NNLL resummation for global observables

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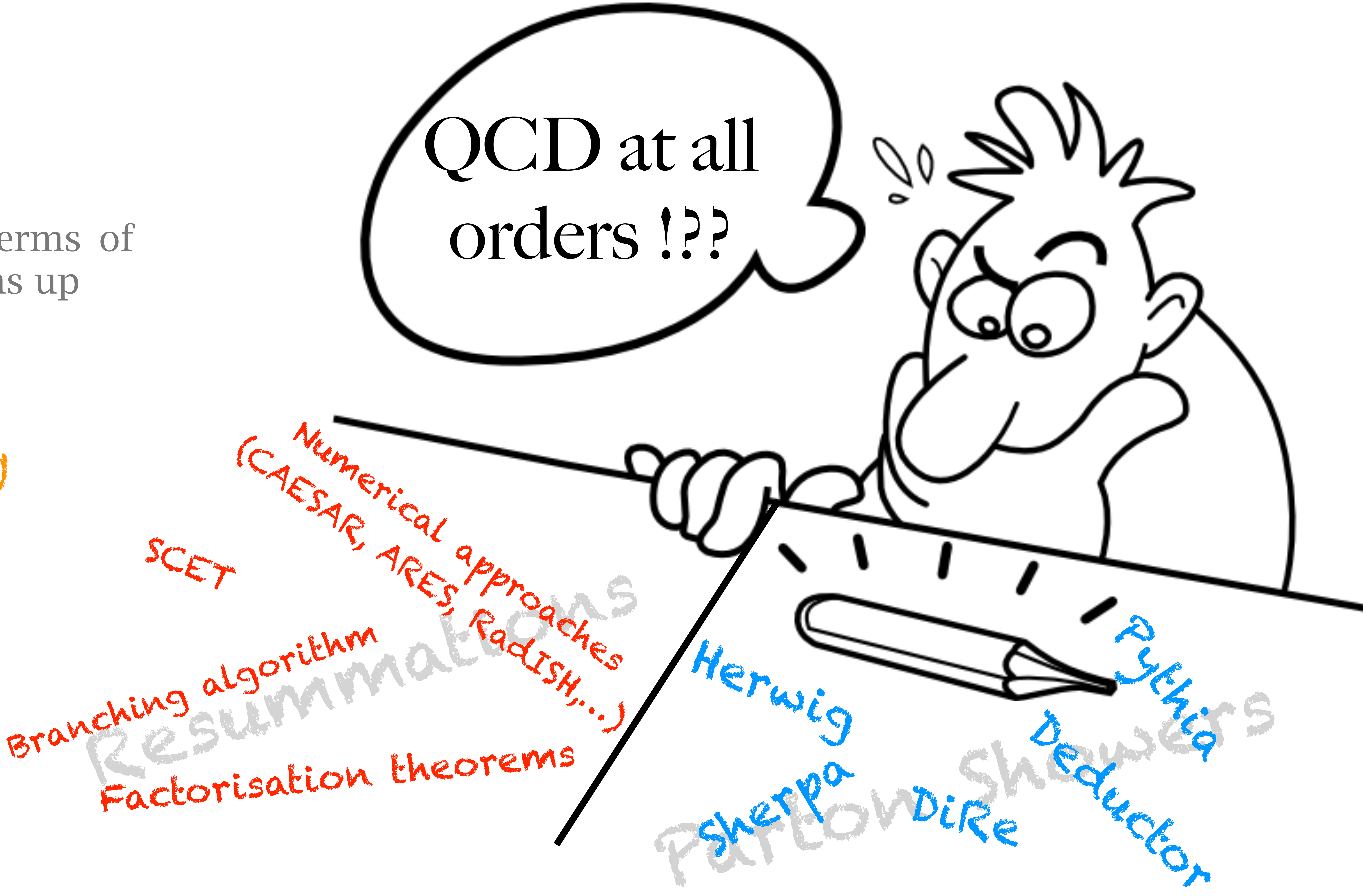
- Perturbative accuracy now defined in terms of how many towers of logarithms one sums up
  - e.g.

LL ~ 100% uncertainty

NLL ~ 20% uncertainty

NNLL ~ 5% uncertainty

...



LL NLL NNLL

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \sim e^{\overbrace{\alpha_s^n L^{n+1}}^{\rm LL} + \overbrace{\alpha_s^n L^n}^{\rm NLL} + \overbrace{\alpha_s^n L^{n-1}}^{\rm NNLL} + \dots}$$

Introduction

- Approach I :

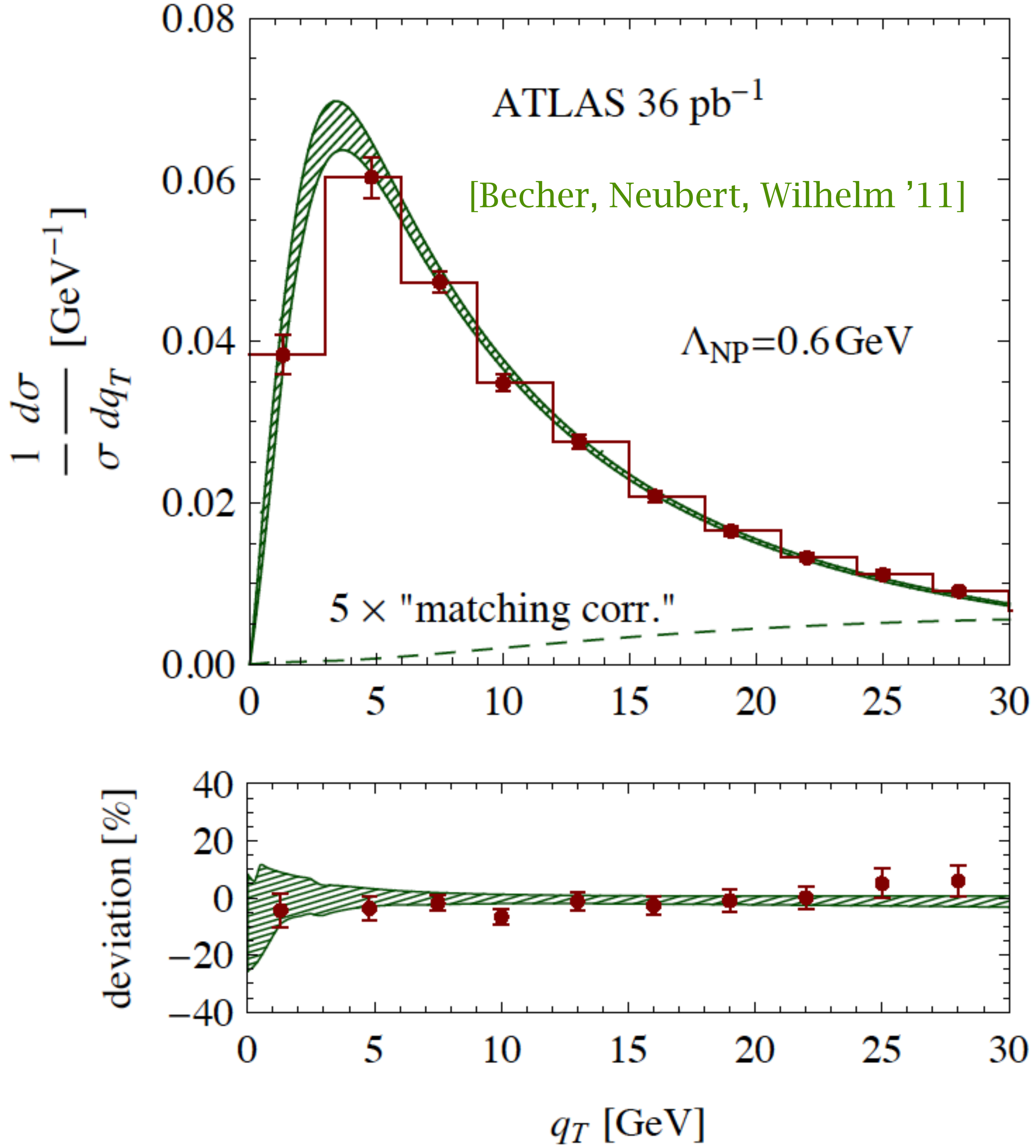
  i) Build an effective field theory of QCD by integrating out the irrelevant d.o.f. (i.e. hard radiation)

$$\mathcal{L}_{\mathrm{QCD}} \longrightarrow \mathcal{L}_{\mathrm{SCET}}$$

$$d\sigma \sim \sum_n C_n \langle \mathcal{O}_n \mathcal{O}_n^\dagger \rangle$$

  ii) **Evolution Equations** arise from RG invariance

$$\frac{d\sigma}{d\ln\mu} = 0$$



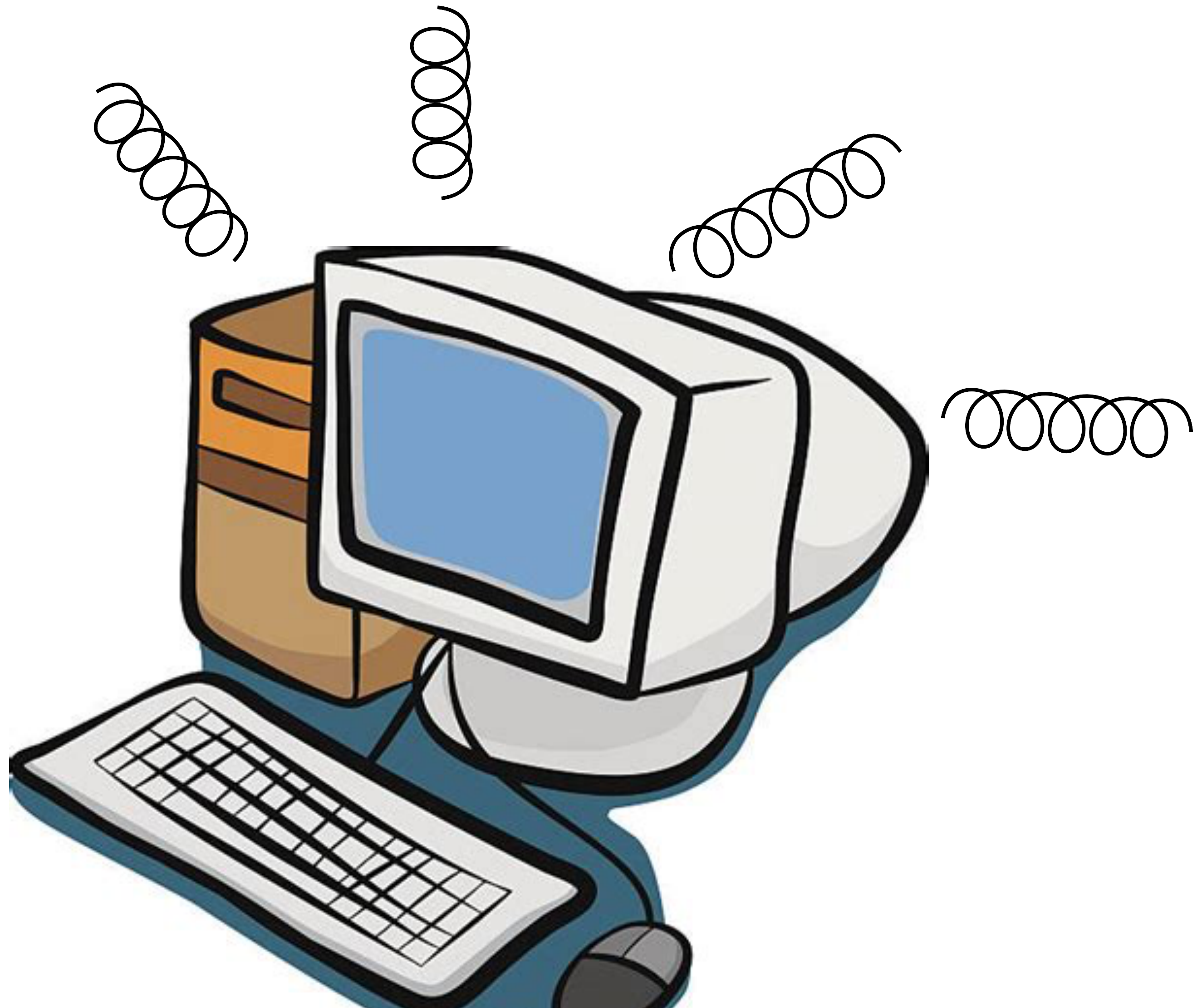
Introduction

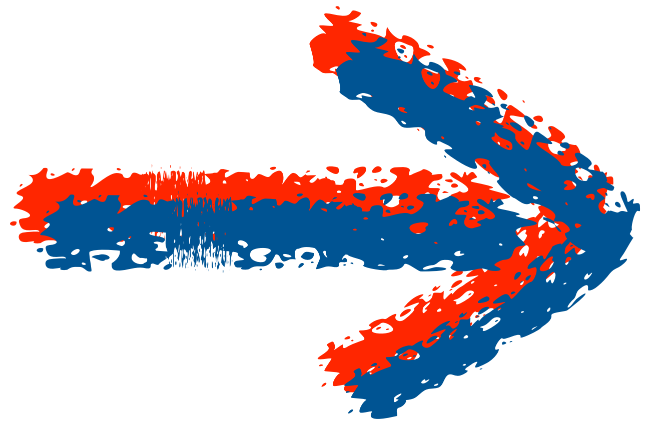
- Approach II :

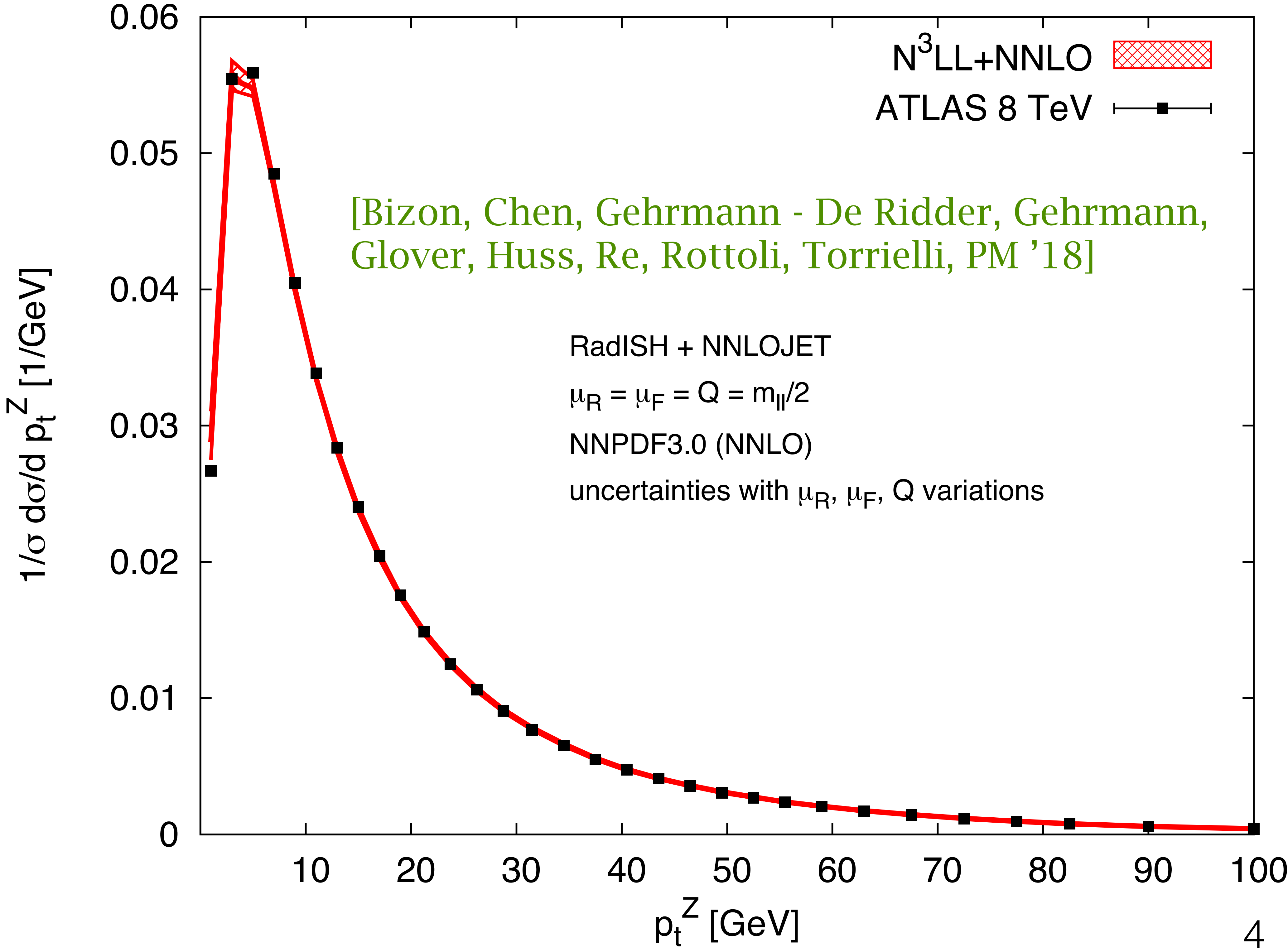
  i) Build a simplified model of QCD amplitudes in soft and collinear limits, simulate radiation at all orders

$$d\sigma \sim \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d\,k_i |\mathcal{M}(k_i)|^2$$

  ii) **Evolution formulated in algorithmic form**







Exploiting synergies in resummation

- Formal link between the two formulations is tricky, some progress recently

[Bauer, PM '18 + mainly ongoing work]

$$\frac{d\sigma}{d\ln\mu} = 0 \quad ?? \quad d\sigma \sim \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d\,k_i|\mathcal{M}(k_i)|^2$$

- A solid connection will shed more light on infrared structure of QCD, and allow us to address some complicated problems from a different angle
  - e.g.
    - multi-leg processes
    - multi-scale problems
    - non-global observables
    - …

Two scale problems: rIRC safe & global

- e.g. e^+e^- —> jets

d-dimensional phase space

observable

$$\Sigma(v) \equiv \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'} = \mathcal{H}(Q)\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{i=1}^{n}[dk_i]\mathcal{M}^2(k_1,...,k_n)\Theta(v - V(\{\tilde{p}\},k_1,\ldots,k_n))$$

Virtual corrections (form factor)

Real (& real-virtual) corrections

Two scale problems: rIRC safe & global

- This problem has a general solution at NNLL ($\lambda = \alpha_s(Q)\beta_0 \ln \frac{1}{v}$)

[Banfi, El Menoufi, PM 1807.11487]
[Banfi, McAslan, Zanderighi, PM 1412.2126, 1607.03111]

NNLL virtual and collinear constants

$$\Sigma_{\mathrm{NNLL}}(v) = e^{-R_{\mathrm{s}}(v)-R_{\mathrm{hc}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(\lambda)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\sum_{\ell=1}^{2}\frac{\alpha_s(Qv^{\frac{1}{a+b_\ell}})}{2\pi}C_{\mathrm{hc},\ell}^{(1)}\right)+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)\right],$$

Sudakov radiator

NLL Transfer function
[Banfi, Salam, Zanderighi ’01 - ’04]

NNLL Transfer function

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{sc}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{clust}}$$

Two scale problems: rIRC safe & global

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Analytical

$$\Sigma_{\rm NNLL}(v) = e^{-R_{\rm s}(v)-R_{\rm hc}(v)}\left[\mathcal{F}_{\rm NLL}(\lambda)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\sum_{\ell=1}^{2}\frac{\alpha_s(Qv^{\frac{1}{a+b_\ell}})}{2\pi}C^{(1)}_{{\rm hc},\ell}\right)+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\rm NNLL}(\lambda)\right],$$

Analytical/Numerical

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm sc} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm clust}$$

Two scale problems: the Radiator

- This problem has a general solution at NNLL ($\lambda = \alpha_s(Q)\beta_0 \ln \frac{1}{v}$)

Analytical

$$\Sigma_{\mathrm{NNLL}}(v) = e^{-R_{\mathrm{s}}(v)-R_{\mathrm{hc}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(\lambda)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\sum_{\ell=1}^{2}\frac{\alpha_s(Qv^{\frac{1}{a+b_\ell}})}{2\pi}C^{(1)}_{\mathrm{hc},\ell}\right) + \frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)\right],$$

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{sc}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{clust}}$$

Sudakov radiator: virtual corrections

- Consider the resummed form factor

Soft Webs in dim. reg.

$$\mathcal{H}(Q) = C(\alpha_s(Q)) \exp\left\{-\int \frac{d^d k}{(2\pi)^d} w(m^2, k_t^2 + m^2; \epsilon)\, \Theta\left(\frac{1}{2}\ln\left(\frac{Q^2}{k_t^2+m^2}\right) - |y|\right) \Theta(Q - k_t)\right\} \times$$

$$\times \exp\left\{-\sum_{\ell=1}^{2} \int^{Q^2} \frac{dk^2}{k^2} \gamma_\ell(\alpha_s(k,\epsilon))\right\}$$

endpoint coefficient of AP splitting functions

strong coupling in dim. reg.

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = -\epsilon\, \alpha_s + \beta^{(\mathrm{d}=4)}(\alpha_s)$$

Sudakov radiator: real (unresolved) corrections

- Decompose squared amplitude in terms of *correlated clusters* of emissions
  - e.g. soft limit

$$M_{\mathrm{s}}^2(k_1) \equiv \tilde{M}_{\mathrm{s}}^2(k_1)$$

two clusters (k_1, k_2) cluster $k_{\mathrm{clust.}} = k_1 + k_2$

- Each cluster is dressed by virtual corrections

nPC(0) nPC(1)

$$\tilde{M}_{\mathrm{s}}^2(k_1, \ldots, k_n) = \tilde{M}_{\mathrm{s},0}^2(k_1, \ldots, k_n) + \frac{\alpha_s(\mu_R)}{2\pi} \tilde{M}_{\mathrm{s},1}^2(k_1, \ldots, k_n) + \ldots$$

Sudakov radiator: real (unresolved) corrections

- Introduce a resolution scale (slicing parameter) such that *unresolved clusters* satisfy

$$V_{\rm sc}(k_{\rm clust.}) < \delta v \qquad V_{\rm sc}(k) \equiv \sum_{\ell=1}^{2} d_\ell \left(\frac{k_t^{(\ell)}}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)})\Theta(\eta^{(\ell)})$$

- Unresolved radiation is unconstrained by the observable —> logarithmic counting

	Unresolved radiation		Resolved radiation	
	$[n\mathrm{PC}^{(j)}]_{\rm sc}$	$[n\mathrm{PC}^{(j)}]_{\not{\rm sc}}$	$[n\mathrm{PC}^{(j)}]_{\rm sc}$	$[n\mathrm{PC}^{(j)}]_{\not{\rm sc}}$
LL	$n+j\leq 1$	–	–	–
NLL	$n+j\leq 2$	$n+j\leq 1$	$n+j\leq 1$	–
NNLL	$n+j\leq 3$	$n+j\leq 2$	$n+j\leq 2$	$n+j\leq 1$
$\mathrm{N}^k\mathrm{LL}$	$n+j\leq k+1$	$n+j\leq k$	$n+j\leq k$	$n+j\leq k-1$

Sudakov radiator at NNLL

- Combination of virtuals and unresolved clusters defines the radiator (cutoff dependence cancels against that of *resolved* radiation)

Starts at LL (double logs)

$$R_{\rm s}(v) = \sum_{\ell=1}^{2} \int \frac{d^4k}{(2\pi)^4} w(m^2, k_t^2 + m^2) \Theta\left(d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)}) - v\right) \Theta(\eta^{(\ell)})$$

Starts at NLL (single logs)

$$R_{\rm hc} = \sum_{\ell=1}^{2} \int_{Q^2 v^{\frac{2}{a+b_\ell}}}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k)}{2\pi} \left[\gamma_\ell^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \gamma_\ell^{(1)}\right]$$

Sudakov radiator at NNLL

- Soft contribution can be further decomposed as follows

$$R_{\mathrm{s}}(v) \simeq \sum_{\ell} \left(R_\ell(v) + R'_\ell(v) \int_0^{2\pi} \frac{d\phi^{(\ell)}}{2\pi} \ln(d_\ell g_\ell(\phi^{(\ell)})) + R''_\ell(v) \int_0^{2\pi} \frac{d\phi^{(\ell)}}{2\pi} \frac{1}{2} \ln^2(d_\ell g_\ell(\phi^{(\ell)})) \right)$$

$$R_\ell(v) \simeq \boxed{R_\ell^0(v)} + \delta R_\ell(v)$$

- *massless term*

$$R_\ell^0(v) = \int \frac{d^4k}{(2\pi)^4} w(m^2, k_t^2 + m^2) \Theta\left(\ln\frac{Q}{k_t} - \eta^{(\ell)}\right) \Theta\left(\left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} - v\right) \Theta(\eta^{(\ell)})$$

Sudakov radiator at NNLL

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$$R_{\rm s}(v) \simeq \sum_{\ell} \left(R_\ell(v) + R'_\ell(v) \int_0^{2\pi} \frac{d\phi^{(\ell)}}{2\pi} \ln(d_\ell g_\ell(\phi^{(\ell)})) + R''_\ell(v) \int_0^{2\pi} \frac{d\phi^{(\ell)}}{2\pi} \frac{1}{2} \ln^2(d_\ell g_\ell(\phi^{(\ell)})) \right)$$

$$R_\ell(v) \simeq R_\ell^0(v) + \boxed{\delta R_\ell(v)}$$

- *mass correction*

$$\begin{aligned}\delta R_\ell(v) &= C_\ell \int_{Qv^{\frac{1}{a+b_\ell}}}^{Q} \frac{dk_t}{k_t} \left(\frac{\alpha_s(k_t)}{\pi}\right)^2 \int_0^\infty \frac{d\mu^2}{\mu^2(1+\mu)} \left(C_A \ln\frac{1+\mu^2}{\mu^4} - 2\pi\beta_0\right) \ln\left(\sqrt{\frac{1}{1+\mu^2}}\right) \\ &= \pi\beta_0\zeta_2 C_\ell \int_{Qv^{\frac{1}{a+b_\ell}}}^{Q} \frac{dk_t}{k_t} \left(\frac{\alpha_s(k_t)}{\pi}\right)^2 .\end{aligned}$$

Physical coupling in the soft limit

- The massless terms defines a physical coupling in the soft limit

$$R_\ell^0(v) = \int \frac{d^4k}{(2\pi)^4} w(m^2, k_t^2+m^2)\Theta\left(\ln\frac{Q}{k_t} - \eta^{(\ell)}\right)\Theta\left(\left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} - v\right)\Theta(\eta^{(\ell)})$$

$$\int_0^\infty dm^2\, w(m^2, k_t^2+m^2) \equiv (4\pi)^2 \frac{2C_\ell}{k_t^2}\alpha_s^{\rm phys}(k_t)$$

$$\alpha_s^{\rm phys} = \alpha_s\left(1+\sum_{n=1}^\infty \left(\frac{\alpha_s}{2\pi}\right)^n K^{(n)}\right)$$

$$K^{(1)} = C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right) - \frac{5}{9}n_f\,,$$ —> [Catani, Marchesini, Webber '91]

$$K^{(2)} = C_A^2\left(\frac{245}{24}-\frac{67}{9}\zeta_2+\frac{11}{6}\zeta_3+\frac{11}{5}\zeta_2^2\right)+C_F n_f\left(-\frac{55}{24}+2\zeta_3\right)$$
$$+C_A n_f\left(-\frac{209}{108}+\frac{10}{9}\zeta_2-\frac{7}{3}\zeta_3\right)-\frac{1}{27}n_f^2+\frac{\pi\beta_0}{2}\left(C_A\left(\frac{808}{27}-28\zeta_3\right)-\frac{224}{54}n_f\right)$$

This is the only, universal, α_s^3 ingredient at NNLL !

Two scale problems: the transfer functions

- The definition of the *unresolved* radiation ensures that all LL are in the Radiator
- One's left with the computation of the transfer functions

$$\Sigma_{\rm NNLL}(v) = e^{-R_{\rm s}(v)-R_{\rm hc}(v)}\left[\mathcal{F}_{\rm NLL}(\lambda)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\sum_{\ell=1}^{2}\frac{\alpha_s(Qv^{\frac{1}{a+b_\ell}})}{2\pi}C_{{\rm hc},\ell}^{(1)}\right)+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\rm NNLL}(\lambda)\right],$$

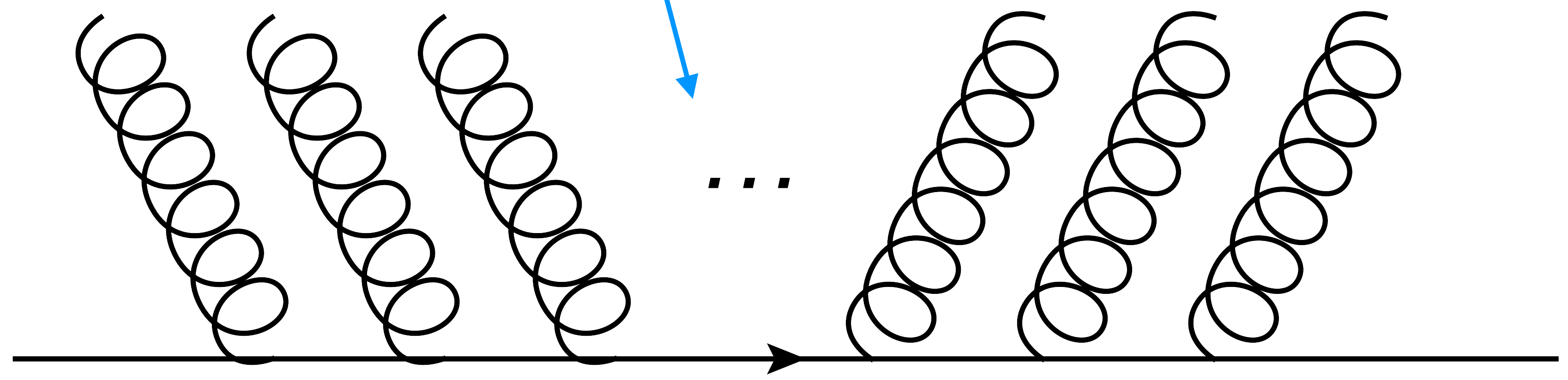
Analytical/Numerical

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm sc} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm clust}$$

Two scale problems: the transfer functions

- Transfer functions describe radiation in well specified kinematical regimes
  - e.g. NLL (CAESAR): ensemble of soft-collinear gluons strongly ordered in rapidity

$$\mathcal{F}_{\rm NLL}(\lambda) = \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}]\,\Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\},\{k_i\})}{v}\right)$$



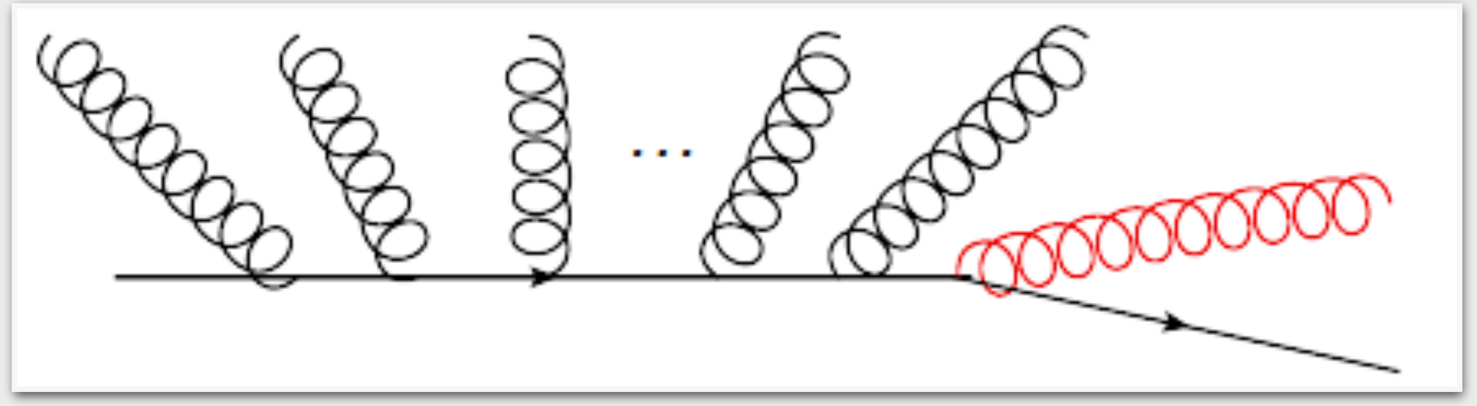
$$\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}]G(\{\tilde{p}\},\{k_i\}) = \epsilon^{R'_{\rm NLL}}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int_{\epsilon}^{\infty}\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1,2}R'_{{\rm NLL},\ell_i}G(\{\tilde{p}\},k_1,\ldots,k_n)$$

Two scale problems: the transfer functions

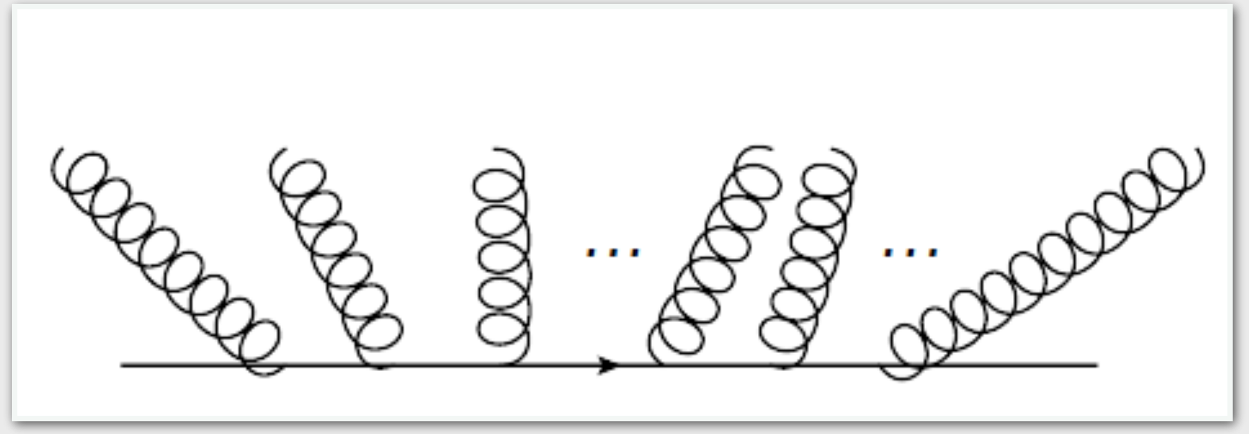
- Transfer functions describe radiation in well specified kinematical regimes
  - e.g. NNLL (ARES): at most one single emission probes less singular kinematics

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm sc} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm clust}$$

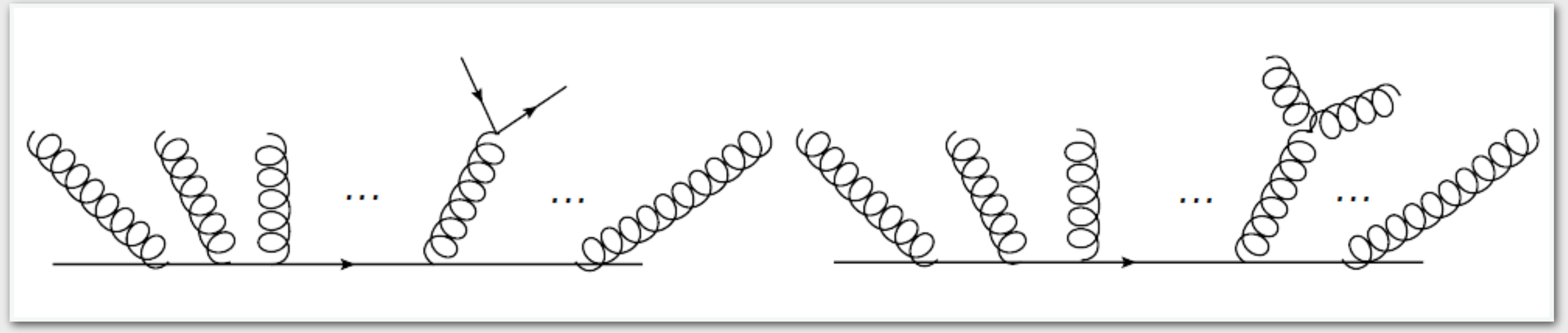
- collinear emission carries a significant energy fraction
  - correction to the amplitude: hard-collinear corrections
  - correction to the observable: recoil corrections



- soft-collinear emission gets close in rapidity to another
  - sensitive to the exact rapidity bounds: rapidity (SC) corrections
  - different clustering history for a jet algorithm: clustering corrections



- insertion of double-soft current and corresponding virtual corrections
  - correlated corrections



- soft emission is allowed to propagate at small rapidities
  - soft-wide-angle corrections

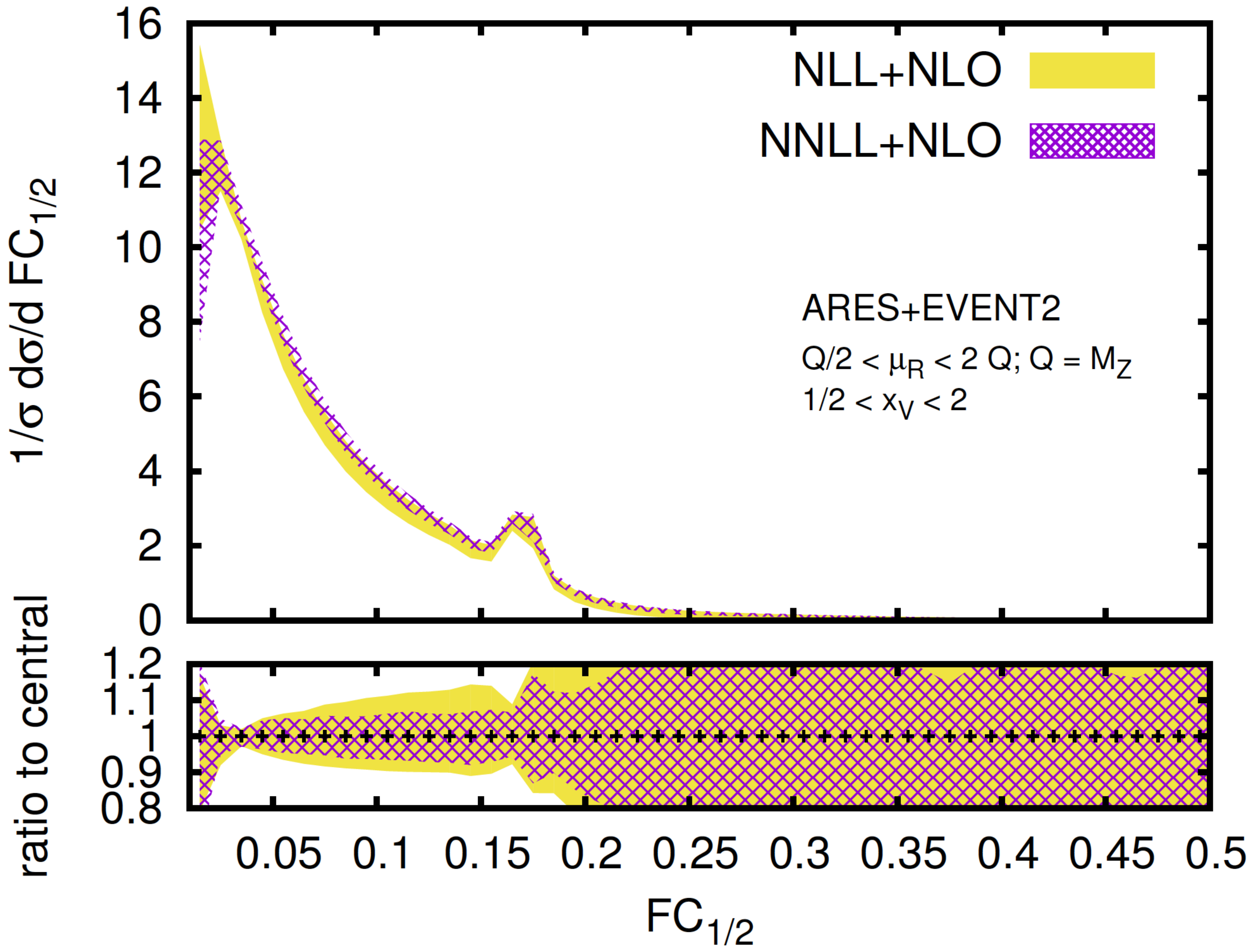
- Restores correct rapidity dependence in observable
- Gets complicated for multi leg case (interference between hard emitters)

Automation

- Suitable for automation in a computer program
    - observable (in various limits) is the only external input
- Agnostic to factorisation structure of the measurement function

correction type	$p_{t,\mathrm{veto}}$	$1-T$	B_T	B_W	C	ρ_H	T_M	O	$y_3^{\mathrm{Dur.}}$	$y_3^{\mathrm{Cam.}}$	p_t
$\mathcal{F}_{\mathrm{NLL}}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\delta\mathcal{F}_{\mathrm{sc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\mathrm{wa}}$	x	x	x	x	✓	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\mathrm{hc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\mathrm{rec}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	x
$\delta\mathcal{F}_{\mathrm{clust}}$	✓	x	x	x	x	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\mathrm{correl}}$	✓	x	✓	✓	x	x	✓	✓	✓	✓	x

Some examples

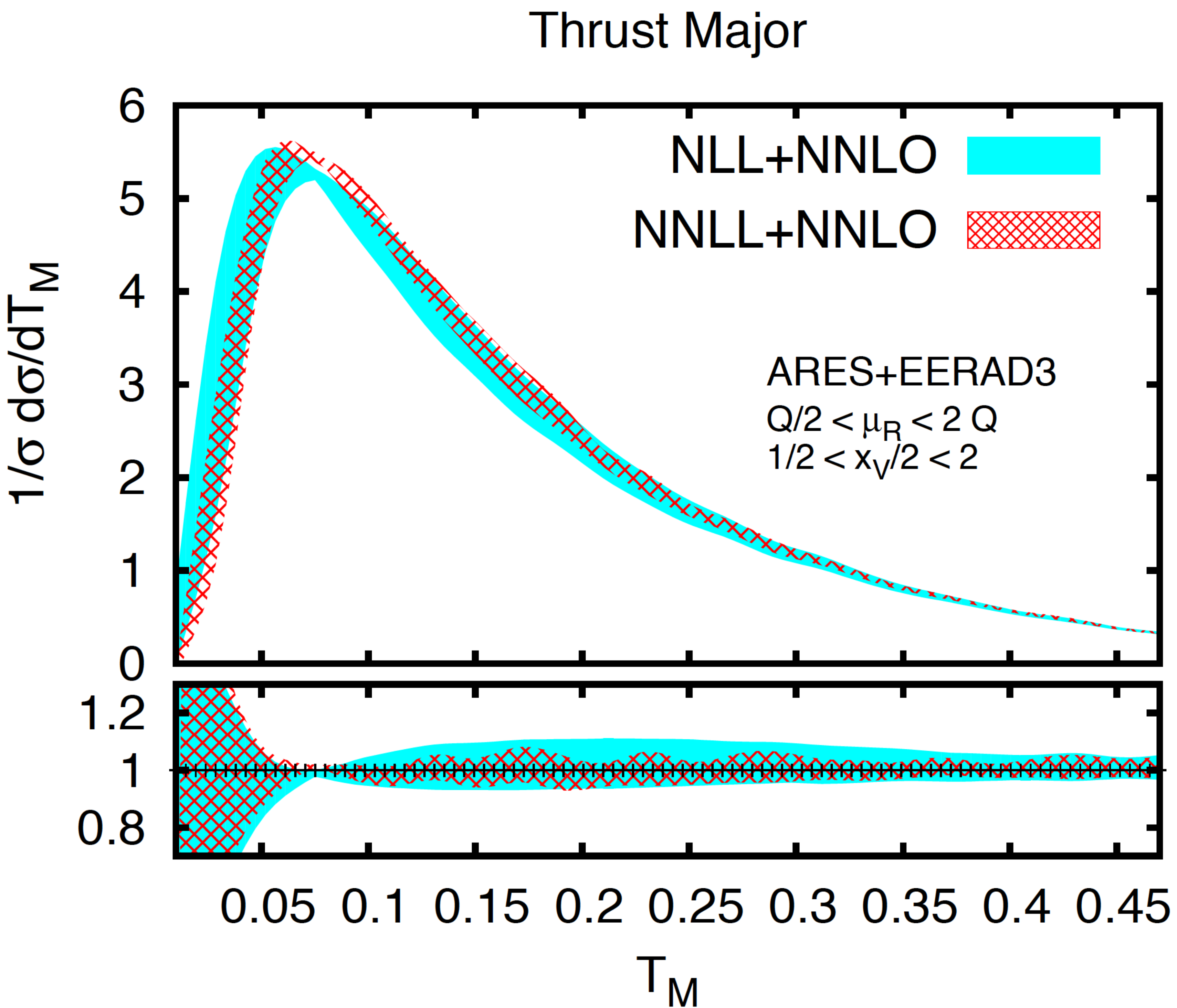


Moments of EEC & angularities

$$FC_x = \sum_{i\neq j} \frac{E_i E_j |\sin\theta_{ij}|^x (1-|\cos\theta_{ij}|)^{1-x}}{\left(\sum_i E_i\right)^2} \Theta\left[(\vec{q}_i\cdot n_T)(\vec{q}_j\cdot\vec{n}_T)\right]$$

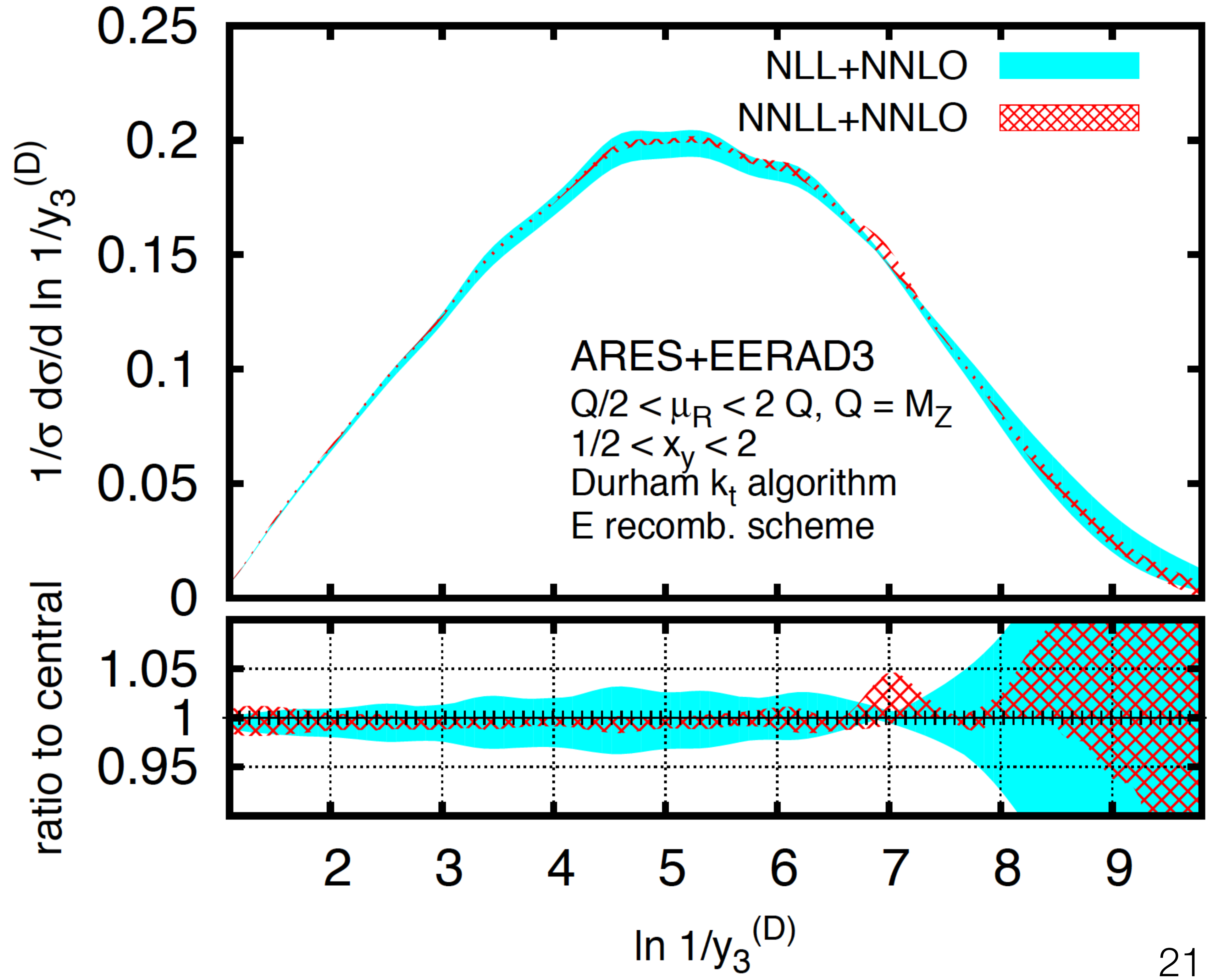
Durham k_t algorithm

$$y_{ij}^{(D)} = v_{ij}^{(D)} = 2\frac{\min\{E_i,E_j\}^2}{Q^2}(1-\cos\theta_{ij})$$



Thrust Major

$$T_M = \frac{1}{\sum_i |\vec{p}_i|} \max_{\vec{n}\cdot\vec{n}_T=0} \sum_i |\vec{p}_i\cdot\vec{n}|$$



Conclusions

- Global problems with two scales have a general solution at NNLL
  - no need for a factorisation theorem
  - suitable for automation
  - can be systematically extended to higher orders if higher precision is needed (e.g. p_T)
- For more complicated observables, more efforts are required to understand the general structure (known at NLL)
  - Multi differential distributions
  - Only a few results available for multi-leg observables at NNLL
  - Full control of IR physics at NNLL requires solution of (next to leading) non-global logarithms
- Matching ambiguity to fixed order ~few-%: pheno impact of subleading power corrections to be established, might be relevant in high precision observables

Backup material

Sudakov radiator: web exponentiation

- Soft unresolved clusters obey non-abelian (web) exponentiation theorem

$$\mathcal{H}(Q)\exp\left\{\int^{Q}\frac{d^dk}{(2\pi)^d}w(m^2,k_t^2+m^2;\epsilon)\Theta(\delta v-V_{\rm sc}(k))\right\}$$

$$=C(\alpha_s(Q))e^{-R_s(\delta v)}\exp\left\{-\sum_{\ell=1}^{2}\int^{Q^2}\frac{dk^2}{k^2}\gamma_\ell(\alpha_s(k,\epsilon))\right\}$$

Soft radiator

Regular terms of the form factor

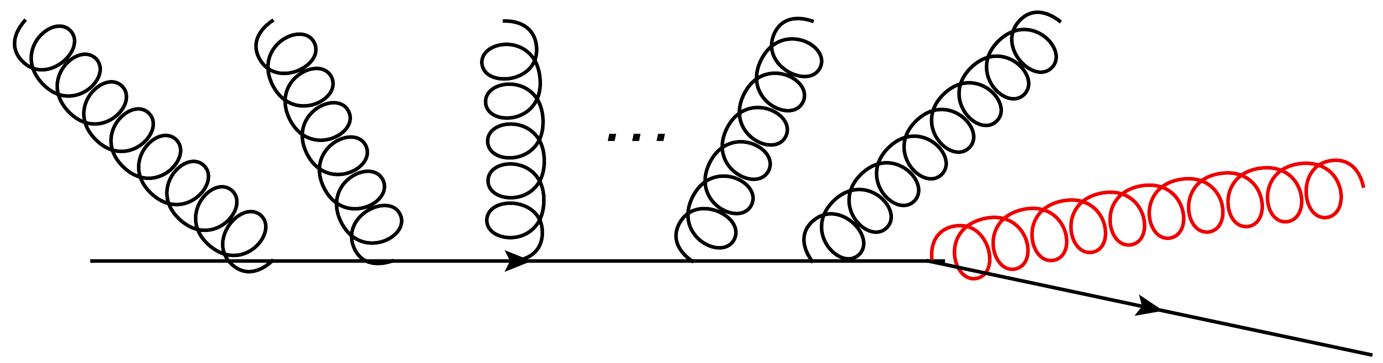
- Cut virtual-collinear at collinear scale, and expand to cancel real divergences

$$\int^{Q^2}\frac{dk^2}{k^2}\gamma_\ell(\alpha_s(k,\epsilon))=\int_{Q^2v^{\frac{2}{a+b_\ell}}}^{Q^2}\frac{dk^2}{k^2}\gamma_\ell(\alpha_s(k))+\int_0^{Q^2v^{\frac{2}{a+b_\ell}}}\frac{dk^2}{k^2}\gamma_\ell(\alpha_s(k,\epsilon))$$

$$\exp\left\{-\int_0^{Q^2v^{\frac{2}{a+b_\ell}}}\frac{dk^2}{k^2}\gamma_\ell(\alpha_s(k,\epsilon))\right\}=1-\int_0^{Q^2v^{\frac{2}{a+b_\ell}}}\frac{dk^2}{k^2}\frac{\alpha_s(k,\epsilon)}{2\pi}\gamma_\ell^{(0)}+\mathcal{O}(\alpha_s^2(Qv^{\frac{1}{a+b_\ell}}))$$

Collinear corrections

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)



  - Corrections affect both matrix element (hard-collinear corrections) and observable (recoil corrections)

$$\delta\mathcal{F}_{\rm hc}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}] \times$$

$$\times \int_0^1 \frac{dz}{z} \left(z p_\ell(z) - 2C_\ell\right) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\}, \{k_i\})}{v}\right)\Theta(1-\zeta)\right]$$

$$\delta\mathcal{F}_{\rm rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}] \times$$

$$\times \int_0^1 dz\, p_\ell(z) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm hc}^{(k')}(\{\tilde{p}\}, k', \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right)\right]$$

Single-soft corrections

- (at most) one soft-collinear emission has the correct rapidity bounds (approximated in the NLL ensemble) - rapidity corrections

$$\delta\mathcal{F}_{\rm sc}(\lambda) = \frac{\pi}{\alpha_s(Q)}\int_0^\infty \frac{d\zeta}{\zeta}\int_0^{2\pi}\frac{d\phi}{2\pi}\sum_{\ell=1,2}\left(\delta R'_{\rm NNLL,\ell} + R''_\ell \ln\frac{d_\ell g_\ell(\phi)}{\zeta}\right)\int d\mathcal{Z}[\{R'_{\rm NLL,\ell_i}, k_i\}]\times$$
$$\times\left[\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},k,\{k_i\})}{v}\right) - \Theta(1-\zeta)\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},\{k_i\})}{v}\right)\right],$$

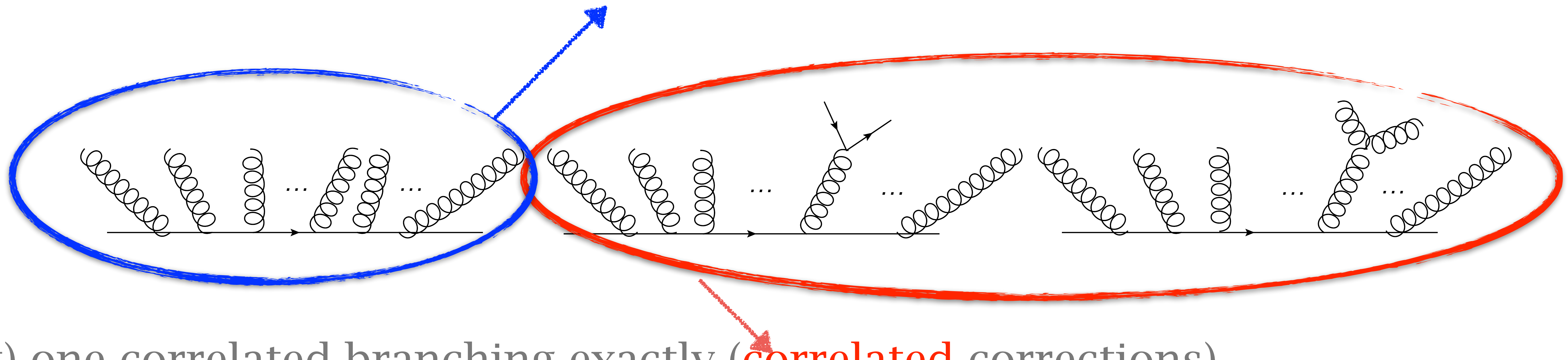
- (at most) one soft emission can propagate at very small rapidities (wide angle corrections)**

$$\delta\mathcal{F}_{\rm wa}(\lambda) = \frac{2C_F}{a}\frac{\alpha_s(v^{1/a}Q)}{\alpha_s(Q)}\int_0^\infty\frac{d\zeta}{\zeta}\int_{-\infty}^{\infty}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\int d\mathcal{Z}[\{R'_{\rm NLL,\ell_i},k_i\}]$$
$$\times\left[\Theta\left(1-\lim_{v\to 0}\frac{V^{(k)}_{\rm wa}(\{\tilde p\},k,\{k_i\})}{v}\right) - \Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},k,\{k_i\})}{v}\right)\right]$$

**With n>2 there are additional contributions due to the quantum interference between hard emitters starting at NLL order, due to soft-wide-angle radiation

Double-soft corrections

- (at most) two soft-collinear emissions get close in rapidity:
  - Relax strong angular ordering (clustering corrections, e.g. jet algorithms)



  - Treat (at most) one correlated branching exactly (correlated corrections)

$$\delta\mathcal{F}_{\rm correl}(\lambda) = \int_0^\infty \frac{d\zeta_a}{\zeta_a}\int_0^{2\pi}\frac{d\phi_a}{2\pi}\sum_{\ell_a=1,2}\left(\frac{2C_{\ell_a}\lambda}{a\beta_0}\frac{R''_{\ell_a}(v)}{\alpha_s(Q)}\right)\int_0^\infty\frac{d\kappa}{\kappa}\int_{-\infty}^{\infty}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{1}{2!}C_{ab}(\kappa,\eta,\phi)\times$$

$$\times\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i},k_i\}]\left[\Theta\left(v - V_{\rm sc}(\{\tilde{p}\},k_a,k_b,\{k_i\})\right) - \Theta\left(v - V_{\rm sc}(\{\tilde{p}\},k_a+k_b,\{k_i\})\right)\right]$$

$$C_{ab}(\kappa,\eta,\phi) = \frac{\tilde{M}^2(k_a,k_b)}{M^2_{\rm sc}(k_a)M^2_{\rm sc}(k_b)}$$