Double-Copy Bootstrap

Henriette Elvang
University of Michigan
Leinweber Center for Theoretical Physics

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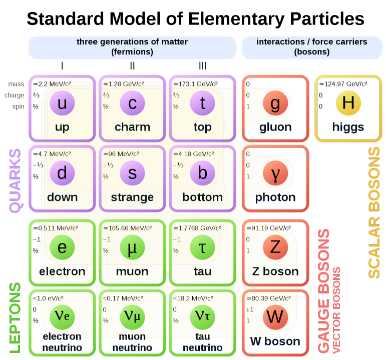
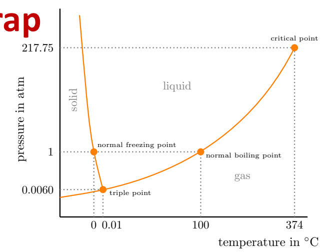
Based on
HH Chi, H.E., A. Herderschee, C. Jones, S. Paranjape, *to appear*

Landscape of Field Theories: vast, rich, interesting, *and* useful in physics!



Conformal Bootstrap



Swampland Conjectures
(which EFTs have a UV completion?)

Soft bootstrap for exceptional EFTs
(Goldstone EFTs)

Double-Copy

(who can be double-copied and to what?)

The double-copy is a map on the space of field theories.

It takes (tree) **amplitudes** in two (possibly distinct) theories and multiply them in a certain way to create the (tree) amplitudes in a third theory.

For example:

(Yang-Mills) x (Yang Mills) = gravity⁺

gluon	$\otimes$ gluon	$\rightarrow$ graviton
$+1$	$+1$	$+2$
-1	-1	-2
$+1$	-1	Z
-1	$+1$	$\bar{Z}$

$Z = \varphi + ia$

dilaton/axion

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For example:

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Many applications:

Explore the UV structure of supergravity theories (finiteness?)

Gravitational radiation (3PM)

Classical double-copy (EOM)

Enhancement of symmetries

Properties of string amplitudes

Generalizations to (A)dS

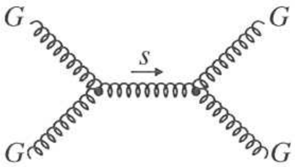
chiPT -> galileons

How?

YM gluon amplitudes can be color-ordered:

$A_4[1^{a_1}2^{a_2}3^{a_3}4^{a_4}]\mathrm{tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4})$

$A_4[1234]$ has s and u channels, but no t-channel.
$A_4[1243]$ has s and t channels, but no u-channel.



Graviton amplitudes have no color-structure, so $M_4(1234)$ has s, t and u channels.

How can a product of A_4's possibly give even the pole structure of M_4???? *And* avoid double-poles?

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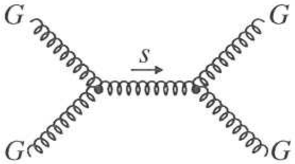

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Answer: need a DOUBLE-COPY **KERNEL**

$$M_4 = -s A_4[1234] A_4[1243]$$

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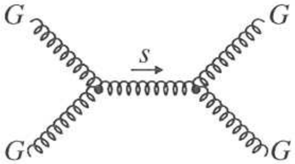

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$$M_4 = -s A_4[1234] A_4[1243]$$

$$M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

These are examples of field theory KLT (Kawai-Lewellen-Tye 1986) formulas at 4-point.

But… if both are true,

$$M_4 = -s A_4[1234] A_4[1243] \qquad\qquad M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

then their difference must be zero, i.e.

$$0 = A_4[1243] - \frac{u}{t} A_4[1234]$$

And this **is** true to YM amplitudes.

This is an example of a **BCJ** (Bern-Carrasco-Johansson) **relation** at 4-point.

Kleiss-Kuijf

$$\text{Trace-reversal:} \quad \mathcal{A}_4[1432] = \mathcal{A}_4[1234]\,, \quad \textit{etc}$$

$$U(1)\text{-decoupling:} \quad \mathcal{A}_4[1234] + \mathcal{A}_4[1243] + \mathcal{A}_4[1423] = 0\,,$$

$$\text{BCJ:} \quad \mathcal{A}_4[1234] - \frac{t}{u}\mathcal{A}_4[1243] = 0\,.$$

Generally, at *n*-point there are KLT relations of the form

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

KLT kernel

and associated **Kleiss-Kuijf and BCJ relations** that ensure that the result is indep. of which color-orders are chosen for the sum.

Field theory double-copy selection criterium

In order to be "double-copyable", a theory's tree amplitudes must obey the Kleiss-Kuijf and BCJ relations.

This reduces the number of color-orderings from (n-1)! to (n-3)!

A new way to explore the space of field theories: which theories can be input/output of the double-copy?

Which theories obey the KK&BCJ relations?

YM theory ✓

Chiral perturbation theory ✓

Super YM theory ✓

Bi-adjoint scalar model ✓

Which theories obey the KK&BCJ relations?

YM theory ✓ Chiral perturbation theory ✓

Super YM theory ✓ Bi-adjoint scalar model ✓

What about higher-derivative operators in EFTs?

Amplitudes offer an efficient systematic way to characterize higher-derivative operators.

YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ ... MHV

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χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓ $\mathrm{tr}\,\partial^4\phi^4$ 2✗ $\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗ $\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗ $\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗ ...

Why are some operators allowed and not others? ***Is this the most general story?***

YM + h.d.

YM + h.d.

Gravity⁺ + h.d.

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

Should also include **higher-derivative corrections to the double-copy kernel**

String theory KLT

KLT originally came from closed string = (open string)^2 at tree-level

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

string KLT kernel

The KLT kernel is deeply linked with the open string amplitudes to ensure correct pole structure in the closed string amps.

Upon expansion in alpha', this translates to very particular higher-derivative corrections of the kernel:
not the most general options and tuned exactly to the alpha' corrections in the open string.

Example: $$S_4[1234|1243] = -\sin(\pi\alpha' s) = -\pi\alpha' s + \frac{1}{6}(\pi\alpha' s)^3 + \ldots$$

Only s-dependence, no t or u; why?

Only odd powers in s; why?

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

What are the rules for generalizing the KLT kernel?

*We present proposal for **generalizing the double-copy**: **a bootstrap for the KLT kernel**.*

- Can systematically solve for higher-derivative corrections to the kernel
- What makes the string kernel special?
- Explore if there are new versions of the double-copy

Forthcoming work with
HuanHang Chi (Michigan)
Aidan Herderschee (Michigan)
Callum Jones (UCLA)
Shruti Paranjape (Michigan -> UC Davis)

The proposal is based on the **KLT algebra** which I'll now introduce

KLT algebra

Double copy is a map FT x FT -> FT

Usual field theory double-copy

FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
YM	gravity+	$\mathcal{N}=4$ SG	BI	YM
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI	$\mathcal{N}=4$ SYM
χPT	BI	$\mathcal{N}=4$ sDBI	sGalileon	χPT
BAS	YM	$\mathcal{N}=4$ SYM	χPT	BAS

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

This map has an *identity element* **1**: the **bi-adjoint scalar model** (BAS)

String KLT also has an identity element and the *same* algebra

KLT algebra

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FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
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χPT	BI	$\mathcal{N}=4$ sDBI	sGalileon	χPT
BAS	YM	$\mathcal{N}=4$ SYM	χPT	BAS

$$\mathrm{L}=\mathrm{L}\otimes\mathbf{1}\,,\qquad \mathrm{R}=\mathbf{1}\otimes\mathrm{R}\,,\qquad \mathbf{1}=\mathbf{1}\otimes\mathbf{1}\,.$$

Generalize the monodromy / KKBCJ relations

KLT Bootstrap Equation

This map has an *identity element* **1**: the **bi-adjoint scalar model** (BAS)

String KLT also has an identity element and the *same* algebra

We propose that the KLT algebra is the fundamental principle for generalizing the double-copy

Bi-Adjoint Scalar Model (BAS)

$$\mathcal{L}_{\mathrm{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g f^{abc} \tilde{f}^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\rm BAS} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g\, f^{abc}\,\tilde{f}^{a'b'c'}\,\phi^{aa'}\,\phi^{bb'}\,\phi^{cc'}$$

Statement BAS = BAS x BAS --- or $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ can be written as

$$m_n[\gamma|\delta] = \sum_{\alpha,\beta} m_n[\gamma|\alpha]\, S_n[\alpha|\beta]\, m_n[\beta|\delta]$$

Double-sum over *(n-3)!* color orderings

or in **matrix form** $m_n = m_n . S_n . m_n$

(n-3)! x *(n-3)! submatrices*

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or in **matrix form** $m_n = m_n . S_n . m_n$

So multiplying from both the left and right with inverses of matrices of BAS amplitudes gives

$$S_n = \left(m_n\right)^{-1}$$

[Cachazo et al]

The KLT kernel is the inverse of an *(n-3)!* x *(n-3)!* submatrix of BAS amplitudes!

The string KLT kernel is also the inverse of a *(n-3)!* x *(n-3)!* submatrix of amplitudes

[Mizera]

Generalize the KLT kernel

BAS + higher-derivative corrections (characterized by on-shell matrix elements)

$$\mathcal{L} = \mathcal{L}_{\mathsf{BAS}} + a_{0,0}\phi^4 + a_{1,i}d^2\phi^4 + a_{2,i}d^4\phi^4 + \ldots$$

KLT bootstrap eq from $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ to determine solution for the coefficients $a_{i,j}$

4-point result

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + f^{abc} f^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$
$$+ \frac{a_L + a_R}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'}) \phi^{cc'} \phi^{dd'}$$
$$+ \frac{a_R}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'}$$
$$+ \frac{a_L}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} + \ldots.$$

$$d^{abc} = \mathrm{Tr}\left[T^a \{T^b, T^c\}\right]$$

- There is no $d^{abc} d^{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$; does not solve the rank 1 bootstrap equations.
- There is no ϕ^4 term; does not solve the rank 1 bootstrap equations
- The d^{abc} terms modify the U(1) decoupling identities that are part of the field theory KK relations and generalize the known strings monodromy relations.

- Known strings kernel has $a_L = a_R$. The generalization allows "heterotic"-type double-copy.

Double-copy of YM + h.d.

Impose generalized KKBCJ relations $\mathbf{1} \otimes \mathbf{R} = \mathbf{R} \quad \mathbf{L} \otimes \mathbf{1} = \mathbf{L}$

on a general ansatz for MHV 4-pt YM + h.d. to find

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2 \left[\frac{(g^L_{\text{YM}})^2}{su} - \frac{1}{\Lambda^4}\left(\frac{(g^L_{\text{YM}})^2}{g^2} a_L + (g^L_{F^3})^2 \frac{t}{s} \right) - \frac{e^L_{3,1}}{\Lambda^6} t + \mathcal{O}\left(\frac{1}{\Lambda^8}\right) \right]$$

Usual YM

tr F^4

Pole term w/ two tr F^3 vertices

tr D^2F^4

Its coefficient is controlled by the generalized KLT kernel

And similarly for the R sector.

Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ ...

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Green checkmark: operator allowed with arbitrary coefficient.
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For chiPT + higher-derivatives

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For FIXED choice of kernel, this LINKS the coefficients of $\mathrm{tr}\,F^4$ with that of one of the $\mathrm{tr}\,\partial^6\phi^4$ operators.

Double-copy of YM + h.d. -> Gravity⁺ + h.d.

Usual Einstein gravity

Pole term from exchanges of dilaton *and* axion!

$$\mathcal{M}_4(1^+2^+3^-4^-) = [12]^4\langle 34\rangle^4 \times \Bigg[-\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^2\Lambda^2}\frac{1}{stu} + \frac{((g^L_{\rm YM})^2(g^R_{F^3})^2 + (g^R_{\rm YM})^2(g^L_{F^3})^2)}{g^2\Lambda^6}\frac{1}{s} + \frac{1}{\Lambda^8}\left(\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^4}a_{2,0} + \frac{1}{g^2}((g^R_{\rm YM})^2 e^L_{3,1} + (g^L_{\rm YM})^2 e^R_{3,1})\right) + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)\Bigg]$$

vanishes in string theory

local R^4 contribution

In the field theory or strings double copy, there is less freedom in the coefficient of R^4.

The result of the double-copy: in all cases checked, *same operators produced but with shifts of their coefficients.*

Higher-point

Necessary to test consistency by going to higher point:

What if the KLT bootstrap at 5-point further fixed some of the 4-point kernel coefficients $a_{i,j}$?
(Then we'd be in trouble!)

For n=5 => (n-1)! = 4! = 24 distinct orderings.

Cyclic symmetry + momentum relabelings => parameterized by 8 functions $g_i(s,t)$, i=1,2,…,8.

We impose the rank (n-3)! = 2 conditions equivalent to $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ on this 24x24 system and solve.

Find consistent solution for the bootstrapped 5pt (BAS+h.d.) amplitudes; **no** constraints placed on 4-pt coefficients; in fact up to quadratic order in Mandelstams, the amplitudes are completely fixed by 4-pt input.
We have consistent 5pt kernel up to 7 orders in Mandelstams.

Tested for 5pt +++++ YM+h.d.

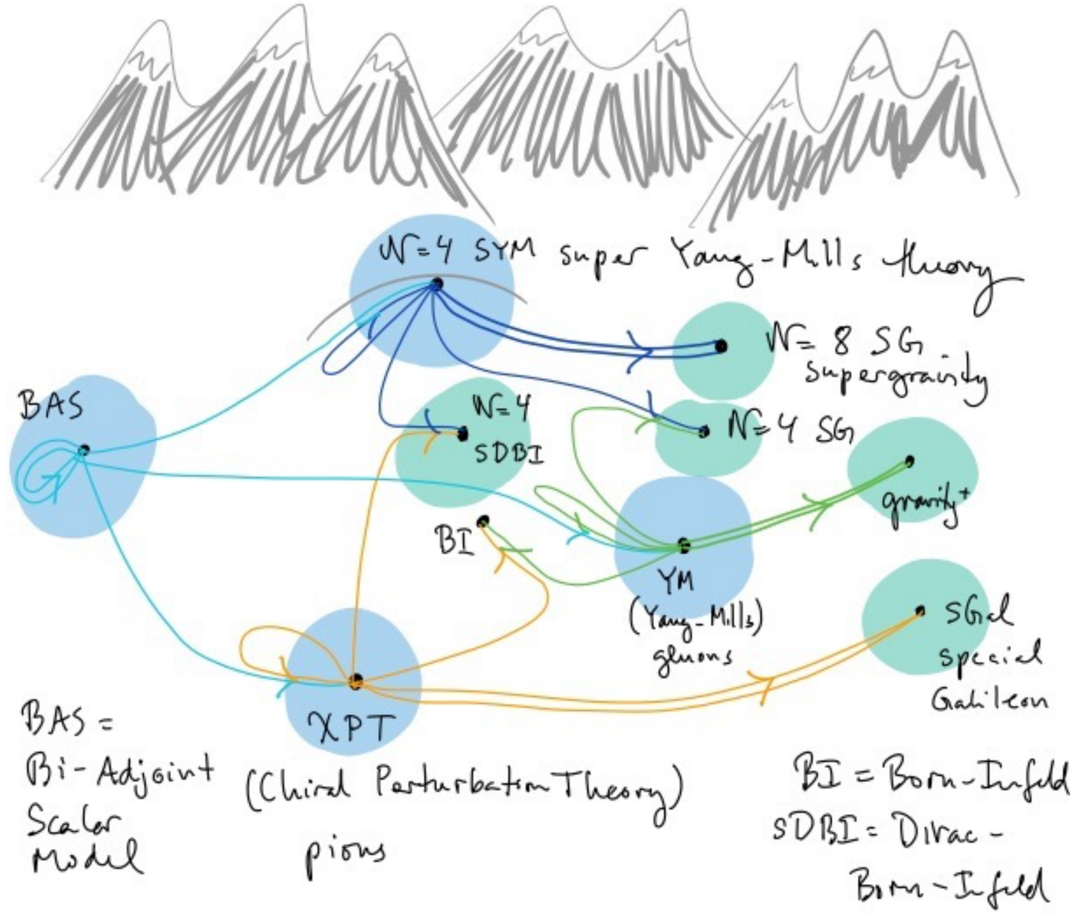
Generalized Double-Copy

generalized KLT kernel

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

“Expand” region of input for
the double-copy via higher-derivative
interactions:

A novel systematic double-copy of Effective Field Theories (EFTs)



The field theory landscape is incredibly rich.

The double-copy is a map among theories that are extremely different:

- **Yang-Mills**: renormalizable theory, part of the Standard Model
- **N=4 SYM**: a conformal field theory, widely used in high energy theory
- **gravity**: non-renormalizable, …. but a phenomenologically amazing EFT!
- **chiral perturbation theory**: low-energy EFT of pions
- **BI** or **sDBI**: low-energy EFT on D-branes
- **special Galileon**: used in cosmology, but by itself a swampland model
- **BAS**: phi^3 theory, potential unbounded from below.

Connected by the "KLT algebra".

The double-copy is part of exploring the space of field theories.

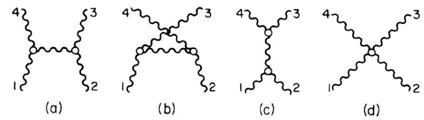
This work is the first systematic study of generalizations of the KLT double-copy kernel.
Other solutions to the KLT bootstrap may exist.

The double-copy is a pretty remarkable relationship!

One thing is 4-point w/ h.d. operators…

… another thing is having it work correctly
at 5-pt with proper factorizations in local 4-pt x 3-pt.

4-point amplitude:



[Sannan (1986)]

$$
\begin{aligned}
(\mathrm{a})_{\text{gravity}} = i\kappa^2 \Bigg[\Bigg[\left| \frac{t}{2} + \frac{su}{4t} \right| (e_1e_4)(e_2e_3) - \frac{t}{2}[(e_1e_4e_2e_3) + (e_1e_2e_3e_4)] \\
&+ [(k_2e_3k_2)(e_1e_2e_4) + (k_3e_2k_3)(e_1e_3e_4) + (k_1e_4k_1)(e_1e_2e_3) + (k_4e_1k_4)(e_2e_3e_4)] \\
&+ \tfrac{1}{2}(e_1e_4)[(k_1e_2e_3k_4) + (k_4e_2e_3k_1) + 2(k_1e_2e_3k_1) + 2(k_4e_2e_3k_4) + 3(k_3e_2e_3k_2)] \\
&+ \tfrac{1}{2}(e_2e_3)[(k_2e_1e_4k_3) + (k_3e_1e_4k_2) + 2(k_2e_1e_4k_2) + 2(k_3e_1e_4k_3) + 3(k_4e_1e_4k_1)] \\
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&- \frac{s}{2t}(e_1e_4)[(k_3e_2e_3k_1) + (k_4e_2e_3k_2) + 2(k_1e_2e_3k_2) + 2(k_3e_2e_3k_4)] \\
&- \frac{u}{2t}(e_2e_3)[(k_1e_4e_1k_2) + (k_3e_4e_1k_4) + 2(k_1e_4e_1k_3) + 2(k_2e_4e_1k_4)] \\
&- \frac{s}{2t}(e_2e_3)[(k_1e_4e_1k_3) + (k_2e_4e_1k_4) + 2(k_1e_4e_1k_2) + 2(k_3e_4e_1k_4)] \\
&- \frac{1}{t}(e_1e_4)[(k_1e_2k_1)(k_2e_3k_2) + (k_3e_2k_3)(k_1e_3k_1) + (k_4e_2k_1)(k_2e_3k_2)
\end{aligned}
$$

+ many more terms

This requires an intricate and fascinating relationship between L and R sector amplitudes and the double-copy kernel.

The new freedom in the kernel deserves further investigation.

- Moving h.d. corrections between kernel & amplitudes via shifts in Wilson coefficients?
- Interplay with positivity constraints from UV completability?
- EFT-hedron?
- What makes the stringy KLT kernel special? (Minimal kernel?)
- Does there exist other new branches of the double-copy?

Collaborators

Callum Jones
Graduated April 2020
Postdoc at UCLA

Shruti Paranjape
5th year graduate student
-> graduating Spring 2021

Aidan Herderschee
3rd year graduate student

HuanHang Chi
Michigan postdoc
PhD Stanford 2019

Home Collaborators


Thank You
for the invitation

Basis Indep & KKBCJ

What ensures independence of choice of *(n-3)!* basis?

For example, compare $M_n = A^{\text{L}}_n.S_n.A^{\text{R}}_n\,,\qquad M_n = A^{\text{L}}_n.S'_n.A^{\text{R}'}_n$

Basis indep. if $0 = S_n.A^{\text{R}}_n - S'_n.A^{\text{R}'}_n$ => $m'_n.S_n.A^{\text{R}}_n = A^{\text{R}'}_n$ => BAS x R = R => $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$

Similarly, independence of the L sector basis choice is ensured by $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$

The relations $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$ and $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$ combine the Kleiss-Kuijf (KK) and BCJ relations.

What happens if…?

Why did we impose "minimal rank" (n-3)! in the bootstrap?

Leading BAS model is rank (n-3)!   (so is the strings kernel)

Double-copy kernel is the inverse of (n-3)! x (n-3)! matrix of BAS + h.d. amplitudes

So if the higher-derivative operators increased the rank of the matrix of (BAS + h.d.) amplitudes, the low-energy limit of the double-copy would be inconsistent.

What about bootstrapping for different versions of the double-copy? With potentially different ranks?

Time to go back and question everything again

What happens if…

We change the identity theory at cubic order: $d^{abc}\tilde{d}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}$

3pt rank 1 => 4-pt rank 3 (no problems) => 5-pt rank 11 (problem: inverse has spurious poles!) ✗

Actually OK with $\mathrm{tr}\,\phi^3$ ✓

but not with $\mathrm{tr}\,\phi F^2$ ✗

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We drop cubic orders and start at 4-pt with leading ϕ^4 ? $d^{abcd}\tilde{d}^{a'b'c'd'}\phi^{aa'}\phi^{bb'}\phi^{cc'}\phi^{dd'}$

4-pt rank 1 (no problems) => 6-pt rank 10 (OK!) => 8-pt rank 273 (spurious poles in the inverse!) ✗

Actually OK with $\mathrm{tr}\,\phi^4$ ✓

Two no-go results, but…

Are there new exact solutions?

Are there new combinations of operators that can give rise to a new form of the double-copy?