Demonstration of moment-kinetics approach for edge modelling

UK Atomic Energy Authority

J. T. Omotani[1], S. L. Newton[1], F. I. Parra[2], M. Barnes[3], M. R. Hardman[4,3]

[1]UK Atomic Energy Authority, Culham Centre for Fusion Energy, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK; [2]Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543, USA; [3]Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford, OX1 3PU, UK; [4]Tokamak Energy Ltd, 173 Brook Drive, Milton Park, Abingdon OX14 4SD, UK

Overview

- Plasma fluctuations in the edge of diverted tokamaks have large amplitude and evolve through regions of strongly varying parameters
- We demonstrate implementation of a consistent coupled set of evolution equations for the low-order moments of plasma ions and neutrals, closed by the kinetic evolution of the full-f 'shape' function
- Electrons are taken here to be adiabatic
- This represents the first step in a system to consistently follow edge evolution on the transport timescale

System

Domain: plasma between two targets

Sources: ionization, charge exchange (CX), neutrals recycled with Knudsen distribution

Helical field: $\boldsymbol{B} = B_z(r)\hat{\boldsymbol{z}} + B_\zeta(r)\hat{\boldsymbol{\zeta}}$, $B_\zeta = \frac{I}{r}$, low pitch $\frac{\rho_i}{L_r} \sim \frac{B_z}{B_\zeta} \ll 1$

Ion moments

$$\frac{\partial n_i}{\partial t} - \frac{\partial}{\partial r}\left(\frac{n_i}{B}\frac{\partial\phi}{\partial z}\right) + \frac{\partial}{\partial z}\left[n_i\left(\frac{u_{i\parallel}B_z}{B} + \frac{1}{B}\frac{\partial\phi}{\partial r}\right)\right] = n_n n_e R_{\mathrm{ion}} + \int S_i\, d^3v$$

$$n_i m_i\left[\frac{\partial u_{i\parallel}}{\partial t} - \frac{1}{B}\frac{\partial\phi}{\partial z}\frac{\partial u_{i\parallel}}{\partial r} + \left(\frac{u_{i\parallel}B_z}{B} + \frac{1}{B}\frac{\partial\phi}{\partial r}\right)\frac{\partial u_{i\parallel}}{\partial z}\right] + \frac{B_z}{B}\frac{\partial p_{i\parallel}}{\partial z} + \frac{e n_i B_z}{B}\frac{\partial\phi}{\partial z}$$
$$= n_i m_i (n_n R_{\mathrm{CX}} + n_e R_{\mathrm{ion}})(u_{n\parallel} - u_{i\parallel}) + \int m_i (v_\parallel - u_{i\parallel}) S_i\, d^3v$$

$$\frac{3}{2}n_i\left[\frac{\partial T_i}{\partial t} - \frac{1}{B}\frac{\partial\phi}{\partial z}\frac{\partial T_i}{\partial r} + \left(\frac{u_{i\parallel}B_z}{B} + \frac{1}{B}\frac{\partial\phi}{\partial r}\right)\frac{\partial T_i}{\partial z}\right] + \frac{B_z}{B}\frac{\partial q_{i\parallel}}{\partial z} + \frac{p_{i\parallel}B_z}{B}\frac{\partial u_{i\parallel}}{\partial z}$$
$$= \frac{3}{2}n_i(n_n R_{\mathrm{CX}} + n_e R_{\mathrm{ion}})(T_n - T_i) + \int\left(\frac{1}{2}m_i\left|\boldsymbol{v} - u_{i\parallel}\hat{\boldsymbol{b}}\right|^2 - \frac{3}{2}T_i\right)S_i\, d^3v$$
$$+ \frac{1}{2}n_i m_i (n_n R_{\mathrm{CX}} + n_e R_{\mathrm{ion}})\left[(u_{n\parallel} - u_{i\parallel})^2 + u_{n\perp}^2\right]$$

Normalised distribution

$$F_i(r, z, w_{i\parallel}, w_{i\perp}, t) = \frac{v_{\mathrm{t},i}}{n_i} f_i(r, z, u_{i\parallel} + v_{\mathrm{t},i}w_{i\parallel}, v_{\mathrm{t},i}w_{i\perp}, t)$$

Enters moment equations via closure

$$\{p_{i\parallel}, q_{i\parallel}\} = 2\pi n_i m_i v_{\mathrm{t},i}^2 \int_{-\infty}^{\infty} dw_{i\parallel} \int_0^\infty dw_{i\perp} w_{i\perp}\left\{w_{i\parallel}^2, \frac{v_{\mathrm{t}i}}{2}w_{i\parallel}\left(w_{i\parallel}^2 + w_{i\perp}^2\right)\right\}F_i$$

Ion drift kinetic equation

$$\frac{\partial F_i}{\partial t} + \dot{r}_i\frac{\partial F_i}{\partial r} + \dot{z}_i\frac{\partial F_i}{\partial z} + \dot{w}_{i\parallel}\frac{\partial F_i}{\partial w_{i\parallel}} + \dot{w}_{i\perp}\frac{\partial F_i}{\partial w_{i\perp}} = \dot{F}_i + \mathcal{C}_{ii} + \mathcal{C}_{i,\mathrm{CX}} + \mathcal{C}_{i,\mathrm{ion}} + S_i$$

$$\dot{r}_i = -\frac{1}{B}\frac{\partial\phi}{\partial z} \quad ; \quad \dot{z}_i = \frac{u_{i\parallel}B_z}{B} + \frac{1}{B}\frac{\partial\phi}{\partial r} + \frac{v_{\mathrm{t},i}B_z}{B}w_{i\parallel}$$

$$\dot{w}_{i\parallel} = \frac{B_z}{n_i m_i v_{\mathrm{t},i} B}\frac{\partial p_{i\parallel}}{\partial z} + \frac{2 w_{i\parallel} B_z}{3 n_i m_i v_{\mathrm{t},i}^2 B}\left[\frac{\partial q_{i\parallel}}{\partial z} + \left(p_{i\parallel} - \frac{3}{2}n_i m_i v_{\mathrm{t},i}^2\right)\frac{\partial u_{i\parallel}}{\partial z}\right] - \frac{w_{i\parallel}^2 B_z}{B}\frac{\partial v_{\mathrm{t},i}}{\partial z}$$

$$\dot{w}_{i\perp} = \frac{2 w_{i\perp} B_z}{3 n_i m_i v_{\mathrm{t},i}^2 B}\left(\frac{\partial q_{i\parallel}}{\partial z} + p_{i\parallel}\frac{\partial u_{i\parallel}}{\partial z}\right) - \frac{w_{i\parallel} w_{i\perp} B_z}{B}\frac{\partial v_{\mathrm{t},i}}{\partial z}$$

$$\dot{F}_i = \frac{B_z}{B}\left[w_{i,\parallel}\left(3\frac{\partial v_{\mathrm{t},i}}{\partial z} - \frac{v_{\mathrm{t},i}}{n_i}\frac{\partial n_i}{\partial z}\right) - \frac{2}{n_i m_i v_{\mathrm{t},i}^2}\left(\frac{\partial q_{i\parallel}}{\partial z} + \left(p_{i\parallel} - \frac{1}{2}n_i m_i v_{\mathrm{t},i}^2\right)\frac{\partial u_{i\parallel}}{\partial z}\right)\right]F_i$$

Neutrals: Similar approach using vector velocity

$$\frac{\partial n_n}{\partial t} + \nabla\cdot(n_n \boldsymbol{u}_n) = -n_n n_e R_{\mathrm{ion}} + \int S_n d^3v$$

Constraints

The last operation in a timestep is to ensure the updated distribution function satisfies the low-order moment constraints.

For 1D1V cases the latest distribution function $\hat{F}_s$ is used to construct the updated normalized, marginalised distribution function $\tilde{F}_s$ such that

$$\tilde{F}_s = A_0\hat{F}_s + A_1 w_{s\parallel}\hat{F}_s + A_2 w_{s\parallel}^2\hat{F}_s$$

$$\{C_n, I_n\} = \pi^{-1/2}\int dw_{s\parallel}\, w_{s\parallel}^n\{\tilde{F}_s, \hat{F}_s\} \quad \Rightarrow \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1/2 \end{pmatrix} = \begin{pmatrix} I_0 & I_1 & I_2 \\ I_1 & I_2 & I_3 \\ I_2 & I_3 & I_4 \end{pmatrix}\begin{pmatrix} A_0 \\ A_1 \\ A_2 \end{pmatrix}$$

Benchmarking

In a straight magnetic field we demonstrate the following benchmarks

Code [1] solves:

- full-f drift kinetic equation, see [2] for numerical scheme
- moment-kinetic approach extracting n_s or $\{n_s, u_{s\parallel}\}$ or full set $\{n_s, u_{s\parallel}, p_{s\parallel}\}$ of moment equations with normalised kinetic equation

Neutral coupling

1D periodic box, plasma and neutrals coupled by charge exchange

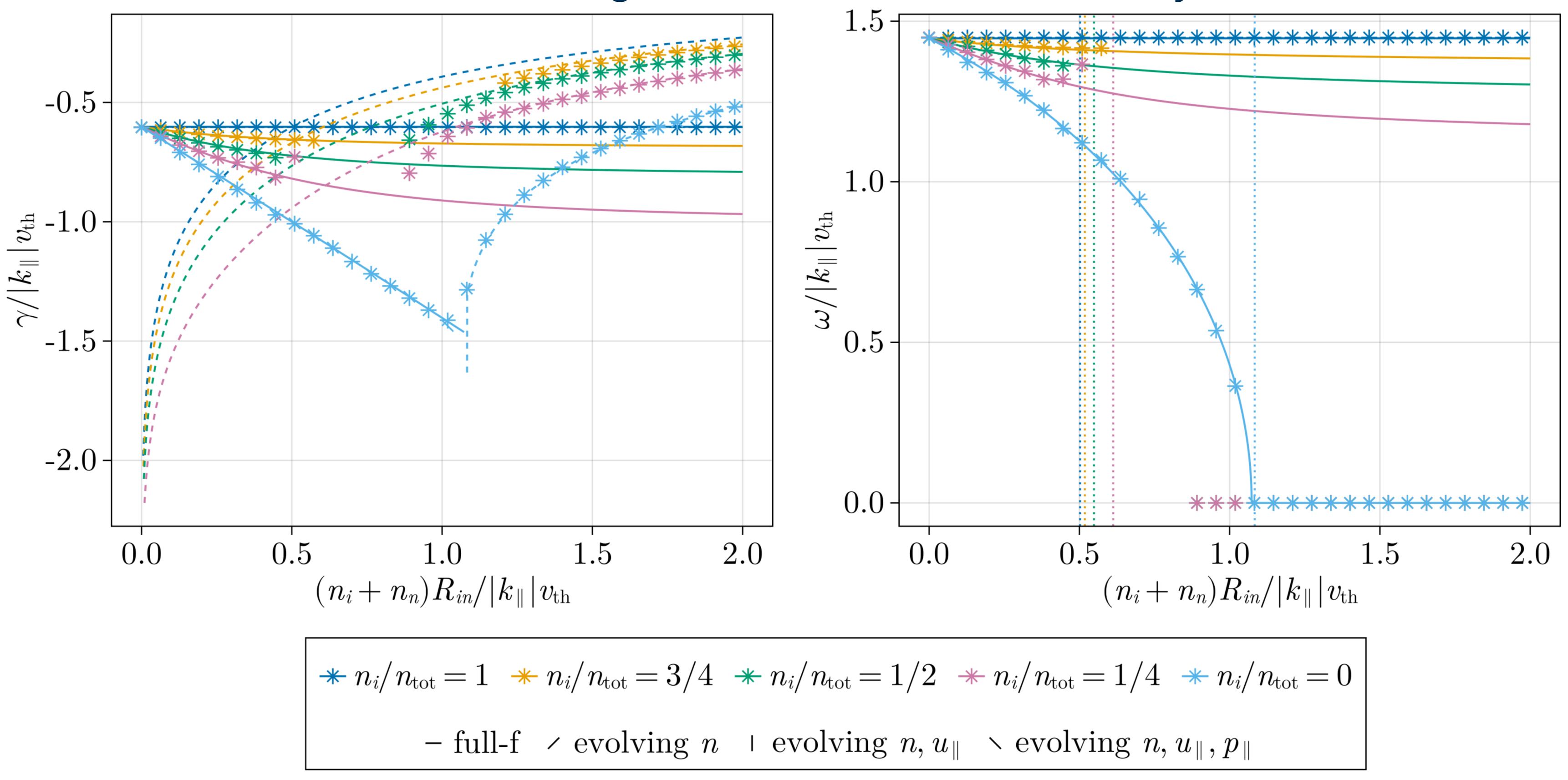
Background Maxwellian for plasma and neutrals, with same temperature

Drift kinetic equation can be solved for linear modes

$$f_s(z, v_\parallel, v_\perp, t) = f_{Ms}(v_\parallel, v_\perp) + f_{s1}(v_\parallel, v_\perp)[\exp(ik_\parallel z - i\omega t) + \mathrm{c.c.}]$$

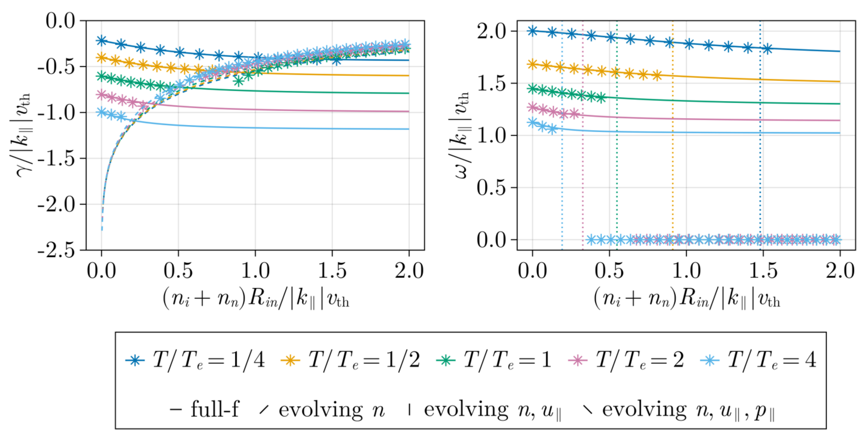
Two types: damped acoustic modes; non-propagating modes with $\omega = 0$

All moment kinetic variants agree well with full-f and analytic solutions



(Above) varying ion/neutral density ratio, (right) varying ion temperature relative to electron temperature

Evaluated plasma dispersion functions in analytic solutions with [3]



1D Scrape-off Layer

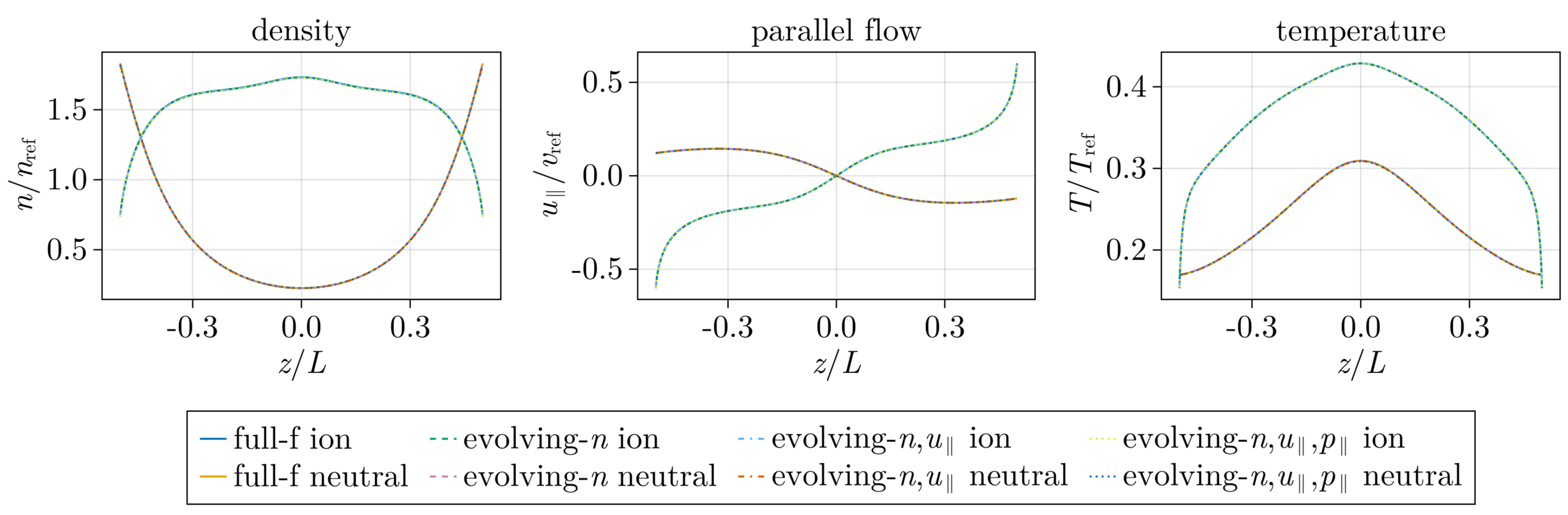
SOL-like benchmark - “attached” conditions

Maxwellian source of plasma at midplane $T_{\mathrm{source}} = 200$ eV

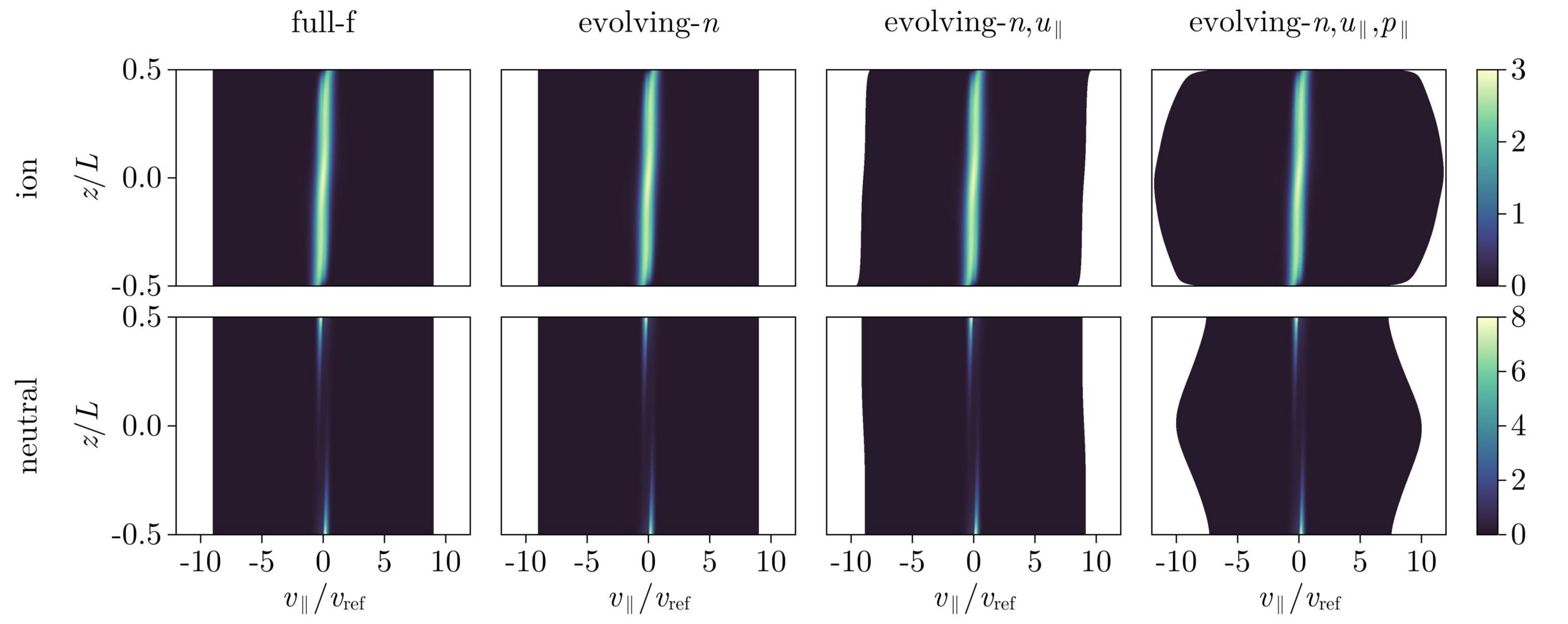
Ions absorbed at targets

Neutral atoms from recycling 50% of ion flux to wall, with $T_{\mathrm{wall}} = 10$eV

Krook collision operator for ions mixes hot and cold populations



All approaches match: moments (above) and distribution functions (below)



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University of Oxford

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References

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[2] M. Hardman et al., “$E{\times}B$ drift physics on open field lines in a drift-kinetic model”, *this conference*

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john.omotani@ukaea.uk