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# Higgs boson decay to $J/\psi$ via $c$ -quark fragmentation

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May 9, 2022

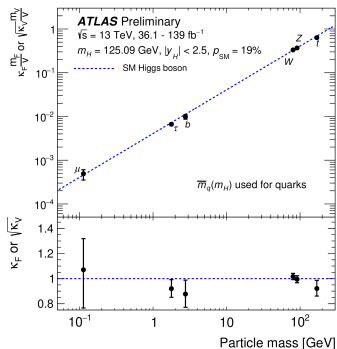
2202.08273, in collaboration with  
T. Han, A. Leibovich (PITT), X. Tan (HIT)

# Why Charm-Higgs coupling?

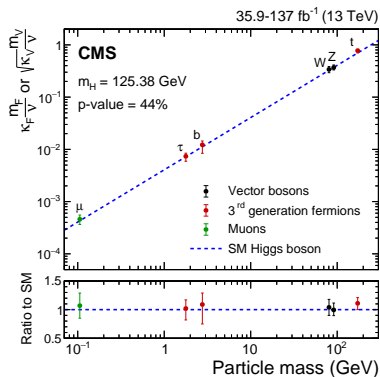
## Higgs is special

- Higgs provides masses to all other elementary particles.
- Higgs is the only known elementary particle with spin 0.
- A portal to new physics beyond the Standard Model.

## Measure the Higgs couplings



[ATLAS-CONF-2021-053]



[2009.04363]

Higgs to light fermion couplings are to be measured

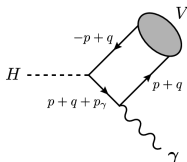
# Measuring Charm-Higgs coupling: current status

## Measuring $Hc\bar{c}$ coupling is not easy

- Smaller mass  $\Rightarrow$  Smaller branching fraction:  $\text{BR}(H \rightarrow c\bar{c}) \simeq 2.9\%$
- Large QCD background at hadron colliders  $\Rightarrow$  Need  $c$ -tagging
- $c$ -tagging is challenging

## Current experimental searching

- $\kappa$  framework: For  $y_c^{\text{SM}} = \sqrt{2}m_c/v$ , set  $y_c = \kappa_c y_c^{\text{SM}}$
- $pp \rightarrow VH(c\bar{c})$ 
  - Need  $c$ -tagging.
  - LHC Run 2: ATLAS  $\kappa_c \leq 8.5$  [ATLAS-CONF-2021-021], CMS  $1.1 < |\kappa_c| < 5.5$  [CMS-PAS-HIG-21-008]
  - Future HL-LHC:  $\kappa_c \leq 3$ . [2201.11428, ATL-PHYS-PUB-2021-039]
- To avoid  $c$ -tagging  $\Rightarrow$  Higgs decay to  $J/\psi$ 
  - Clean final states  $J/\psi \rightarrow \mu^+\mu^-$ , may avoid  $c$ -tagging
  - Use an addition photon as trigger:  $H \rightarrow J/\psi + \gamma$
  - The rate is too low:  $BR \sim 10^{-6}$ . [1306.5770, 1407.6695]
  - Result is less sensitive:  $\kappa_c \leq 100$ . [1807.00802, 1810.10056]



# Higgs decay to charmonia (I)

## The Nonrelativistic QCD framework

- The Higgs decay width in NRQCD factorization

$$\Gamma = \sum_{\mathbb{N}} \hat{\Gamma}_{\mathbb{N}}(H \rightarrow (Q\bar{Q})[\mathbb{N}] + X) \times \langle \mathcal{O}^h[\mathbb{N}] \rangle, \quad d\hat{\Gamma}_{\mathbb{N}} = \frac{1}{2m_H} \frac{|\mathcal{M}|^2}{\langle \mathcal{O}^{Q\bar{Q}} \rangle} d\Phi_3$$

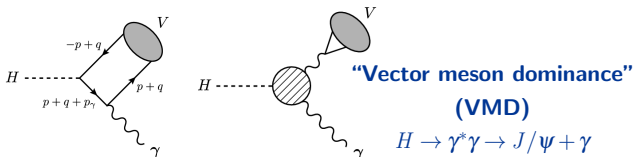
- Long distance matrix element (LDME) are related to the wave function at origin

$$\begin{aligned} \langle \mathcal{O}^{J/\psi}[{}^3S_1^{[1]}] \rangle &= \frac{3N_c}{2\pi} |R(0)|^2, & \langle \mathcal{O}^{\eta_c}[{}^1S_0^{[1]}] \rangle &= \frac{N_c}{2\pi} |R(0)|^2 \\ \langle \mathcal{O}^{Q\bar{Q}} \rangle &= 6N_c, \text{ for } {}^3S_1^{[1]}, & \langle \mathcal{O}^{Q\bar{Q}} \rangle &= 2N_c, \text{ for } {}^1S_0^{[1]} \end{aligned}$$

## Higgs decay to $J/\psi$ and a photon

- $Hc\bar{c}$  diagram is suppressed  $\Rightarrow$  Small branching fraction
- The dominant contribution is from  $H\gamma\gamma$  diagram  $\Rightarrow$  Less sensitive to  $\kappa_c$

$$\Gamma_{H\gamma\gamma^*} \simeq 1.32 \times 10^{-8} \text{ GeV}, \quad \Gamma_{\text{SM}} \simeq 1.00 \times 10^{-8} \text{ GeV} \quad [1306.5770, 1407.6695]$$



# Higgs decay to charmonia (II)

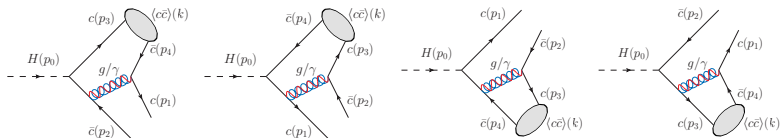
## Our idea

- Take advantage of the clean  $J/\psi \rightarrow \mu^+ \mu^-$  decay
- Look for a process

$$H \rightarrow c + \bar{c} + J/\psi \text{ (or } \eta_c)$$

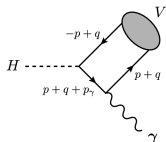
- The rate is larger than that of  $H \rightarrow J/\psi + \gamma$
- The  $Hc\bar{c}$  channel dominates over possible contaminations

**Color-singlet mode:** Charm quark fragmentation to  $^3S_1^{[1]}(J/\psi)$  and  $^1S_0^{[1]}(\eta_c)$



## Compare with $H \rightarrow J/\psi + \gamma$

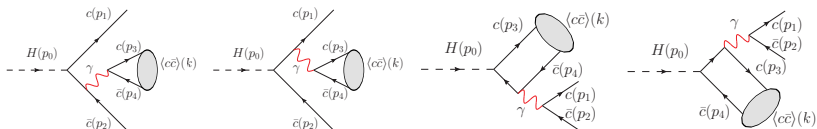
- Enhancement from the quark fragmentation  
 $\Rightarrow$  Larger rate
- The decay width is more sensitive to  $\kappa_C$



# More corrections from QED and EW sector

## Pure QED diagrams: sizable correction to $^3S_1^{[1]}(J/\psi)$ production

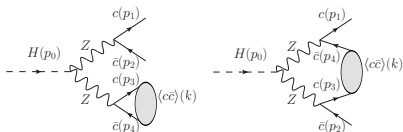
The photon propagator  $1/q^2 = 1/m_{J/\psi}^2$



Single photon fragmentation (SPF)  $\Rightarrow$  **logarithmic enhancement**

## Electroweak correction from the $HZZ$ diagrams

This may be the contamination for Charm-Higgs coupling determination



One of the  $Z$  can be on shell  $\Rightarrow$  **resonance enhancement**

- The resonance peak can be seen in the  $J/\psi(\eta_c)$  energy distribution.
- Sizable for  $^1S_0^{[1]}(\eta_c)$  due to the larger axial  $Zc\bar{c}$  coupling.

# Charmonia production via color-octet states

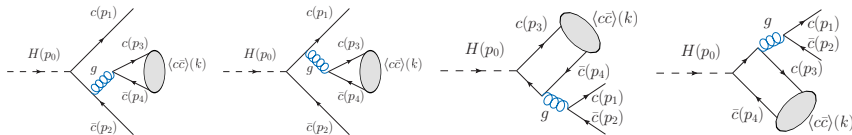
## A key property of NRQCD: color-octet states also contribute

- A quarkonium can also be produced through color-octet  $Q\bar{Q}$  Fock states
- New states involved:  $^3S_1^{[8]}$ ,  $^1S_0^{[8]}$ ,  $^3P_J^{[8]}$ , and  $^1P_1^{[8]}$
- The LDMEs  $\langle \mathcal{O}^h [^{2S+1}L_J^{\text{color}}] \rangle$  need to be fitted from experimental data

Reference	$\langle \mathcal{O}^{J/\psi} [^1S_0^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [^3S_1^{[8]}] \rangle$	$\langle \mathcal{O}^{J/\psi} [^3P_0^{[8]}] \rangle / m_c^2$
G. Bodwin,	$(9.9 \pm 2.2) \times 10^{-2}$	$(1.1 \pm 1.0) \times 10^{-2}$	$(4.89 \pm 4.44) \times 10^{-3}$
K.T. Chao,	$(8.9 \pm 0.98) \times 10^{-2}$	$(3.0 \pm 1.2) \times 10^{-3}$	$(5.6 \pm 2.1) \times 10^{-3}$
Y. Feng,	$(5.66 \pm 4.7) \times 10^{-2}$	$(1.77 \pm 0.58) \times 10^{-3}$	$(3.42 \pm 1.02) \times 10^{-3}$

## New diagrams for $^3S_1^{[8]}$

Similar to the SPF: The gluon propagator  $1/q^2 = 1/m_J^2/\psi$



Single gluon fragmentation (SGF)  $\Rightarrow$  **logarithmic enhancement**

# Standard Model results (I)

## Numerical parameters

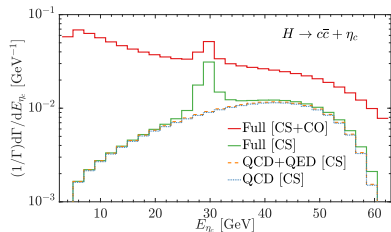
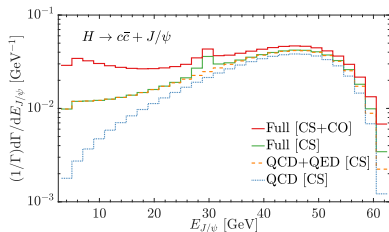
$$\alpha = 1/132.5, \quad \alpha_s(2m_c) = 0.235, \quad m_c^{\text{pole}} = 1.5 \text{ GeV}, \quad m_c(m_H) = 0.694 \text{ GeV}, \\ m_H = 125 \text{ GeV}, \quad m_W = 80.419 \text{ GeV}, \quad m_Z = 91.188 \text{ GeV}, \quad v = 246.22 \text{ GeV}.$$

$$y_c^{\text{SM}} = \frac{\sqrt{2}m_c(m_H)}{v} \approx 3.986 \times 10^{-3},$$

## Decay width and branching fraction

	QCD [CS]	QCD+QED [CS]	Full [CS]	Full [CO]	Full [CS+CO]
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$4.8 \times 10^{-8}$	$5.8 \times 10^{-8}$	$6.1 \times 10^{-8}$	$2.2 \times 10^{-8}$	$8.3 \times 10^{-8}$
$\text{BR}(H \rightarrow c\bar{c} + J/\psi)$	$1.2 \times 10^{-5}$	$1.4 \times 10^{-5}$	$1.5 \times 10^{-5}$	$5.3 \times 10^{-6}$	$2.0 \times 10^{-5}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$4.9 \times 10^{-8}$	$5.1 \times 10^{-8}$	$6.3 \times 10^{-8}$	$1.8 \times 10^{-7}$	$2.4 \times 10^{-7}$
$\text{BR}(H \rightarrow c\bar{c} + \eta_c)$	$1.2 \times 10^{-5}$	$1.2 \times 10^{-5}$	$1.5 \times 10^{-5}$	$4.5 \times 10^{-5}$	$6.0 \times 10^{-5}$

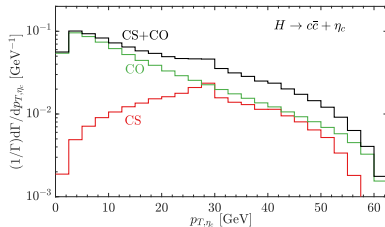
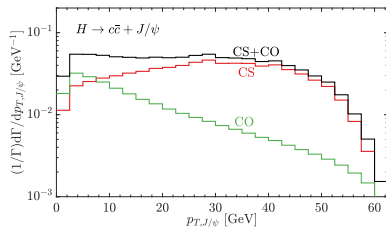
## Charmonium energy distributions



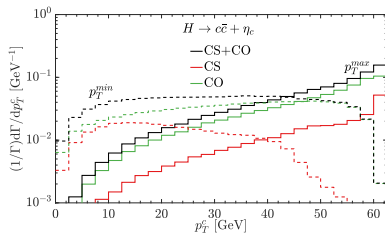
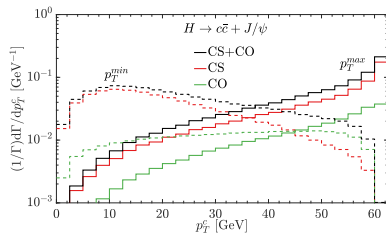


# Standard Model results (II)

## Charmonium transverse momentum distribution

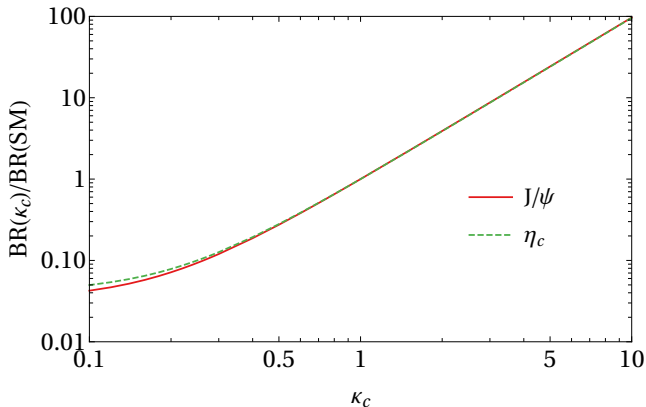


## Transverse momentum distribution for the free charm quark



## Probe the $Hc\bar{c}$ coupling (I)

Use the  $\kappa$  framework  $y_c = \kappa_c y_c^{\text{SM}}$ ,  $\text{BR} \approx \kappa_c^2 \text{BR}^{\text{SM}}$



**Note there are small contaminations:**

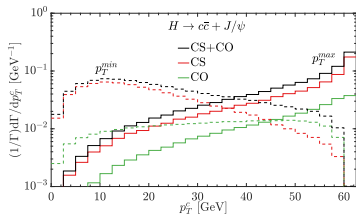
- $HZZ$  diagrams
- The  $H \rightarrow g^* g^* / \gamma^* \gamma^* \rightarrow J/\psi + c\bar{c}$  channel

# Probe the $Hc\bar{c}$ coupling (II)

## Some rough analysis:

- Higgs production cross section at LHC  $\sigma_H \sim 50$  pb
- Expect HL-LHC  $L \sim 3$   $\text{ab}^{-1}$  at ATLAS and CMS and  $L \sim 0.3$   $\text{ab}^{-1}$  at LHCb
- Detection efficiency  $\varepsilon$  for the final state  $c\bar{c} + \ell^+\ell^-$
- $\text{BR}(J/\psi \rightarrow \ell^+\ell^-) \sim 12\%$ ,  $\text{BR}(H \rightarrow J/\psi + c\bar{c}) \sim 2 \times 10^{-5}$
- Event number  $N = L\sigma_H \varepsilon \text{BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{\text{ab}^{-1}} \times \frac{\varepsilon}{10\%}$
- Considering the statistical error only  $\delta N \sim \sqrt{N}$  gives

$$\Delta\kappa_c \approx 15\% \times \left( \frac{L}{\text{ab}^{-1}} \times \frac{\varepsilon}{10\%} \right)^{-1/2}$$

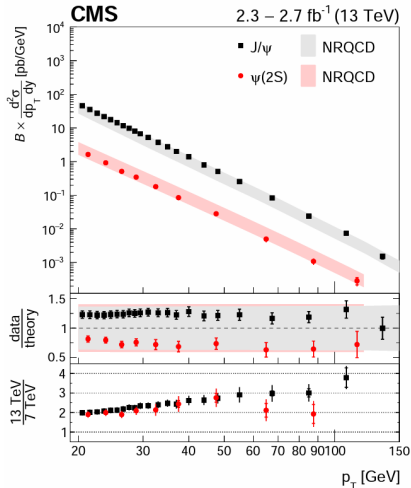


## Detection efficiency $\varepsilon$ :

- Double charm-tagging  $(40\%)^2 \sim 16\%$
- Kinematic acceptance 50%
- Assume  $\varepsilon \sim 10\% \Rightarrow \Delta\kappa_c \sim 15\%$

# Probe the $Hc\bar{c}$ coupling (III)

Background:  $pp \rightarrow J/\psi + X$

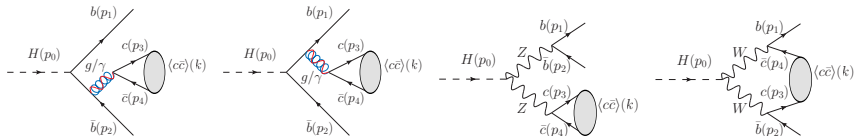


- Prompt  $J/\psi$  production  $\text{BR}(J/\psi \rightarrow \mu^+\mu^-) \times \sigma(pp \rightarrow J/\psi) \simeq 860 \text{ pb}$  [1710.11002]
- Estimate 75000 events for  $pp \rightarrow J/\psi + c\bar{c} \Rightarrow \sim 25 \text{ fb}$  for a  $3 \text{ ab}^{-1}$  HL-LHC [2012.14161]
- Charm-tagging is needed. • Some kinematic cuts may help.

# Probe the $Hc\bar{c}$ coupling (IV)

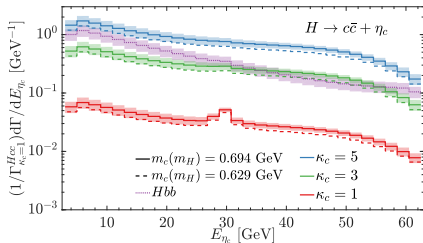
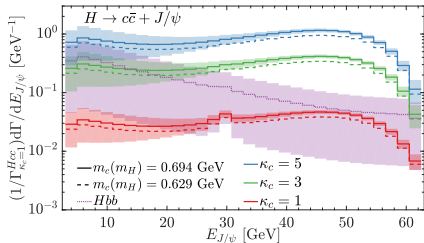
**Background:**  $H \rightarrow J/\psi + b\bar{b}$

Color-octet contribution dominates



## Charmonium energy distributions

Take the  $^3S_1^{[8]}$  LDME uncertainty for error estimation



- Need to determine charm from bottom  $\Rightarrow$  Charm-tagging is needed.
- Large uncertainty from LDME  $\Rightarrow$  More work on LDMEs fitting is needed.

## Probe the $Hc\bar{c}$ coupling (V)

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- **If there were no background:**  $\Delta\kappa_c \sim 15\%$
- **However, there is background in the real world:**
  - Assume 10,000 background events after the election cuts at the HL-LHC
  - Assume the detection efficiency  $\varepsilon \sim 10\%$
  - The signal event number is given by

$$N = L\sigma_H \varepsilon \text{BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{\text{ab}^{-1}} \times \frac{\varepsilon}{10\%}$$

- Sensitivity  $S \simeq N_{\text{signal}}/\sqrt{N_{\text{Background}}}$   
 $\Rightarrow$  It is possible to reach  $2\sigma$  for  $\kappa_c \approx 2.4$ .
- systematic effect  $N_{\text{signal}}/N_{\text{Background}} = 2\%$  for  $\kappa_c \approx 2.4$ .

# Conclusion

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- **Higgs is special and important**

- The Higgs sector is the portal to new physics beyond SM.
- Testing the SM mass generation mechanism helps BSM physics searches.
- The Yukawa couplings of the 3rd generation fermions are precisely measured  
⇒ Charm quark is the next target.

- **For the current determination of the Charm-Higgs coupling**

- $pp \rightarrow VH(c\bar{c})$ ,  **$c$ -tagging is challenging**

ATLAS:  $\kappa_c < 8.5$ , CMS:  $1.1 < |\kappa_c| < 5.5$ , Future 3  $\text{ab}^{-1}$  HL-LHC:  $\kappa_c < 3$

- $H \rightarrow J/\psi + \gamma$ , no need for  $c$ -tagging but insensitive to  $\kappa_c$  ATLAS:  $\kappa_c < 100$

- **Another possible approach:  $H \rightarrow J/\psi + c\bar{c}$**

- The rate is larger due to the fragmentation enhancements
- There are both color-singlet and color-octet contributions
- The QED and EW corrections can be sizable, so need to be included
- The SM prediction gives  $BR \sim 2 \times 10^{-5}$
- For a possible 3  $\text{ab}^{-1}$  HL-LHC, with a 10% final state detection efficiency  
⇒  $\Delta\kappa_c \sim 10\%$
- Assume there are 10,000 background events ⇒  $2\sigma$  for  $\kappa_c \simeq 2.4$

- **More work in progress:**

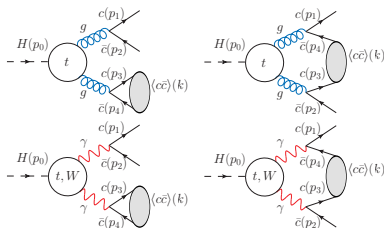
- Background analysis, detector/systematic effects
- Better LDMEs fittings, higher order calculations/resummation ...

# Worry about VMD ?

$$H \rightarrow J/\psi + c\bar{c}$$

- Larger decay rate  
 $\text{BR}(H \rightarrow J/\psi + c\bar{c}) \simeq 2 \times 10^{-5}$
- Sensitive to  $Hc\bar{c}$  coupling  
 QCD and QED dominates
- Other diagrams

$$H \rightarrow g^* g^* / \gamma^* \gamma^* \rightarrow J/\psi + c\bar{c}$$



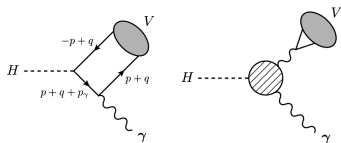
$$\text{BR}(g^* g^*) \sim 2.5 \times 10^{-6}, \text{BR}(\gamma^* \gamma^*) < 2 \times 10^{-7}$$

- **No need to worry about VMD**

$$H \rightarrow J/\psi + \gamma$$

- Small decay rate  
 $\text{BR}(H \rightarrow J/\psi + \gamma) \simeq 2.8 \times 10^{-6}$
- Insensitive to  $Hc\bar{c}$  coupling  
 $\Rightarrow \kappa_c \leq 100$

## VMD dominates



- $\gamma^* \rightarrow J/\psi$  dominates over  $Hc\bar{c}$   
 Two orders of magnitude larger.



# Color-singlet VS color-octet

## Recall the NRQCD factorization formalism

$$\Gamma = \sum_{\mathbb{N}} \hat{\Gamma}_{\mathbb{N}}(H \rightarrow (Q\bar{Q})[\mathbb{N}] + X) \times \langle \mathcal{O}^h[\mathbb{N}] \rangle$$

## Long distance: the color-octet LDMEs are suppressed

They are in higher orders of  $v$  than the color-singlet one

$$\frac{\langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^3), \quad \frac{\langle \mathcal{O}^{J/\psi}(3S_1^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^4), \quad \frac{\langle \mathcal{O}^{J/\psi}(3P_J^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^4),$$
$$\frac{\langle \mathcal{O}^{\eta_c}(3S_1^{[8]}) \rangle}{\langle \mathcal{O}^{\eta_c}(1S_0^{[1]}) \rangle} \sim \mathcal{O}(v^3), \quad \frac{\langle \mathcal{O}^{\eta_c}(1P_1^{[8]}) \rangle}{\langle \mathcal{O}^{\eta_c}(1S_0^{[1]}) \rangle} \sim \mathcal{O}(v^4)$$

## Short distance coefficient (SDC)

- The color factors are different for color-singlet and color-octet states

	Charm fragmentation			SPF	SGF
	QCD	QED	QCD×QED	QED	QCD
CS	16/9	1	4/3	9	-
CO	2/9	8	-4/3	-	2

- There may appear new diagrams for color-octet state production

The SGF diagrams result in large  $3S_1^{[8]}$  SDC

⇒ Sizable color-octet contribution (mainly from  $3S_1^{[8]}$ )

# Standard Model results: some details

## Color-octet contributions

	$3S_1^{[8]}$	$1S_0^{[8]}$	$1P_1^{[8]}$	$3P_J^{[8]}$	Total
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$2.0 \times 10^{-8}$	$9.8 \times 10^{-10}$	-	$2.2 \times 10^{-10}$	$2.2 \times 10^{-8}$
BR( $H \rightarrow c\bar{c} + J/\psi$ )	$5.0 \times 10^{-6}$	$2.4 \times 10^{-7}$	-	$5.3 \times 10^{-8}$	$5.3 \times 10^{-6}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$1.8 \times 10^{-7}$	$3.6 \times 10^{-11}$	$1.0 \times 10^{-10}$	-	$1.8 \times 10^{-7}$
BR( $H \rightarrow c\bar{c} + \eta_c$ )	$4.5 \times 10^{-5}$	$8.9 \times 10^{-9}$	$2.5 \times 10^{-8}$	-	$4.5 \times 10^{-5}$

## Contributions with respect to QCD

$\hat{\Gamma}_N/\hat{\Gamma}_N^{\text{QCD}}$	$1S_0^{[1]}$	$3S_1^{[1]}$	$1S_0^{[8]}$	$3S_1^{[8]}$	$1P_1^{[8]}$	$3P_0^{[8]}$	$3P_1^{[8]}$	$3P_2^{[8]}$
QCD	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
QED	$1.1 \times 10^{-4}$	0.077	0.0073	$1.1 \times 10^{-5}$	0.0068	0.0073	0.0073	0.0073
QCD $\times$ QED	0.021	0.14	-0.17	0.0012	-0.15	-0.17	-0.17	-0.17
EW	0.24	0.051	0.28	$2.6 \times 10^{-4}$	1.4	0.29	0.33	1.5

## Some observations

- QCD is dominant in most of the Fock states
- SPF brings sizable QED correction to  $3S_1^{[1]}$ , but it is forbidden for  $1S_0^{[1]}$
- SGF makes  $3S_1^{[8]}$  super large
- For  $1S_0^{[8]}$  and  $3P_J^{[8]}$ , charm-quark fragmentation is the only production channel, so that QED and QCD differ by a universal factor
- EW correction is large since  $Z$  is closed to its mass shell

# Color-octet uncertainties from the LDMEs

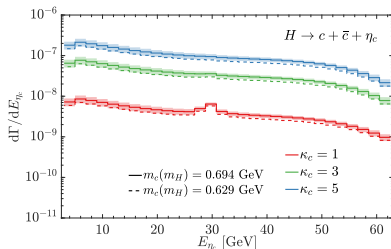
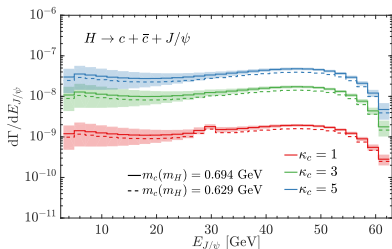
Color-octet contributions:  ${}^3S_1^{[8]}$  dominates

	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^1P_1^{[8]}$	${}^3P_J^{[8]}$	Total
$\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV)	$2.0 \times 10^{-8}$	$9.8 \times 10^{-10}$	-	$2.2 \times 10^{-10}$	$2.2 \times 10^{-8}$
$\text{BR}(H \rightarrow c\bar{c} + J/\psi)$	$5.0 \times 10^{-6}$	$2.4 \times 10^{-7}$	-	$5.3 \times 10^{-8}$	$5.3 \times 10^{-6}$
$\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV)	$1.8 \times 10^{-7}$	$3.6 \times 10^{-11}$	$1.0 \times 10^{-10}$	-	$1.8 \times 10^{-7}$
$\text{BR}(H \rightarrow c\bar{c} + \eta_c)$	$4.5 \times 10^{-5}$	$8.9 \times 10^{-9}$	$2.5 \times 10^{-8}$	-	$4.5 \times 10^{-5}$

Take the  ${}^3S_1^{[8]}$  LDME for the uncertainty estimation

$$\text{BR}(H \rightarrow c\bar{c} + J/\psi) = (2.0 \pm 0.5) \times 10^{-5},$$

$$\text{BR}(H \rightarrow c\bar{c} + \eta_c) = (6.0 \pm 1.0) \times 10^{-5}.$$



# When is $y_c$ not related to the charm mass?

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## Higgs Effective Field Theory (HEFT)

$SU(2)$  doublets of the global  $SU(2)_{L,R}$  symmetries:

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ E_L \end{pmatrix}, \quad L_R = \begin{pmatrix} 0 \\ E_R \end{pmatrix}.$$

Define

$$U(x) \equiv \exp(i\sigma_a \pi^a(x)/v)$$

so that the Lagrangian contains

$$\mathcal{L} \supset -\frac{v}{\sqrt{2}} \bar{Q}_L U y_Q(h) Q_R - \frac{v}{\sqrt{2}} \bar{L}_L U y_L(h) L_R + h.c.$$

The functions  $y_Q(h)$  and  $y_L(h)$  control the Yukawa couplings

$$y_Q(h) \equiv \text{diag} \left( \sum_n y_U^{(n)} \frac{h^n}{v^n}, \sum_n y_D^{(n)} \frac{h^n}{v^n} \right)$$

$$y_L(h) \equiv \text{diag} \left( 0, \sum_n y_\ell^{(n)} \frac{h^n}{v^n} \right) L$$

$n = 0$  is for mass term,  $n = 1$  is for Yukawa coupling.

# Fragmentation formalism

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## The decay width is written as a convolution

Define  $z \equiv 2E_\psi/m_H$

$$\frac{d\Gamma}{dz}(H \rightarrow \psi(z)q\bar{q}) = 2C_q \otimes D_q + C_g \otimes D_g, C \otimes D \equiv \int_z^1 C(y)D(z/y)\frac{dy}{y}$$

## Hard coefficient

$$C_q(\mu^2, z) = \Gamma(H \rightarrow q\bar{q})\delta(1-z)$$

$$C_g(\mu^2, z) = \frac{4\alpha_s}{3\pi}\Gamma(H \rightarrow q\bar{q}) \left[ \frac{(z-1)^2+1}{z} \log\left(\frac{(1-z)z^2 m_H^2}{\mu^2}\right) - z \right]$$

## Fragmentation functions

$$D_{c \rightarrow J/\psi}^{(1)}(\mu^2, z) = \frac{128\alpha_s^2}{243m_{J/\psi}^3} \frac{z(1-z^2)}{(2-z)^6} (16 - 32z + 72z^2 - 32z^3 + 5z^4) \langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$$

$$D_{q \rightarrow \psi}^{(8)}(\mu^2, z) = \frac{2\alpha_s^2}{9m_\psi^3} \left[ \frac{(z-1)^2+1}{z} \log\left(\frac{\mu^2}{m_\psi^2(1-z)}\right) - z \right] \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$$

$$D_{g \rightarrow \psi}(\mu^2, z) = \frac{\pi\alpha_s}{3m_\psi^3} \delta(1-z) \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$$