String-inspired Infinite Derivative Non-local QFT: Non-perturbative results

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Introduction

- An important aspect of string theory is the finitess of the loop amplitudes (Polchinski, 1998).
- In this context, an infinite derivative non-local approach was motivated starting from string field theory.
- Attempts where made to address the divergence problem in QFT by generalizing the kinetic energy operators of the Standard Model (SM) to an infinite series of higher order derivatives suppressed by a scale of non-locality.
- They have been explicitly shown to be ghost-free (Buoninfante, 2018), predicting conformal invariance in the UV, trans-planckian scale transmutation and dark matter phenomenology (Ghoshal, 2018, Buoninfante, 2018).
- We study them in a strong coupling regime with recently devised techniques (**Frasca**&Ghoshal, 2020, 2021).
- Recently, a reciprocity principle has been proposed to get non-local theories from first principles (Buoninfante, 2021).

Scalar field theory

We consider the following non-local scalar field theory

$$L = -\phi(x)e^{f(\Box)}\Box\phi(x) - \frac{\lambda}{4}\phi^4(x) + j(x)\phi(x).$$

Then, we perform the change of field variable

$$\tilde{\phi}(x) = e^{-\frac{1}{2}f(\Box)}\phi(x)$$

that yields our working action

$$L = -\phi(x)\Box\phi(x) - \frac{\lambda}{4}\left[e^{\frac{1}{2}f(\Box)}\phi(x)\right]^4 + j(x)e^{\frac{1}{2}f(\Box)}\phi(x).$$

The non-locality introduces a mass-scale factor M that grants an UV-finite theory.

In the limit of the mass-scale factor running to infinity, the local theory is properly recovered.

SU(N) Gauge Theory

We also consider the following SU(N) gauge theory (Tomboulis, 1997)

$$L = -\frac{1}{4}\,\mathrm{tr}\,F^{a\mu\nu}e^{f(D^2)}F^a_{\mu\nu} + \bar{c}^a D^{ab}_\mu \partial^\mu c^b + \bar{\eta}^a c^a + \bar{c}^a \eta^a + j^a_\mu A^{a\mu}.$$

We have set for the covariant derivative

$$D^{ab}_\mu = \delta_{ab}\partial_\mu + igA^c_\mu (T^c)^{ab}.$$

and for the gauge potentials one has

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu.$$

The mass-scale M here is a scale for both the magnitude of the gauge fields and of the space-time gradients.
In the limit of the mass-scale factor running to infinity, the local theory is properly recovered.

Bender-Savage-Milton Technique and Dyson-Schwinger equations

- Bender-Savage-Milton technique (1999) is a methodology to derive the Dyson-Schwinger set of equations for the correlation functions of a given theory in a differential form.
- In the local case, both the scalar field theory and the SU(N) gauge theory can be solved exactly yielding, in principle, the full set of correlation functions (**Frasca**, 2017). LSZ theorem grants the observables of the given theories.
- 1P- and 2P-functions are exactly evaluated but nP-functions can be computed exactly as well.
- For our cases, this amounts to work with a background field being the solution of the given equation for the 1P-function. Different choices are also possible.

Exact solutions for the local scalar field theory: G_1

1P-function for the local $\lambda\phi^4$ theory is given by

$$G_1(x) = \mu\left(\frac{2}{\lambda}\right)\mathrm{sn}(p\cdot x+\theta, i),$$

where μ and θ are two integration constants and given the following dispersion relation

$$p^2 = \mu^2\sqrt{\frac{\lambda}{2}}.$$

At this stage we have omitted a mass shift arising from mass renormalization. Anyway, this becomes important when one needs to evaluate the spectrum of the theory.

Exact solutions for the local scalar field theory: G_2

Similarly, for the 2P-function one gets in momenta space

$$G_2(p) = \frac{4\pi^3}{K^3(i)} \sum_{n=0}^{\infty} (2n+1)^2 \frac{e^{-\left(n+\frac{1}{2}\right)\pi}}{1+e^{-(2n+1)\pi}} \frac{1}{p^2 - m_n^2 + i\epsilon}$$

with a mass spectrum

$$m_n = (2n+1)\frac{\pi}{2K(i)} \left(\lambda/2\right)^{\frac{1}{4}} \mu.$$

Again, we are neglecting a mass shift in the spectrum. This will yield a gap equation that is not essential for our aims.

1P-function for non-local scalar theory

Bender-Milton-Savage extends naturally to the non-local case. In this case, Dyson-Schwinger equations cannot be solved exactly but just around the local solution. So, one us or the 1P-function

$$\phi_{NL}(x) = \phi(x) + \int d^4 y G_2(x-y)\delta\phi(y) + \ldots$$

where

$$\delta\phi(x) = -\mu^3 \left(\frac{2\lambda}{9}\right)^{\frac{1}{4}} \frac{4\pi^3}{3K^3(i)} \left[\sum_{n=1}^{\infty} C_n(x)\right] + \mu^3 \left(\frac{2\lambda}{9}\right)^{\frac{1}{4}} \left[1 - \frac{4\pi^3}{3K^3(i)} \frac{e^{-\frac{3\pi}{2}}}{(1+e^{-\pi})^3} e^{3f\left(-\frac{\pi^2}{4K^2(i)}p^2\right)}\right] \sin^3\left(\frac{\pi}{2K(i)}(p\cdot x+\theta)\right).$$

$C_n(x)$ are some coefficients obtained through product of series of known terms (**Frasca**&Ghoshal, 2020).

2P-function for non-local scalar theory

2P-function can be written in the form

$$G_2(k) = G_2^{(c)}(k)\frac{1}{1+\delta m_0^2 e^{f(-k^2)} G_2^{(c)}(k)},$$

where the shift δm_0^2 can then be computed by the gap equation

$$\delta m_0^2 = 3\lambda \int \frac{d^4k}{(2\pi)^4} G_2(k).$$

and

$$G_2^{(c)}(k) = \frac{e^{f(-k^2)}}{k^2 + m_0^2 e^{2f(-k^2)}} \frac{1}{1-\Pi(k)}.$$

Mass gap gets diluted by non-locality and higher order excitations are moved far away in the spectrum making them possibly not observable.

Non-local mass gap

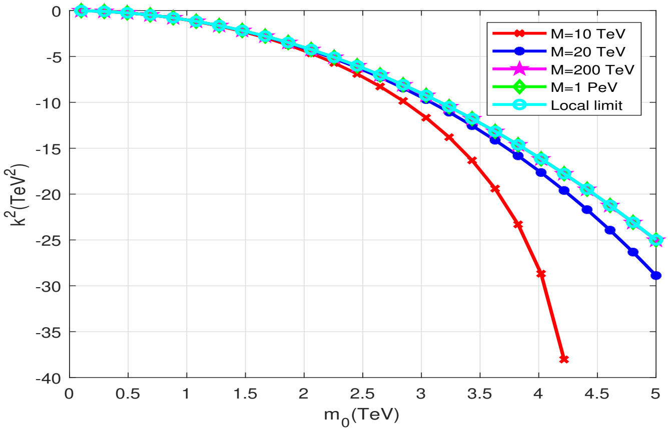

Figure: *Plot of the mass gap solution as a function of m_0. The mass gap gets damped in the UV. We see for M being $O(10$ TeV$)$, the curves do not change appreciably reaching the local limit $M \to \infty$.*

1P- and 2P-function for non-local gauge theory

We apply the mapping theorem between scalar field theory and gauge theory, taking into account that $\lambda \to Ng^2$, therefore

$$G^a_{1\mu}(x) = \eta^a_\mu G_1(x)$$

where we introduced the η-symbols. Similarly, in the Landau gauge,

$$G^{ab}_{2\mu\nu}(k) = \delta_{ab}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) G_2(k)$$

provided the gap equation

$$\delta m_0^2 = 2Ng^2 \int \frac{d^4k}{(2\pi)^4} G_2(k).$$

Again, we have a diluted mass gap and higher excited states moved far away in the spectrum.

Confinement

- Confinement can be proven for non-local gauge theory (**Frasca**, Ghoshal, Okada, in preparation).
- The beta function is obtained with the technique devised in (**Frasca**&Chaichian, 2018) based on BRST and Kugo-Ojima confinement criterion.
- We aim to apply this technique to quantum gravity to confine the ghost states like in R^2 theories, impeding them to propagate.
- Also, this would have implications for the behavior of cosmological and black hole singularities.

Future perspectives

- These results can find straight-forwardly wide applications in modern cosmology, like one particular application for the scale free theory comesin the context of cosmological inflation.
- The mass gap together with the mass-scale M will play a significant role in breaking the scale invariance, as well as creating the observed density fluctuations in the cosmic microwave background radiation.
- For dark energy models with scalar fields, the mass gap in the theory acts as source of current cosmic acceleration but it could have been damped in early universe, due to the exponentially damping from the presence of non-local scale M in the UV, thereby offering an explanation for the fine-tuning problem.

Conclusions

- The mass gap and the spectrum of the states is completely changed in the non-local theory with respect to the local scenario.
- Dyson-Schwinger approach can be straightforwardly extended from the local field theory to the infinite-derivative case.
- We showed that the non-local scale M is responsible for no-extra poles in the propagator even in the non-perturbative regime.
- In the UV, beyond the scale of non-locality M, the mass gap generated gets exponentially suppressed and the theory and the other higher excited states are far detached in the UV, possibly beyond the non-locality limit.
- In the limit of the mass scale going to infinity, known results are recovered.