV}^{-3}$, $6.5 \times 10^{-21} \text{ m}^{-2}$, 55.6 eV (H gas)
 $1.6 \times 10^{-6} \text{ eV}^{-3}$, $1.1 \times 10^{-20} \text{ m}^{-2}$, 66.4 eV (H_2/D gas)

All simulations indicate that average electron energy $e(t)$ evolves linearly with time. For example, the rate is about 34 eV/ms and 75 eV/ms for atomic hydrogen at pressure 10 mPa, loop voltage 0.7 V/m and for 0 kW and 600 kW RF power respectively. The respective numbers for molecular hydrogen are 28 eV/ms and 40 eV/ms.

Normalizing in time we can calculate the approximation

$$c_f(\tau) = C_0 \frac{p\sigma_{max}}{[1-v(\tau)]^\lambda} \left\{ 1 - \beta [\tau_0^2 - \delta^2 \tau_{RF} \tau]^3 \right\}$$

Where

$$\kappa = \frac{\omega}{\omega_{ce}}, \quad \beta = C_1 \left(\frac{m_e c^2}{2} \right)^3, \quad \tau_0 = \sqrt{\frac{2e_{opt}}{m_e c^2}}, \quad \tau = t\omega_{ce} \quad \delta^2 = \kappa^2 e_{RF} \varepsilon_{RF}^2 + e_{LV} \varepsilon_{LV}^2$$

$$e_{RF} = \frac{7r_{RF}}{\sqrt{2} \sin \vartheta_0} / 2\pi R, \quad r_{RF}: \text{beam nominal radius}, \vartheta_0: \text{beam inclination}, R: \text{major radius}$$

$$e_{LV} = \tau_{LV} / \tau_{RF}, \quad (e_{LV}, \tau_{RF}, \text{determined through fitting from simulations})$$

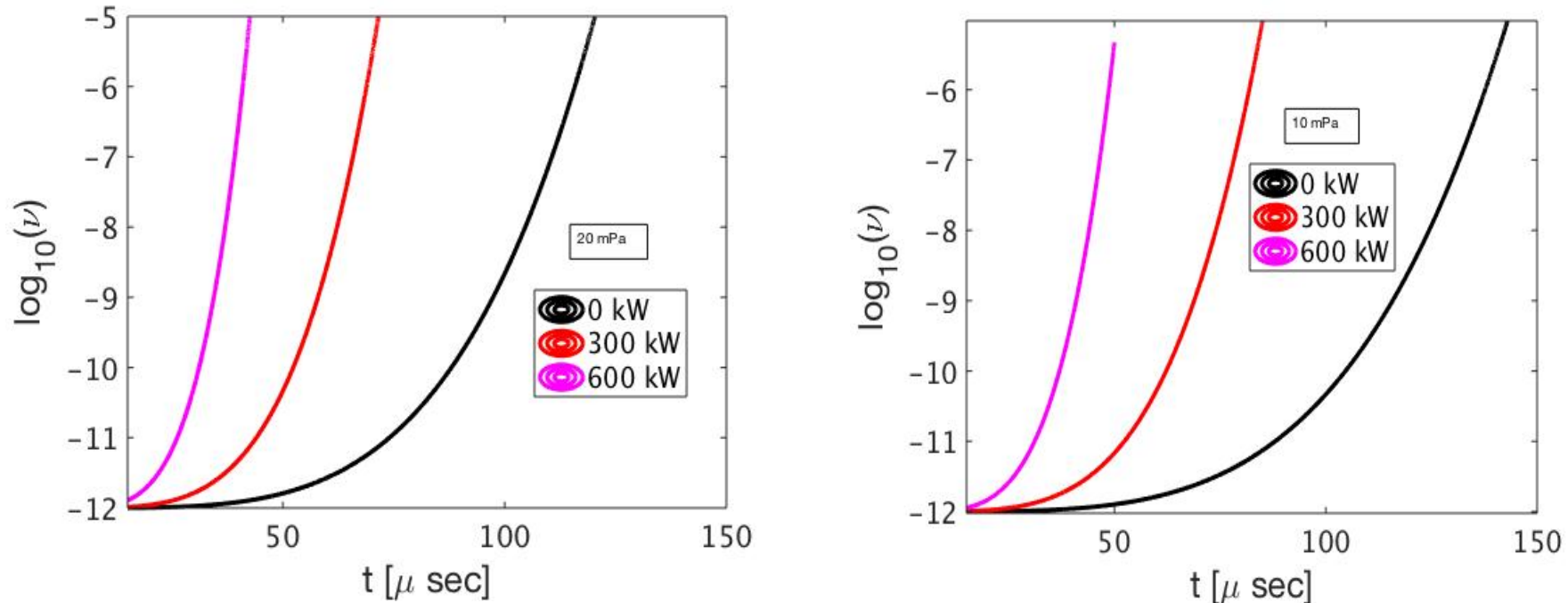
$$\varepsilon_{RF}^2 = \langle E_{RF}^2 \rangle (m_e c \omega / q)^{-2}, \quad \varepsilon_{LV}^2 = E_{LV}^2 (m_e c \omega_{ce} / q)^{-2}, \quad \langle E_{RF}^2 \rangle, E_{LV}^2 : \text{average RF/Loop Voltage intensity}$$

Rearranging (1),(3), one can end with the following equation

$$\int_{v(0)=ns_{eed}/nth}^{v_{BD}} \frac{[1-v(\tau)]^\lambda}{v(\tau)} dv(\tau) = C_0 p\sigma_{max} \int_{\tau_0=0}^{\tau_{BD}} \left[1 - b(\tau_0^2 - \delta^2 \tau_{RF} \tau)^3 \right] d\tau$$

The integrals can be calculated analytically on the left hand side from the initial density of seed electrons to the density assigned to the end of the avalanche process and on the right hand side from time zero until breakdown time.

Fixing $\lambda=1$ and fitting $C_0 e_{LV}$, τ_{RF} , to the simulated evolution of $v_{sim} < v_{BD}$ ($\tau_{sim} < \tau_{BD}$) one can extrapolate the analytical results till $v_{final}=v_{BD}$, and estimate the breakdown time τ_{BD}



Normalized electron density evolution obtained analytically for pressures 10mPa (right) and 20 mPa (left), with (red, mangenta) or without (black) RF assistance. ($B_\phi = 1.475 \text{ T}$, X2-mode)

Acknowledgement - References



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