Influence of asymmetry of potential on stability of domain walls

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What are domain walls?

- Domain walls (DWs) are sheet-like topological defects.
- A potential with two (or more) local minima is necessary for the existence of DWs.
- Cosmological DWs could be produced in the early Universe during spontaneous symmetry breaking.
- DWs are formed at boundaries of regions (domains) where a scalar field has different vacuum expectation values (VEVs).
- Cosmological domain walls form networks whose dynamics is non-linear.

Toy model

Let us consider the model given by the potential of the form:

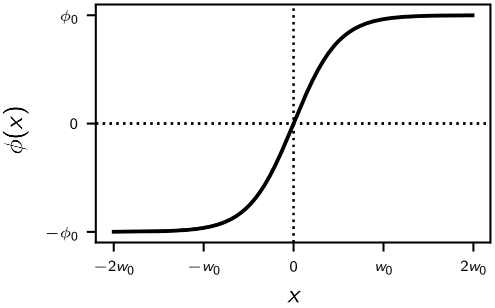
$$V(\phi) = V_0 \left(\left(\frac{\phi}{\phi_0} \right)^2 - 1 \right)^2 .$$

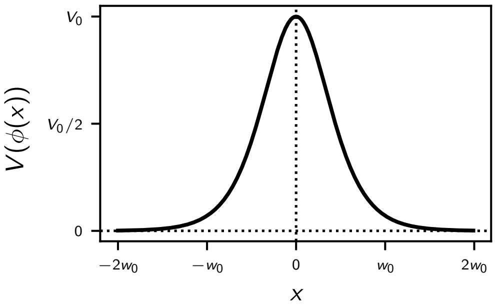
The EOM has the time independent, planar solution

$$\phi(x) = \phi_0 \tanh \left(\frac{x}{w_0} \right) ,$$

where $w_0 = \frac{\phi_0}{\sqrt{2V_0}}$ is the width of the domain wall.

Profile of walls in the toy model

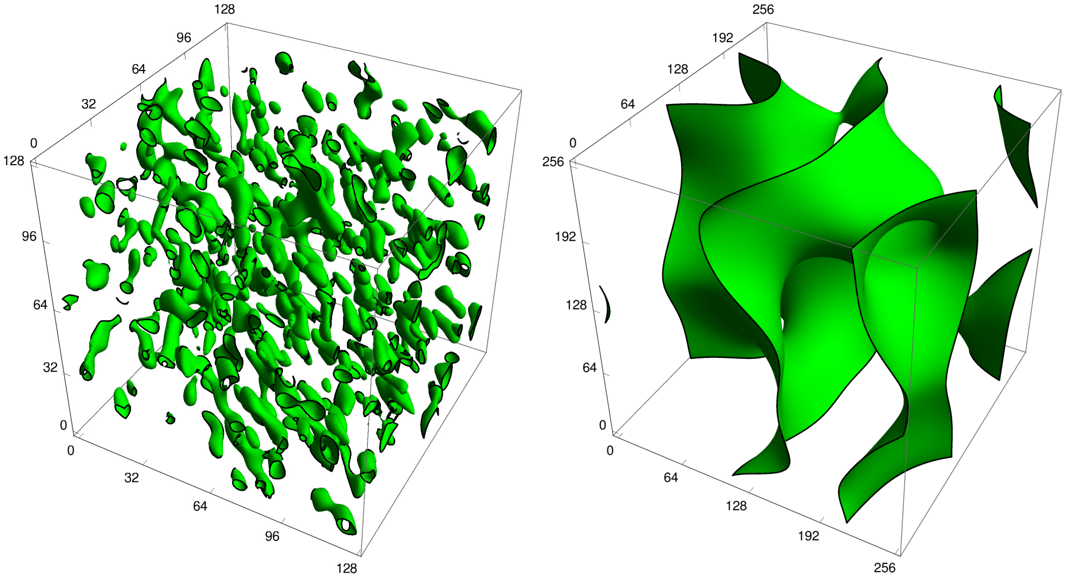




Networks of domain walls

Networks of cosmological domain walls could have twofold topologies:

- finite bubbles of one vacuum in a sea of the other,
- an infinite network spreading through whole Universe.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Motivation

1. We have studied Higgs domain walls in series of publications [1, 2, 3].
2. We observed that networks of Higgs domain walls were more stable than one can expect from shape of the potential.
3. In our recent publication [4] we have studied the influence of shape of a potential on the stability of cosmological domain walls.

[1] Tomasz Krajewski et al. In: *JCAP* 1612.12 (2016), p. 036. arXiv: 1608.05719 [astro-ph.CO].

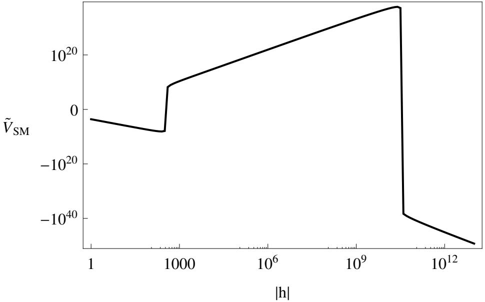
[2] Tomasz Krajewski et al. In: *JCAP* 05 (2018), p. 007. arXiv: 1709.10100 [hep-ph].

[3] Tomasz Krajewski et al. In: *Phys. Dark Univ.* 26 (2019), p. 100347. arXiv: 1902.05560 [hep-ph].

[4] Tomasz Krajewski et al. In: (Mar. 2021). arXiv: 2103.03225 [astro-ph.CO].

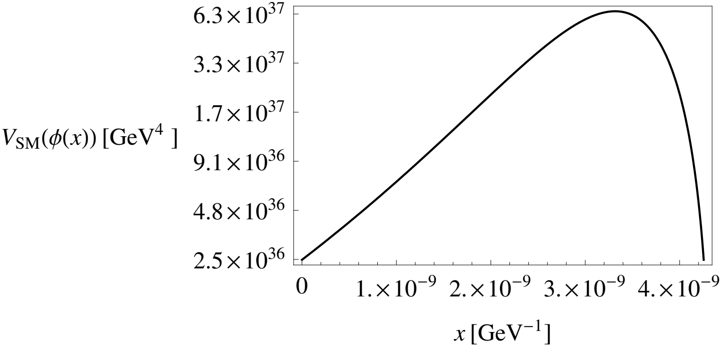
RG improved effective potential of the SM

The quantitative study of the RG improved effective SM potential has revealed an existence of local maxima at field strengths of the order of 10^{10} GeV and new minima at superplanckian field strengths. Domain walls can interpolate over local maximum between high field strength minimum and electroweak minimum.



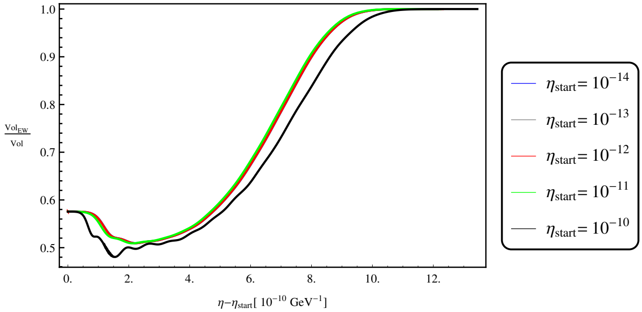
The profile of the SM domain wall

The profile of domain wall reflects the asymmetry of the potential around its local maximum.



(In)sensitivity to asymmetry of SM potential

For nearly equal contributions of both vacua at the initialization, Higgs domain walls decay into EW vacuum despite the high asymmetry of the SM potential.



Modeling asymmetry of the potential

In order to investigate the influence of the asymmetry of the potential we needed the family of potentials satisfying following conditions:

1. given in analytic form,
2. with exactly known positions of extrema,
3. independently controlled scales of asymmetry around local maximum and the difference of values in minima δV,
4. constant width of domain walls.

Model of asymmetric potentials

It is convenient to define the potential V_{AS} by its derivative:

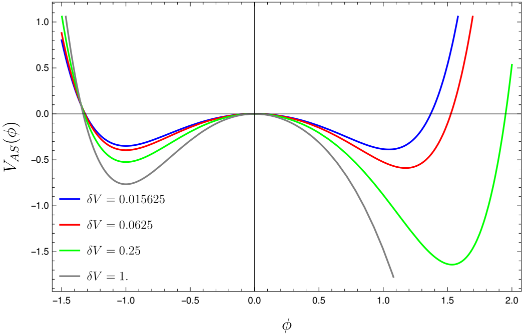
$$\frac{\partial V_{AS}}{\partial \phi}(\phi) := V_0(\phi - a)(\phi - b)(\phi - c)\left(e^2(\phi - d)^2 + 1\right),$$

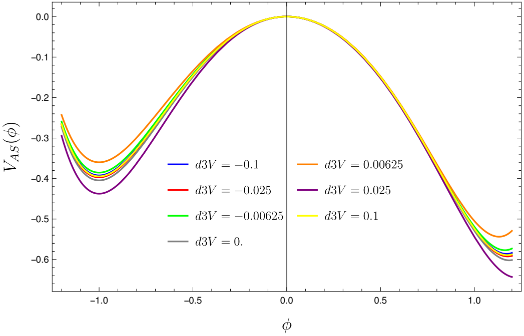
where a, b, c determine positions of the extrema of the potential and parameters e, d controls the shape of the potential. We quantitatively estimate the asymmetry of the potential around local maximum as a value of the third derivative of the potential:

$$\frac{\partial^3 V_{AS}}{\partial \phi^3}(\phi) = 2V_0 \left[e^2(a - \phi)(\phi - b)(c + 2d - 3\phi)\right.$$
$$\left. +(\phi - c)\left(e^2(d - \phi)^2 + 1\right) + (-a - b + 2\phi)\left(e^2(d - \phi)(2c + d - 3\phi) + 1\right)\right]$$

at the maximum: $\frac{\partial^3 V_{AS}}{\partial \phi^3}(c) := d3V$.

Families of potentials





The probability distribution of field's strengths

According to general considerations the probability distribution of field's strengths of fluctuations produced during inflation is approximately gaussian:

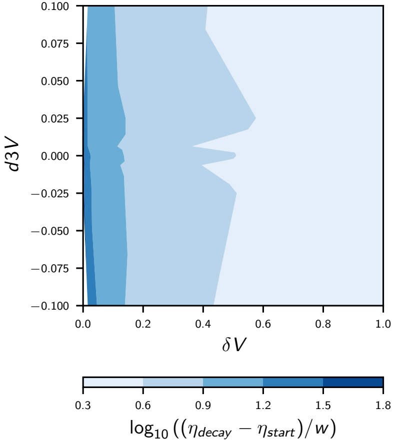
$$P(\phi) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(\phi-\theta)^2}{2\sigma_I^2}},$$

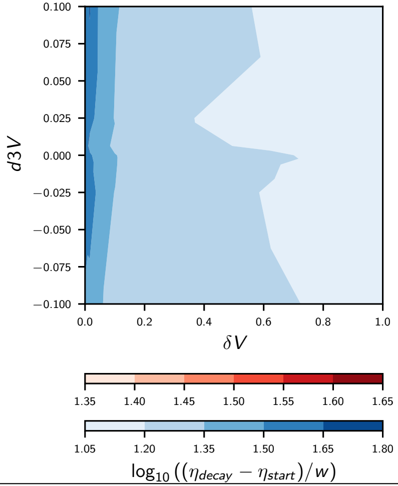
with standard deviation of the order

$$\sigma_I \sim \frac{\sqrt{N} H_I}{2\pi},$$

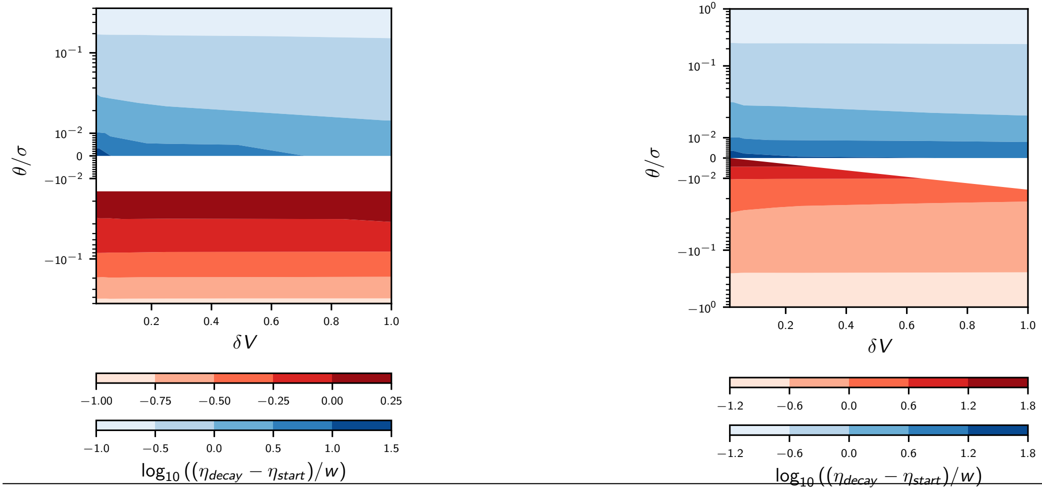
where H_I is value of the Hubble parameter and N is number of e-folds.

Dependence on δV and $d3V$ with $\theta = 0$

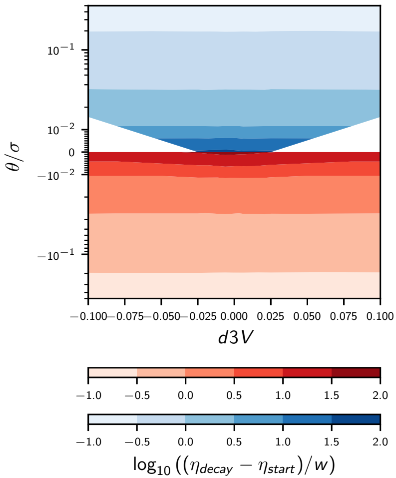


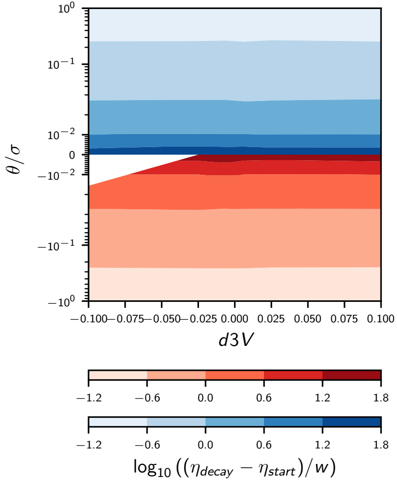


Dependence on δV and θ



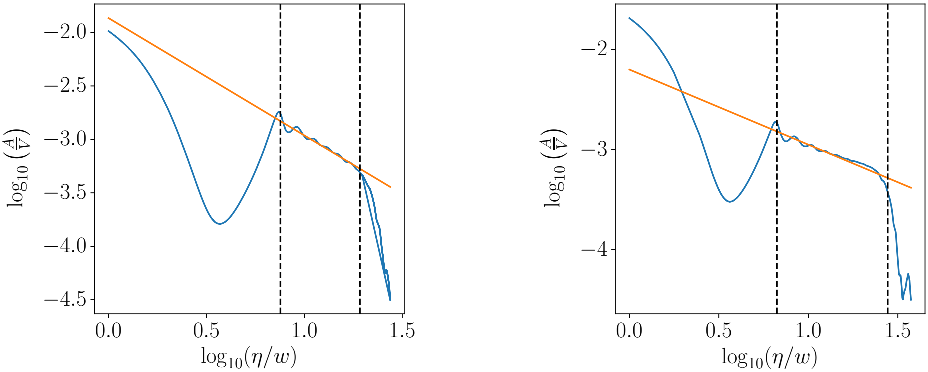
Dependence on $d3V$ and θ





Scaling regime

So called scaling regime when $\frac{\sigma_{wall}}{\rho_{wall}} \propto t$ is an attractor of evolution of metastable networks.



Spectrum of emitted GWs

For nearly degenerate minima the energy density of GWs at the peak can be estimated using:

$$\Omega_{GW}(\eta_{dec})|_{peak} = \frac{\tilde{\epsilon}_{GW}\mathcal{A}^2{\sigma_{wall}}^2}{24\pi {H_{dec}}^2 {M_{Pl}}^4}, \tag{1}$$

where $\tilde{\epsilon}_{GW} \simeq 0.7$ and

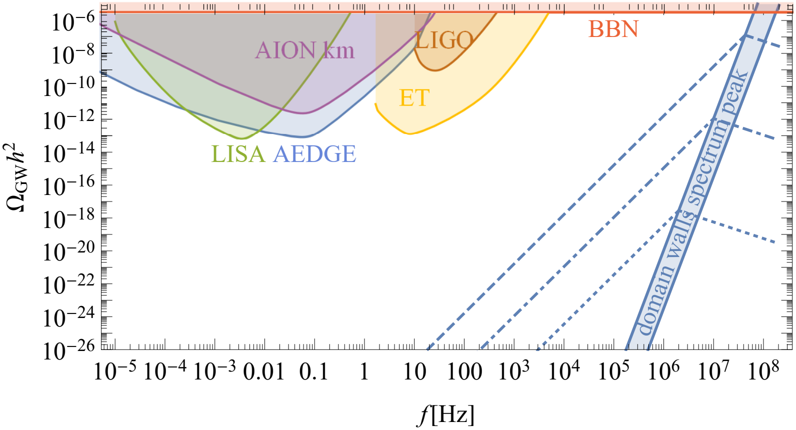
$$\mathcal{A} = \frac{\rho_{wall}}{\sigma_{wall}} t \tag{2}$$

is a scaling parameter $\mathcal{A} \sim 0.8$ [1] (or lower as in our numerical simulations). Value of the Hubble parameter at the time of the decay H_{dec} determines the peak frequency:

$$f_0|_{peak} \sim H_{dec}^{-1} \tag{3}$$

[1] Takashi Hiramatsu et al. In: *JCAP* 1402 (2014), p. 031. arXiv: `1309.5001 [astro-ph.CO]`.

Present spectrum of GWs



Summary

1. Domain walls in models with asymmetric potentials could be formed in the early Universe.
2. Results of our recent simulations suggest that role of the asymmetry of a potential is not as important as of the initial distribution.
3. The (non)degeneracy of minima of the potential are more important than details of its shape.
4. Networks of cosmological domain walls are unstable if formed from biased initial distributions.
5. Observation of GWs from decaying domain walls is highly unlikely.

Thank you for your attention.