Gravitational waves from first-order phase transition during inflation

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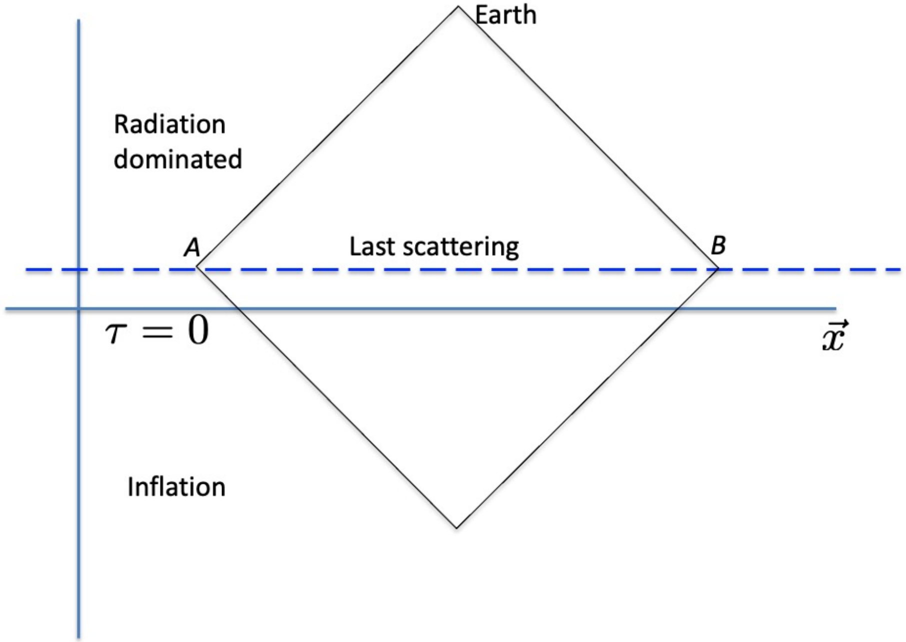
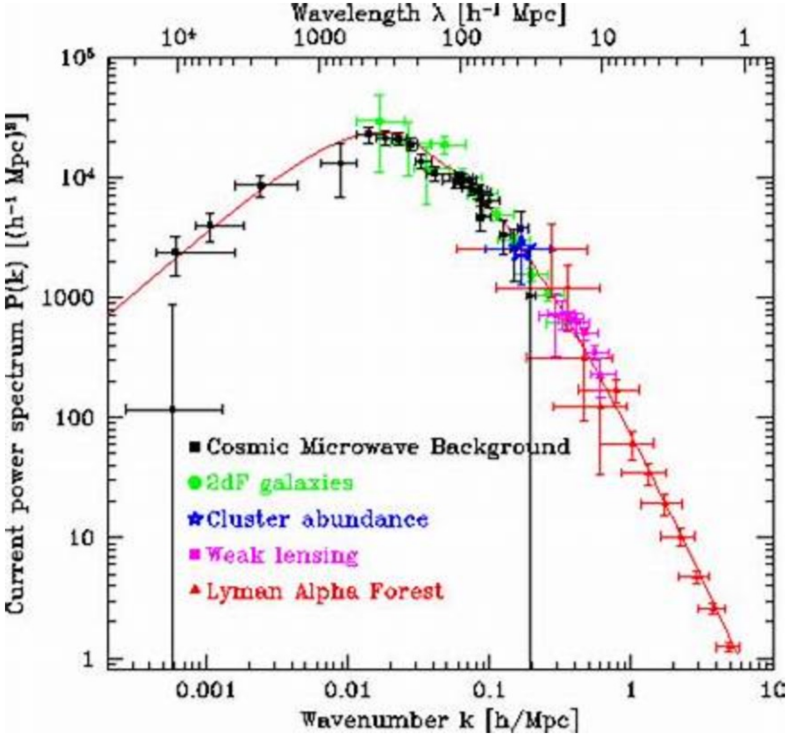
2009.12381, 210X.XXXXX

Very brief history of our universe

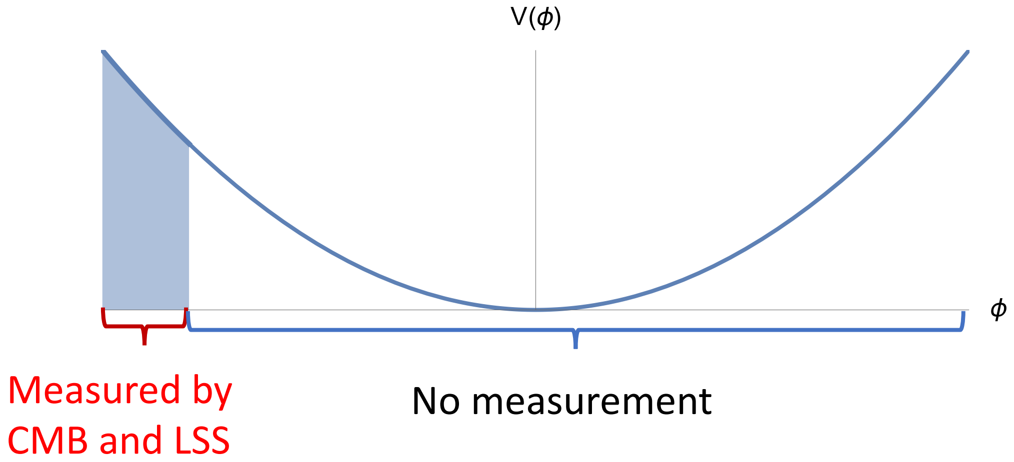




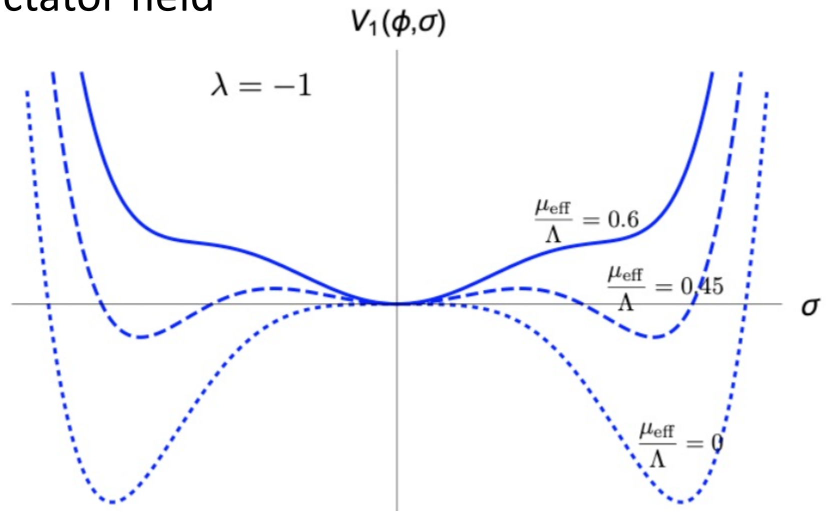
- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

First order phase transition driven by the evolution of the inflaton

- ϕ: inflaton field



σ: spectator field



$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

- It is generic to expect the inflaton to couple to some spectator sector.
- The masses or couples in the spectator sector can be changed drastically due to the evolution of the inflaton field.
- It may induce first order phase transitions in the spectator sector so that classical gravitational waves might be generated.

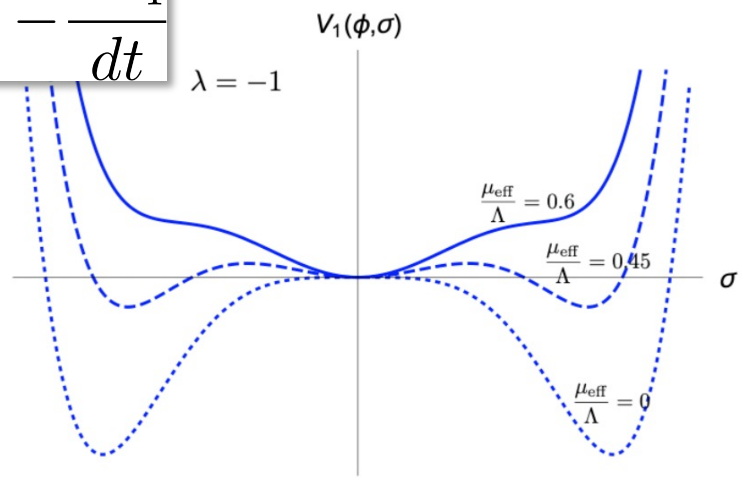
First order phase transition during inflation

- Bubble nucleation rate: $\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$
- Phase transition starts: $\mathcal{O}(1) = \int_{-\infty}^{t} dt' H^{-3} I_0 m_\sigma^4 e^{-S_4(t')}$
- The bounce: $S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm pl}} \frac{m_\sigma^4}{H^4}\right)$
- First order phase transiton: $S_4 \gg 1$

$$H^4 \ll m_\sigma^4 \ll 3M_{\rm pl}^2 H^2$$

First order phase transition during inflation

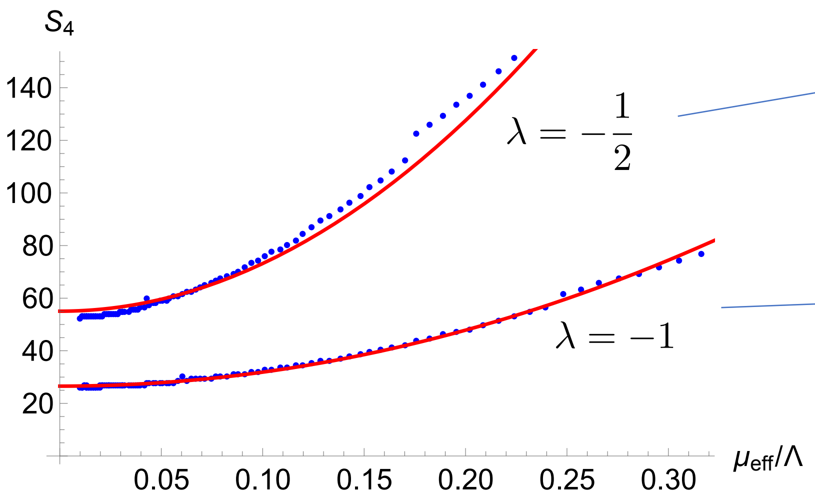
$$\beta = -\frac{dS_4}{dt}$$



$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

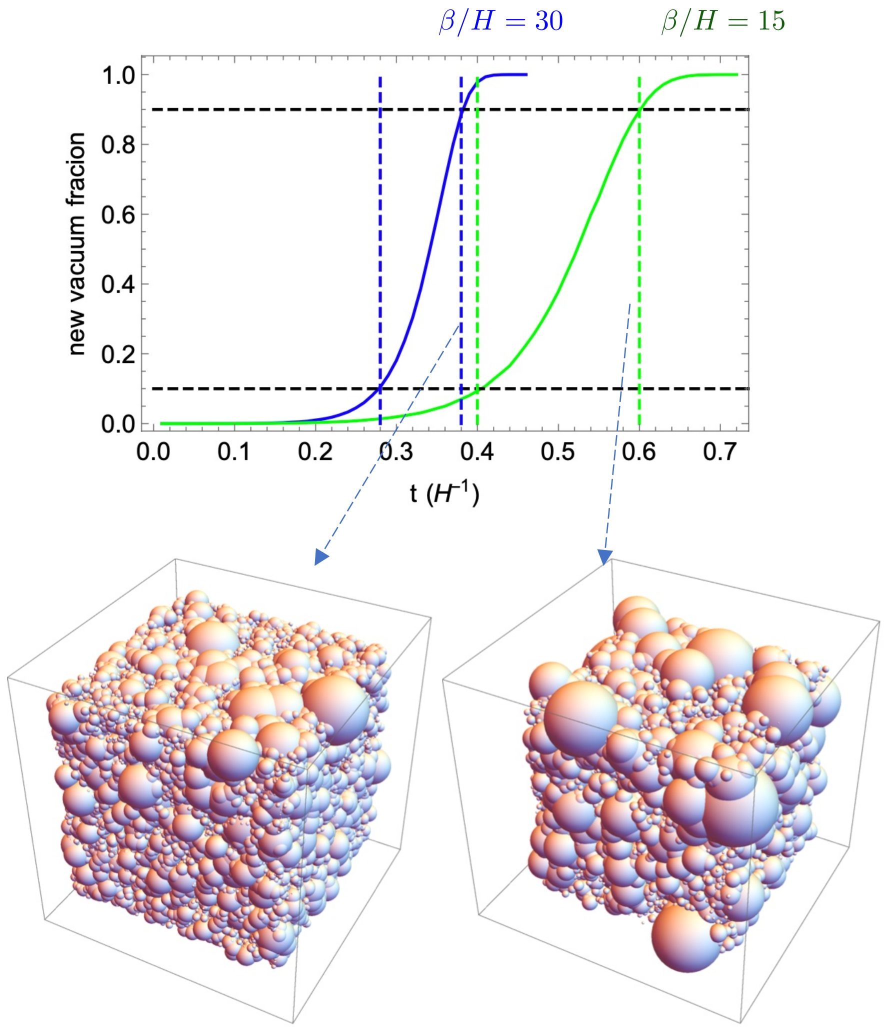
$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

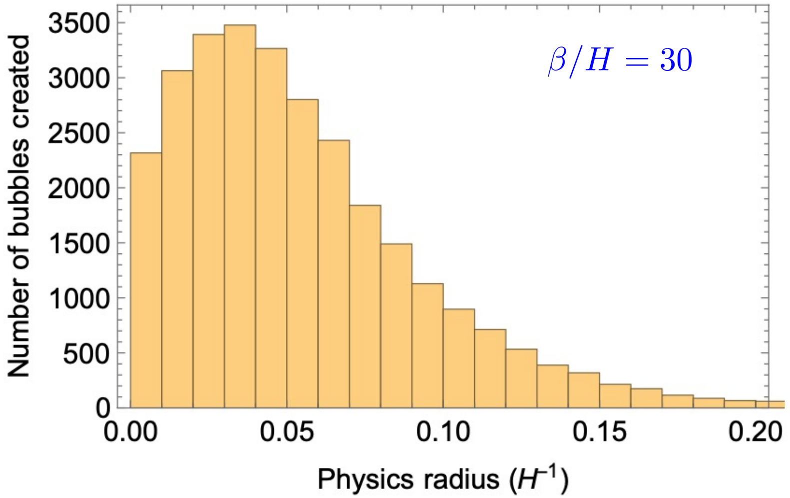
$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

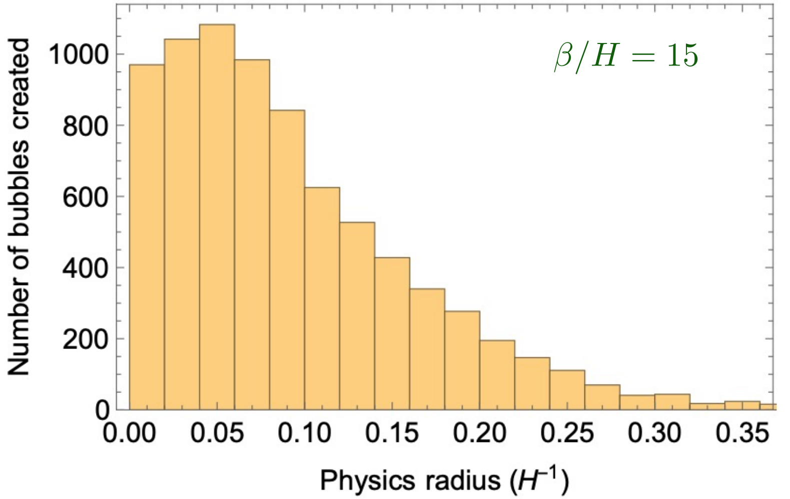
CosmoTransitions

Numerical results

$$R_{\text{bubble}} \approx \beta^{-1} \ll H^{-1}$$

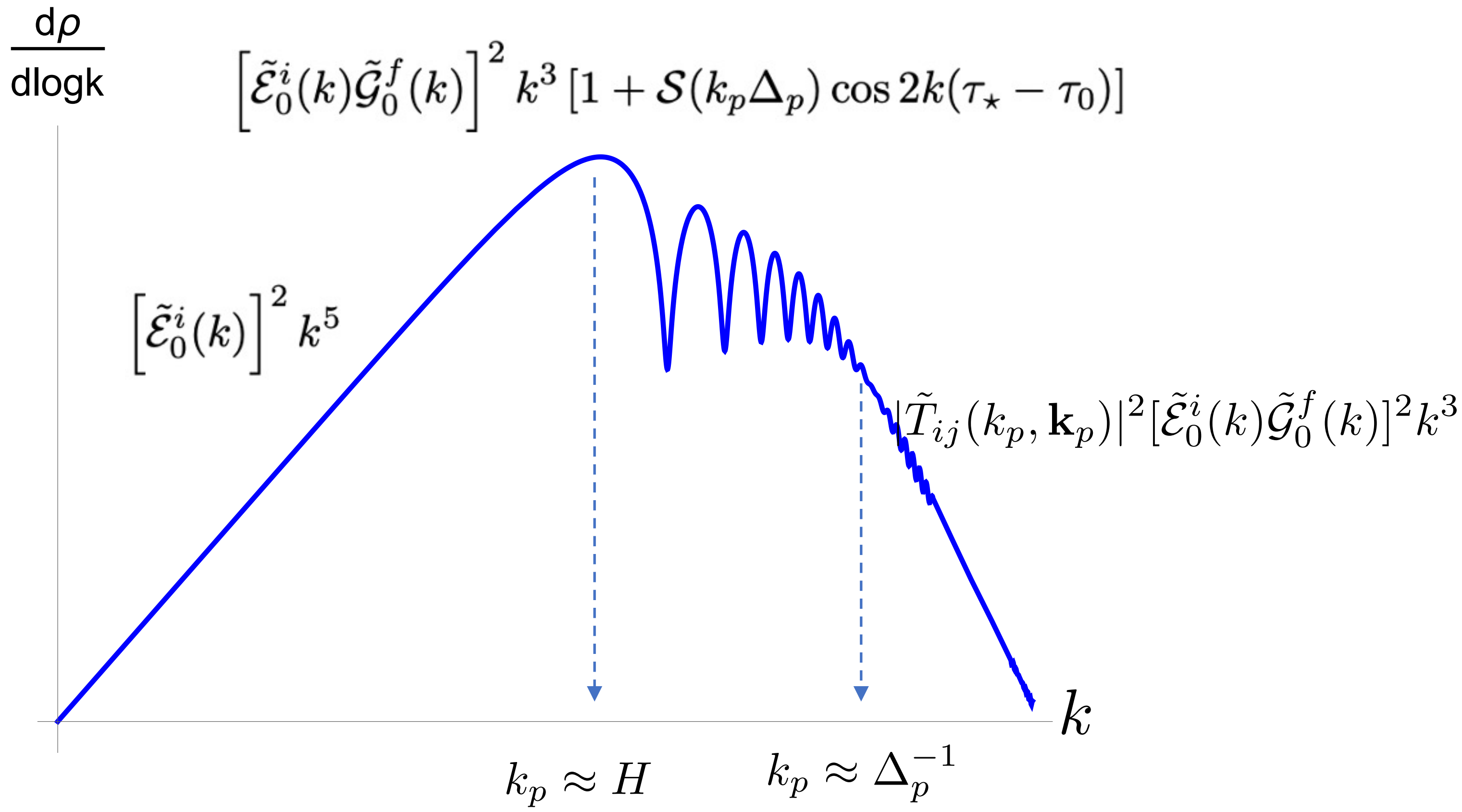






Generic features of GW spectrum

- Shape of the spectrum



How to calculate GW?

- In E&M: $$\partial_\mu F^{\mu\nu} = J^\nu$$
    - We solve the Green's function first.
    - We convolute the Green's function with the source.
- In GR:

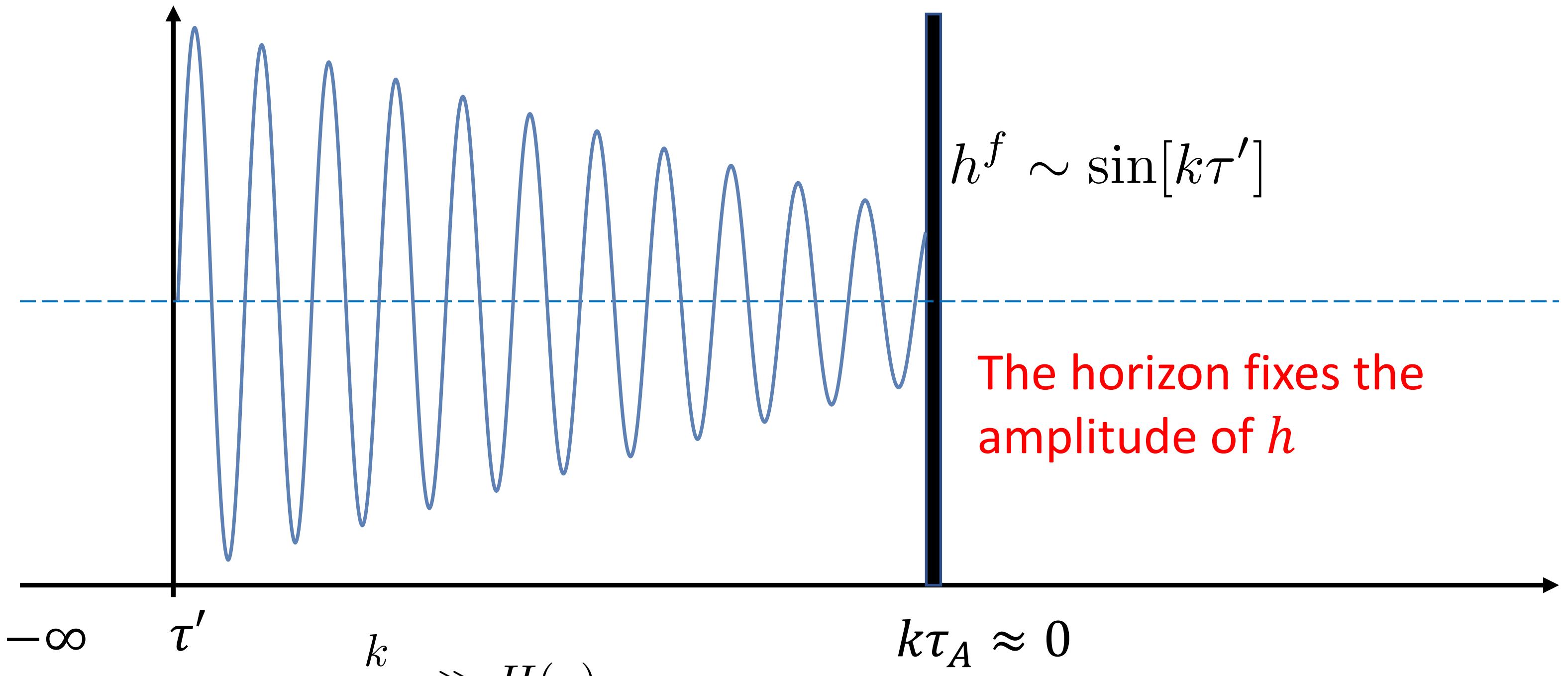
$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

    - Green's function in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$



$$\frac{k}{a(\tau)} \gg H(\tau)$$

$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

GW from instantaneous and local sources (qualitative study)

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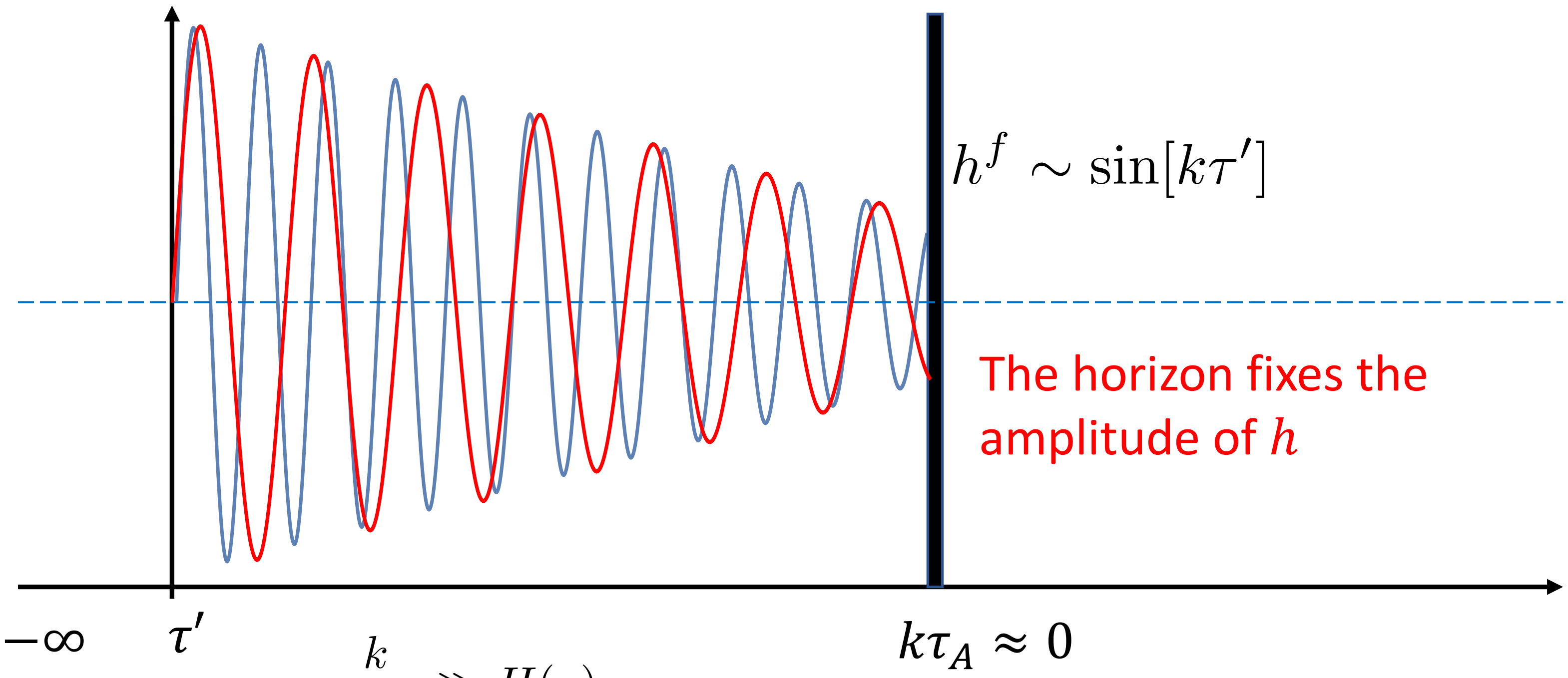
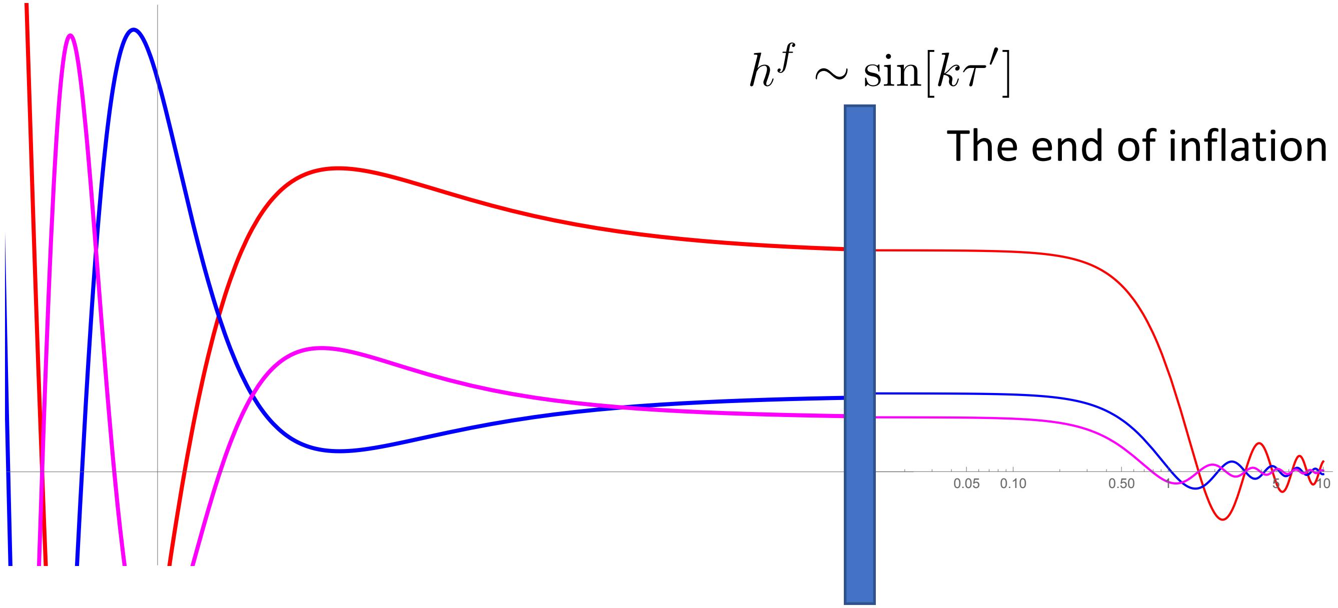


$$\frac{k}{a(\tau)} \gg H(\tau)$$

$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

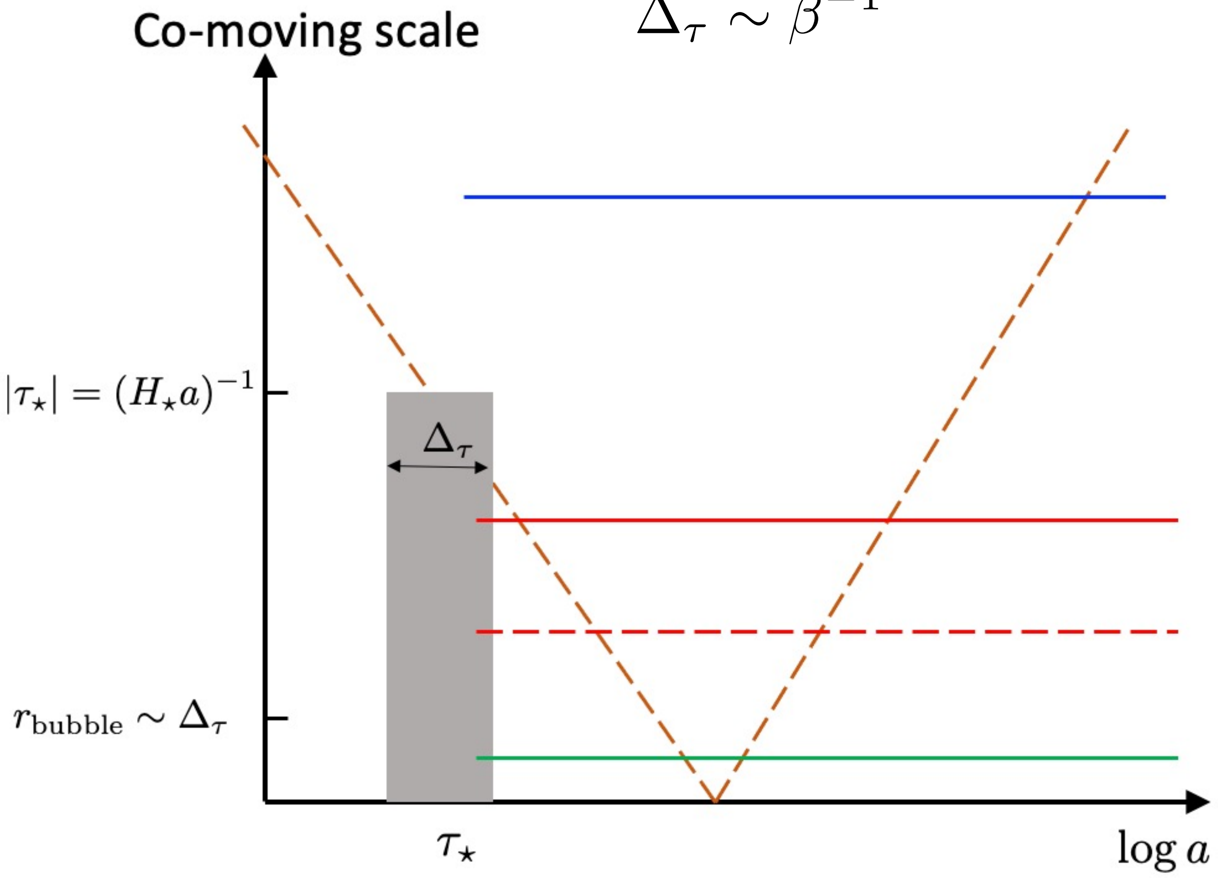
After inflation

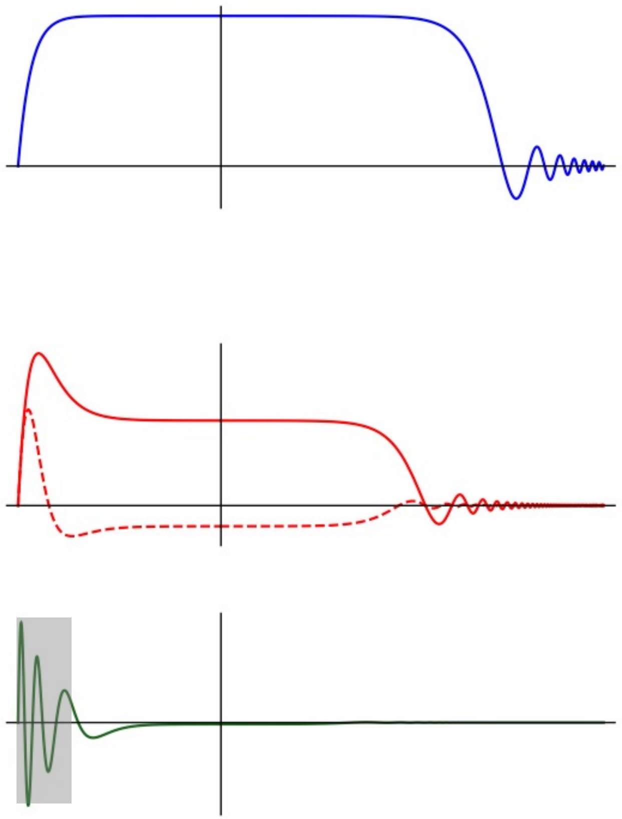
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



GW spectrum for a real source

$$\Delta_\tau \sim \beta^{-1}$$





Generic features of GW spectrum

- Inflation models
  - de Sitter inflation $\tilde{\mathcal{G}}_0^f \sim \frac{1}{k}$
  - t^p inflation $\tilde{\mathcal{G}}_0^f \sim k^{\frac{p}{1-p}}$
- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
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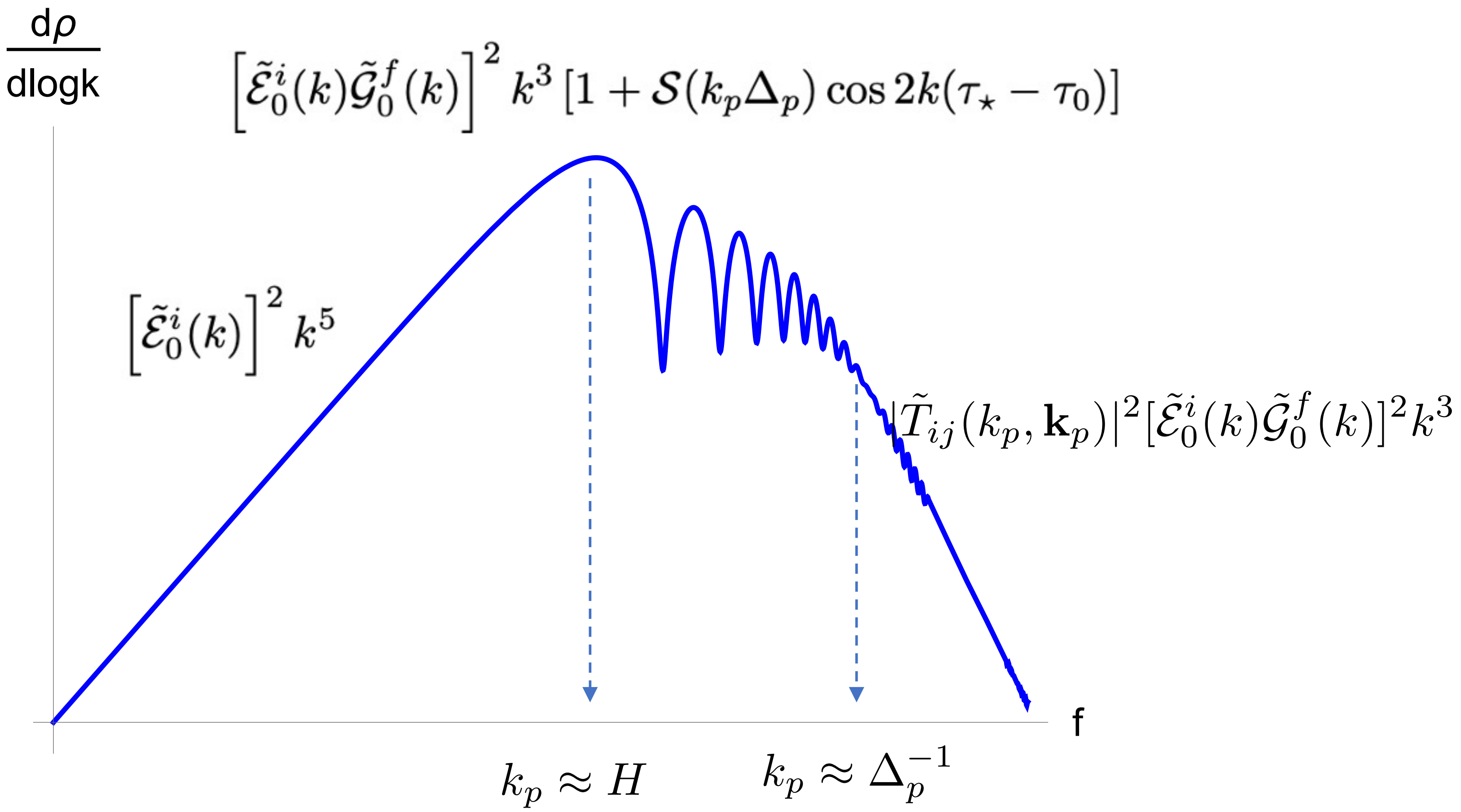
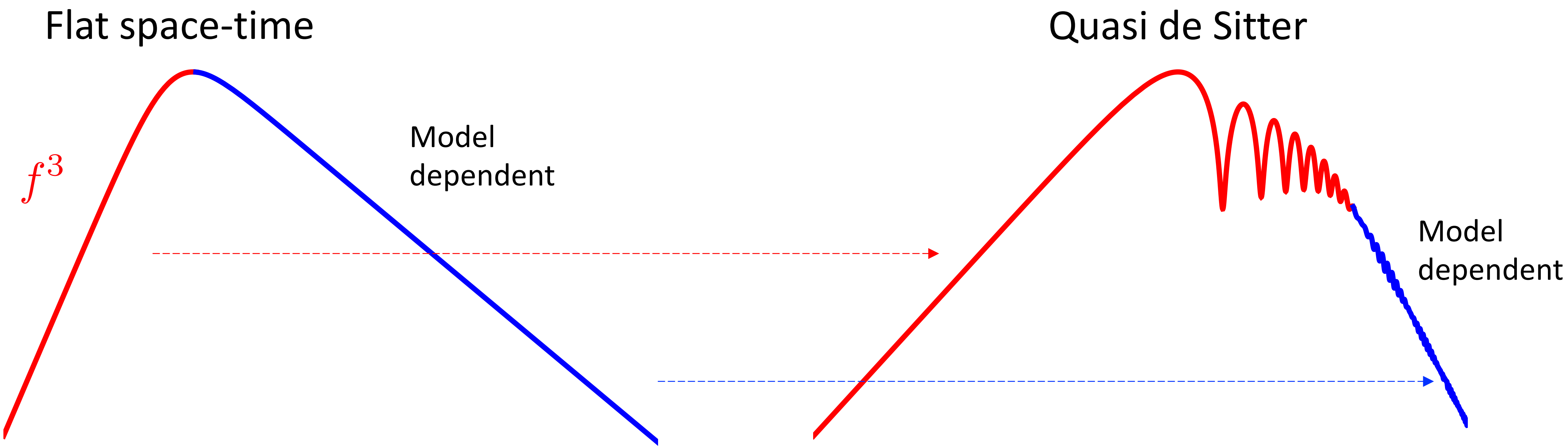


Spectrum distortion



Cai, Pi, Sasaki, 1909.13728

GW Power spectrum

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac} d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Flat space spectrum

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$$e^{-N_e}$$

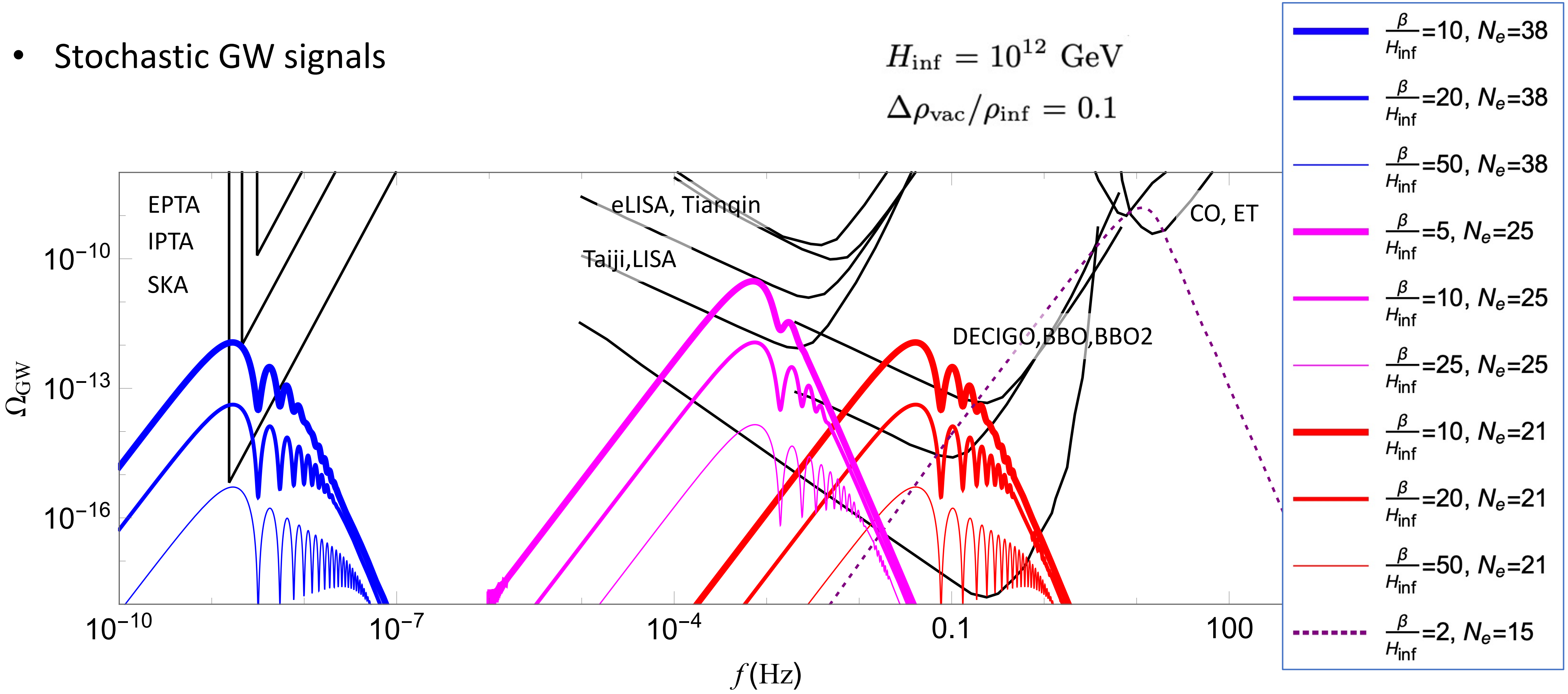
N_e: e-folds before the end of inflation

First order phase transition during inflation

- Stochastic GW signals

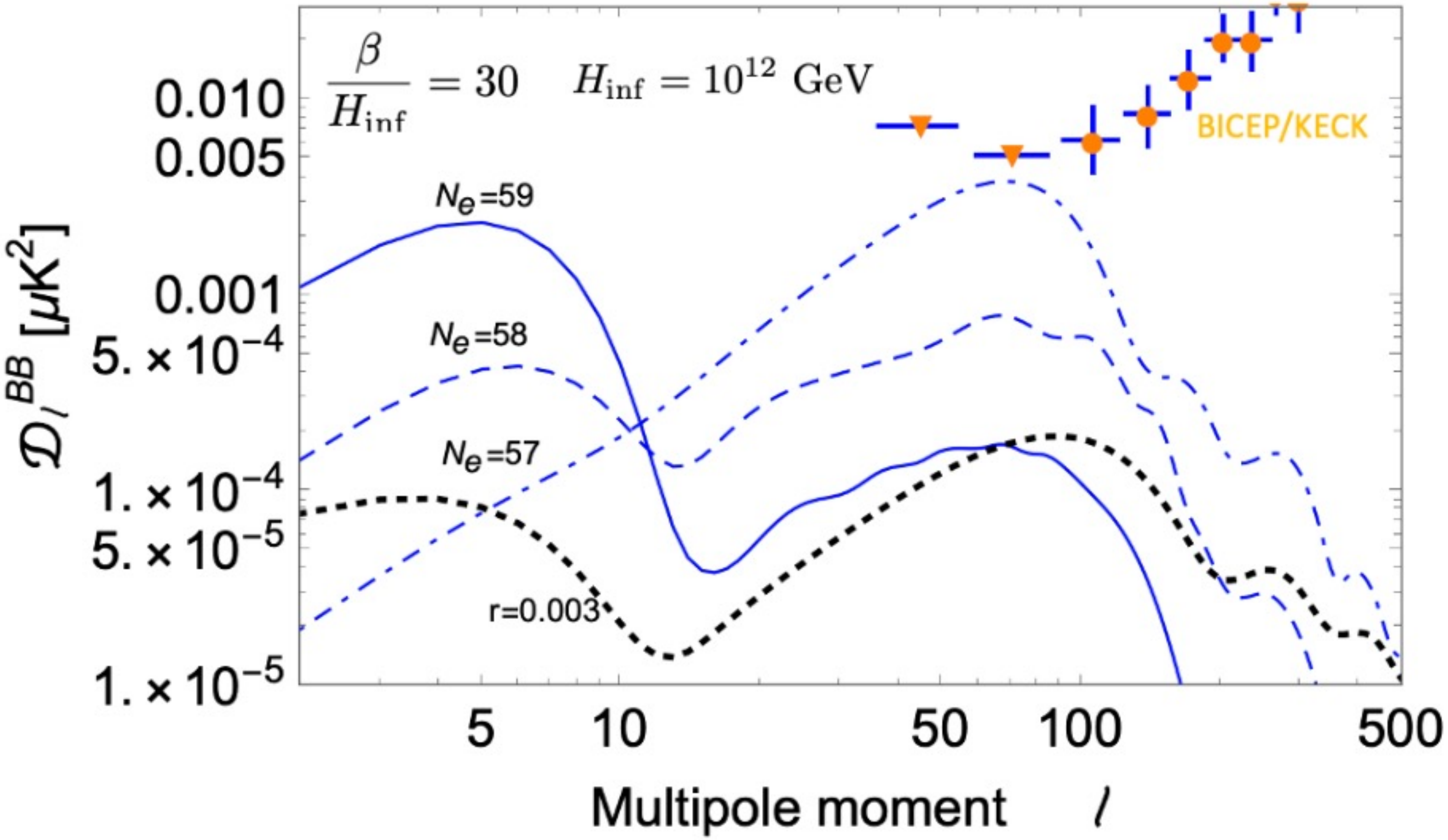
$$H_{\rm inf} = 10^{12}\ \text{GeV}$$

$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.1$$



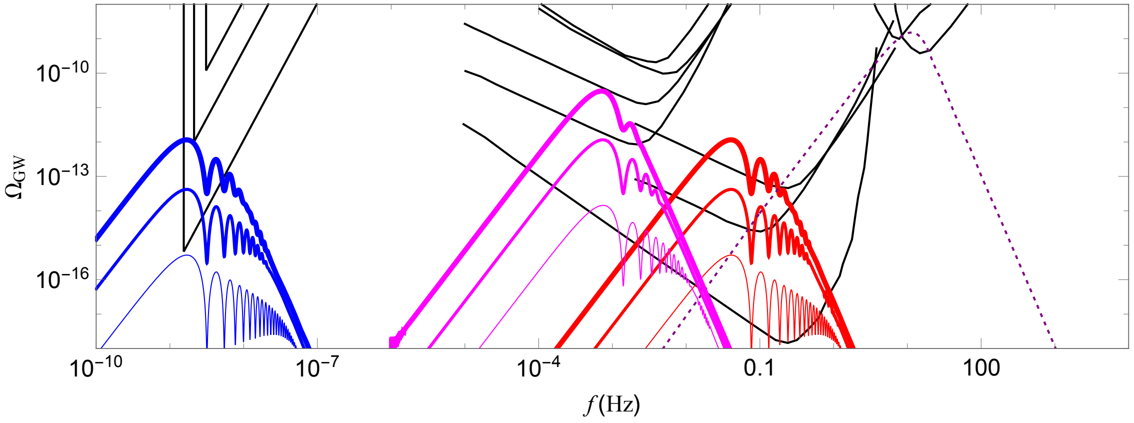
First order phase transition during inflation

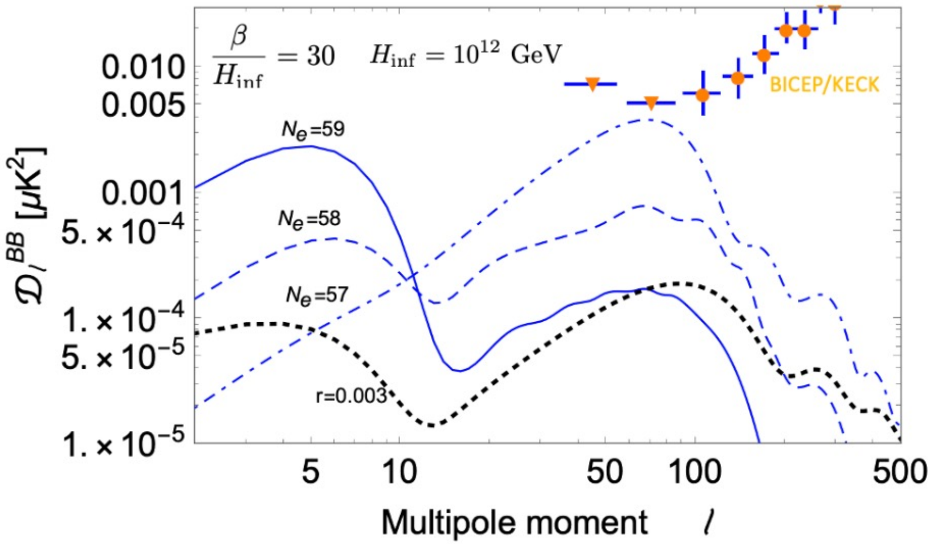
- CMB B modes



Summary

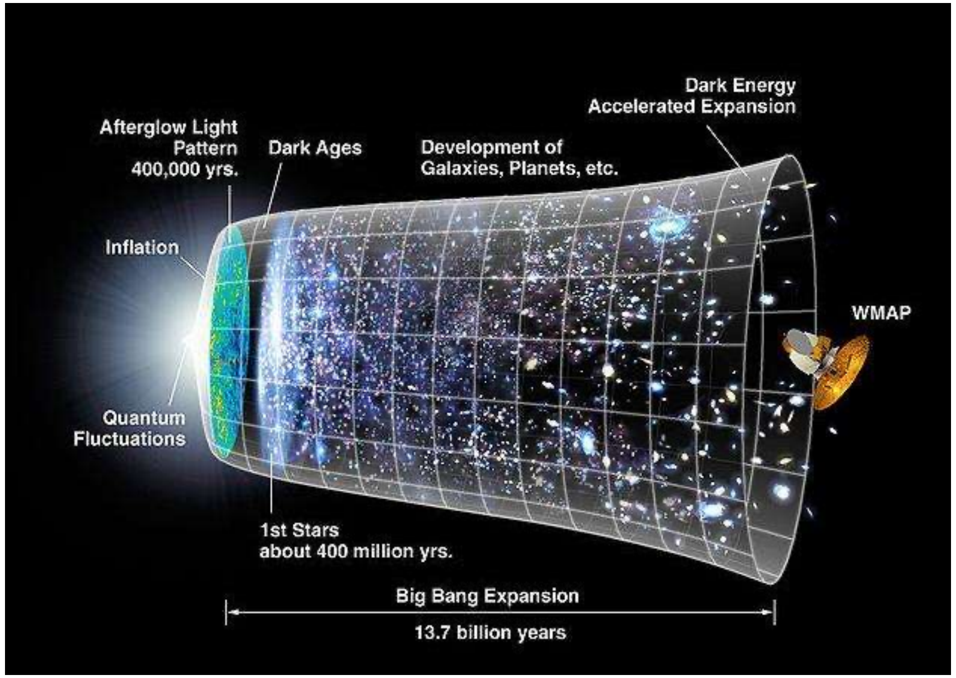
- We show that there is an oscillatory feature in the spectrum.
- The slopes of the spectrum can tell us information about the inflation model and evolution of the universe when the modes re-enter the horizon.
- First order phase transition during inflation can be realized with simple models.
- If we are lucky enough, such a signal can be detected by future GW detectors.





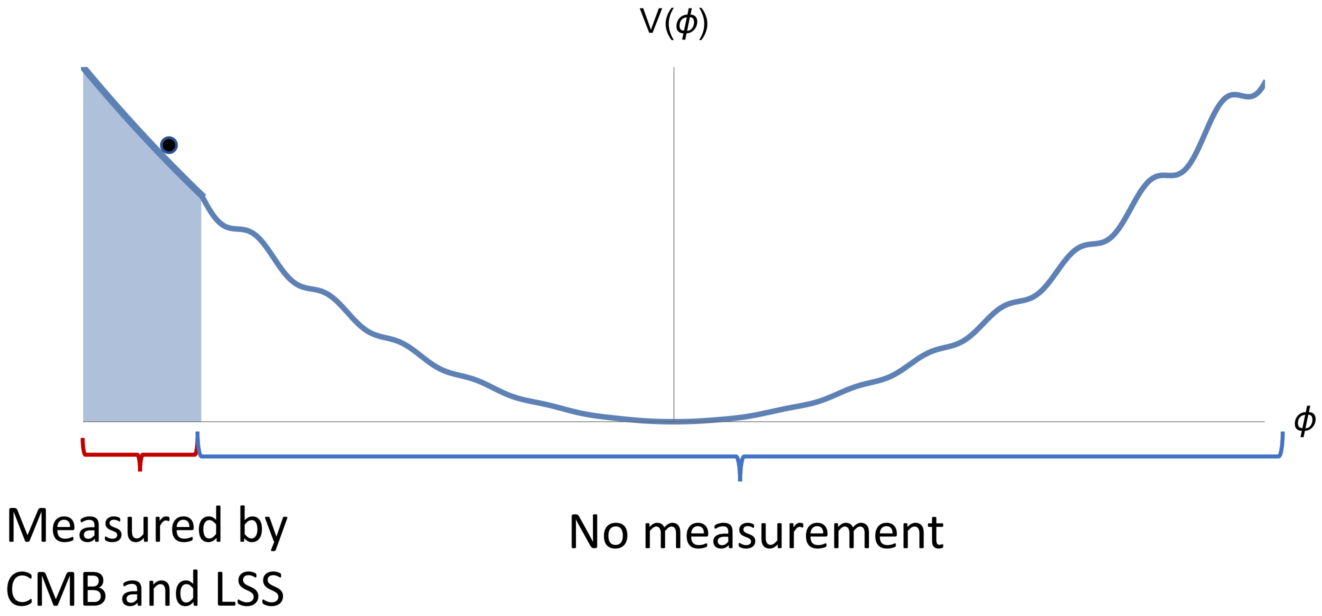
Very brief history of our universe

1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure



Slow roll models

- We usually assume a potential.
- Use it to calculate n_s, r ...

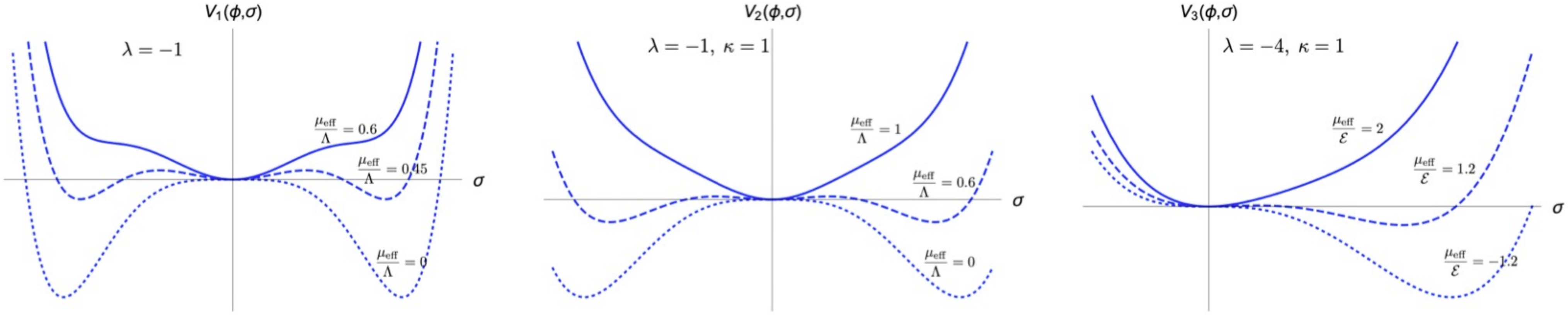


- It is generic to expect the inflaton to couple to some spectator sector.
- The masses or couples in the spectator sector can be changed drastically due to the evolution of the inflaton field.
- It may induce first order phase transitions in the spectator sector so that classical gravitational waves might be generated.

Models of first order phase transition during inflation

- Simple models:

$$\phi: \text{ inflaton field}, \quad \sigma: \text{ spectator}$$



$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

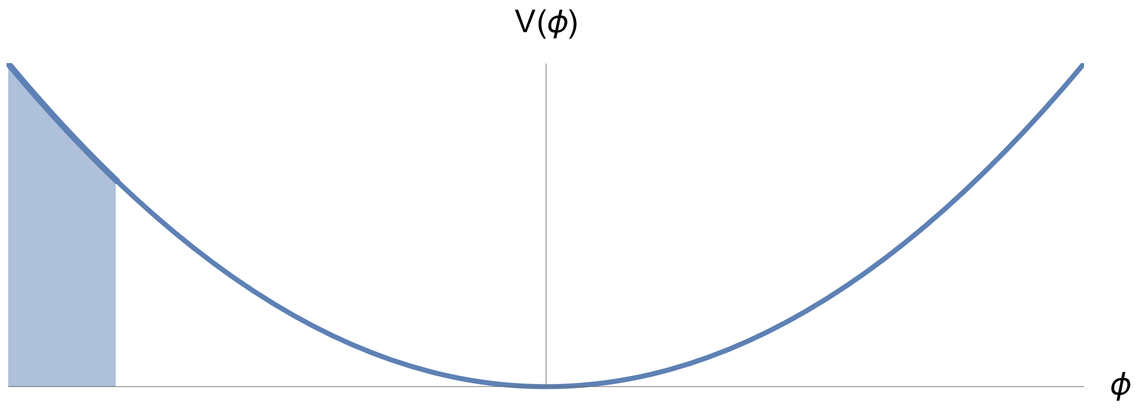
$$V_2(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{4}\sigma^4 \log\frac{\sigma^2}{\Lambda^2}$$

$$V_3(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{3}\mathcal{E}\sigma^3 + \frac{\kappa}{4}\sigma^4 \ .$$

$$\mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$

First order phase transition driven by the evolution of the inflaton

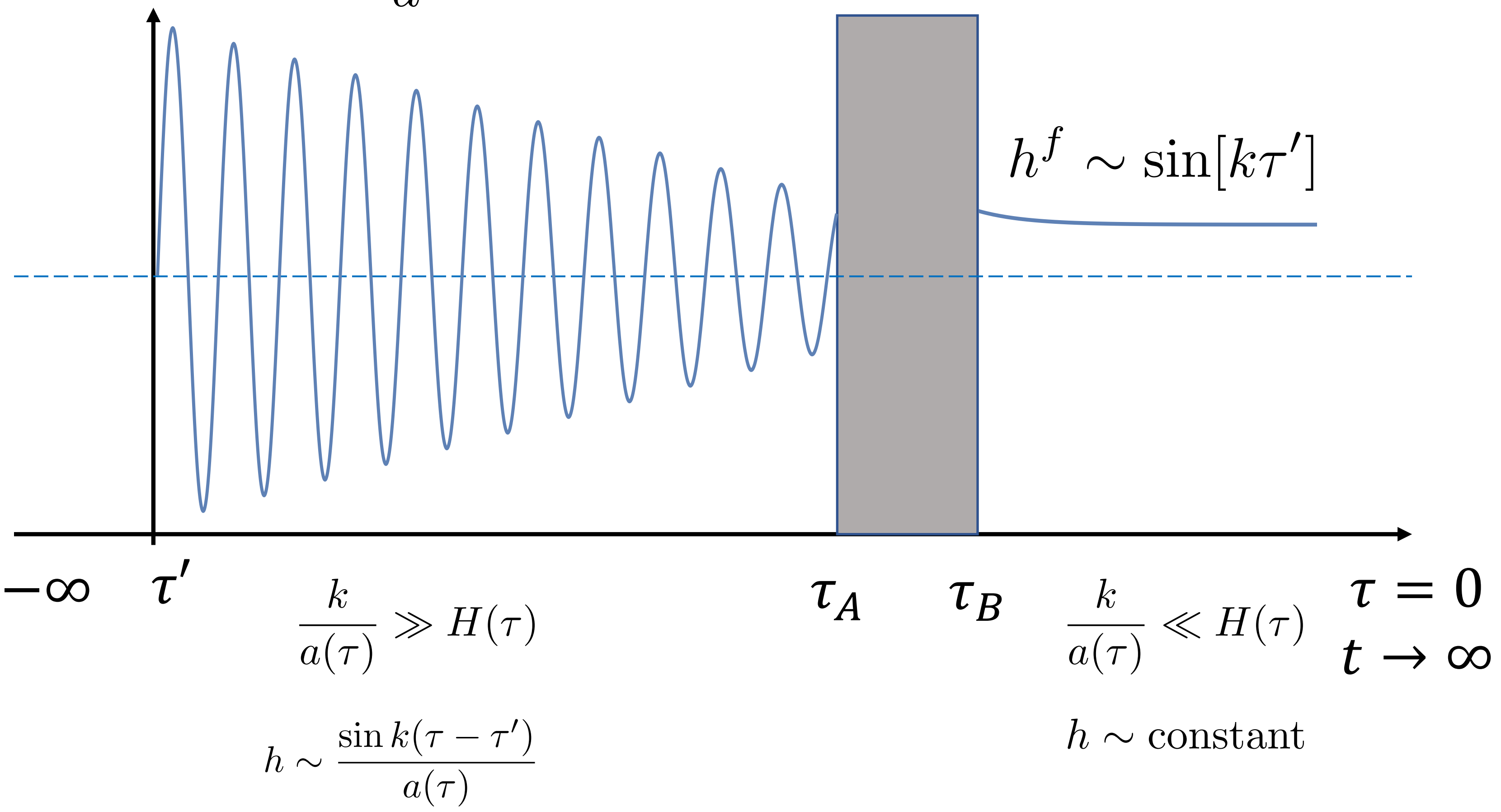
- ϕ: inflaton field $\qquad$ G: strong interacting spectator field



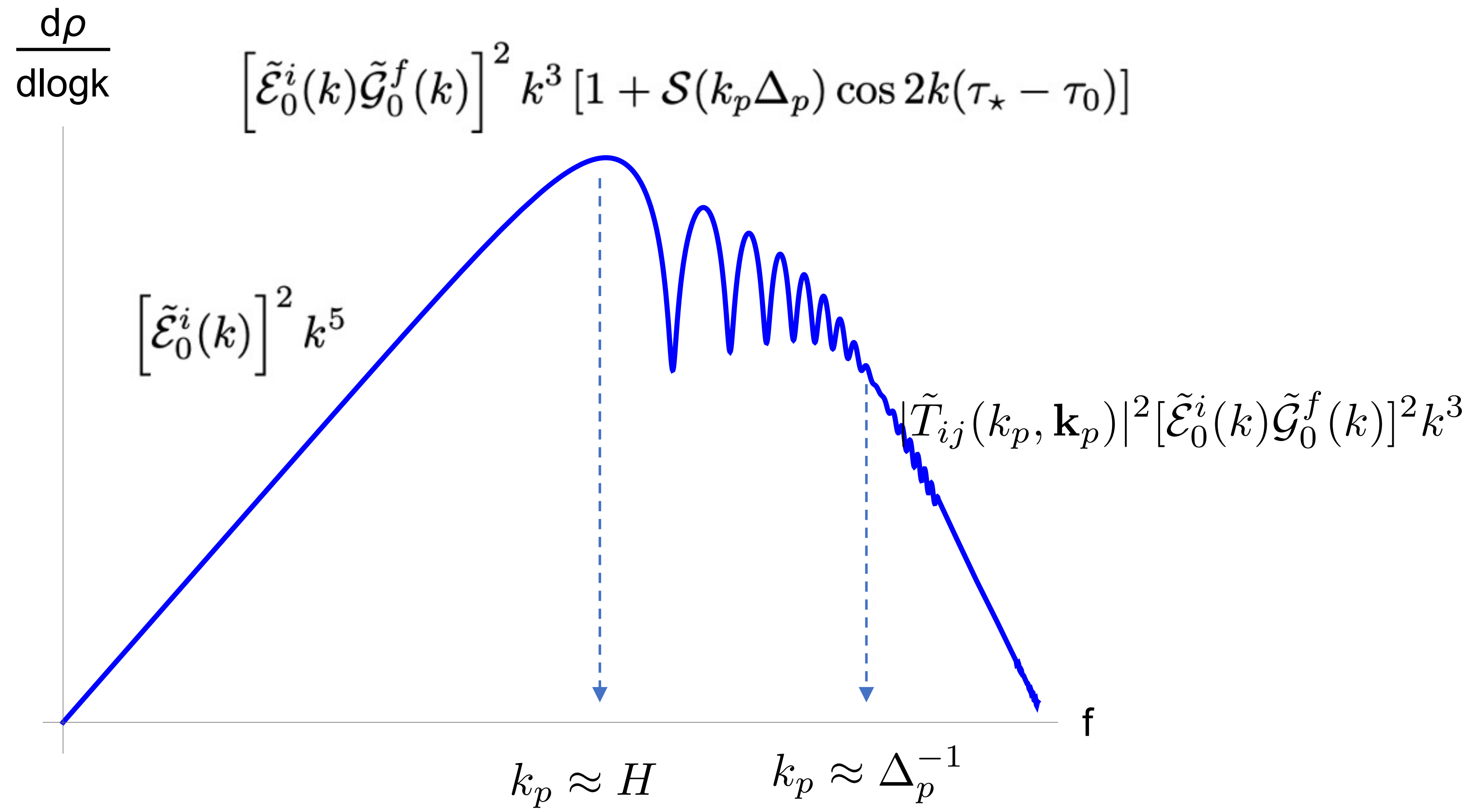
$$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$



Generic features of GW spectrum



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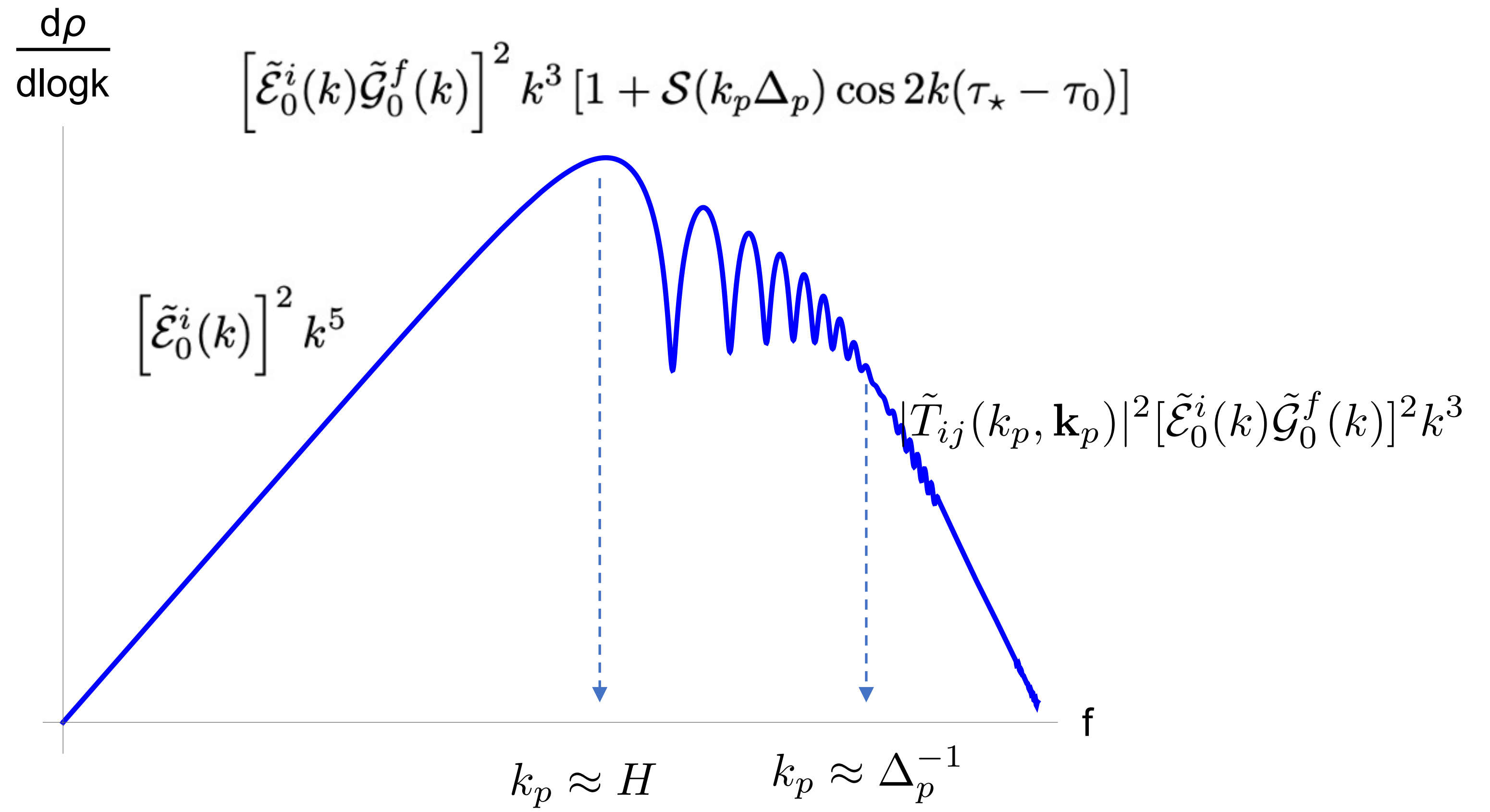

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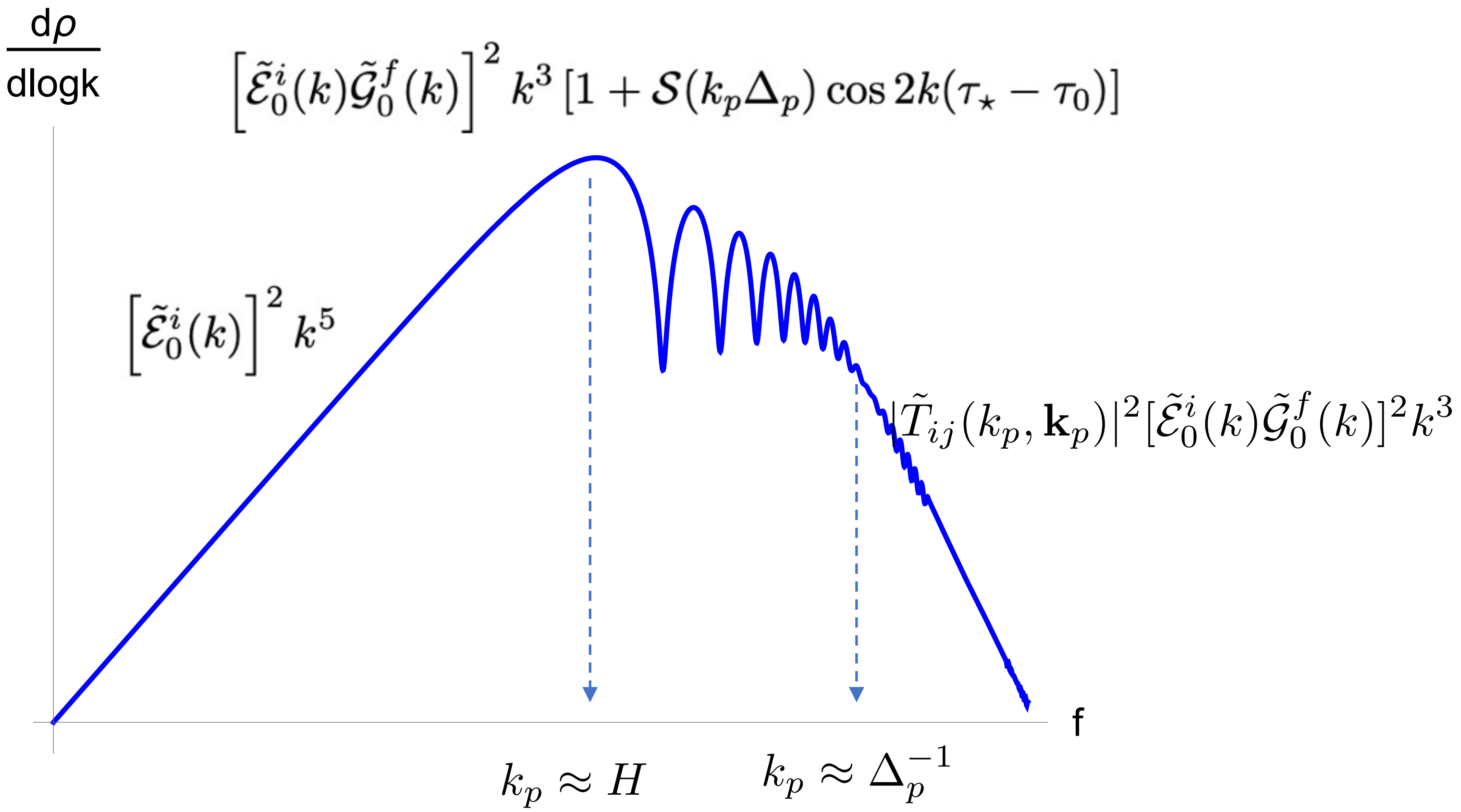

Examples

- Inflation models
  - Quasi-de Sitter inflation $\tilde{\mathcal{G}}_0^f \sim \frac{1}{k}$
  - t^p inflation $\tilde{\mathcal{G}}_0^f \sim k^{\frac{p}{1-p}}$
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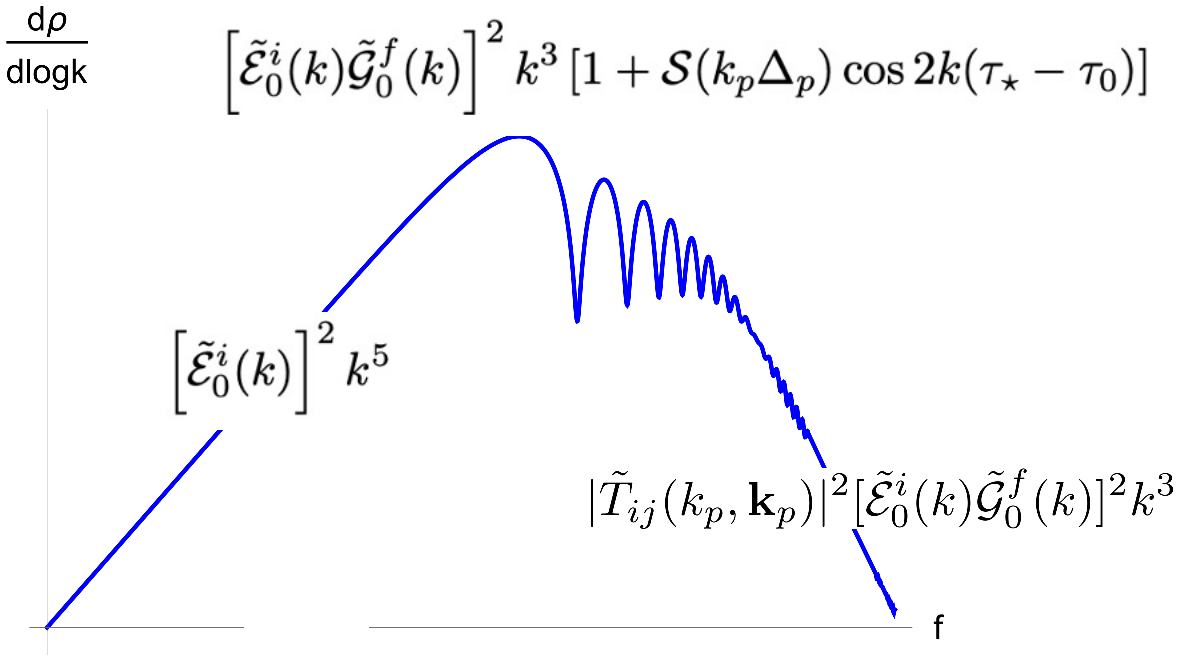
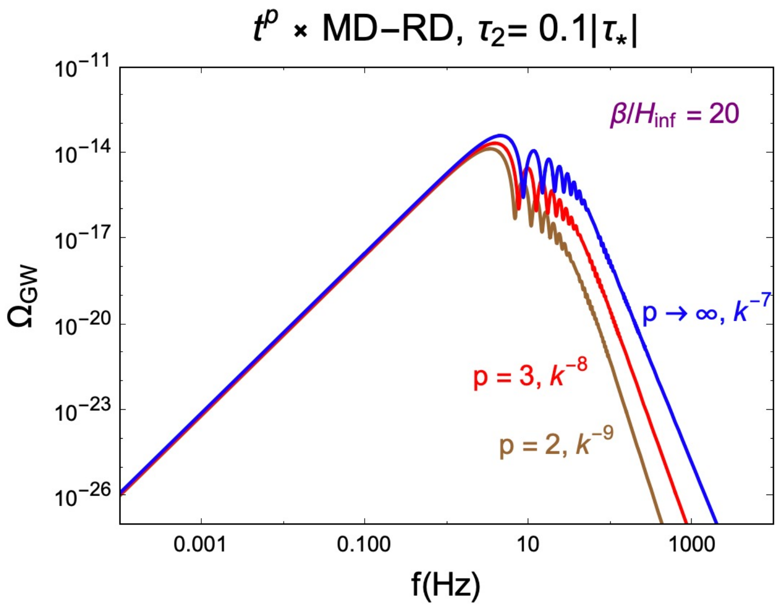
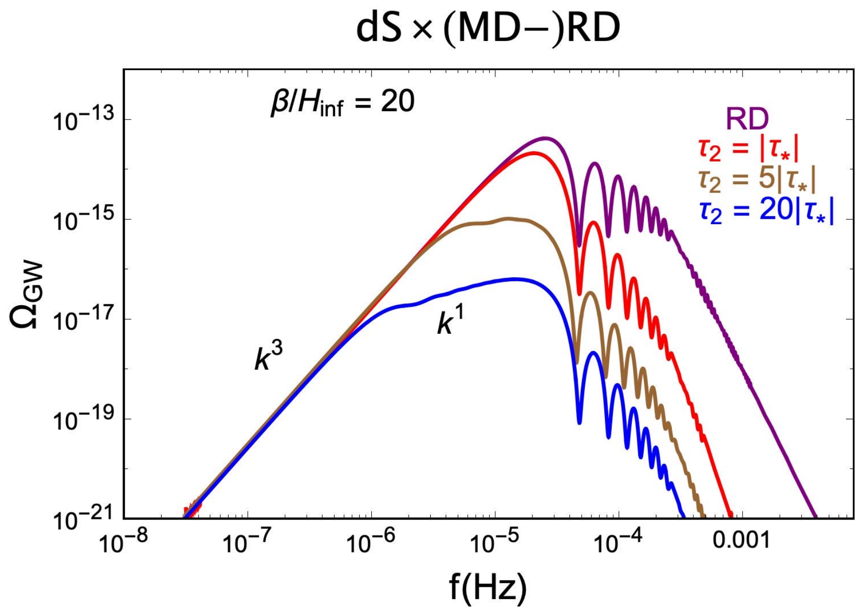


	w	$\rho(a)$	$\tilde{p}$
MD	0	a^{-3}	2/3
RD	1/3	a^{-4}	1/2
Λ	-1	a^0	∞
Cosmic string	-1/3	a^{-2}	1
Domain wall	-2/3	a^{-1}	2
kination	1	a^{-6}	1/3

Comparing scenarios





Different inflation scenarios

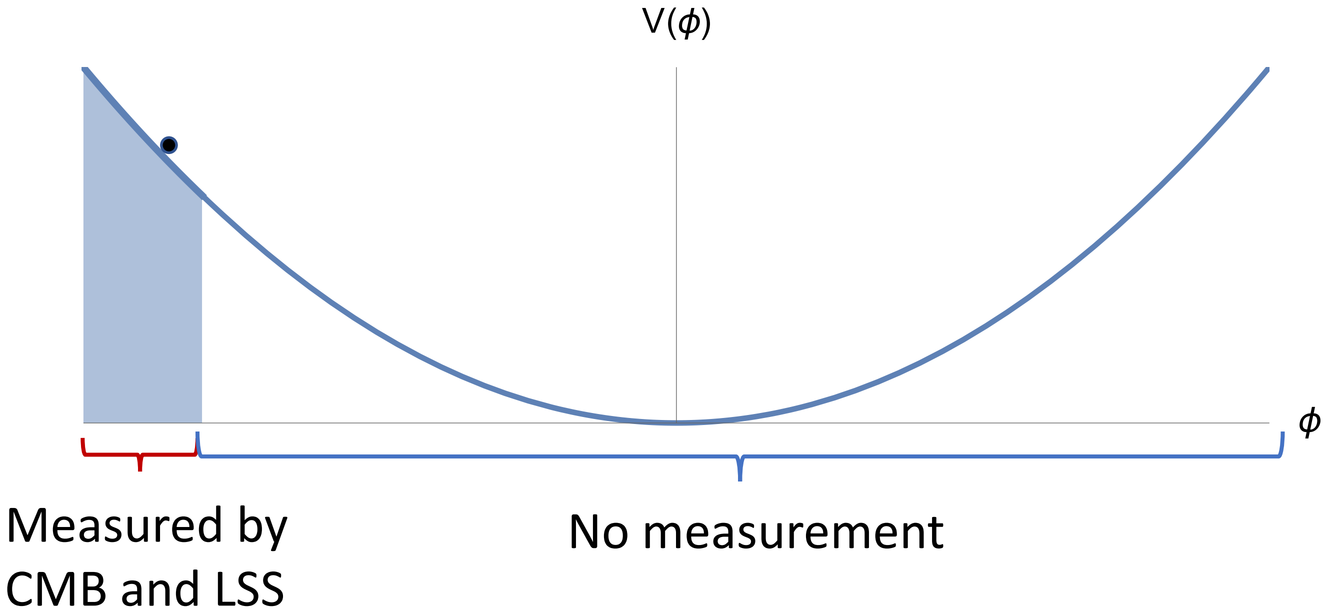
→ Different slopes in the UV and oscillatory parts

Temporary MD betwteen inflation and RD

τ_2: MD-RD transition

Slow roll models

- We usually assume a potential.
- Use it to calculate n_s, r …



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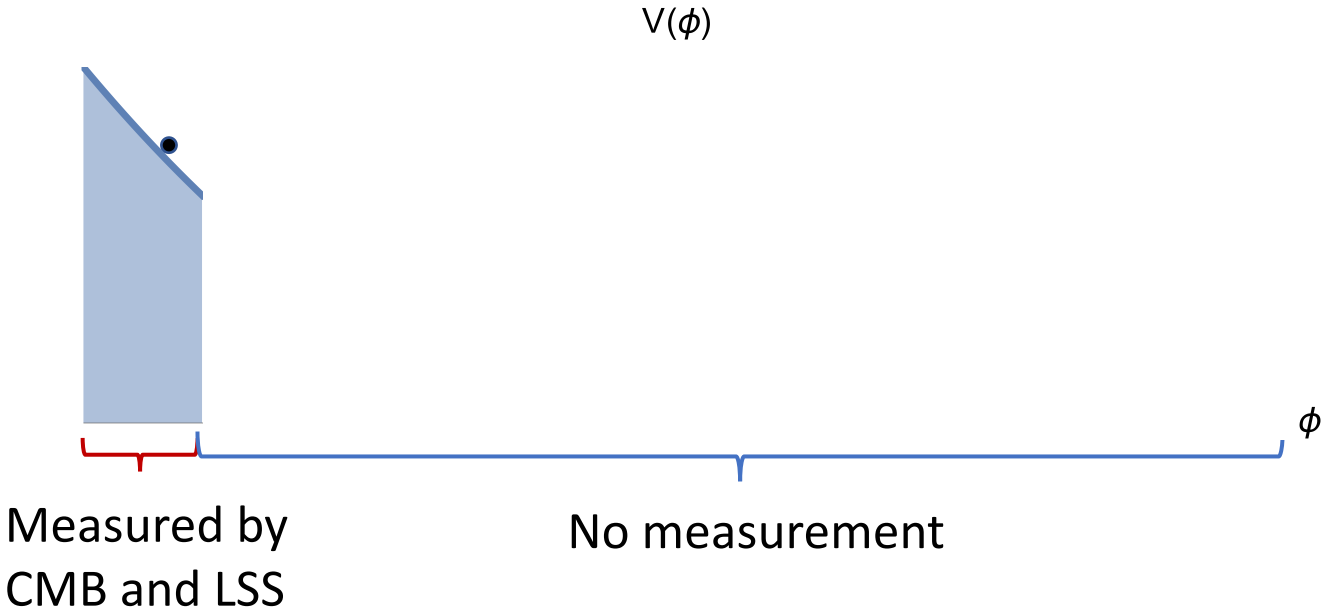

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