Analytical instruments that can be useful to plasma theory: axial anomaly, conformal invariance, effective Larmor radius

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Outline

- Axial anomaly; vorticity concentration [fluids and plasmas]
- Intrinsic length (Hasegawa Mima) vs. conformal invariance (Euler)
- Effective ion-Larmor radius; vortex filaments as precursors of ELMs

Experiments and numerical simulations:
the emergence of coherent structures in turbulence involves separation and clusterization of vortices. There is attraction between like-sign, - and repulsion of opposite sign vortices.

The most studied system: ideal, incompressible, two-dimensional (Euler) fluid.

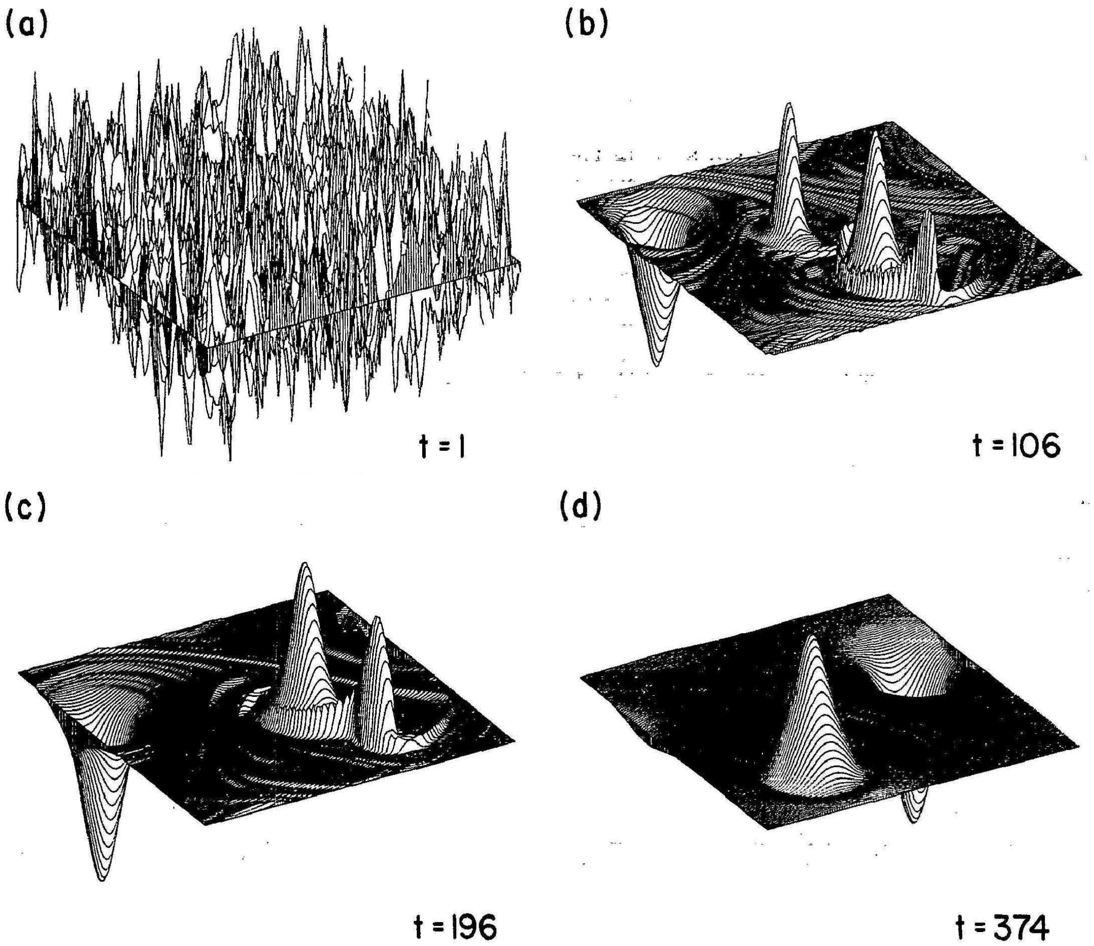
A We show that a field theoretical (FT) formulation provides exact, pure analytical, derivation of the equation verified by the asymptotic, highly coherent relaxed states (sinh-Poisson eq.)

B We then show that inside this FT model, a subsystem of dynamical equations provides the analytical description of the interaction of a small vortex (vorticity perturbation) with a large vortex. This will confirm analytically that a large vortex acts upon the turbulent sea around by attracting elements of vorticity of like sign and repelling elements of vorticity of opposite sign.

C The continuous increase of a large vortex by inflow of smaller vortices still has a deeper theoretical description; we show how it results from our FT. It is a purely classical manifestation of the (famous) axial anomaly.
The basic equation of the axial anomaly is the well known ion-polarization equation in plasma.

We first present the Field Theoretical formulation of an ideal $2D$ Euler fluid, since on this basis we derive the attraction/repulsion and the axial anomaly

Coherent structures of the Euler fluid at asymptotic relaxation



Numerical simulations of the Euler equation.

D. Montgomery, W.H. Matthaeus, D. Martinez, S. Oughton, Phys. Fluids A4 (1992) 3.

The equation for the $2D$ Euler fluid

$$\frac{d\omega}{dt} = \frac{\partial}{\partial t}\Delta\psi + \left[\left(-\nabla\psi \times \widehat{\mathbf{e}}_z\right)\cdot\nabla\right]\Delta\psi = 0. \tag{1}$$

An equivalent discrete model for the Euler equation

$$\frac{dr_k^i}{dt} = \varepsilon^{ij}\frac{\partial}{\partial r_k^j}\sum_{n=1,n\neq k}^{N}\omega_n G\left(\mathbf{r}_k - \mathbf{r}_n\right)\ ,\ i,j=1,2\ ,\ k=1,N \tag{2}$$

the Green function of the Laplacian

$$G\left(\mathbf{r},\mathbf{r}'\right) \approx -\frac{1}{2\pi}\ln\left(\frac{\left|\mathbf{r}-\mathbf{r}'\right|}{L}\right) \tag{3}$$

The statistical physics of the system of point-like vortices interacting in plane by a long range potential: the phase space is finite, the temperature is negative. The extremum of the entropy leads to

$$\Delta\psi + |\lambda| sinh(\psi) = 0 \tag{4}$$

The Lagrangian

of the $2D$ *Euler fluid (e.g water): Non-Abelian* $SL(2,\mathbf{C})$*, Chern-Simons,* 4^{th} *order*

$$\begin{aligned}\mathcal{L} &= -\varepsilon^{\mu\nu\rho}Tr\left(\partial_\mu A_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho\right) + \\ &\quad iTr\left(\phi^\dagger D_0\phi\right) - \frac{1}{2}Tr\left((D_i\phi)^\dagger D_i\phi\right) + \frac{1}{4}Tr\left(\left[\phi^\dagger,\phi\right]\right)^2\end{aligned} \tag{5}$$

where

$$D_\mu\phi = \partial_\mu\phi + [A_\mu,\phi]$$

The equations of motion are

$$iD_0\phi = -\frac{1}{2}\mathbf{D}^2\phi - \frac{1}{2}\left[\left[\phi,\phi^\dagger\right],\phi\right] \tag{6}$$

$$F_{\mu\nu} = -\frac{i}{2}\varepsilon_{\mu\nu\rho}J^\rho \tag{7}$$

The Hamiltonian density is

$$\mathcal{H} = \frac{1}{2} Tr \left((D_i \phi)^\dagger (D_i \phi) \right) - \frac{1}{4} Tr \left(\left[\phi^\dagger, \phi \right]^2 \right) \tag{8}$$

Using the notation $D_\pm \equiv D_1 \pm i D_2$

$$\begin{aligned} Tr \left((D_i \phi)^\dagger (D_i \phi) \right) &= Tr \left((D_- \phi)^\dagger (D_- \phi) \right) + \\ & \frac{1}{2} Tr \left(\phi^\dagger \left[\left[\phi, \phi^\dagger \right], \phi \right] \right) \end{aligned}$$

Then the energy density is

$$\mathcal{H} = \frac{1}{2} Tr \left((D_- \phi)^\dagger (D_- \phi) \right) \geq 0 \tag{9}$$

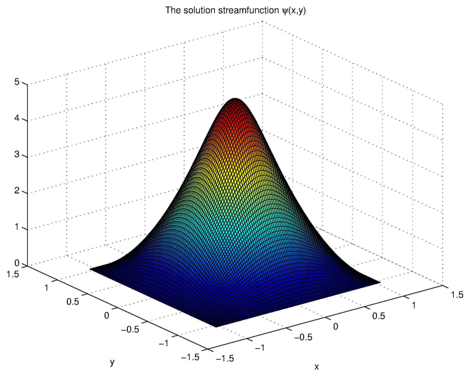
and the Bogomol'nyi inequality is saturated at *self-duality*

$$D_- \phi = 0 \tag{10}$$

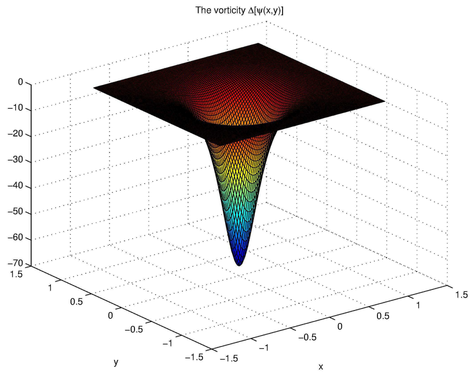
$$\partial_+ A_- - \partial_- A_+ + [A_+, A_-] = \left[\phi, \phi^\dagger \right] \tag{11}$$

With an algebraic ansatz, $\Delta \psi + \gamma \sinh(\beta \psi) = 0.$

Solutions of the *sinh*-Poisson eq.: regular solutions

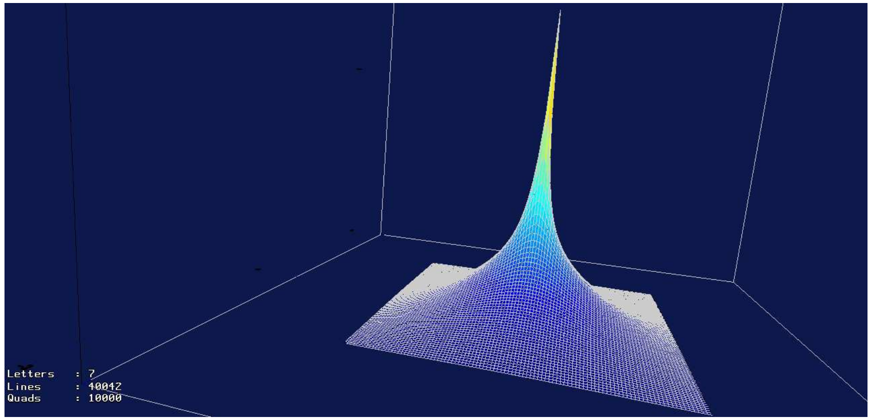


Solution streamfunction

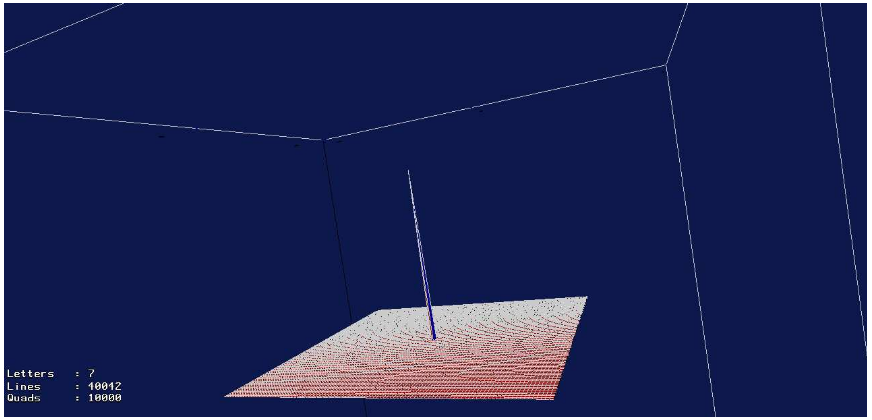


Solution vorticity.

Solutions of *sinh*-Poisson eq.: strange solutions, extreme concentration of vorticity



Solution streamfunction



Solution vorticity.

The problem of generation of large scale flows and coherent structures is reformulated as the problem of separation and clusterization of vorticities of opposite signs.
In particular, the late phase of organization of the flow (late phase in the emergency of coherent structures) is characterized by encounters and coalescences of like-sign (individualized) vortices.
The natural question: like-sign vortices: are-they attracting each other ? opposite-sign vortices: are-they repelling each other ?

Then we formulate the following problem: assume there is a strong, localized vortex, in a sea of low-amplitude turbulent field of vorticity. We want to find what is the effect of the vortex on the field of vorticity.

We dispose of a very powerful field theoretical framework. We found that it is able to give a response to this problem: *indeed a large, fixed vortex (1) attracts elements of vorticity of the same sign and (2) repels the elements of vorticity of opposite sign, - from the turbulent vorticity field around it.*

To show this we have to place more emphasis on the *fermion content* of our field theoretical model.

We then start by showing that the general FT formulation contains a system of fermions interacting with an external magnetic field.

The FT matter field is a mixed spinor

$$\phi \sim x^{\alpha\dot{\beta}} = \phi_1 E_+ + \phi_2 E_- = \begin{pmatrix} 0 & \phi_1 \\ \phi_2 & 0 \end{pmatrix}$$

The equation of Self-Duality derived in Field Theory is $D_-\phi = 0$, or

$$\frac{\partial \phi_1}{\partial z} + a\phi_1 = 0 \quad \text{and} \quad \frac{\partial \phi_2}{\partial z} - a\phi_2 = 0$$

The algebraic ansatz for the potential in FT $A_- = aH$.

The equation for the fermion fields, in external gauge potential $\mathbf{A}^f(\mathbf{x})$, at zero energy, is

$$\alpha \cdot \left(\mathbf{p} - e\mathbf{A}^f\right) \phi^f_{E=0} = 0$$

The matrices are $\alpha^1 = -\sigma^2$ and $\alpha^2 = \sigma^1$ and the column matrix

$\phi^f = \begin{pmatrix} u \\ v \end{pmatrix}$. On components

$$\frac{\partial u}{\partial z} - ieA_-^f u = 0 \quad \text{and} \quad -\frac{\partial v}{\partial \overline{z}} + ieA_+^f v = 0$$

where $A_\pm^f$ are scalar function.

The two sets of equations become identical if

$$\begin{aligned} u &\to \phi_1 \\ v &\to \phi_2^* \\ A_-^f &\to -\frac{1}{ie} a \\ A_+^f &\to \frac{1}{ie} a^* \end{aligned}$$

Check: We can map the two sets equations of fermions and the equations of FT. The first Jackiw equation becomes after replacements

$$\frac{\partial u}{\partial z} - ieA_{-}^{f} u = 0 \rightarrow \frac{\partial \phi_1}{\partial z} + a\phi_1 = 0$$

The second Jackiw equation becomes

$$-\frac{\partial v}{\partial \overline{z}} + ieA_{+}^{f} v = 0 \rightarrow -\frac{\partial \phi_2^*}{\partial \overline{z}} + a^* \phi_2^* = 0$$

and after taking the conjugate

$$\frac{\partial \phi_2}{\partial z} - a\phi_2 = 0$$

This shows that we have the correct mapping. The suggested conclusion is that the Field Theoretical model (Schrodinger, CS, 4-degree nonlinear self-interaction) is locally equivalent with a system of massless fermions in interaction with an external gauge field.

The effect of a strong, localized vortex on the vorticity field around (equivalently: the effect of a tube of magnetic field on the fermion current)

We start from the gauged Dirac equation

$$\left(i\partial\!\!\!/ - eA\!\!\!/ - m\right)\psi = 0$$

and multiply

$$\left(i\partial\!\!\!/ - eA\!\!\!/ + m\right)\left(i\partial\!\!\!/ - eA\!\!\!/ - m\right)\psi = 0$$

with the result

$$\left[\left(i\partial_\mu - eA_\mu\right)^2 - \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu} - m^2\right]\psi = 0$$

We note the occurrence of

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$$

which for the *magnetic field* F_{12} becomes

$$\sigma^{12} = \frac{i}{2}\left(\gamma^1\gamma^2 - \gamma^2\gamma^1\right) = \sigma_3$$

Then the term with the magnetic field is

$$\sigma_3 B$$

We know that this term will multiply

$$\phi = \begin{pmatrix} \Phi\left(x\right) \\ 0 \end{pmatrix} \quad \text{for} \quad E > 0$$

and

$$\phi = \begin{pmatrix} 0 \\ \chi\left(x\right) \end{pmatrix} \quad \text{for } E < 0$$

and will give

$$\sigma_3 B \begin{pmatrix} \Phi(x) \\ 0 \end{pmatrix} = \begin{pmatrix} B\Phi(x) \\ 0 \end{pmatrix}$$

and respectively

$$\sigma_3 B \begin{pmatrix} 0 \\ \chi(x) \end{pmatrix} = \begin{pmatrix} 0 \\ -B\chi(x) \end{pmatrix}$$

Therefore the sign of this term is opposite for $E > 0$ *and respectively for* $E < 0$.

We conclude that

1. *the states with* $E < 0$ *will be REPELLED from the magnetic field*
2. *the states with* $E > 0$ *will be ATTRACTED by the tube of magnetic flux*

Since these are negative vorticity and respectively positive vorticity, we have that a tube of vorticity (a vortex) induces separation of the vorticities and, by attraction and repulsion, clusterization of like sign vorticity.

Axial anomaly and vorticity concentration

(a) the FT is able to prove the concentration of vorticity
(b) There is separation and clusterization of elementary vortices: consider a large initial vortex; from all directions the small vortices (from perturbations) flow radially (1) toward the large vortex if they are like-sign; (2) flow radially outward as repelled from the large vortex, if they have opposite-sign.
Note: the same geometry of flows exists in the Axial Anomaly

Brief presentation: (1) Three fields: scalar ϕ, gauge A_μ, massless fermions $\overline{\psi}, \psi$;

Topology (of gauge field): the Chern Simons term, which is the change in time (for all the volume) of the helicity of the gauge field.

(2) the anomaly equation, The divergence of the fermion current is not zero. It is the amount of topology that the gauge field injects in the system: div-j = topology;

$$\partial_\mu j_5^\mu = \frac{1}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}F_{\mu\nu}F_{\alpha\beta} = \frac{1}{8\pi^2}\mathrm{tr}F\widetilde{F}$$

(this is also $n_+ - n_-$ of the number of zero modes along the vortex) where

$$j_5^\mu = \overline{\psi}\gamma^\mu\gamma_5\psi$$

and

$$F_{\mu\nu}^a\widetilde{F}_{\mu\nu}^a \sim \partial_\mu\left[\varepsilon^{\mu\nu\rho\sigma}\left(A_\nu\partial_\rho A_\sigma + \frac{2}{3}A_\nu A_\rho A_\sigma\right)\right]$$

It is as if the fermions and the Chern-Simons gauge field winding number had a similar nature. It is possible to transfer this common content from one to the other.

In brief, axial anomaly is Conversion of gauge field topology into fermions.

conversion of winding number into fermions and

conversion of fermions into winding number

So how can help us the axial anomaly in the study of vorticity concentration?

Answer: It is the nature of the fields in our FT
While in basic Axial Anomaly there is conversion of gauge field winding number into fermions and the two fields have different natures (electromagnetic field respectively matter) [and you need quantum theory to do that]
in our purely classical FT the gauge field and the matter field are two different representations of a single physical object, - the vorticity. Then Axial Anomaly converts vorticity from its representation as topology of flow velocity into vorticity resulting from the density of elementary vortices.
Nothing is quantum here.

The point-like vortices (represented in the FT for $2D$ Euler as mixed spinors $\phi^{\alpha\dot{\beta}}$) are spinors i.e. the elementary vortices correspond to the fermions [classical spinors, no quantum flip, only "pure" i.e. no intermediate states between up and down].

The topology (in Axial Anomaly theory) is for the fluid the winding number of the velocity field, which, in our FT formulation, is the Chern Simons term in Lagrangian, i.e. the helicity, i.e. the vorticity.

In our FT model for Euler $2D$ the gauge field A_μ comes from the physical velocity field and the matter field $\phi^{\alpha\dot{\beta}}$ (spinors) comes from the density of the elementary vortices.

The winding of the velocity field (i.e. the topology of the gauge field) is the vorticity, because it results from the helicity (CS); on the other hand the $\phi^{\alpha\dot{\beta}}$ (i.e. the matter) is also vorticity as density of elementary vortices. Then for the fluid FT there is no mistery in transmutation from gauge winding to matter with spin. It is transmutation between vorticity from winding of the velocity field to vorticity from collection of elementary vortices. It is effectively the same physical object (vorticity) represented by two fields A_μ and $\phi^{\alpha\dot{\beta}}$. This is why the axial anomaly is the adequate theory for the concentration of vorticity in fluid/plasma.

What can-we learn from the axial anomaly (which we have seen is another view of our FT for Euler) ?

The flows of small vortices toward (like-sign) or outward (opposite sign) relative to an initial large vortex. The flows are components of the axial current ($j_5 \equiv j_{Axial}$)

$$\begin{aligned} j_{Axial}^{\rho} &= -q\frac{F_{0z}}{4\pi}\frac{1}{\rho} \quad \text{(radial current, going into the vortex)} \\ j_{Axial}^{z} &= -q\frac{1}{4\pi}A_0\delta^{(2)}(\rho) \quad \text{(along the vortex)} \end{aligned}$$

(where we have made explicit the "charge" q). The quantity F^{0z} is the *electric field* along the z axis, E^z, $F^{0z} = -\frac{\partial A_z}{\partial t} - \frac{\partial A_0}{\partial z}$. At stationarity $E^z = -\partial\varphi/\partial z$,

$$\partial_i \; j^i_{Axial} = q\frac{E^z}{4\pi}$$

which means that the charge is accumulated on the filament at a rate

$$q^2\frac{E^z}{4\pi}$$

per unit time and unit length. As explained "charge q" = vorticity.

The presence of a large vortex has effect

(1) produces separation of vortices (attraction of like- and repulsion of opposite, - sign), which is polarization of "charge" (= vortices).

(2) produces a flow of "charges" = like-sign vortices toward the large vortex, increasing its amplitude, $\partial\omega/\partial t$. The static accumulation of "charges" = vortices on the main vortex leads to its increase in

magnitude

This is vorticity concentration.

Since $A_\mu \sim \mathbf{v}^{phys}$, one has $A_0 \equiv \varphi \equiv$ "electrostatic potential" with variation on z, which is v_z the *physical* fluid velocity along the axis. Then the *electric field* E^z is

$$E^z = -\frac{\partial A_0}{\partial z} \equiv -\frac{d\varphi}{dz} \sim \frac{dv_z}{dz}$$

and the accumulation of charges is the axial anomaly equation: "divergence of the axial current, $\partial_i \ j^i_{Axial}$ is $q\frac{E^z}{4\pi} \sim E^z$ " . The divergence $\partial_i \ j^i_{Axial}$ reduces to the time variation of the 0-component $\frac{\partial j^0_{Axial}}{\partial x^0} \equiv \frac{\partial Q}{\partial t}$ where Q is the total charge (=vorticity) accumulated, $Q \sim \omega$. Then $\partial_i \ j^i_{Axial} = q\frac{E^z}{4\pi}$ becomes (ignoring factors)

$$\frac{\partial \omega}{\partial t} \sim \frac{dv_z}{dz}$$

In tokamak plasma the polarization current balances the parallel current and the evolution of the vorticity (**Horton,** RMP1999) is $\frac{m_i n \ c^2}{B_0^2} \frac{d}{dt} \nabla^2 \varphi = \nabla_{\parallel} j_{\parallel}$,

$$\frac{\partial \omega}{\partial t} \sim \frac{\partial j_{\parallel}^{plasma}}{\partial l_{\parallel}}$$

which is the same since, $v_z \sim j_{\parallel}^{plasma}$.

We find in the polarization current in tokamak
$\mathbf{j}_p = \frac{c^2 n_i m_i}{B^2} \frac{d\mathbf{E}_{\perp}}{dt} = \widehat{\mathbf{n}} \times \frac{c^2 n_i m_i}{B_0} \frac{d}{dt} \left(\frac{-\nabla \varphi \times \widehat{\mathbf{n}}}{B_0} \right)$ all elements of the Axial Anomaly

- the axial current j_{Axial}^{μ} is a physical filament of vorticity ω;

- the divergence of the axial current j_{Axial}^{μ} is non-zero because new "charges" flow into it; the 0-component $j_{Axial}^{0} \sim Q$ is the total "charge" (vorticity ω) accumulated on the vortex due to this inflow,

$Q \sim \omega =$ Laplacian of the electrostatic plasma potential, $\Delta\varphi^{phys}$; in the physical language vortices flow toward the filament and accumulate on it, thus increasing its magnitude: $\frac{\partial j^0_{Axial}}{\partial x^0} \equiv \frac{\partial Q}{\partial t}$; this is the physical $\partial\omega/\partial t$.

- the radial inflow is determined by the gauge field $E^z = -\frac{\partial A_0}{\partial z}$ along the direction of the axial current, i.e. along the filament of vorticity; since $A_0 \sim v_z^{phys}$, we have $\frac{\partial \omega}{\partial t} \sim \frac{\partial v_z^{phys}}{\partial z} \sim \frac{\partial j_\parallel^{phys}}{\partial z}$ which is the equation for ion polarization in tokamak.

The effective Larmor radius

Experimental observations reveal the equal importance (for the quality of confinement) of the ion poloidal rotation u and the ion diamagnetic rotation, $v_{dia,i}$. In H mode they are observed to have the same direction and with comparable magnitude (**Burrell 1994,1996**).

We connect this with a particular combination $\left(1-\frac{v_{dia}}{u}\right)$ that occurs frequently in drift wave theory. For reference we mention one (**Horton Morrison 1991**).

The equations $\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}=-\frac{e}{m}\nabla\phi+\mathbf{v}\times(\Omega_{ci}\widehat{\mathbf{n}})$ (for ions); the continuity $\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0$, with the assumption of a Boltzmann electron response $n=n_0\exp\left(\frac{e\phi}{T_e}\right)$. The result of these three equations is the Ertel's theorem $\frac{d}{dt}\left(\frac{\omega+\mathbf{\Omega}_{ci}}{n}\right)=0$ where $\omega\equiv\nabla\times\mathbf{v}$. It is assumed that the plasma rotates with the velocity u in poloidal direction. The equation can be reduced to

$$\nabla_{\perp}^{2}\phi=\left(1-\frac{v_{dia,i}}{u}\right)\phi+\ \mathcal{O}\left(\phi^{2}\right)$$

The parameter $k_0^2\equiv 1-\frac{v_{dia,i}}{u}$ appears currently in the theory of drift waves (**Petviashvili**, **Horton, Spatschek, Nycander, etc**). This suggests to introduce

$$\frac{1}{\left(\rho_s^{eff}\right)^2}\equiv\frac{1}{\rho_s^2}\left(1-\frac{v_{dia,i}}{u}\right)$$

After the onset of the Transport Barrier at the edge of the plasma, the gradients increase and the ion diamagnetic velocity approaches from below the plasma rotation velocity $v_{dia,i}\rightarrow u$ and the "effective Larmor radius" becomes very large, $\rho_{eff}\rightarrow\infty$.

The consequences are:

A. Transition from a regime where there is an intrinsic space scale (ρ_s, Charney Hasegawa Mima) to *conformal invariance*, Euler fluid $2D$, absence of any intrinsic spatial scale. Except for imposing bounday conditions (which defines a box = a spatial length) no structure is possible, since the system scales-up or down at no energy cost.

B. Loss of Ertel's theorem, from $\frac{d}{dt}\left(\frac{\omega+\boldsymbol{\Omega}_{ci}}{n}\right)=0$ to $\frac{d\omega}{dt}=0$, the vorticity and the density are decoupled.

C. The polarization drift has zero divergence. Take electric velocity $\mathbf{V}_E=-\frac{1}{B_0}\frac{d\Phi_0}{dr}\widehat{\mathbf{e}}_\theta\equiv u$ and diamagnetic velocity:

$$\nabla_\perp\cdot\mathbf{v}_p\approx-\frac{1}{B_0\Omega_{ci}}\left[\left(-V_E+v_{dia,i}\right)\frac{\partial}{\partial y}\right]\nabla_\perp^2\phi=\frac{1}{\rho_s^2}\left(1-\frac{v_{dia,i}}{V_E}\right)\frac{V_E}{\Omega_{ci}}\frac{\partial}{\partial y}\left(\frac{\rho_s^2\nabla_\perp^2\phi}{B_0}\right)$$

The divergence of the ion polarization drift depends on $\frac{1}{\left(\rho_s^{eff}\right)^2} \equiv \frac{1}{\rho_s^2}\left(1 - \frac{v_{dia,i}}{u}\right)$ and it is noted that when $\rho_s^{eff} \to \infty$ the divergence is zero and the compressibility of $\mathbf{v}_p$ is suppressed.

The meaning is that $\rho_{eff} \to \infty$ is actually the suppression of the transversal divergence of the ion polarization drift. From $\nabla_\perp \cdot \mathbf{j}_p = -\nabla_\| j_\|$, we find that when $\rho_{eff} \to \infty$ the parallel direction is decoupled. This is a radical change in the condition for drift wave excitation.

D. The balance of contributions to the radial electric field at the plasma edge, also depends on ρ_{eff}. When the toroidal term $v_\varphi B_\theta$ is imposed, the only possibility to decrease E_r results from $E_r^{(1)} = -v_\theta B_\varphi \frac{\rho_s^2}{\left(\rho_s^{eff}\right)^2} \to 0$.

Vortices as precursors of ELMs

The possibility of *electron vortices* (cf. **Petviashvili and Pogutse**) with section much less than ρ_i and with frequency much less than Ω_i has been demonstrated. They propagate at an angle α relative to the magnetic field and are *monopolar*.

These vortices have negative energy (they are fed by inverse Landau damping).

Since the Larmor radius that occurs in calculation is *effective ion Larmor radius* ρ_{eff} and since this is very large when $v_{dia,i} \rightarrow u$, these vortices are possible in the pedestal layer and can become precursors of ELM.

The ions have Boltzmann density

$$n = n_0 \left(1 + \kappa x - \frac{T_e}{T_i}\phi\right)$$

The continuity $N = n_0 + n_0 \frac{e\phi}{T_e}$, (with $\kappa \sim v_{dia,i}$ and $\tau = T_i/T_e$)

$$\frac{\partial N}{\partial t} + (-\nabla\phi \times \widehat{\mathbf{e}}_z) \cdot \nabla N = \frac{\partial}{\partial z}\Delta A + (-\nabla A \times \widehat{\mathbf{e}}_z) \cdot \nabla (\Delta A)$$

and equation of parallel motion of electrons

$$(1+\nu) E_\| = \frac{\partial N}{\partial z} + (-\nabla A \times \widehat{\mathbf{e}}_z) \cdot \nabla N$$

where $E_\| = -\frac{\partial A}{\partial t} - \left(\frac{\partial \phi}{\partial z} + (-\nabla A \times \widehat{\mathbf{e}}_z) \cdot \nabla\phi\right)$and $\nu \equiv$viscosity operator, $\nu_{\mathbf{k},\omega} = i\sqrt{\pi}\frac{\omega+\kappa\ k_y}{|k_z|\ v_{th,e}}$. The equations for nondimensional variables and ignoring the dissipation,
$\tau\frac{\partial\phi}{\partial t} + \kappa\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial z}\Delta A + (-\nabla A \times \widehat{\mathbf{e}}_z) \cdot \nabla (\Delta A)$ and

$\frac{\partial A}{\partial t} - \kappa \frac{\partial A}{\partial y} = -\left(1+\tau\right)\left(\frac{\partial \phi}{\partial z} + \left(-\nabla A \times \widehat{\mathbf{e}}_z\right) \cdot \nabla \phi\right)$. After the transformation $\eta = y - ut + \alpha z$, it results $\Delta A + \alpha s^2 x = f\left(A - \alpha x\right)$ with f arbitrary and

$$s^2 = \frac{\left(\kappa + u\right)\left(\kappa - \tau u\right)}{\alpha^2 \left(1+\tau\right)}$$

For ion diamagnetic velocity $< u$, $s^2 > 0$ and the solution is localized.

The condition for existence of vortices was "Boltzmann response of ions" which is possible if ρ_{eff} is large.

Conclusions

Problems that are typical for fluid and plasma are actually found in different formulation in other fields (especially in field theory). Methods and analytical technics that have been developed there can also be useful for fluid and plasma theory. Many applications can result.