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Pitch Angle Averaged Quasi-Linear Operator for ICRH

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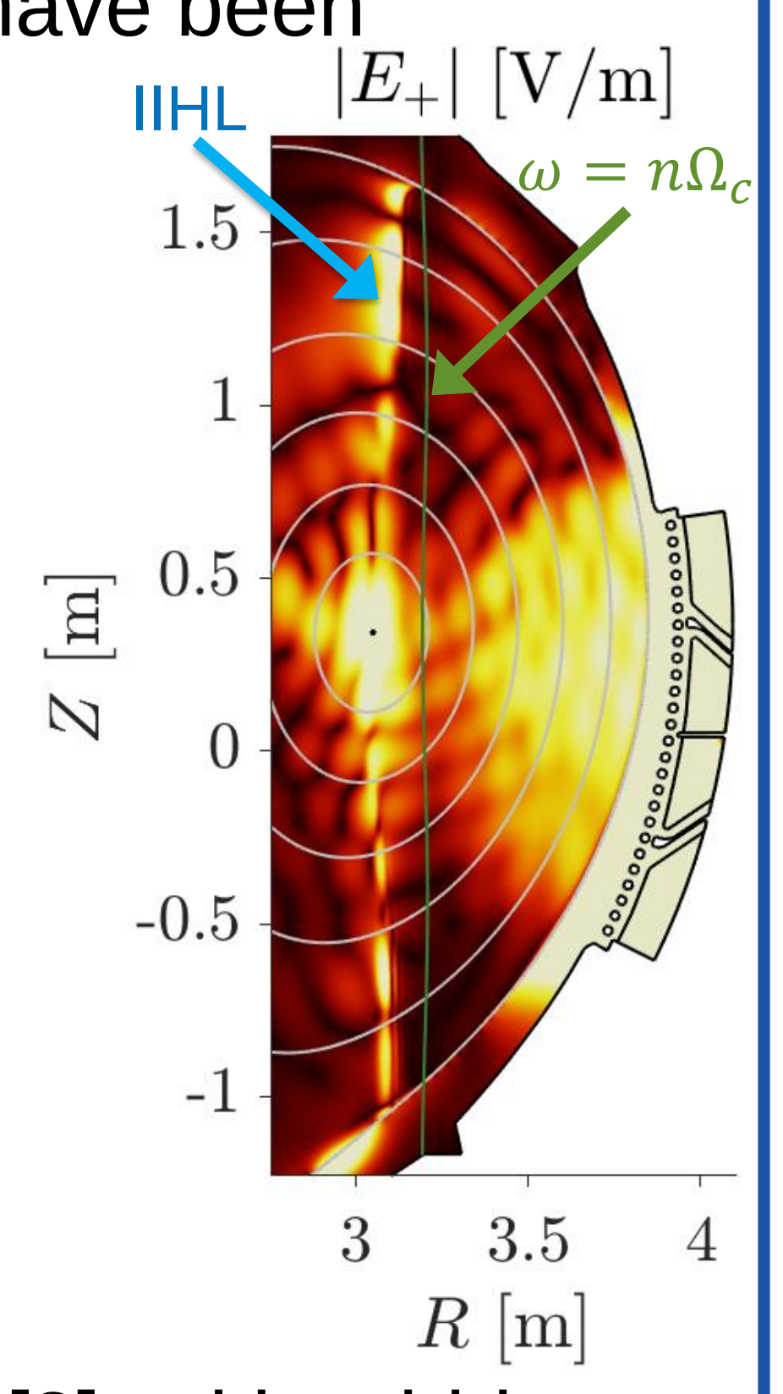
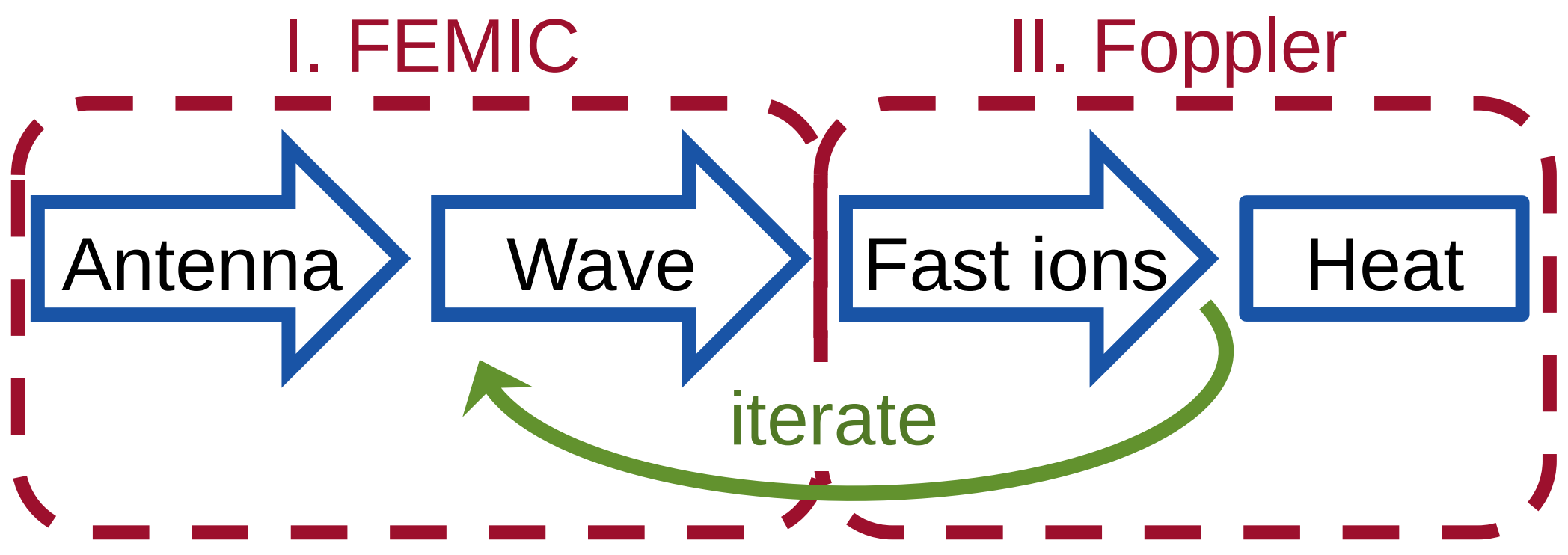
Introduction

- Rapid ICRH modelling is needed for integrated modelling, scenario development and analysis, machine design etc.



- 1D Fokker-Planck models have been successful [1,2]
- Doppler physics is missing in 1D model:
  - Important for interaction with transport and MHD
  - Doppler shifted absorption at the ion-ion hybrid layer (IIHL) enhances the absorption, see Fig.
- **Foppler**, new 1D-FP code including Doppler physics
- Combines the Stix-solution [3], with rabbit ears from Dendy [4].
- Uses waves from FEMIC [5]. Self-consistent coupling with FEMIC is under development.



I. FEMIC

II. Foppler

Antenna

Wave

Fast ions

Heat

iterate

Our goals are:

I. Present 1D Fokker-Plank model Foppler

II. Ad-hoc pitch angle distribution, from Dendy model, enables pitch angle averaging.

III. Clarify impact of Doppler shift on quasilinear diffusion and distribution fcn.

IV. Quantify importance of Doppler shifts in JET D-T plasmas

Fokker-Planck model

- Fokker-Planck (FP) eq. predicts generation of fast ions and heating of thermal plasma
- Quasilinear time scales can be described by bounce averaged FP, for $f(v, \lambda, p_\phi, \sigma)$, where $\lambda = B_0\mu/E$

$$C(f) + Q(f) = 0$$

  - $C(f)$ is the Coulomb collision operator
  - $Q(f)$ is the quasilinear diffusion operator
- Simplifications required for rapid simulations

  I. Thin orbit, $p_\phi \rightarrow \psi$

  II. Ansatz $f = f_s(v)\, f_D(v,\lambda)$, $\langle f_D\rangle_\lambda = 1$

  III. Solve pitch angle averaged FP equation for f_s

  IV. Use ad-hoc model for f_D

References

[1] L.-G. Eriksson et al., Nucl. Fusion 33, 1037 (1993)
[2] D. Van Eester, Journal of Plasma Physics 87, 855870202 (2021)
[3] T. H. Stix, *Waves in Plasmas* (New York: American Institute of Physics, 1992)
[4] R. O. Dendy et al., *Phys. Plasmas* **2** 1623 (1995)
[5] P. Vallejos et al., *Nuclear Fusion* **59,** 076022 (2019)
[6] L.-G. Eriksson and P. Helander, *Physics of plasmas* **1**, 308–314 (1994)
[7] M.J. Mantsinen et al., to be submitted to *Nuclear Fusion*

Pitch angle averaged FP

- Pitch angle average, $\langle X\rangle_\lambda = I(X)/I(1)$, where

$$I(X) = \int_\psi^{\psi+d\psi} d^3r \int_v^{v+dv} d^3v\ X \propto \int \tau_b X\, \mathrm{d}\lambda$$

- $\langle C\rangle$ includes energy diffusion, β, and slowing down, κ

$$\langle C\rangle = \frac{1}{v^2}\frac{\partial}{\partial v}v^2\left(-\kappa f_s + \frac{\beta}{2}\frac{\partial f_s}{\partial v}\right)$$

- Quasilinear operator

$$\langle Q\rangle = \frac{1}{v^2}\frac{\partial}{\partial v}v^2\langle f_D D\rangle\ \frac{\partial f_s}{\partial v}$$

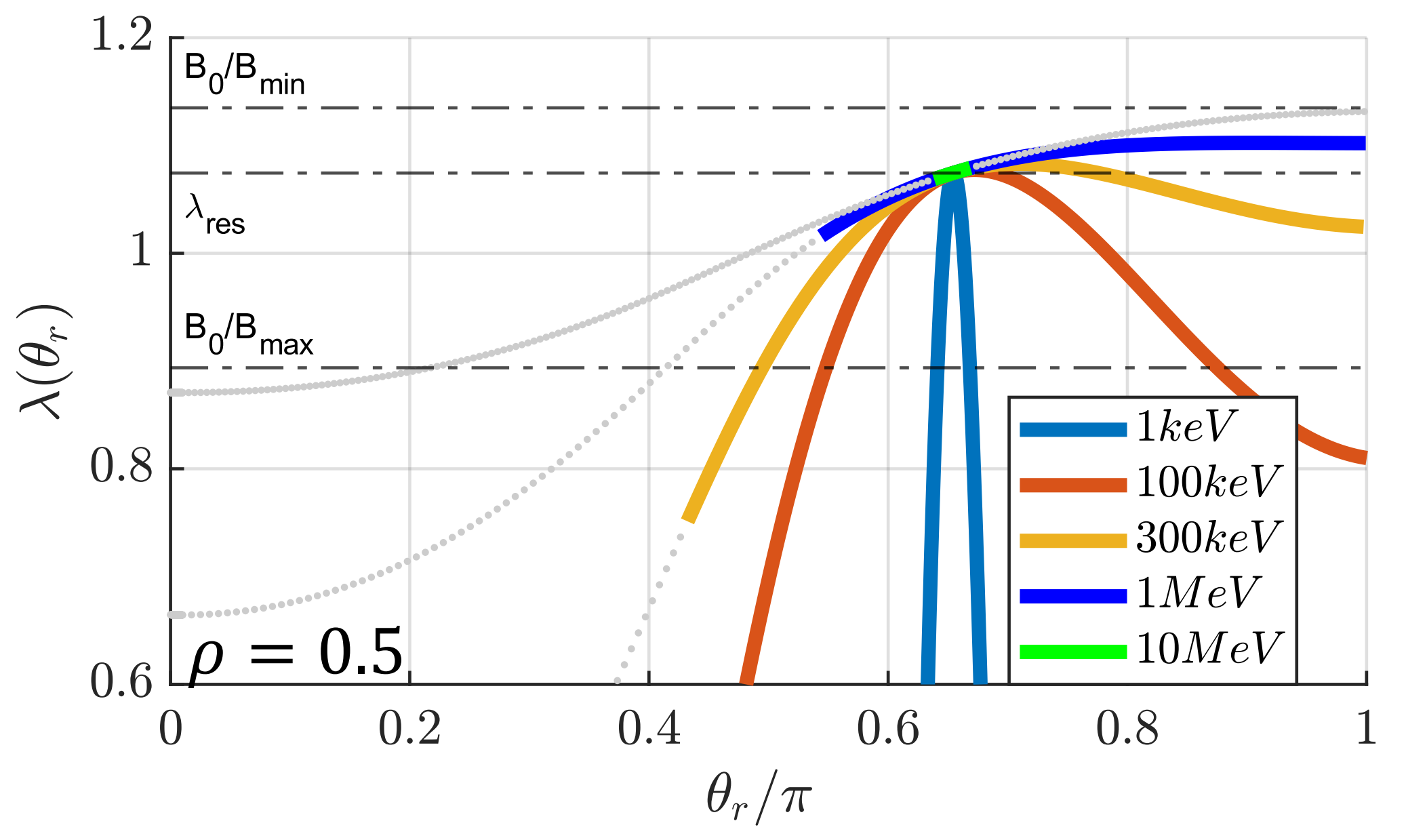
  Here $D(v,\lambda)$ is bounce averaged diffusion coefficient [6]

- Integral $\int \tau_b f_D D\, \mathrm{d}\lambda$ has singularity at tangential resonance that is removed when integrating in poloidal angle θ_r, where

$$\omega - k_\parallel(\theta_r)\, v_\parallel(\theta_r) - n\,\Omega_c(\theta_r) = 0$$

$$\langle f_D D\rangle = \sum_{n.n_\phi,\omega} \frac{Z^2e^2L}{8vm^2\ell}\int_0^{2\pi} \frac{f_D\,\lambda\,|E_+J_{n-1} + E_-J_{n+1}|^2}{k_\parallel}d\theta_r$$

where $\ell = v\int \tau_b\, \mathrm{d}\lambda$ and L is circumference of flux surface.



Pitch angle distribution

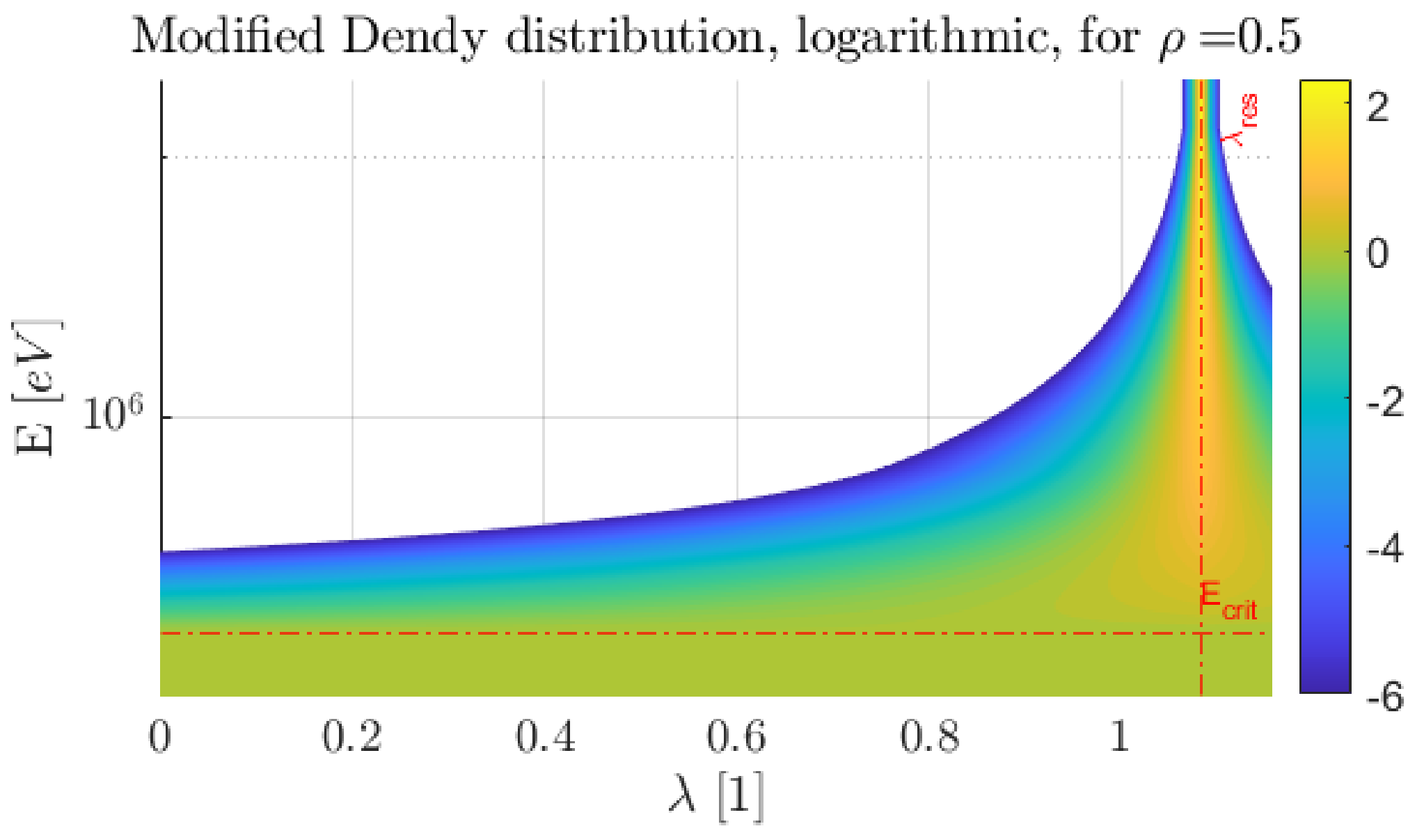
- The solution to the 1D Fokker-Planck equation as proposed by Stix [3]:

$$f_s(v) = f(0)\exp\left[-\int_0^v \frac{2\,\kappa}{\beta + \langle D\rangle}dv'\right]$$

- We propose a modified Dendy model [4]

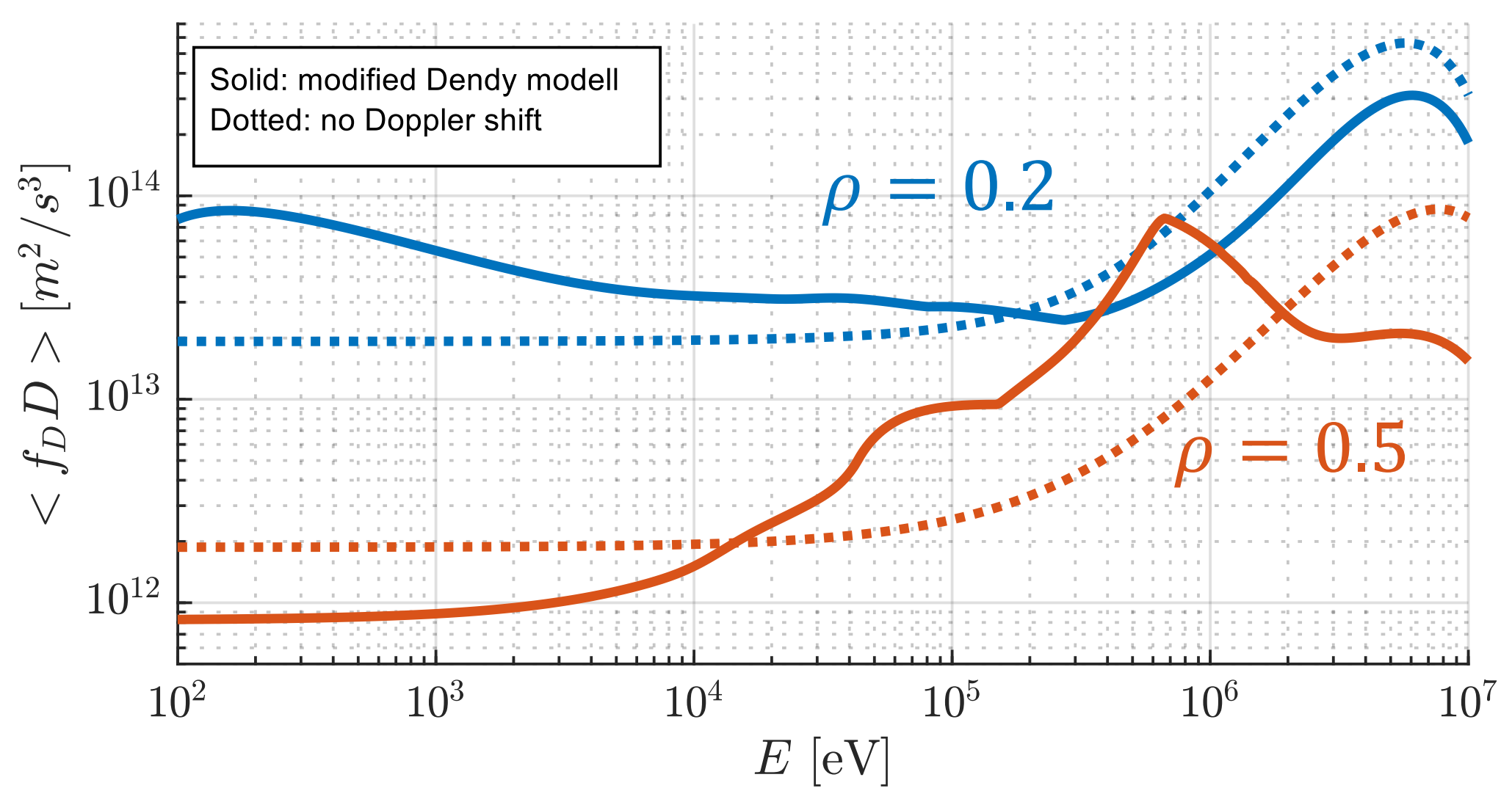
$$f_D(v,\lambda) \propto \exp\left[-\frac{E_\perp}{T_\perp} - \frac{|E_\parallel|}{T_\parallel} - \frac{E_\parallel - |E_\parallel|}{T_\perp}\right]$$

where $E_\perp = B_r\mu$ and $E_\parallel = E - E_\perp$ and $B_r = m\omega/(nZe)$, $T_\perp = -[\partial_E \ln(f_s)\,]^{-1}$, $T_\perp = \max\left(T_\perp, E_\gamma\right)$, and E_γ is the critical energy.



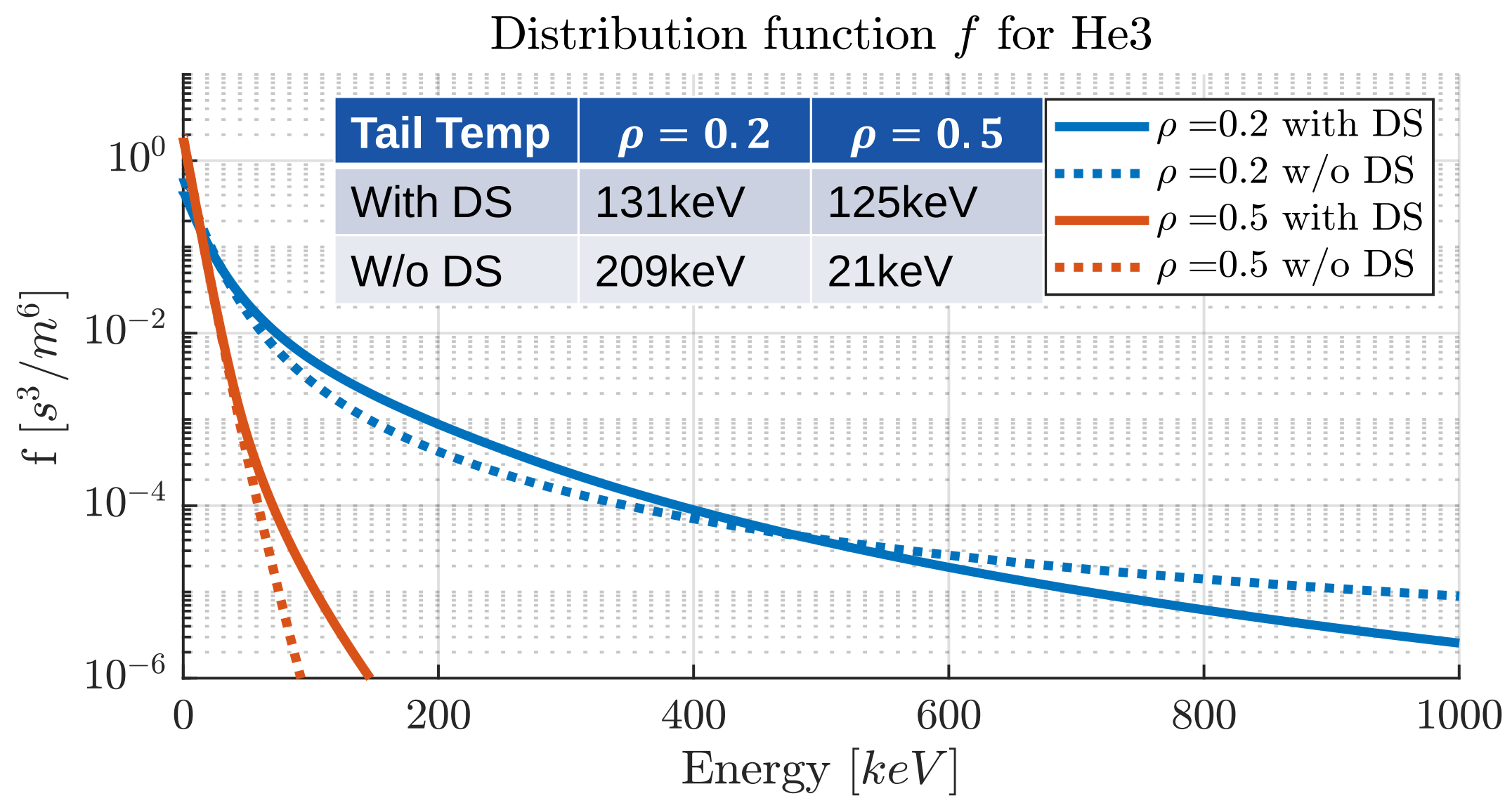
Diffusion coefficient $\langle f_D D(v)\rangle$

- Example, JET D-T plasmas with fast ^{3}He (similar to #99629) [7]
- Doppler shift changes absorption by spreading out resonances to regions with different polarization and phase integral
- Pitch distribution narrows Doppler shifted resonance region for high energies



Distribution $f(v)$

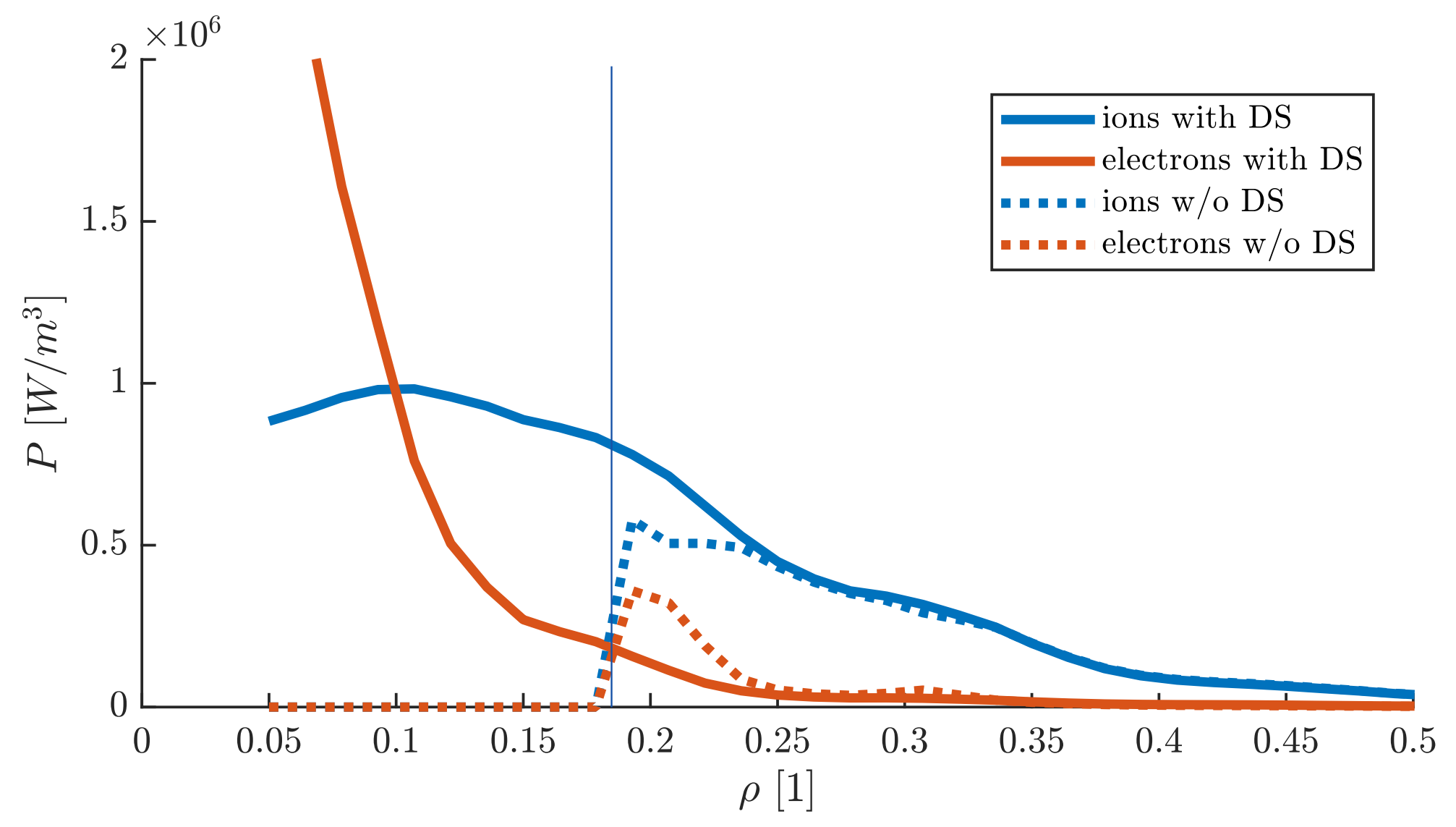
- Including Doppler shift (DS) predicts increased or decreased fast ion temperatures in different regions of plasma
- Without DS localised high fast ion temperature around tangential resonance



Tail Temp	$\rho = 0.2$	$\rho = 0.5$
With DS	131keV	125keV
W/o DS	209keV	21keV

Power Absorption

- $\rho < 0.2$: strong ^{3}He absorption at IIHL
- $\rho < 0.2$: No resonances w/o Doppler shift
- $\rho \sim 0.2$: Increased electron absorption w/o Doppler shift
- $\rho \sim 0.2$: Doppler shift pushes tangential resonances away from equatorial plane



Discussion and Conclusions

1. 1D Fokker-Planck model is advantageous in simplicity, transparency and rapidity
2. Despite being a 1D model, we include Doppler shift and a pitch angle distribution
3. Singularities in RF diffusion coefficient have been resolved
4. Fast ion temperature modified by DS
5. LFS resonance, strong damping at IIHL requires Doppler shifts to model.

EUROfusion

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.