Parton Showers for the LHC and Beyond

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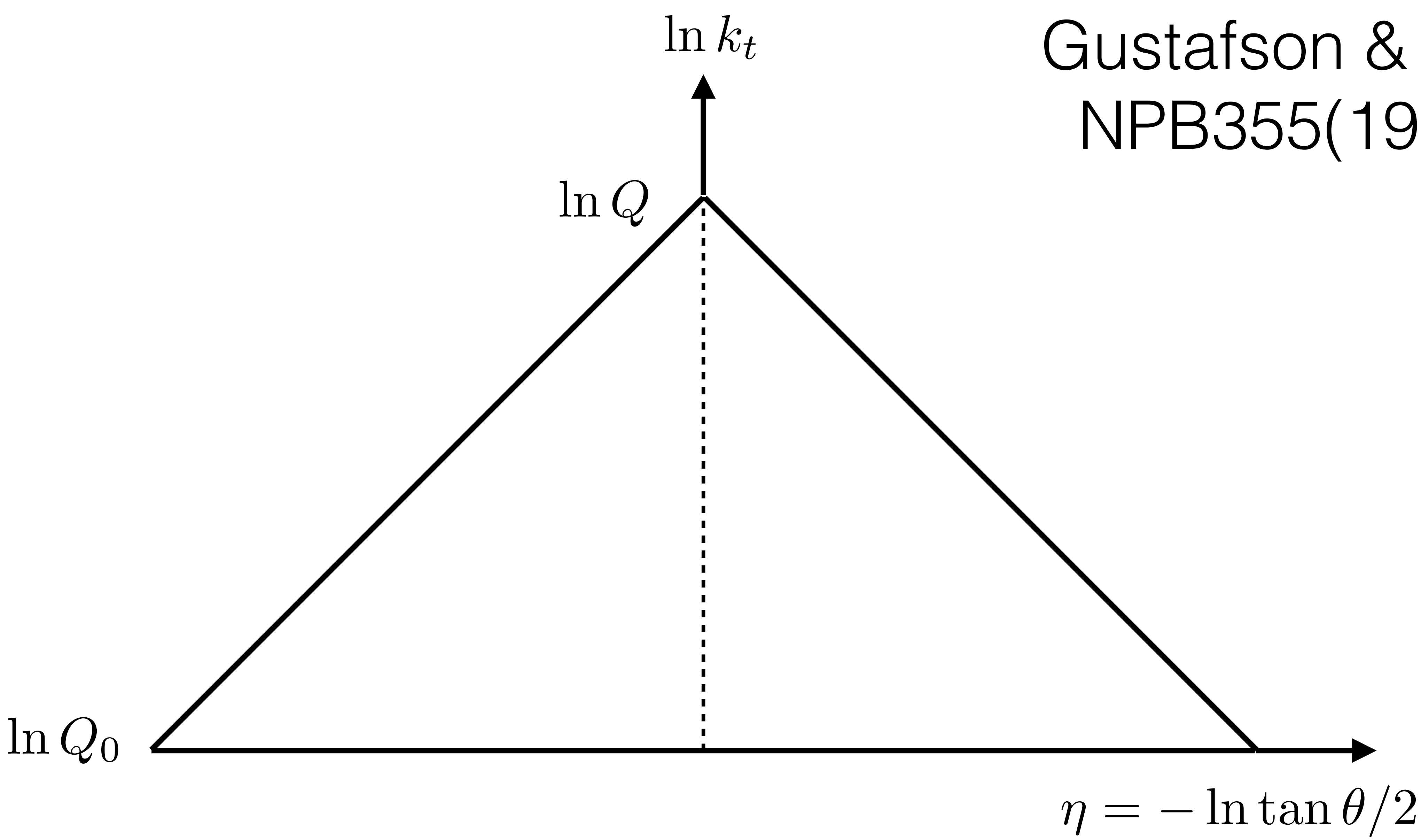
Cambridge

- Dipole vs parton showers
- Fractal structure of showers
- Spin effects in showers
- Including electroweak showering

Dipole vs Parton Showers

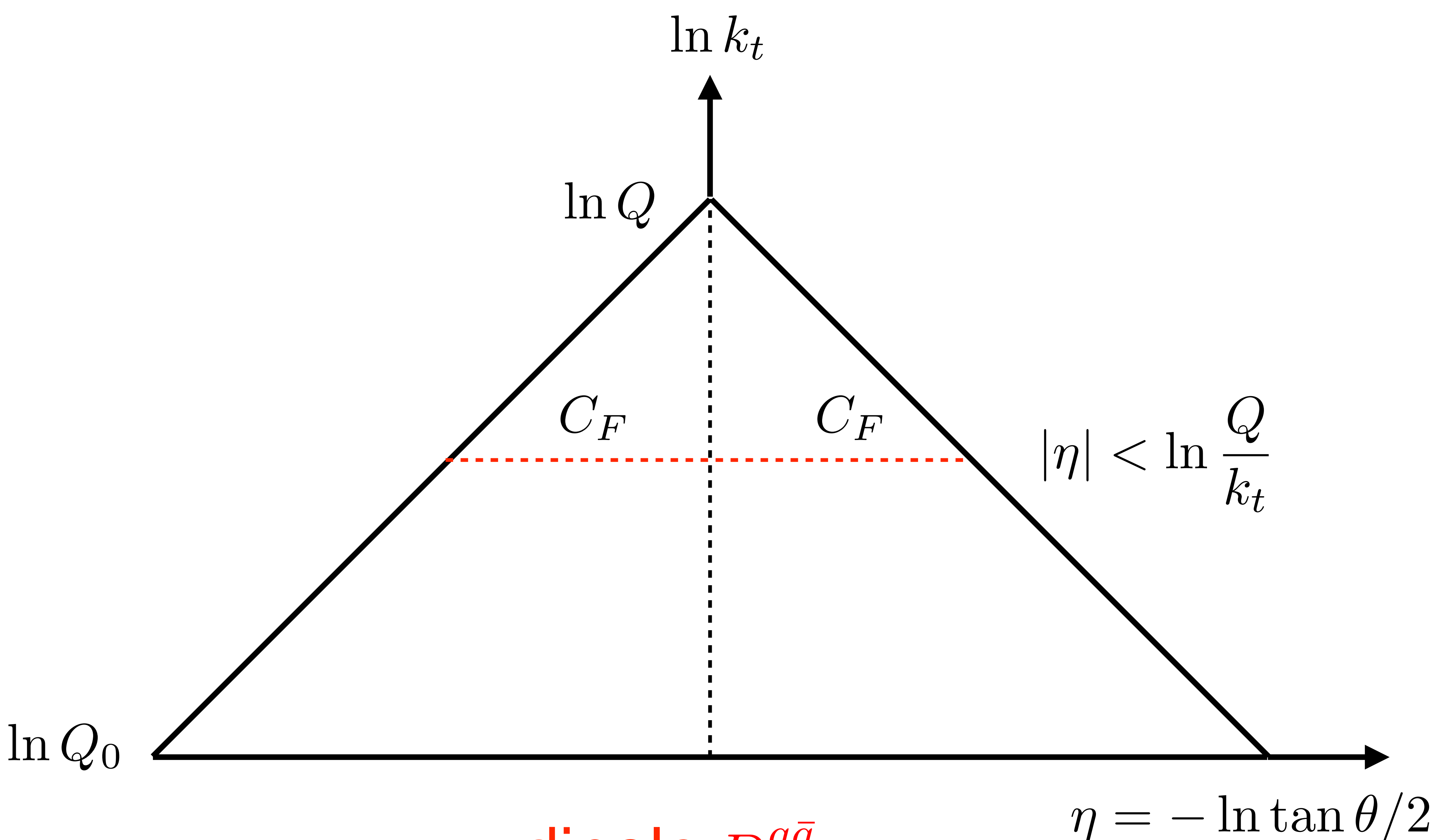
Lund origami diagram

Gustafson & Nilsson, NPB355(1991)106



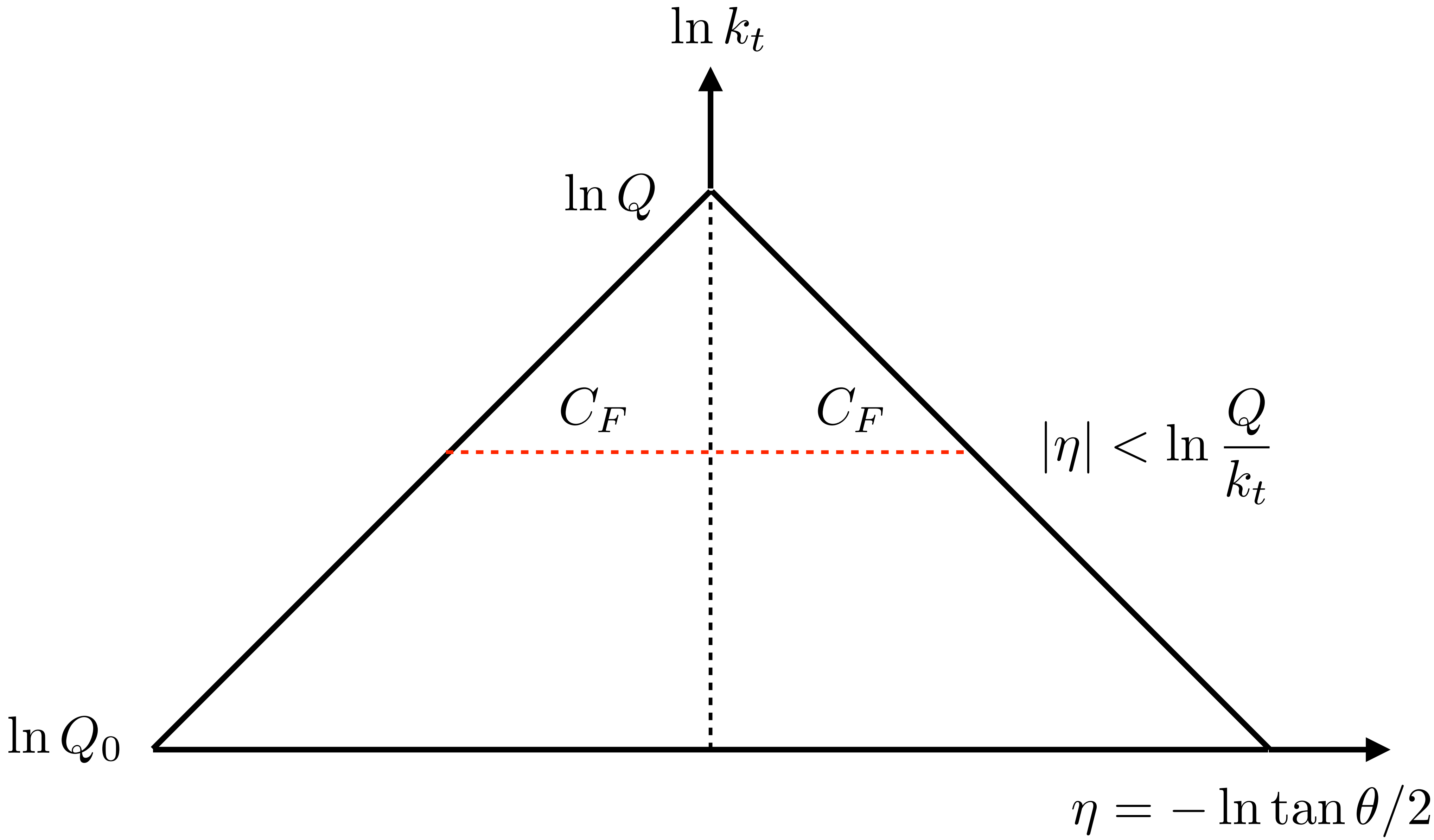
$$dP(q\bar{q}g) \simeq C_F \frac{\alpha_s}{2\pi} \frac{p_q \cdot p_{\bar{q}}}{p_q \cdot k \, p_{\bar{q}} \cdot k} dk_t^2 d\eta = 2C_F \frac{\alpha_s}{\pi} \frac{dk_t}{k_t} d\eta$$

k_t-ordered dipole shower

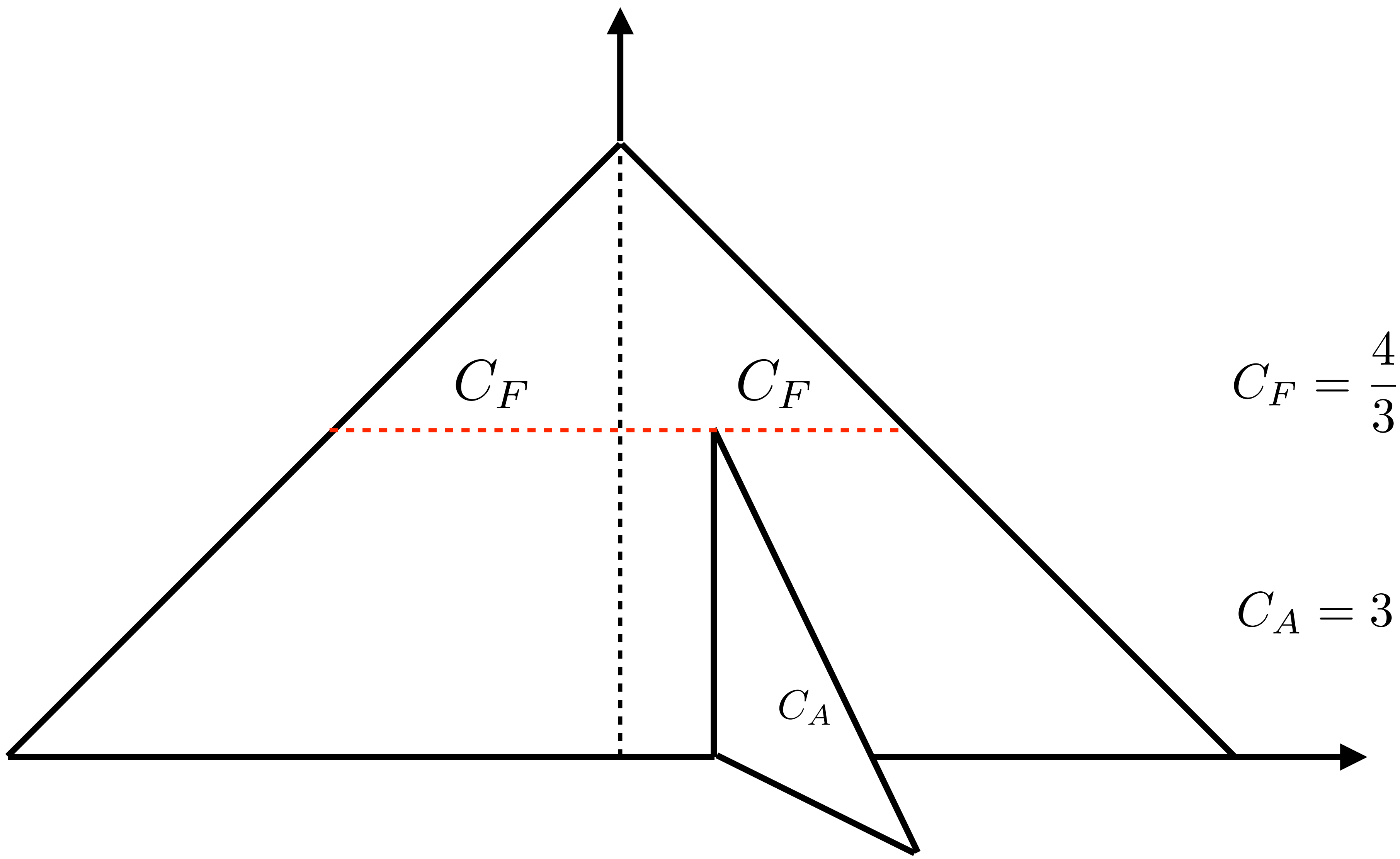


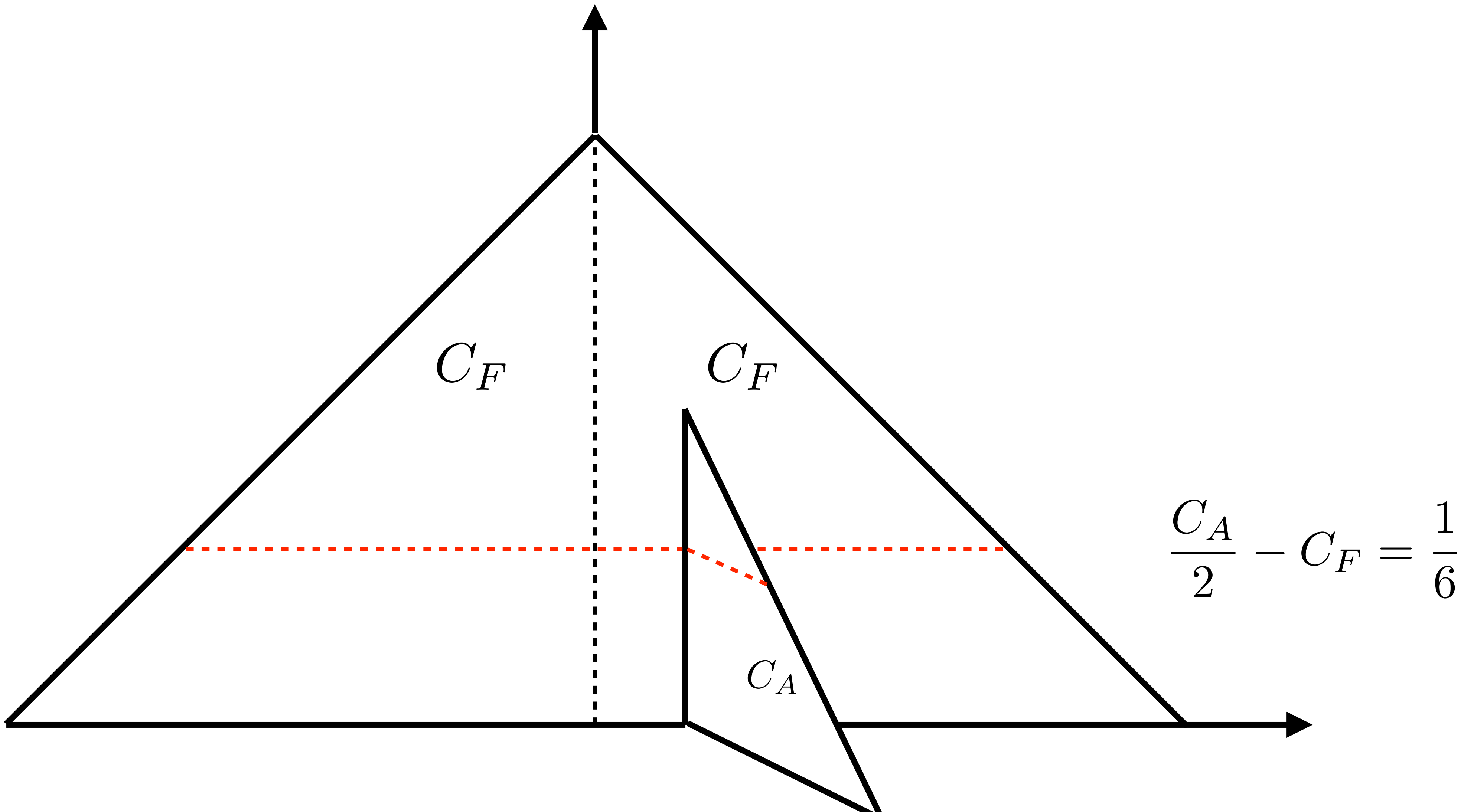
dipole $D_k^{q\bar{q}}$

$$dP(q\bar{q}g) \simeq C_F \frac{\alpha_s}{2\pi} \frac{p_q \cdot p_{\bar{q}}}{p_q \cdot k \, p_{\bar{q}} \cdot k} dk_t^2 d\eta = 2C_F \frac{\alpha_s}{\pi} \frac{dk_t}{k_t} d\eta$$



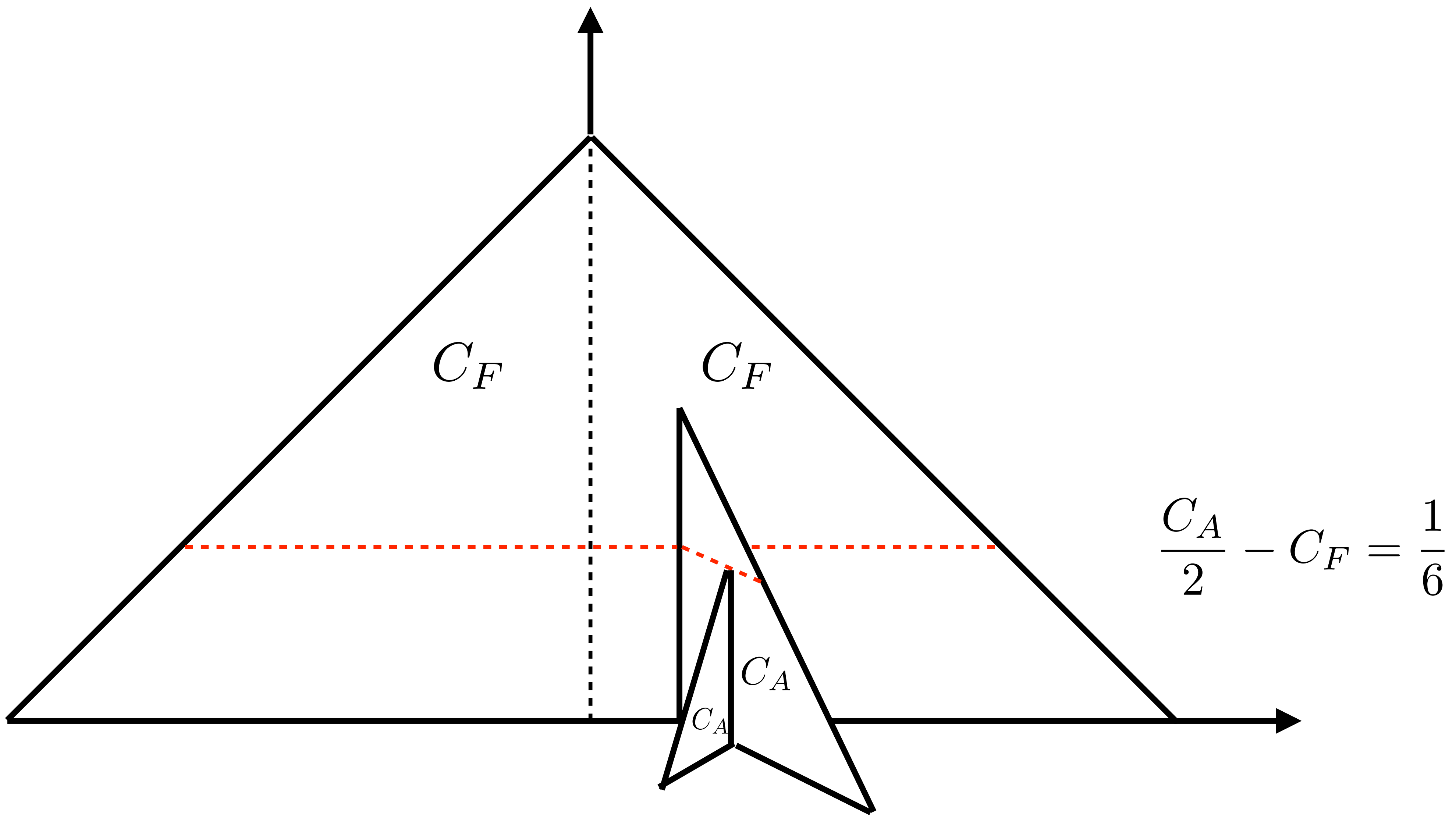
$$P(q\bar{q}g) \sim 4C_F \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_t}{k_t} \int_0^{\ln\frac{Q}{k_t}} d\eta = 2C_F \frac{\alpha_s}{\pi} L^2 \qquad \left(L = \ln \frac{Q}{Q_0}\right)$$

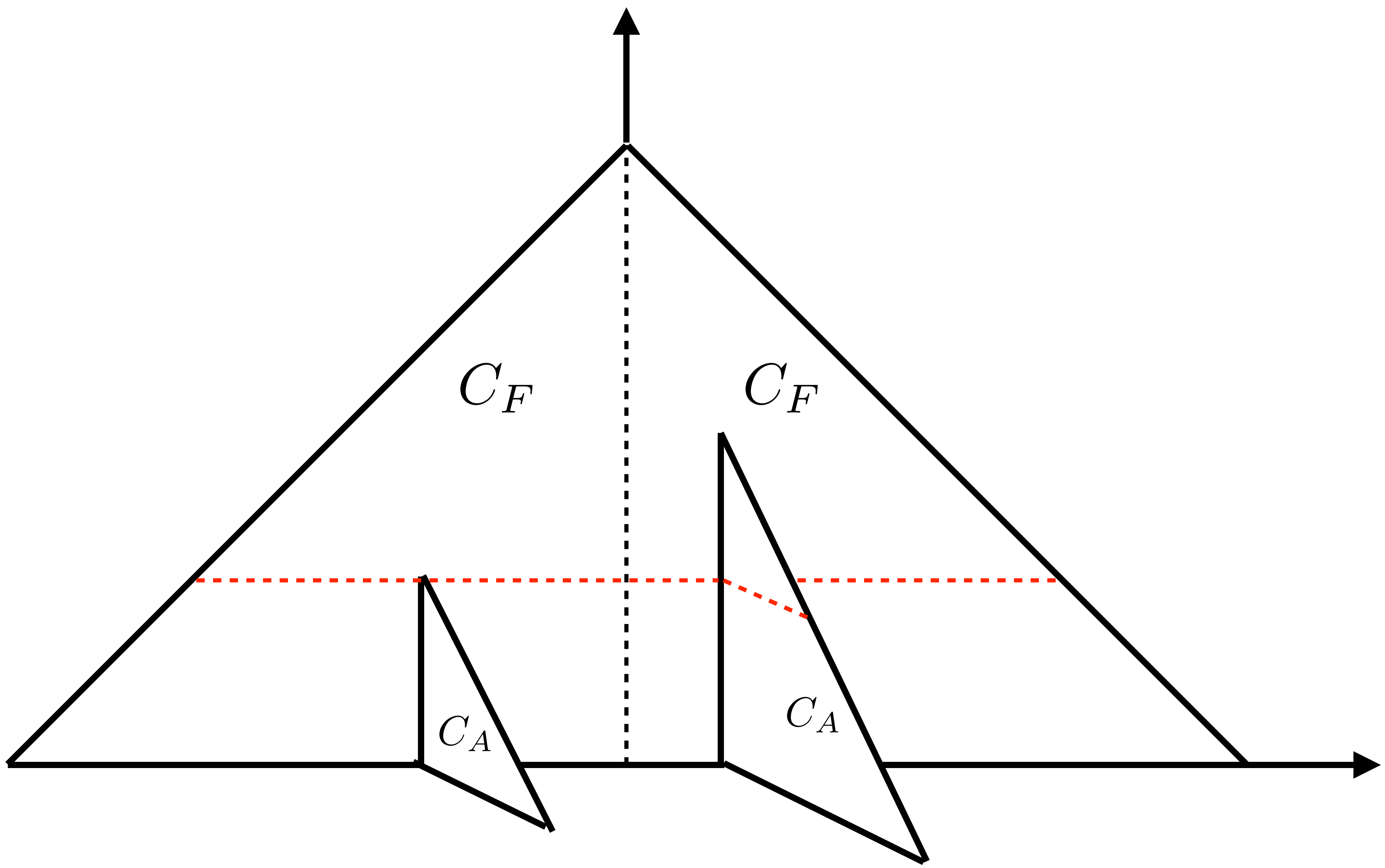

C_F
C_F
$C_F = \frac{4}{3}$
$C_A = 3$
C_A



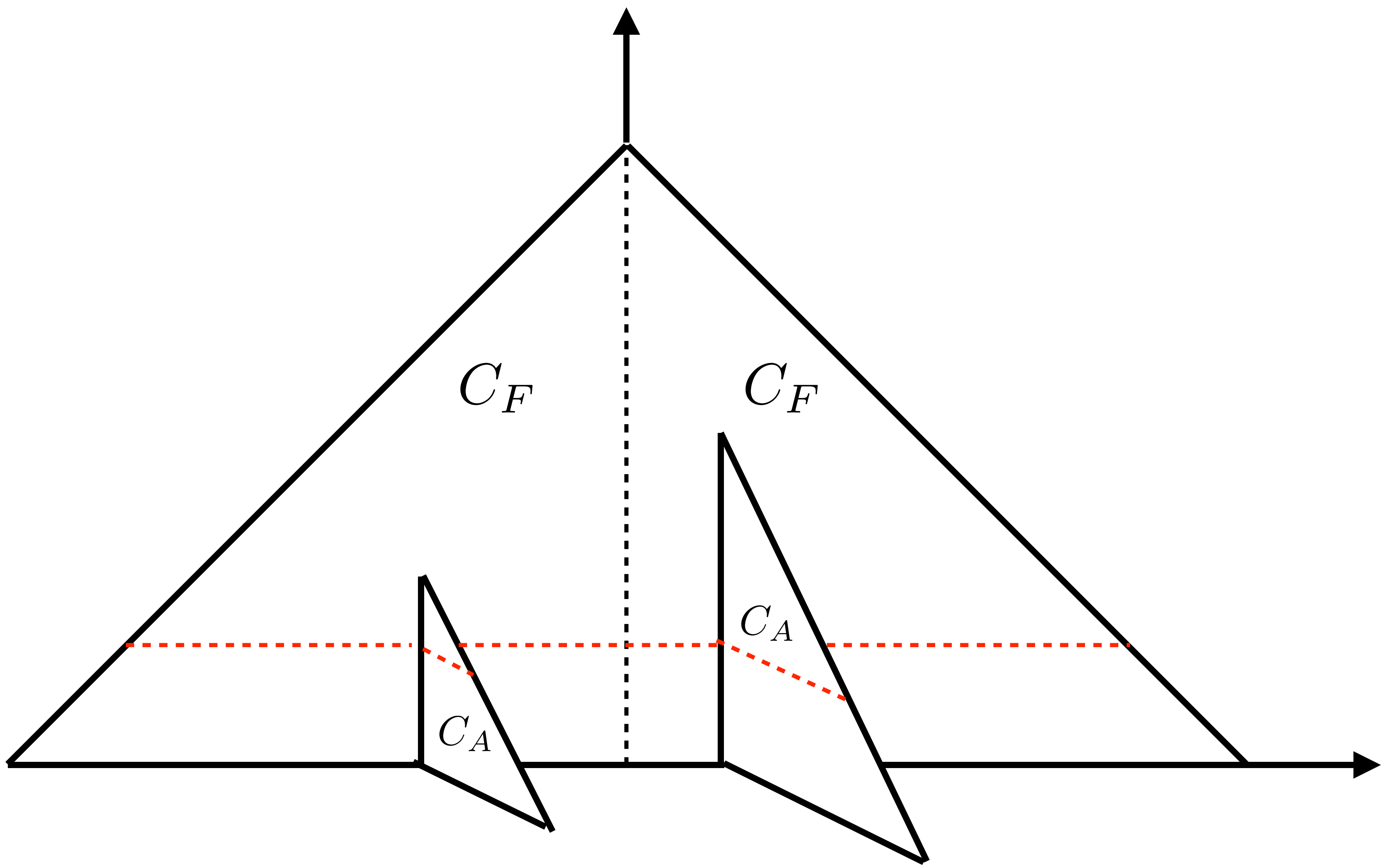
$$dP(q\bar{q}gg) \simeq C_F \left(\frac{\alpha_s}{2\pi}\right)^2 D_{k_1}^{q\bar{q}} \left[\frac{C_A}{2}\left(D_{k_2}^{qk_1} + D_{k_2}^{\bar{q}k_1}\right) - \left(\frac{C_A}{2} - C_F\right) D_{k_2}^{q\bar{q}}\right] dk_{t1}^2 dk_{t2}^2 \, d\eta_1 \, d\eta_2$$

$$P(q\bar{q}gg) \sim 2C_F \left(\frac{\alpha_s}{\pi}\right)^2 \left(C_F + \frac{C_A}{6}\right) L^4$$


C_F
C_F
$\frac{C_A}{2} - C_F = \frac{1}{6}$
C_A
C_A

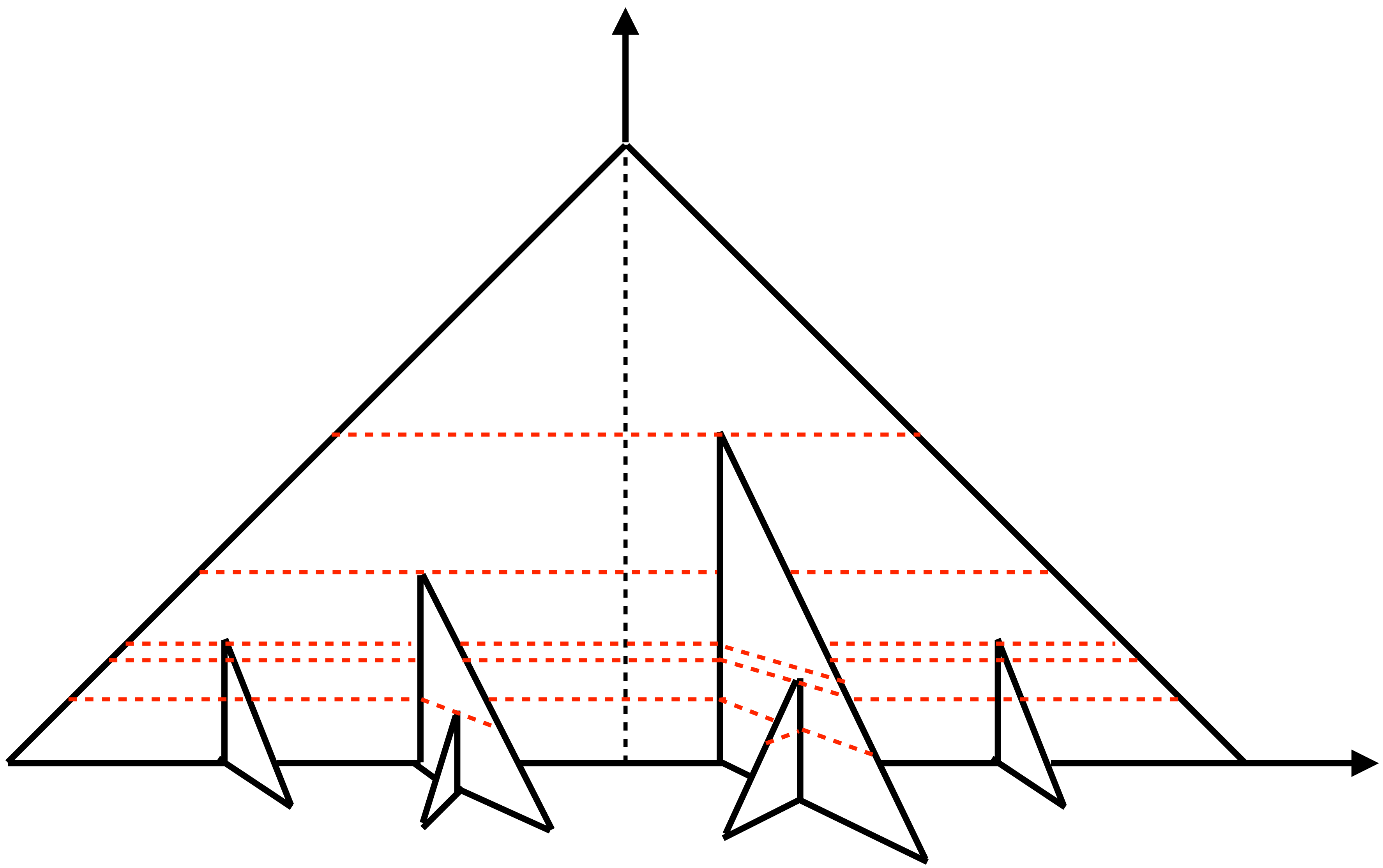


C_F
C_F
C_A
C_A

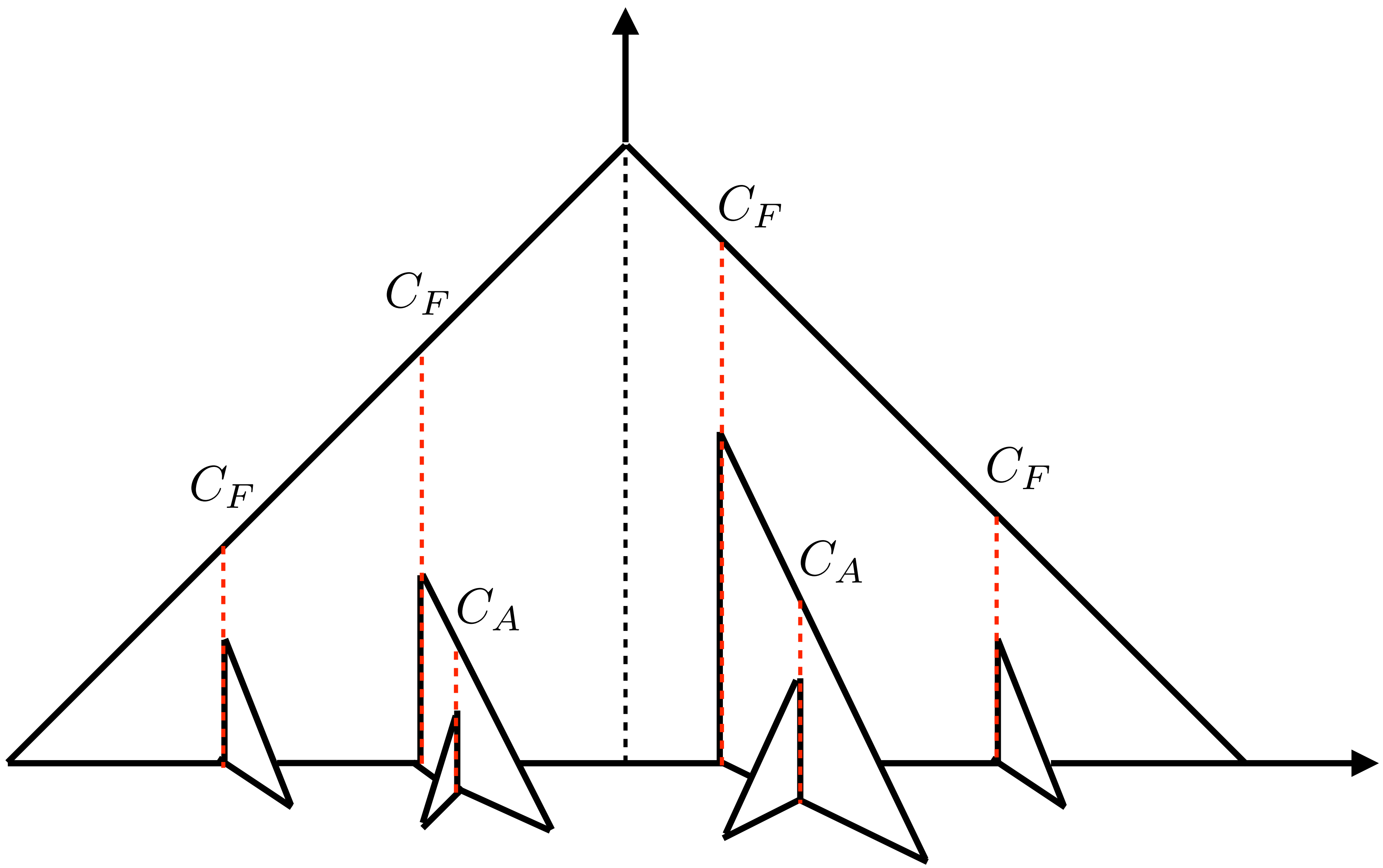


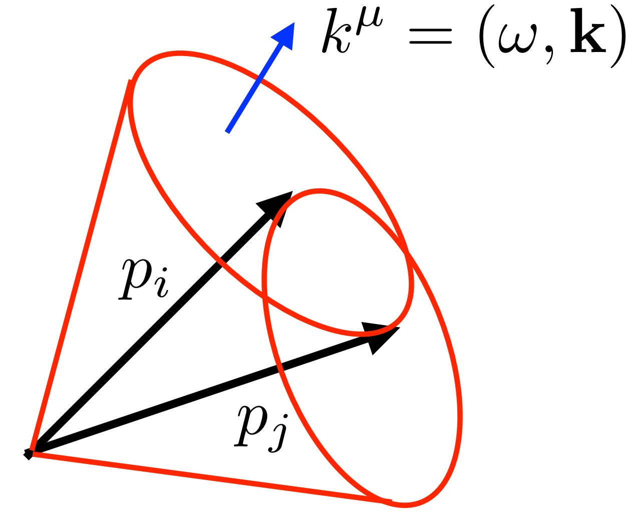
$$P(q\bar{q}ggg) \sim \frac{4}{3} C_F \left(\frac{\alpha_s}{\pi}\right)^3 \left(C_F^2 + \frac{C_F C_A}{2} + \frac{C_A^2}{15}\right) L^6$$

k_t-ordered dipole shower

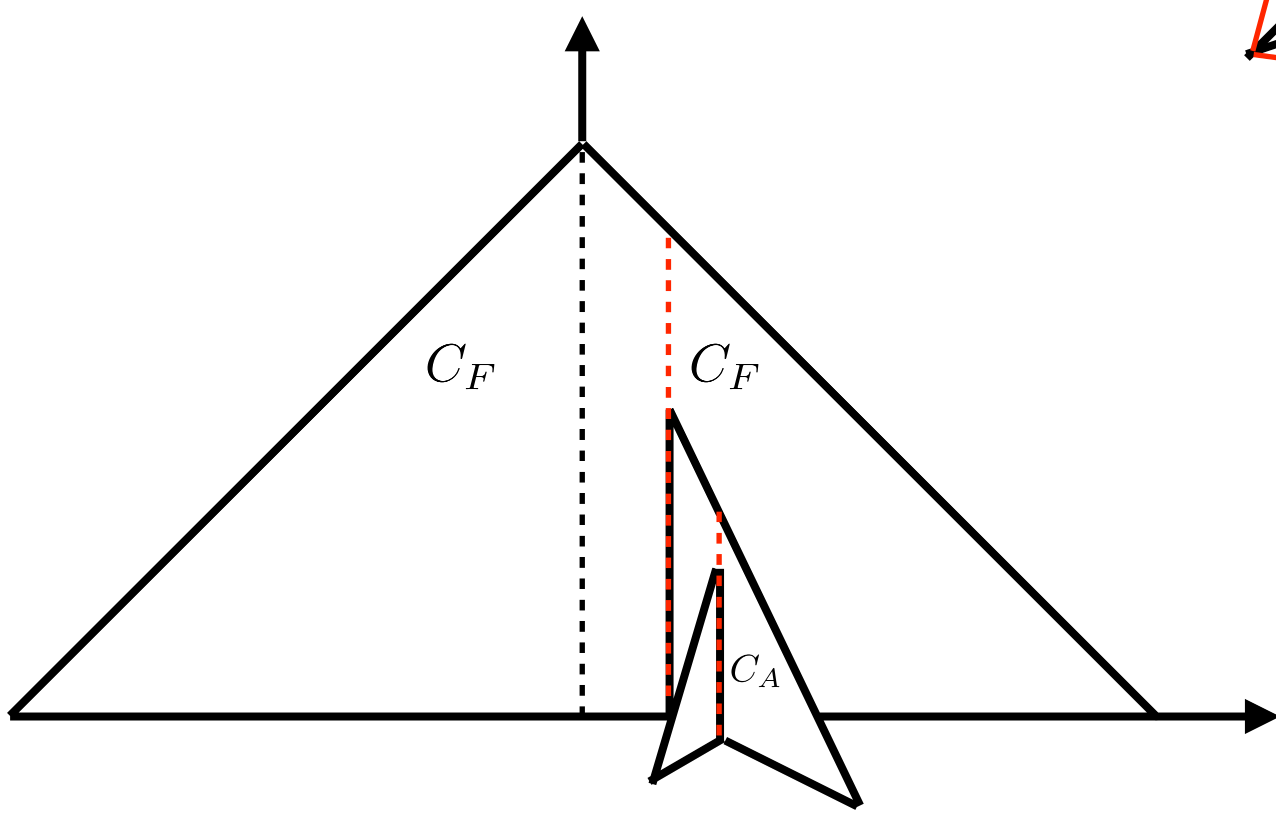


Angular-ordered parton shower



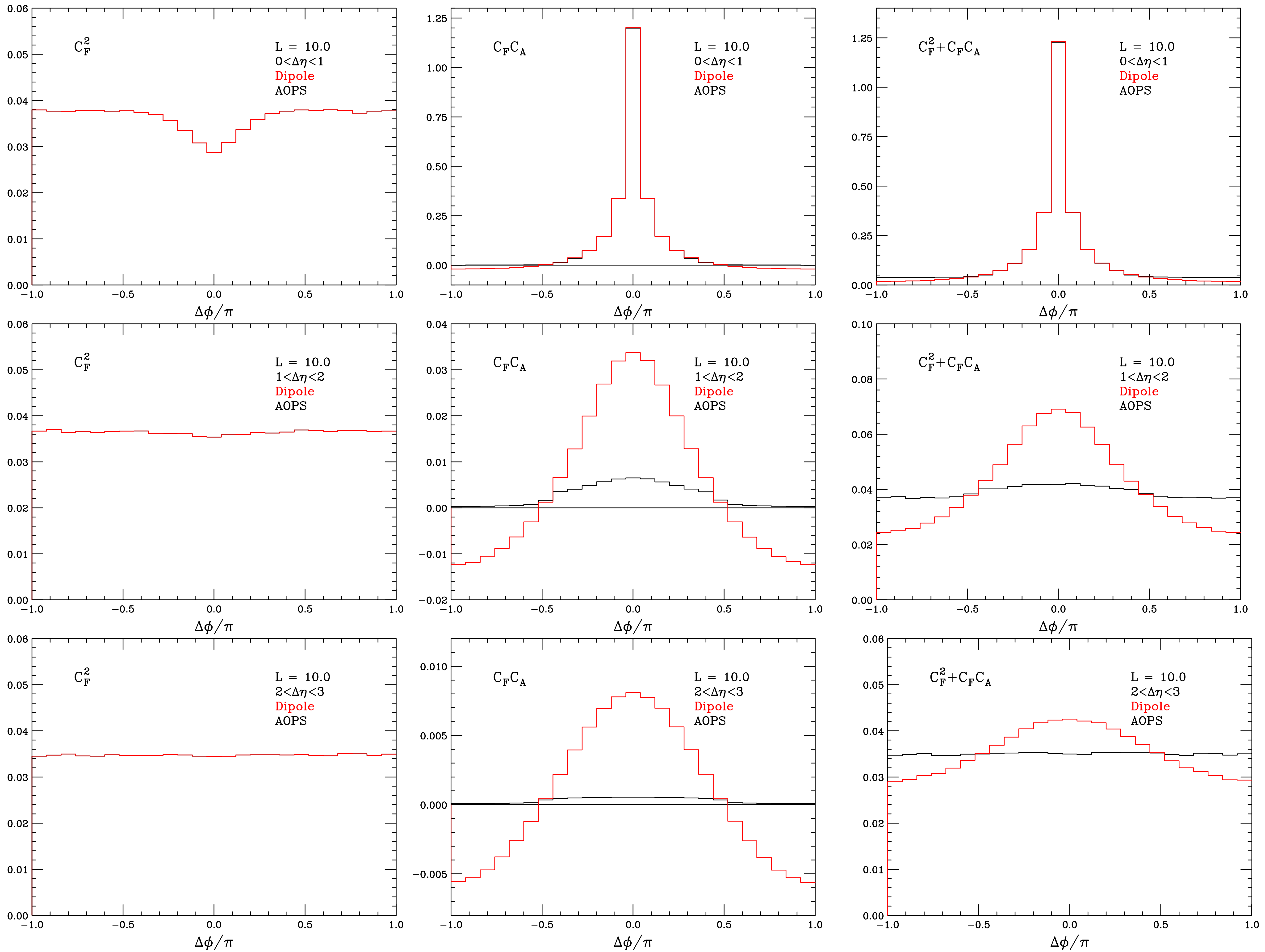


$$\int D_k^{ij} \frac{d\phi_k}{2\pi} = \frac{1}{\omega^2}\left[\frac{\Theta(\theta_{ij} > \theta_{ik})}{1-\cos\theta_{ik}} + \frac{\Theta(\theta_{ij} > \theta_{jk})}{1-\cos\theta_{jk}}\right]$$

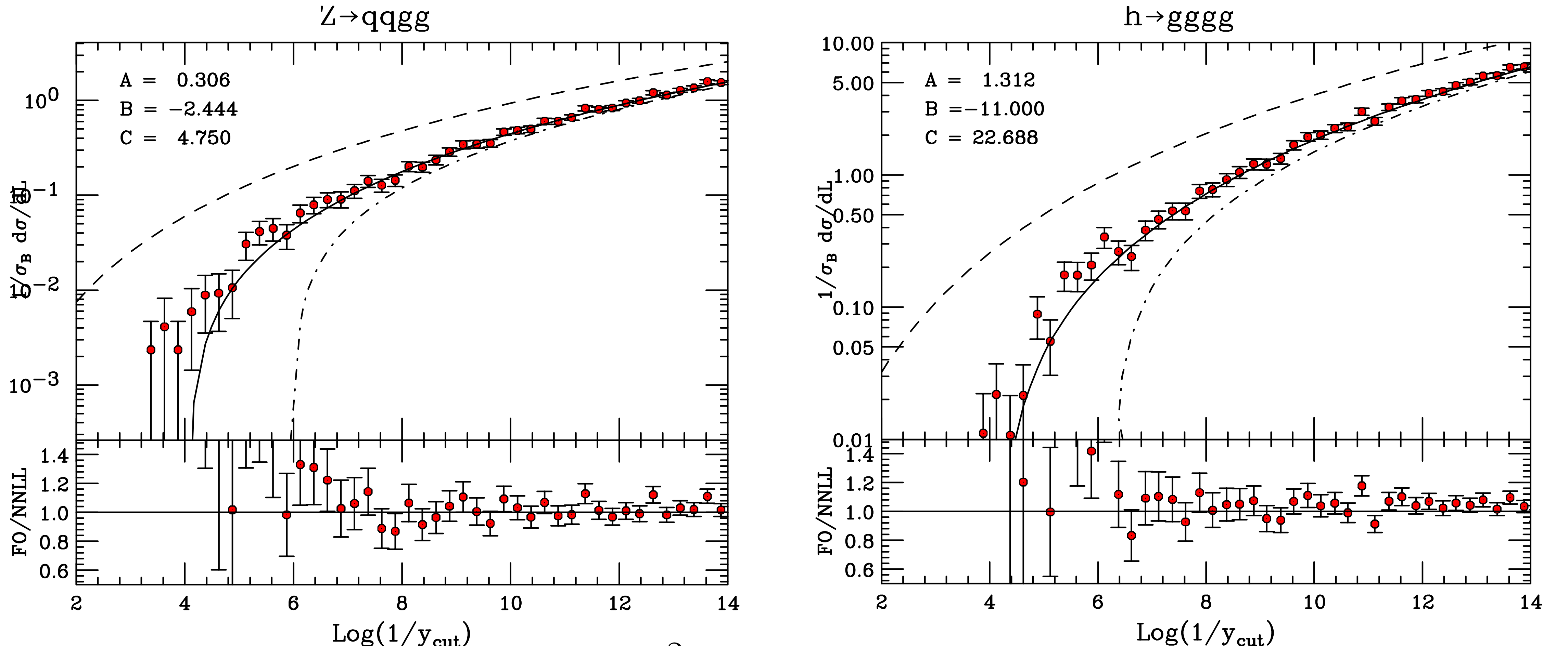


$$dP(q\bar{q}gg) \simeq C_F C_A \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\Theta(\theta_{q\bar{q}} > \theta_{q1})}{1-\cos\theta_{q1}} \frac{\Theta(\theta_{q1} > \theta_{12})}{1-\cos\theta_{12}} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} d\cos\theta_1 \, d\cos\theta_2$$

- Some azimuthal correlations lost through averaging



C_F^2
$C_F C_A$
$C_F^2+C_F C_A$
L = 10.0
0<Δη<1
1<Δη<2
2<Δη<3
Dipole
AOPS
Δφ/π



$$P(q\bar{q}gg) \simeq \left(\frac{\alpha_s}{\pi}\right)^2 \left(AL^4 + BL^3 + CL^2\right)$$

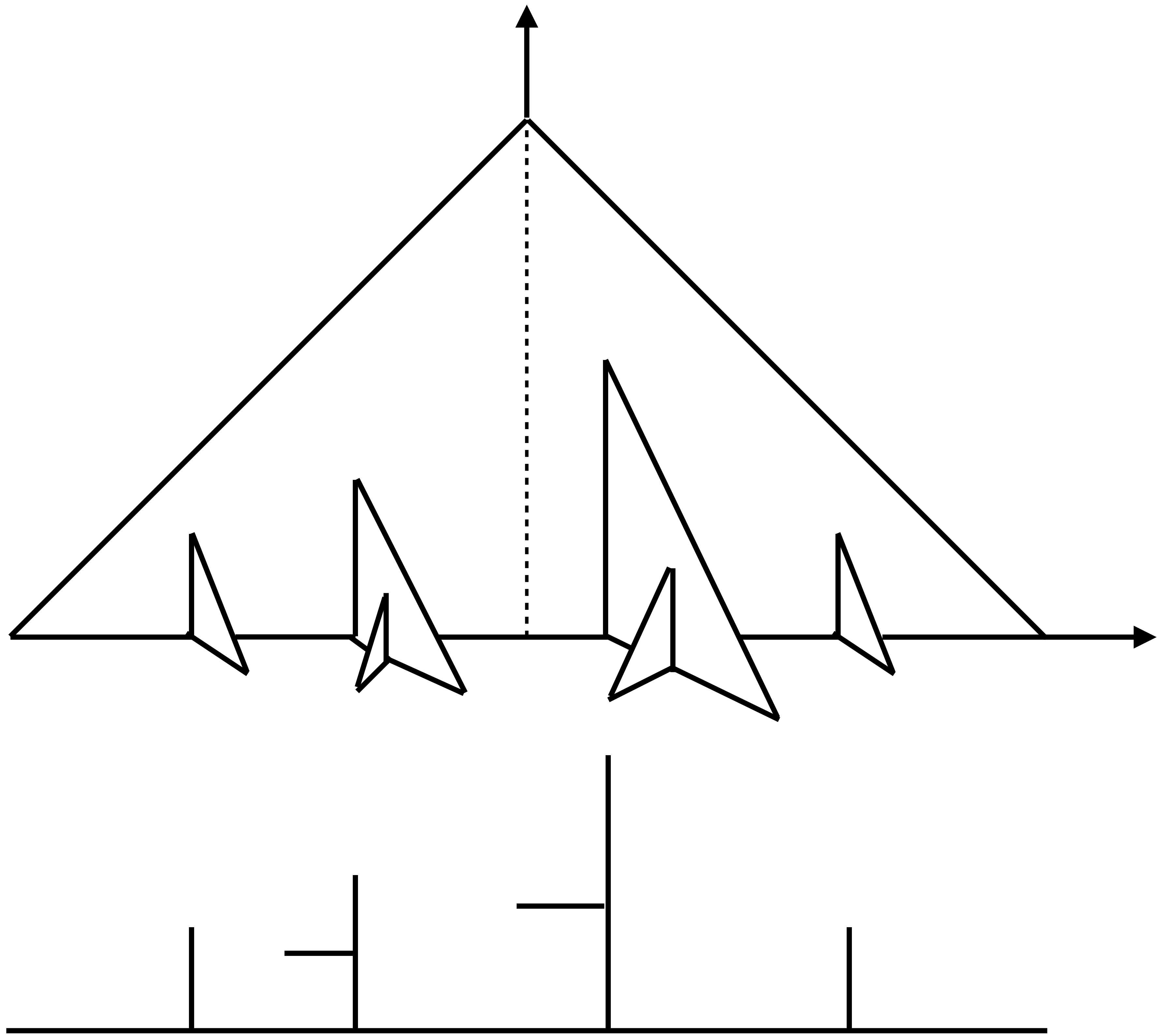
$$\frac{dP}{dL} \simeq \left(\frac{\alpha_s}{\pi}\right)^2 \left(4AL^3 + 3BL^2 + 2CL\right)$$

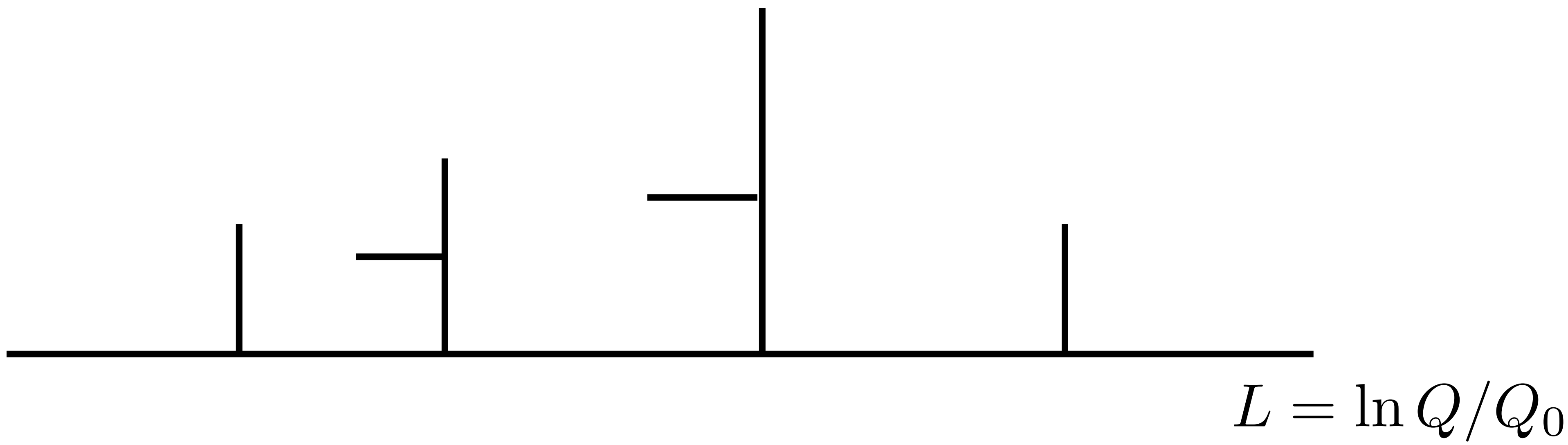
- AOPS vs Exact LOME (Madgraph)
  - A=collinear-soft, B=collinear-nonsoft
  - C not reliable (but improves agreement)

Parton vs Dipole Showers

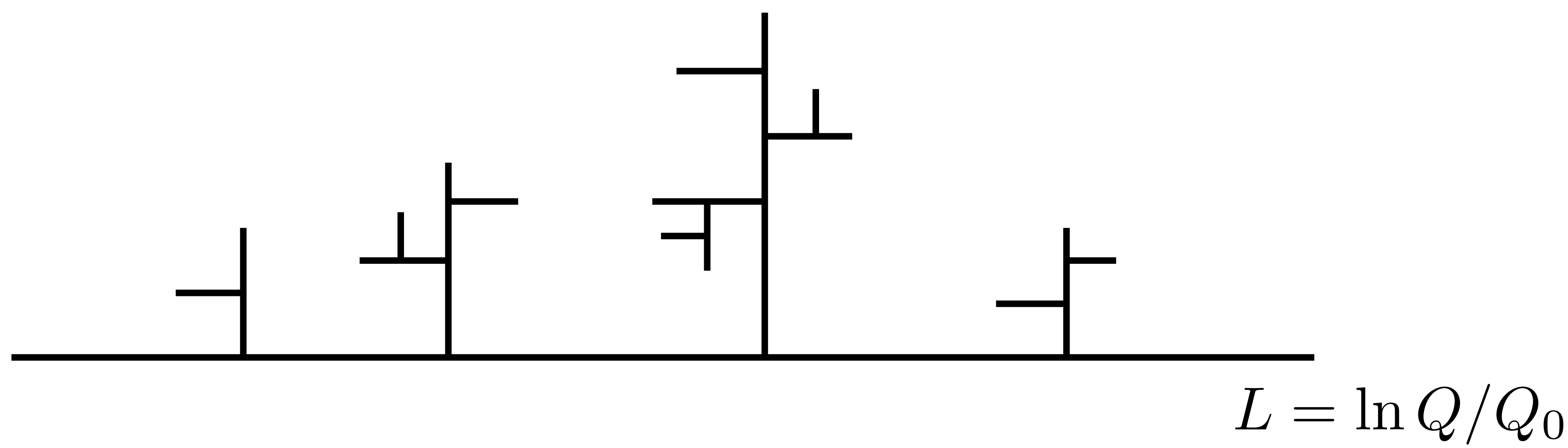
- Parton Shower
  - Simple 1-to-2 splittings: fewer recoil ambiguities
  - Colour structure simple at DL, NDL
  - Soft azimuthal correlations missing
- Dipole shower
  - 2-to-3 splittings mean more recoil ambiguities
  - Colour structure more difficult, even at DL
  - Azimuthal correlations included

Fractal Structure of Showers

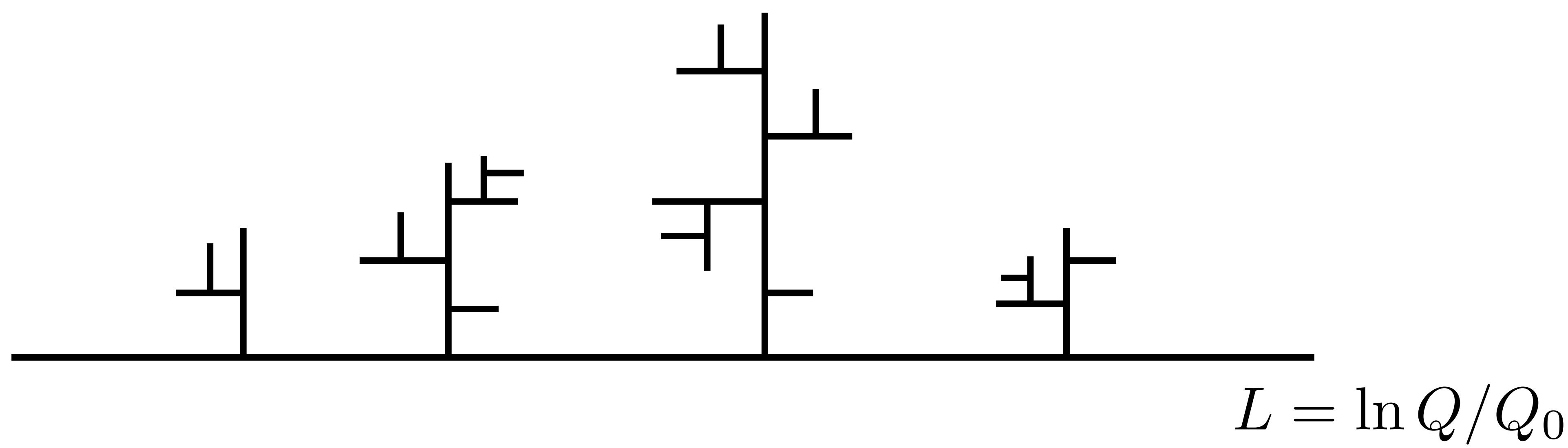



$L = \ln Q/Q_0$

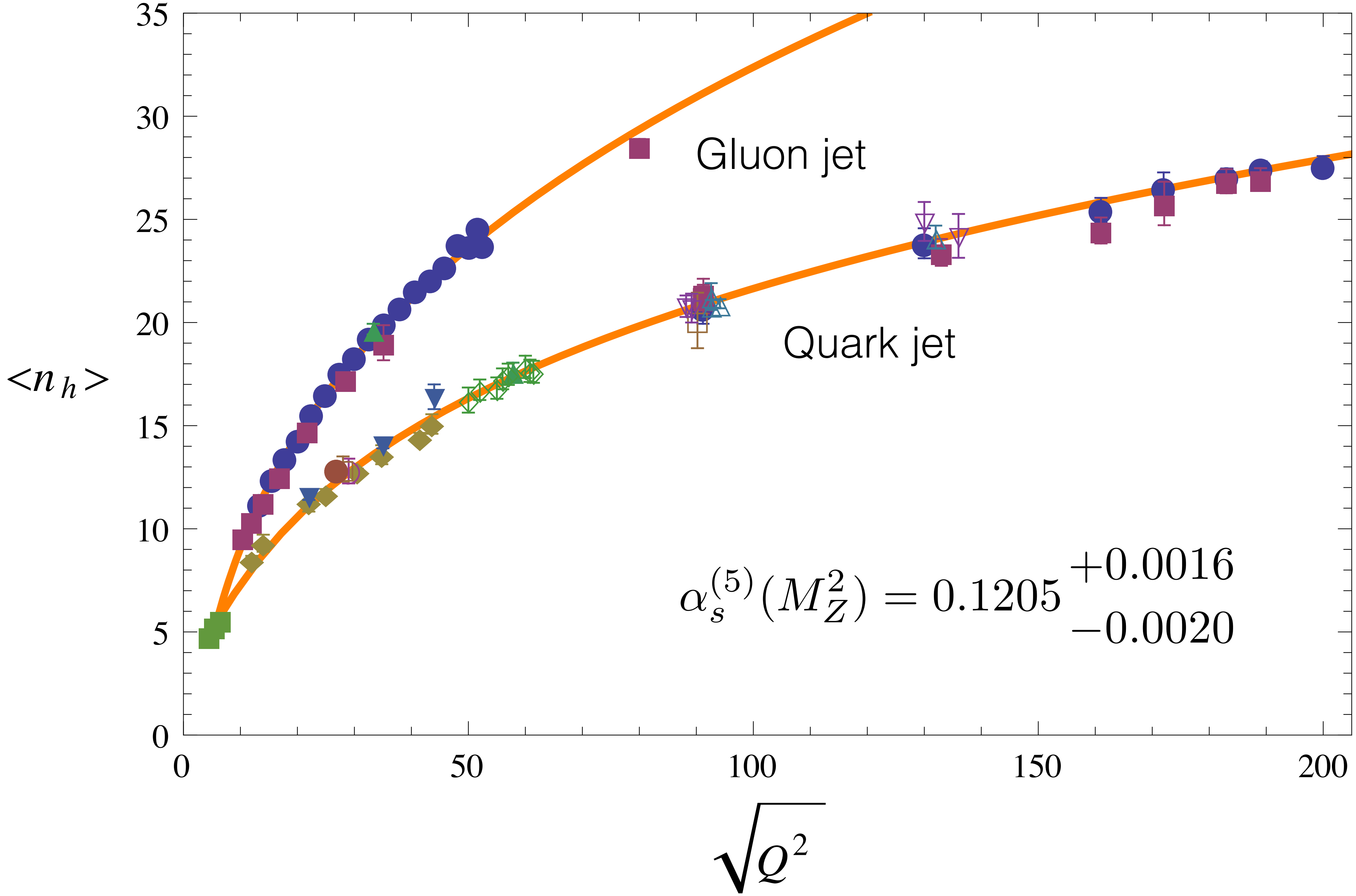
Increasing Q:



Increasing Q:



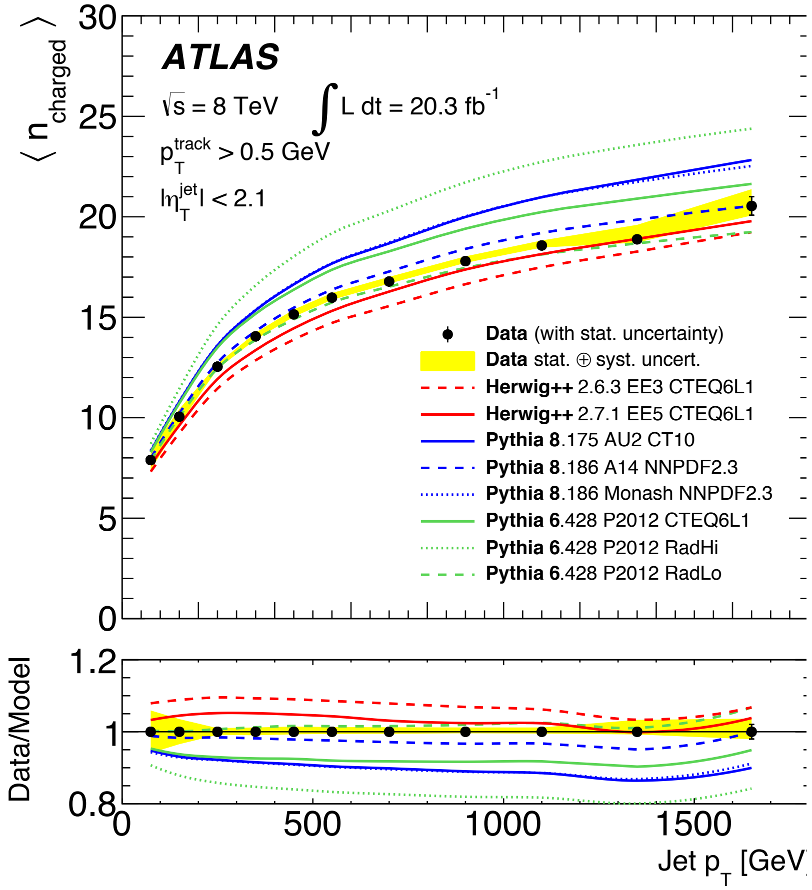
- Fractal curve length $\langle S \rangle \sim \exp\left(c\sqrt{L}\right)$
- Hadron multiplicity $\propto S$

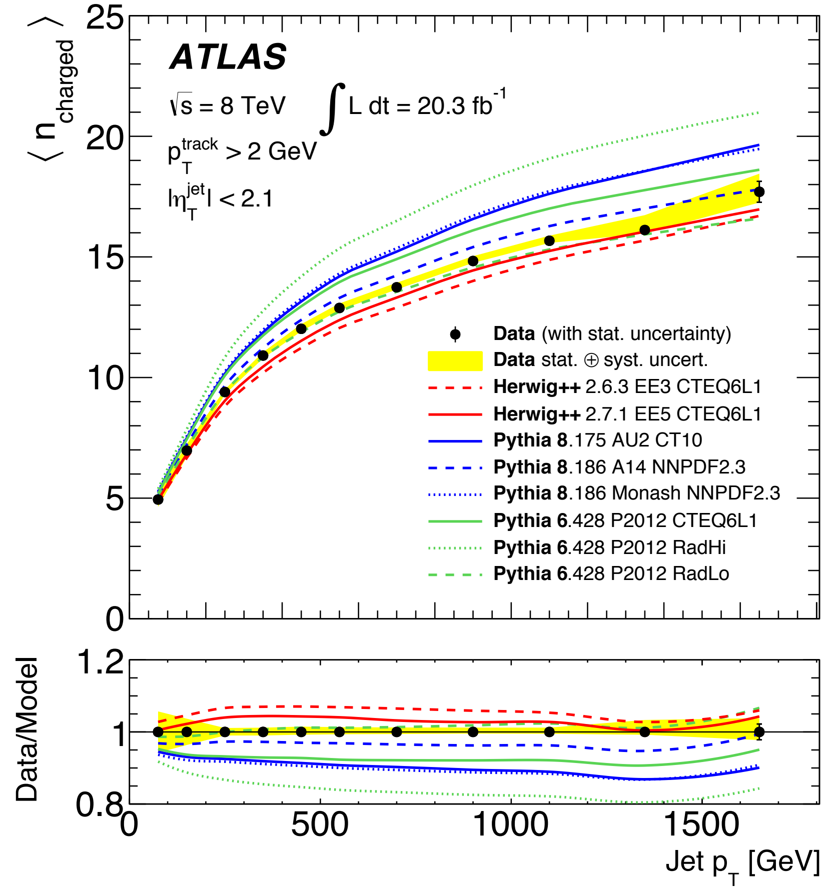


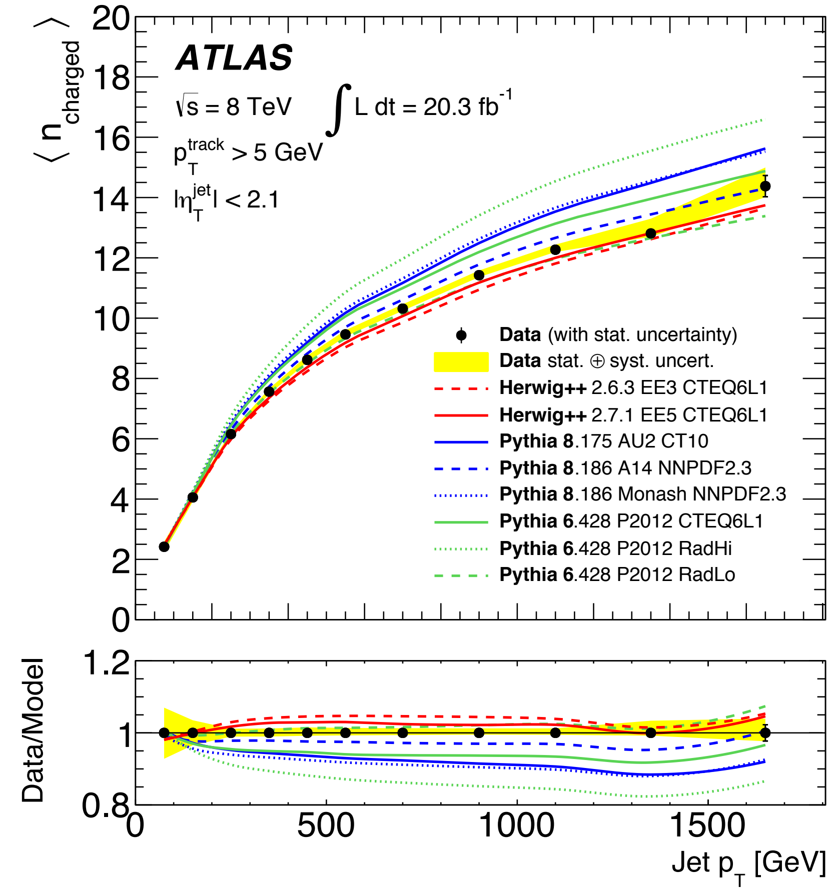
Kniehl & Kotikov, 1702.03193 (NNLL)

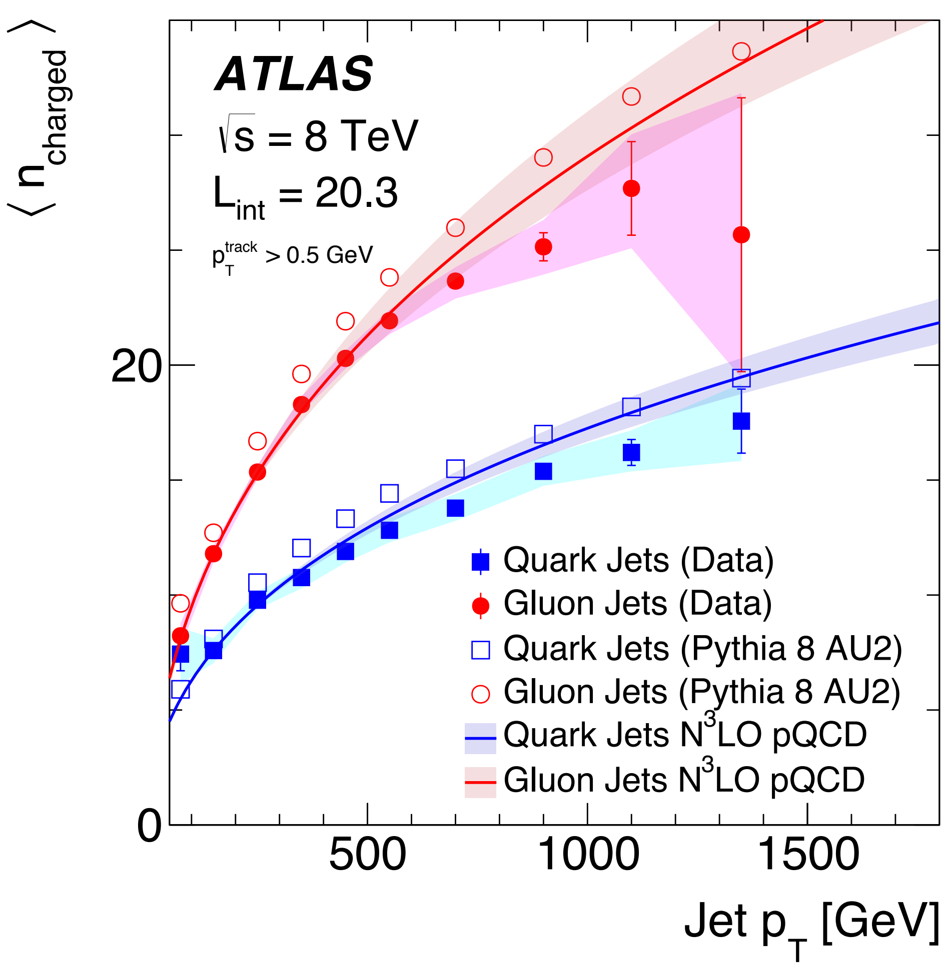
ATLAS, EPJC76(2016)322

Dijets, radius R=0.4



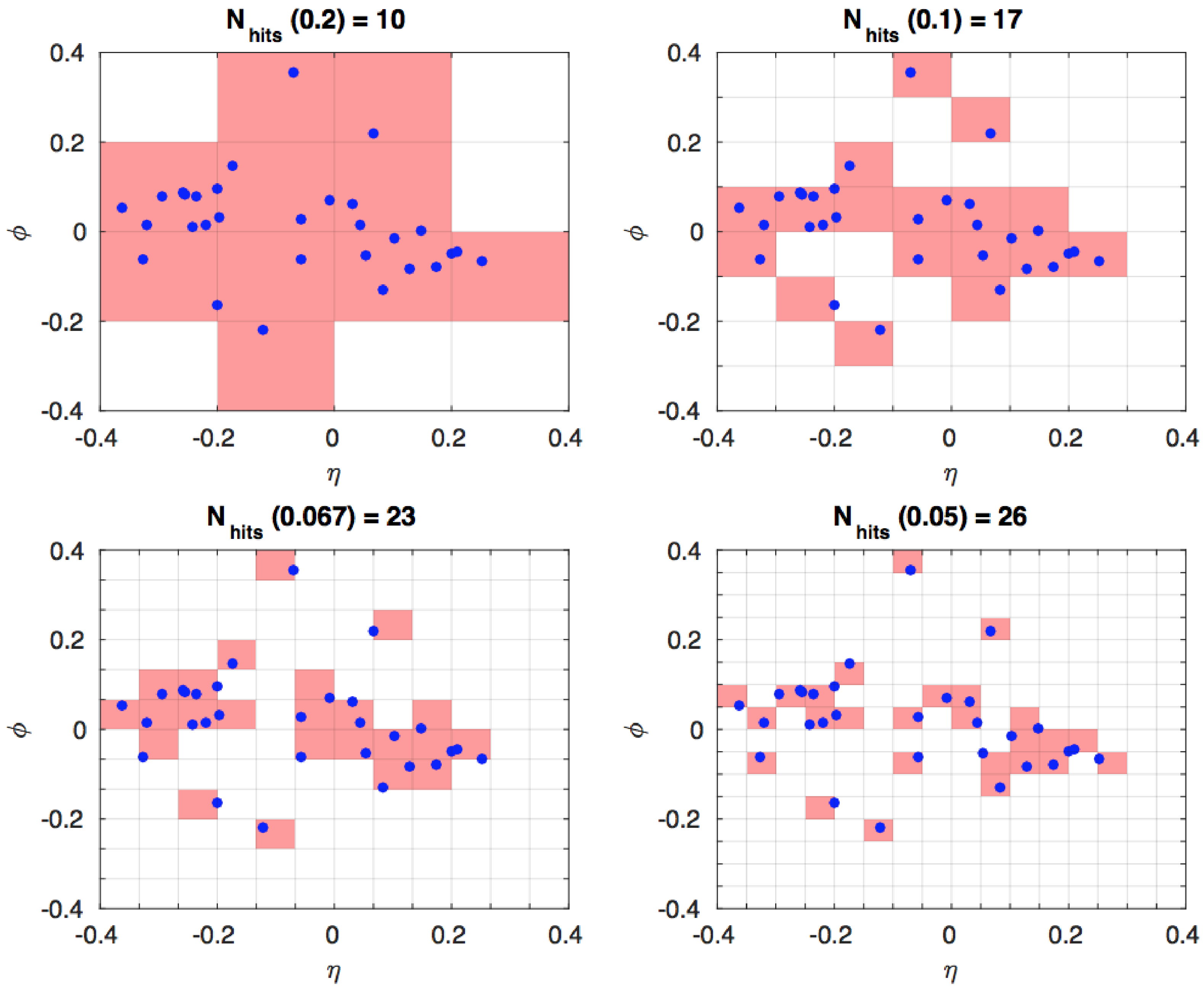




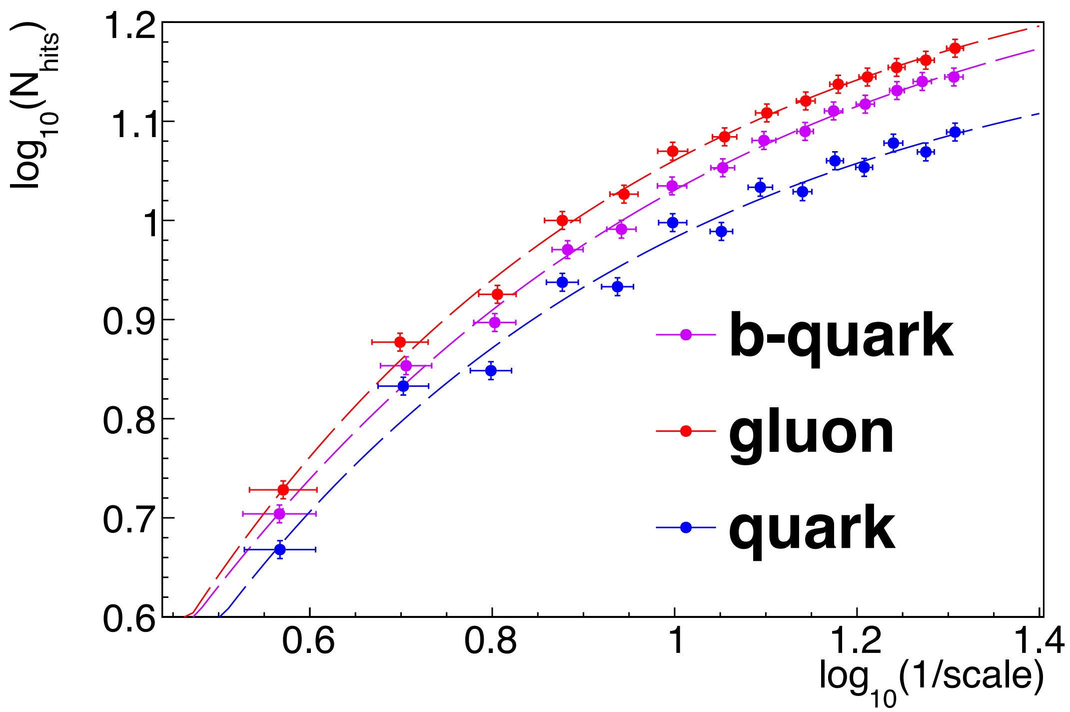


- Quark/gluon fraction from Pythia 8
- $N^3LO = N^3LL_{approx}$
- Scale $Q=Rp_T$
- Normalized at $100<p_T<200$ GeV

Fractal Structure: Box Counting



- Decreasing Q_0



Davighi & Harris,1703.00914

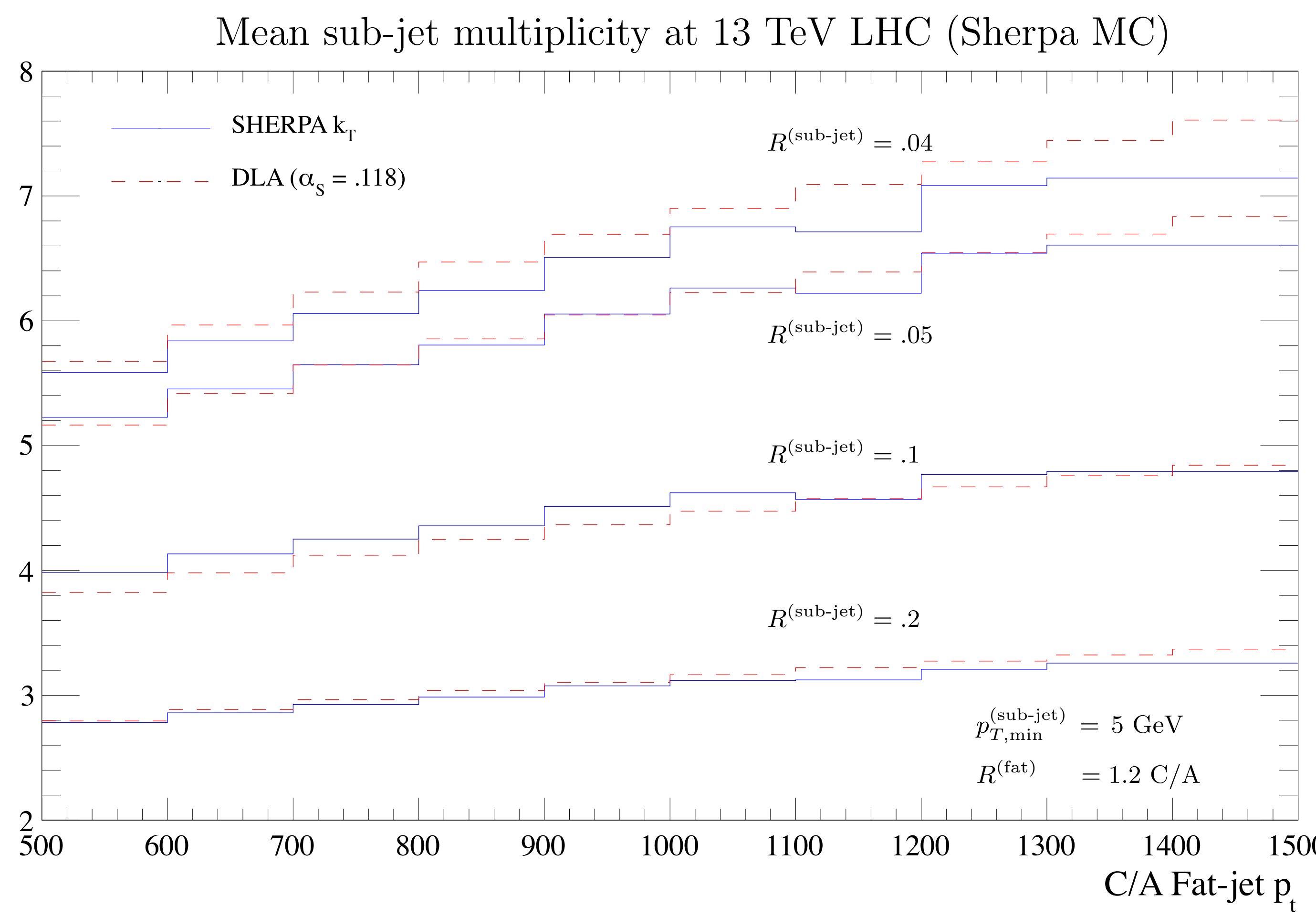
Fractal Structure: Subjets

- Summing leading double logs

$$\langle n_{\rm sub}\rangle_g \sim I_0(\sqrt{z})$$

$$z = 24\frac{\alpha_S}{\pi}\ln\left(\frac{p_{T\rm fat}}{p_{T\rm sub}}\right)\ln\left(\frac{R_{\rm fat}}{R_{\rm sub}}\right)$$

$$\langle n_{\rm sub}\rangle_q \sim \frac{5}{9}+\frac{4}{9}I_0(\sqrt{z})$$

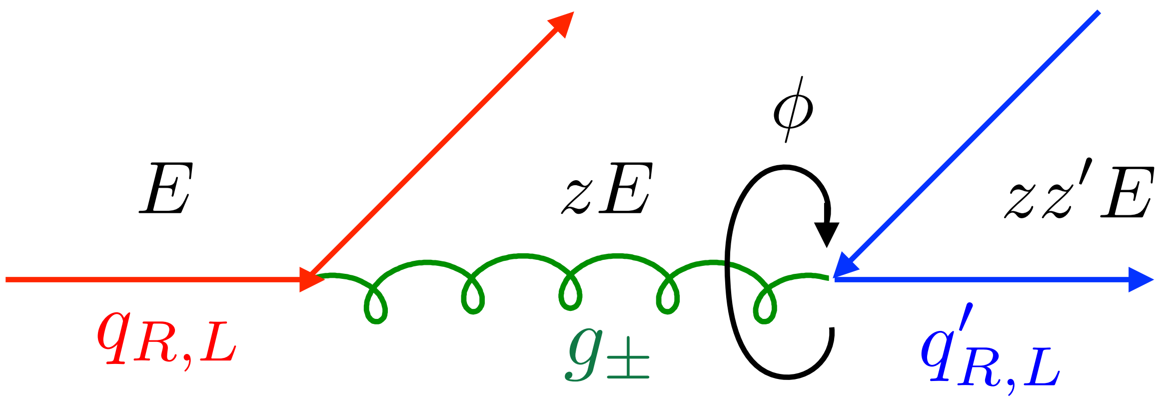


- Agrees quite well with SHERPA MC

Gerwick, Gripaios, Schumann, BW,1212.5235

Spin Effects in Showers

Azimuthal Correlations



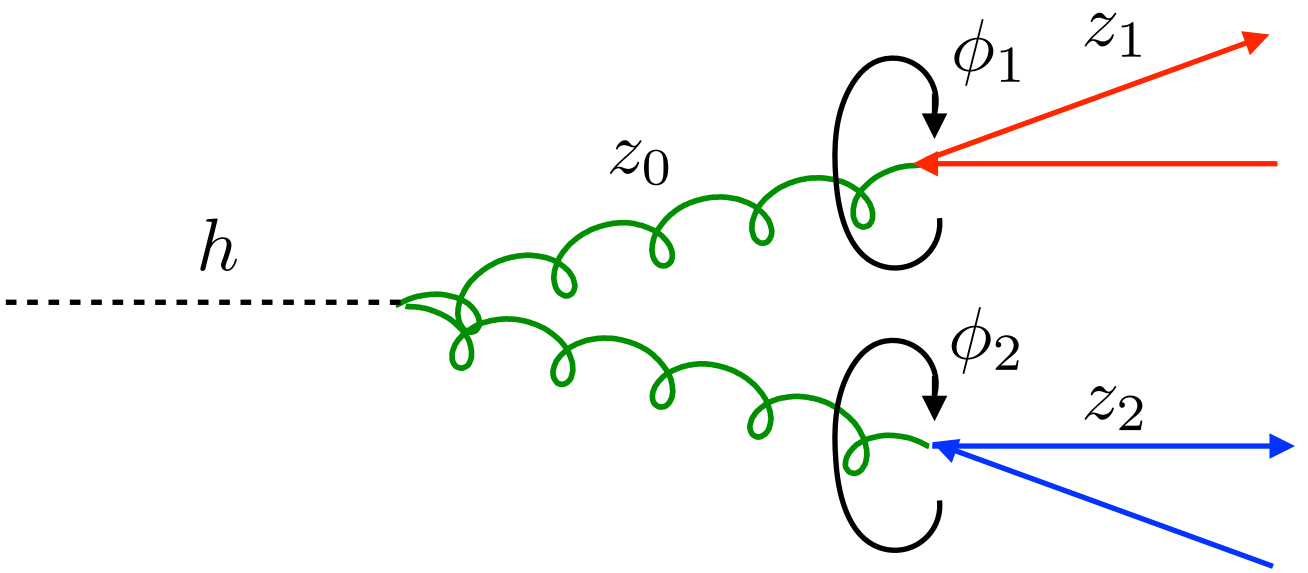
h_1	h_2	h_3	$\mathcal{M}_{h_1 h_2 h_3}$
R	+	R	$z^{-\frac{1}{2}} \mathrm{e}^{i\phi} z'$
R	+	L	$z^{-\frac{1}{2}} \mathrm{e}^{i\phi} (1 - z')$
R	-	R	$-(1-z) z^{-\frac{1}{2}} \mathrm{e}^{-i\phi} (1 - z')$
R	-	L	$-(1-z) z^{-\frac{1}{2}} \mathrm{e}^{-i\phi} z'$

$$|\mathcal{M}_{R+R} + \mathcal{M}_{R-R}|^2 = \frac{1}{z}\big[z'^2 + (1-z)^2(1-z')^2 - 2(1-z)z'(1-z')\cos 2\phi\big]$$

$$|\mathcal{M}_{R+L} + \mathcal{M}_{R-L}|^2 = \frac{1}{z}\big[(1-z')^2 + (1-z)^2 z'^2 - 2(1-z)z'(1-z')\cos 2\phi\big]$$

$$\sum_{h_3} \Big| \sum_{h_2} \mathcal{M}_{h_1 h_2 h_3} \Big|^2 = \frac{1+(1-z)^2}{z}[z'^2 + (1-z')^2] - 4\frac{(1-z)}{z} z'(1-z')\cos 2\phi$$

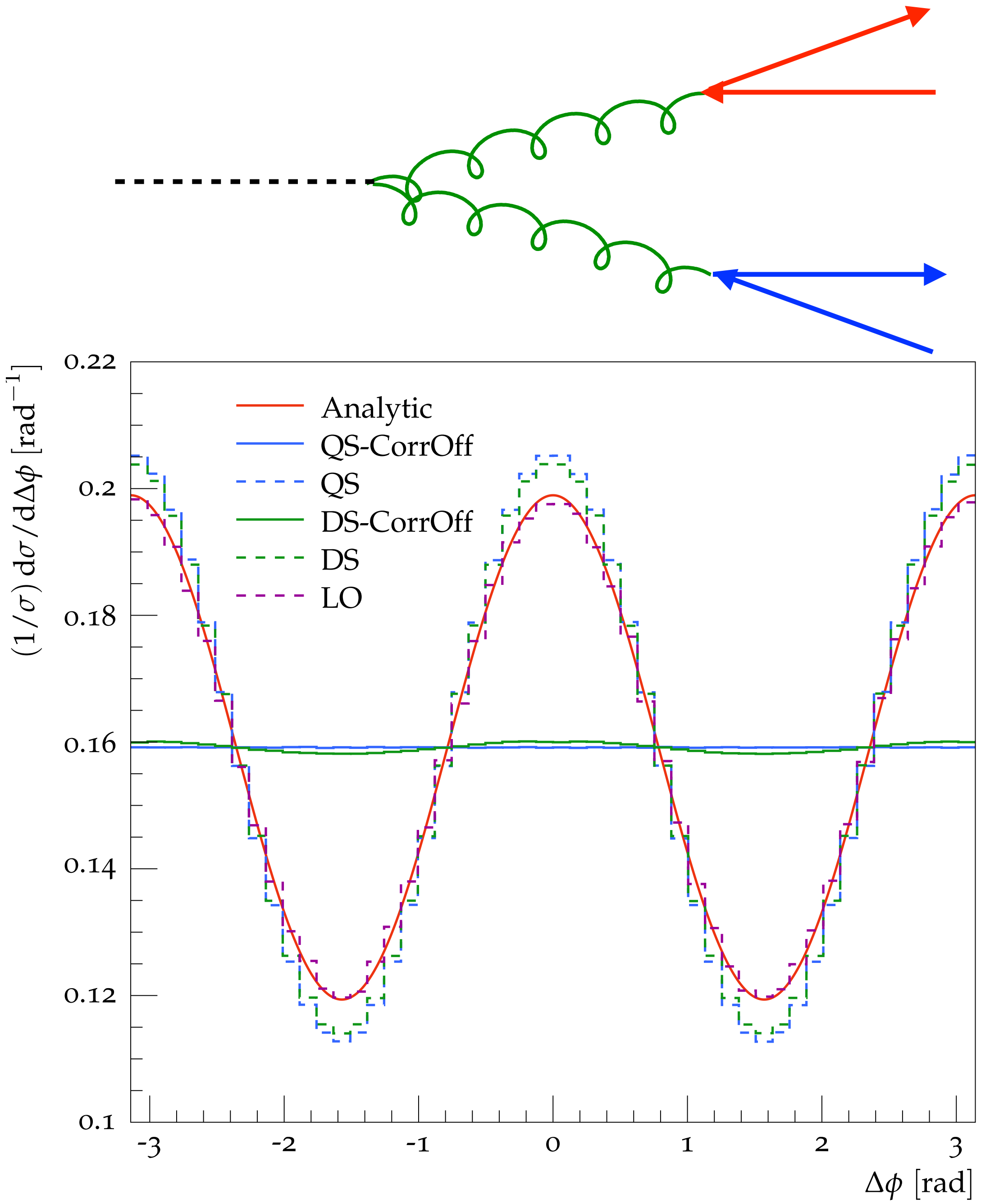
EPR Correlations

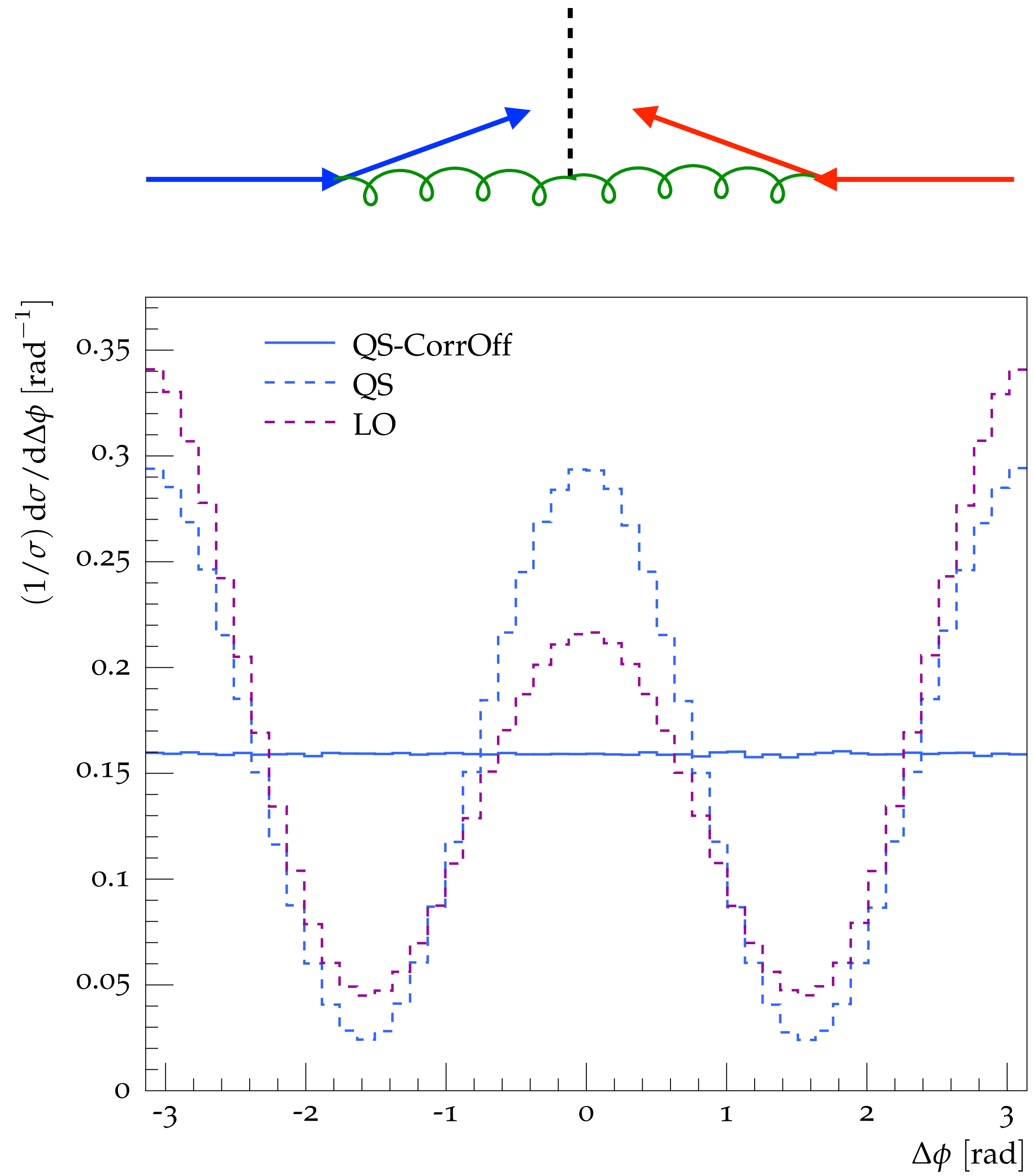


$$P(h \to q\bar{q}q\bar{q}) \propto 1 + a(z_1)a(z_2)\cos 2(\phi_1 - \phi_2)$$

- where $a(z) = \dfrac{2z(1-z)}{1-2z(1-z)}$
- Fully included in Herwig (CKR method)

Collins, NPB304(1988)794
Knowles,CPC58(1990)271
Richardson, JHEP111(2001)029

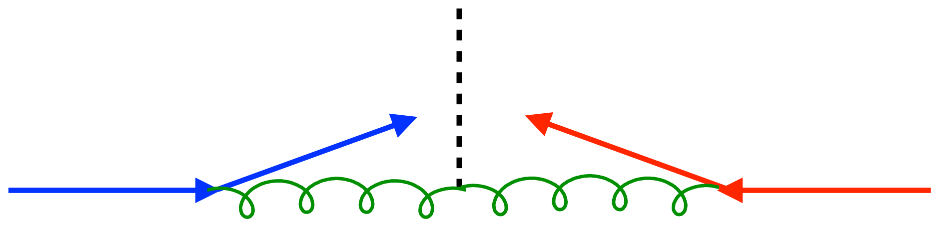


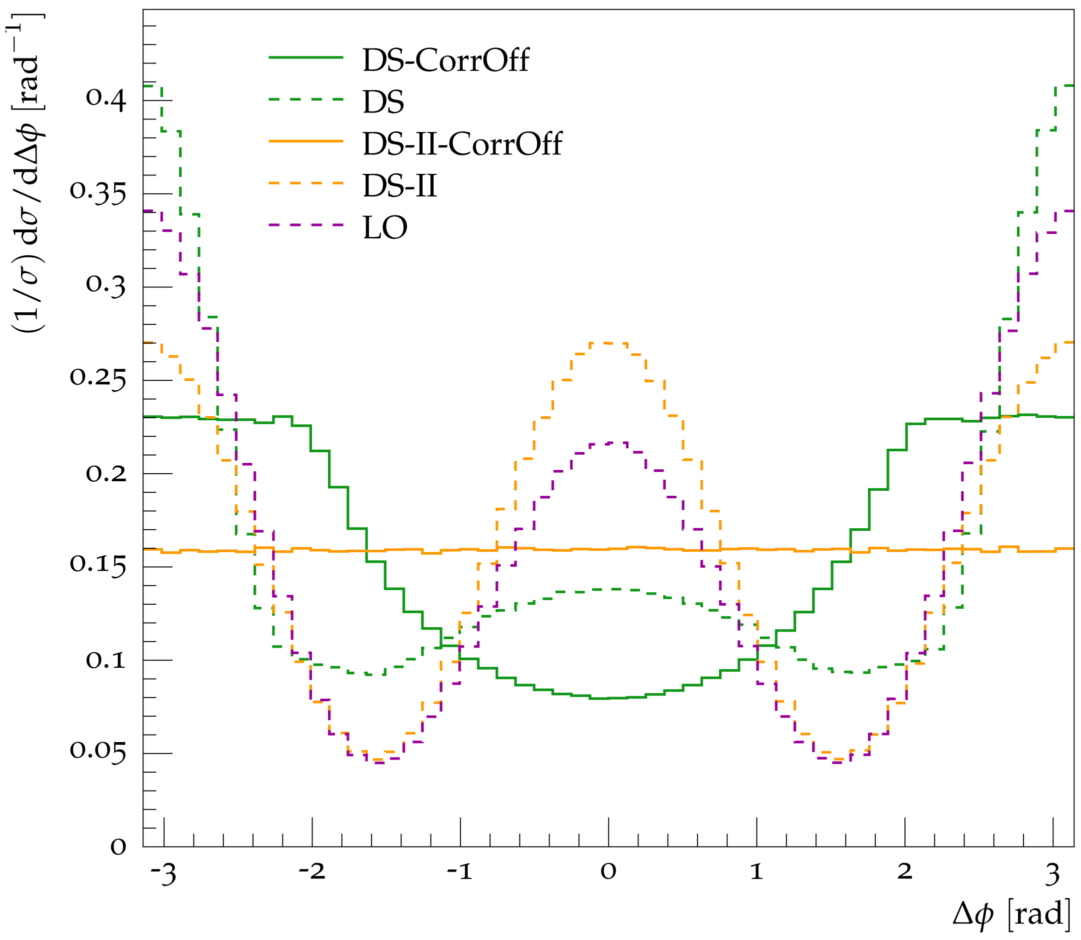


- LO=MadGraph5, QS=Herwig7AO, DS=Herwig7DS

Richardson & Webster, 1807.01955

- Different dipole options illustrate recoil ambiguity

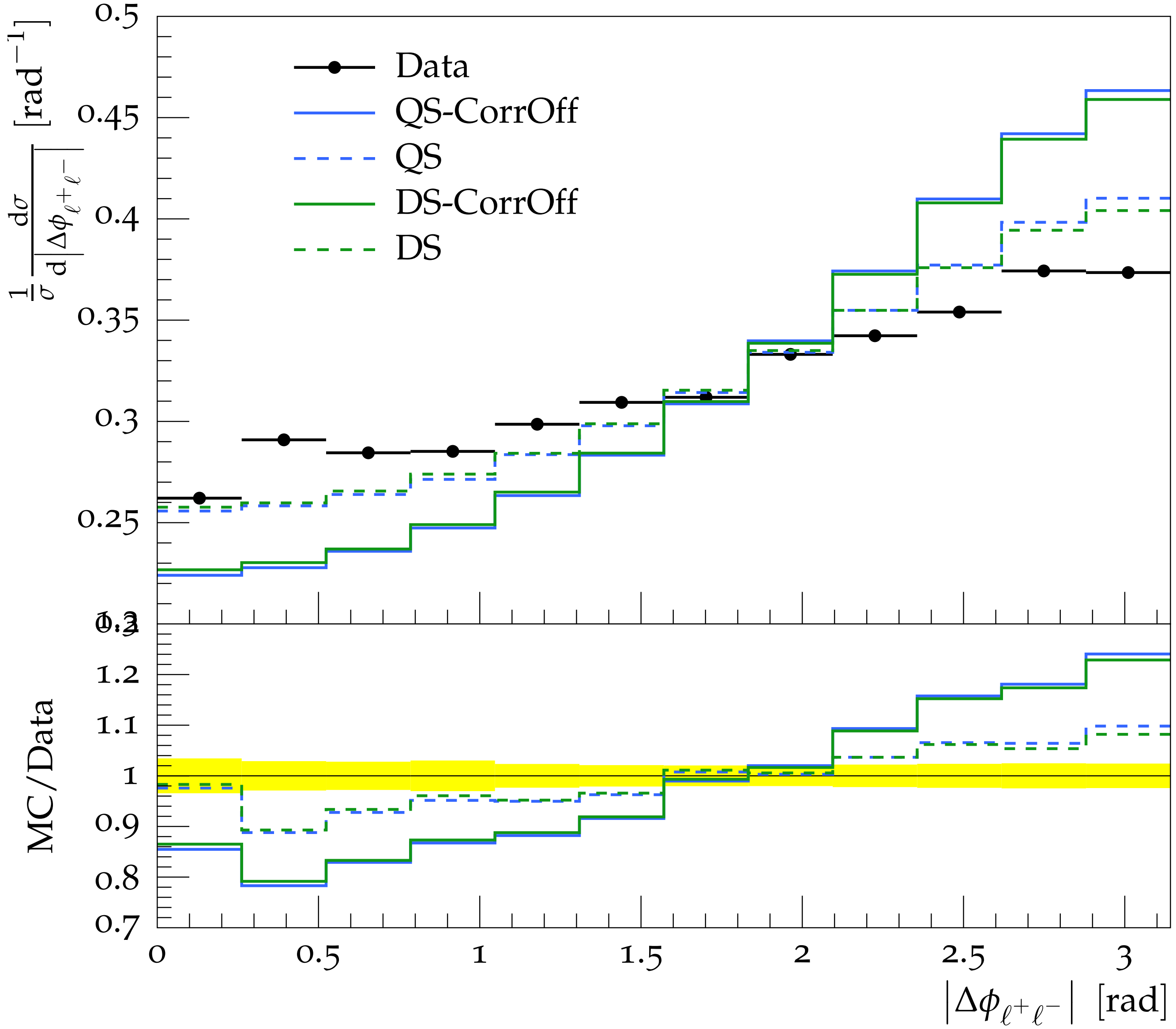




Richardson & Webster, 1807.01955

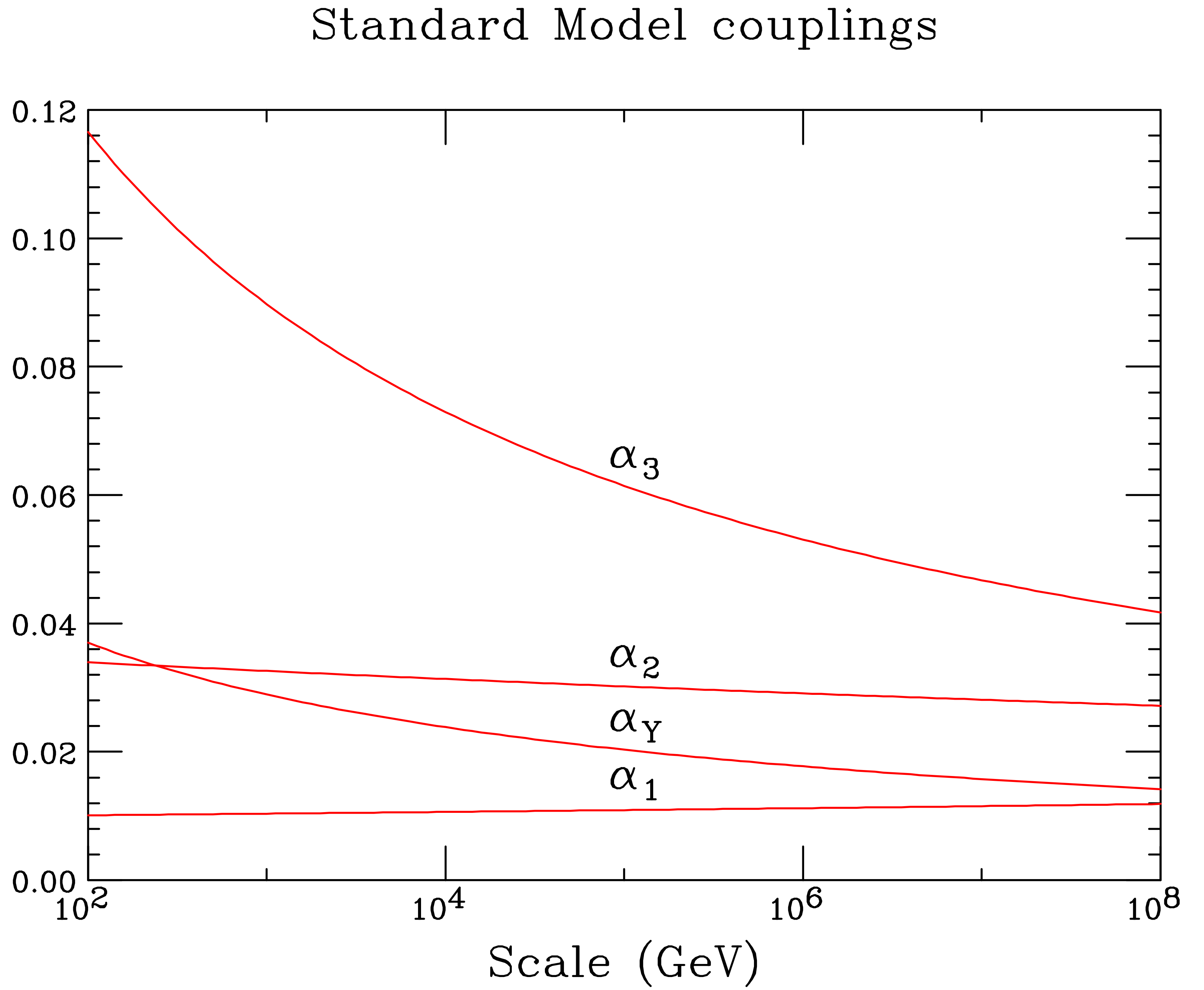
- Dilepton correlation in top decays

$$pp \to t\bar{t} \to b\bar{b}\,\ell^+\ell^-\nu_\ell\bar{\nu}_\ell$$



Richardson & Webster, 1807.01955

Electroweak Showers



- Far above EW scale, at $q \gg m_W$, we have approximately unbroken SU(3)xSU(2)xU(1)

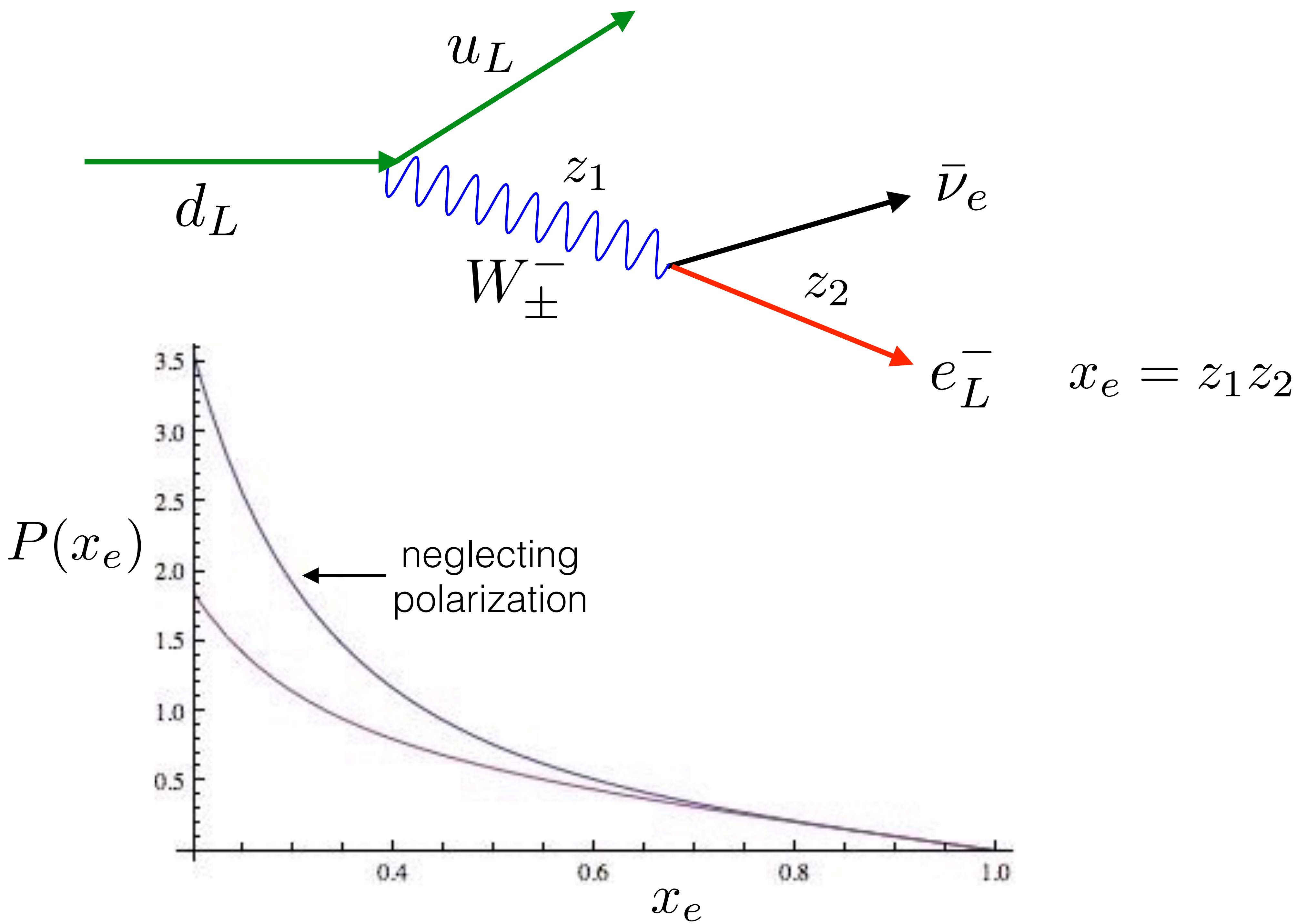
- Corrections ~ m_W/q

Polarized Splitting Functions

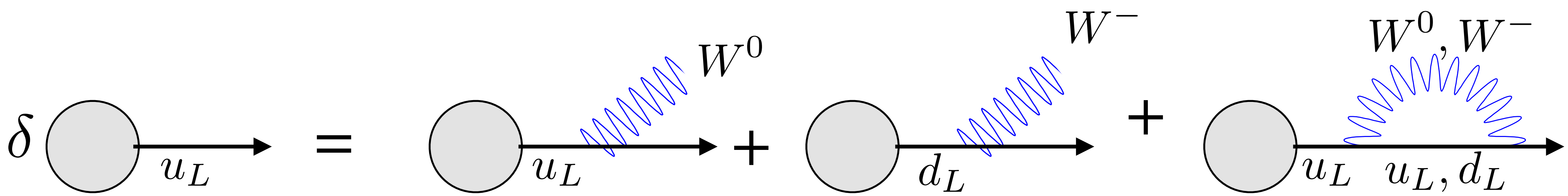
- For any gauge interaction G=SU(3), SU(2), U(1) (neglecting azimuthal correlations)

$$
\begin{aligned}
P^R_{f_L f_L,G}(z) &= P^R_{f_R f_R,G}(z) = \frac{2}{1-z} - (1+z)\,, \\
P^R_{V_+ f_L,G}(z) &= P^R_{V_- f_R,G}(z) = \frac{(1-z)^2}{z}\,, \\
P^R_{V_- f_L,G}(z) &= P^R_{V_+ f_R,G}(z) = \frac{1}{z}\,, \\
P^R_{f_L V_+,G}(z) &= P^R_{f_R V_-,G}(z) = \frac{1}{2}(1-z)^2\,, \\
P^R_{f_L V_-,G}(z) &= P^R_{f_R V_+,G}(z) = \frac{1}{2}z^2\,, \\
P^R_{V_+ V_+,G}(z) &= P^R_{V_- V_-,G}(z) = \frac{2}{1-z} + \frac{1}{z} - 1 - z(1+z)\,, \\
P^R_{V_+ V_-,G}(z) &= P^R_{V_- V_+,G}(z) = \frac{(1-z)^3}{z}\,, \\
P^R_{HH,G}(z) &= \frac{2}{1-z} - 2\,,
\end{aligned}
$$

- Parity violation implies large polarization effects



- Real-virtual emission mismatch leads to double logarithms of q/m_W



$$q\frac{\partial}{\partial q}u_L(x,q) = \frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q}\frac{dz}{z}P_{ff}(z)\left[\frac{1}{3}u_L(x/z,q)+\frac{2}{3}d_L(x/z,q)-zu_L(x,q)\right]$$

$$q\frac{\partial}{\partial q}d_L(x,q) = \frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q}\frac{dz}{z}P_{ff}(z)\left[\frac{1}{3}d_L(x/z,q)+\frac{2}{3}u_L(x/z,q)-zd_L(x,q)\right]$$

- Define $Q^{\pm} = \frac{1}{2}(u_L \pm d_L)$

$$q\frac{\partial}{\partial q}Q^+(x,q) = \frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q}\frac{dz}{z}P_{ff}(z)\left[Q^+(x/z,q)-zQ^+(x,q)\right]$$

$$q\frac{\partial}{\partial q}Q^-(x,q) = -\frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q}\frac{dz}{z}P_{ff}(z)\left[\frac{1}{3}Q^-(x/z,q)+zQ^-(x,q)\right]$$

  - Q^+ has DGLAP (single-log) evolution
  - Q^- has double-log damping (asymptotic symmetry)

$$q\frac{\partial}{\partial q}Q^-(x,q) = -\frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q}\frac{dz}{z}P_{ff}(z)\left[\frac{1}{3}Q^-(x/z,q) + zQ^-(x,q)\right]$$

- Define $F(q) = \int_0^1 dx\, x\, Q^-(x,q) = \int_0^1 dx\, x\, [u_L(x,q) - d_L(x,q)]$
  - Then $q\frac{dF}{dq} = -\frac{\alpha_2}{\pi}C_F\int_0^{1-m_W/q} dz\, P_{ff}(z)\frac{4}{3}F(q)$

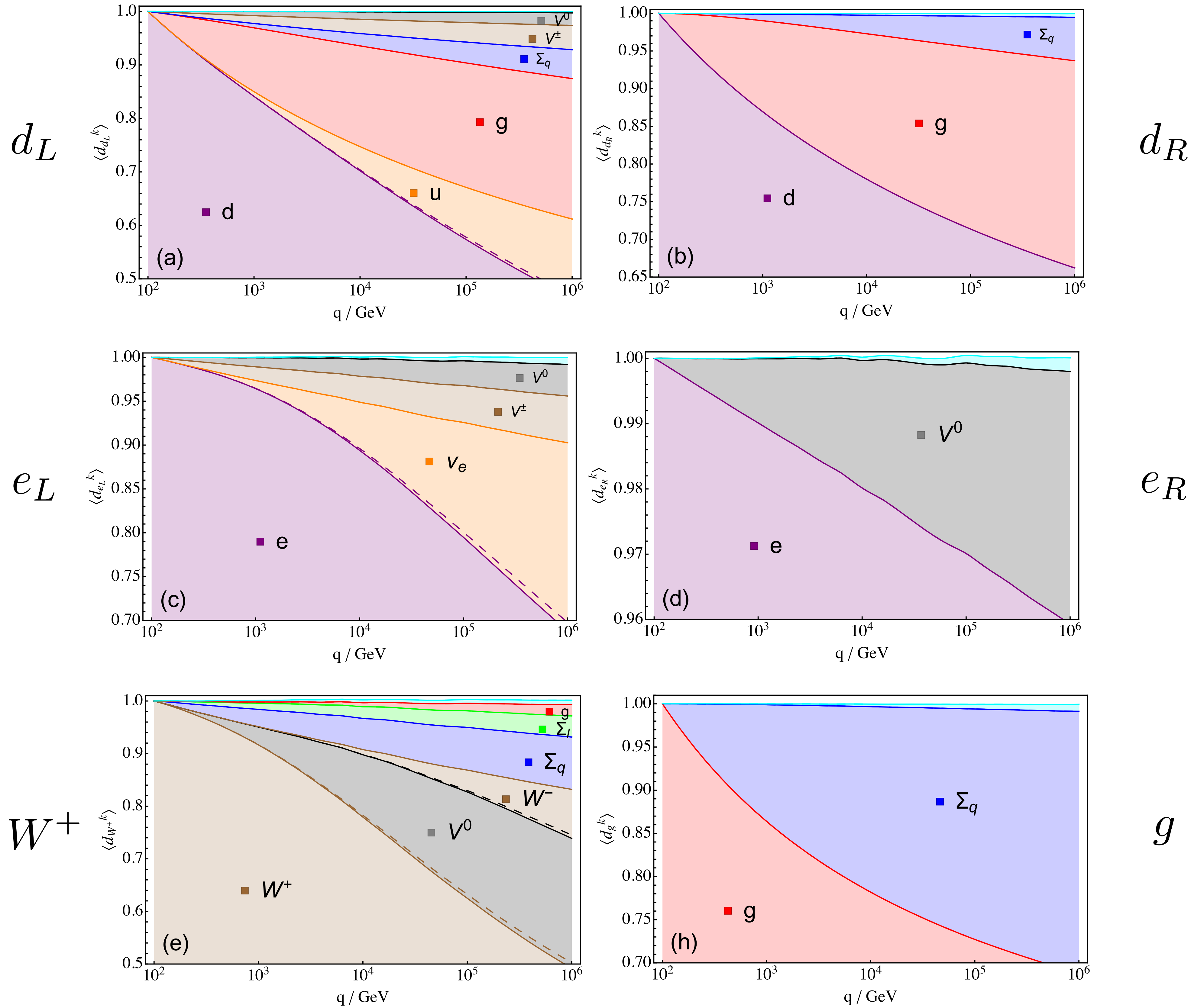
    where $C_F\int_0^{1-m_W/q} dz\, P_{ff}(z) \sim \frac{3}{2}\ln\left(\frac{q}{m_W}\right)$ $[C_F = 3/4 \text{ for SU}(2)]$

- Hence

$$F(q) \sim F(m_W)\exp\left[-\frac{\alpha_2}{\pi}\ln^2\left(\frac{q}{m_W}\right)\right]$$

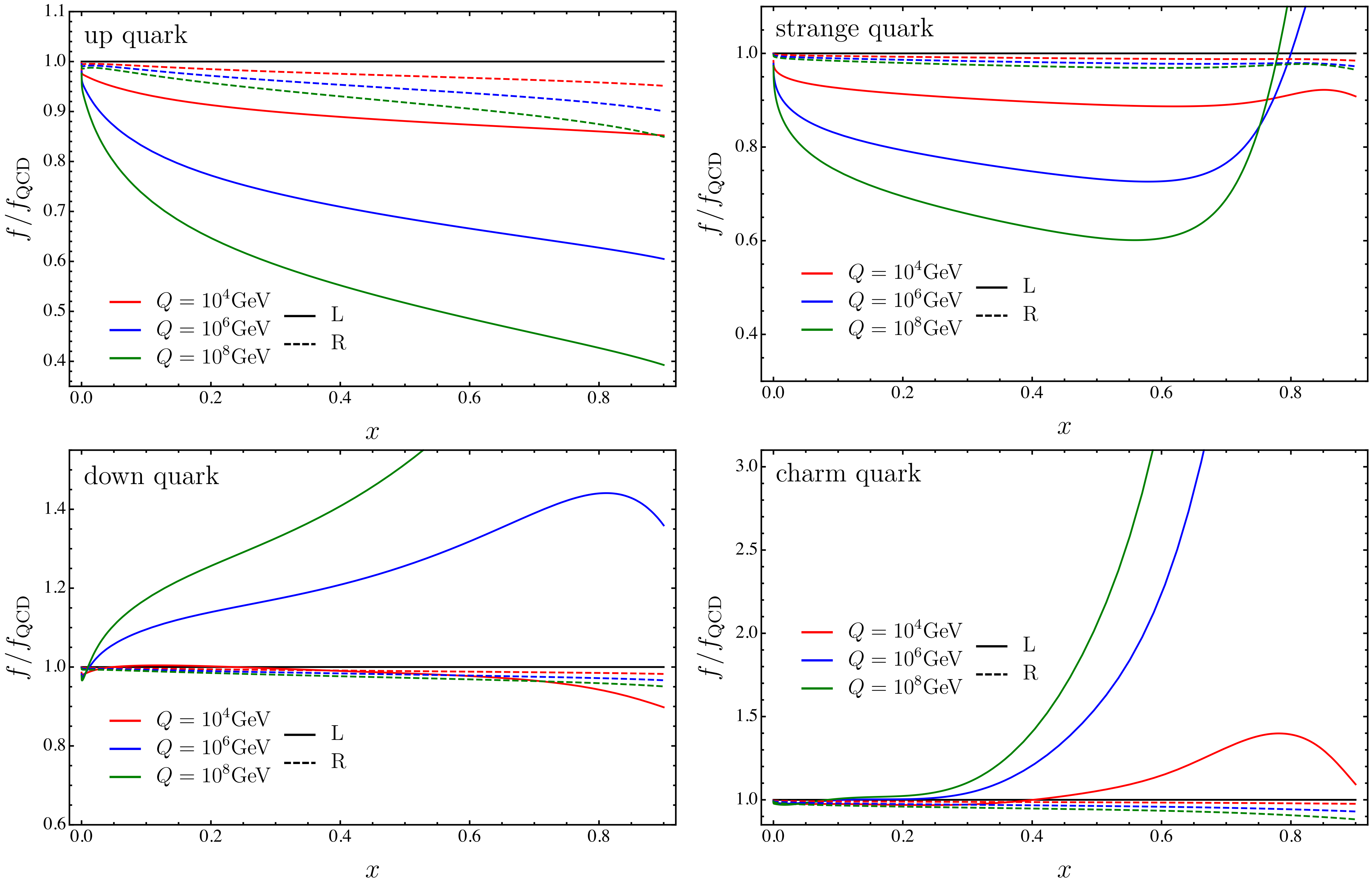
- For LLA resummation: $\alpha_2 \to \alpha_2(q(1-z))$

Momentum fractions in jets



Bauer, Provasoli, BW, 1808.08831

- Similarly in initial-state showering (PDF evolution)
  - u_L-d_L (& s_L-c_L) has double-log damping



Bauer, Ferland, BW, 1703.08562

Summary

- Dipole and parton showers have complementary features
- Azimuthal correlations can be important
  - CKR method for spin correlations
  - PS vs DS studies probe soft correlations (& recoils)
- Electroweak showering introduces novel features
  - Self-polarization, double log evolution