Two-loop Electroweak Corrections to the Top-Quark Contribution to ϵ_K

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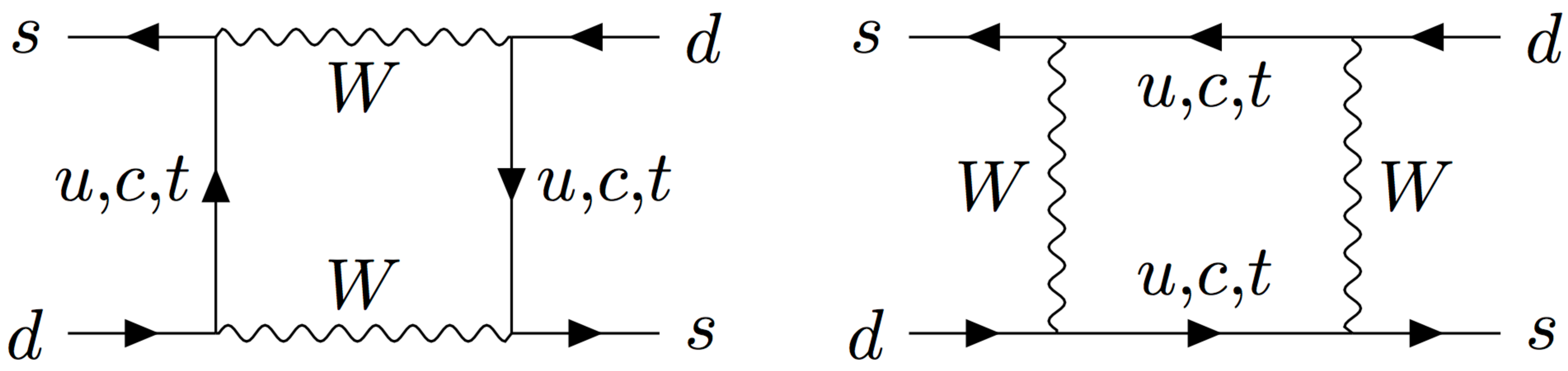
Introduction

- Indirect CP violation is defined as

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | T | K_L \rangle}{\langle (\pi\pi)_{I=0} | T | K_S \rangle}$$

- It arises from the kaon mixing via the diagrams



as the mass eigenstates are admixtures of CP eigenstates

- It is used to constrain the CKM matrix and unitarity triangle

- Experimentally: $|\epsilon_K|_{exp} = (2.228 \pm 0.011) \times 10^{-3}$ (PDG 2020)

Introduction

- Can be written as

$$\epsilon_K \equiv e^{i\phi_\epsilon} \sin \phi_\epsilon \frac{1}{2} \arg \left(\frac{-M_{12}}{\Gamma_{12}} \right)$$

where $\phi_\epsilon = \arctan(2\Delta M_K / \Delta\Gamma_K)$ and $M_{12} = -\langle K^0 | \mathscr{L}_{f=3}^{\Delta S=2} | \bar{K}^0 \rangle / (2\Delta M_K)$

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- At scales around $\mu = 2$ GeV, effective $|\Delta S| = 2$ Lagrangian is given by

$$\mathscr{L}_{f=3}^{|\Delta S|=2} = -\frac{G_F^2 M_W^2}{4\pi^2} \left[\lambda_u^2 C_{S2}^{''uu}(\mu) + \lambda_t^2 C_{S2}^{''tt}(\mu) + \lambda_u \lambda_t C_{S2}^{''ut}(\mu) \right] Q_{S2}'' + \text{h.c.} + \dots$$

where $Q_{S2}'' = \left(\bar{s}_L^\alpha \gamma_\mu d_L^\alpha \right) \otimes \left(\bar{s}_L^\beta \gamma^\mu d_L^\beta \right)$ and $\lambda_i \equiv V_{is}^* V_{id}$

EFTs and Matching

Effective field theory and matching

- EFT Lagrangian: $\mathscr{L} = \sum_i C_i^{(0)} \mathscr{O}_i^{(0)}$

- Define renormalized Wilson Coefficients: $C_i^{(0)} = Z_{ij} C_j$

- RGEs determine the scale-dependence of renormalized Wilson Coefficients in terms of ADM and evolution matrix

$$\frac{dC_i}{d\log(\mu)} = C_j(\mu)\gamma_{ji} \;\; \rightarrow \;\; C_i(\mu) = C_j(\mu_0) U_{ji}(\mu_0, \mu)$$

- The initial conditions for $C_i(\mu)$ are found by requiring

$$\mathscr{A}_{full}(\mu_{match}) = \mathscr{A}_{EFT}(\mu_{match})$$

- At LO this is just $\mathscr{A}_i^{(0)} = C_i^{(0)}$

Evanescent operators

- Use dimensional regularization $d = 4 - 2\epsilon$
- γ_5 is not well defined
- Introduce evanescent operators which vanish as $d \to 4$

$$E_{S2}^{(1)} = \left(\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} d_L^\beta\right) - (16 - a_{11}\epsilon - 4\epsilon^2) Q_{S2}$$

$$E_{S2}^{(2)} = \left(\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} d_L^\beta\right) - \left(256 - a_{21}\epsilon - \frac{108816}{325}\epsilon^2\right) Q_{S2}$$

- Results in scheme dependent Wilson coefficient

Factorization of evolution matrix

- QCD amplitude factorizes into two separately scheme and scale independent pieces

$$C^{ij}(\mu)U(\mu,\mu_0)\langle Q_{S2}\rangle(\mu_0) = \underbrace{\left[C^{ij}(\mu)J^{-1}(\mu)U^{(0)}(\mu)\right]}_{\eta_{ij}S(x_i,x_j)} \underbrace{\left[(U^{(0)}(\mu_0))^{-1}J(\mu_0)\langle Q_{S2}\rangle(\mu_0)\right]}_{\hat{B}_K}$$

perturbative non-perturbative

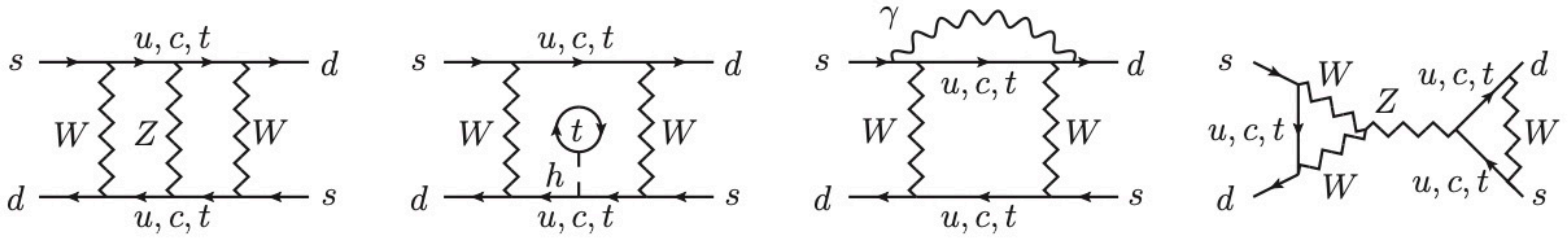
- For QED amplitude $J(\mu)$ is just a number, hence it is not possible to factorise the amplitude in such way

- As QED ADM is scheme independent, dependence must cancel against the matrix element:

$$\langle Q_{S2}\rangle \equiv \left[1+\frac{\alpha}{4\pi}\left(\frac{1}{9}a_{11}-\frac{4}{3}\log\frac{\mu_t}{\mu_{\text{had}}}\right)+\ldots\right]\langle Q_{S2}\rangle^{(0)}$$

Results

Calculation



- $\mathcal{O}(30{,}000)$ two-loop diagrams - independently cross-checked
- Renormalized by counterterm insertions and by replacing bare parameters with renormalised ones in addition to expanding in α
- UV and IR divergences cancel in the matching
- Obtain the Wilson coefficient at matching scale and run to 2 GeV

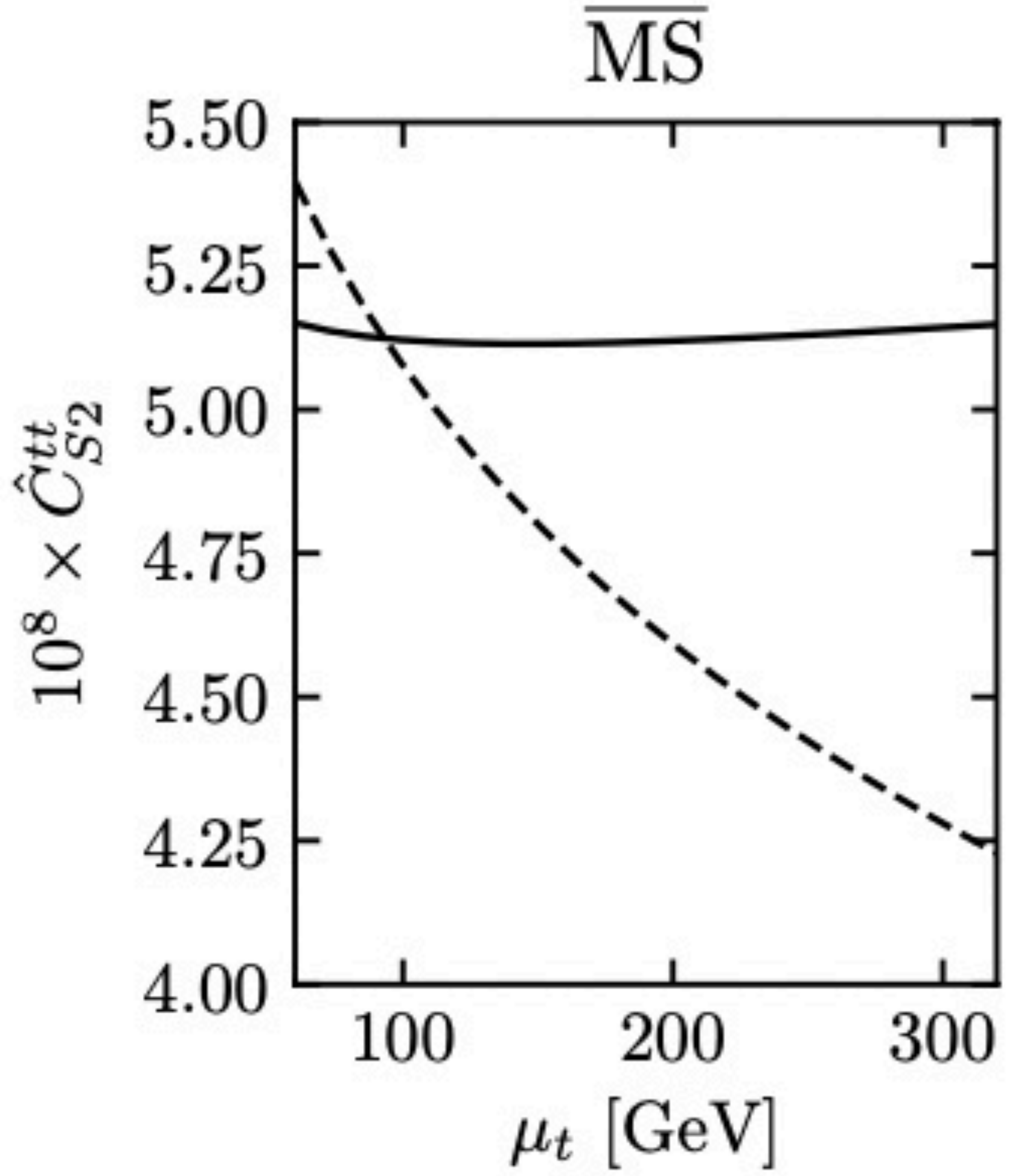
EW renormalization schemes

- Study residual theory uncertainty w.r.t. the higher order ew corrections.
- Three schemes: $\overline{\text{MS}}$, on-shell and hybrid
- For $\overline{\text{MS}}$ scheme:

$$\hat{C}^{tt}_{S2} = \frac{\alpha^2(\mu)}{8m_W^2(\mu)\left(s_w^{\overline{\text{MS}}}(\mu)\right)^4} C^{tt}_{S2}(\mu)\langle Q_{S2}\rangle(\mu)$$

- matching scale dependence is reduced from

$\pm 12\,\%$ at LO to $\pm 0.4\,\%$ at NLO

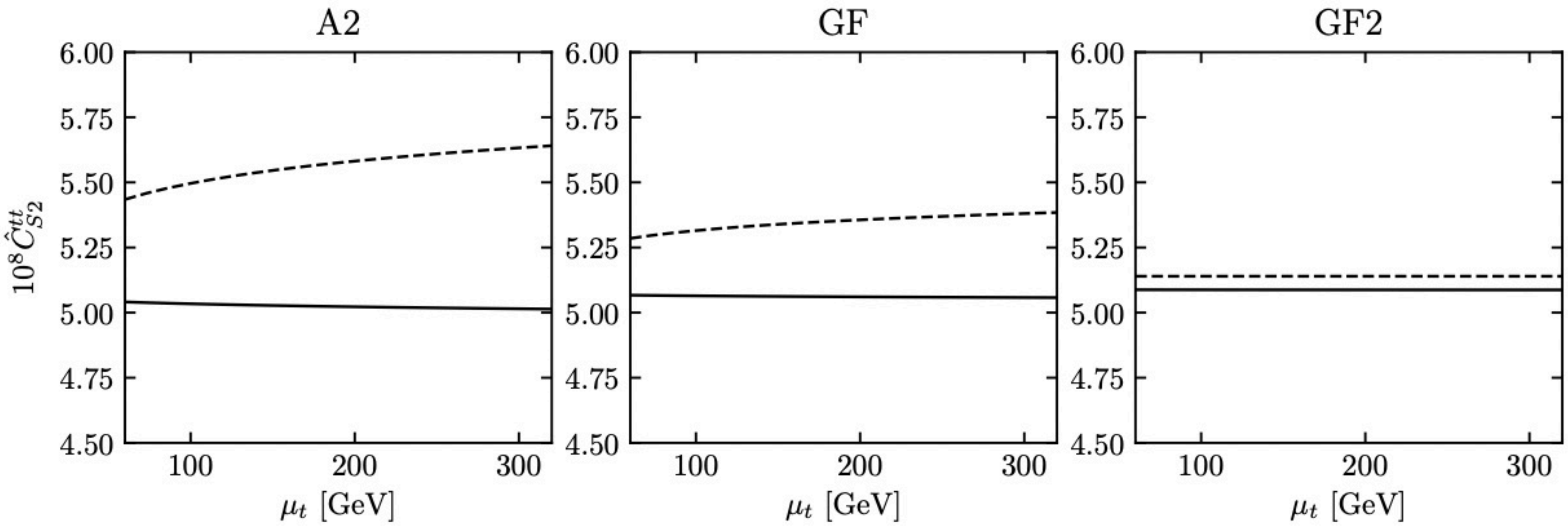


On-shell scheme

- Weak mixing angle defined in terms of physical boson masses: $\sin^2\theta_w^{\mathrm{OS}} = 1 - M_W^2/M_Z^2$

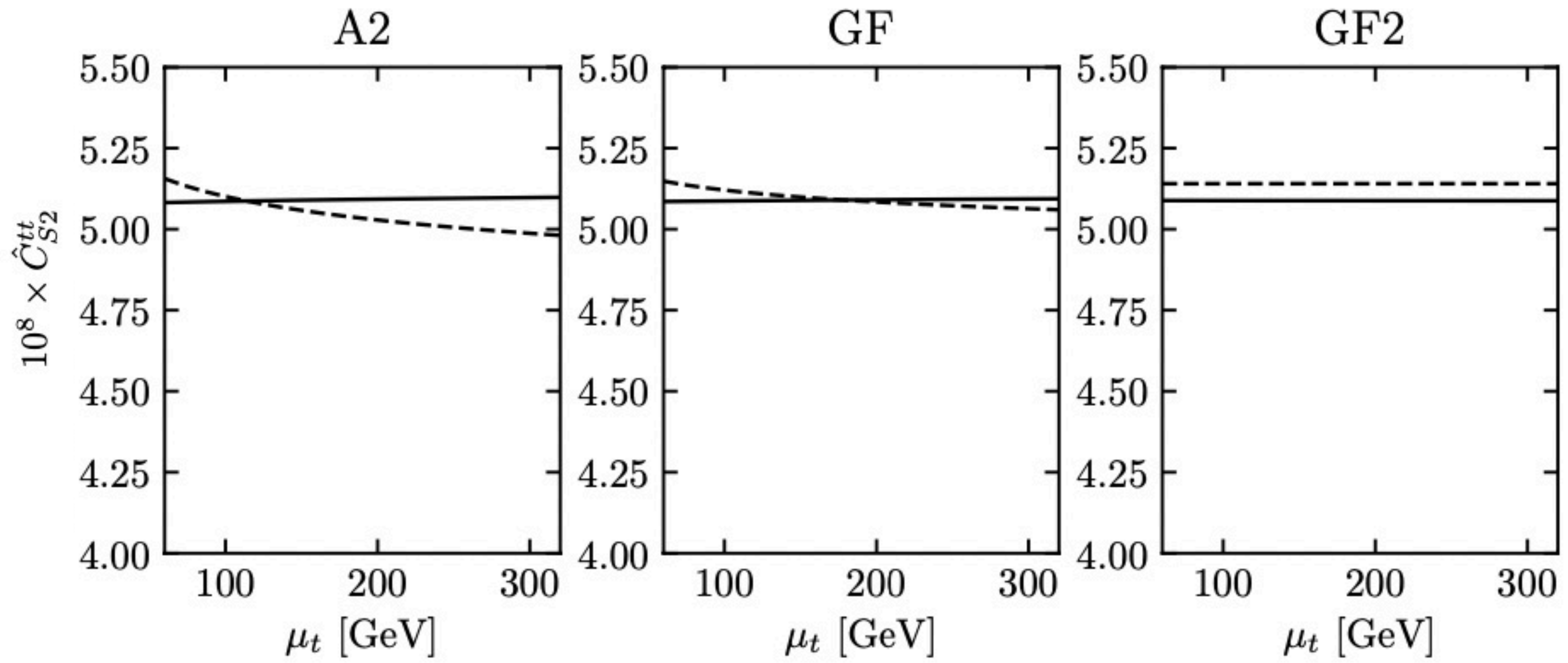
$$\hat{C}_{S2}^{tt} = \frac{\alpha^2(\mu)}{8M_W^2\left(s_w^{\mathrm{OS}}\right)^4} C_{S2}^{tt}(\mu)\langle Q_{S2}\rangle(\mu),\ \hat{C}_{S2}^{tt} = \frac{\alpha(\mu)G_F}{4\sqrt{2}\left(s_w^{\mathrm{OS}}\right)^2} C_{S2}^{\prime tt}(\mu)\langle Q_{S2}\rangle(\mu),\ \hat{C}_{S2}^{tt} = \frac{G_F^2 M_W^2}{4\pi^2} C_{S2}^{\prime\prime tt}(\mu)\langle Q_{S2}\rangle(\mu)$$

- NLO corrections are large, indicating slow convergence of the perturbation series



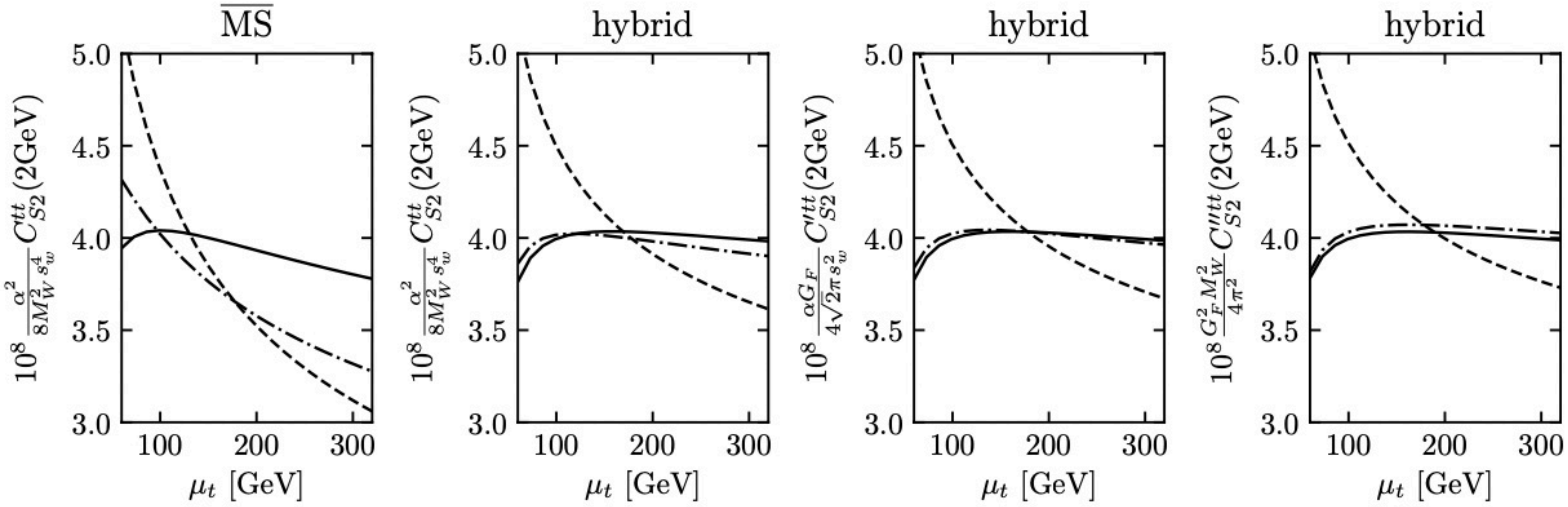
Hybrid scheme

- $s_w^{\overline{\text{MS}}}(\mu)$ is defined in the $\overline{\text{MS}}$ scheme, while M_W is defined in the on-shell scheme.
- GF norm. shows better perturbative convergence than conventionally used GF2
- Matching scale dependence $\pm 0.4\,\%$ at NLO



QCD+EW

- Resummation of QED logs: $\alpha\alpha_s^n \log^{n+1}(\mu_t/\mu_{\text{had}})$ and $\alpha\alpha_s^n \log^{n}(\mu_t/\mu_{\text{had}})$
- Two-loop ADM: $\gamma_{S2} = \frac{\alpha_s}{\pi} + \left(\frac{4N_f}{9} - 7\right)\frac{\alpha_s^2}{16\pi^2} + \frac{\alpha}{3\pi} - \frac{148}{9}\frac{\alpha\alpha_s}{16\pi^2}$



- Small scheme dependence: $C_{S2}^{\prime\prime tt}(2\,\text{GeV}) = (3.90 - 0.0003a_{11}) \times 10^{-8}$

Results

	NLL QCD	NLL QCD & NLL QED
$\alpha^2/(8M_W^2 s_w^4)C_{S2}^{tt}(2\,\mathrm{GeV}) \times 10^8$	3.96(6)	3.98(6)
$\alpha G_F/(4\sqrt{2}\pi s_w^2)C_{S2}^{\prime tt}(2\,\mathrm{GeV}) \times 10^8$	4.00(4)	3.98(5)
$G_F^2 M_W^2/(4\pi^2)C_{S2}^{\prime\prime tt}(2\,\mathrm{GeV}) \times 10^8$	4.02(5)	3.98(5)

- Central values of all three normalizations perfectly coincide
- Need the matrix element to cancel scheme dependence
- Temp. solution: choose GF2 norm. and multiply η_{tt} by $(1 - \Delta_{tt})$, with $\Delta_{tt} = 0.01 \pm 0.004$

$$|\epsilon_K| = 2.15(6)_{pert}(7)_{non-pert}(15)_{param} \times 10^{-3}$$

$$|\epsilon_K|_{exp} = (2.228 \pm 0.011) \times 10^{-3}$$

Conclusions

- Presented NLO EW corrections to top-quark contributions to ϵ_K
- Discussed scheme-dependence and the need for non-perturbative ME including QED
- See –1.0% shift in central value of Wilson coefficient
- Upcoming three-loop QCD top contributions, two-loop EW charm contributions, two-loop matching for $\hat{B}_K$ and possible updated lattice calculations will give more accurate prediction of ϵ_K