Multi-Boson Production and the Muon Yukawa Coupling

Yang Ma

PITT PACC, University of Pittsburgh

PIKIMO 11
December 4, 2021

arXiv:2108.05362, in collaboration with T. Han, K. Xie (PITT),
W. Kilian, N. Kreher, T. Striegl (U. Seigen), and J. Reuter (DESY)

The Dream Machine

A high-energy muon collider

A possible high-energy lepton collider: Why?

Why lepton colliders?

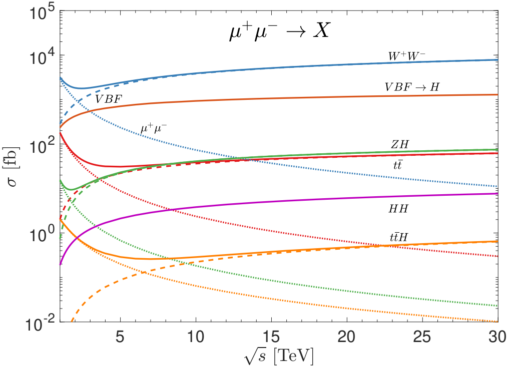
- **Leptons** are the ideal probes of short-distance physics
  - Cleaner background comparing to hadron colliders
  - High-energy physics probed with much smaller collider energy
- **ee colliders**
  - A glorious past: discovery of charm, τ, and gluon
  - Important future: Precision EW constraints on BSM physicss, Higgs physics
- **Muon colliders**
  - A s-channel Higgs factory: Higgs production enhanced by $m_\mu^2/m_e^2 \sim 40000$
    - Direct measurements on y_μ and Γ_H
  - Multi-TeV muon colliders: Less radiations then electron
    - Center of mass energy $3-15$ TeV and the more speculative $E_{\rm cm} = 30$ TeV
    - New particle mass coverage $M \sim (0.5-1)E_{\rm cm}$
    - Great accuracies for WWH, $WWHH$, H^3, H^4
    - $\cdots$

Muon Collider Physics Potential Pillars

Direct search of heavy particles	High rate indirect probes	High energy probes
SUSY-inspired, WIMP, VBF production, 2->1	Higgs single and self-couplings, rare Higgs decays, exotic decays	difermion, diboson, EFT, Higgs compositeness

A possible high-energy muon collider: The full picture

Just like in hadronic collisions: $\mu^+\mu^- \to$ exclusive particles + remnants



[T. Han, Y. Ma, K.Xie 2007.14300]

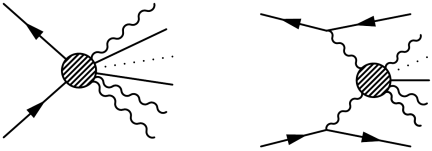
Some observations:

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln(s/m_\mu^2)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$.

Multi-boson physics

New phenomenology at a multi-TeV lepton collider:

1. Multi-boson production (annihilation)
2. . . . and vector boson fusion to multi-bosons,
   leading to multi-fermion final states with resonance structure.

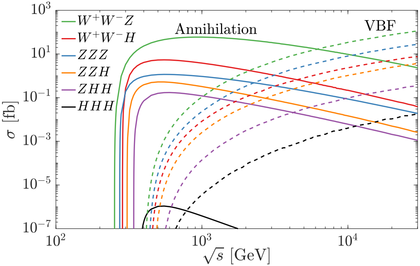


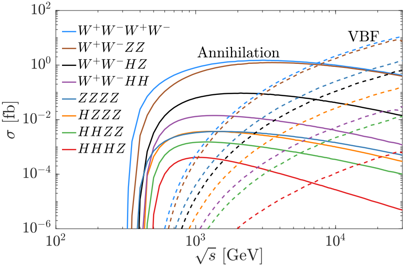
[Barger, Cheung, Han, Phillips 1995]

[Boos, He, Kilian, Pukhov, Yuan, Zerwas 1998]

Task:
Measure all interactions of multiple SM particles exclusively and with precision, from threshold to up to 2 orders of magnitude above EW scale.

Annihilation vs VBF: Properties (SM)





VBF:

- Increases rapidly
- Most of cross section at threshold
- Highly boosted final state (forward/backward)

Annihilation:

- Decreases slowly
- Most of cross section at highest energy
- One Boson highly off-shell
- Final state in rest frame (central)

Annihilation processes important for analysis at all energies.

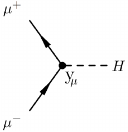
Muon-Higgs Coupling

- Physics: We actually do not know whether the SM mass-generation mechanism applies just to the heavy particles, or also to the 1st/2nd generations.
- Logical possibility: Muon mass not (only) generated by SM Higgs.
  $\Rightarrow$ **Why not hava an arbitrary Yukawa coupling?**

Muon Yukawa coupling

In SM:

- One of the fundamental parameters of the SM: $y_\mu = \sqrt{2} m_\mu / v$



- Recently confirmed to have the predicted order of magnitude, but results are not yet at the 5σ level

[arXiv:2007.07830,2009.04363]

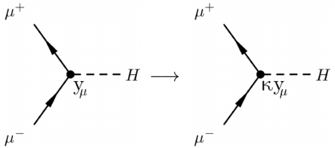
- HL-LHC predicts a measurement of y_μ within an accuracy of $\mathcal{O}(10\%)$

[ATL-PHYS-PUB-2014-016]

- High beam qulity is required for directly measuring $\mu^+\mu^- \to H$.

If BSM exists:

- Correction from BSM: $y_\mu = \kappa y_\mu^{\mathrm{SM}}$



- The delicate gauge cancellation in SM will be spoiled if $\kappa \neq 1$, resulting in huge cross sections.

[arXiv:hep-ph/0106281]

- An experimental bound on $\Delta\kappa_\mu$ translates to a bound on the scale of new physics

$$\Lambda > 10\ \mathrm{TeV}\sqrt{\frac{g}{\Delta\kappa_\mu}} \quad .$$

EFT parameterizations

- Nonlinear HEFT [Coleman et al., PR1969, Weinberg, PLB1980, ⋯]

$$\mathscr{L}_{UH} = \frac{v^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right]F_U(H) + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H)$$
$$- \frac{v}{2\sqrt{2}}\left[\bar{\ell}_L^i \tilde{Y}_\ell^{ij}(H)\, U\,(1-\tau_3)\,\ell_R^j + \text{ h.c.}\right]$$

with $F_U, V, \tilde{Y}$ expanded as

$$F_U(H) = 1 + \sum_{n\geq 1} f_{U,n}\left(\frac{H}{v}\right)^n, V(H) = v^4 \sum_{n\geq 2} f_{V,n}\left(\frac{H}{v}\right)^n,$$
$$\tilde{Y}_\ell^{ij}(H) = \sum_{n\geq 0} \tilde{Y}_{\ell,n}^{ij}\left(\frac{H}{v}\right)^n$$

which gives muon-Higgs effective coupling $\kappa_\mu = \frac{v}{\sqrt{2}m_\mu} y_1$.

- Linear SMEFT [Weinberg PRL1979, Abbott & Wise PRD1980, ⋯]

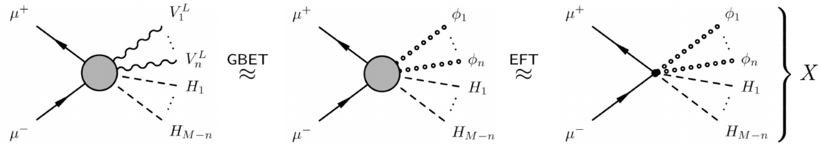
$$\mathscr{L} = \mathscr{L}_{\mathrm{EW}} + \left[\sum_{n=1}^{N} \frac{\tilde{C}_{\ell\varphi}^{(n)ij}}{\Lambda^{2n}}\left(\varphi^\dagger\varphi\right)^n \bar{\ell}_L^i \varphi\, e_R^j + \text{ h.c.}\right]$$

giving

$$M_\ell^{(6)} = \frac{v}{\sqrt{2}}\left(Y_\ell - \frac{v^2}{2}C_{\ell\varphi}\right), \text{ and } \kappa_\mu^{(6)} = 1 - \frac{v^3}{\sqrt{2}m_\mu}c_{\ell\varphi}^{(1)}$$

EFT + GBET

- For a multi boson final state X the **longitudinal polarizations** will dominate the high energy asymptotics.
- **Longitudinal mode** can be approximated using Goldstone bosons. **(GBET)**.
- EFT introduces contact terms that dominate the high energy asymptotics.



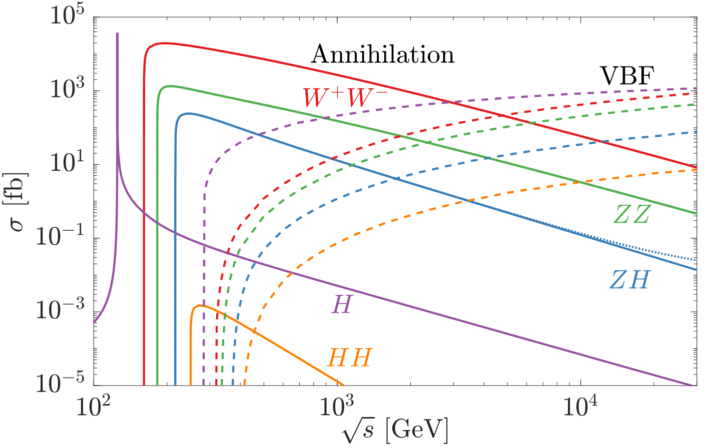
The corresponding phase space is

$$\Phi_M^{X_i}(k_1+k_2;p_1,\ldots,p_M) = \frac{1}{(2\pi)^{4M}}\left(\frac{\pi}{2}\right)^{M-1}\frac{s^{M-2}}{\Gamma(M)\Gamma(M-1)} \quad ,$$

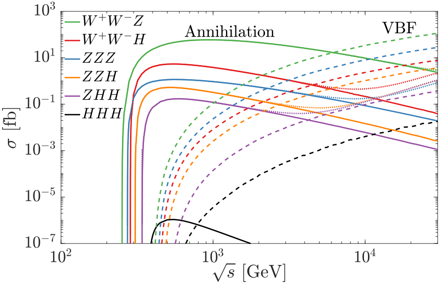
The ratio between different cross section is

$$R^{X_i} := \frac{\sigma^{X_i}}{\sigma^{X_{\text{ref}}}} = \frac{|C_{X_i}|^2\left(\prod_{j\in J_{X_i}}\frac{1}{n_j!}\right)}{|C_{X_{\text{ref}}}|^2\left(\prod_{j\in J_{X_{\text{ref}}}}\frac{1}{n_j!}\right)} \quad .$$

Two-boson final states

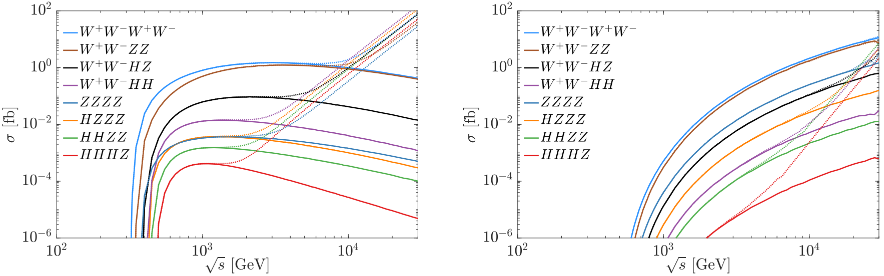


Three-boson final states



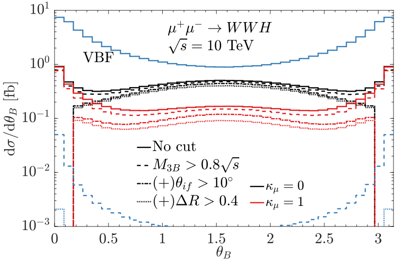
	$\Delta\sigma^X/\Delta\sigma^{W^+W^-H}$					
	SMEFT				HEFT	
$\mu^+\mu^- \to X$	dim_6	dim_8	$\text{dim}_{6,8}$	$\text{dim}_{6,8}^{\text{matched}}$	dim_∞	$\text{dim}_\infty^{\text{matched}}$
WWZ	1	1/9	$R_{(3),1}^{\text{SMEFT}}$	1/4	$R_{(3),1}^{\text{HEFT}}/9$	1/4
ZZZ	3/2	1/6	$3\,R_{(3),1}^{\text{SMEFT}}/2$	3/8	$R_{(3),1}^{\text{HEFT}}/6$	3/8
WWH	1	1	1	1	1	1
ZZH	1/2	1/2	1/2	1/2	1/2	1/2
ZHH	1/2	1/2	1/2	1/2	$2\,R_{(3),2}^{\text{HEFT}}$	1/2
HHH	3/2	25/6	$3\,R_{(3),2}^{\text{SMEFT}}/2$	75/8	$6\,R_{(3),3}^{\text{HEFT}}$	0

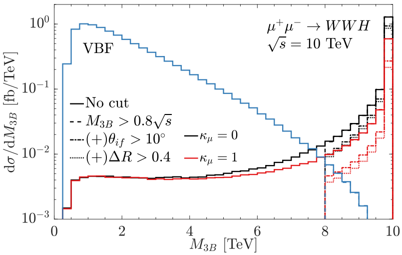
Four-boson final states

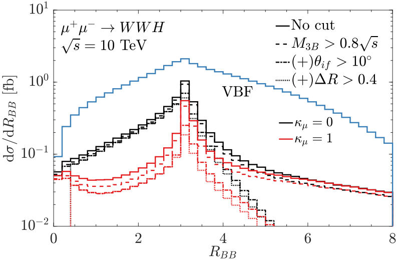


	$\Delta\sigma^X/\Delta\sigma^{WWHH}$					
	SMEFT				HEFT	
$\mu^+\mu^- \to X$	$\dim_{6,8}$	$\dim_{10}$	$\dim_{6,8,10}$	$\dim^{\text{matched}}_{6,8,10}$	$\dim_\infty$	$\dim^{\text{matched}}_\infty$
$WWWW$	2/9	2/25	$2\,R^{\text{SMEFT}}_{(4),1}/9$	1/2	$R^{\text{HEFT}}_{(4),1}/18$	1/2
$WWZZ$	1/9	1/25	$R^{\text{SMEFT}}_{(4),1}/9$	1/4	$R^{\text{HEFT}}_{(4),1}/36$	1/4
$ZZZZ$	1/12	3/100	$R^{\text{SMEFT}}_{(4),1}/12$	3/16	$R^{\text{HEFT}}_{(4),1}/48$	3/16
$WWZH$	2/9	2/25	$2\,R^{\text{SMEFT}}_{(4),1}/9$	1/2	$R^{\text{HEFT}}_{(4),2}/8$	1/2
$WWHH$	1	1	1	1	1	1
$ZZZH$	1/3	3/25	$R^{\text{SMEFT}}_{(4),1}/3$	3/4	$R^{\text{HEFT}}_{(4),2}/12$	3/4
$ZZHH$	1/2	1/2	1/2	1/2	1/2	1/2
$ZHHH$	1/3	1/3	1/3	1/3	$3\,R^{\text{HEFT}}_{(4),3}$	1/3
$HHHH$	25/12	49/12	$25\,R^{\text{SMEFT}}_{(4),2}/12$	1225/48	$12\,R^{\text{HEFT}}_{(4),4}$	0

WWH at a 10 TeV muon collider: Kinematics







- Background (VBF) is much larger than signal (annihilation)
- VBF events accumulate around threshold, and mostly forward
- Annihilation in the rest frame (central, and $M \sim \sqrt{s}$ spread by ISR)
- Annihilation also has forward dominance, due to the gauge splitting $W \to WH$

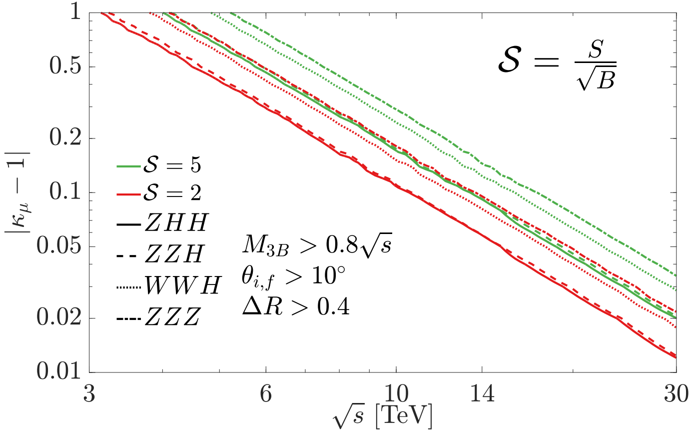
WWH at a 10 TeV muon collider: Cuts

Cut flow	$\kappa_\mu = 1$	w/o ISR	$\kappa_\mu = 0$ (2)	CVBF	NVBF
σ [fb]	WWH				
No cut	0.24	0.21	0.47	2.3	7.2
$M_{3B} > 0.8\sqrt{s}$	0.20	0.21	0.42	$5.5 \cdot 10^{-3}$	$3.7 \cdot 10^{-2}$
$10^\circ < \theta_B < 170^\circ$	0.092	0.096	0.30	$2.5 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
$\Delta R_{BB} > 0.4$	0.074	0.077	0.28	$2.1 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$
# of events	740	770	2800	2.1	2.4
S/B	2.8				

- Integrated luminosity $\mathscr{L} = (\sqrt{s}/10\ \ \mathrm{TeV})^2 \cdot 10\ \mathrm{ab}^{-1}$ [1901.06150]
- $S = N_{\kappa_\mu} - N_{\kappa_\mu=1},\ B = N_{\kappa_\mu=1} + N_{\mathrm{VBF}}$.
- VBF and ISR are mostly excluded by invariant mass cut.
- Angular cut also weaken VBF further.

Test the muon Yukawa: statistical sensitivity

- The most sensitive channels are ZHH and ZZH, similar probes due to GBET.
- Taking $S = 2$ criterion, we can test the muon-Higgs coupling up to 10% (1%) precision at a 10 (30) TeV muon collider, corresponding to new physics scale $\Lambda_{\rm NP} \sim 30 - 100$ TeV.



Conclusion

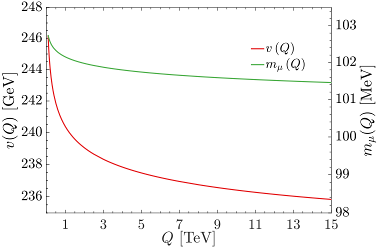
- A high-energy muon collider is a dream machine for new physics search, both for energy and precision frontiers.
- Multi-boson production at a multi-TeV muon collider accounts for a rich phenomenology: frequent resonant 8, 10, . . . jet events
  $\Rightarrow$ **Complete account of EW interactions**
- Annihilation may dominate sensitivity over VBF.
- The sensitivity reach to anomalous muon-Higgs couplings rises with the number of gauge bosons.
  - Isolation of signal from background (VBF) can be achieved by investigation of kinematical distributions.
  - A multi-TeV muon collider can provide us a great chance to test the muon-Higgs coupling, up to percision of 10% (1%) at a 10 (30) TeV machine, probing new physics scale at 30 (100) TeV.
- Many improvements can be performed: different channels, multiplicities, multivariate analysis, polarization information.

Muon Yukawa coupling: running

- In SM

$$m_\mu(Q) = y_\mu(Q) v(Q)/\sqrt{2}$$

$$\begin{aligned}
\beta_{y_t} &= \frac{\mathrm{d}y_t}{\mathrm{d}t} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2\right),\\
\beta_{y_\mu} &= \frac{\mathrm{d}y_\mu}{\mathrm{d}t} = \frac{y_\mu}{16\pi^2}\left(3y_t^2 - \frac{9}{4}(g_2^2 + g_1^2)\right),\\
\beta_v &= \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v}{16\pi^2}\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right),\\
\beta_{g_i} &= \frac{\mathrm{d}g_i}{\mathrm{d}t} = \frac{b_i g_i^3}{16\pi^2},
\end{aligned}$$

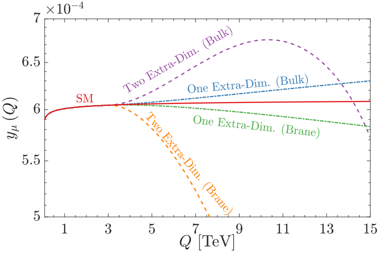


- In potential new physics (NP)

$$\beta_\lambda = \beta_\lambda^{\mathrm{SM}} + \sum_{s\in\mathrm{NP}} \Theta(Q - M_s) \times N_s \beta_{s,\lambda}^{\mathrm{NP}}$$

One example: the Bulk and Brane extra-dimensional scenarios

[Cornell et al. 1110.1942, 1209.6239, 1306.4852]



[We choose $1/R = 3$ TeV for illustration, Han et al. 2108.05362]

Unitarity bounds on a nonstandard Yukawa sector

Inclusive inelastic cross section $\mu^+\mu^- \to X$ for multiple Goldstone and Higgs-boson production in the GBET approximation