



WHAT IS A PARTON SHOWER?

ZOLTÁN NAGY
DESY-HH

In collaboration with Dave Soper

Perturbative Cross Section

The main focus of this workshop is to calculate the pQCD cross sections as precise as possible, thus we have a pretty integral

$$\begin{aligned}
 \sigma[O_J] = & \sum_m \frac{1}{m!} \sum_{\{a,b,f_1,\dots,f_m\}} \int_0^1 d\eta_a \overbrace{\int_{\eta_a}^1 \frac{dz}{z} \Gamma_{aa'}^{-1}(z, \mu^2) f_{a'/A}(\eta_a/z, \mu^2)}^{\text{Bare PDF}} \\
 & \times \int_0^1 d\eta_b \int_{\eta_b}^1 \frac{d\bar{z}}{\bar{z}} \Gamma_{bb'}^{-1}(\bar{z}, \mu^2) f_{b'/A}(\eta_b/\bar{z}, \mu^2) \\
 & \times \int d\phi(\eta_a \eta_b s, \{p, f\}_m) \langle M(\{p, f\}_m) | \underbrace{O_J(\{p, f\}_m)}_{\text{IR safe measurement operator}} | M(\{p, f\}_m) \rangle \\
 & + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mu_J^2}\right)
 \end{aligned}$$

Partonic matrix element

Error of the factorization

(Cannot be beaten by calculating higher and higher order.)

and here the MSbar parton in parton renormalised PDF is

$$\Gamma_{aa'}(z, \mu^2) = \delta(1-z)\delta_{aa'} - \frac{\alpha_s(\mu^2)}{2\pi} \frac{1}{\epsilon} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} P_{aa'}(z) + \dots$$

Motivation

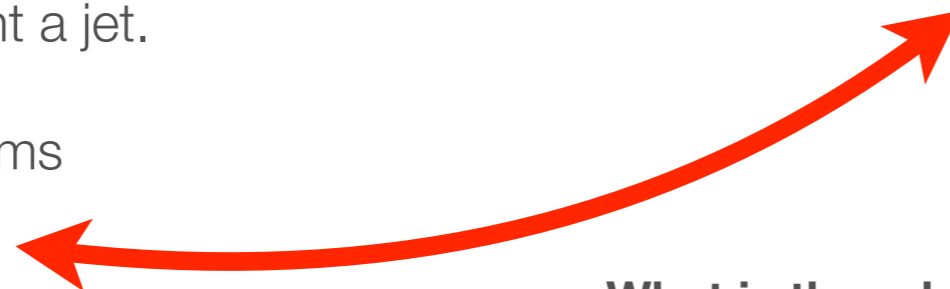
For a generic IR safe observable we can do either **fixed order** or **parton shower** calculations

Fixed order calculations

- ✓ Systematically improvable by working to higher order.
 - ▶ The procedure is well defined and can be carried out order by order. The definition of cross section tells us what to do.
 - ▶ The **subtraction procedure regularizes the α_s series** and turns the $d=4-2\epsilon$ dimensional expression to a $d=4$ dimensional one.
 - ▶ Counter-terms are defined order by order
 - ▶ The result is **independent of the ambiguities of the counter-terms** order by order.
- ✗ Only few partons represent a jet.
- ✗ Suffers from large logarithms

Parton shower algorithms

- ✗ But what about parton showers?
 - ▶ Are they just QCD inspired or fit into a scheme that can be systematically improved by working to higher order?
 - ▶ Is the (*all order*) shower cross section equal to the pQCD (*all order*) cross section?
 - ▶ Is there a shower way to regularize α_s series?
- ✓ A jet consists of many partons
- ✓ Sums up logarithms (only for some observable).



What is the relation between fixed order and parton shower?

Motivation

Fixed order NLO PDF is a well defined and systematically improvable approximation of the usual LO PDF:

$$f(\eta, \mu^2) = f_{1\text{GeV}}(\eta) + \int_{1\text{GeV}^2}^{\mu^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2} \frac{\alpha_s(\tilde{\mu}^2)}{2\pi} \int_{\eta}^1 \frac{dx}{x} P^{(1)}(x) f_{1\text{GeV}}(\eta/x) + \dots$$

But we never use this and we prefer the **fully exponentiated** solution of the DGLAP equation

$$f(\eta, \mu^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \eta^{-N} \exp\left(\int_{1\text{GeV}^2}^{\mu^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2} \frac{\alpha_s(\tilde{\mu}^2)}{2\pi} \int_0^1 dx x^{N-1} P^{(1)}(x) \right) \\ \times \int_0^1 dx x^{N-1} f_{1\text{GeV}}(x)$$

A strong statement: The fixed order NLO, NNLO and N^kLO calculations are just approximations to the fully exponentiated LO, NLO and N^{k-1}LO calculations.

A brave claim: The parton shower provides the fully exponentiated LO, NLO and N^{k-1}LO calculations.

Statistical Space

Introducing the statistical space we can represent the QCD density operator as a vector

$$\sigma[O_J] = \underbrace{(1|}_{\text{All the initial and final state sums and integrals}} \mathcal{O}_J \overbrace{[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)]}^{\text{Bare PDFs for both incoming hadrons}} \underbrace{|\rho(\mu^2)\rangle}_{|M\rangle\langle M|}$$

QCD density operator
Describes the fully exclusive partonic final states.

The physical cross section is RG invariant as well as the QCD density operator and the bare PDF.

$$\mu^2 \frac{d}{d\mu^2} |\rho(\mu^2)\rangle = \mu^2 \frac{d}{d\mu^2} [\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)] = 0 + \mathcal{O}(\alpha_s^{k+1})$$

Perturbative expansion of the density operator

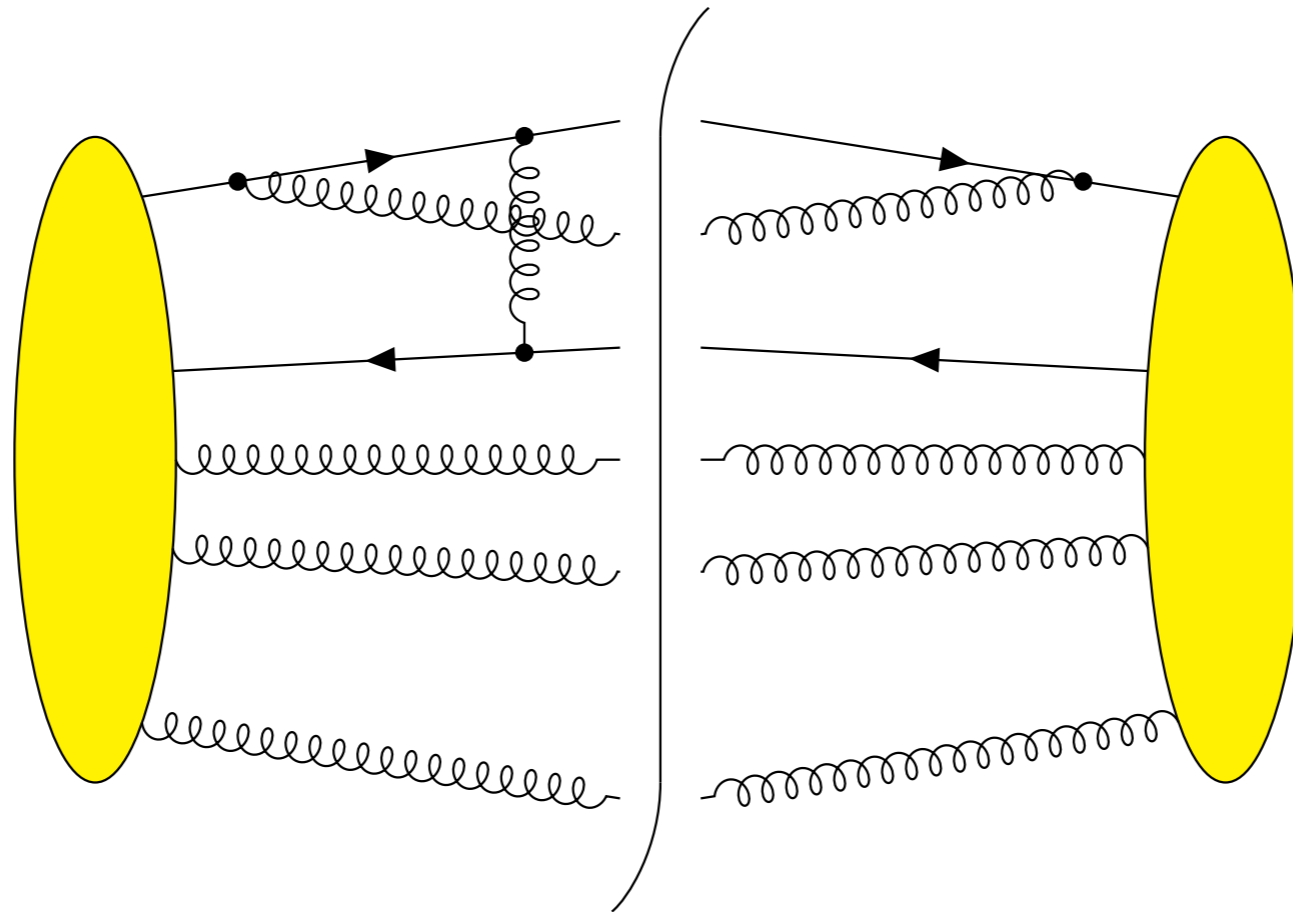
$$|\rho(\mu^2)\rangle = \sum_{n=0}^k \left[\frac{\alpha_s(\mu^2)}{2\pi} \right]^n \sum_{\substack{n_R=0 \\ n_V=0 \\ n_R+n_V=n}}^n \sum_{n_V=0}^n |\rho^{(n_R, n_V)}(\mu^2)\rangle$$

Number of real radiations

Number of loops

Infrared Sensitive Operator

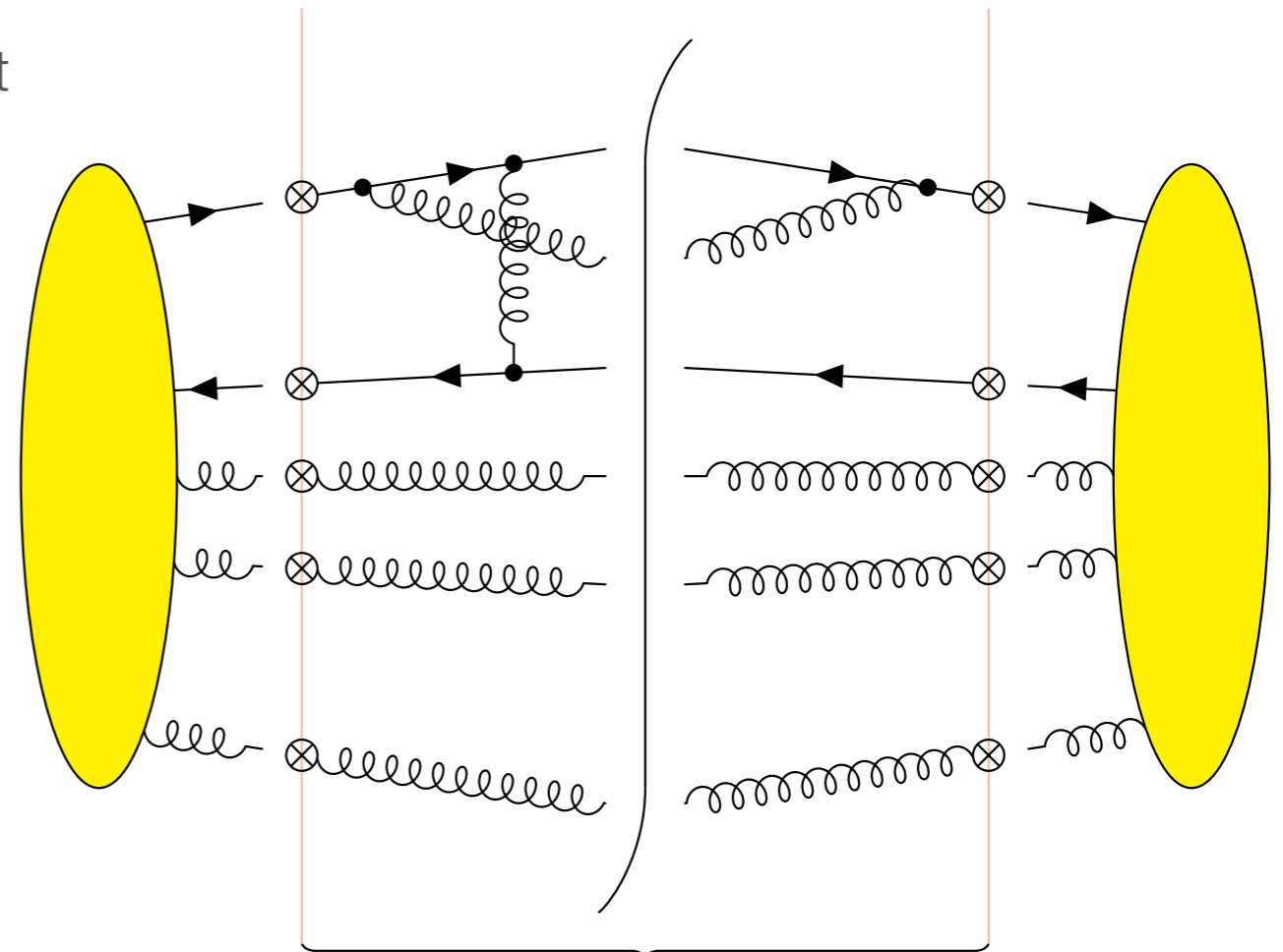
Amplitudes have **soft or collinear singularities** and they have **divergences $1/\epsilon$** from the loops



- ➡ We want to describe the singularity structure in **process independent way**.
- ➡ Everything in the yellow blobs is considered hard.

Infrared Sensitive Operator

Consider the momenta coming from the hard part as fixed and on shell.



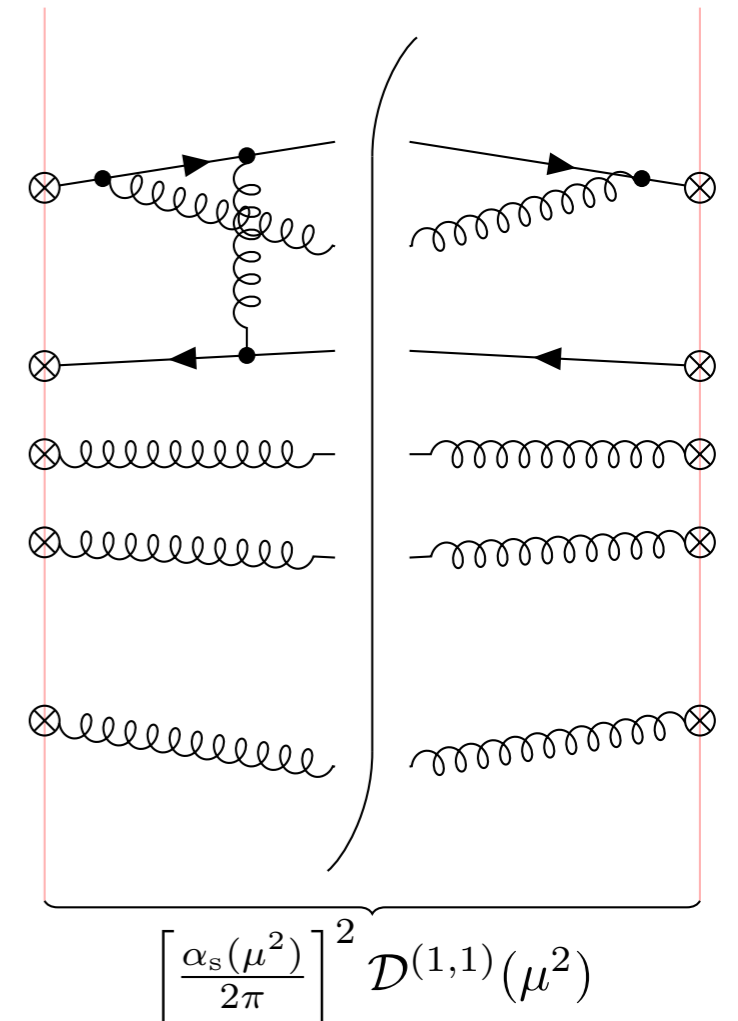
This gives us an operator as

$$\begin{aligned}
 & \left(\{ \hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}' \}_{m+n_R} \mid \rho(\mu^2) \right) \\
 & \sim \frac{1}{m!} \int [d\{p\}_m] \sum_{\{f\}_m} \sum_{\{s, s', c, c'\}_m} \\
 & \quad \times \left(\{ \hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}' \}_{m+n_R} \mid \mathcal{D}(\mu^2) \mid \{p, f, s, s', c, c'\}_m \right) \\
 & \quad \times \left(\{p, f, s, s', c, c'\}_m \mid \rho_{\text{hard}}(\mu^2) \right)
 \end{aligned}$$

Infrared Sensitive Operator

We can consider a more constructive approach to build the full infrared sensitive operator. This operator basically represents the QCD density operator of a $m \rightarrow X$ (anything) process.

$$\mathcal{D}(\mu^2) = 1 + \sum_{n=1}^k \left[\frac{\alpha_s(\mu^2)}{2\pi} \right]^n \sum_{\substack{n_R=0 \\ n_V=0 \\ n_R+n_V=n}}^n \sum_{n_V=0}^n \mathcal{D}^{(n_R, n_V)}(\mu^2)$$



The structure is rather straightforward:

$$\begin{aligned} & (\{\hat{p}, \hat{f}, \hat{s}', \hat{c}', \hat{s}, \hat{c}\}_{m+n_R} | \mathcal{D}^{(n_R, n_V)}(\mu^2, \mu_S^2) | \{p, f, s', c', s, c\}_m) \\ &= \sum_{G \in \text{Graphs}} \int d^d \{\ell\}_{n_V} \langle \{\hat{s}, \hat{c}\}_{m+n_R} | \mathbf{V}_L(G; \{\hat{p}, \hat{f}\}_{m+n_R}, \{\ell\}_{n_V}, \mu^2) | \{s, c\}_m \rangle \\ & \quad \times \langle \{s, c\}_m | \mathbf{V}_R^\dagger(G; \{\hat{p}, \hat{f}\}_{m+n_R}, \{\ell\}_{n_V}, \mu^2) | \{\hat{s}, \hat{c}\}_{m+n_R} \rangle_D \\ & \quad \times \sum_{I \in \text{Regions}(G)} (\{\hat{p}, \hat{f}\}_{m+n_R} | \mathcal{P}_G(I) | \{p, f\}_m) \underbrace{\Theta_G(I; \{\hat{p}, \hat{f}\}_{m+n_R}, \{\ell\}_{n_V}; \mu_S^2)}_{\text{Constrains the off-shellness of the hard partons}} \end{aligned}$$

Constrains the off-shellness of the hard partons

Infrared Sensitive Operator

- ➡ We have to introduce an **ultraviolet cutoff to capture only the IR part** of the amplitudes. At first order level in the real graphs it is just a cut on an infrared sensitive variable of the splitting:

$$\Theta_G(I; \{\hat{p}, \hat{f}\}_{m+n_R}, \{\ell\}_{n_V}; \mu_S^2) \sim \theta(k_{\perp}^2 < \mu_S^2)$$

- ➡ The D operator depends on two scales (renormalization scale μ and the **shower scale** μ_S) but we always set them equal.

$$\mu_S^2 = \mu^2$$

- ➡ We don't do eikonal approximation in the soft gluon exchange between two external lines because that messes up the **Glauber region**.
- ➡ We also need a **momentum mapping**. This can be tricky at higher order level and not necessary the simpler is the better. We prefer “global” momentum mapping.

N^kLO calculations

$$\begin{aligned}
 \sigma[O_J] = & \overbrace{\left(1 \mid \left[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\right] \mathcal{D}(\mu^2)\right)}^{\text{Singularities cancel each other here}} \underbrace{\mathcal{D}^{-1}(\mu^2) O_J \mid \rho(\mu^2))}_{\substack{\text{Subtractions} \\ = |\rho_H(\mu^2)) \\ \text{Hard part, finite in d=4 dimension}}} \\
 & + \mathcal{O}(\alpha_s^{k+1} L^{2k+2}) \\
 & + \mathcal{O}(\Lambda_{QCD}^2 / \mu_J^2)
 \end{aligned}$$

Normally $\mathcal{D}^{-1}(\mu^2)$ is constructed by hand and $\mathcal{D}(\mu^2)$ is its inverse.

N^kLO calculations

Collecting all the singularities in an operator,

$$\mathcal{X}(\mu^2) = [\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)] \mathcal{D}(\mu^2) \mathcal{F}^{-1}(\mu^2)$$

Then we have found that $(1 | \mathcal{X}(\mu^2) | \{p, f, c', c, s', s\}_m) = \text{finite}$. Now **define a finite operator** that **leaves the momenta and flavors unchanged** in such a way that

$$(1 | \underbrace{\mathcal{V}(\mu^2)}_{\text{IR finite operator}} | \mathcal{X}(\mu^2) |$$

With this we have the usual fixed order cross section structure:

$$\begin{aligned} \sigma[O_J] &= (1 | \mathcal{X}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \mathcal{O}_J | \rho(\mu^2)) \\ &\quad + \mathcal{O}(\alpha_s^{k+1} L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2 / \mu_J^2) \end{aligned}$$

$$\begin{aligned} &= (1 | \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \mathcal{O}_J | \rho(\mu^2)) \\ &\quad + \mathcal{O}(\alpha_s^{k+1} L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2 / \mu_J^2) \end{aligned}$$

Shower Cross Section

At this point we have everything to **derive** the shower cross section. Let us do it!

Start with the fixed order (all order) cross section

$$\sigma[O_J] = (1|\mathcal{O}_J \mathcal{X}(\mu^2) \mathcal{F}(\mu^2) \underbrace{\mathcal{D}^{-1}(\mu^2)}_{=|\rho_H(\mu^2))} |\rho(\mu^2))$$

Insert an unit operator

$$\sigma[O_J] = (1|\mathcal{O}_J \mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2) \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) |\rho_H(\mu^2))$$

and restructure the expression

$$\sigma[O_J] = (1|\mathcal{O}_J [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) |\rho_H(\mu^2))$$

Shower Cross Section

Insert another unit operator

$$\sigma[O_J] = (1 | \mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) | \rho_H(\mu^2))$$

and restructure the expression

$$\sigma[O_J] = (1 | \mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) | \rho_H(\mu^2))$$

Shower Cross Section

Let us play this game one more time!

Insert another unit operator

$$\sigma[O_J] = (1|\mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \mathcal{V}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2) \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) |\rho_H(\mu^2))$$

and restructure the expression

$$\sigma[O_J] = (1|\mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \mathcal{V}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2) \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) |\rho_H(\mu^2))$$

Shower Cross Section

We can simplify this further introducing the evolution operators. Thus we have

$$\sigma[O_J] = (1|\mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] \overbrace{[\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} [\mathcal{X}(\mu^2) \mathcal{V}^{-1}(\mu^2)]}^{\mathcal{U}(\mu_f^2, \mu^2)} \underbrace{\mathcal{V}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2) \mathcal{V}(\mu^2)}_{\mathcal{U}_\mathcal{V}(\mu_f^2, \mu^2)} \mathcal{F}(\mu^2) | \rho_H(\mu^2))$$

and with this notation the cross section is

$$\sigma[O_J] = (1|\mathcal{O}_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] \mathcal{U}(\mu_f^2, \mu^2) \mathcal{V}(\mu_f^2) \mathcal{U}_\mathcal{V}(\mu_f^2, \mu^2) \mathcal{F}(\mu^2) | \rho_H(\mu^2))$$

Shower Cross Section

We have to deal with the singular part.

$$\sigma[O_J] = (1|O_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] \mathcal{U}(\mu_f^2, \mu^2) \mathcal{V}(\mu_f^2) \mathcal{U}_\nu(\mu_f^2, \mu^2) \mathcal{F}(\mu^2) | \rho_H(\mu^2))$$

When $\mu_f^2 \sim \Lambda_{\text{QCD}}^2$

$$(1|O_J [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)] | \{p, f, \dots\}_m) = (1|O_J | \{p, f, \dots\}_m) + \mathcal{O}\left(\frac{\mu_f^2}{\mu_J^2}\right)$$

and

$$\mathcal{V}(\mu_f^2) \approx 1$$

since this operator is finite.

$$\sigma[O_J] = (1|O_J \underbrace{\mathcal{U}(\mu_f^2, \mu^2)}_{\text{Resummation of threshold effects}} \underbrace{\mathcal{U}_\nu(\mu_f^2, \mu^2)}_{\text{Resummation of threshold effects}} \mathcal{F}(\mu^2) | \rho_H(\mu^2)) + \mathcal{O}(\underbrace{\alpha_s^{k+1} L^n}_{\text{Hopefully } n \ll 2k+1}) + \mathcal{O}(\mu_f^2 / \mu_J^2)$$

Threshold Logs

The threshold operator is defined by

$$\mathcal{U}_{\mathcal{V}}(\mu_f^2, \mu_H^2) = \mathcal{V}^{-1}(\mu_f^2) \mathcal{V}(\mu_H^2) = \mathbb{T} \exp \left(\int_{\mu_f^2}^{\mu_H^2} \frac{d\mu^2}{\mu^2} \mathcal{S}_{\mathcal{V}}(\mu^2) \right)$$

where the generators are

$$\begin{aligned} \frac{1}{\mu^2} \mathcal{S}_{\mathcal{V}}(\mu^2) &= \mathcal{V}^{-1}(\mu^2) \frac{d\mathcal{V}(\mu^2)}{d\mu^2} \\ &= \mathcal{V}^{-1}(\mu^2) \frac{\partial}{\partial \mu_S^2} \mathcal{V}(\mu^2, \mu_S^2) \Big|_{\mu_S^2 = \mu^2} - \underbrace{\frac{d\mathcal{F}(\mu^2)}{d\mu^2} \mathcal{F}^{-1}(\mu^2)}_{\text{pure DGLAP evolution}} \end{aligned}$$

- ▣▣▣▣ ➔ Doesn't create new partons.
- ▣▣▣▣ ➔ Provides perturbative corrections to the hard part.
- ▣▣▣▣ ➔ **Sums up threshold logarithms**

Unitary Shower

The unitary shower operator is

$$\mathcal{U}(\mu_f^2, \mu_H^2) = [\mathcal{X}(\mu_f^2) \mathcal{V}^{-1}(\mu_f^2)]^{-1} \mathcal{X}(\mu_H^2) \mathcal{V}^{-1}(\mu_H^2) = \mathbb{T} \exp \left(\int_{\mu_f^2}^{\mu_H^2} \frac{d\mu^2}{\mu^2} \mathcal{S}(\mu^2) \right)$$

where the generators are

$$\begin{aligned} \frac{1}{\mu^2} \mathcal{S}(\mu^2) &= \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \frac{d}{d\mu^2} [\mathcal{D}(\mu^2) \mathcal{F}^{-1}(\mu^2) \mathcal{V}^{-1}(\mu^2)] \\ &= \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \frac{\partial \mathcal{D}(\mu^2, \mu_s^2)}{\partial \mu_s^2} [\mathcal{V}(\mu^2) \mathcal{F}(\mu^2)]^{-1} \Big|_{\mu_s^2 = \mu^2} \\ &\quad - \frac{\partial \mathcal{V}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \mathcal{V}^{-1}(\mu^2) \Big|_{\mu_s^2 = \mu^2} \end{aligned}$$

|||➔ Creates new partons.

|||➔ Preserves probabilities: $(1 | \mathcal{U}(\mu_f^2, \mu_H^2) = (1 |$

|||➔ **Sums up “visible” logarithms** (accuracy can depend on the observable)

Shower Kernel

The generators of the unitary shower can be expanded in the coupling:

$$S(\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} S^{(1)}(\mu^2) + \left[\frac{\alpha_s(\mu^2)}{2\pi} \right]^2 S^{(2)}(\mu^2) + \dots$$

and the first order term is rather simple

$$\begin{aligned} \frac{1}{\mu_s^2} S^{(1)}(\mu^2) &= \frac{\partial}{\partial \mu_s^2} \left[\mathcal{F}(\mu^2) \mathcal{D}^{(1,0)}(\mu^2, \mu_s^2) \mathcal{F}^{-1}(\mu^2) + \mathcal{D}^{(0,1)}(\mu^2, \mu_s^2) \right]_{\mu_s^2 = \mu^2} - \frac{\partial \mathcal{V}^{(1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \Big|_{\mu_s^2 = \mu^2} \\ &= \left[\underbrace{\mathcal{F}(\mu^2) \frac{\partial \mathcal{D}^{(1,0)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \mathcal{F}^{-1}(\mu^2)}_{\text{Real operator}} - \underbrace{\frac{\partial \mathcal{F}(\mu^2)}{\partial \mu_s^2} \circ \overline{\mathcal{D}^{(1,0)}}(\mu^2, \mu_s^2) \mathcal{F}^{-1}(\mu^2)}_{\text{Integrated real operator}} + \underbrace{\text{Im} \frac{\partial \mathcal{D}^{(0,1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2}}_{\text{Glauber gluon}} \right]_{\mu_s^2 = \mu^2} \end{aligned}$$

Real operator
 all the quantum numbers of the emitted parton is **resolved**

Integrated real operator
 - all the quantum numbers of the emitted parton is **integrated out**
 - it is **not** the contribution of the virtual graphs

Glauber gluon
 imaginary part of the virtual graphs
 $\sim i\pi$

Note, the first order kernel is **independent of** the real part of the virtual graphs.

Shower Kernel

At second order level we are not that lucky. The shower kernel is much more complicated:

$$\begin{aligned} \frac{1}{\mu_s^2} S^{(2)}(\mu^2) = & \mathcal{F}(\mu^2) \left(\frac{\partial \mathcal{D}^{(2)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} - \mathcal{D}^{(1)}(\mu^2) \frac{\partial \mathcal{D}^{(1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \right)_{\mu_s^2 = \mu^2} \mathcal{F}^{-1}(\mu^2) \\ & - \left(\frac{\partial \mathcal{V}^{(2)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} - \mathcal{V}^{(1)}(\mu^2) \frac{\partial \mathcal{V}^{(1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \right)_{\mu_s^2 = \mu^2} \\ & + \left[\mathcal{V}^{(1)}(\mu^2), \frac{1}{\mu^2} \mathcal{S}^{(1)}(\mu^2) \right] \end{aligned}$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

$$\mathcal{D}^{(2)}(\mu^2, \mu_s^2) = \overbrace{\mathcal{D}^{(2,0)}(\mu^2, \mu_s^2)}^{\text{Double real}} + \underbrace{\mathcal{D}^{(1,1)}(\mu^2, \mu_s^2)}_{\text{Real-virtual}} + \overbrace{\mathcal{D}^{(0,2)}(\mu^2, \mu_s^2)}^{\text{Double virtual}}$$

and

$$\mathcal{D}^{(1)}(\mu^2, \mu_s^2) = \overbrace{\mathcal{D}^{(1,0)}(\mu^2, \mu_s^2)}^{\text{Single real}} + \underbrace{\mathcal{D}^{(0,1)}(\mu^2, \mu_s^2)}_{\text{Single virtual}}$$

Summary

- **Fixed order** calculations

$$\sigma[O_J] = (1 | \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \mathcal{O}_J | \rho(\mu^2)) \\ + \mathcal{O}(\alpha_s^{k+1} L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2 / \mu_J^2)$$

can be systematically improved by working to higher order.

- **Parton shower** calculations

$$\sigma[O_J] = (1 | \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu^2) \mathcal{U}_{\mathcal{V}}(\mu_f^2, \mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) | \rho(\mu^2)) \\ + \mathcal{O}(\alpha_s^{k+1} L^n) + \mathcal{O}(\mu_f^2 / \mu_J^2)$$

can be systematically improved by working to higher order.

Implementation

- **DEDUCTOR** is designed to do a better job with color, spin and resummation of large logarithms compared to other shower generators.
 - Lambda, kT and angular ordering
 - LC+ color treatment. It allows us to do color evolution at amplitude level
 - Threshold log resummation
 - Spin correlations are not yet computed
- Next version is available soon...
 - Fully exponentiated Glauber (Coulomb) gluon effects
 - Wide angle soft gluon effects perturbatively.
- It is available from

<http://www.desy.de/~znagy/deductor>

<http://pages.uoregon.edu/soper/deductor>

Threshold Effect in Jet Production

Ratios to NLO jet cross section, $R = 0.4$

