Precision Calculation of Dark Radiation from Spinning Primordial Black Holes and Early Matter Dominated Eras

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Outline

- Formation and evaporation of spinning primordial black holes (PBHs) in an early matter dominated era (EMDE)

- Precision calculation of dark radiation (graviton) emitted by spinning PBHs

- Constraints on PBHs from BBN, CMB, and the projected sensitivity of CMB Stage 4

PBHs: motivated by many different scenarios

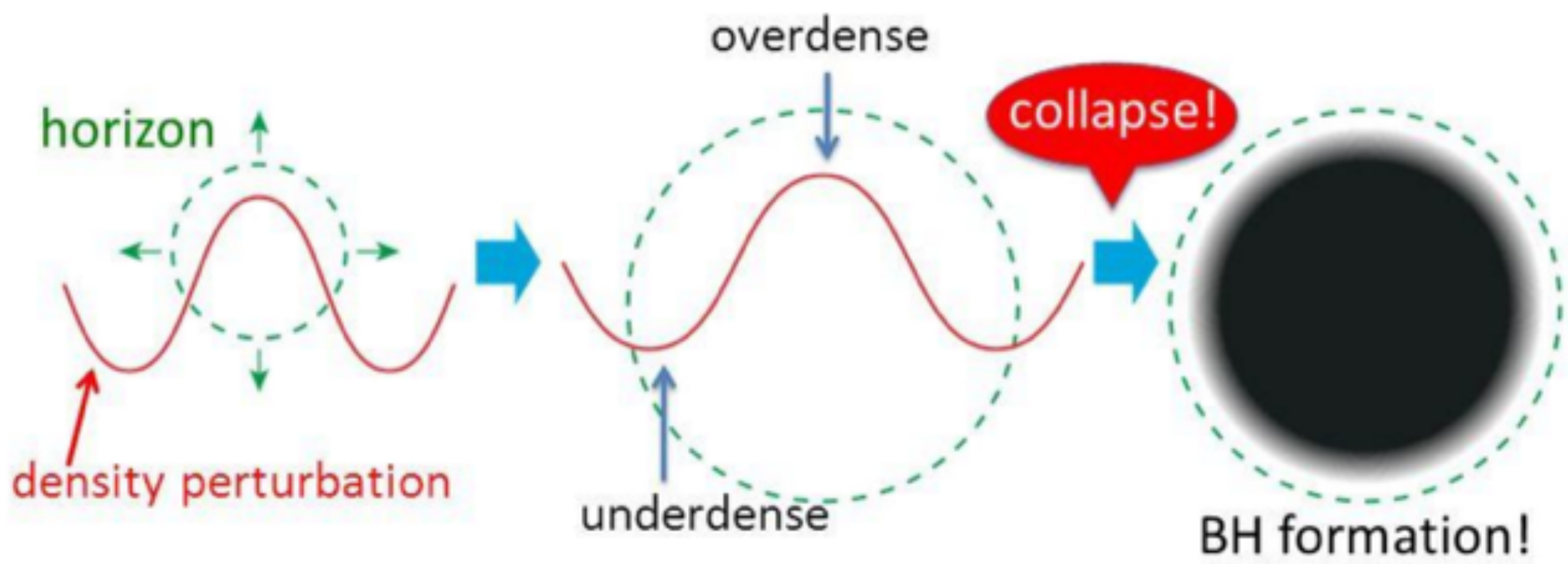


image credit: https://slideplayer.com/slide/7773485/

BHs can be fully characterized by their mass, spin, and charge

mass:

$$M_i \sim \frac{4}{3}\pi H(t_i)^{-3}\rho(t_i)$$

mass distribution:

1) monochromatic, e.g., if the PBH production occurs at a precise time.

2) extended, e.g., if the power spectrum of primordial inhomogeneities embeds a wide peak around some spatial scale

spin (more involved):

Radiation-dominated: small angular momentum

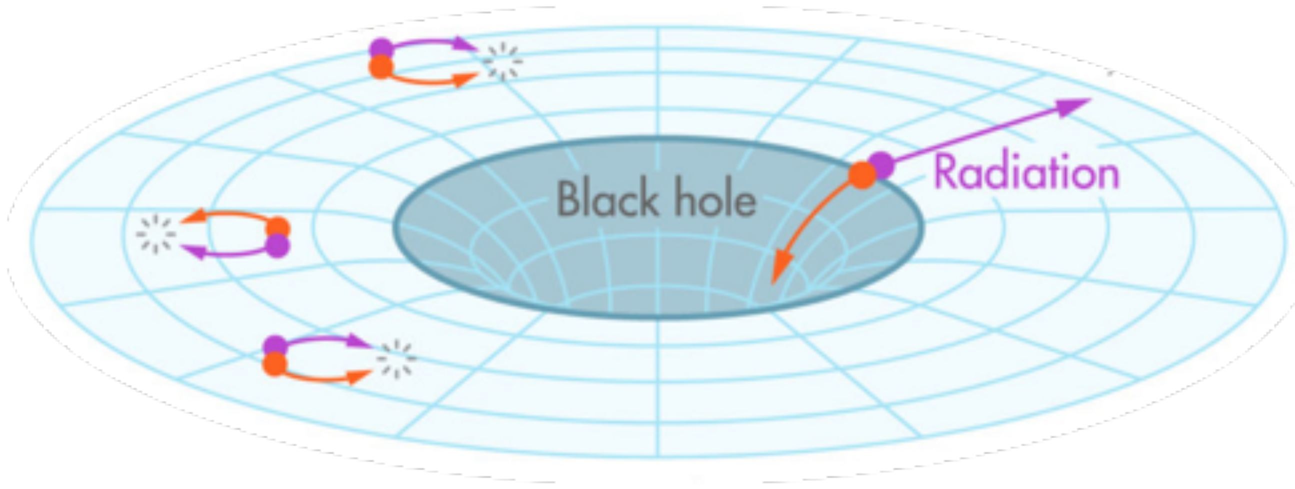
Matter-dominated: large angular momentum

can also gain spin either through early accretion or hierarchical merger

Hawking evaporation of Schwarzschild BHs:

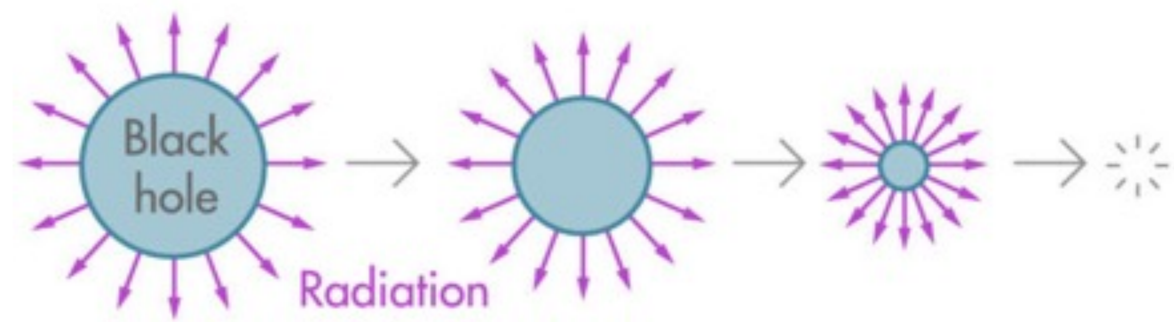
$$T_{\rm BH} = \frac{M_{\rm Pl}^2}{8\pi M_{\rm BH}}$$

image credit: Quantamagazine



$$\frac{d^2 u_i(E,t)}{dt dE} = \frac{g_i}{8\pi^2}\frac{E^3}{e^{E/T_{\rm BH}} \pm 1}$$

$$M(t) = M_i\left(1 - \frac{(t-t_i)}{\tau}\right)^{1/3}, \qquad \tau = \frac{10240\pi}{g_*(T_{\rm BH})}\frac{M_i^3}{M_{\rm Pl}^4}$$



$$N_i = \frac{120\,\zeta(3)}{\pi^3}\frac{g_i}{g_*(T_{\rm BH})}\frac{M_{\rm BH}^2}{M_{\rm Pl}^2}, \qquad T_{\rm BH} > m_i$$

$$N_i = \frac{15\,\zeta(3)}{8\pi^5}\frac{g_i}{g_*(T_{\rm BH})}\frac{M_{\rm Pl}^2}{m_i^2}, \qquad T_{\rm BH} < m_i$$

Hawking evaporation of Kerr BHs:

$$T_{\rm K} = \frac{1}{2\pi}\left(\frac{r_+ - M_{\rm BH}}{r_+^2 + a^{*2}M_{\rm BH}^2}\right)$$

$$a^* \equiv L/M_{\rm BH}^2 \qquad\qquad r_+ \equiv M_{\rm BH}(1+\sqrt{1-a^{*2}})$$

$$\frac{d^2N_i}{dtdE} = \frac{1}{2\pi}\frac{\Gamma_{s_i}^{l,m}}{e^{E'/T_{\rm K}} - (-1)^{2s_i}} \qquad\qquad E' \equiv E - m\Omega = E - ma^*/2r_+$$

$$\frac{dM_{\rm BH}}{dt} = -\frac{f(M_{\rm BH},a^*)}{M_{\rm BH}^2}\,,$$

$$\frac{da^*}{dt} = \frac{a^*\left[2f(M_{\rm BH},a^*) - g(M_{\rm BH},a^*)\right]}{M_{\rm BH}^3}$$

$$f(M_{\rm BH},a^*) \equiv M_{\rm BH}^2\int_0^{+\infty}\sum_i\sum_{\rm dof}\frac{E}{2\pi}\frac{\Gamma_{s_i}^{l,m}(E,M_{\rm BH},a^*)}{e^{E'/T_{\rm K}} - (-1)^{2s_i}}dE\,,$$

$$g(M_{\rm BH},a^*) \equiv \frac{M_{\rm BH}}{a^*}\int_0^{+\infty}\sum_i\sum_{\rm dof}\frac{m}{2\pi}\frac{\Gamma_{s_i}^{l,m}(E,M_{\rm BH},a^*)}{e^{E'/T_{\rm K}} - (-1)^{2s_i}}dE$$

`BlackHawk` Arbey, Auffinger1905.04268

Kerr BHs have enhanced emission of particles with higher spin.

Gravitons: impact the effective number of relativistic degrees of freedom $\Delta N_{\rm eff}$

$\Delta N_{\rm eff}$

assuming PBHs are abundant enough to initiate an EMDE
and reheat the universe via Hawking evaporation:

$$\Delta N_{\rm eff} = \frac{\rho_{\rm DR}(t_{\rm EQ})}{\rho_{\rm R}(t_{\rm EQ})}\left[N_\nu + \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\right]$$

$$\frac{\rho_{\rm DR}(t_{\rm EQ})}{\rho_{\rm R}(t_{\rm EQ})} = \frac{\rho_{\rm DR}(t_{\rm RH})}{\rho_{\rm R}(t_{\rm RH})}\left(\frac{g_*(T_{\rm RH})}{g_*(T_{\rm EQ})}\right)\left(\frac{g_{*,S}(T_{\rm EQ})}{g_{*,S}(T_{\rm RH})}\right)^{4/3} \qquad N_\nu = 3.046$$

Hooper, Krnjaic, McDermott 1905.01301

current experimental limits and projected sensitivities of future experiments:
CMB(two are taken from the Planck)

CMB^1 (TT+low E, conservative)

CMB^2 (TT,TE,EE+low E, more stringent)

BBN: AlterBBN
the sensitivity of the future CMB Stage 4 (CMB-S4) experiment

precision calculation:

- extended spin distribution

$$\int_0^1 \frac{dn}{da^*}\, da^* = 1$$

$$\rho_{\mathrm{DR/SM}}(t_{\mathrm{RH}}) = \int_0^1 da^* \, \frac{dn}{da^*} \int_0^{t_{\mathrm{RH}}} dt \int_0^{+\infty} dE\, E\, \frac{d^2 N_{\mathrm{DR/SM}}}{dt dE}(M, a^*)$$

- reheating temperature

1) RH time = time at which last PBH (with lowest spin evaporates)

2) RH time = the average PBH lifetime, weighted by the spin distribution

- degrees of freedom

$g_*(T),\ g_{*,S}(T)$ `SuperIso Relic`

benchmark spin distributions:

1) Early matter domination era (EMDE):

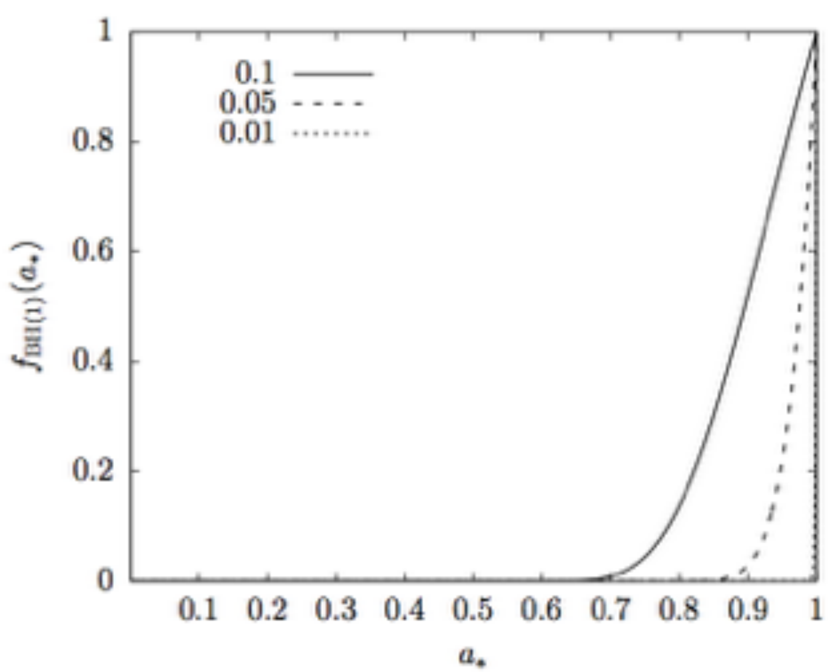
spin source:

- the first-order contribution: from deviation of the boundary of the volume from a sphere.

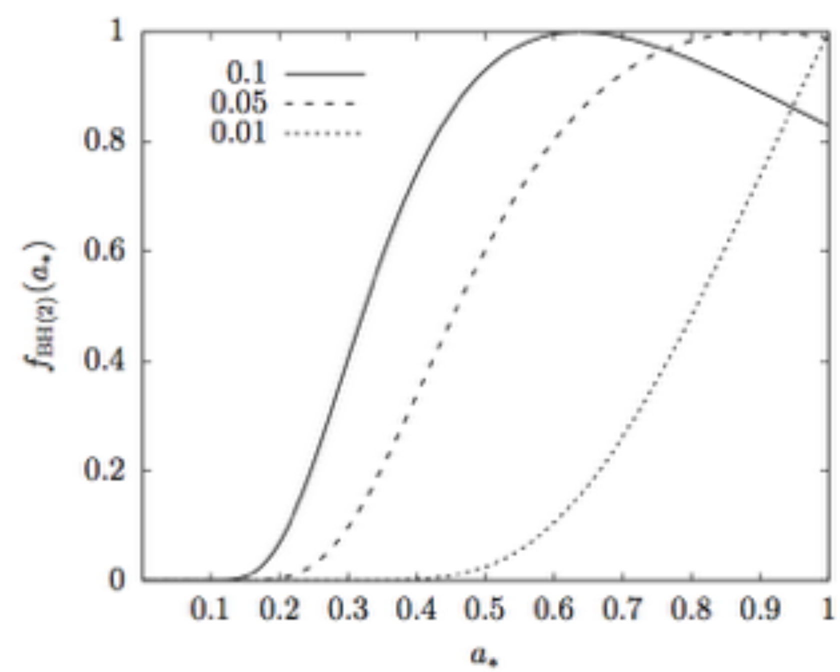
- the second-order contribution: density fluctuations in the comoving region.

The first-order effect usually dominates
(when the initial deviation of the boundary of collapsing region from a sphere is large)

EMDE: 1st order



EMDE: 2nd order



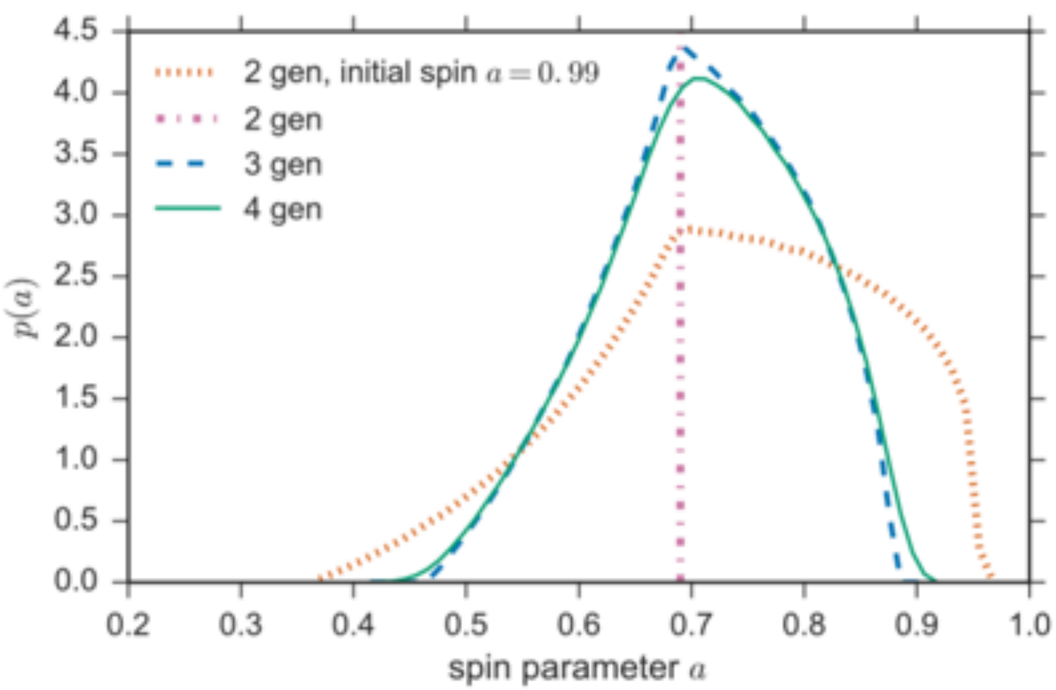
Harada,Yoo, Kohri, Nakao 1707.03595

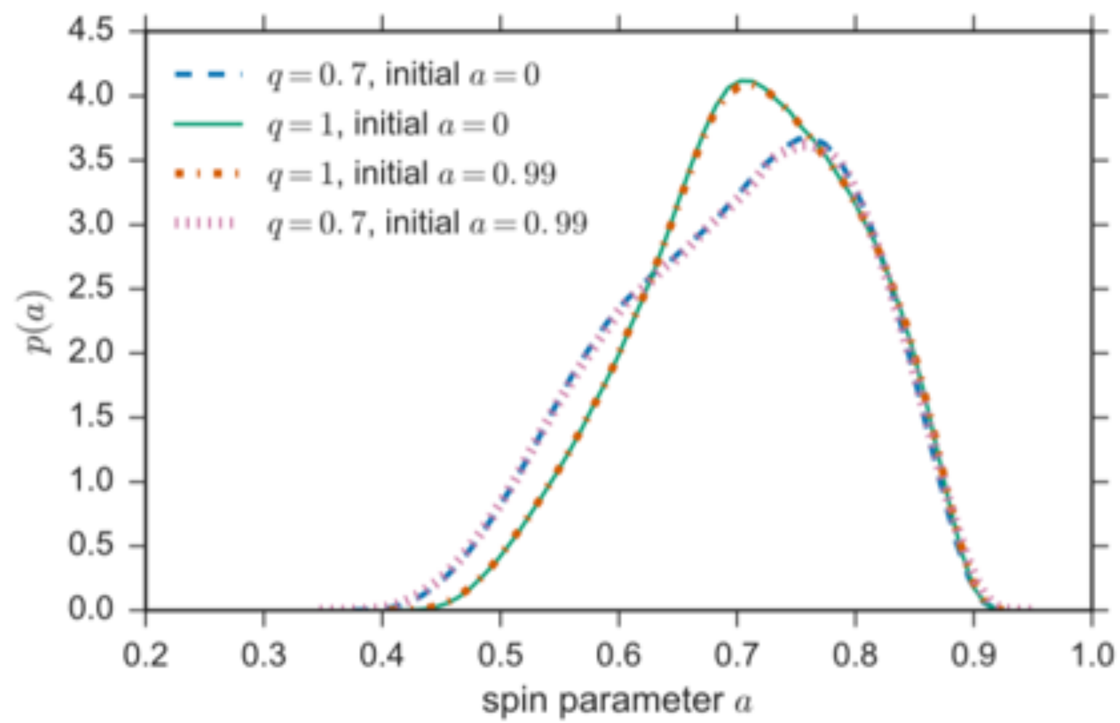
In both cases, spin distribution is a function
of the mean variance of the density perturbations at horizon entry

$$\sigma_{\mathrm{H}} = \langle \delta_s(t_{\mathrm{H}})^2 \rangle^{1/2}$$

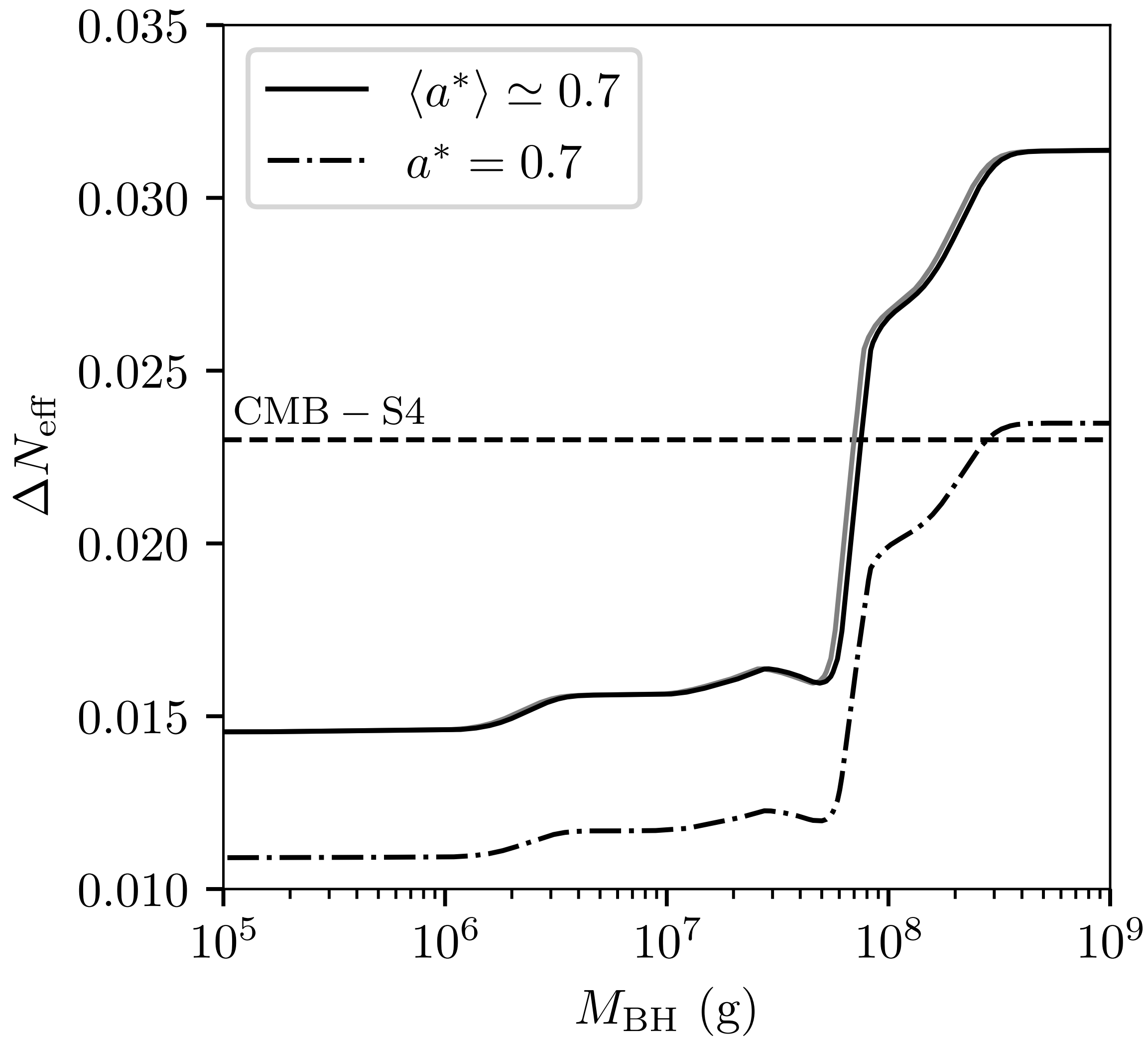
2) distribution from inspirals:

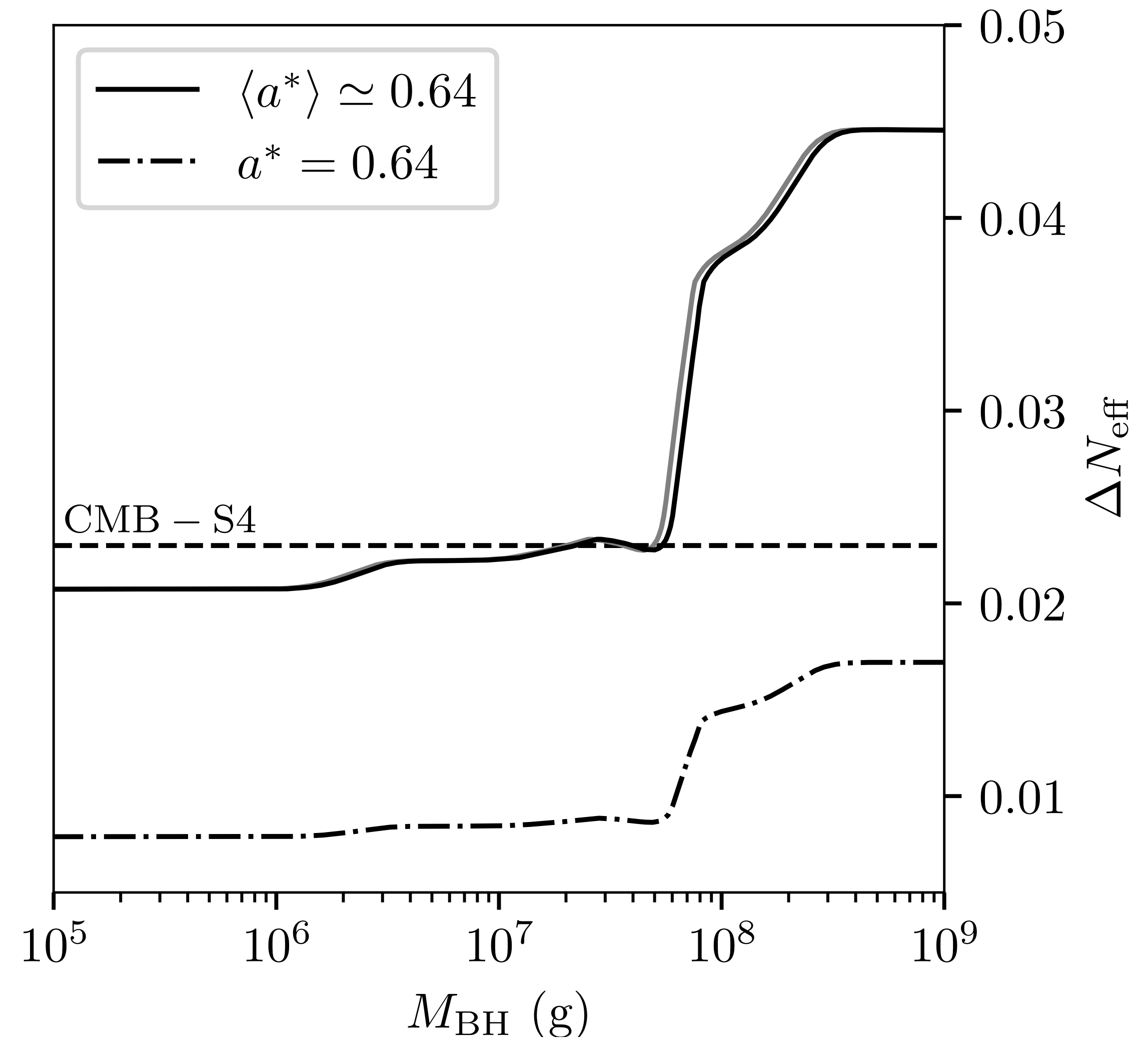
Fishbach, Holz, Farr 1703.06869

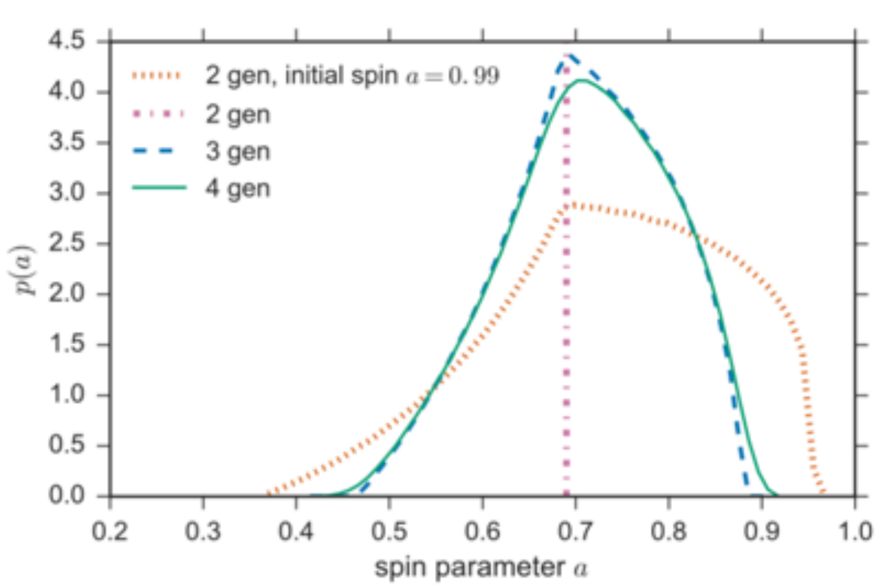




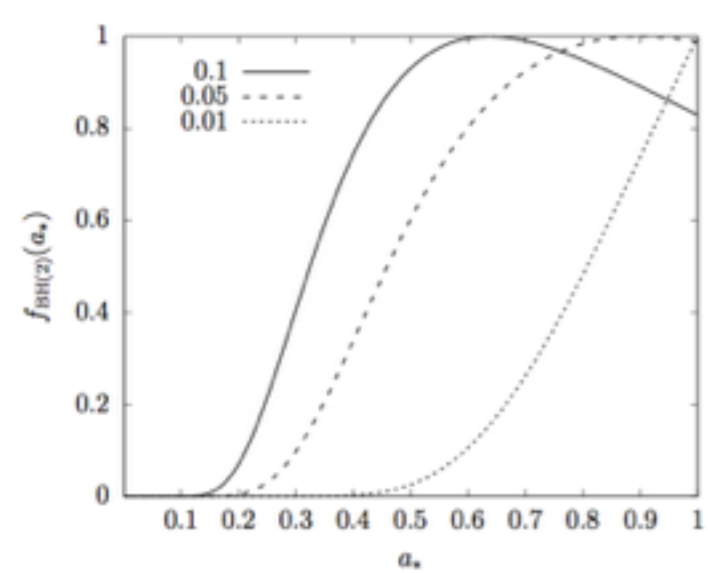
PBH spins compares to the monochromatic approximation







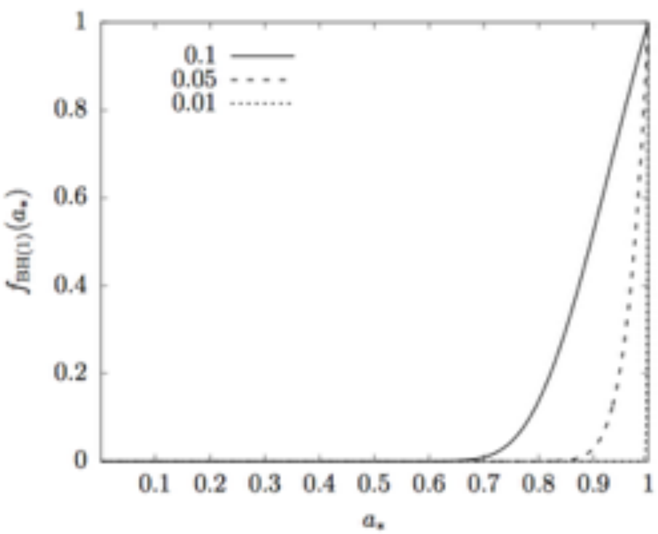
mergers

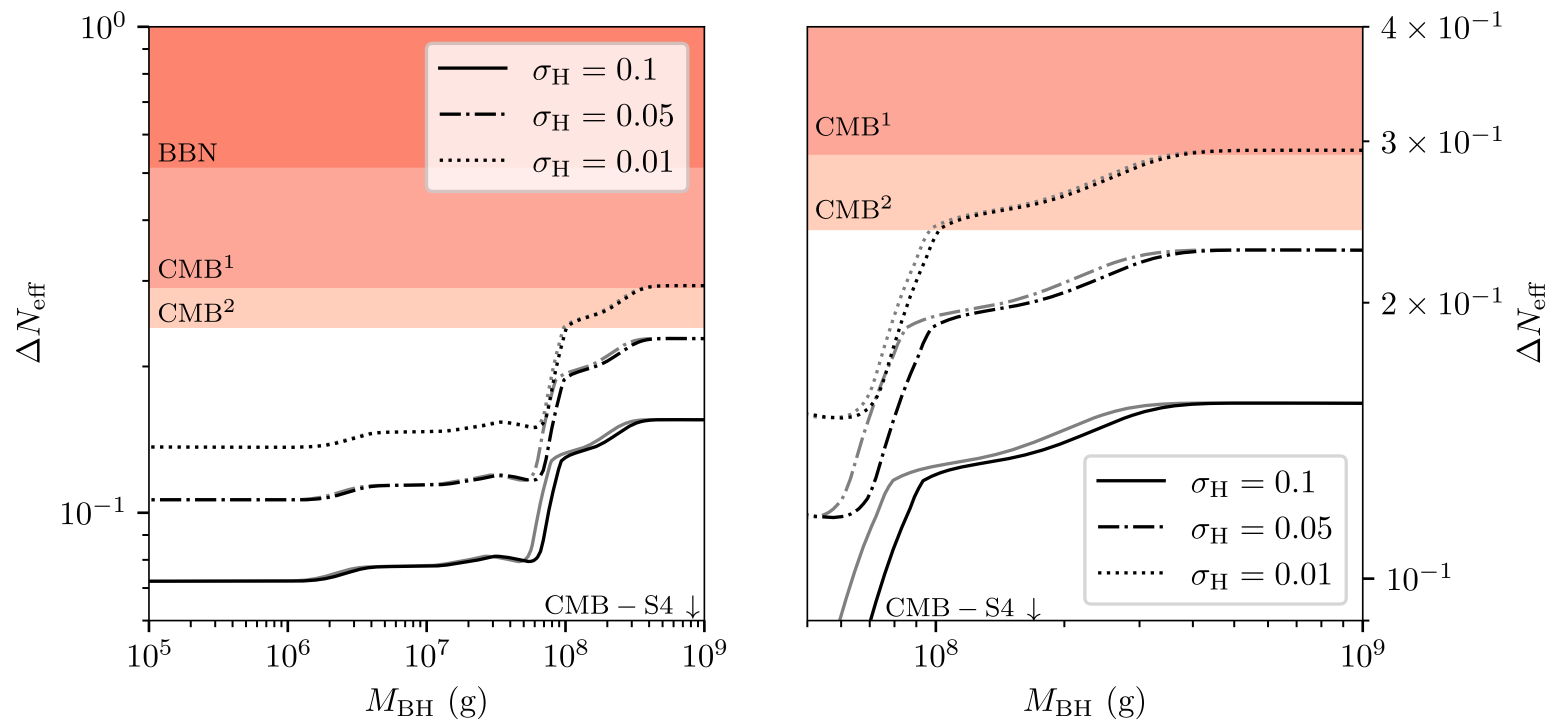


EMDE: 2nd order

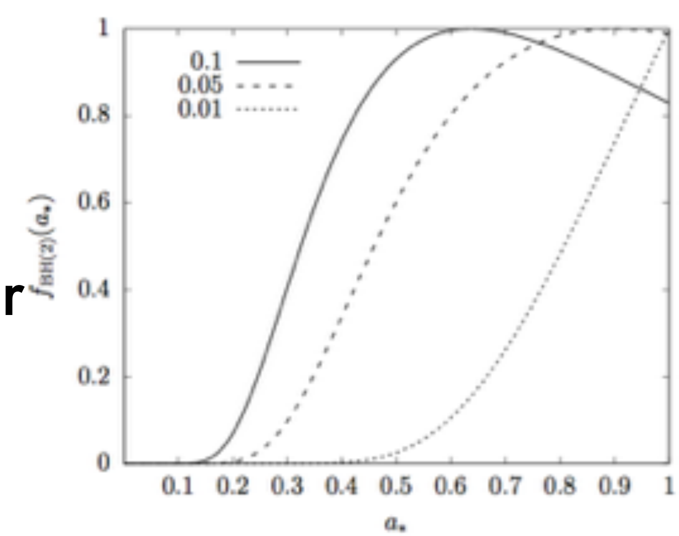
Early Matter domination:

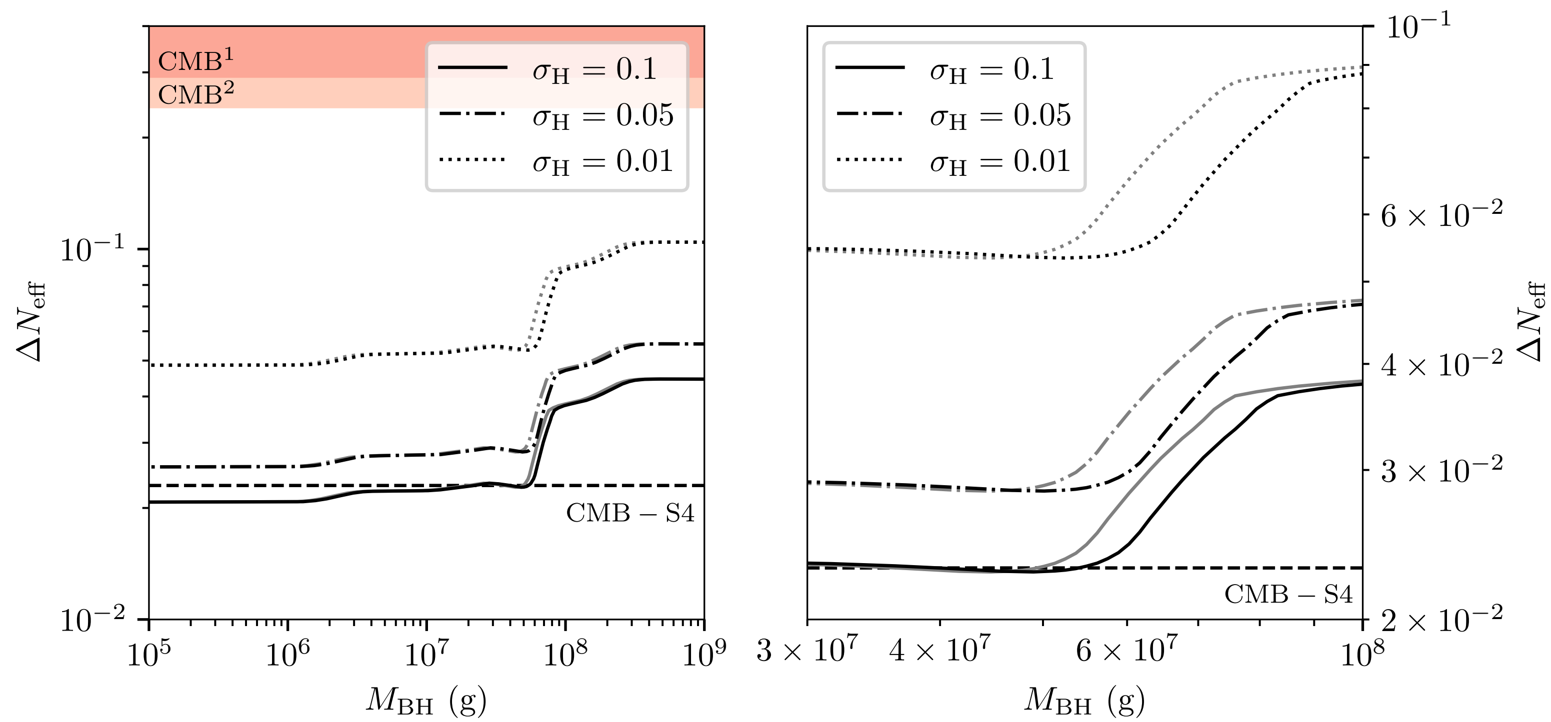
EMDE:
1st order





EMDE:
2nd order

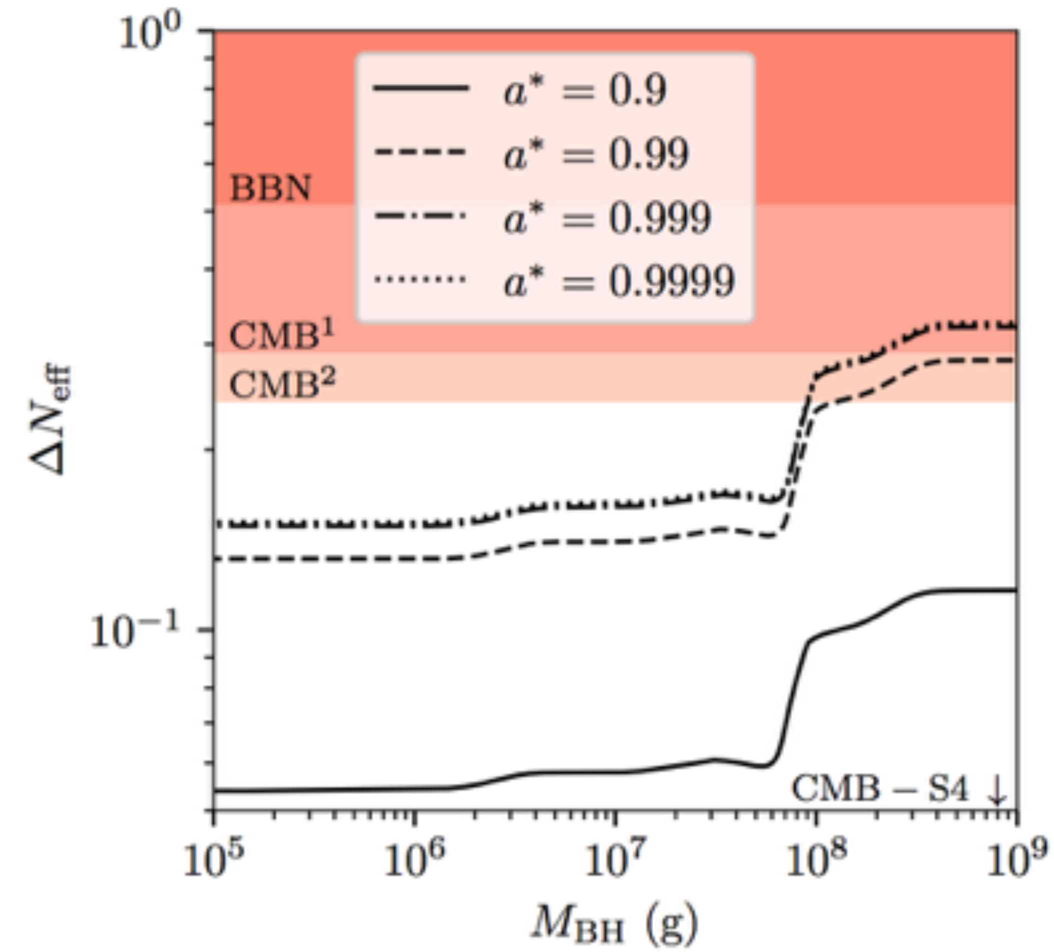




near-extremal PBH spins (monochromatic spin distribution):

- Larget PBH masses already excluded.

- CMB-S4 will probe all near-extremal monochromatic spin distributions.



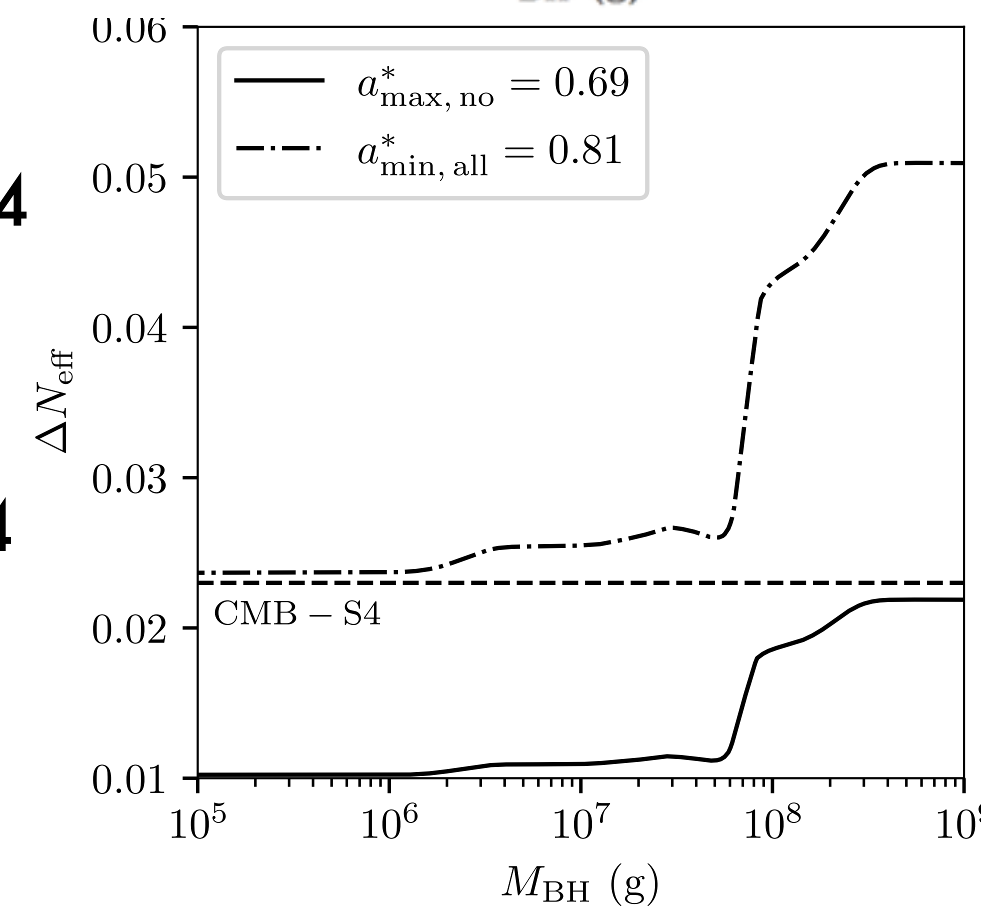
Low and high cut-off for CMB -S4:

- The smallest monochromatic spin for which CMB-S4 will be sensitive to the entire mass rang:

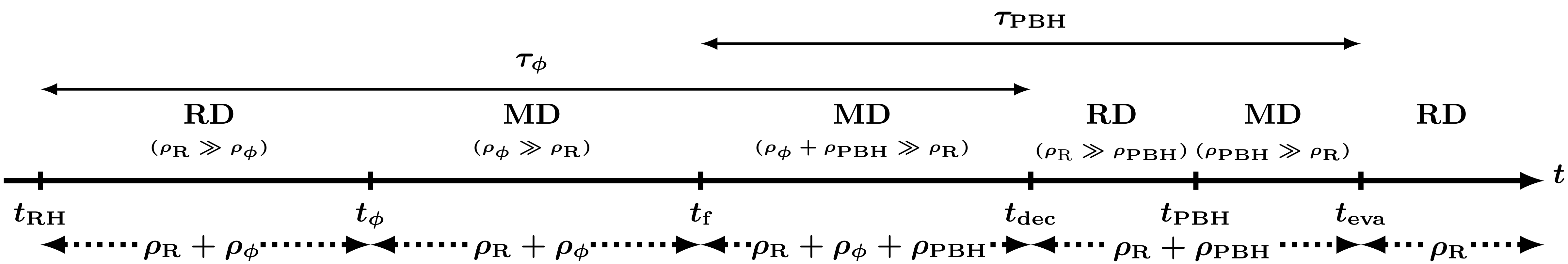
$$a^*_{\rm min,\ all} = 0.81$$

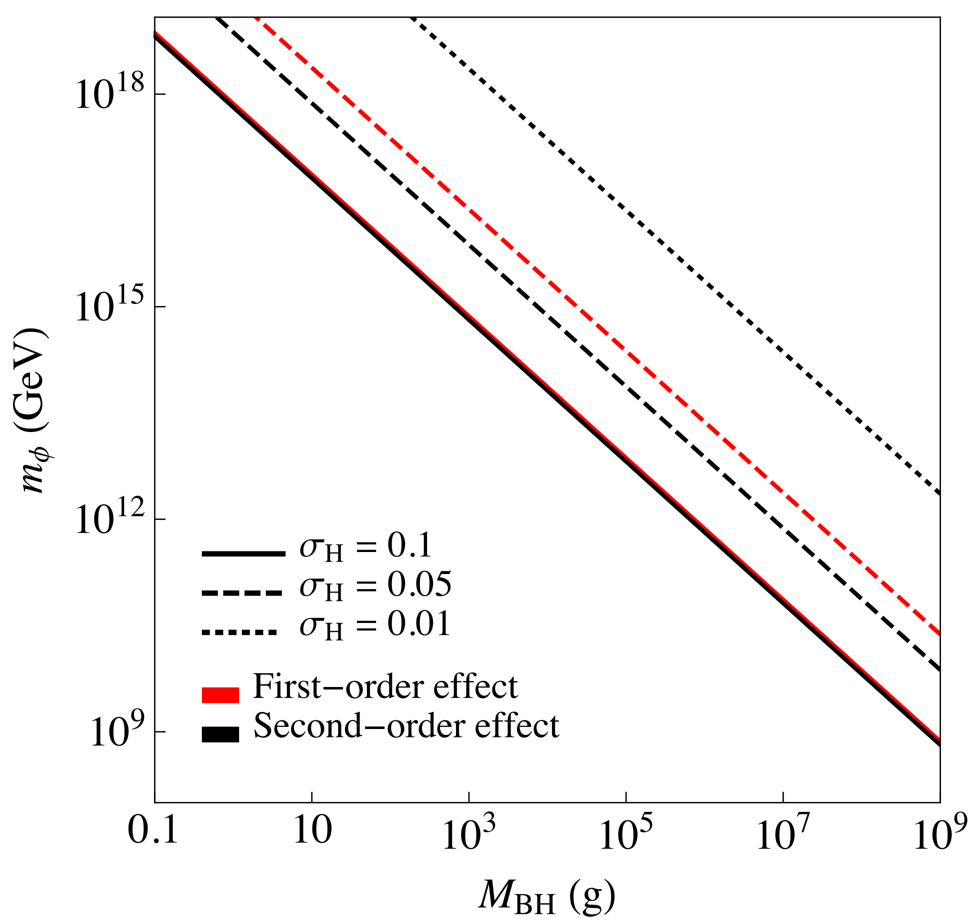
- The largest monochromatic spin for which CMB-S4 will not be sensitive to any part of the mass range:

$$a^*_{\rm max,\ no} = 0.69$$



Early matter domination era leading to a PBH-dominated era: a gravitationally coupled modulus field





Conclusions:

- The first precision study of dark radiation emission by Kerr PBHs

- Precision study includes: extended spin distributions, careful prescription for reheating time, precise evaluation of degrees of freedom.

- precision results for PBHs formed during an EMDE.

- Provided PBHs dominate the energy density of the Universe before evaporation, CMB-S4 will be sensitive to nearly all EMDE distributions and monochromatic spins greater than 0.69.