Impact of 3D velocity boundary conditions consistent with Ohm's law on the stability of free-boundary modes

Luca Spinicci[1,2], Daniele Bonfiglio[1,3], Susanna Cappello[1,3] , Marco Veranda[1,3]

[1]Consorzio RFX, Euratom-ENEA Association, Padova, Italy
[2]University of Padova, Padova, Italy
[3]ISTP-CRN, Milano-Padova-Bari, Italy
Luca.Spinicci@igi.cnr.it

General context:

Resistive wall modules [1] allow realistic magnetic boundary formulation in most nonlinear MHD codes.

However, the fluid boundary is usually simplified: $\vec{v}\cdot\hat{\boldsymbol{n}} = 0$, or possibly, $\vec{v}\cdot\hat{\boldsymbol{n}} = v^{0,0}$.
Such an assuption is both unphysical and inconsistent with the magnetic boundary.

A 3D velocity boundary was identified as crucial for modelling Vertical Displacement Events (VDEs) [2] and have been recently included in the DEBS [3], NIMROD [4] and JOREK [5] codes for better modelling of the scrape-off layer in simulations of VDEs.

Despite this, all nonlinear MHD studies on free-boundary modes leverage a high-resistivity low-density «pseudo-vacuum» region around the hot and denser plasma core, enforcing boundary conditions at some analytical boundary.

Specific premises:

Resistive-wall boundary conditions, with fluid boundary consistent with Ohm's law, have been recently implemented in SPECYL [1,6] and PIXIE3D [1,7].

A very thorough nonlinear verification benchmark has been performed between the two codes [1].

This poster, a proof of principle:

A fully consistent boundary must be capable of reproducing a free plasma-vacuum interface in the «transparent-wall» limit, by setting:

$$\textit{analytical domain} \;\equiv\; \textit{plasma boundary}$$

W.r.t. the pseudo-vacuum approach:
1) More robust convergence (asymptotic!!) to analytical models
2) Wider applicability to several initial equilibria
3) No waste of computational time in modelling vacuum

Limitation: plasma surface deformation must be negligible for the dynamics $\Longrightarrow$ we study linear perturbations

1. The SpeCyl code

The **SpeCyl** code [1,6] advances in time t the magnetic field $\boldsymbol{B}$ and the velocity $\boldsymbol{v}$, according to a visco-resistive scheme:

$$\rho\partial_t \boldsymbol{v} + \rho\boldsymbol{v}\cdot\nabla\boldsymbol{v} = \boldsymbol{J}\times\boldsymbol{B} - \rho\nu\nabla^2\boldsymbol{v}$$
$$\partial_t\boldsymbol{B} = -\nabla\times(\eta\boldsymbol{J} - \boldsymbol{v}\times\boldsymbol{B})$$
$$\boldsymbol{J} = \nabla\times\boldsymbol{B} \qquad \nabla\cdot\boldsymbol{B} = 0$$

Main assumptions:
1. Cylindrical geometry
2. Negligible pressure gradients ($\beta \to 0$)
3. Time-const. density $\rho(r)$, resistivity $\eta(r)$ and viscosity $\nu \nrightarrow r$

The new and more realistic boundary will be crucial for proper modelling of reversed-field pinch helical states [6,8].

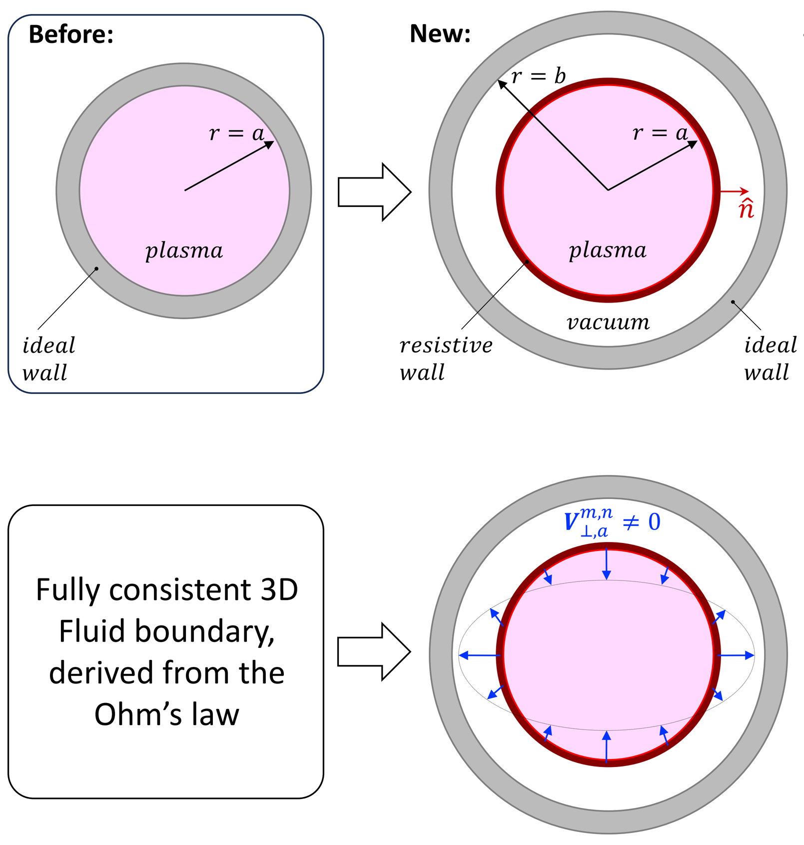
2. External kink mode in the straight tokamak [9]

straight tokamak $\equiv$ $R \gg a$ (cylindrical approx.)
$\Rightarrow$ no pressure driven dynamics

ideal MHD $\equiv$ $\eta, \nu \to 0$ inside the plasma

external kink mode $(m,n) = (2,1)$:
- ✓ External mode ($\boldsymbol{v}_{edge}\cdot\hat{\boldsymbol{n}} \neq 0$)
- ✓ Needs vacuum outside the plasma to get unstable
- ✓ Linear perturbation $m = 2$ on top of axisymm. equilibrium: $\propto e^{i2\theta - \frac{iz}{R} - i\omega t}$; $\mathcal{Re}(-i\omega) = \gamma$

3. Resistive wall boundary with 3D flow consistent with Ohm's law



Thin-shell relations [1] yield:

$$\frac{\partial B_{n,a}}{\partial t} = \frac{1}{\tau_w}\left[\frac{\partial(rB_n)}{\partial r}\right]_-^+$$
$$\boldsymbol{E}_{t,a}(\boldsymbol{B}_a,\boldsymbol{v}_a) = \frac{a}{\tau_w}[\hat{\boldsymbol{n}}\times\boldsymbol{B}_t]_-^+$$

Resistive wall time scale τ_w (input param.)
- $\tau_w \ll \tau_{dyn.}$ → transparent-wall
- $\tau_w \geq \tau_{dyn.}$ → resistive wall
- $\tau_w \gg \tau_{sim.}$ → ideal wall

Scan for a GIF

$\tau_{dyn.}$ = dynamic time-scale; $\tau_{sim.}$ = simulation length

Enforcing Ohm's law:

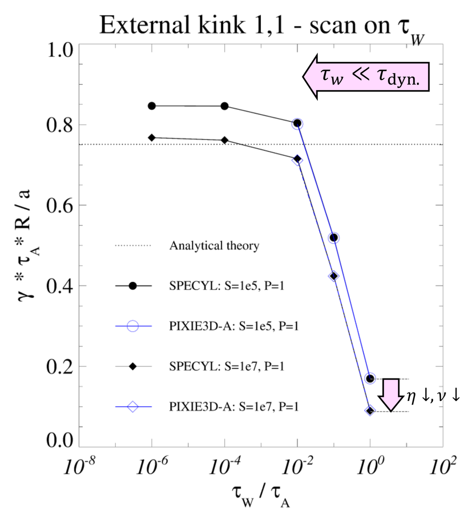
$$\boldsymbol{E}_{t,a}(\boldsymbol{B}_a,\boldsymbol{v}_a) \equiv \frac{\eta}{\mu_0}\nabla\times\boldsymbol{B}_a - \boldsymbol{v}_a\times\boldsymbol{B}_a$$

$$\boldsymbol{v}_a = \boldsymbol{v}_{\perp,a} = \frac{\boldsymbol{E}_{t,a}\times\boldsymbol{B}_a}{B^2}; \qquad \boldsymbol{v}_{\parallel,a} = 0$$

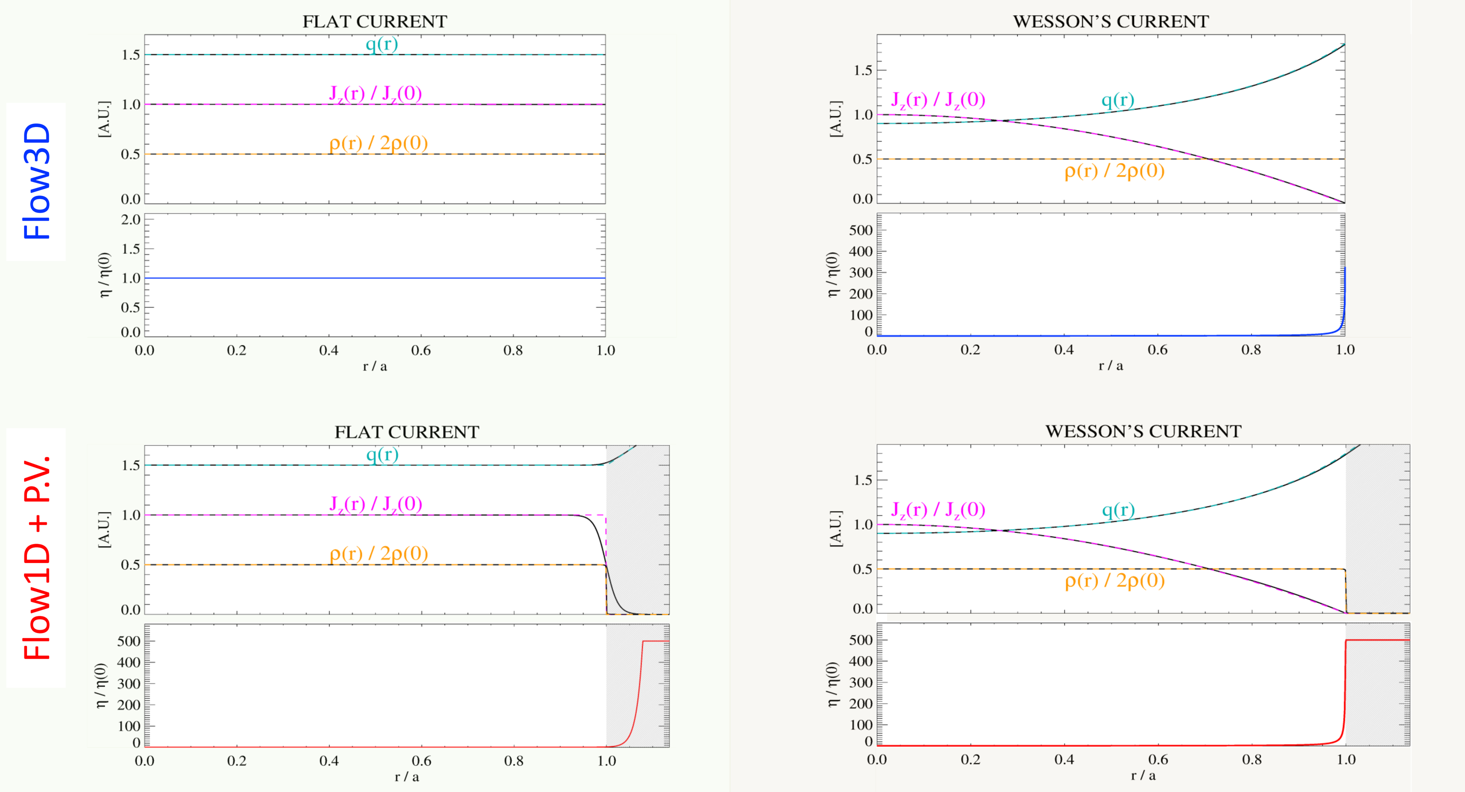
Purely perpendicular flow

The plasma can impinge on / suck particles from the wall

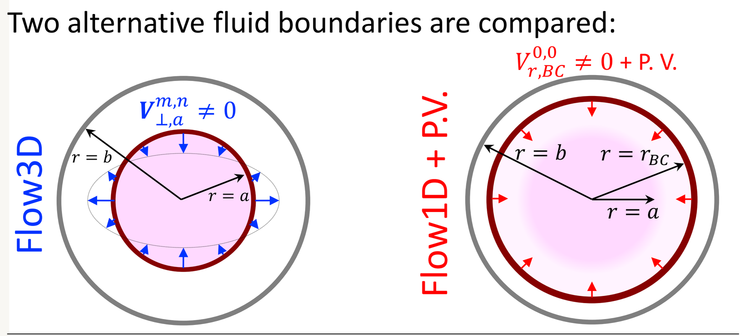
Fluid BCs could easily adapt to accomodate also finite $v_{\parallel,a}$



4. Comparative study: external kink mode 2,1 with two alternative velocity boundaries

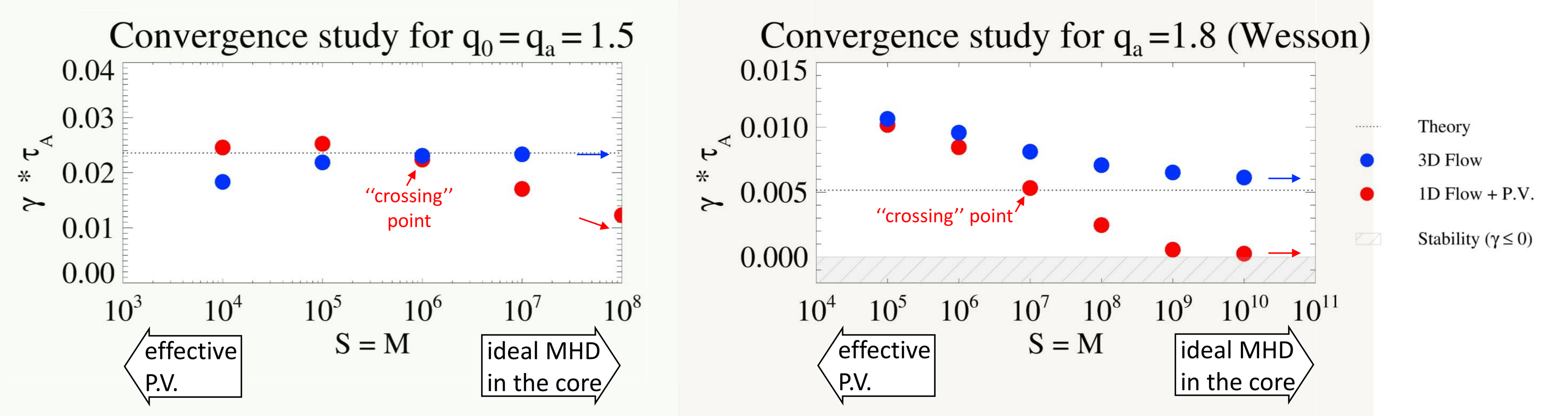


Two alternative fluid boundaries are compared:



Pseudo vacuum (P.V.) $\Longrightarrow$ $\begin{cases}\rho(r>a) \approx 10^{-4}\rho(0)\\ \eta(r>a) \approx 500\,\eta(0)\end{cases}$
best possible, numerically stable

Two initial axisymmetric Ohmic equilibria ($J_z \propto \eta^{-1}$):
1) Flat-current model [9]:
$$J_z(r) = \begin{cases}J_z(0), & \text{if } r \leq a\\ 0, & \text{if } r > a\end{cases}$$
2) Wesson's model [10]:
$$J_z(r) = \begin{cases}J_z(0)[1-(r/a)^2], & \text{if } r \leq a\\ 0, & \text{if } r > a\end{cases}$$

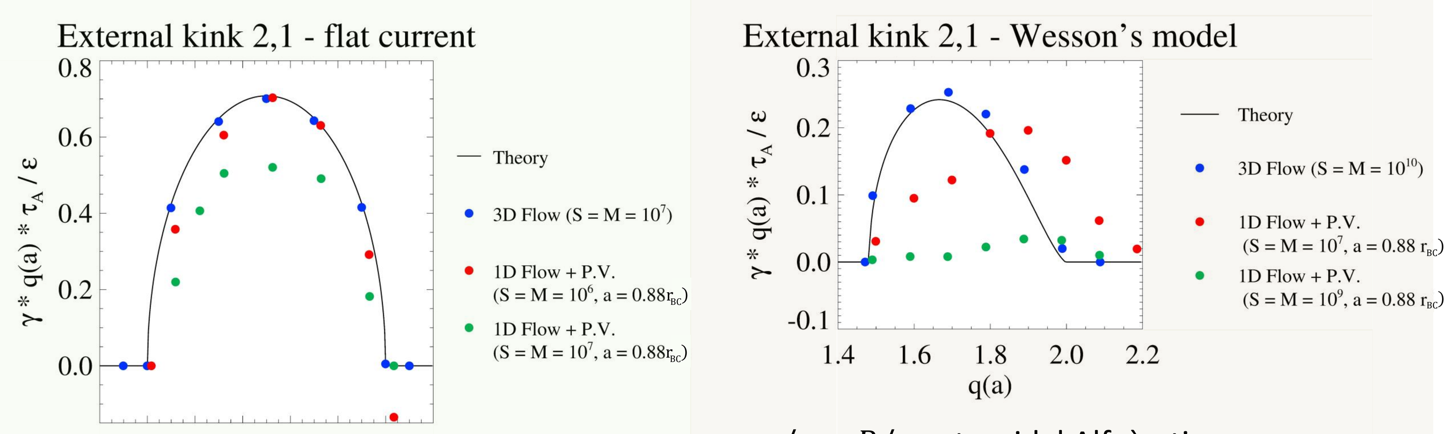


Robust convergence

Flow3D: asymptotic convergence to γ_{theory}

Flow1D+P.V "crosses" γ_{theory} and is asymptotically stable

$$S = \frac{1}{\eta}; \qquad M = \frac{1}{\nu}$$

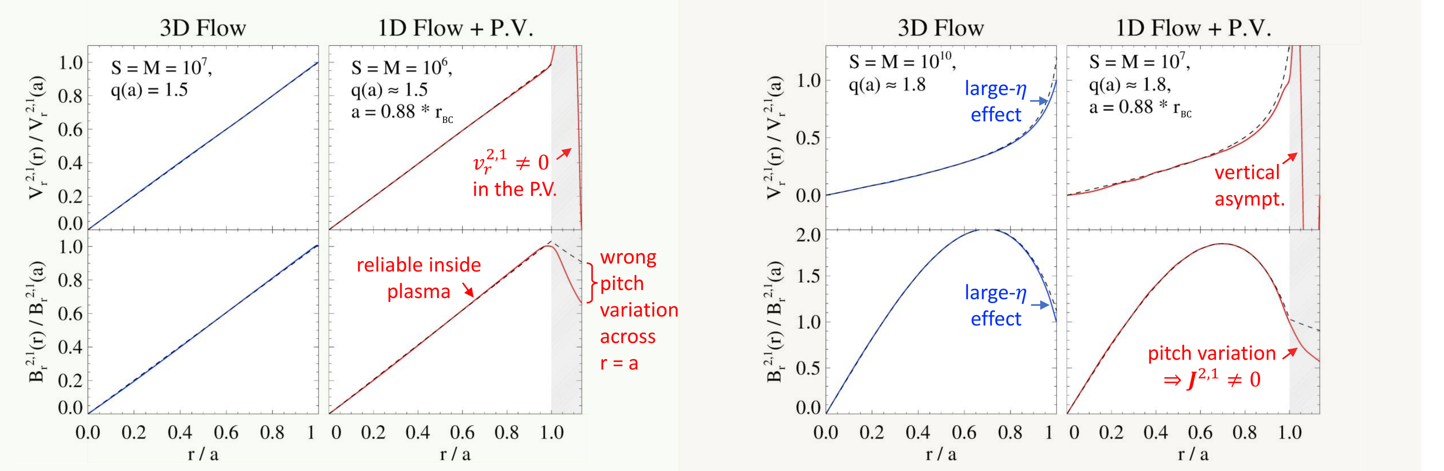


$\tau_A/\varepsilon = R/v_A$ toroidal Alfvèn time

Wide range of initial equilibria

Flow3D: sticks to theoretical expectations

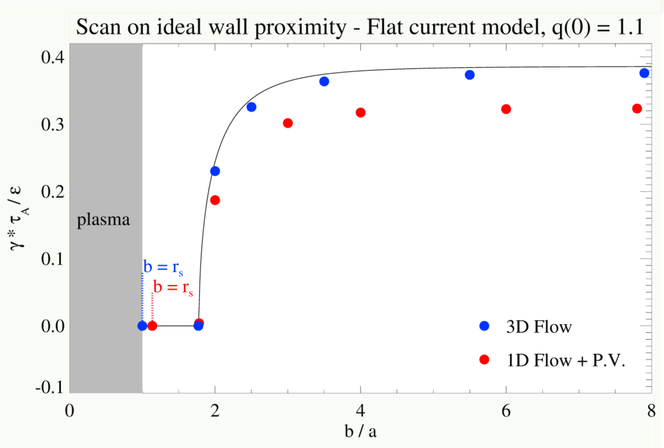
Flow1D+P.V. ("crossing" point) & Flow1D+P.V. (more conductive): can only deal with a flat current. Finite $\partial_r\eta$ inside the plasma completely spoils the stability boundaries in the Wesson's equilibrium case study.



P.V. is not a reliable vacuum

Flow3D: sticks to theoretical profiles (----) (little disturbed by larger η_{edge} in Wesson's case study).

Flow1D+P.V. ("crossing" point): competition between MHD ideality in the core and effective vacuum behaviour of the P.V.
In the Wesson's case there is evidence of a spike in $\boldsymbol{J}^{2,1}$ at resonance radius (inside P.V.!!!) $r_{q=2} \approx 1{,}05\,a$



Stabilising effect of an ideal wall

Only for the flat-current model:

Flow3D: sticks to theoretical expectations, within a 1% tolerance.

Flow1D+P.V. ("crossing" point): Sticks to theoretical expectations, within a 15-20% tolerance.

IN CONCLUSION:

- P.V. is unavoidable when the boundary conditions are not fully self-consistent
- Our fully consistent boundary conditions can reproduce a free boundary when $\tau_w \ll \tau_{dyn.}$
- Also, convergence is more robust and general
- Thorough modelling of vacuum: as we know, the first time with a nonlinear MHD code!

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