Jet Pull:
Resummation for the Pull Magnitude and Beyond

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Andrew Larkoski‡

In Preparation:
Theory Predictions for Pull
[arXiv:18xx.xxxx]
To appear soon... hopefully!

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Outline

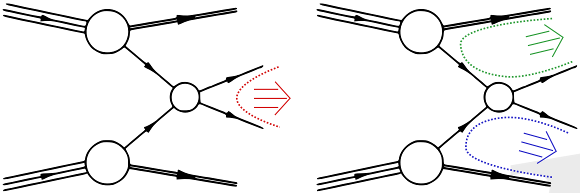
1. *Introduction and Motivation*
   - colour flow
   - definition of pull
2. *Fixed Order Calculation*
   - set up for the calculation
   - leading order result
   - check with EVENT2
3. *NLL Resummation for the Magnitude of Pull*
   - resummation formalism
   - fixed-order vs resummation
   - improvement for the fixed-order results
4. *Beyond the Magnitude*

Precision era of Higgs Physics

- With the high accuracy data from the LHC Run-2, entering a precision era
- The main higgs decay channel: $H \to b\bar{b}$, branching ratio about 58%
- Recently CMS and ATLAS have just reported the first direct measurements of Higgs decay to bottom quarks
- Need to seperate the signal ($H \to b\bar{b}$) from background ($g \to b\bar{b}$)

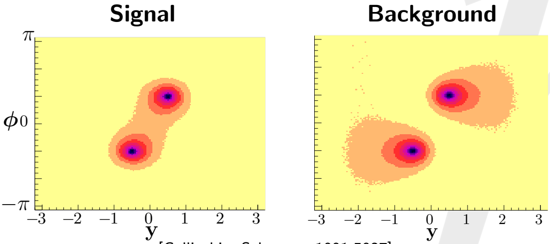
Colour Flow

- difference between colour singlet (signal) and colour octet (background)



[Gallicchio, Schwartz, 1001.5027]

- From MC shower picture: (signal) radiation pulled toward each other



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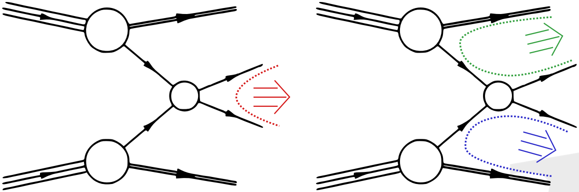

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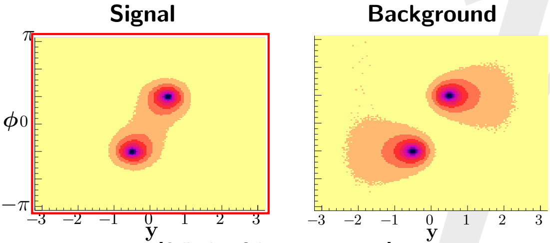

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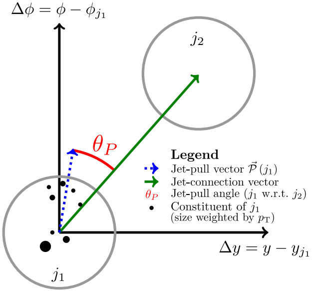
Definition of Pull

- Diagram for the construction of the jet pull:



[ATLAS, 1506.05629]

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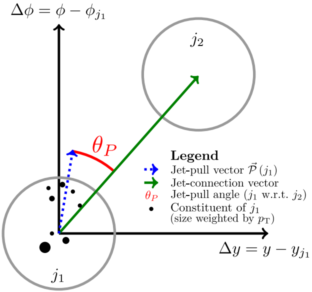

[ATLAS, 1506.05629]

- Definition of pull: jet constituents with transverse momentum p_T^i and location $\vec{r_i}$

$$\vec{t} = \sum_{i \in jet} \frac{p_T^i |r_i|}{p_T^{jet}} \vec{r_i}$$

Properties of Pull

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- Properties of pull:
  - IRC safety: soft or collinear emissions don't change the hard jets

$$O(p_1...p_i, p_j...p_n) \rightarrow O(p_1...(p_i + p_j)...p_n), \ \ if \ \ (p_i + p_j)^2 = 0$$

  - Additive observable: separate the contributions from soft and collinear emissions
  - Pull angle: IRC unsafe

$$\phi_p = cos^{-1} \frac{t_x}{t}$$

Modification for Pull

- Modified version of the pull vector in $e_+ \; e_-$

$$\vec{t}_{modified} = \sum_{i \in jet} \frac{E_i sin^2\theta_i}{E_J} \left(cos\phi_i, sin\phi_i \right)$$

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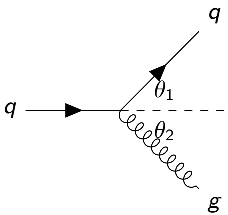
- Example: LO (2 parton), collinear splitting



$$z \text{ particle: } \vec{t}_z = z \cdot (1-z)^2 \theta^2 (\cos\phi, \sin\phi)$$

$$1-z \text{ particle: } \vec{t}_{1-z} = (1-z) \cdot z^2 \theta^2 (-\cos\phi, -\sin\phi)$$

$$\vec{t} = z(1-z)(1-2z)\theta^2 (\cos\phi, \sin\phi)$$

Compare Pull Magnitude with Jet Mass

- Magnitude of pull:

$$t = z(1-z)(1-2z)\theta^2 \leqslant 0.1R^2$$

- Definition of jet mass:

$$\rho = \frac{m_J^2}{E_J^2} \sim z(1-z)\theta^2 \leqslant \frac{R^2}{4}$$

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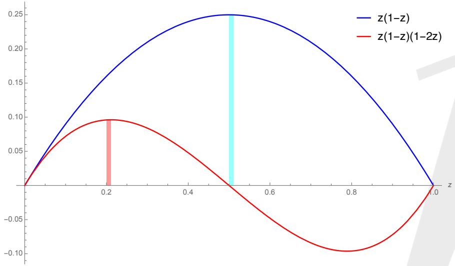
- End point:



EVENT2: $\rho_{max} = 0.431, \ t_{max} = 0.204$

Set Up For the Calculation

- Set up:

$$\frac{d^2\sigma}{d\vec{t}} = S\left(t, \phi_p, \mu\right) + J\left(t, \phi_p, \mu\right)$$

soft function:
eikonal factor

$$\mathcal{J}(q) = C_F \frac{2k_1 k_2}{(qk_1)(qk_2)}$$

jet function:
collinear splitting functions

$$P_{qq} = C_F \frac{1+(1-z)^2}{z}$$

LO Result

- Double differential distribution

$$\frac{d^2\sigma}{dt\,d\phi_p} = \frac{\alpha_s}{\pi^2}\frac{C_F}{t}[\log\frac{4\tan^2\frac{R}{2}}{t} - \frac{3}{4}$$

$$+ 2\cot\phi_p\ \tan^{-1}\frac{\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\sin\phi_p}{1-\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\phi_p} - \log(1+\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}} - 2\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\phi_p)]$$

- Magnitude of pull

$$\frac{d\sigma}{dt} = \frac{\alpha_s}{\pi}\frac{C_F}{t}\left[\log\frac{1}{t} - \frac{3}{4} - \log\left(\frac{1-\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}}}{4\tan^2\frac{R}{2}}\right)\right].$$

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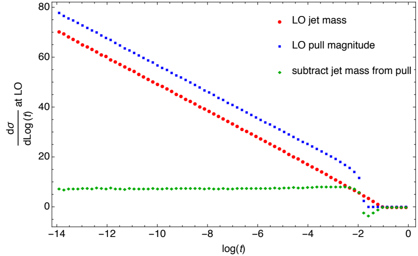
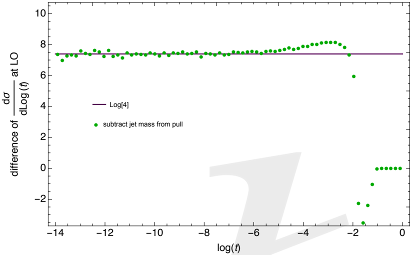
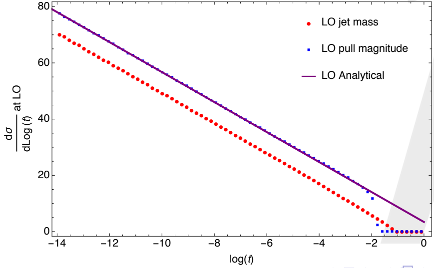
LO with EVENT2

- Check pull magnitude with jet mass:





- Compare with fixed-order calculation:



Resummation Formalism

- Q_T resummation formalism: (detail see: Giancarlo Ferrera' talk)

$$\frac{d^2\sigma}{d\vec{t}} = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] P(z_i)\, \delta^{(2)} \left(\vec{t} - \sum_{i=1}^{n} \tilde{t}_i \right) \mathcal{V}$$

$$\frac{d^2\sigma}{d\vec{t}} = \frac{1}{4\pi^2} \int \mathrm{d}^2\underline{b}\, e^{i\underline{b}\cdot\vec{t}} \exp\left[- \int [dk] P(z) \left(1 - e^{-i\underline{b}\cdot\tilde{\vec{t}}} \right) \right]$$

$$\equiv \frac{1}{4\pi^2} \int \mathrm{d}^2\underline{b}\, e^{i\underline{b}\cdot\vec{t}} e^{-R(b)}$$

$$\frac{d\sigma}{tdt} = \int_0^{\infty} bdbJ_0(bt)e^{-R(b)}$$

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- Structure of NLL resummation

$$\Sigma(t) = (1 + \alpha_s C_1^{(q)}) S(\alpha_s L) e^{-R_q(\alpha_s C_F, L)} + \alpha_s C_1^{(g)} e^{-R_g(\alpha_s C_A, L)}$$

- Non Global Logarithms [Dasgupta, Salam, hep-ph/0104277]

$$S(\alpha_s L) = 1 + \sum_{n=2} S_n(\bar{\alpha}_s L)^n$$

$$\rightarrow 1 - (\bar{\alpha}_s L)^2 \cdot C_F C_A \frac{\pi^2}{3}$$

Radiator in b-space

- Radiator in b-space: $-R = Lf_1 + f_{2c} + f_{2s},\ \lambda = \alpha_s \beta_0 \bar{L},\ B_q = \frac{3}{4}$

$$f_1(\lambda) = -\frac{C_F}{2\pi\beta_0\lambda}\left[(1-2\lambda)\log(1-2\lambda) - 2(1-\lambda)\log(1-\lambda)\right]$$

$$f_{2c}(\lambda) = -\frac{C_F B_q}{\pi\beta_0}\log(1-\lambda) - \frac{C_F K}{4\pi^2\beta_0^2}\left[2\log(1-\lambda) - \log(1-2\lambda)\right]$$
$$-\frac{C_F\beta_1}{2\pi\beta_0^3}\left[\log(1-2\lambda) - 2\log(1-\lambda) + \frac{1}{2}\log^2(1-2\lambda) - \log^2(1-\lambda)\right]$$

$$f_{2s}(\lambda) = -\frac{C_F}{2\pi\beta_0}\log\left(\frac{1-\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}}}{4\tan^2\frac{R}{2}}\right)\log(1-2\lambda)$$

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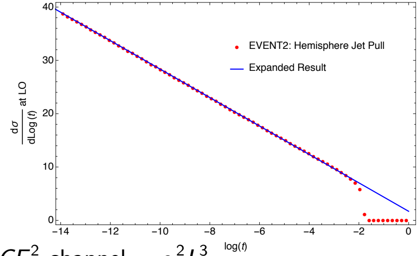
- Expansion of resummed result:

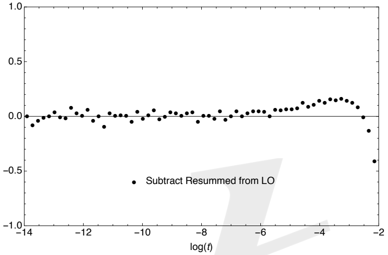
$$\frac{d\sigma^{exp}}{tdt} = \int_0^\infty bdbJ_0(bt)(1-R+\frac{R^2}{2})$$

$$\frac{d\sigma^{LO}}{dlogt} = -\frac{\alpha_s C_F}{\pi}\left(log\frac{t}{4\tan^2\frac{R}{2}} + \frac{3}{4}\right)$$

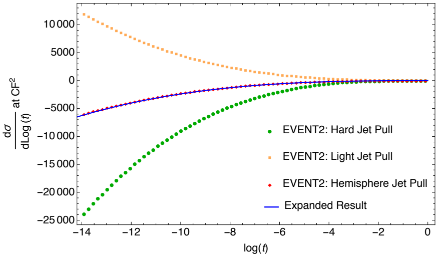
Compare with EVENT2

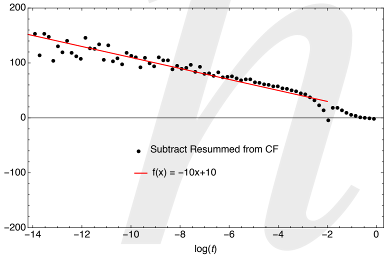
- LO channel





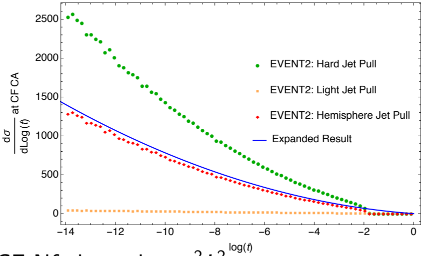
- CF^2 channel $\sim \alpha_s^2 L^3$

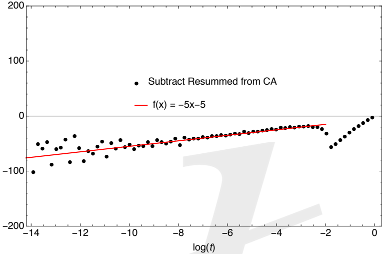




Compare with EVENT2

- CF CA channel (with NGL) $\sim \alpha_s^2 L^2$





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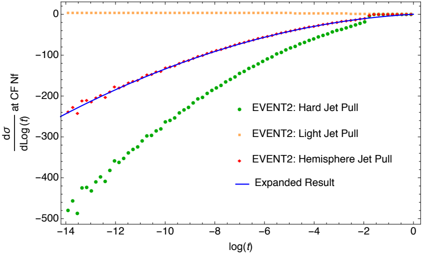

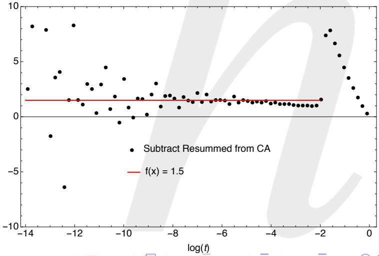

All-Order Resumed Result

- All-order numerical integration:

$$\frac{d\sigma^{all}}{tdt} = \int_{b_{min}}^{b_{max}} bdbJ_0(bt)e^{-R(\alpha_s \bar{L})}$$

$$b_{min} = 2e^{-\gamma_E}, \ \lambda \leq \frac{1}{2}$$

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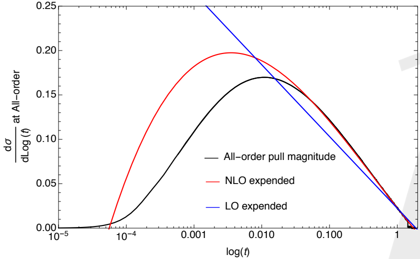

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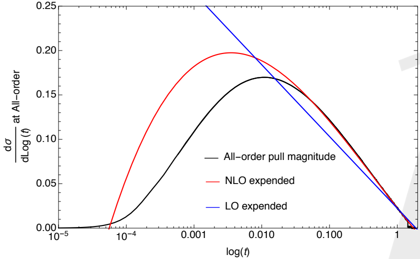

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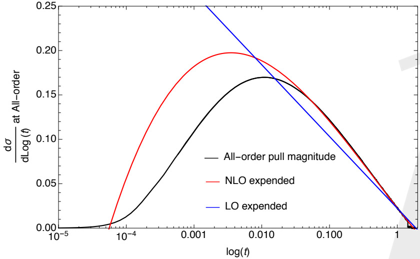
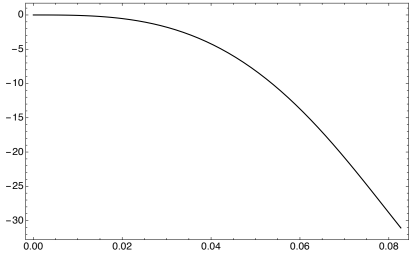
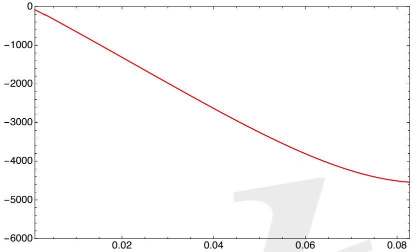


- Cross check: subtract expanded result from all-order, as a function of α_s

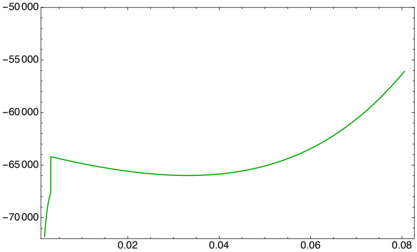
Cross Check: NLO



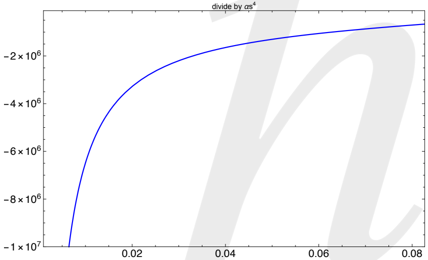
subtract NLO from all-order



divide by α_s^2



divide by α_s^3



divide by α_s^4

Improvement for the Fixed-Order Results

- Method of matching

$$\frac{1}{\sigma}\frac{d\sigma_{NLL+LO}}{dlogt} = \frac{1}{\sigma}\left[\frac{d\sigma_{LO}}{dlogt} + \frac{d\sigma_{NLL}}{dlogt} - \frac{d\sigma_{NLL,\alpha_s}}{dlogt}\right]$$

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- Matched end-point

$$log\frac{1}{t} \rightarrow log\left(\frac{1}{t} - \frac{1}{t_{max}} + \frac{e^{Bq}}{4}\right)$$

$$with\ t_{all} = 4e^{-Bq},\ \ t_{max} \sim 0.2$$

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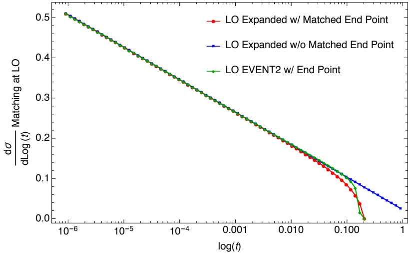
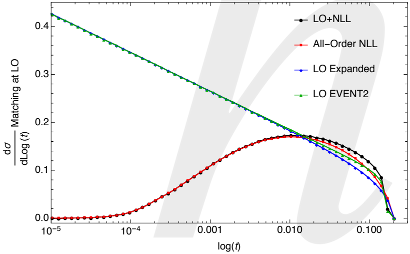
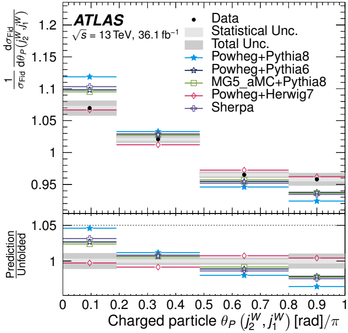
- Result





Pull Angle: IRC unsafety

- Result from ATLAS



- Definition of pull angle

$$\phi_p = cos^{-1}\frac{t_x}{t}$$

Pull Angle: IRC unsafety

- Joint probability approach:

$$p(\phi_p) = \int dt\; p(t) p(\phi_P|t)$$
$$\sim \int dt\; e^{-R(t)} \frac{d^2\sigma^{Fo}}{dtd\phi_p}$$

exponential suppression at soft-collinear region

- In the region of soft-collinear, fix order calculation spoiled by large logarithm $log^m(t)$ enhancement

$$\frac{d\sigma}{dt} = \frac{d\sigma^{Res}}{dt} + \frac{d\sigma^{Fo}}{dt}$$

Calculation Technique (sketch)

- Formalism for the pull angle:
  - Full double differential resummation approach ✗

$$\frac{d\sigma}{d\phi_p} = \frac{1}{2\pi}\int_0^\infty dt\, t \int_0^\infty bdb\, J_0(bt) e^{-R(b)}$$

  - Sudakov safety approach ✓

$$\frac{d\sigma}{d\phi_p} = \int dt\; e^{-R(t)} \frac{d^2\sigma^{Fo}}{dtd\phi_p}$$

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$$\frac{d\sigma}{d\phi_p} = \frac{1}{2\pi}\int_0^\infty dt\, t \int_0^\infty bdb\, J_0(bt)e^{-R(b)}$$

  - Sudakov safety approach ✓

$$\frac{d\sigma}{d\phi_p} = \int dt\; e^{-R(t)}\frac{d^2\sigma^{Fo}}{dtd\phi_p}$$

- New (IRC safe) observable: $t_x = \sum_i \frac{E_i sin^2\theta_i}{E_J} cos\phi_i$

$$\frac{d\sigma}{dt_x} = \frac{1}{\pi}\int_0^\infty db\cos(bt_x)e^{-R(b)}.$$

Next Step

- Sudakov safety calculation
- Full double differential resummation
- Evaluate non-perturbative corrections, with Pythia

Future Works

- Push to higher accuracy (NNLL) and compare with ATLAS (**next project**)

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