The MHD dynamo effect in reversed-field pinch and tokamak plasmas: indications from nonlinear 3D MHD simulations

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This work is dedicated to the memory of our colleague and friend Paolo Piovesan

Abstract

- The MHD dynamo effect is an intrinsic and fundamental feature of reversed-field pinch (RFP) plasmas [1-3]. It plays an important role in the tokamak as well (commonly as referred to as the "flux pumping" mechanism) in particular for the hybrid scenario with central safety factor close to one [4-5].
- In this contribution, we review results based on nonlinear 3D MHD theory and simulations, and related experiments. Such results allow to identify the underlying physics of the MHD dynamo effect common to tokamak and RFP configurations: a helical core displacement modulates parallel current density along flux tubes, which requires a helical electrostatic potential to build up, giving rise to a helical MHD dynamo flow.
- Similarities between the MHD dynamo at play in the reversed-field pinch and tokamak configuration will be discussed, with the aim of providing a common theoretical framework for the two configurations. Both the quasi-periodic sawtoothing regime and the stationary helical regime (obtained either with application of magnetic perturbations [6, 7] or at high plasma pressure [8]) will be considered as result of the nonlinear 3D MHD codes SpeCyl [9] and PIXIE3D [10].

References

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3. S. Cappello, et al., *Nuclear Fusion* **51**, 103012 (2011)
4. S. C. Jardin, N. Ferraro, and I. Krebs, *Physical Review Letters* **115**, 215001 (2015)
5. P. Piovesan, et al., *Nuclear Fusion* **57**, 076014 (2017)
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8. D. Bonfiglio, et al., *Plasma Physics and Controlled Fusion* **57**, 044001 (2017)
9. S. Cappello, *Plasma Physics and Controlled Fusion* **46**, B313 (2004)
10. L. Chacón, *Physics of Plasmas* **15**, 056103 (2008)

Outline of the poster

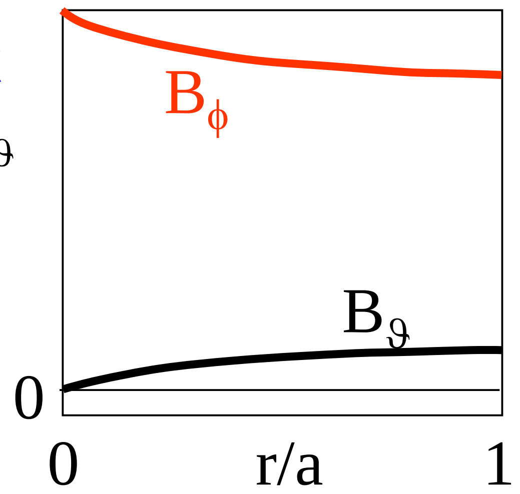
- The reversed-field pinch and its dynamo effect
- Numerical tools: the SpeCyl and PIXIE3D nonlinear 3D MHD codes
- Physical interpretation of the RFP dynamo based on nonlinear 3D MHD simulations
- The dynamo effect (a.k.a. flux pumping) in the tokamak
- Nonlinear 3D MHD simulations show that the internal kink mode in the tokamak is able to produce a significant dynamo effect
- Final remarks

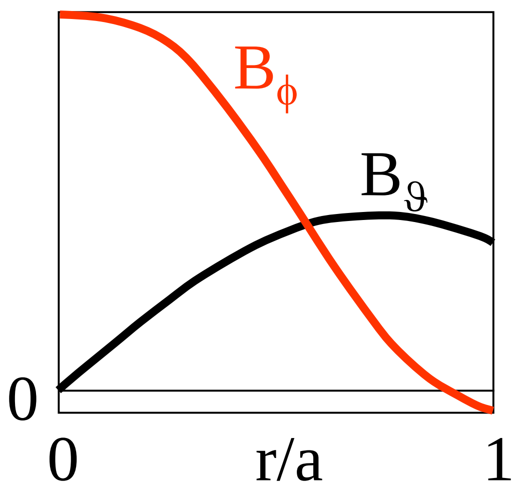
The reversed-field pinch configuration

- The RFP is a toroidal device like the tokamak, but for the same core toroidal field:
  - the plasma current is larger (more Ohmic heating)
  - the edge toroidal field is small (superconductive coils not needed) and reversed

Tokamak

$B_\phi >> B_\vartheta$





RFP

$B_\phi \approx B_\vartheta$

$B_\vartheta(a) \sim I_P$

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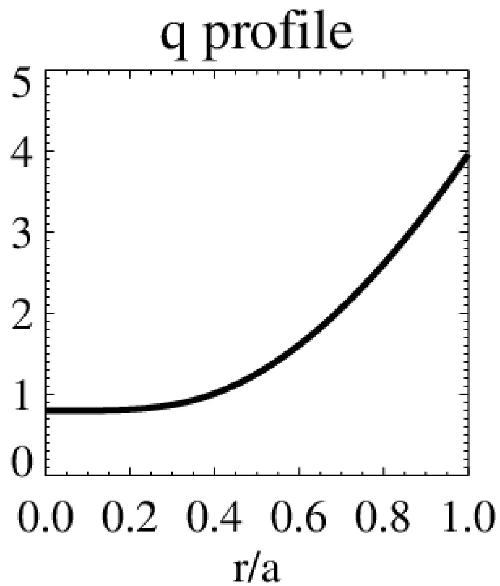
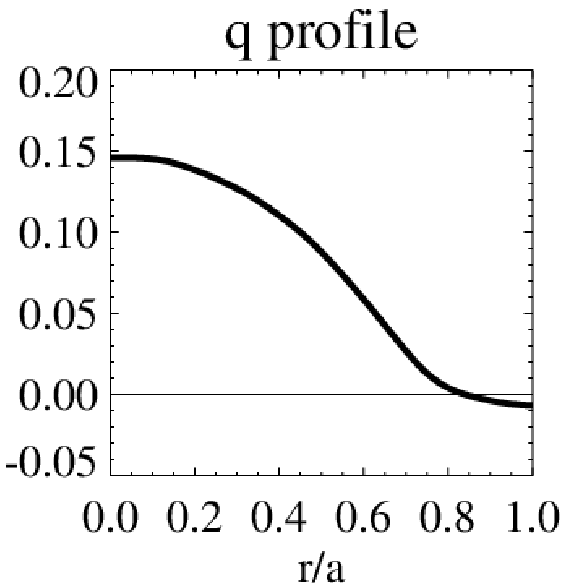




RFP

$B_\phi \approx B_\vartheta$

$B_\vartheta(a) \sim I_P$

- RFP edge reversal due to saturated resistive kink-tearing instabilities
- Helical self-organization = one dominant mode, improved magnetic topology

Dynamo effect in the RFP

- In the RFP, $\eta \mathbf{J}_{\|}$ does not match the parallel electric field $\mathbf{E}_{\|}$ estimated from the externally applied electric field $\mathbf{E}_{ext}$ alone.

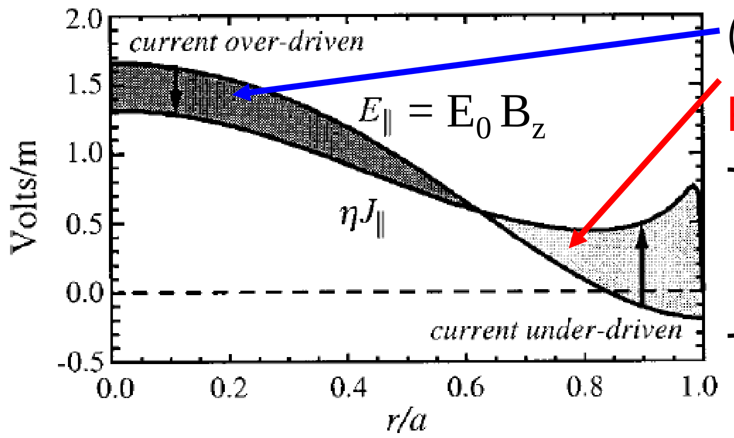


FIG. 2. An illustration showing that $\langle E \rangle_{\|} \neq \eta \langle J \rangle_{\|}$ for an equilibrium modeled using typical MST parameters.

Example from the RFP experiment in Madison [D. Den Hartog *et al.*, PoP 1999]

- A dynamo electric field is produced by the plasma itself:

$\eta \mathbf{J} = \mathbf{E}_{ext} + \mathbf{E}_{dyn}$ $\quad$ $\mathbf{E}_{dyn} = \langle \mathbf{v} \times \mathbf{B} \rangle$ $\quad$ $\langle \cdot \rangle \equiv$ average at fixed radius

- Where does the associated flow come from?
- This process was investigated in nonlinear MHD simulations

[D. Bonfiglio *et al.*, PRL 2005; S. Cappello *et al.*, PoP 2006; S. Cappello *et al.*, NF 2011; P.. Piovesan *et al.*, NF 2017]

Numerical tools: the SPECYL code

- **SPECYL** code [S. Cappello and D. Biskamp, NF 1996]:
  - Solves the equations of the nonlinear visco-resistive MHD model

    $$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = \mathbf{J}\times\mathbf{B} + \nu\nabla^2\mathbf{v} \qquad \leftarrow \text{momentum balance}$$

    $$\partial_t \mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\mathbf{J}) \qquad \leftarrow \text{Faraday-Ohm eq.}$$

    $$\nabla\cdot\mathbf{B} = 0, \quad \mathbf{J} = \nabla\times\mathbf{B}$$

    - Resistivity $\eta = \tau_A/\tau_R \equiv S^{-1}$ (inverse Lundquist number)
    - Viscosity $\nu = \tau_A/\tau_V \equiv M^{-1}$ (inverse viscous Lundquist number)
      - Fixed η and $\boldsymbol{\nu}$ radial profiles

  - Approximations: cylindrical geometry, $\rho = \text{const}$, $\beta = 0$
  - Finite differences in r, spectral in θ, z, semi-implicit time advance
  - Magnetic BCs:
    - ideal wall $B_r(a)=0$
    - helical MPs $B_r^{m,n}(a)=\text{const}$ [D. Bonfiglio *et al.*, NF 2011]

- **PIXIE3D** code [L. Chacón, PoP 2008 and refs. therein]:
  - Takes into account additional MHD terms

$$\partial_t \rho + \nabla\cdot(\rho \mathbf{v}) = 0 \quad \leftarrow \text{continuity equation}$$

$$\partial_t(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = \mathbf{J}\times\mathbf{B} - \nabla p + \nabla\cdot(\rho\nu\nabla\mathbf{v}) + \mathbf{S}_M \quad \leftarrow \text{momentum balance}$$

$$\partial_t T + \mathbf{v}\cdot\nabla T + (\gamma-1)[T\nabla\cdot\mathbf{v} - (\nabla\cdot\kappa\nabla T + S_H)/2\rho] = 0 \quad \leftarrow \text{temperature eq.}$$

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\mathbf{J} - d_i/\rho\,(\mathbf{J}\times\mathbf{B} - \nabla p_e)) \quad \leftarrow \text{Faraday-Ohm eq.}$$

$$\nabla\cdot\mathbf{B} = 0, \quad \mathbf{J} = \nabla\times\mathbf{B}$$

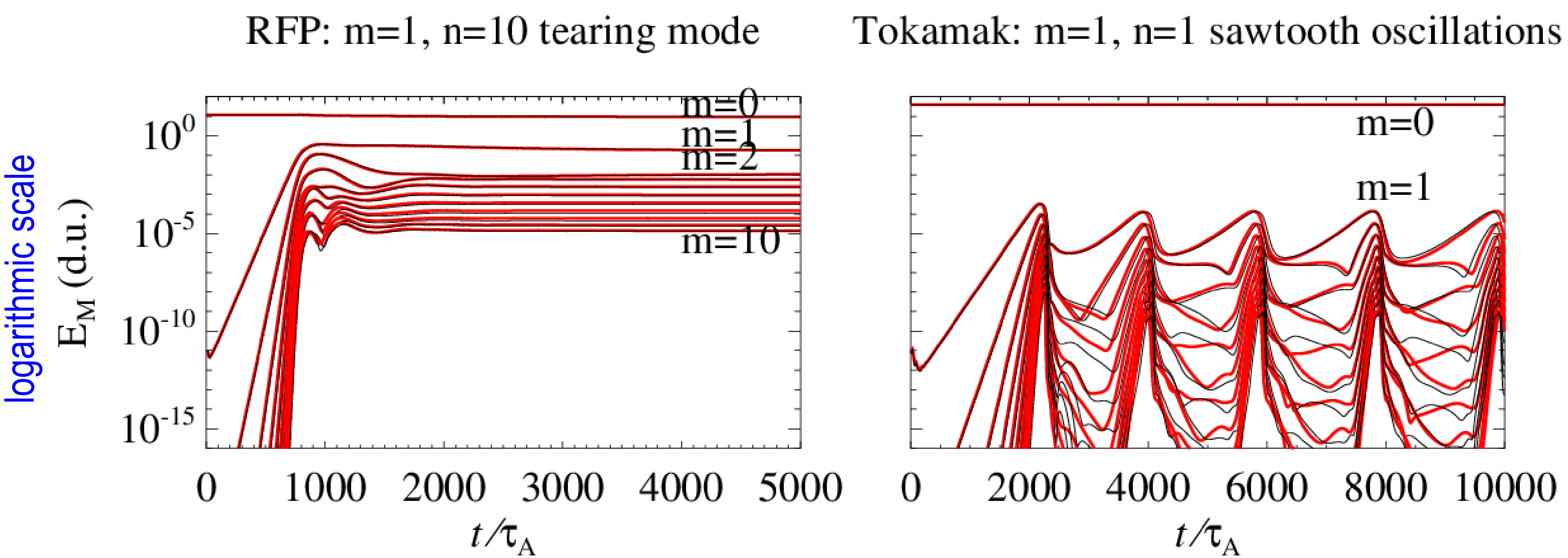
- Finite volume, fully implicit, general curvilinear formulation:
  - Both cylindrical and toroidal geometries allowed
- Same magnetic BCs as **SPECYL**: ideal wall or helical MPs

- In this talk just a couple of **PIXIE3D** applications:
  - as a visco-resistive code in toroidal geometry
  - as a finite-β cylindrical code

Nonlinear verification benchmark: SPECYL – PIXIE3D

- Nonlinear verification benchmark performed in the common limit of application of the two codes [D. Bonfiglio, L. Chacón and S. Cappello, PoP 2010]

- Examples: helical (2D) simulations in cylindrical geometry
  - Temporal evolution of the magnetic energy associated with helical harmonics
  - **SPECYL** (black) and **PIXIE3D** (red curves) superimposed

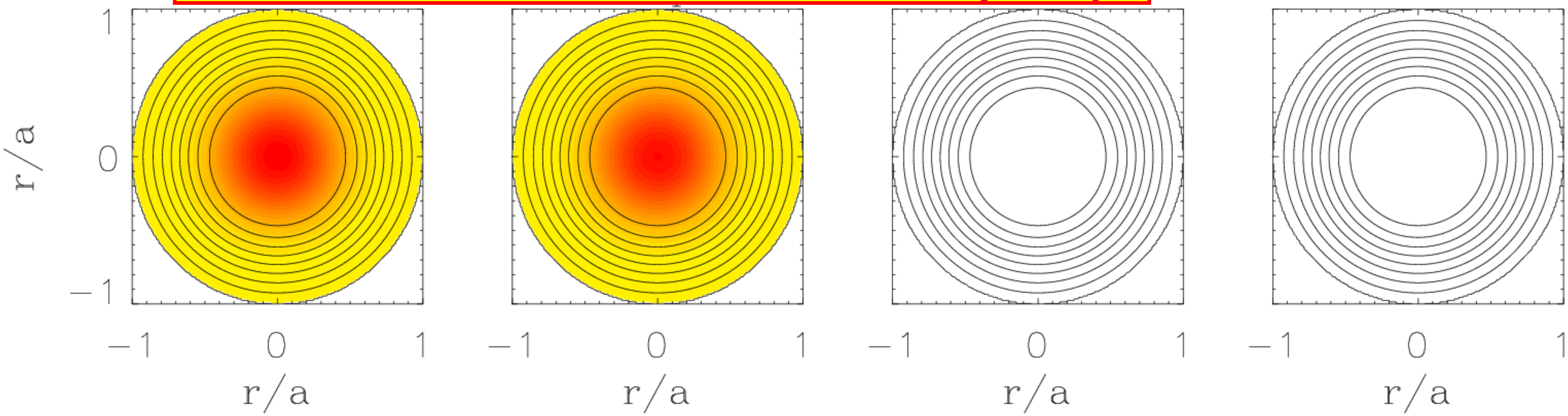


RFP dynamo: stationary conditions

- In stationary conditions, the electric field is irrotational: $\mathbf{E} = E_0\,\hat{\mathbf{z}} - \nabla\phi$
- Consider parallel Ohm's law: $\eta\mathbf{J}\cdot\mathbf{B} = \mathbf{E}\cdot\mathbf{B} = E_0\,B_z - \nabla\phi\cdot\mathbf{B}$
- Initial axisymmetric equilibrium (unstable, not reversed):
  - Parallel current density balanced by parallel applied electric field
  - No need of additional electrostatic fields

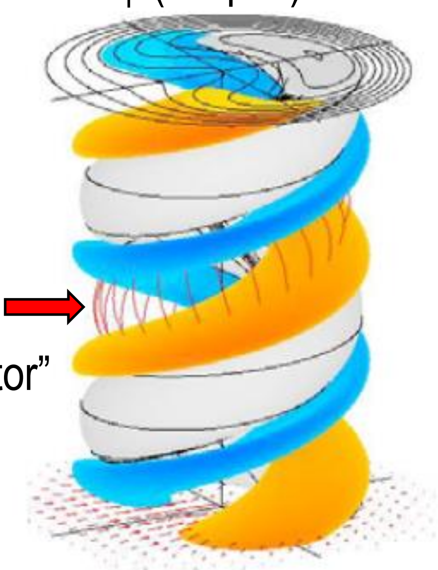
Parallel Ohm's law:

$$\boxed{\eta\mathbf{J}\cdot\mathbf{B} \quad = \quad E_0\,B_z \quad - \quad \cancel{\nabla\phi\cdot\mathbf{B}}} \qquad \phi$$



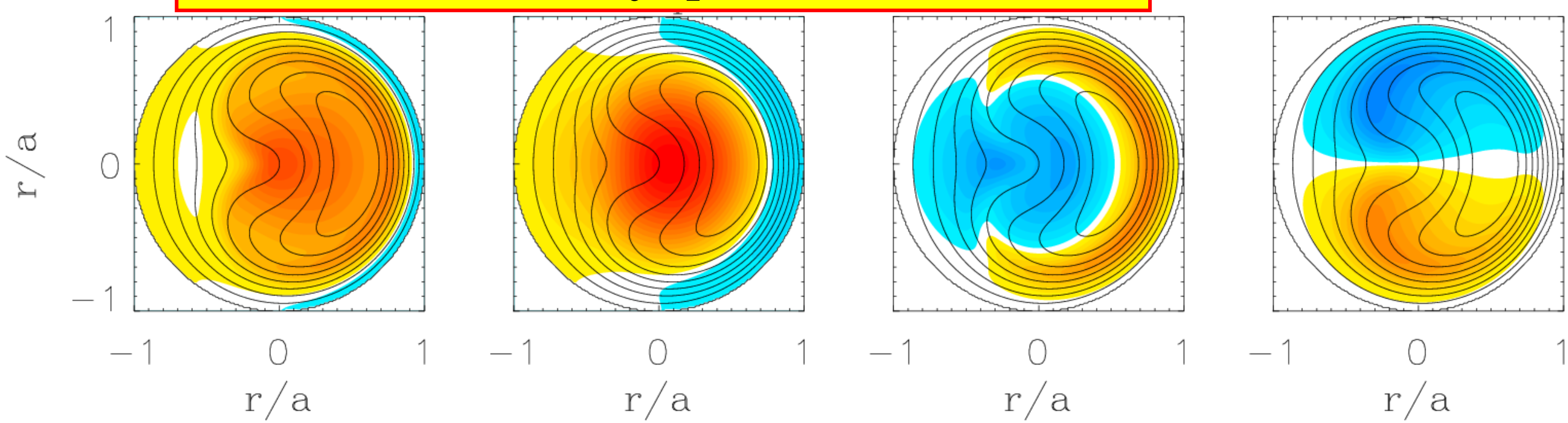
Black curves: flux surfaces. Colors: parallel Ohm's law contributions.

- Final helical equilibrium (stable):
  - Helical deformation of flux surfaces causes modulation of parallel current density and applied electric field along field lines
  - The electrostatic potential builds up to account for the modulation along field lines of the difference $\eta \mathbf{J}\cdot\mathbf{B} - E_0 B_z$



Parallel Ohm's law:

$$\eta \mathbf{J}\cdot\mathbf{B} = E_0 B_z - \nabla\phi\cdot\mathbf{B}$$



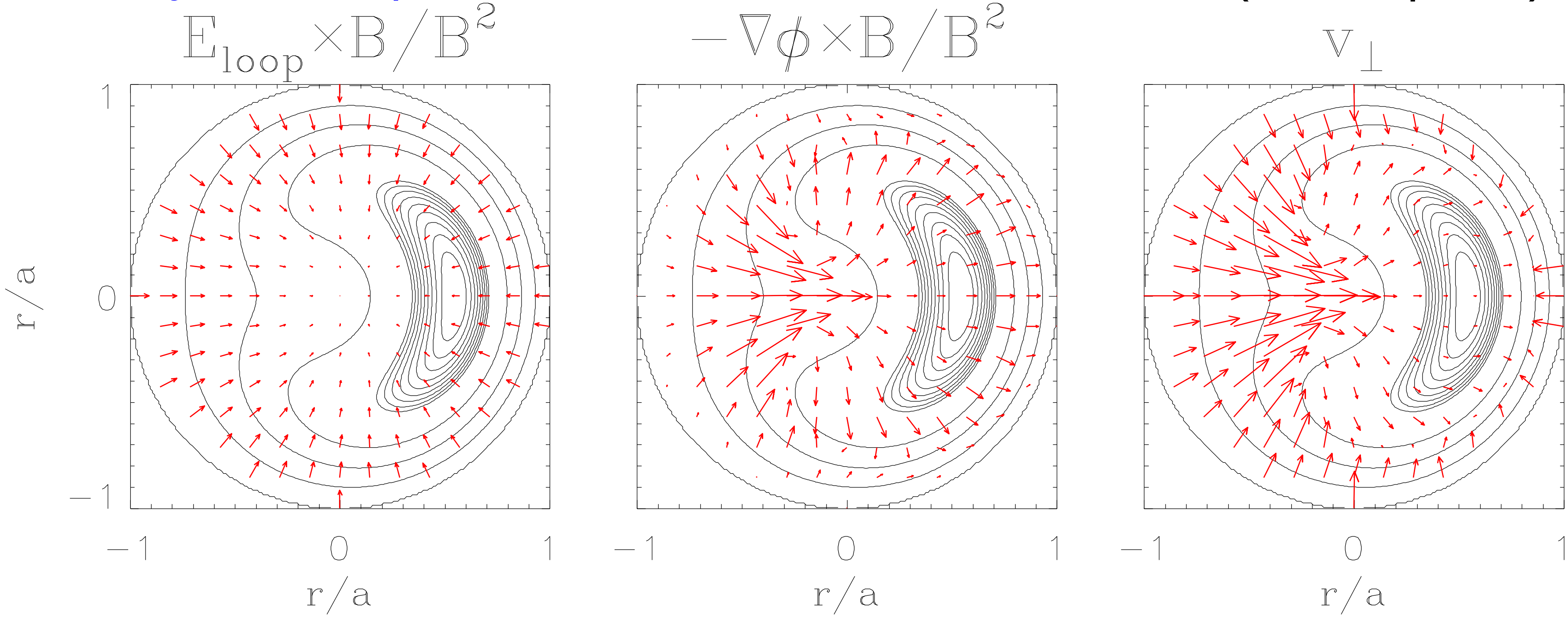
Black curves: flux surfaces. Colors: parallel Ohm's law contributions.

RFP dynamo: stationary conditions

- Dynamo velocity field: pinch + electrostatic drift

$$\mathbf{v}_\perp = \boxed{\mathbf{E}_{\text{loop}} \times \mathbf{B}/B^2} \boxed{-\nabla\phi \times \mathbf{B}/B^2} - \eta \mathbf{J} \times \mathbf{B}/B^2 = \boxed{\mathbf{v}_{\text{pinch}}} + \boxed{\mathbf{v}_{\text{drift}}} + \mathbf{v}_{J\times B}$$

Axisymmetric pinch: Electrostatic drift: total (helical pinch):



- Electrostatic drift: essential contribution to the dynamo.

RFP dynamo: stationary conditions

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- We use to say that the velocity field is slave to the magnetic field
- Indeed, in principle one could solve the Ohmic helical equilibrium equations:
  [J.M. Finn, R. Nebel, C. Bathke, Ph. Fluids B 1992]

$$\mathbf{J}\times\mathbf{B} = 0 \quad \leftarrow \text{helical Grad-Shafranov}$$

$$\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle_\psi = E_0\langle \mathbf{B}_z\rangle_\psi \quad \leftarrow \text{Ohm's law averaged over helical flux surfaces}$$

$$\langle \mathbf{B}\cdot\nabla\phi\rangle_\psi \equiv 0$$ [Book by W. D'haeseleer]

- Then, the electrostatic potential is computed from:

$$\mathbf{B}\cdot\nabla\phi = R = E_0\mathbf{B}_z - \eta\mathbf{J}\cdot\mathbf{B} \quad \leftarrow \text{magnetic differential equation}$$

- Finally, the perpendicular flow is just the corresponding electrostatic drift:
  [D. Bonfiglio *et al.*, PRL 2005, S. Cappello *et al.*, PoP 2006, S. Cappello *et al.*, NF 2011]

$$\mathbf{v}_\perp = \mathbf{E}\times\mathbf{B} / B^2 = \underbrace{E_0\, \hat{\mathbf{z}}\times\mathbf{B} / B^2}_{\text{radial inward pinch}} - \underbrace{\nabla\phi\times\mathbf{B} / B^2}_{\text{helical dynamo flow}}$$

RFP dynamo: non-stationary conditions

- In non-stationary conditions, the electric field is no longer curl-free

- The inductive contribution to the electric field must be included:
  $\mathbf{E} = E_0\,\hat{\mathbf{z}} - \nabla\phi - \partial\mathbf{A}/\partial t$

- Before, ϕ computed simply by integrating Ohm's law $E_0\,\hat{\mathbf{z}} - \nabla\phi = \mathbf{E} = \eta\mathbf{J} - \mathbf{v}\times\mathbf{B}$

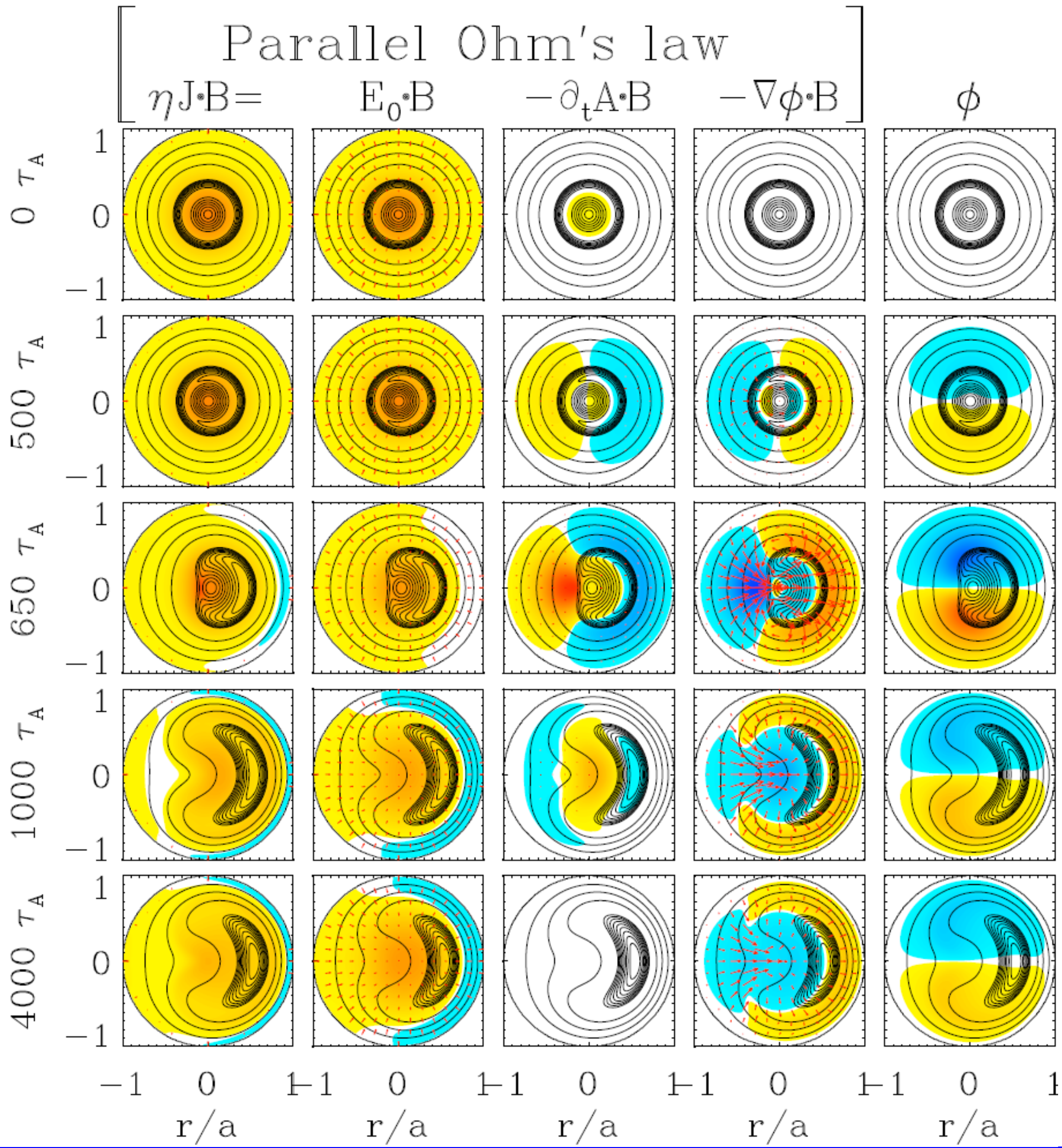
- Now, to obtain ϕ the charge separation ρ_c is computed from Gauss's law:
  $\rho_c = \varepsilon_0\,\nabla\cdot\mathbf{E}$

- ϕ follows through inversion of Poisson's equation in Coulomb gauge ($\nabla\cdot\mathbf{A}=0$):
  $\nabla^2\phi = -\,\rho_c$

- ρ_c turns out to be consistent with the quasi-neutrality condition [D. Bonfiglio *et al.*, PRL 2005]
  $\rho_c\,/\,e = n_i - n_e \ll n_e \cong n_i \cong n$

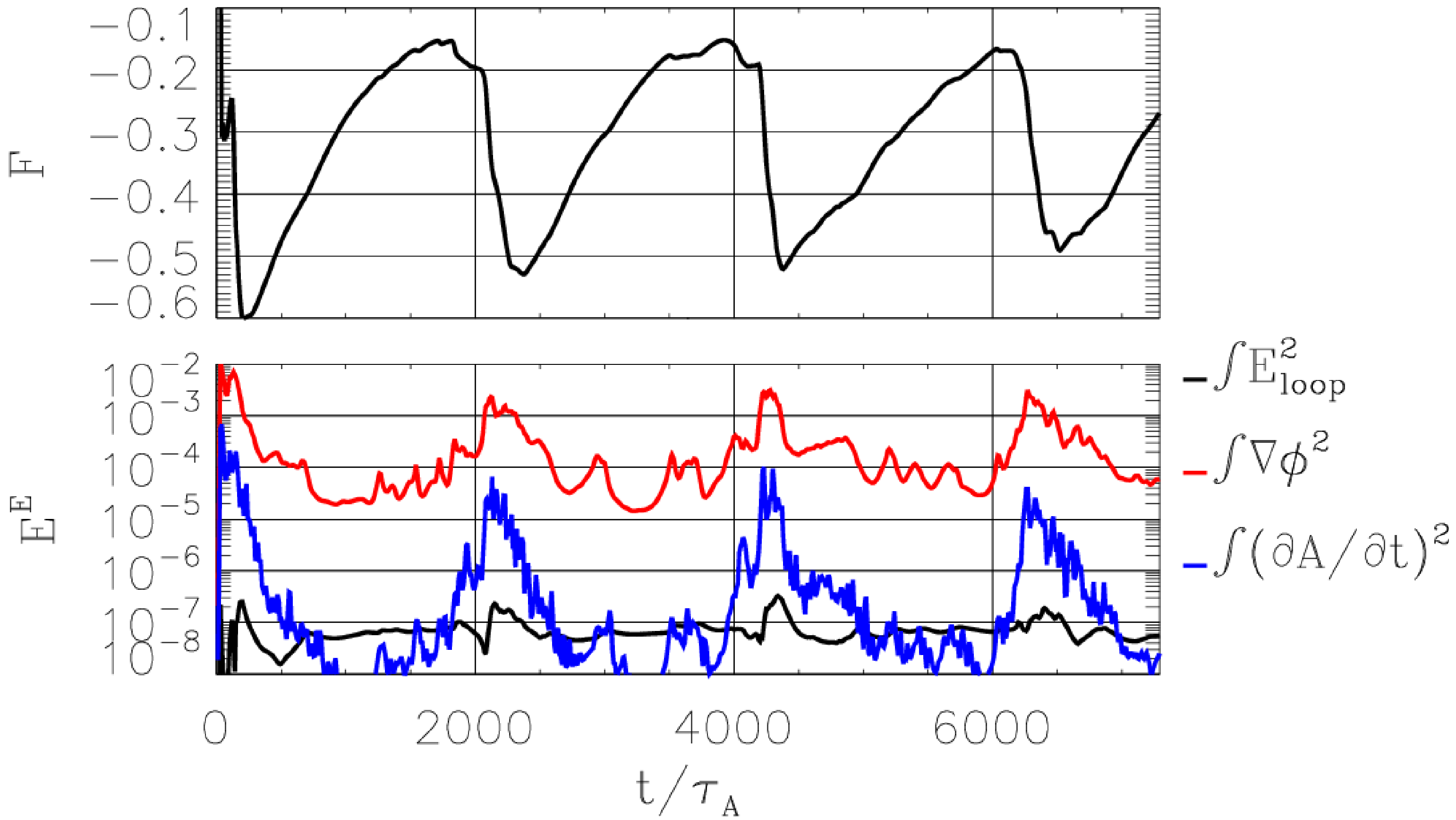
RFP dynamo: non-stationary conditions

- Different contributions to parallel Ohm's law in 2D (helical) simulation:
  [D. Bonfiglio *et al.*, Invited Varenna 2006]



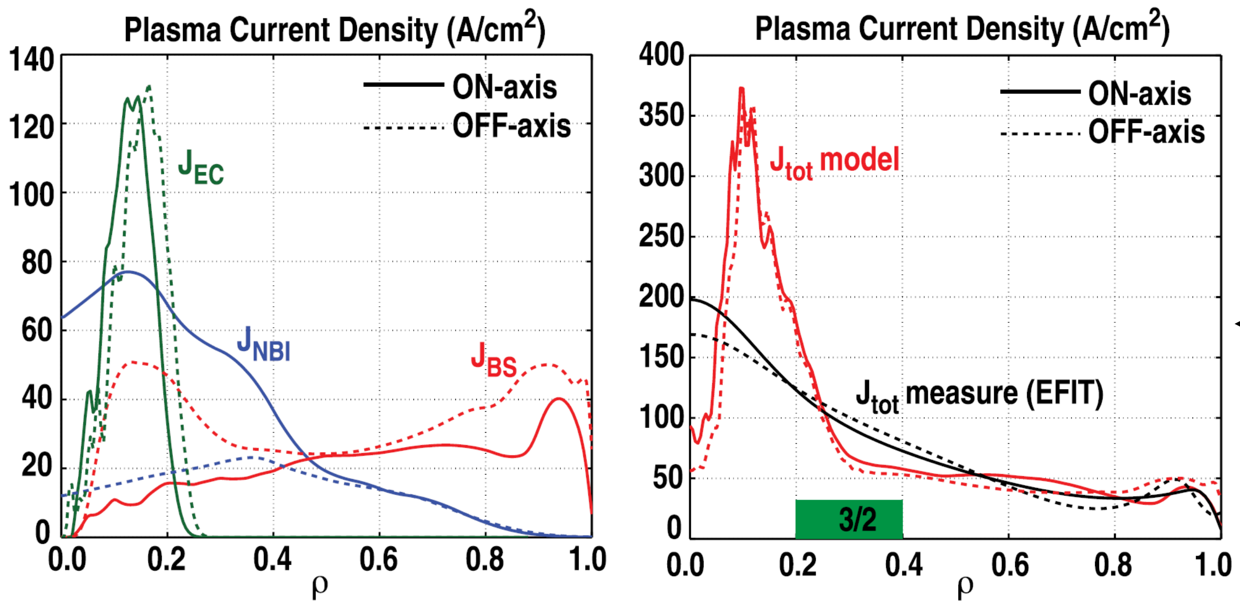
RFP dynamo: non-stationary conditions

- Fully 3D sawtoothing simulation [D. Bonfiglio *et al.*, Invited Varenna 2006]
- Electrostatic contribution to the electric field energy remains dominant [D. Bonfiglio *et al.*, PRL 2005; S. Cappello *et al.*, PoP 2006]
  - Peaks at magnetic reconnection events: role for particle acceleration



Dynamo effect in the tokamak

- Is the dynamo effect at work in the tokamak, too?
- In the DIII-D hybrid scenario, a flux pumping mechanism is required to redistribute current and flatten the central q profile [C. C. Petty *et al.*, PRL 2009; F. Turco *et al.*, PoP 2015].



The calculated total current density does not match the measured one

- Is this just a dynamo effect?
  - As speculated in [M. R. Wade *et al.*, PoP 2001; T. C. Luce *et al.*, NF 2003]
  - Investigated numerically with **M3D-C^1** [S. C. Jardin *et al.*, PRL 2015; I. Krebs *et al.*, PoP 2017] and recent ongoing work with **JOREK** [I. Krebs, invited talk at this conference]
- High-β hybrid scenario of interest for ITER and DEMO [H. Zohm, EFPW 2020]

Dynamo effect in the tokamak: **SPECYL** simulations

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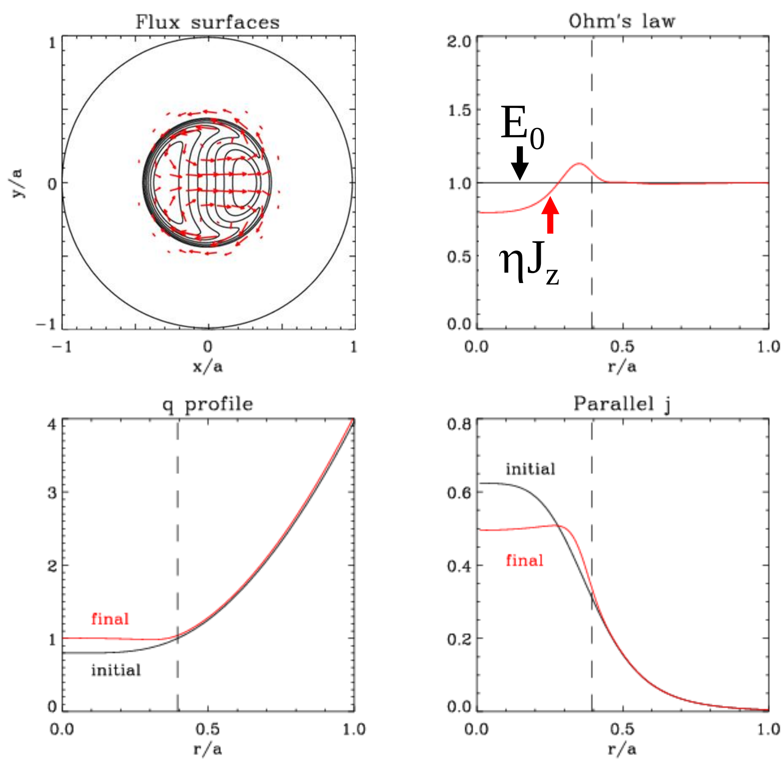
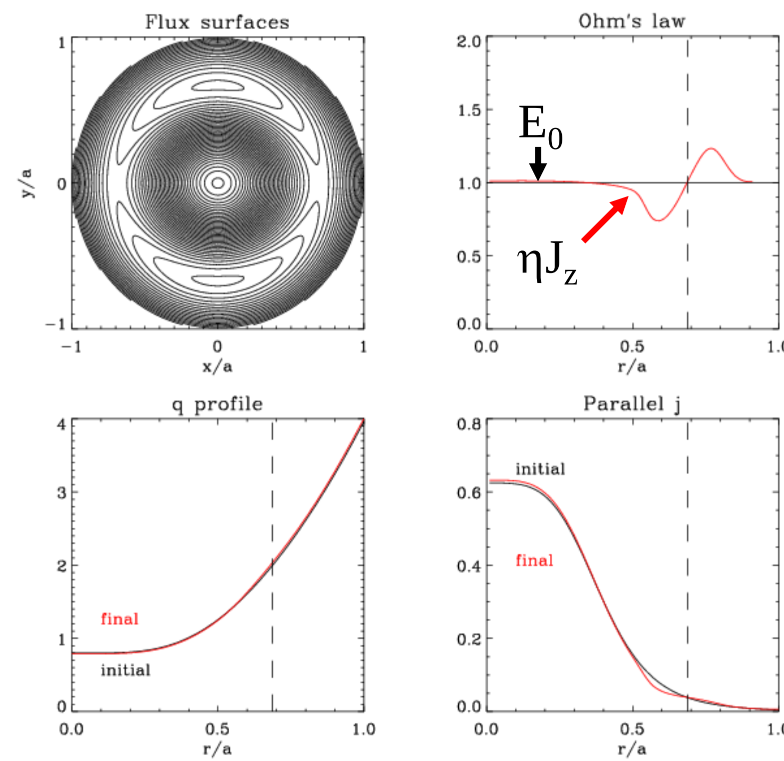
- The flux pumping/dynamo effect in DIII-D hybrid scenario is usually identified with the suppression of sawtoothing 1/1 mode by the 3/2 tearing mode [M. R. Wade *et al.*, NF 2005]
- Another possibility is that the 1/1 mode itself becomes stationary and keeps the central q close to 1 [S. C. Jardin *et al.*, PRL 2015; W. Cooper *et al.*, NF 2013; D. Brunetti *et al.*, NF 2014]
- **SPECYL** simulations show that saturated 1/1 makes enough dynamo to keep $q_0 \cong 1$, TMs only make a local dynamo around the resonant surface [P.. Piovesan *et al.*, NF 2017]

saturated 1/1 internal kink



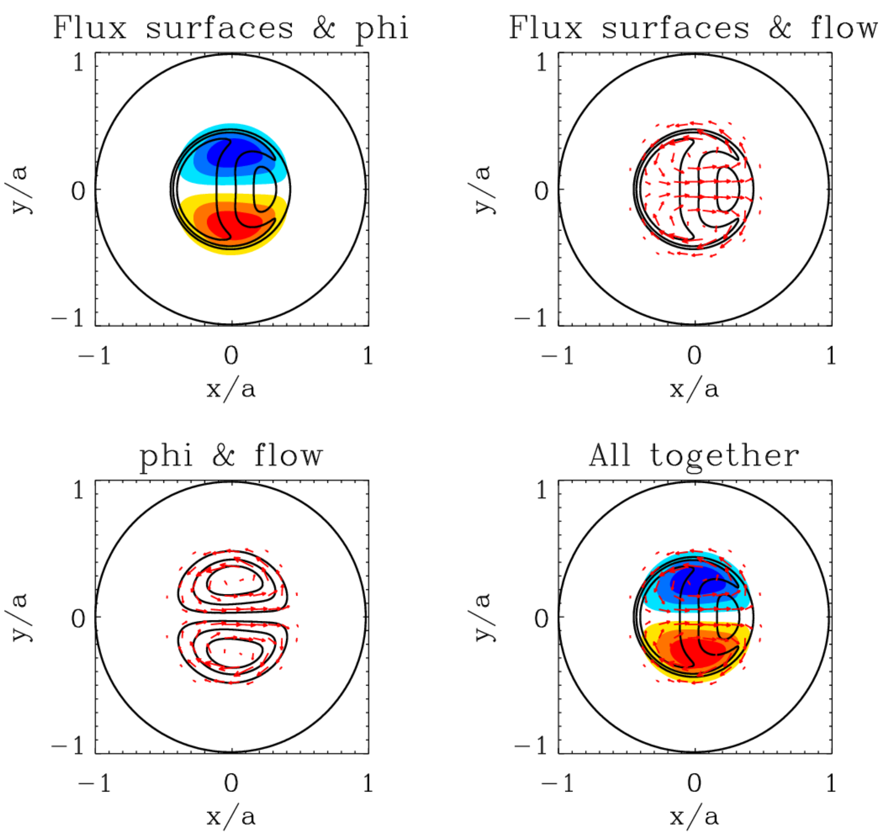
(huge) saturated 2/1 tearing mode



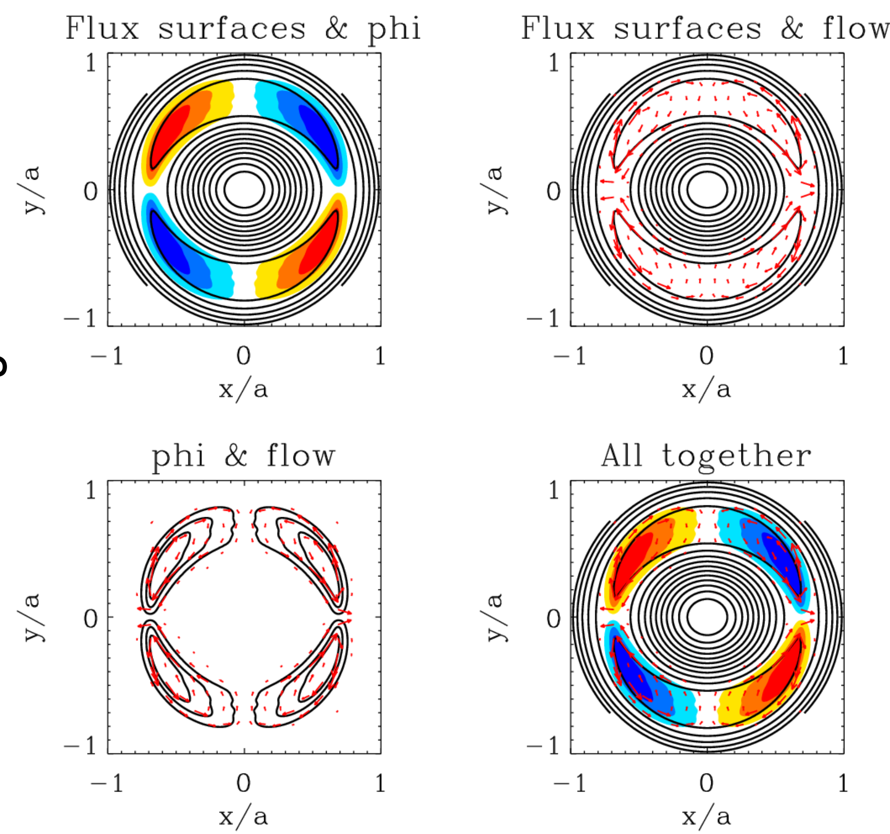
Dynamo effect in the tokamak: **SPECYL** simulations

- Electrostatic potential and flow from **SPECYL** simulations [P.. Piovesan *et al.*, NF 2017]
- Same patterns as in (for instance) reduced MHD simulations of [W. Park *et al.*, PoF 1984]
- In reduced MHD codes such as **JOREK**, the importance of ϕ is made explicit by the fact that ϕ is the stream function for the perpendicular velocity: $\mathbf{v}_\perp = \hat{\mathbf{z}} \times \nabla\phi$

saturated 1/1 internal kink

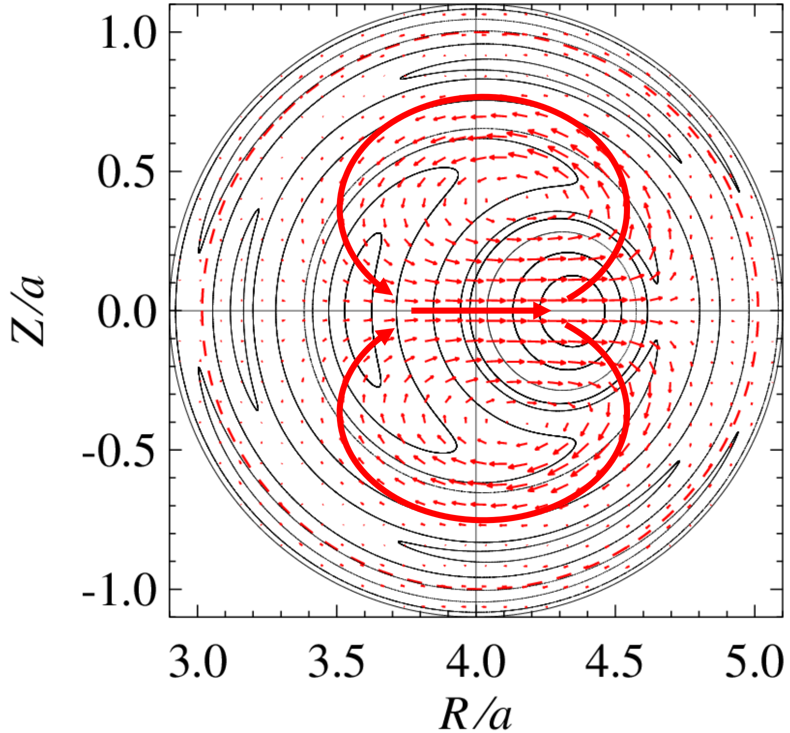


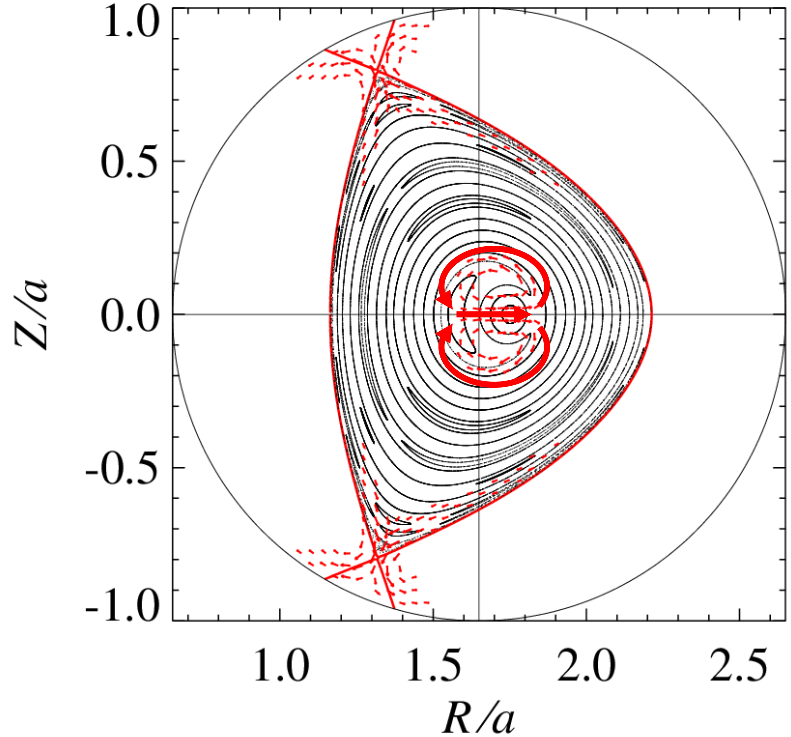
saturated 2/1 tearing mode



Dynamo effect in the tokamak: PIXIE3D simulations

- Same effect of saturated 1/1 is observed in toroidal geometry:
- Core convective cell makes $\mathbf{v}\times\mathbf{B}$ dynamo e.m.f. that flattens central current density
  - In both with circular (left) and D-shaped plasmas (right) [D. Bonfiglio *et al.*, PPCF 2017]
- Similar patterns of ϕ and $\mathbf{v}_\perp$ reconstructed for RFX-mod and DIII-D tokamak experiments with helical core [P. Piovesan *et al.*, NF 2017]

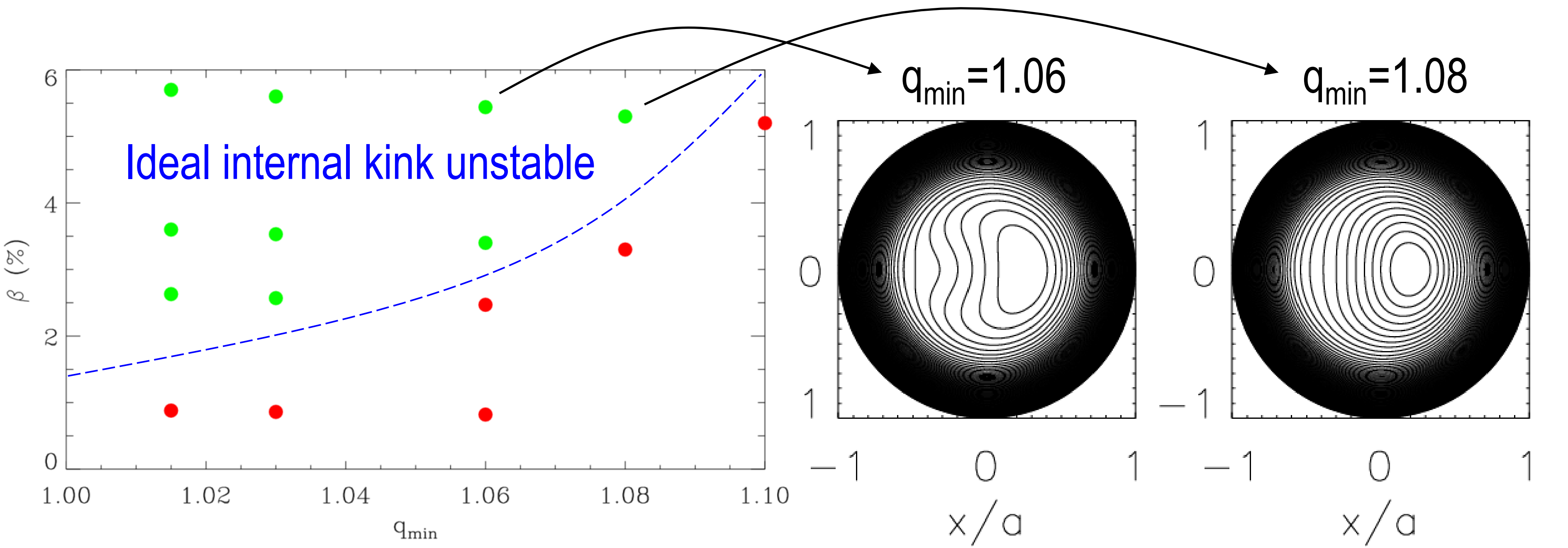




Finite-β simulations of the ideal internal kink

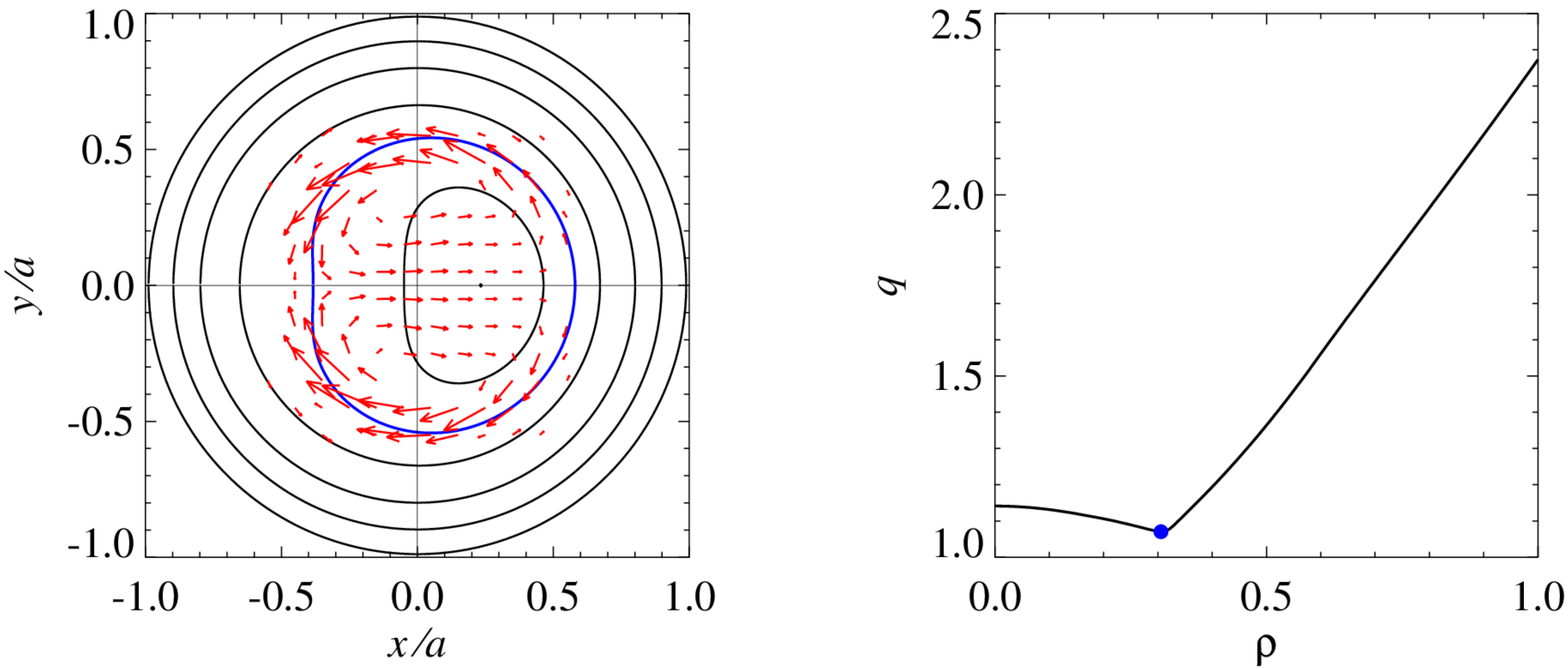
- Saturated pressure driven internal kink in tokamak with hollow q profile and $q_{min} \gtrsim 1$
- Main **PIXIE3D** results (in agreement with **XTOR** simulations [D. Brunetti *et al.*, NF 2014]):
  - internal kink linearly unstable provided q_{min} is close to 1 and β is large
  - saturated state: helical core with largest displacement when q_{min} is close to 1

- **PIXIE3D** simulations with varying total β and q_{min}:



Finite-β simulations of the ideal internal kink

- Dynamo flow and q profile for with a saturated ideal kink mode in **PIXIE3D**
  - A $m=1$ convection cell is present like for the current-driven internal kink
  - The poloidal flow is maximum on the minimum q surface (blue curve)
- This pressure driven kink mode is also called ideal interchange (quasi-interchange if q_0 just below 1). Dynamo effect by quasi-interchange mode at high β found to prevent sawteeth in **M3D-C**^{1} simulations [S. C. Jardin *et al.*, PRL 2015; I. Krebs *et al.*, PoP 2017]



Final remarks

- A physical interpretation of the RFP dynamo effect in given:
  - *a helical core displacement*
    modulates parallel current density
    along flux tubes, which requires
    a helical electrostatic potential to build up,
    giving rise to a helical MHD dynamo flow

Paolo talking about the dynamo at the IAEA FEC in Kyoto (October 2016)

- The same effect is working in the tokamak core as well, as associated to a stationary internal kink mode

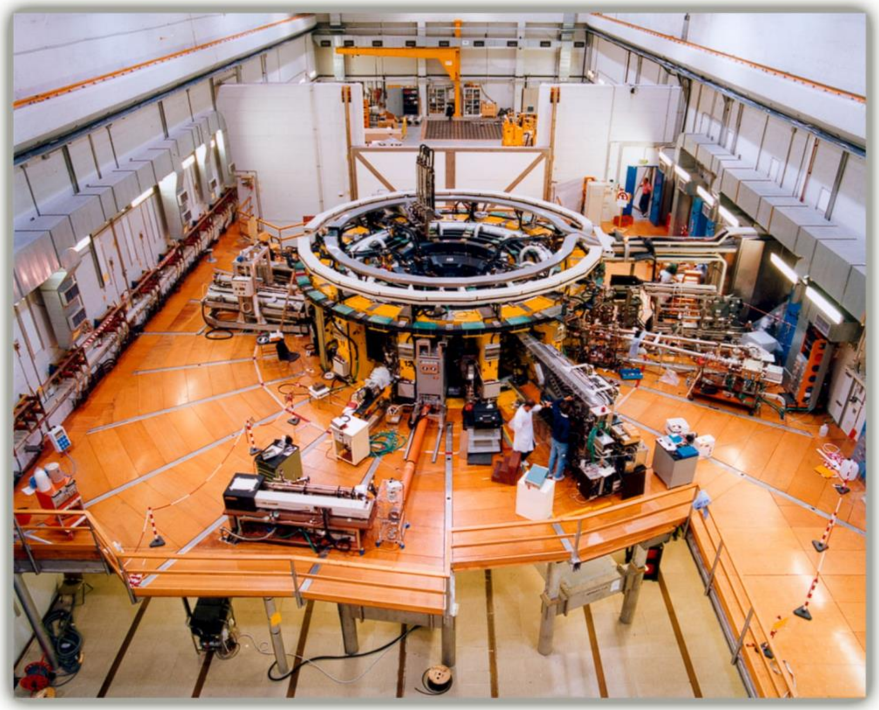
- It remains to be understood why and how, depending on plasma conditions, the internal kink in the tokamak can be either sawtoothing or stationary
  - ongoing PIXIE3D simulations at finite-β to address this issue

Spare slides

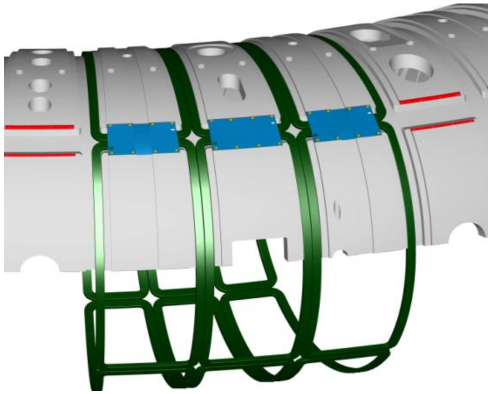
The RFX-mod device in Padova

Largest RFP device:

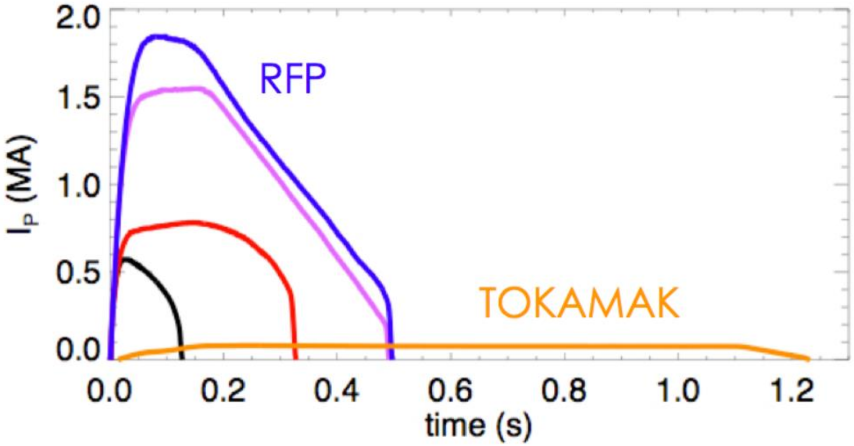
- $R_0 = 2$ m, $a = 0.46$ m
- Max $I_P = 2$ MA
- Max $B_\phi = 0.7$ T
- $n_e \cong 1 \div 5 \times 10^{19}$ m^{-3}



- Fully covered by saddle coils for MHD control and applied MPs:

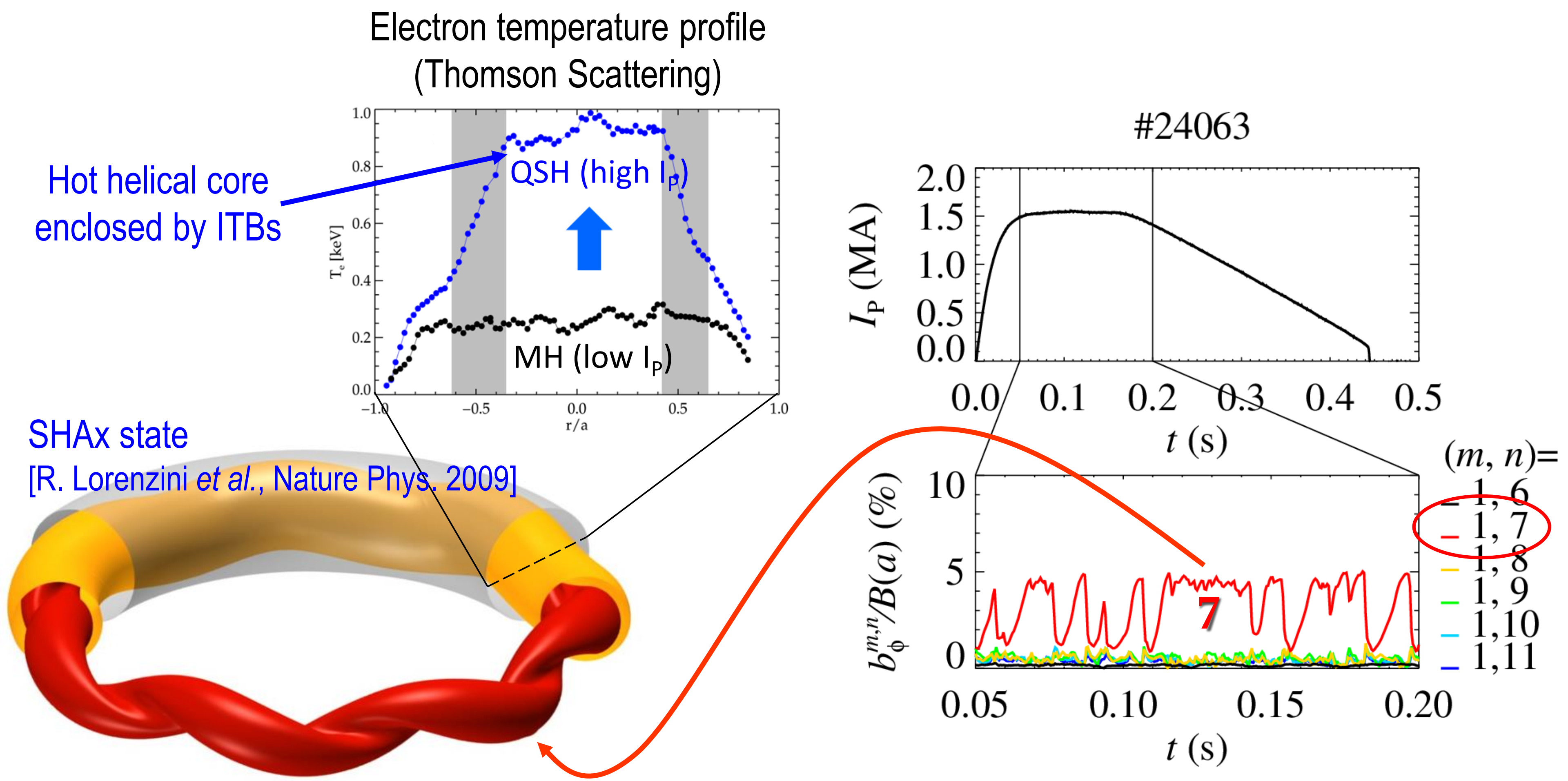


- Also operated as Ohmic tokamak:



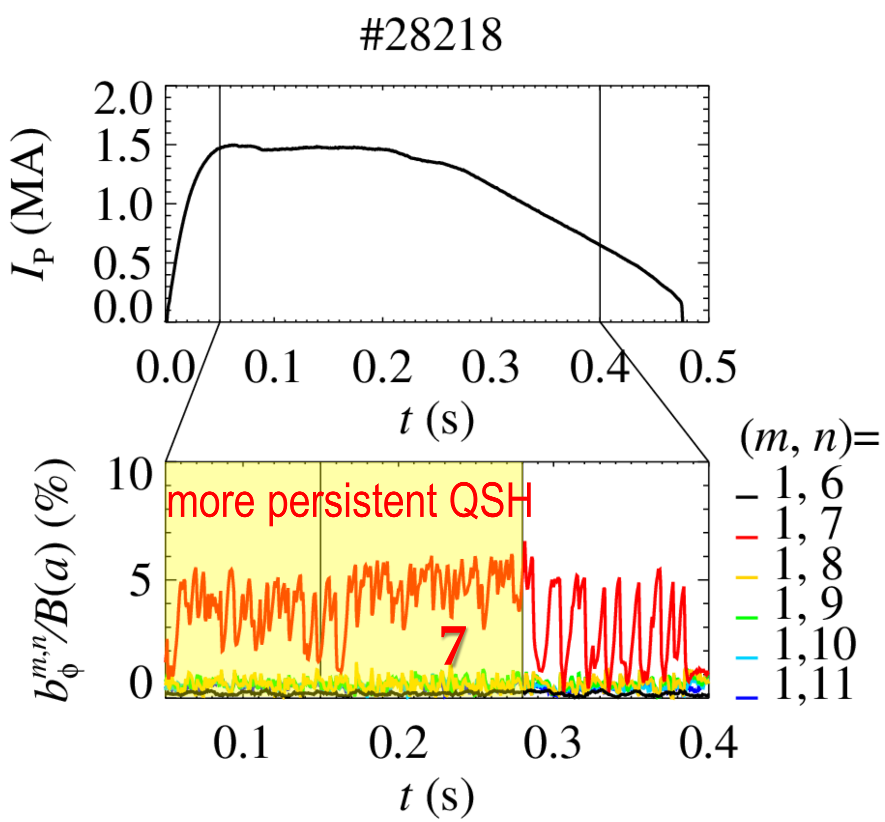
- Major upgrade ongoing (RFX-mod2) [L. Marrelli *et al.*, NF 2019]

- **High plasma current** regime: intermittent quasi-single helicity states (QSH) with improved thermal properties



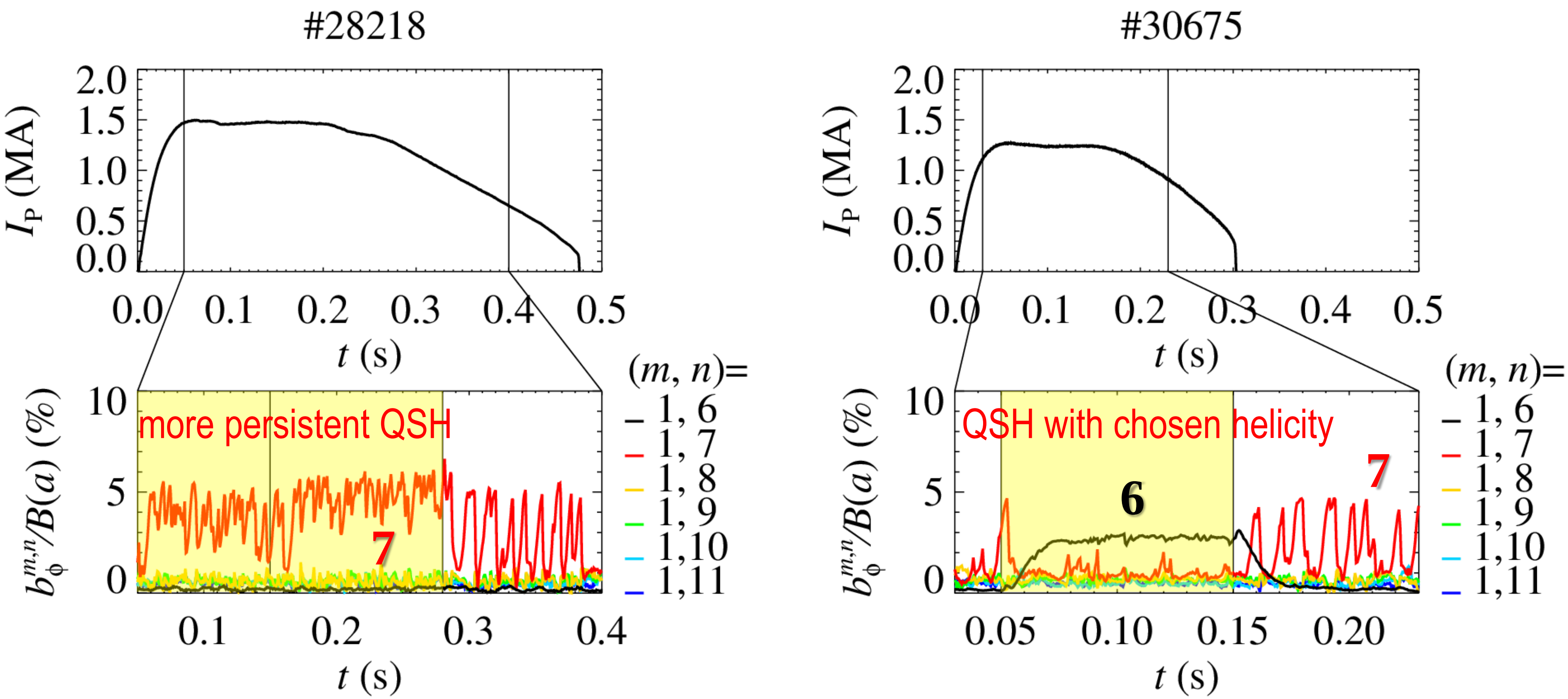
RFP experiments: effect of MPs

- Experiments with applied MPs show high flexibility of RFP plasmas
- MPs with m=1, n=7: more persistent n=7 QSH [P. Piovesan et al., PPCF 2011]



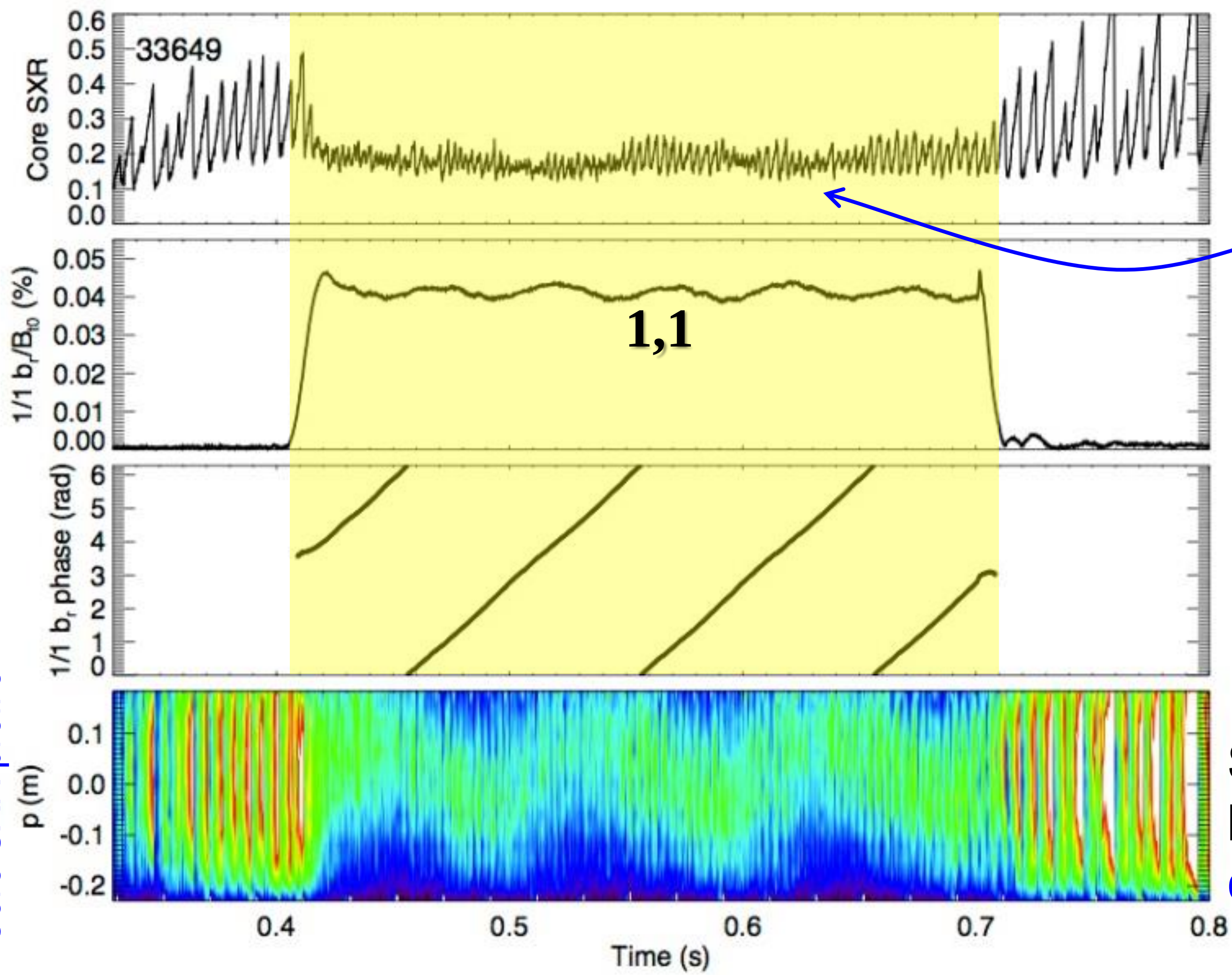
RFP experiments: effect of MPs

- Experiments with applied MPs show high flexibility of RFP plasmas
- MPs with m=1, n=7: more persistent n=7 QSH [P. Piovesan et al., PPCF 2011]
- Stimulated helicities: MPs with m=1, n=6 (non-resonant) helicity ⇒ n=6 QSH states (improved chaos healing predicted by MHD) [S. Cappello, IAEA 2012; M. Veranda *et al.*, NF 2017]



Tokamak experiments: effect of n=1 MPs

- In RFX-mod (reproduced in DIII-D) sawtooth oscillations are mitigated by n=1 MPs
  [P. Martin et al., IAEA 2014; C. Piron et al., NF 2016]



Sawtooth period and amplitude reduced

Amplitude of the applied 1,1 MP

Phase of the applied 1,1 MP

Sawtoothing 1,1 internal kink replaced by a more continuous 1,1 helical core