Stellar Rotation

Introducing stellar rotation in PARSEC

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Outline

- Part I

  \+ Why is it important to model rotation?

- Part II

  Effect of rotation:

  \+ Geometrical distortion

  \+ Transport of angular momentum and chemical species through the star

- Part III

  \+ State of the art of my work

  \+ Future perspectives

I - Why is it important the rotation in the stellar physics?

- All the stars rotate!

Examples:

➢ Sun, $\odot$

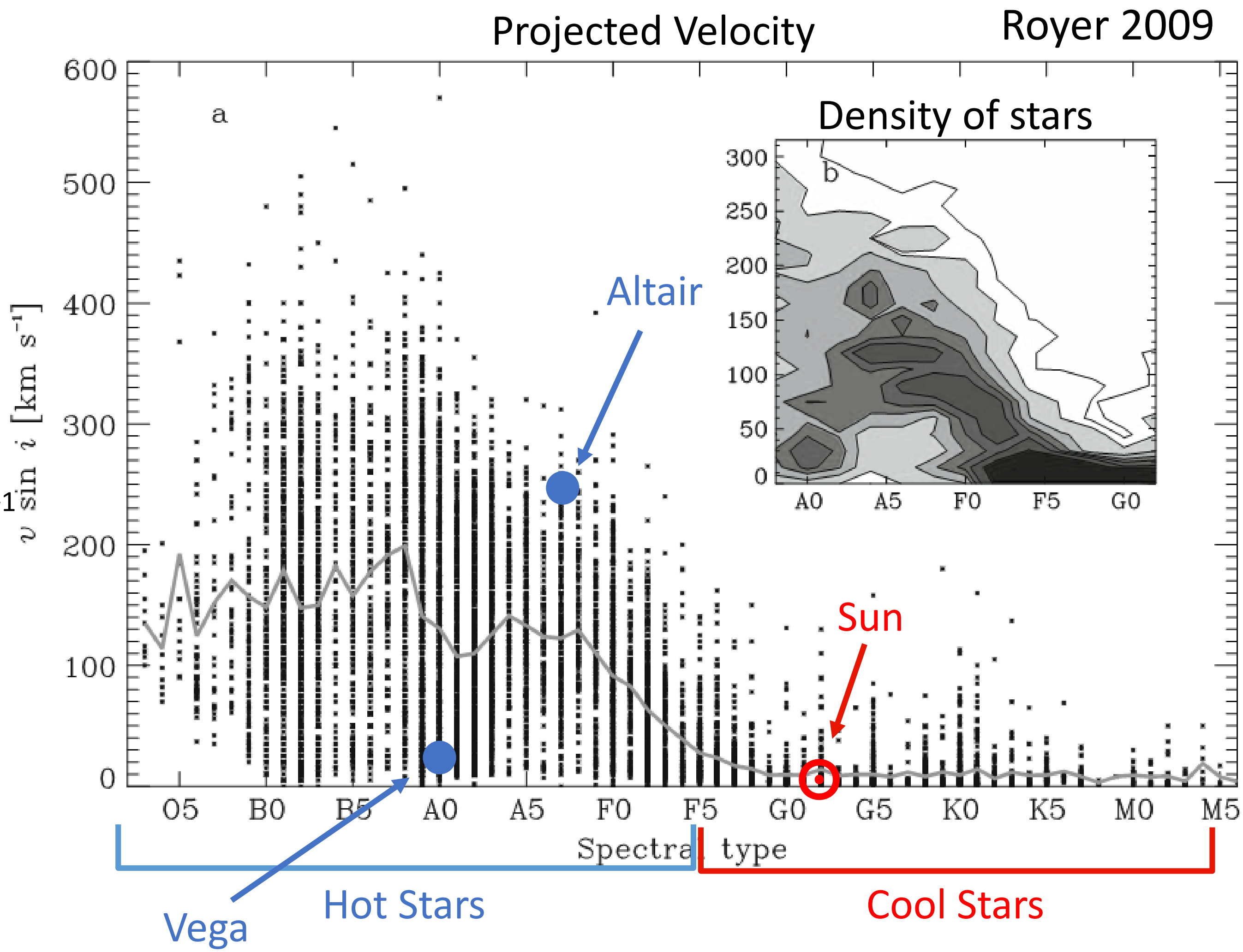
- Spectral Type G2V
- T_{eff} = 5770 K
- $v_{rot} \sim$ 2 km s^{-1}

➢ Vega, α Lir

- Spectral Type A0V
- T_{eff} = 9550 K
- $v_{rot}\sin i = 25 \pm 2$ km s^{-1}
- $v_{rot} = 274 \pm 14$ km s^{-1}
- $i = 4.54° \pm 0.33°$

➢ Altair, α Aql

- Spectral Type A7IV-V
- T_{eff} = 7550 K
- $v_{rot}\sin i = 217$ km s^{-1}
- $v_{rot} = 273 \pm 13$ km s^{-1}
- $i = 63.9° \pm 1{,}7°$



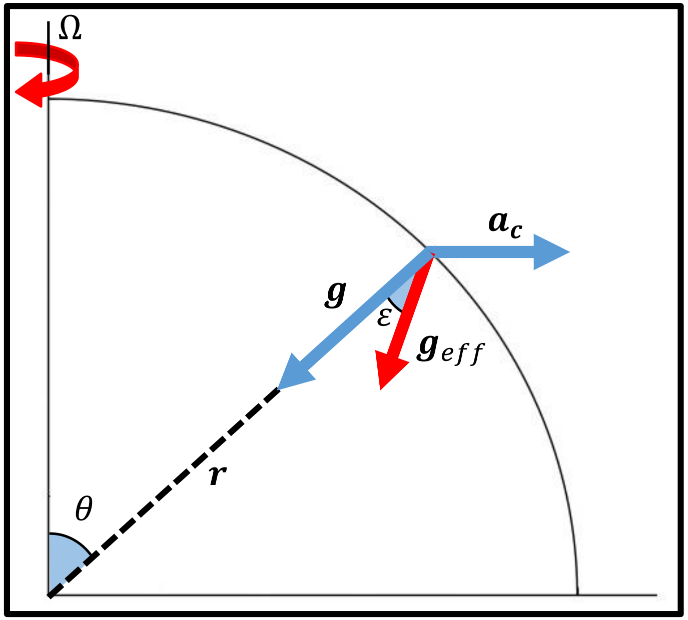
II – Effects of stellar rotation

The effect of rotation on stellar models has been studied for *decades* (Kippenhahn & Thomas 1970 ; Endal & Sofia 1976; Zahn 1992; Heger et al. 2000; Maeder & Meynet 2000; Palacios et al. 2003; Chieffi & Limongi 2013, etc…),

but it remains as one of the most challenging and uncertain problems in stellar astrophysics.

Effects of rotation in stars (e.g. Kippenhahn & Weigert 2012) :

- **Centrifugal force reduces** the **effective gravity** at any point, not at the poles;
- **Departure** from **spherical shape of surfaces**;
- **Effective Temperature depends** on **colatitude (θ)**.
- Rotation may induce same **mixing processes**.



Effective gravity
$$g_{eff} = \mathbf{g} + \mathbf{a_c}$$

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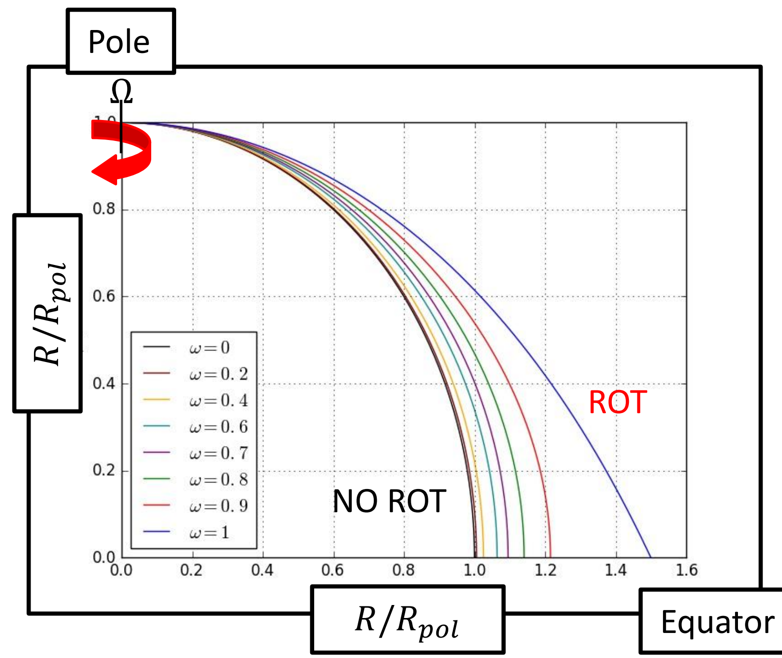
Effects of rotation in stars (e.g. Kippenhahn & Weigert 2012) :

- **Centrifugal force reduces** the **effective gravity**;
- **Departure** from **spherical shape of surfaces**.
  Since the centrifugal force is not, in general, parallel to the force of gravity, equipotential surfaces are no longer spheres;
- **Effective Temperature depends** on **colatitude (θ)**.
- Rotation may induce same **mixing processes**.



Ω_{crit} maximal angular velocity;

$\omega = \dfrac{\Omega}{\Omega_{crit}}$ dimensionless “angular velocity”.

II - Von Zeipel Effect

- The rotation affect the surface temperature of the star because to the **gravity darkening** due to the **Von Zeipel effect**, that changes the flux distribution on the stellar surface due to temperature gradient.

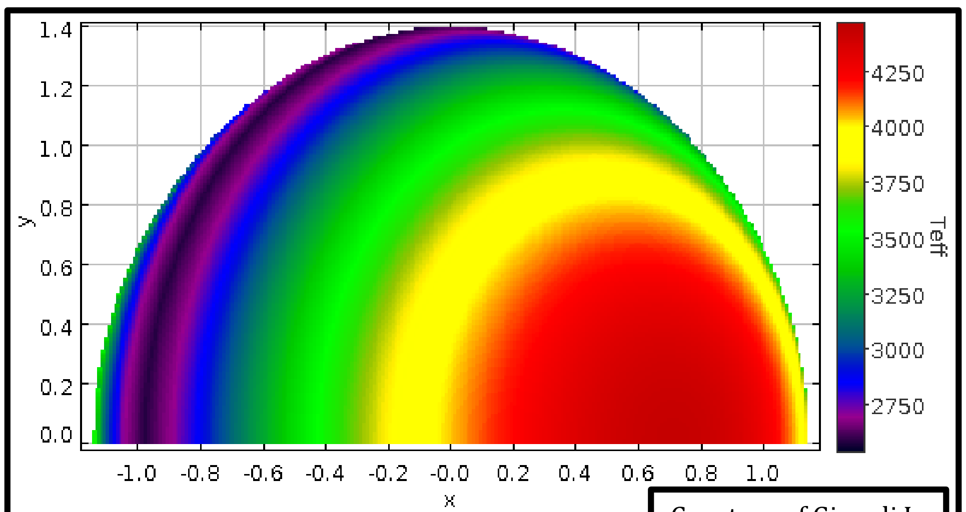
The ***Von Zeipel theorem*** (1924) is

$$\boldsymbol{F}(\omega,\theta) = -\frac{L}{4\pi\ G\ M}\ \boldsymbol{g}_{eff}(\omega,\theta)$$

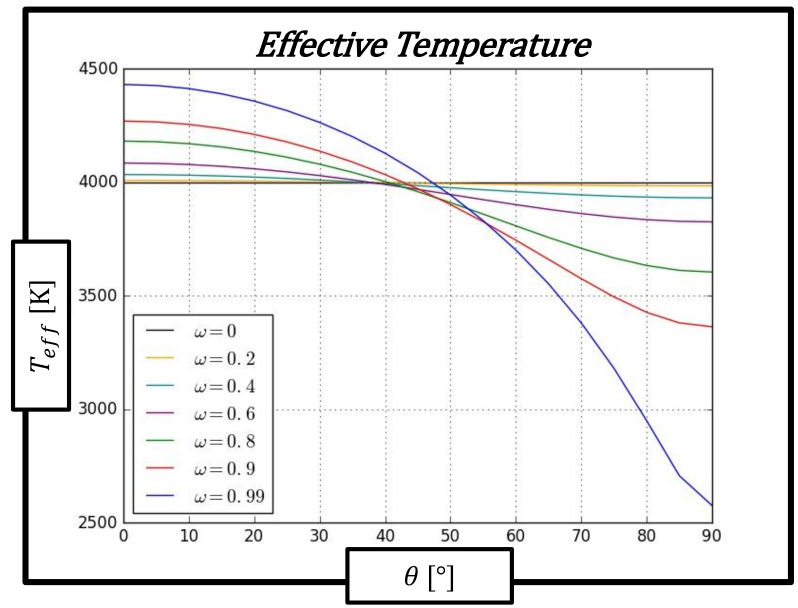
θ is the co-latitude.
ω dimensionless "angular velocity".
σ is the Stefan-Boltzmann constant.

Then, the ***effective surface temperature*** is

$$T_{eff}(\omega,\theta) = -\left(\frac{L}{4\pi\ \sigma\ G\ M}\right)^{\frac{1}{4}} \left[g_{eff}(\omega,\theta)\right]^{\frac{1}{4}}$$



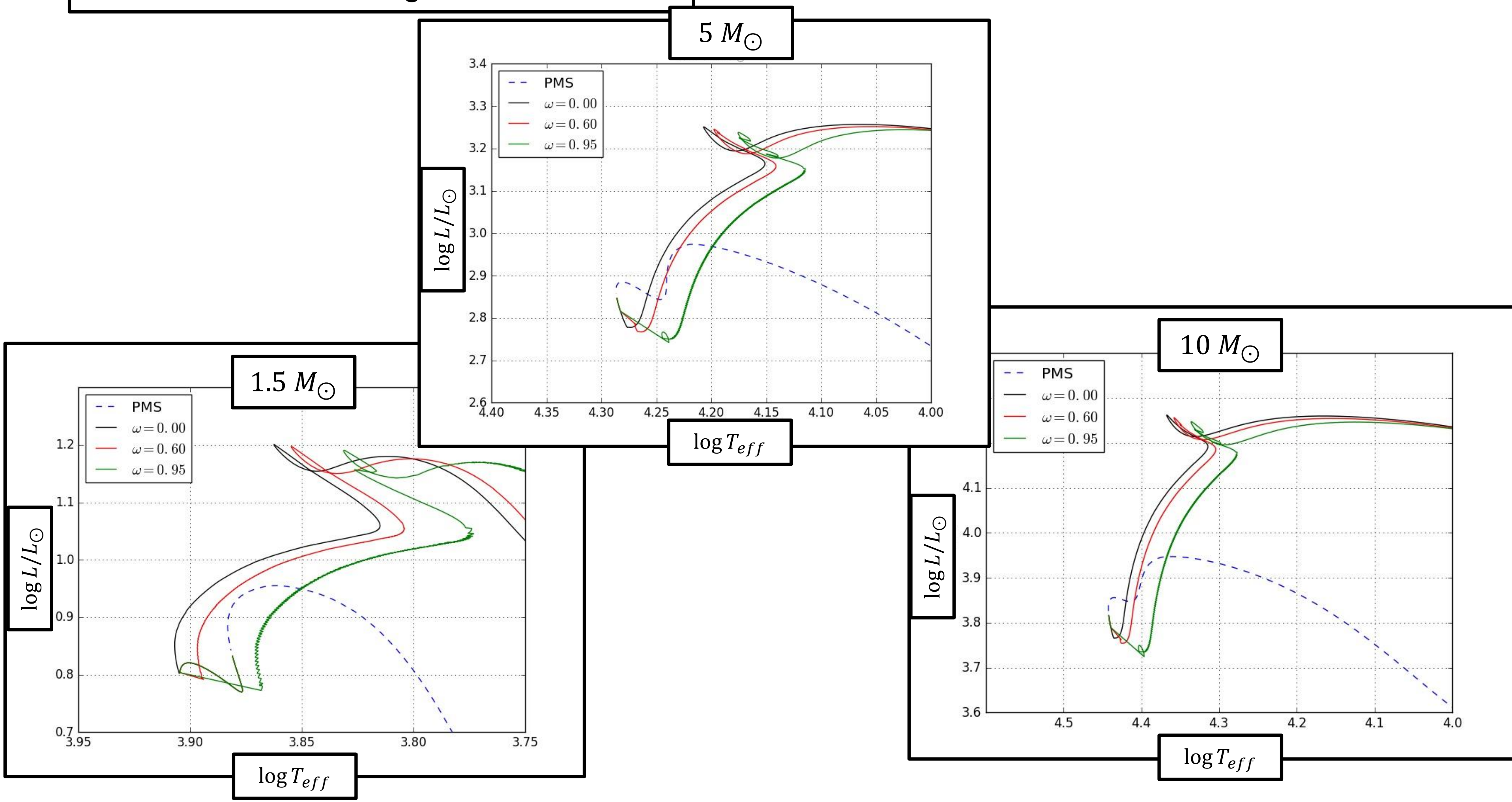
Courtesy of Girardi L.



II – Implementing rotation in PARSEC

- Updating equations
- Introducing the angular velocity
- Conservation of angular momentum

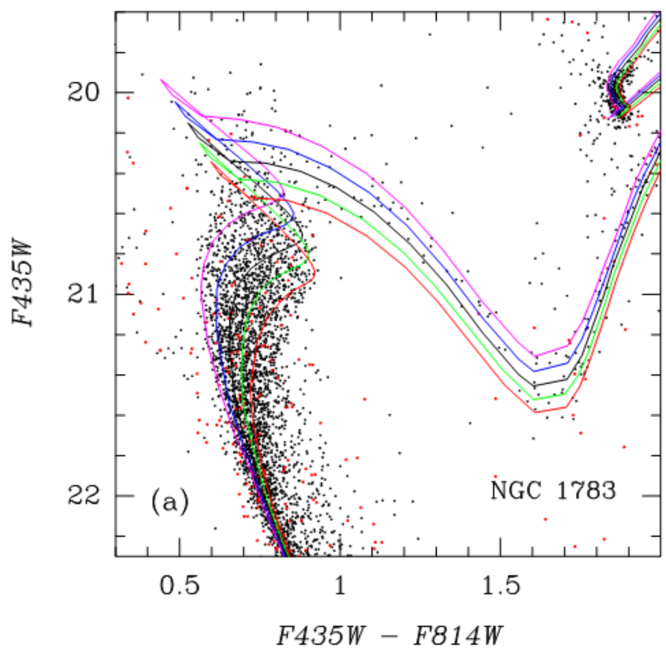
Evolution tracks of rotating stars



II - Von Zeipel Effect a simple application

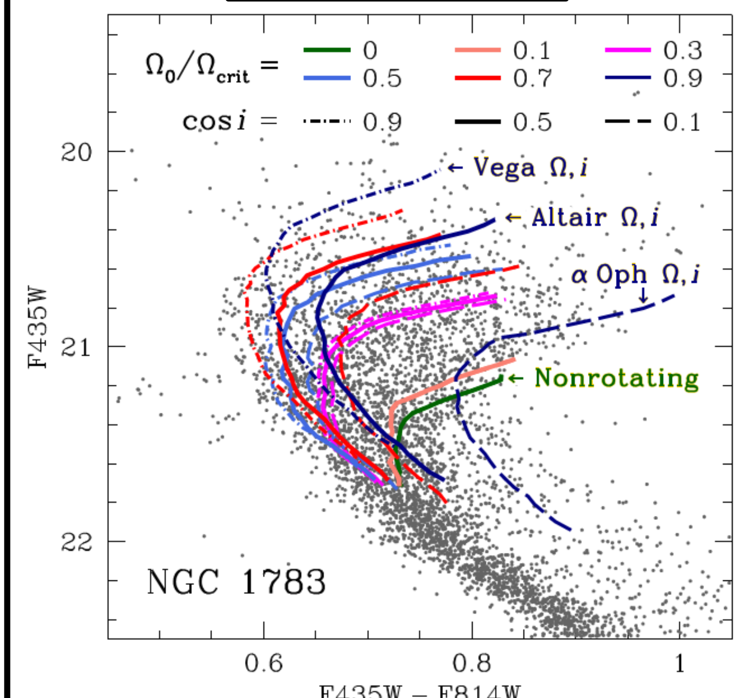
Extended Main Sequence Turn-Off (eMSTO) in intermediate age cluster.

From Goudfrooij et al. (2011a)



Different stellar population

From Brandt et al,. 2015



Different stellar rotation and inclination

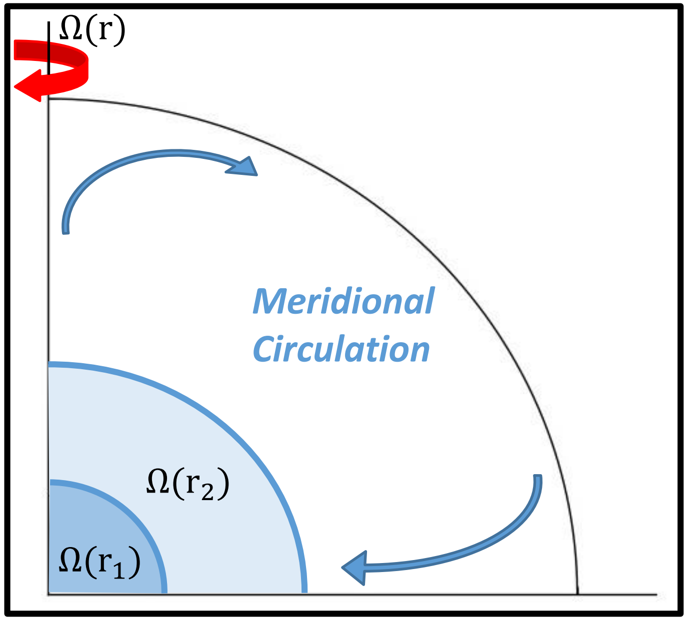
II – Effects of stellar rotation

The effect of rotation on stellar models has been studied for *decades* (Kippenhahn & Thomas 1970 ; Endal & Sofia 1976; Zahn 1992; Heger et al. 2000; Maeder & Meynet 2000; Palacios et al. 2003; Chieffi & Limongi 2013, etc…),

<u>but it remains as one of the most challenging and uncertain problems in stellar astrophysics.</u>

Effects of rotation in stars (e.g. Kippenhahn & Weigert 2012) :

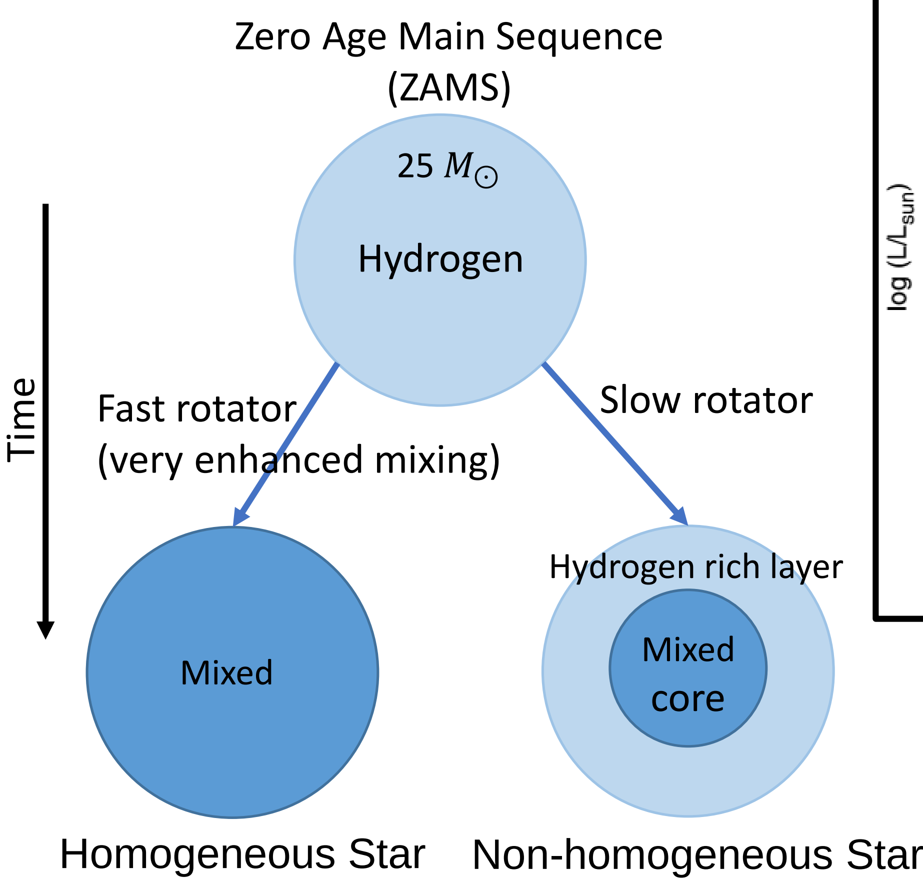
- **Centrifugal force reduces** the **effective gravity**;
- **Departure** from **spherical shape of surfaces**;
- **Effective Temperature depends** on **colatitude (θ)**;
- Rotation may induce same **mixing processes,** due to two main phenomena:
  - The **meridional circulation,** caused by the departure from hydrodynamical and radiative equilibrium;
  - The **shear friction,** caused by the differential angular velocity;



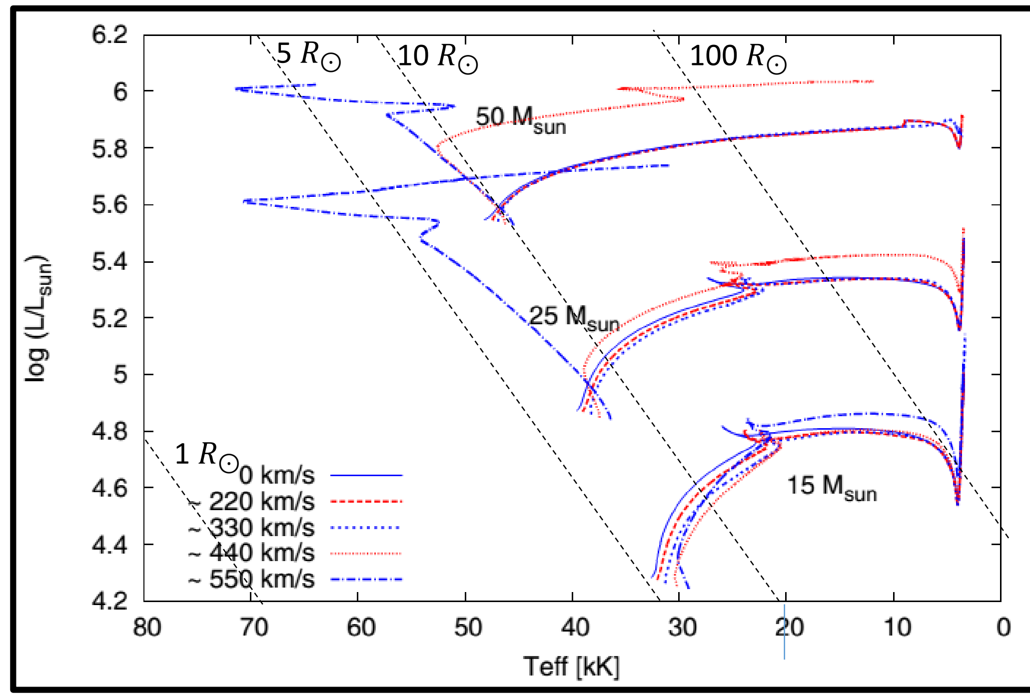
$\boxed{\Omega(r_1) \neq \Omega(r_2)}$

II – Effects of the enhanced mixing in star evolution

- Rotation significantly alter the evolution of stars :
  I. Lifetimes, in particular the stars live longer;
  II. Structure and surface abundances;
  III. Evolutionary fates;



Evolutionary tracks (Brott et al. 2011)



Homogeneous evolution takes
- More massive remnants
- Binary evolution without Roche lobe overflow (without common envelope)

II – Effects of the enhanced mixing in star evolution

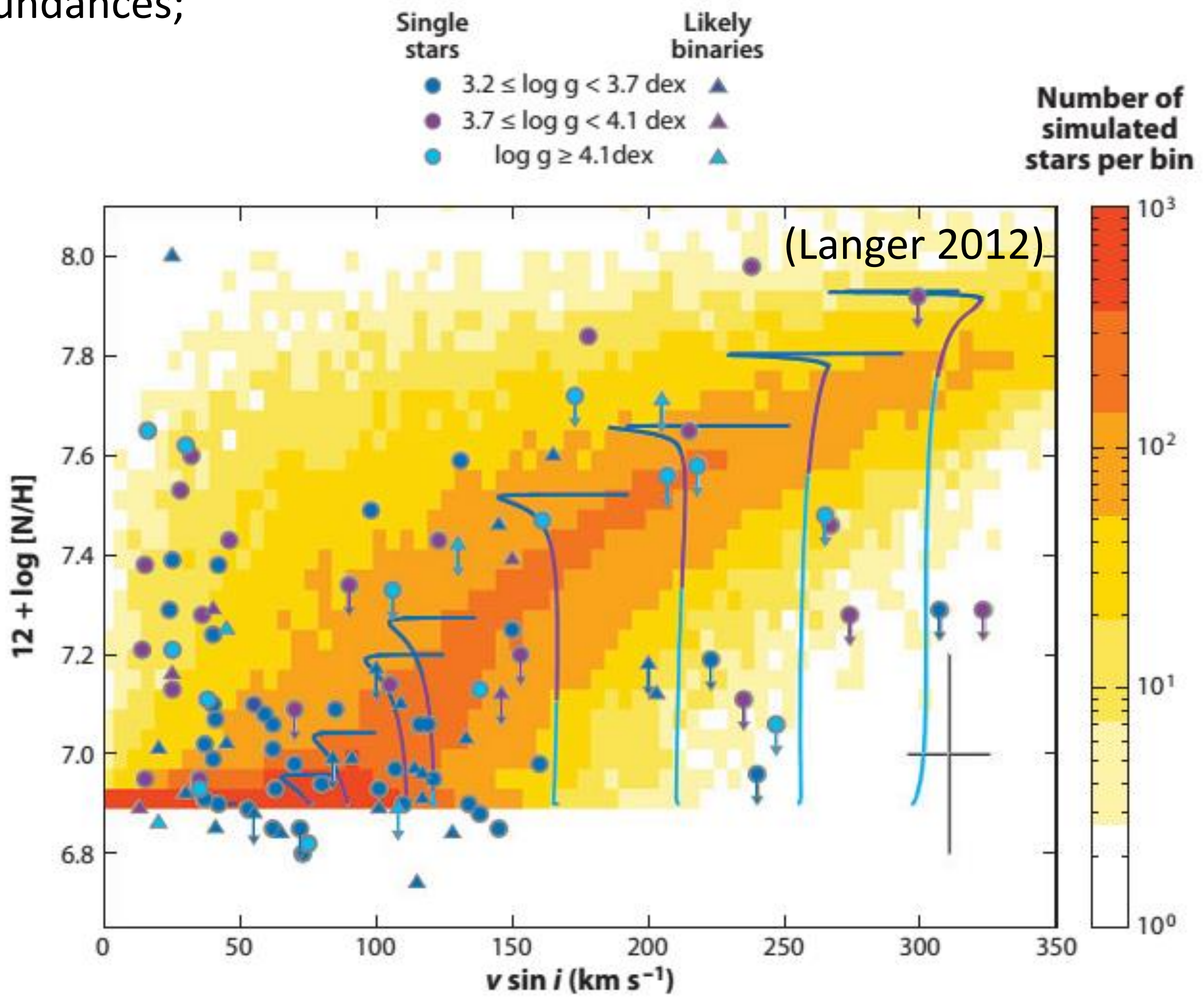
- Rotation significantly alter the evolution of stars :
  I. Lifetimes, in particular the stars live longer;
  II. Structure and surface abundances;
  III. Evolutionary fates;

Observational effect:

- The surface enhancement of N in MS stars (Mokiem et al. 2006, 2007; Hunter et al. 2009; Langer 2012);

Momentum Transport:

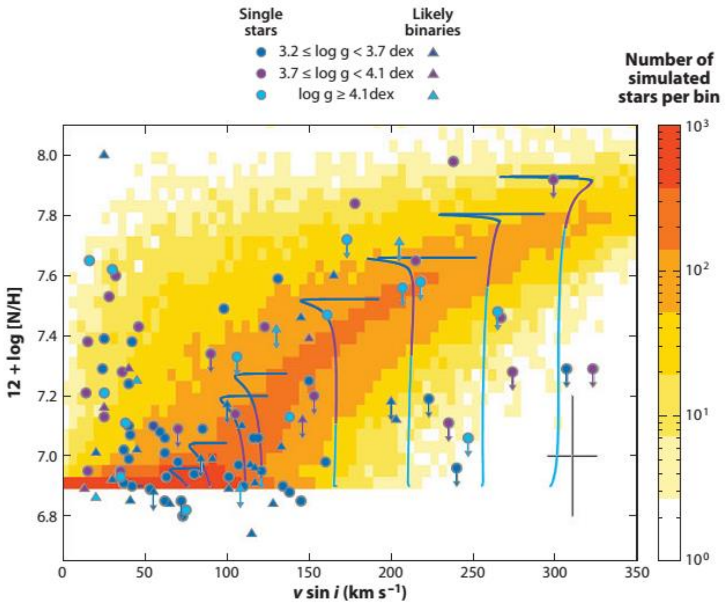
in systems in which there is angular momentum exchange, e.g. binaries or star-disk systems (or star-planets systems)



III – Goals and future perspectives

Actual Goals

- Complete the implementation of rotation features in PARSEC;
- Calibrate the parameters;
- Reproduce the N enrichment;



Future

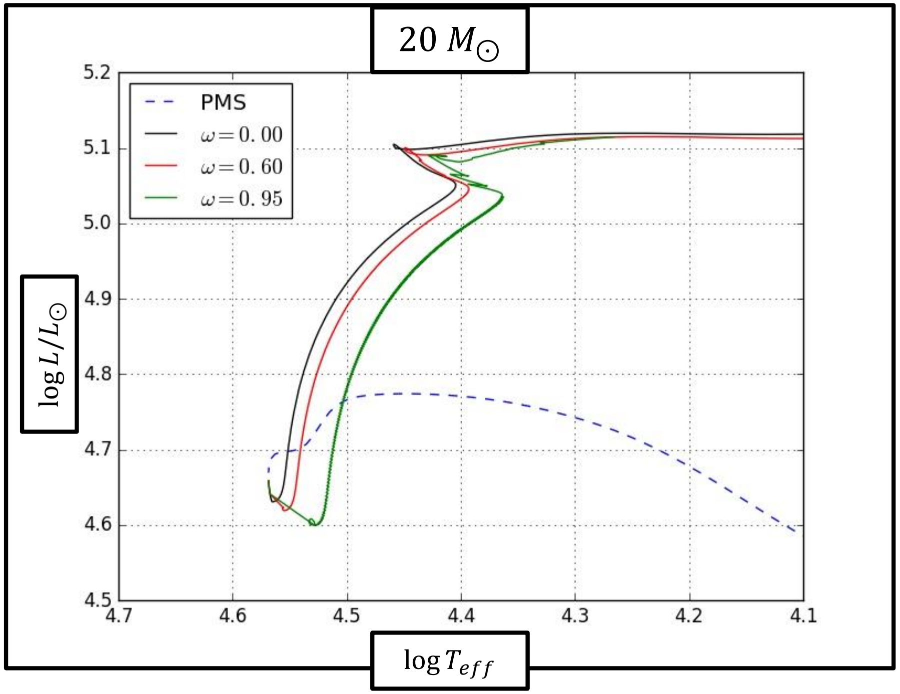
Possible application of the code in the study of:

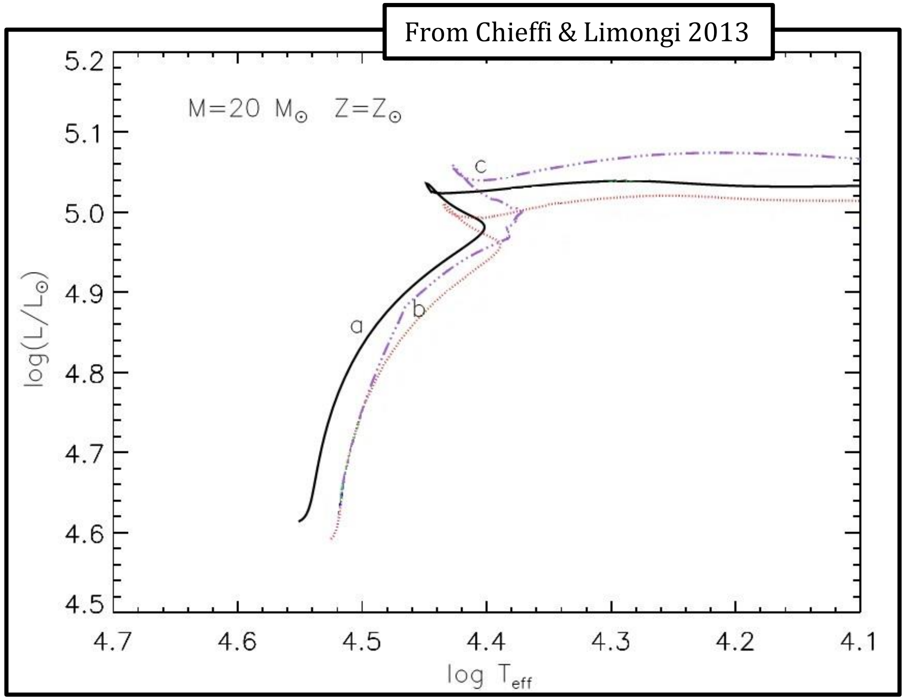
- Mixing enhancement in rotating star at different metallicities (Z);
- Star structure, with astro-sysmology (G. Mirouh);
- Evolution of Binaries systems;
- Homogeneous evolution;
- Clusters turn off;
- Etc ....

Thank you for your attention

II – Implementing rotation in PARSEC

Comparing results with a different star evolution code, e.g. FRANEC





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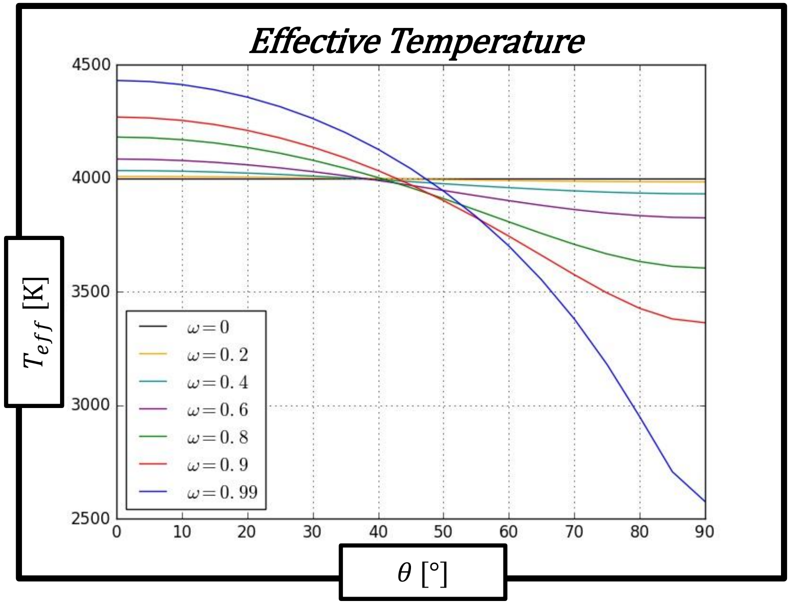
but it remains as one of the most challenging and uncertain problems in stellar astrophysics.

Effects of rotation in stars (e.g. Kippenhahn & Weigert 2012) :

- **Centrifugal force reduces** the **effective gravity**;
- **Departure** from **spherical shape of surfaces**;
- **Effective Temperature depends** on **colatitude (θ)**.
  Because the radiative flux varies with the local effective gravity (the **Von Zeipel effect**, 1924), the radiative flux is not constant on an equipotential surface;
- Rotation may induce same **mixing processes**.

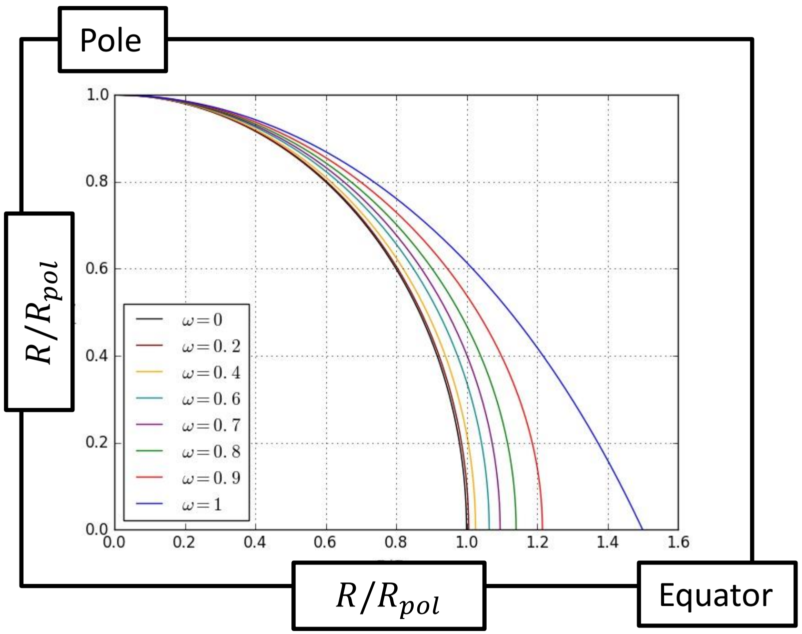


III – Shape of the star

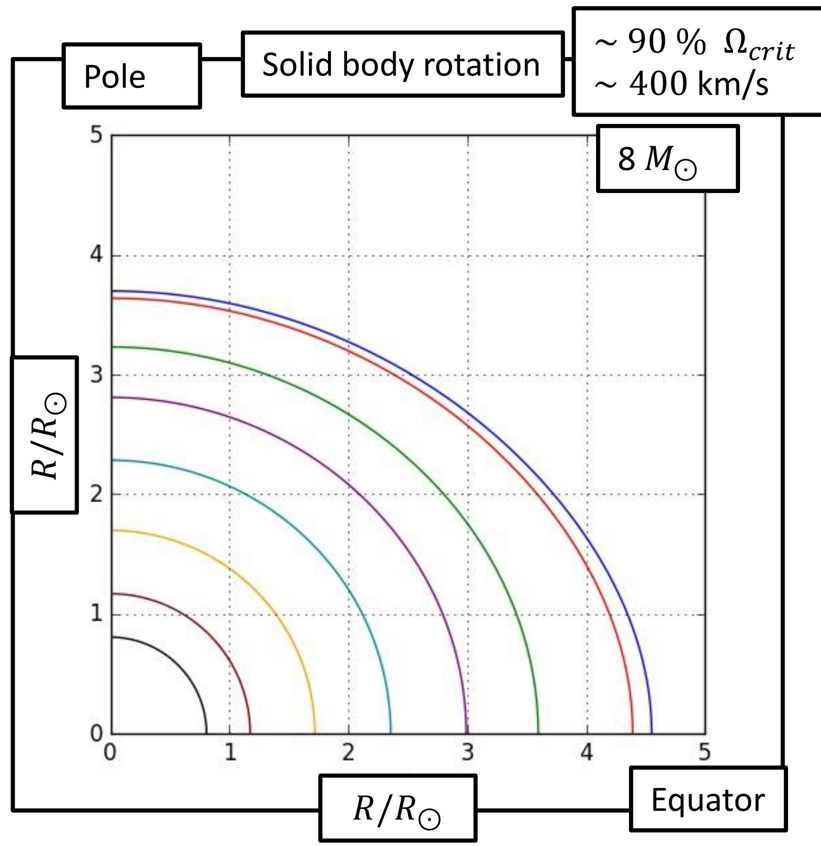
R_{pol} is the polar radius of an isobar.

$\Omega_{crit} = \left(\frac{2}{3}\right)^{\frac{3}{2}} \sqrt{\frac{G\,M_P}{R_{pol}^3}}$ is the critical angular velocity.

$\omega = \frac{\Omega}{\Omega_{crit}}$ is the dimensionless "angular velocity".



ω	$\frac{R_{eq}}{R_{pol}}$
0.0	1.000
0.2	1.006
0.4	1.026
0.6	1.064
0.8	1.141
0.9	1.216
1.0	1.500



II – Transport of angular momentum and mixing Work in progress

To insert the mixing effects due to rotation we need to treat the transport of angular momentum; Because in the general case the *stars rotate in differential way*.

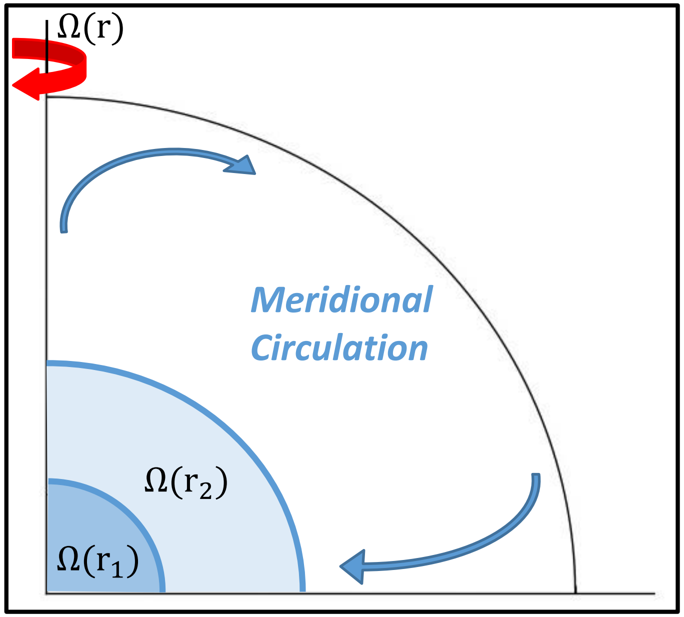
Transport of angular momentum (only diffusion) (Heger et al. 2000; Yoon & Langer 2005; Chieffi & Limongi 2013):

$$\rho r^2 \frac{dr^2\Omega}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4\, D\, \frac{\partial\Omega}{\partial r}\right)$$

$$D = \mathrm{D_{shear}} + \mathrm{D_{m.c.}}$$

Transport of chemical species:

$$\rho \frac{dX_i}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 D_i\, \frac{\partial \mathrm{X_i}}{\partial r}\right)$$



$\Omega(r_1) \neq \Omega(r_2)$

II – Stellar rotation in simulations

In principle rotation is a full **3D** problem, but thanks to **Kippenhahn & Thomas (1970), Endal & Sofia (1976), Zahn (1992)** and **Meynet & Maeder (1997)** studies, under proper assumptions we can simulate the effect of rotation in **1D** simulations.
Those are:

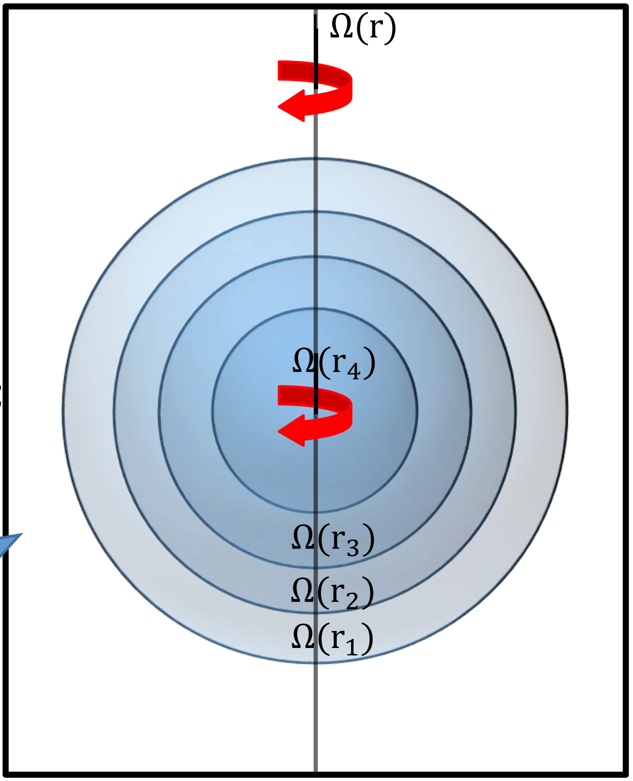
1. Roche approximation;
2. Change the spherical stratification in a

   rotationally deformed stratification (**isobars**);
3. $\Omega = const$ along the isobars (internal "*shellular*" rotation law);

   Tangenzial Diffusion >> Horizontal Diffusion

$$D_h \gg D_v$$

The **final scheme** is that the star has a *shellular* structure in which the angular velocity is constant in each shell (isobars).



This scheme is actually used by
Kepler (Heger et al. 2000); **STERN** (Yoon & Langer 2005); **Geneva code** (Eggenberger 2008); **RoSE** (Potter 2012b); **FRANEC** (Chieffi 2013); **MESA** (Paxton 2015); and now **PARSEC**.

Why it is important the rotation in the stellar physics?

- All the stars rotate! (Show the image)
- Rotation in stars significantly alter their evolution:

  I. Lifetimes;

  II. Structure and surface abundances;

  III. Evolutionary fates;

- The rotation affect the surface temperature of the star because to the gravity darkening due to the Von Zeipel effect (show the image), that changes the flux distribution on the stellar surface, due to temperature gradient
- It is fundamental to study the momentum flows through the star, in particular in systems in which there is angular momentum exchange, e.g. binaries or star-disk systems (or star-planets systems)

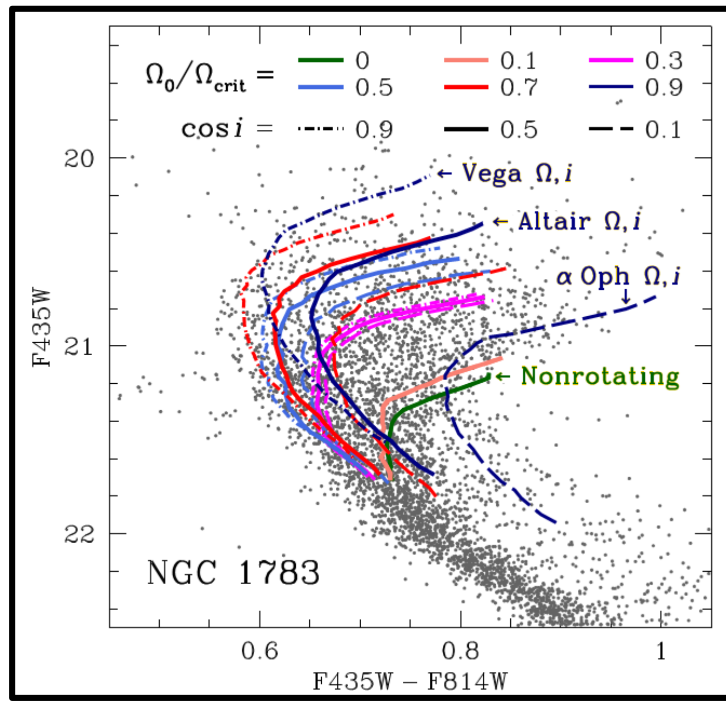
I – Introduction to Stellar Rotation

The effect of rotation on stellar models has been studied for *decades* (Kippenhahn & Thomas 1970 ; Endal & Sofia 1976; Zahn 1992; Heger et al. 2000; Maeder & Meynet 2000; Palacios et al. 2003; Chieffi & Limongi 2013, etc…),

<u>but it remains as one of the most challenging and uncertain problems in stellar astrophysics.</u>

Rotation in stars significantly alter their evolution:

- <u>Lifetimes;</u>
- <u>Surface abundances;</u>
- <u>Evolutionary fates;</u>



Possible observational effects:

- Extendend main sequence turn-off (eMSTO) in intermediate age cluster (Brandt 2015);
- The surface enhancement of He and N in MS stars (Mokiem et al. 2006, 2007; Hunter et al. 2009);
- The peculiar chemical composition of some very metal-poor stars (Limongi & Chieffi 2012);

I – Introduction to Stellar Rotation

The effect of rotation on stellar models has been studied for *decades* (Kippenhahn & Thomas 1970 ; Endal & Sofia 1976; Zahn 1992; Heger et al. 2000; Maeder & Meynet 2000; Palacios et al. 2003; Chieffi & Limongi 2013),

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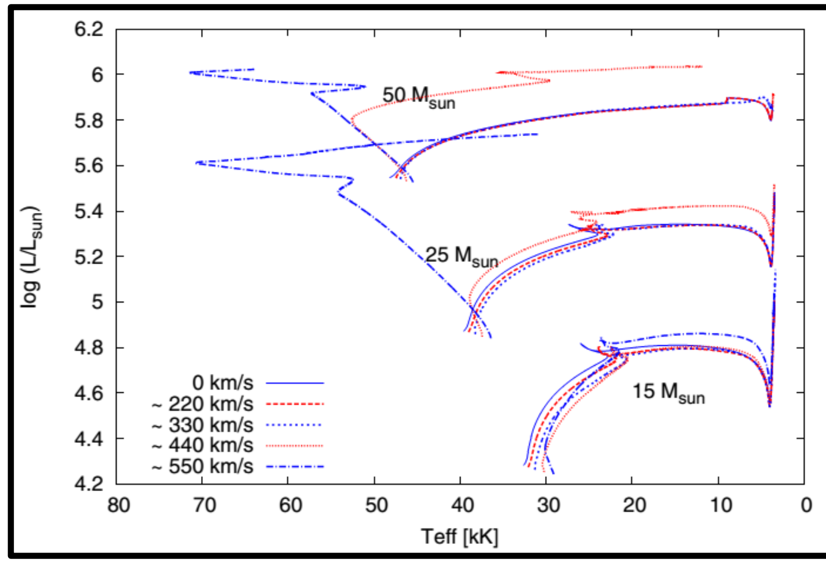
Rotation in stars significantly alter their evolution:

- Lifetimes;
- Surface abundances;
- Evolutionary fates;

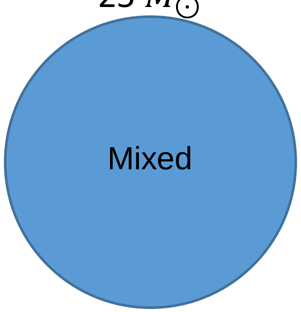
Non-homogeneous vs. homogeneous evolution.

- More massive remnants
- Binary evolution without Roche lobe overflow (without common envelope)



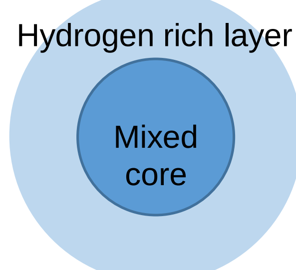
The "quasi-chemically homogeneous evolution" (Brott et al. 2011).

$25\ M_\odot$



Homogeneous Star

$25\ M_\odot$



Non-homogeneous Star

Homegeneous evolution might play a key role in questions as the progenitors of the long soft GRB's and on the production of massive binary Black Holes whose merging produce detectable gravitational wave signals Meynet et al. 2017

II - Stellar Structure Equations

Continuity equation:

$$\frac{\partial r}{\partial M} = \frac{1}{4\pi\, r^2\, \rho}$$

Hydrostatic equilibrium equation:

$$\frac{\partial P}{\partial M} = -\frac{G\ M}{4\pi\, r^4}$$

Energy conservation equation:

$$\frac{\partial L}{\partial M} = \varepsilon_n + \varepsilon_g - \varepsilon_\nu$$

Energy transport equation:

$$\frac{\partial \ln T}{\partial M} = -\frac{G\ M}{4\pi\, r^4}\frac{1}{P}\min[\nabla_{ad}, \nabla_{rad}]$$

Adiabatic gradient:

$$\nabla_{ad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad} = \frac{P\delta}{T\rho c_p}$$

Radiative gradient:

$$\nabla_{rad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{rad} = \frac{3}{16\ \pi\ ac\ G}\frac{\kappa L\ P}{M}$$

Density gradient:

$$\delta = \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$$

II - Stellar Structure + Rotation

Continuity equation:

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi\, r_P^2\, \bar{\rho}}$$

Hydrostatic equilibrium equation:

$$\frac{\partial P}{\partial M_P} = -\frac{G\, M_P}{4\pi\, r_P^4}\, f_P$$

Energy conservation equation:

$$\frac{\partial L_P}{\partial M_P} = \varepsilon_n + \varepsilon_g - \varepsilon_\nu$$

Energy transport equation:

$$\frac{\partial \ln \bar{T}}{\partial M_P} = -\frac{G\, M_P}{4\pi\, r_P^4}\frac{1}{P} f_P \min\left[\nabla_{ad}, \frac{f_T}{f_P}\nabla_{rad}\right]$$

Adiabatic gradient:

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Density gradient:

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Form Factors:

$$f_P = \frac{4\pi\, r_P^4}{G\, M_P\, S_P}\frac{1}{< g_{eff}^{-1} >}$$

$$f_T = \left(\frac{4\pi\, r_P^2}{S_P}\right)^2 \frac{1}{< g_{eff}^{-1} >< g_{eff} >}$$

II - Re-definition of variables

We define r_P that is the "volumetric" radius, given by

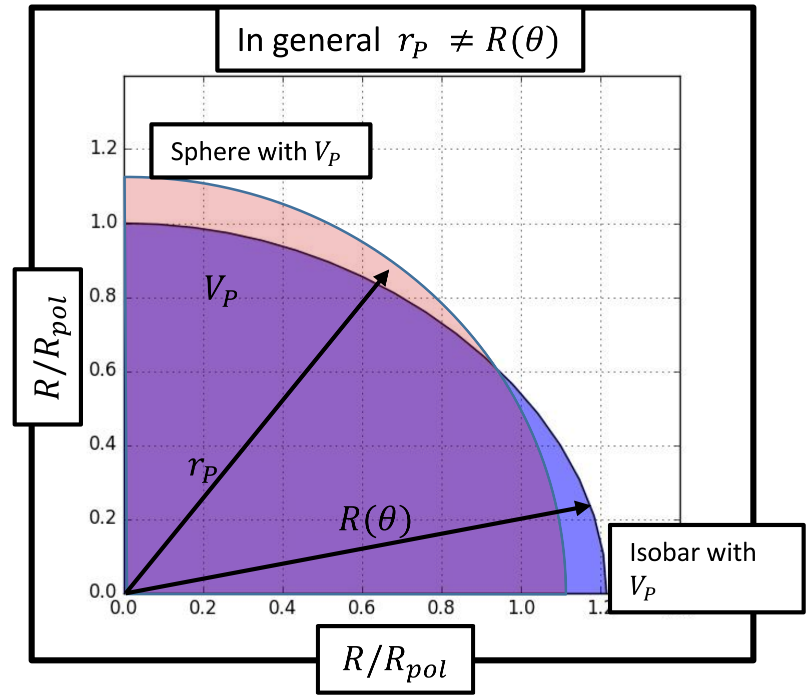
$$V_P = \frac{4\pi}{3}\, r_P,$$

where V_P is the volume inside an isobar.

For any quantity, q, which is not constant over an isobaric surface, a mean value is defined by

$$< q > = \frac{1}{S_P} \int_{\Psi = const} q \, d\sigma \,,$$

where $S_P = \int_{\Psi=const} d\sigma$ is the total surface of the isobar and $d\sigma$ is an element of that surface.



II - Properties of isobars

Roche model assumption:

- $\Phi = -\frac{GM_r}{\boldsymbol{r}}$
- For each shell it is like as the mass is all in the center.

Φ is the gravitational potential
$\boldsymbol{r}$ is the radius
Ω is the angular velocity
θ is the colatitude

Surface equation:

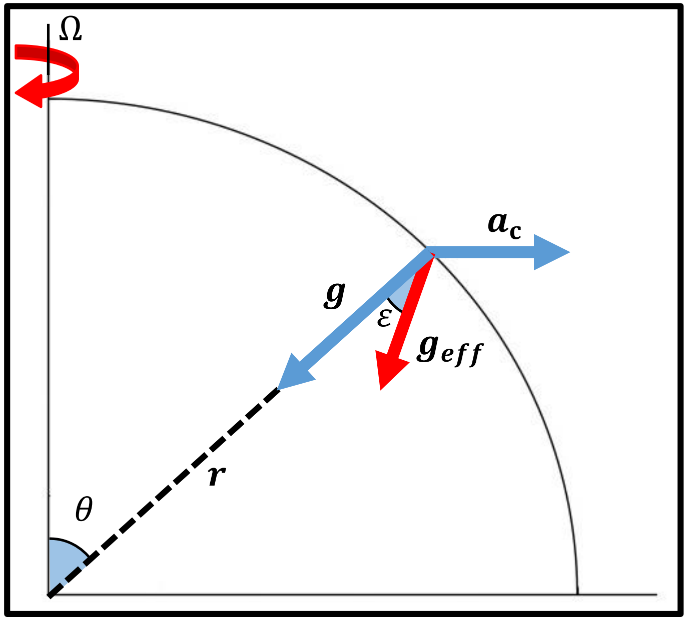
$$\Psi = \Phi - \frac{1}{2}\,\Omega^2 r^2 \,\sin^2\theta = const$$

From hydrostatic equilibrium:

$$\boldsymbol{\nabla} P = -\rho\, \boldsymbol{g_{eff}} = -\rho(\boldsymbol{\nabla}\Psi - r^2 \,\sin^2\theta\, \Omega\boldsymbol{\nabla}\Omega).$$

That implies

$$\boldsymbol{\nabla} P \parallel \boldsymbol{\nabla}\Omega \parallel \boldsymbol{\nabla}\Psi$$



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- For each shell it is like as the mass is all in the center.

Surface equation:

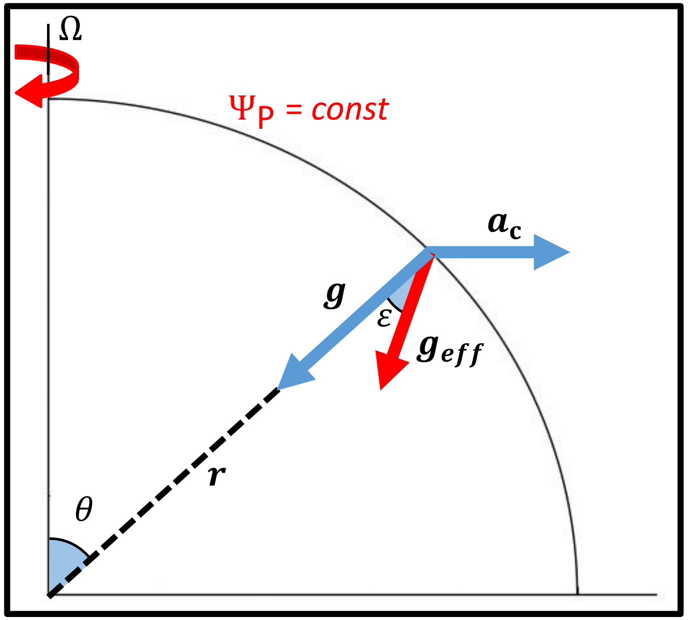
$$\Psi = \Phi - \frac{1}{2}\,\Omega^2 r^2 \sin^2\theta = const$$

Isobaric surfaces

Hydrostatic equilibrium :

$$\nabla P = -\rho(\nabla\Psi - r^2 \sin^2\theta\ \Omega\nabla\Omega).$$

Non-conservative case.



The isobaric surface are not equipotentials and the star is said to be ***baroclinic*** *(non-conservative case)*.
In case of solid body rotation, isobars and equipotentials coincide and the star is ***barotropic*** *(conservative case)*.

II - Hydrostatic equilibrium

The classical hydrostatic equation in Lagrangian form is

$$\frac{\partial P}{\partial M} = -\frac{G\ M}{4\pi\ r^4}$$

In the rotating star, we obtain

$$\frac{\partial P}{\partial M_P} = \frac{-1}{< g_{eff}^{-1} > S_P}$$

If we define the "form" factor as

$$f_P = \frac{4\pi\ r_P^4}{G\ M_P\ S_P} \frac{1}{< g_{eff}^{-1} >}$$

In the end we obtain

$$\frac{\partial P}{\partial M_P} = -\frac{G\ M_P}{4\pi\ r_P^4}\ f_P$$

II - Continuity equation

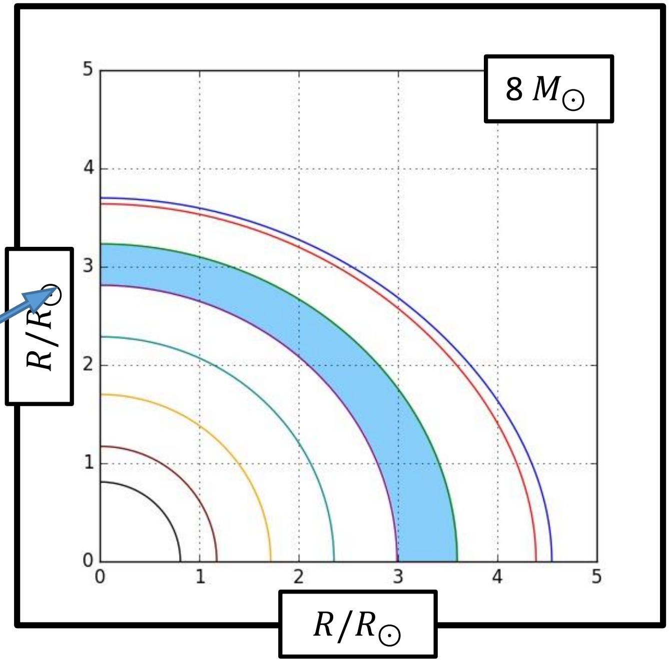
In the same way we derive the conservation of mass

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi\, r_P^2\, \bar{\rho}}$$

Where

$$\bar{\rho} = \frac{\rho(1 - r^2\, \sin^2\theta\, \Omega\, \alpha) < g_{eff}^{-1} >}{< g_{eff}^{-1} > - < g_{eff}^{-1} r^2 \sin^2\theta > \Omega\, \alpha}$$

Is the density averaged in the shell between two isobars.



In general $< \rho > \neq \bar{\rho}$,

but when the mass steps are very small the difference becomes negligible. And so

$$< \rho > \cong \bar{\rho}.$$

II - Stellar Structure + Rotation

Continuity equation:

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi\, r_P^2\, \bar{\rho}}$$

Hydrostatic equilibrium equation:

$$\frac{\partial P}{\partial M_P} = -\frac{G\, M_P}{4\pi\, r_P^4}\, f_P$$

Energy conservation equation:

$$\frac{\partial L_P}{\partial M_P} = \varepsilon_n(\bar{\rho},\bar{T}) + \varepsilon_g(\bar{\rho},\bar{T}) - \varepsilon_\nu(\bar{\rho},\bar{T})$$

Energy transport equation:

$$\frac{\partial \ln \bar{T}}{\partial M_P} = -\frac{G\, M_P}{4\pi\, r_P^4}\frac{1}{P} f_P \min\left[\nabla_{ad}, \frac{f_T}{f_P}\nabla_{rad}\right]$$

Adiabatic gradient:

$$\nabla_{ad} = \left(\frac{\partial \ln \bar{T}}{\partial \ln P}\right)_{ad} = \frac{P\delta}{\bar{T}\bar{\rho}c_p}$$

Radiative gradient:

$$\nabla_{rad} = \left(\frac{\partial \ln \bar{T}}{\partial \ln P}\right)_{rad} = \frac{3}{16\,\pi\, ac\, G}\frac{\kappa L_P P}{M_P}$$

Density gradient:

$$\delta = \left(\frac{\partial \ln \bar{\rho}}{\partial \ln \bar{T}}\right)_{P,\mu}$$

Form Factors:

$$f_P = \frac{4\pi\, r_P^4}{G\, M_P\, S_P}\frac{1}{< g_{eff}^{-1} >}$$

$$f_T = \left(\frac{4\pi\, r_P^2}{S_P}\right)^2 \frac{1}{< g_{eff}^{-1} >< g_{eff} >}$$

II - Calculating f_P and f_T

To calculate the form factors, we used the same variables defined before, and obtain

$$f_P = \left(\frac{r_P}{R_{Pol}}\right)^4 \frac{1}{\Sigma_P < g_{ad}^{-1} >}$$

$$f_T = \left(\frac{r_P}{R_{Pol}}\right)^4 \frac{1}{\Sigma_P^2 < g_{ad} >< g_{ad}^{-1} >}$$

$$V_P = \frac{4}{3}\pi\, r_P^3 = \frac{4}{3}\pi\, R_{pol}^3 V'$$

$$\frac{r_P^3}{R_{pol}^3} = V'$$

$$\rho_M = \frac{M_P}{V_P}$$

$$\frac{\Omega^2}{2\pi G\, \rho_M} = \omega^2 V'$$

$R(\theta)$ is the radius for an isobar only function of θ.

$\Omega_{crit} = \left(\frac{2}{3}\right)^{\frac{3}{2}} \sqrt{\frac{G\, M_P}{R_{pol}^3}}$ is the critical angular velocity.

$\omega = \frac{\Omega}{\Omega_{crit}}$ is the dimensionless “angular velocity”.

$x(\theta) = \frac{R(\theta)}{R_{pol}}$ is the dimensionless radius for an isobar.

Dimensionless quantities

$S_P = 4\pi\, R_{pol}^2 \Sigma_P$ $\quad\quad \Sigma_P = \int_{\Psi=const} d\sigma$

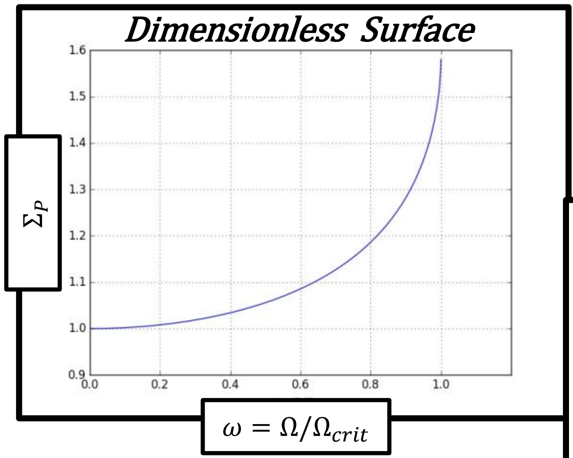
$$< g_{eff} > = \frac{GM_P}{R_{pol}^2} < g_{ad} >$$

$$< g_{eff}^{-1} > = \frac{R_{pol}^2}{GM_P} < g_{ad}^{-1} >$$

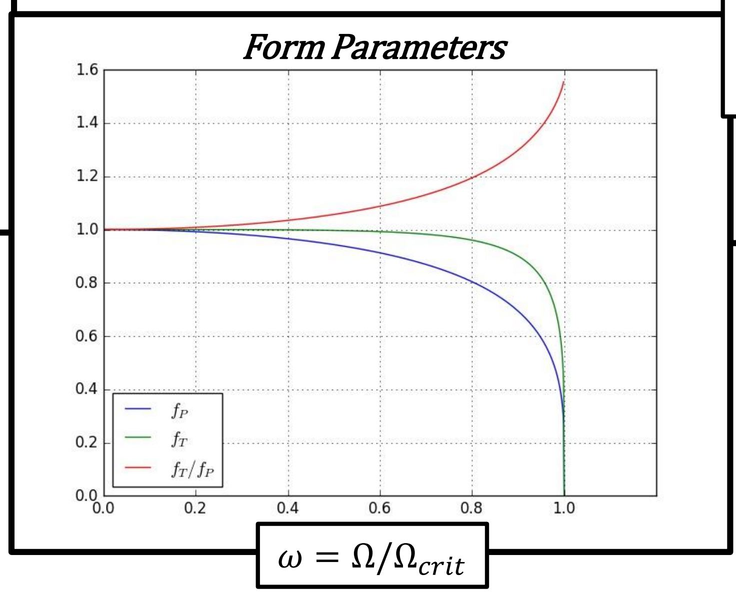
$V_P = \frac{4}{3}\pi\, R_{pol}^3 V'$ $\quad\quad V' = \int_0^{\Psi} dn\, d\sigma$

II - Calculating f_P and f_T

Quantities <u>independent</u> from any physical variables of the star, they are only function of ω.
<u>We can compute them analytically</u>.

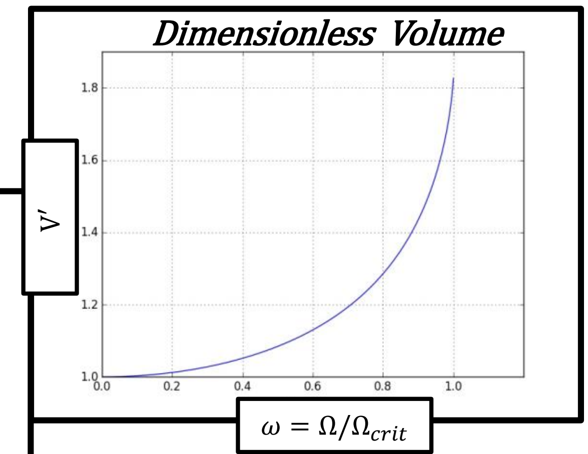


$\Sigma_P\ (\omega)$



$f_P\ (\omega)$

$f_T\ (\omega)$

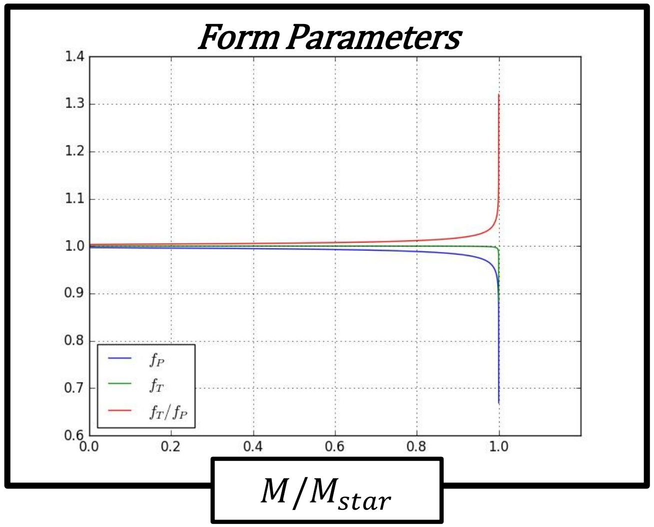


$V'\ (\omega)$

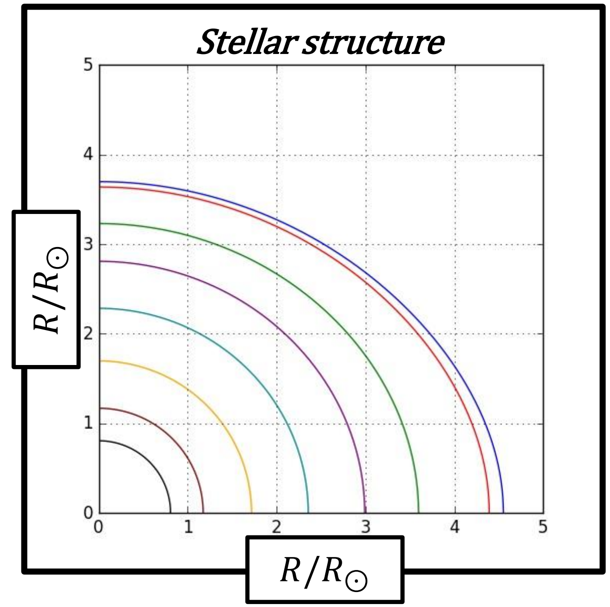
II - Implementing rotation in PARSEC

Physical variables → $$\frac{\Omega^2}{2\pi G\ \rho_M} = \omega^2 V'$$ ← *Dimensionless variables*

ω	$\frac{\Omega^2}{2\pi G\ \rho_M}$
0.0	0.000
0.2	0.009
0.4	0.035
0.6	0.085
0.8	0.165
0.9	0.223
1.0	0.361



$$f_P\ (M/M_{star})$$
$$f_T\ (M/M_{star})$$



II - Von Zeipel Effect

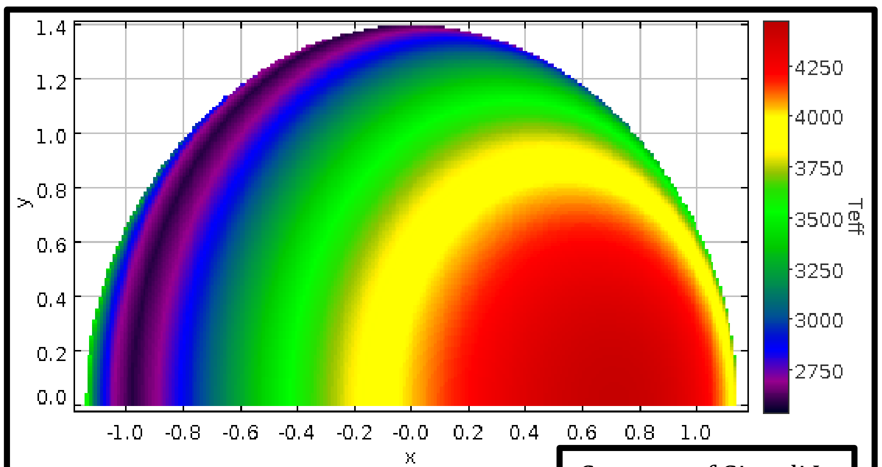
The ***Von Zeipel theorem*** (1924) is

$$\boldsymbol{F}(\Omega,\theta) = -\frac{L}{4\pi\ G\ M^*}\ \boldsymbol{g}_{eff}(\Omega,\theta)$$

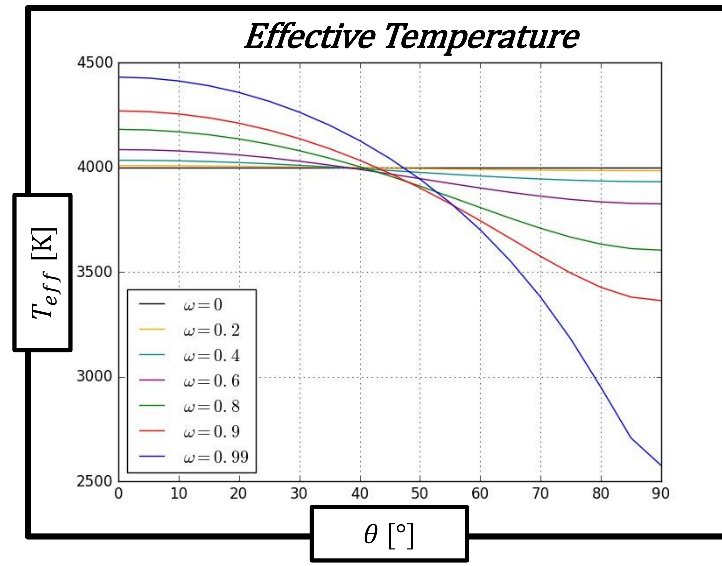
We can easily find the ***effective surface temperature*** at

$$T_{eff}(\Omega,\theta) = -\left(\frac{L}{4\pi\ \sigma\ G\ M^*}\right)^{\frac{1}{4}} \left[g_{eff}(\Omega,\theta)\right]^{\frac{1}{4}}$$

$M^* = M\left(1 - \frac{\Omega^2}{2\pi G\ \rho_M}\right)$ is the effective mass.
σ is the Stefan-Boltzmann constant.



Courtesy of Girardi L.



II – Simple application

Hubble Space Telescope		Cycle 25 AR Proposal		**<ID>**

Looking for Photometric Signatures of Fast Rotation in Intermediate-Age Star Clusters in the Magellanic Clouds

Investigators:

	Investigator	Institution	Country
	A Bellini	Space Telescope Science Institute	USA/MD
*	A Bressan	Scuola Internazionale Superiore di Studi Avanzati	ITA
*	Y Chen	Universita degli Studi di Padova	ITA
	M Correnti	Space Telescope Science Institute	USA/MD
*	G Costa	Scuola Internazionale Superiore di Studi Avanzati	ITA
*	L Girardi	Osservatorio Astronomico di Padova	ITA
	P Goudfrooij	Space Telescope Science Institute	USA/MD

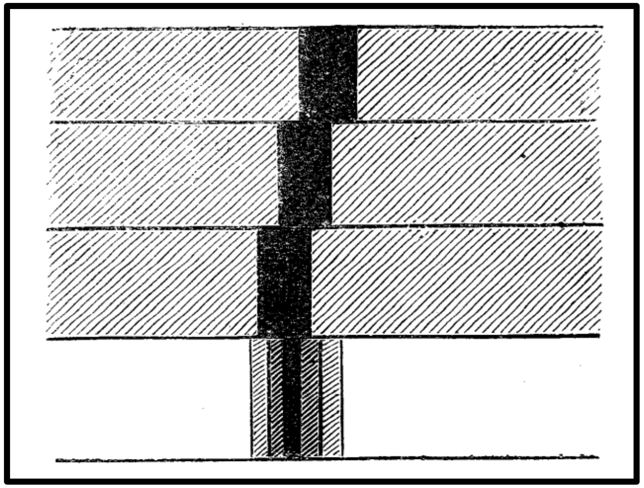
I – Introduction to Stellar Rotation

Some historical events:

At the times of **Galileo (1600)** was discovered that the Sun rotates.

In **1877 Cpt. Abney W.** supposed that we can observe the rotation in stars through the Doppler shift of the spectrum lines.

Finally in **1909 Schlesinger** was able to detect the shift, and derive the rotation of the star δ Librae.



From **Cpt. Abney W. 1877.**
Conclusions:
"I am convinced that from a good photograph much might be determined."

First stellar evolution models:

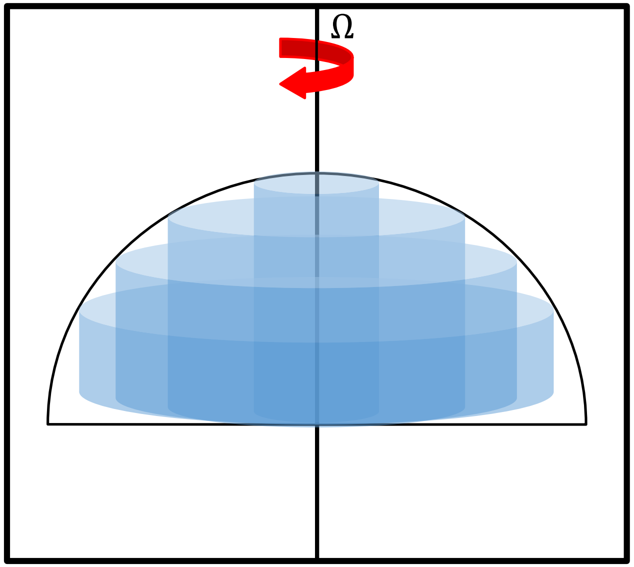
Henyey (1964) and later **Kippenhahn & Weigert (1967)** developed the first models of stellar evolution that are the basis of all the *new generation codes*, like

- **PARSEC** (Bressan et al. 2012)
- **Geneva code** (Eggenberger et al. 2008)
- **FRANEC** (Chieffi & Limongi 2013)
- **MESA** (Paxton et al. 2013).
- **RoSE** (Potter et al. 2012b)
- **STERN** (Yoon & Langer 2005)
- **Kepler** (Heger et al. 2000)

II - Stellar Rotation in simulations

In principle rotation is a full **3D** problem, but thanks to **Kippenhahn & Thomas (1970)** and **Endal & Sofia (1976)** studies, under proper assumptions we can simulate the effect of rotation in **1D** simulations. Those are:

1. Change the spherical stratification with a rotationally deformed stratification (**equipotentials**);
2. Cylindrical symmetry for the angular velocity;
3. $\boldsymbol{\Omega = \mathrm{const}}$ along the **equipotential** surfaces;
4. **Roche approximation** (for simplicity).



With this scheme the **surface of the star coincides with the equipotential surface** (conservative case).

But the combination of cylindrical symmetry (2.) and $\Omega = const$ (3.) necessarily implies a **solid body rotation**, and this would **limit** the application of the Kippenhahn & Thomas (1970) scheme to differentially rotating stars.

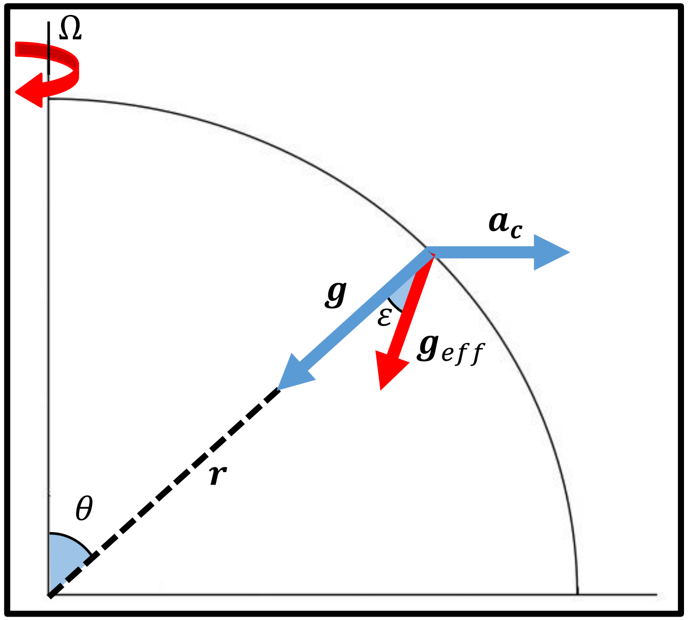
II - Stellar Rotation in simulations

In principle rotation is a full **3D** problem, but thanks to **Kippenhahn & Thomas (1970)** and **Endal & Sofia (1976)** studies, under proper assumptions we can simulate the effect of rotation in **1D** simulations. Those are:

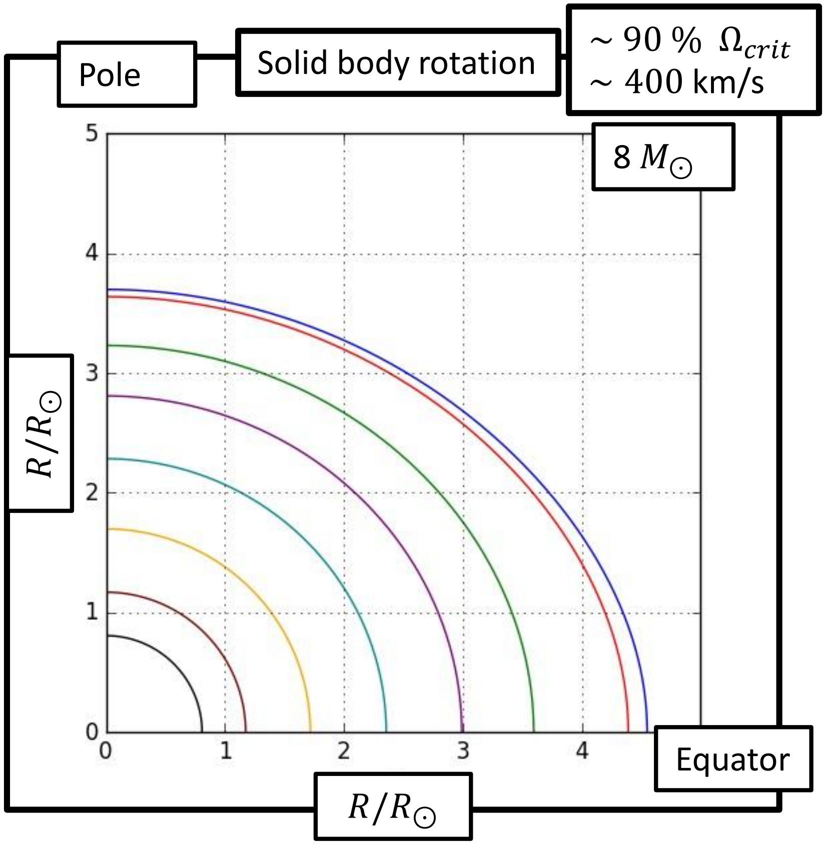
1. Change the spherical stratification with a rotationally deformed stratification (**isobars**);
2. Cylindrical symmetry for the angular velocity;
3. $\Omega = const$ along the **isobars** ("shellular" rotation law);
4. Roche approximation.

Zahn (1992) supports the 3rd assumption with the introduction of the internal "shellular" rotation law.

The stellar shells do not coincide anymore with equipotentials, but with **isobars**.



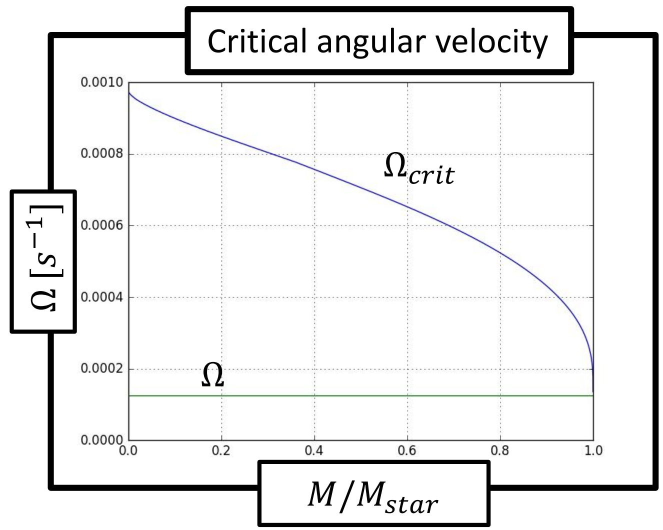
II - Shape of the star



$$\Omega_{crit} = \left(\frac{2}{3}\right)^{\frac{3}{2}} \sqrt{\frac{G\,M_P}{R_{pol}^3}}$$

$$\omega = \frac{\Omega}{\Omega_{crit}}$$

$$x(\theta) = \frac{R(\theta)}{R_{pol}}$$

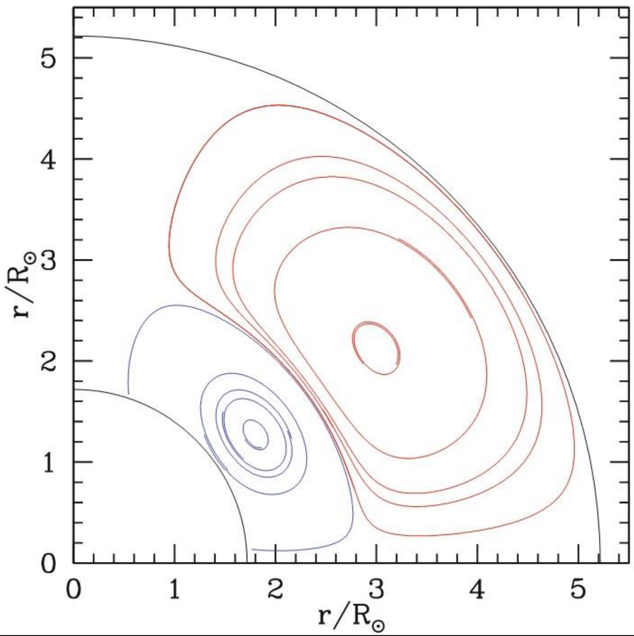


III - Meridional Circulation

$$U(r) \propto \left(1 - \frac{\Omega^2}{2\pi G\,\rho}\right)$$

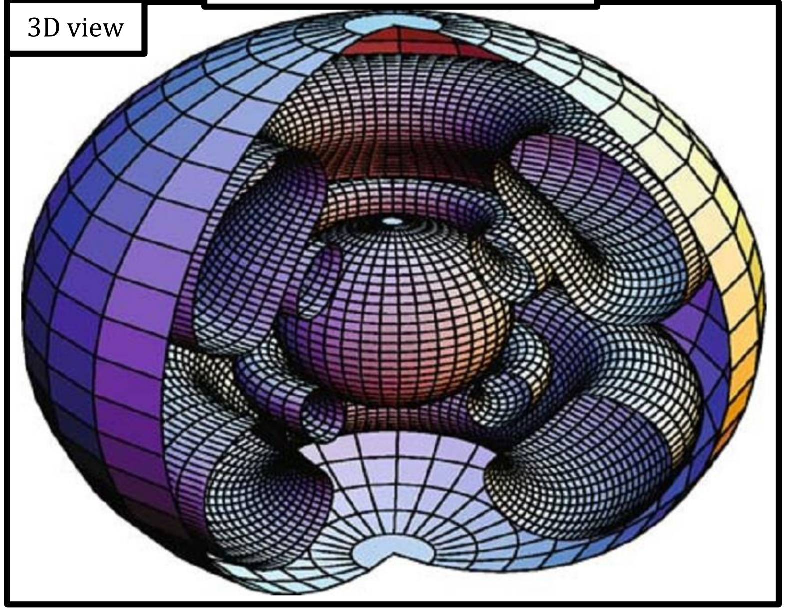
local density

From Maeder 2009



From Maeder & Meynet 2002

3D view



III - Transport of angular momentum work in progress

To insert the mixing effects due to rotation we need to treat the transport of angular momentum;
Because in the general case the *stars rotate in differential way*.

Transport of angular momentum:

$$\rho r^2 \frac{dr^2\Omega}{dt} = \underbrace{\frac{1}{5}\frac{\partial}{\partial r}(\rho r^4 \Omega \mathrm{U}(r))}_{\textbf{advection}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4\, D_{shear}\frac{\partial\Omega}{\partial r}\right)}_{\textbf{diffusion}}$$

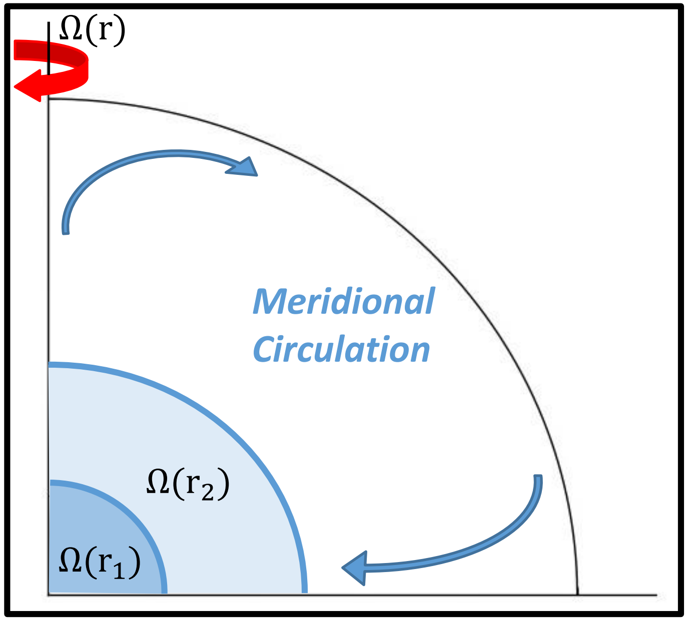
Transport of angular momentum (only diffusion) (Heger et al. 2000; Yoon & Langer 2005; Chieffi & Limongi 2013):

$$\rho r^2 \frac{dr^2\Omega}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4\, D\frac{\partial\Omega}{\partial r}\right)$$

$$D = \mathrm{D_{shear}} + \mathrm{D_{m.c.}}$$

Transport of chemical species:

$$\rho \frac{dX_i}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 D_i\, \frac{\partial \mathrm{X_i}}{\partial r}\right)$$



$\Omega(\mathrm{r_1}) \neq \Omega(\mathrm{r_2})$

III - Meridional Circulation

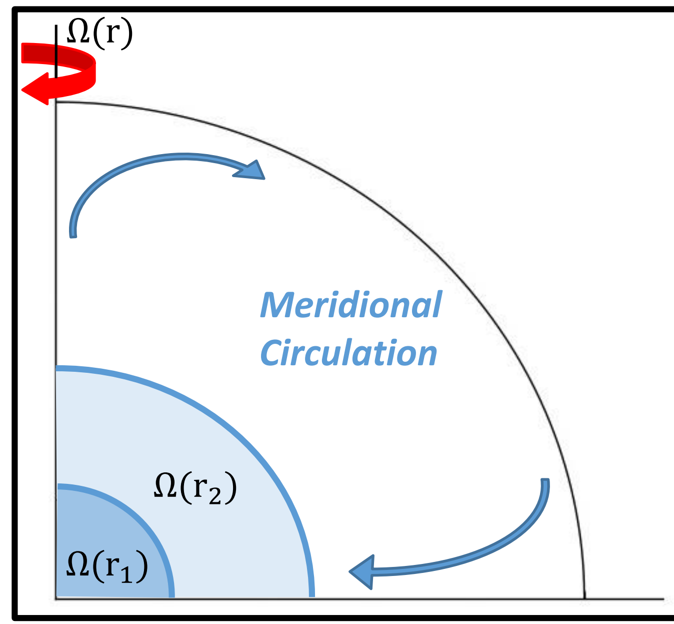
Maeder & Zahn 1998 developed the theory of **Zahn** 1992, they expand all the physical variables in a *radial* (vertical) component and in a *horizontal* one.

The general expression for the radial component of the velocity is (Eggenberger 2008 , Chieffi & Limongi 2013):

$$U(r) = \frac{P}{\bar{\rho}\bar{g}c_p\bar{T}\left[\nabla_{ad} - \nabla_{rad} + \left(\frac{\varphi}{\delta}\right)\nabla_\mu\right]}\left\{\frac{L}{M^*}\left(E_\Omega + E_\mu\right)\right\}$$

$$\nabla_{ad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad} \; ; \; \nabla_{rad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{rad} \; ; \; \nabla_\mu = \frac{d \ln \mu}{d \ln P}$$

$$\delta = \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu} \; ; \quad \varphi = \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T}$$



$\Omega(r_1) \neq \Omega(r_2)$

In case of solid body one could use following **Kippenhahn & Weigert** 1990, (Heger 2000, Chieffi & Limongi 2013, Paxton 2013):

$$U(r) = \frac{8}{3}\frac{\Omega^2 r}{g}\frac{L}{M^* g}\frac{\gamma - 1}{\gamma}\frac{1}{\nabla_{ad} - \nabla}\left(1 - \frac{\Omega^2}{2\pi G \rho}\right)$$