

Signals of primordial black holes at gravitational wave interferometers

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Background and Motivation

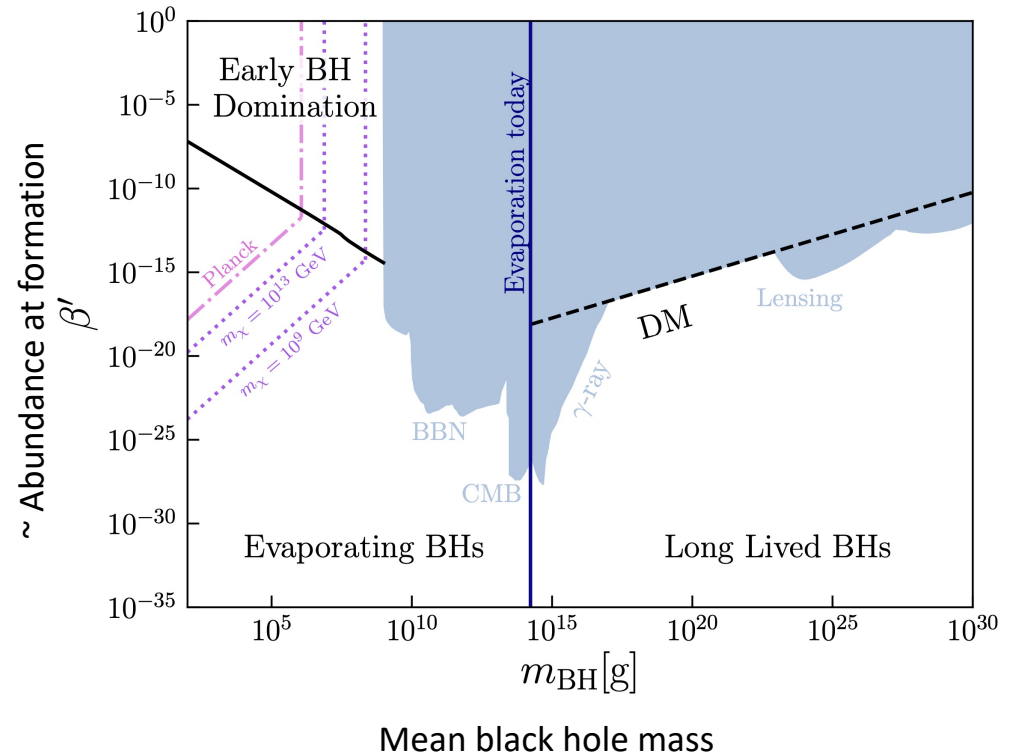
Primordial black holes → Dark Matter

- **Long lived** – PBHs themselves constitute a component of DM
- **Short lived** – Hawking evaporation produces particle DM, stable Planck-scale relics?

Interesting cosmology, early black hole domination?

Goal: Investigate this parameter space through gravitational waves

Constraints from
B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, (2020),
arXiv:2002.12778,
D. Hooper, G. Krnjaic, and S. D. McDermott, JHEP08, 001
(2019), arXiv:1905.01301



PBH Formation

A. D. Gow, C. T. Byrnes, P. S. Cole, and S. Young, JCAP02, 002 (2021), arXiv:2008.03289

Inflation may generate large primordial curvature perturbations at small scales

- These curvature perturbations induce large overdensities that will form primordial black holes (PBHs)

PBH mass and abundance



Curvature perturbation amplitude, scale, width

- PBH mass \sim Horizon mass at k_*
- Abundance \leftrightarrow curvature perturbation amplitude + width

$$\mathcal{P}(k) = \frac{A}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\log^2(k/k_*)}{2\sigma^2}\right)$$

$$\beta = \frac{\rho_{\text{BH},*}}{\rho_{r,*}}$$

$$m_{\text{BH}} \propto \frac{4\pi}{3} \rho_{r,*} H_*^{-3}$$

Ex:

$$A = 0.016, \sigma = 1 \rightarrow \beta = 10^{-7}$$

$$A = 0.0029, \sigma = 1 \rightarrow \beta = 10^{-35}$$

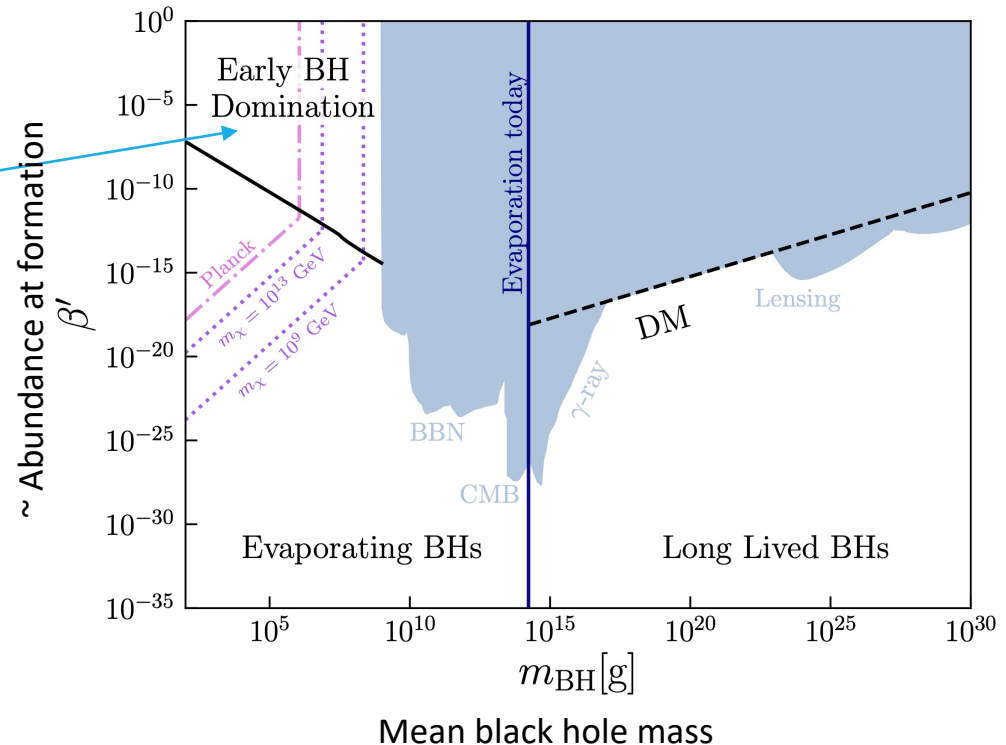
PBH parameter space

Gravitational waves indirectly probe deviations to standard cosmology

- Matter density grows with scale factor
- RD era → PBH dominated era → RD → Usual MD

$$\frac{\rho_{\text{PBH}}}{\rho_{\text{r}}} \propto a$$

- GWs from primordial perturbations at formation
- GWs from early PBH fluid



Predicted GWs at second order

Formalism due to
K. Kohri and T. Terada, Phys. Rev. D97, 123532 (2018), arXiv:1804.08577,
T. Papanikolaou, V. Vennin, and D. Langlois, arXiv:2010.11573v2

We want to calculate the GW spectrum today
from early curvature perturbations

$$h_{\mathbf{k}}''(\eta) + 2\mathcal{H}h_{\mathbf{k}}'(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

$$S_{\mathbf{k}}(\eta) \sim \int d^3q (\Phi_{\mathbf{q}} \Phi_{\mathbf{k}-\mathbf{q}} + \text{quadratic terms in } \Phi, \Phi')$$

$$\mathcal{P}_{\text{GW}}(\eta, k) \delta^3(\mathbf{k} + \mathbf{k}') = \frac{k^3}{2\pi^2} \langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}'}(\eta) \rangle,$$
$$\mathcal{P}(\eta, k) \delta^3(\mathbf{k} + \mathbf{k}') = \frac{k^3}{2\pi^2} \langle \Phi_{\mathbf{k}}(\eta) \Phi_{\mathbf{k}'}(\eta) \rangle.$$

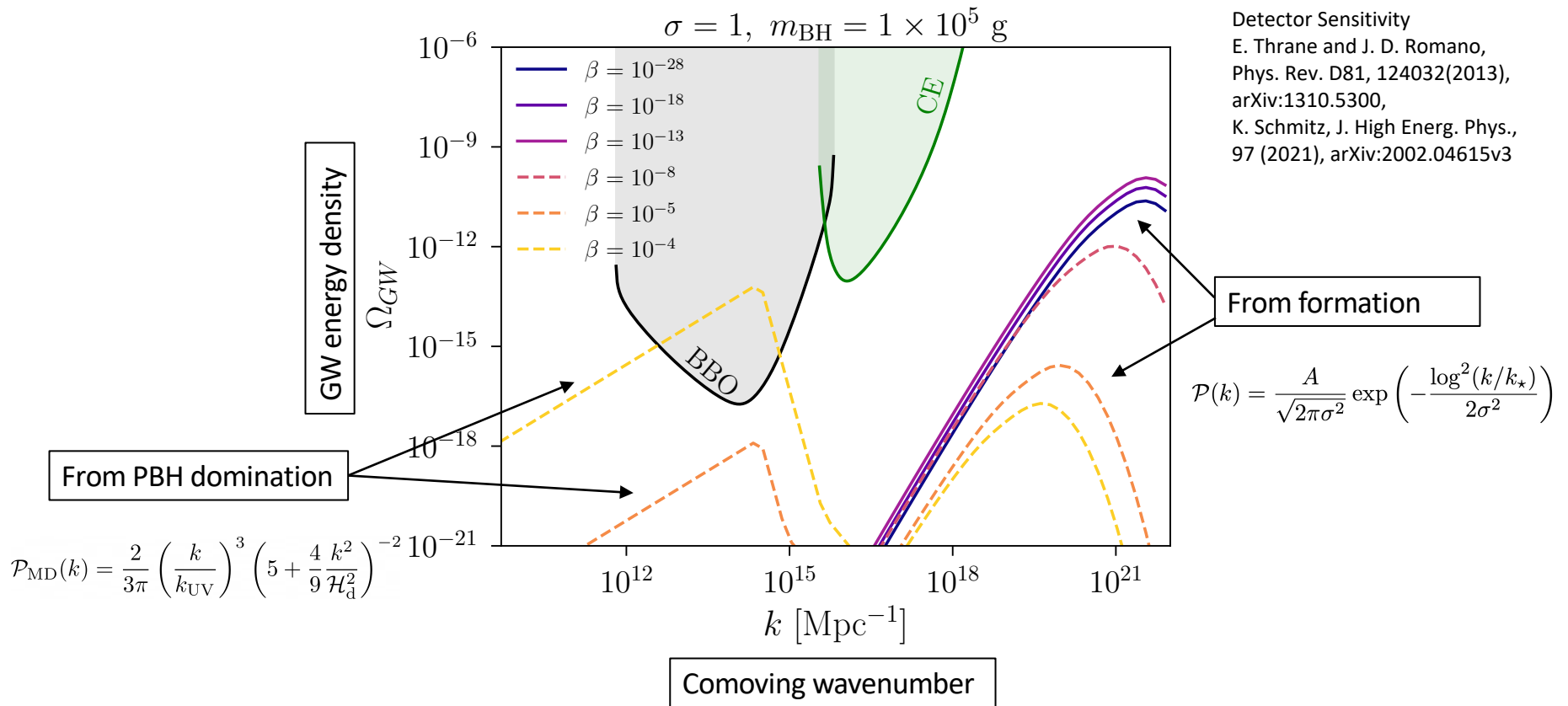
Initial curvature perturbation in PBH
dominated era:

$$\mathcal{P}_{\text{MD}}(k) = \frac{2}{3\pi} \left(\frac{k}{k_{\text{UV}}} \right)^3 \left(5 + \frac{4}{9} \frac{k^2}{\mathcal{H}_d^2} \right)^{-2}$$

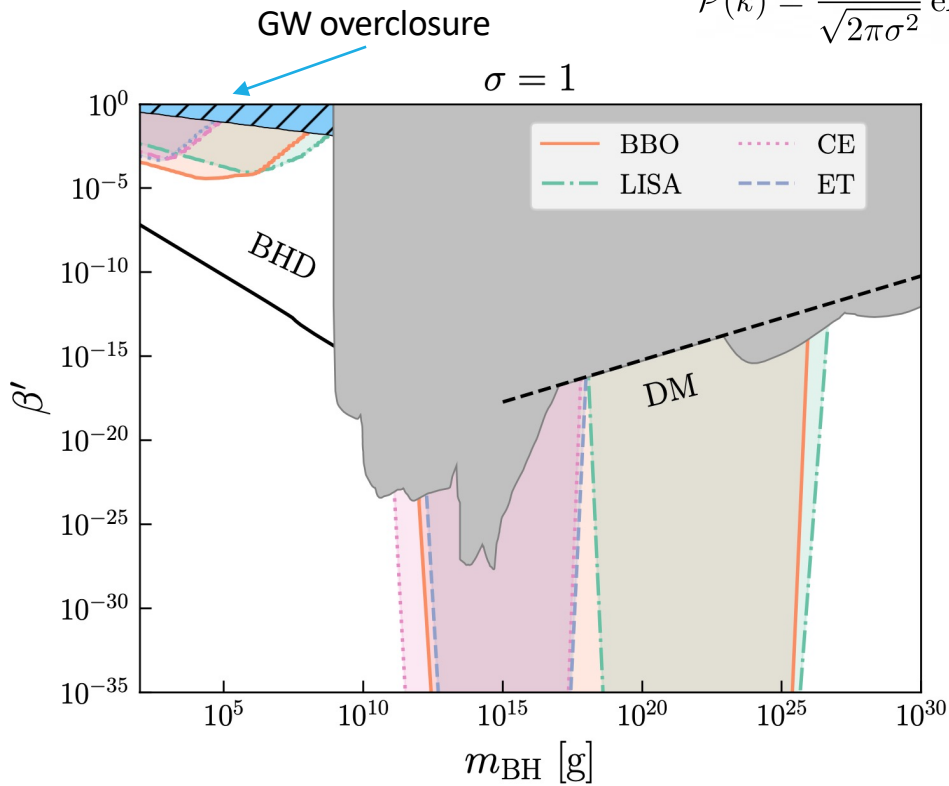
Primordial curvature perturbation
entering horizon during RD:

$$\mathcal{P}(k) = \frac{A}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\log^2(k/k_*)}{2\sigma^2}\right)$$

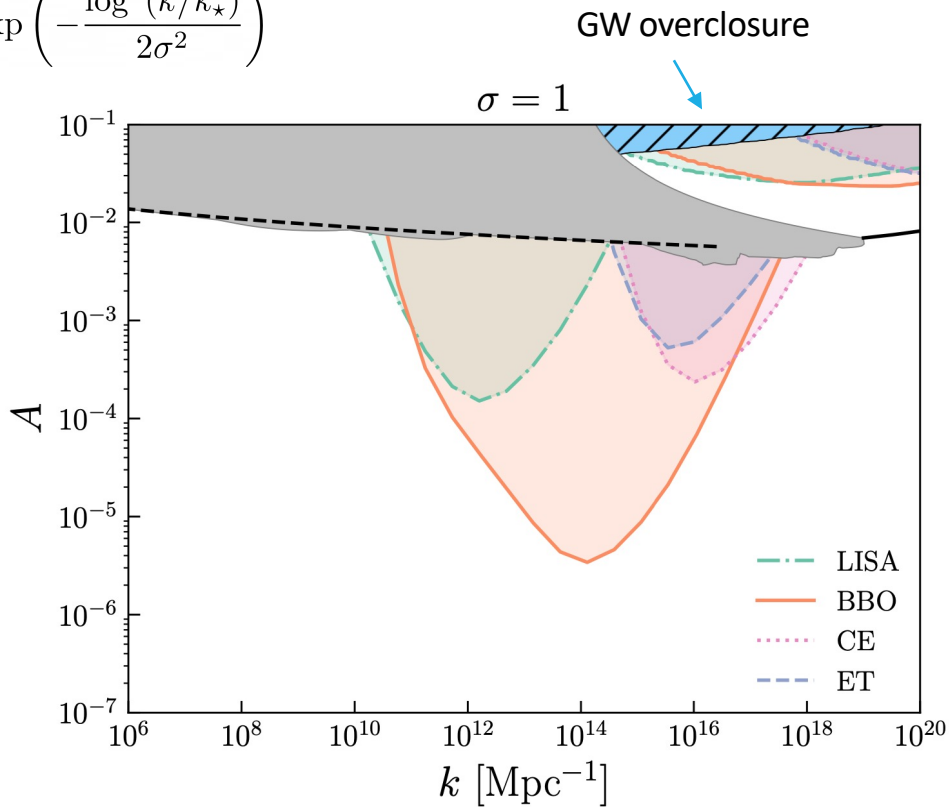
Resulting GW spectra today



$$\mathcal{P}(k) = \frac{A}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\log^2(k/k_*)}{2\sigma^2}\right)$$



Existing bounds, reaches in black hole mass – abundance plane



Existing bounds, reaches in curvature perturbation peak frequency – amplitude plane

Observation Time = 1 yr
SNR = 1

Assuming perfect
subtraction of foreground

Conclusions

Next generation GW observatories may indirectly PBHs in the early universe

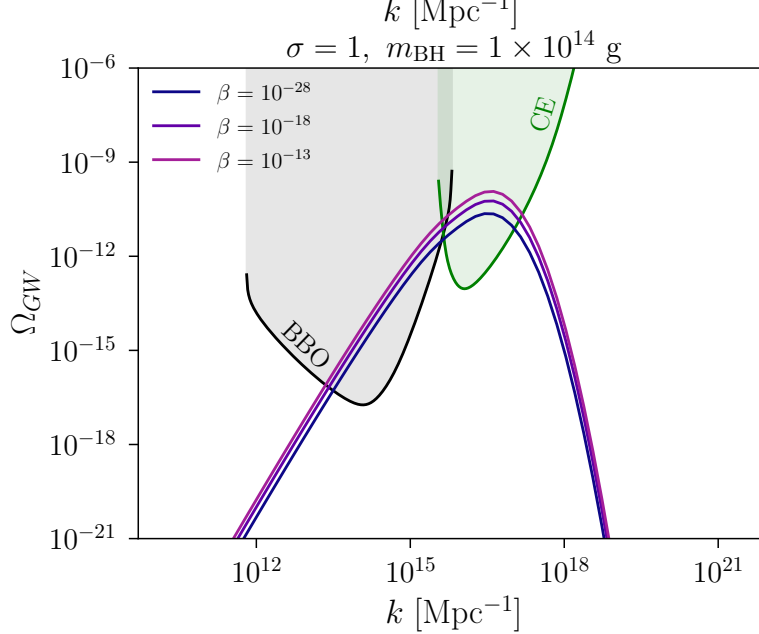
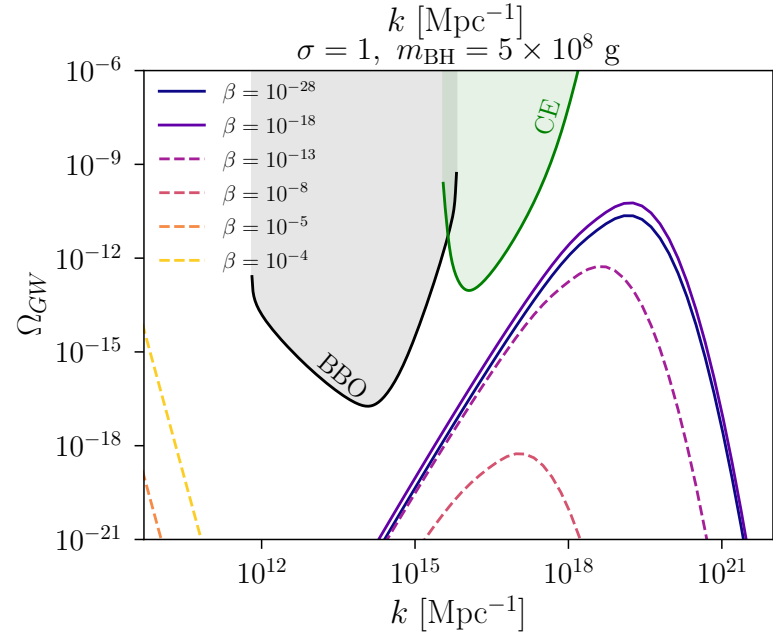
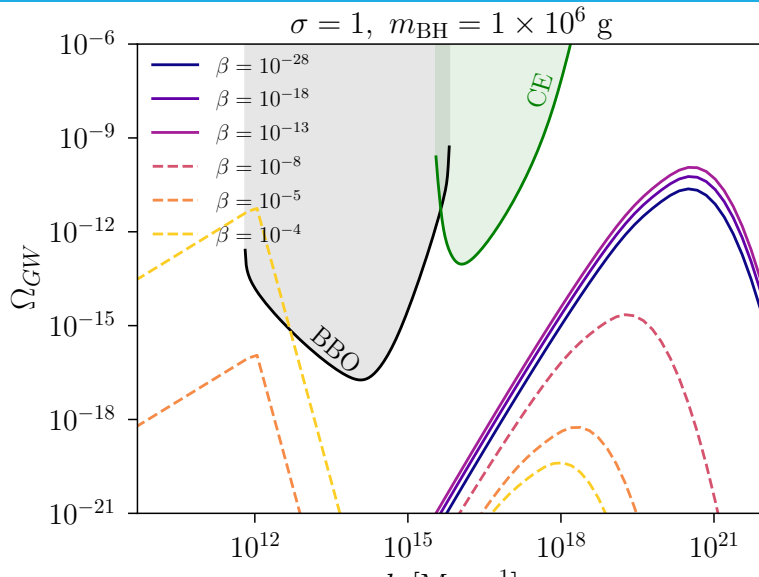
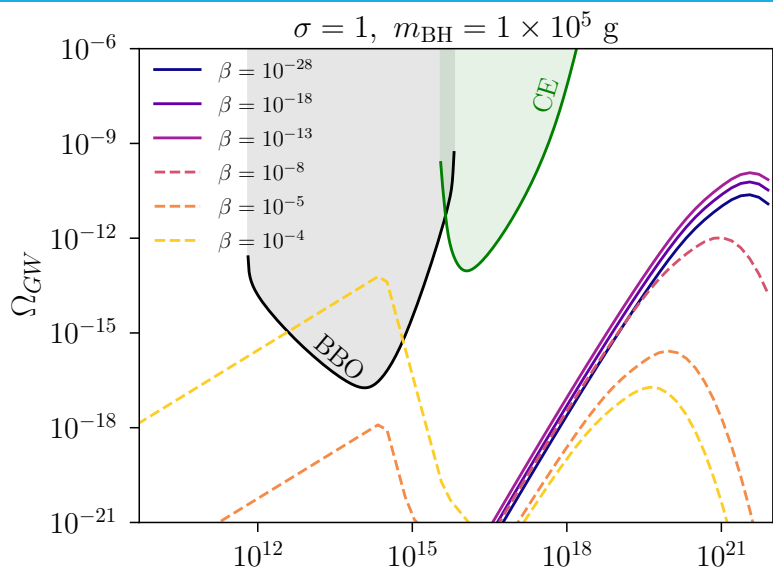
These observatories will indirectly detect long-lived black holes in almost the entire allowed parameter space (depending on the primordial curvature perturbation shape)



Existing bounds from BBN, EGB, etc. (current observational bounds on long lived black holes) can be used to indirectly bound the parameter space of primordial curvature perturbations

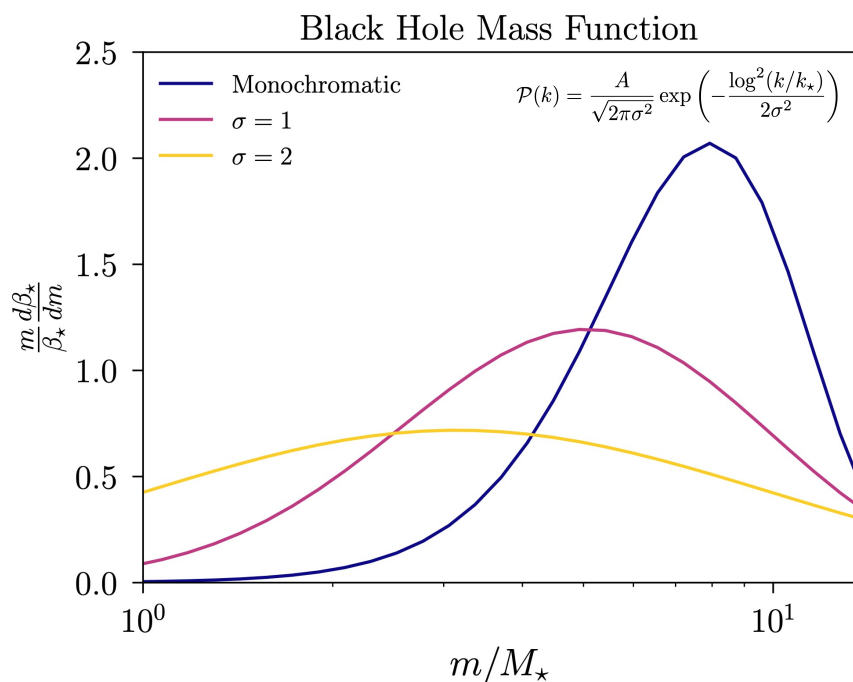
We can also probe primordial perturbations at small scales and high amplitude, *because of the induced black hole dominated era.*

Extras



Primordial Black Hole Formation

We work in the Press-Schechter formalism – PBHs are formed when the density contrast exceeds a certain threshold (various papers have calculated this numerically)

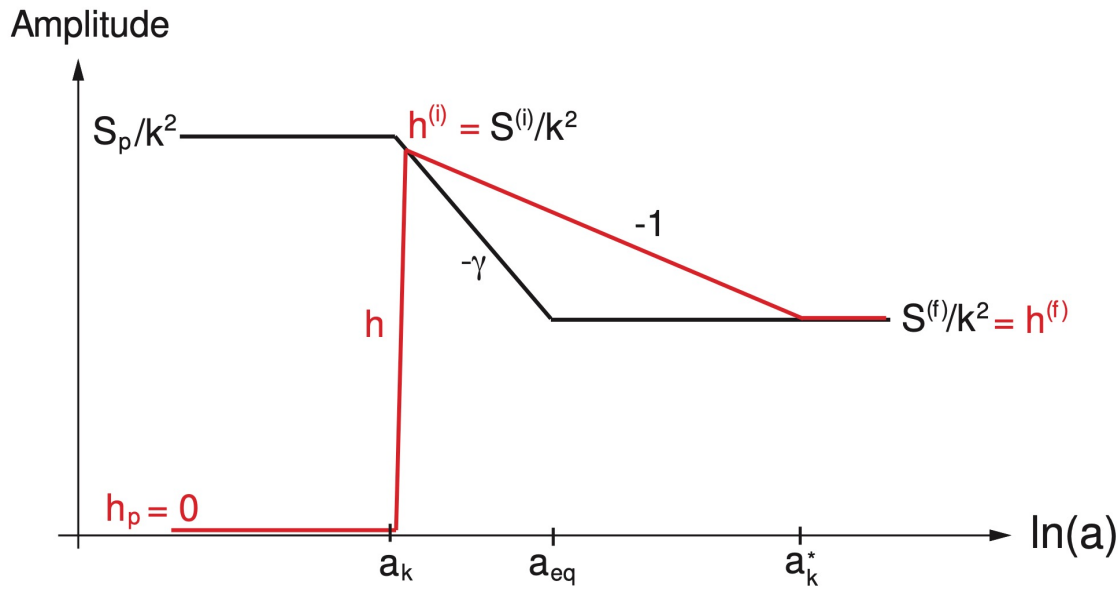


$$m_{\text{BH}} \propto \frac{4\pi}{3} \rho_{r,*} H_*^{-3}$$

$$\beta \propto A^b \text{Erfc} \left[\frac{c}{\sqrt{A}} \right]$$

A monochromatic curvature perturbation with $A = 0.01$ gives an abundance of $\beta = 4 \times 10^{-5}$

Evolution of the source term



$$\Phi_{\mathbf{k}}''(\eta) + \frac{6(1+w)}{(1+3w)\eta} \Phi_{\mathbf{k}}'(\eta) + wk^2 \Phi_{\mathbf{k}}(\eta) = 0$$

$$\text{In RD: } \Phi \sim \frac{1}{(k\eta)^2}$$

$$\text{In MD: } \Phi \sim \text{const.}$$

Second order perturbation theory

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

$$S_{\mathbf{k}} = \int \frac{d^3q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k}) q_i q_j \left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \frac{4}{3(1+w)} (\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}) (\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}) \right)$$

$$\Phi''_{\mathbf{k}}(\eta) + \frac{6(1+w)}{(1+3w)\eta} \Phi'_{\mathbf{k}}(\eta) + wk^2\Phi_{\mathbf{k}}(\eta) = 0.$$

$$\mathcal{P}_h(\eta, k) = 4 \int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1+v^2 - u^2)^2}{4vu} \right)^2 I^2(v, u, x) \mathcal{P}_\zeta(kv) \mathcal{P}_\zeta(ku)$$

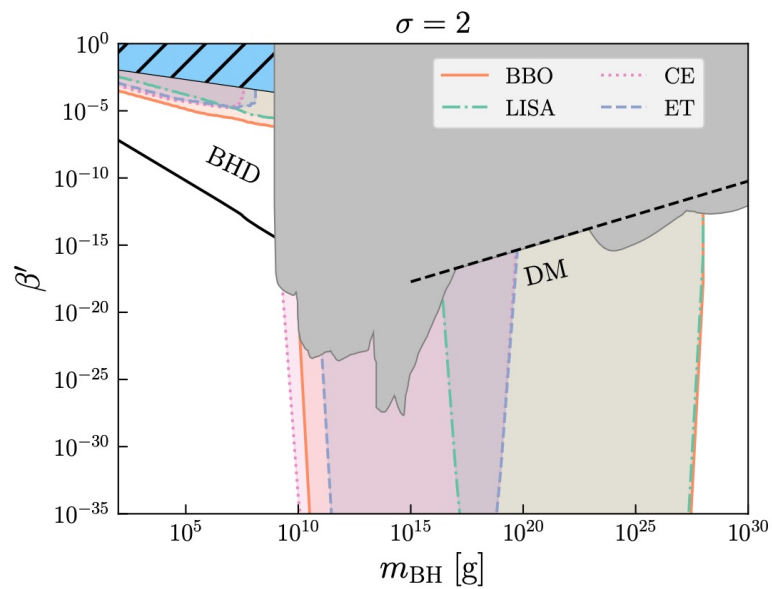
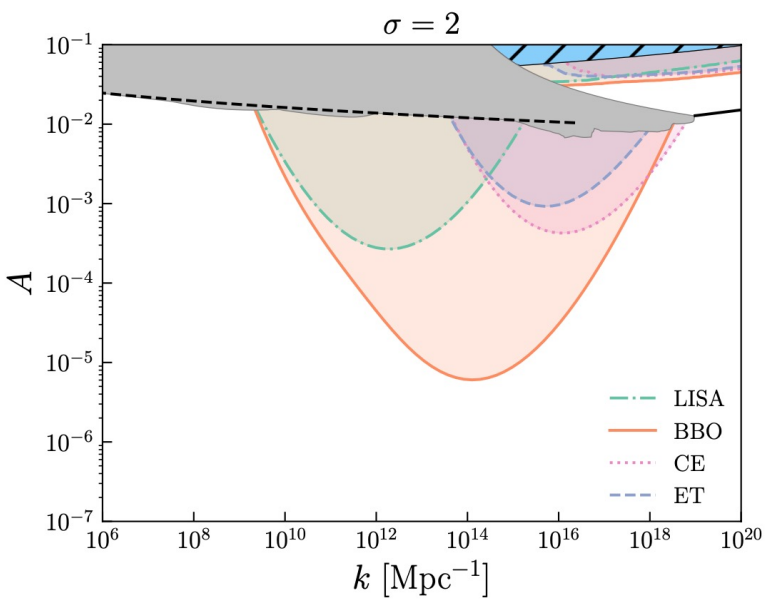
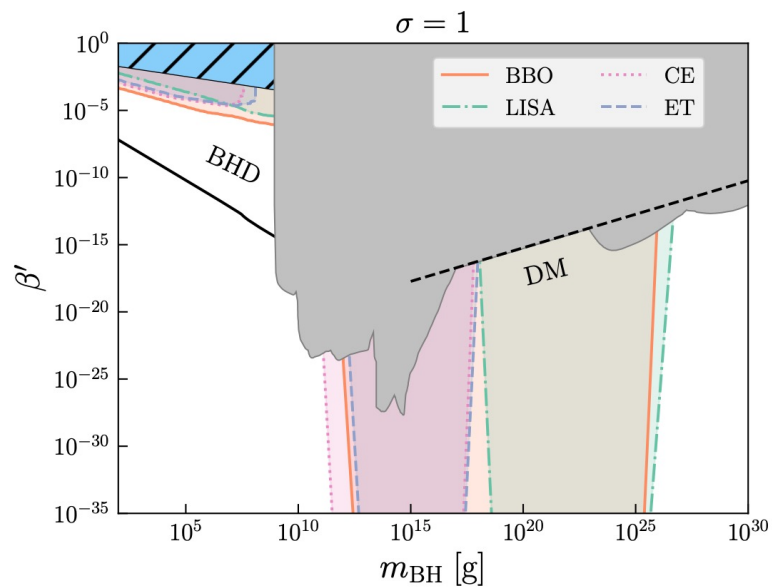
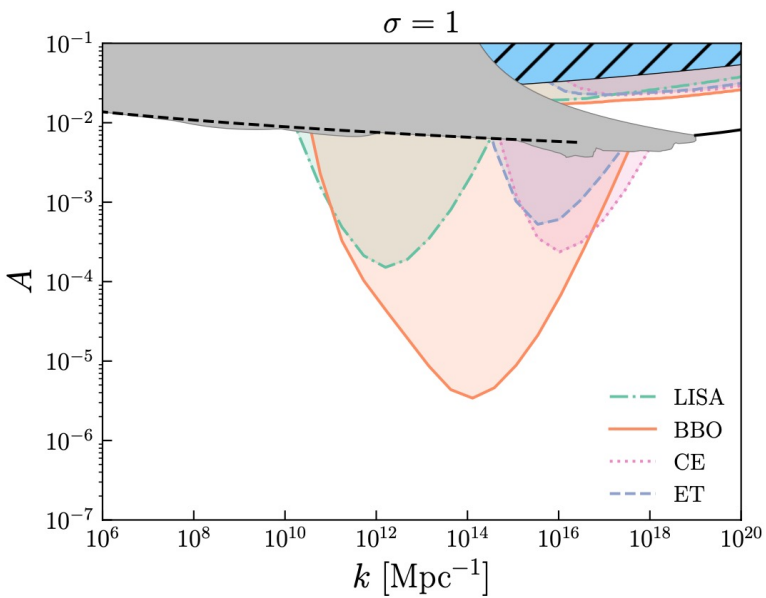
$$\Omega_{\text{GW}}(\eta, k) = \frac{\rho_{\text{GW}}(\eta, k)}{\rho_{\text{tot}}(\eta)} = \frac{1}{24} \left(\frac{k}{a(\eta)H(\eta)} \right)^2 \overline{\mathcal{P}_h(\eta, k)}$$

Nonlinearities from MD era

Since matter density perturbations grow with the scale factor, in a long enough MD era, they will eventually become non-linear, what happens then?

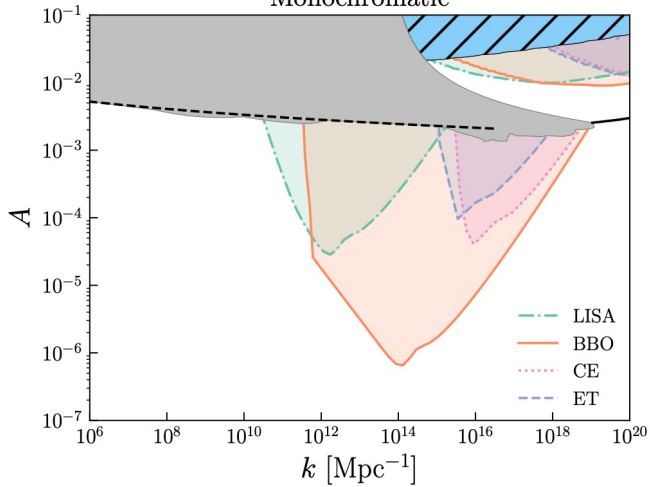
- Possible clustering effects, PBHs form halos and transfer power to smaller scales?

In our results, we apply a conservative estimate: apply a cutoff to the source when the matter perturbations become nonlinear.

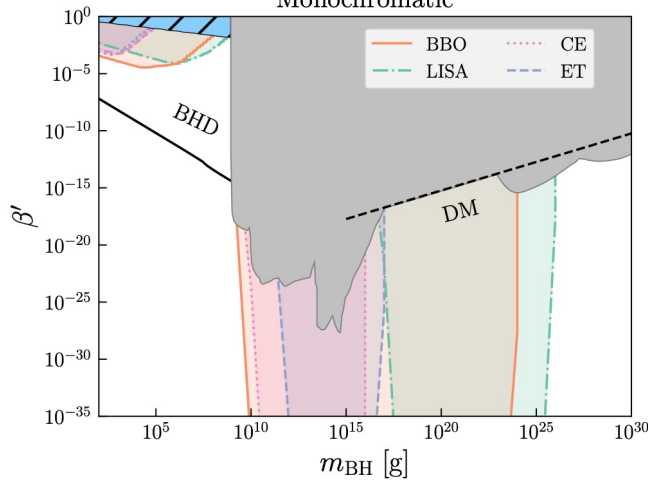


Reaches
without
non-linear
cutoff

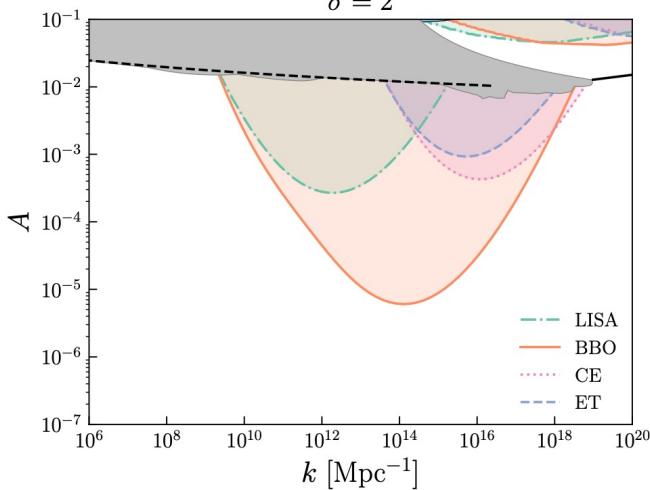
Monochromatic



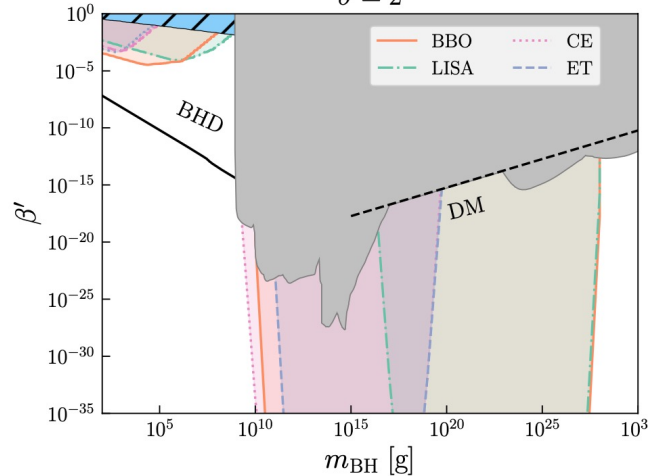
Monochromatic



$\sigma = 2$



$\sigma = 2$



With NL cutoff, monochromatic and $\sigma = 2$ widths

Note that the relationship between amplitude and abundance is highly dependent on the curvature perturbation width

Additional Sources of GWs

- Mergers
- Hawking radiation
- Rapid reheating from black hole dominated \rightarrow radiation dominated

We don't take into account the mergers + Hawking radiation, as the scales are too small to appear in our reaches.

Rapid reheating also requires a very narrow mass function, so we don't consider this case

