Moduli Stabilisation and the Statistics of SUSY Breaking in the Landscape

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Content

Review of Statistical Approach

- SUSY is a central idea in Pheno and Theory (Hierarchy probl., DM candidates, etc.)
- Can String Theory give guidance in the search for SUSY?
  [Baer,Sengupta,Salam,Sinha 20]
- Landscape is large, no vacuum is preferred (yet), many vacua at least roughly match SM → Statistical analysis
- First studies found a preference for high scale SUSY, due to a uniform distribution of SUSY breaking scale
  [Douglas, 04], [Denef,Douglas, 04], [Denef,Douglas, 05]
- These studies focused on the dilaton and complex structure F-terms and neglected the Kähler moduli F-terms, since these fields are stabilized beyond tree-level → only sub-leading correction?
- Based on dynamical SUSY breaking arguments a logarithmic behavior of the SUSY breaking scale was also expected (BUT: for KKLT)
  [Dine,Gorbatov,Thomas, 04],[Dine, 05],[Dine,O'Neil,Sun, 05],[Dine, 04]

→ What is the origin for the power-law / logarithmic scaling?

Importance of the Kähler moduli

Short summary of the results of D.D.:

- The Kähler potential is: $K_{\text{tree}} = -2\ln\mathcal{V} - \ln\left(S+\bar{S}\right) - \ln\left(-i\int_X \Omega(U)\wedge\bar{\Omega}(\bar{U})\right),$
- The Superpotential is: $W_{\text{tree}} = \int_X G_3 \wedge \Omega(U)$
- Using the standard expression for the SUGRA scalar potential, one can write down the tree level potential as

$$\rightarrow V_{\text{tree}} = |F^S|^2 + |F^U|^2 + |F^T|^2 - 3m_{3/2}^2 \approx |F^S|^2 + |F^U|^2 - 3m_{3/2}^2$$

- Where the gravitino mass is given by: $m_{3/2} = e^{K/2}|W|$
- Kähler moduli not stabilized at tree-level → only a small correction to leading order?

Importance of the Kähler moduli

- Distribution of SUSY breaking vacua was assumed to be:

$$dN_{\Lambda=0}(F) = \prod d^2F^S d^2F^U d\hat{\Lambda}\rho(F,\hat{\Lambda})\delta\left(|F^S|^2 + |F^U|^2 - \hat{\Lambda}\right)$$

- With the AdS vacuum depth: $\hat{\Lambda} = 3m_{3/2}^2$

- Assuming that the distribution of the SUSY breaking scale is decoupled from the distribution of the cosmological constant we can write: $\rho(F,\hat{\Lambda}) = \rho\left(F\right)$

- Assuming the vanishing of the cosmological constant:

$|F|^2 = 3m_{3/2}^2$ and $d^2F \simeq |F|d|F| \simeq m_{3/2}dm_{3/2}$

$\rightarrow$ $\boxed{dN_{\Lambda=0}(m_{3/2}) \sim \rho(m_{3/2})m_{3/2}dm_{3/2}}$ $\boxed{\rho(m_{3/2}) \sim m_{3/2}^{\beta},\ \beta = 0}$ [Douglas, 04]

Importance of the Kähler moduli

- **BUT:** Using the famous 'no-scale' relation $K_{\bar{T}} K^{\bar{T}T} K_T = 3$ the scalar potential can be rewritten as

$$V_{\rm tree} = |F^S|^2 + |F^U|^2 + m_{3/2}^2 \left(K_{\bar{T}} K^{\bar{T}T} K_T - 3 \right) = \frac{e^{K_{cs}}}{\mathcal{V}^2 (S+\bar{S})} \left(|D_S W|^2 + |D_U W|^2 \right)$$

→ any vacuum with $D_i W \neq 0$ is unstable since it gives rise to a run-away for the volume mode. Hence a stable solution requires $F^S = F^U = 0$

→ at tree-level the gravitino mass is set by the F-terms of the T-moduli since 'no-scale' implies $|F^T|^2 = 3 m_{3/2}^2$

→ soft terms are of order $m_{3/2}$ only for matter located on D7 branes, not for D3. For instance, gaugino masses for D3's are set by F^S , which is non-zero due to sub-leading corrections beyond tree-level. In order to determine F^S one needs to stabilise the Kähler moduli [Jockers, 05]

→ SUSY statistics should be driven by the Kähler moduli

Stabilisation mechanism - KKLT

- Purely non-perturbative stabilisation: $W = W_0 + Ae^{-aT}$
  [Kachru,Kallosh,Linde,Trivedi, 03]
- Here the Kähler modulus is $T = \tau + i\Theta$ and $a = 2\pi/\mathfrak{n}$ is a parameter that determines the nature of the non-perturbative effect.
- Minimizing the scalar potential leads to: $e^{a\langle\tau\rangle} \sim \frac{2Aa\langle\tau\rangle}{3W_0} \Leftrightarrow \langle\tau\rangle \sim \frac{1}{a}|\ln W_0|$
- The gravitino mass at the minimum is: $$\boxed{m_{3/2} = \frac{\pi g_s^{1/2}}{\mathfrak{n}^{3/2}} \frac{|W_0| M_p}{|\ln W_0|^{3/2}}}$$

→ In order to be able to neglect stringy corrections to the effective action and pert. corrections to K one needs: $\mathrm{W}_0 \ll 1$

→ the gravitino mass in KKLT is mainly driven by W_0

Stabilisation mechanism - LVS

[Balasubramanian,Berglund,Conlon,Quevedo, 05]
[Cicoli,Conlon,Quevedo, 08]

- Perturbative and non-perturbative stabilisation:

  → perturbative: $$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}\right) + \ldots$$

  → non-perturbative: $$W = W_0 + A_s e^{-a_s T_s}$$

- Minimizing the scalar potential leads to: $$\langle\mathcal{V}\rangle \sim \frac{3\sqrt{\langle\tau_s\rangle}|W_0|}{4a_s A_s} e^{a_s\langle\tau_s\rangle}, \ \langle\tau_s\rangle \sim \frac{1}{g_s}\left(\frac{\xi}{2}\right)^{2/3}$$

- The gravitino mass at the minimum is: $$\boxed{m_{3/2} = c_1 \frac{g_s M_p}{\mathfrak{n}} e^{-\frac{c_2}{g_s \mathfrak{n}}}}$$

- Where c_1 and c_2 are numerical coefficients

  → the gravitino mass in LVS is mainly driven by g_s

SUSY breaking statistics

- Gravitino mass is mainly determined by $W_0, \ g_s, \ \mathfrak{n}$

→ The distribution of $|W_0|^2$ as a complex variable is assumed to be uniform:
[Douglas, 04]

$$\boxed{dN \sim |W_0| d|W_0|}$$

→ The distribution of g_s was checked to be uniform for rigid CY, and was shown to hold in more general cases:
[Shok,Douglas, 04][Denef,Douglas, 04]
[Blanco-Pillado,Sousa,Urkiola,Wachter, 20]

$$\boxed{dN \sim dg_s}$$

→ The distribution of the rank of the condensing gauge group is still poorly understood. We expect the number of states N to decrease when $\mathfrak{n}$ increases, since D7-tadpole cancellation is more difficult to satisfy

$$\boxed{dN \sim -\mathfrak{n}^{-r} d\mathfrak{n}}$$

SUSY breaking statistics - LVS

- Using the scaling of the underlying parameters, we can compute the scaling behavior of the gravitino in LVS:

$$\to dm_{3/2} \sim \mathfrak{n} m_{3/2} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^2 \left[1 - \frac{c_2 \mathfrak{n}^{r-2}}{\ln \left(\frac{M_p}{m_{3/2}} \right)} \right] dN$$

- For any value of the exponent r the leading order result is given by

$$\to \boxed{\rho_{\text{LVS}}(m_{3/2}) \sim \frac{1}{\mathfrak{n} m_{3/2}^2} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^{-2}} \qquad \boxed{N_{\text{LVS}} \sim \ln \left(\frac{m_{3/2}}{M_p} \right)}$$

- In LVS we have: $m_{3/2} \sim M_{\text{soft}}^{1/p}$, where the value of p depends on the specific model (D3, D7, sequestered)

→ LVS vacua feature a logarithmic distribution of soft terms

SUSY breaking statistics - KKLT

- Using the scaling of the underlying parameters, we can compute the scaling behavior of the gravitino in KKLT:

$$dm_{3/2} \sim \frac{M_p^2}{m_{3/2}} \left[\frac{g_s}{\mathfrak{n}^3 |\ln W_0|^3} + \frac{m_{3/2}^2}{2M_p^2} \left(\frac{1}{g_s} + 3\mathfrak{n}^{r-1} \right) \right] dN$$

- For any value of the exponent r the leading order result is given by

$$\rightarrow \boxed{\rho_{\mathrm{KKLT}}(m_{3/2}) \sim \frac{1}{M_p^2} \left(\frac{\mathfrak{n}^3 |\ln W_0|^3}{g_s} \right) \sim \mathrm{const.}} \qquad \boxed{N_{\mathrm{KKLT}} \sim \left(\frac{m_{3/2}}{M_p} \right)^2}$$

- In KKLT we have: $m_{3/2} \sim M_{\mathrm{soft}}$

→ KKLT vacua feature a power-law distribution of soft terms

SUSY breaking statistics - KKLT

- The derivation of the previous result relied heavily on the assumption of a uniform distribution of the tree-level superpotential

- However, recent constructions of explicit KKLT models where the crucial relation $W_0 \ll 1$ is satisfied, showed a correlation between the tree-level superpotential and the string coupling of the form

$$\frac{W}{\sqrt{2/\pi}} = \sum_{\vec{q}} \frac{A_{\vec{q}} \vec{M} \cdot \vec{q}}{(2\pi i)^2} e^{2\pi i \tau \vec{p} \vec{q}}$$

[Demirtas, Kim, McAllister, Moritz 20]

- The procedure is based on the neglection of non-pert. correc. at the prepotential level and solving for fluxes, which produce a vanishing superpotential. A subsequent inclusion of the corrections, preserves the exponentially small value of the superpotential

- The exponential dependence of the superpotential on the string coupling lead to a logarithmic scaling in KKLT as well

- How general these constructions are is currently under investigation

Implications for Axion Physics

- Our landscape studies are very general and not restricted to SUSY breaking
- Other phenomenologically interesting quantities such as axion masses, photon-axion couplings, axion decay constants, reheating temperatures etc. were also studied
- For LVS models we observe a logarithmic distribution for all these quantities, e.g. for the axion decay constant:

$$\boxed{N_{\mathrm{LVS}}(f_a) \sim \ln\left(\frac{f_a}{M_p}\right)}$$

- Is a logarithmic distribution a general feature of low energy string constructions?

Conclusion

- We have stressed that Kähler moduli stabilisation is a critical requirement for a proper treatment of the statistics of SUSY breaking
- Different no-scale breaking effects used to fix the Kähler moduli lead to a different dependence of $m_{3/2}$ on the flux dependent microscopic parameters
- In LVS models the distribution of the gravitino mass and soft terms are **logarithmic**
- In KKLT the distribution are **power-law (?)**
- Determining which distribution is more representative of the structure of the flux landscape translates into the question of which vacua are more frequent, LVS or KKLT?
- LVS needs less tuning → larger parameter space → LVS models favoured?
- Definite answer requires more detailed studies

BACKUP SLIDES

Stabilisation mechanism – perturbative

- Purely perturbative stabilisation:
  [Berg,Haack,Kors, 06]

$$K_{g_s^0\alpha'^3} = -\frac{\xi}{g_s^{3/2}\mathcal{V}}, \quad K_{g_s^2\alpha'^2} = g_s\frac{b(U)}{\mathcal{V}^{2/3}}, \quad K_{g_s^2\alpha'^4} = \frac{c(U)}{\mathcal{V}^{4/3}}.$$

- The functions $b(U), c(U)$ are known explicitly only for simple toroidal orientifolds but are expected to be $\mathcal{O}(1-10)$
- Minimizing the scalar potential leads to: $\langle\mathcal{V}\rangle \sim 26 g_s^{9/2}\left(\frac{c(U)}{|\xi|}\right)^3$
- The gravitino mass at the minimum is: $\boxed{m_{3/2} = \lambda\frac{|W_0|M_p}{g_s^4 c(U)^3}}$
- Consistency of the stabilisation requires $\langle\mathcal{V}\rangle \gg 1, \; g_s \ll 1$

→ the gravitino mass in pert. stabilisation is mainly driven by $c(U)$

SUSY breaking statistics - perturbative

- Using the scaling of the underlying parameters, we can compute the scaling behavior of the gravitino in pert. stabilisation:

$$dm_{3/2} \sim m_{3/2} \left(3c^{k-1} - \frac{4}{g_s}\right) dN$$

- Control over the effective field theory requires $k > 1$

$$\rightarrow \boxed{\rho_{\rm PERT}(m_{3/2}) \sim \frac{1}{M_p^2} \left(\frac{m_{3/2}}{M_p}\right)^{\frac{k-7}{3}}} \qquad \boxed{N_{\rm PERT} \sim \left(\frac{m_{3/2}}{M_p}\right)^{\frac{k-1}{3}}}$$

- Qualitatively similar to KKLT (equal for k=7)

- Soft masses are expected to behave as in LVS

→ pert. stabilised vacua feature a power-law distribution of soft terms