

Lorentz Invariance from Locality of Massless Spin 2

Jacob Litterer

Institute of Cosmology
Tufts University

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Based on [2005.01731], [2005.01744], [2008.06510].

A few fundamental principles

- 1 Rotation and translation symmetries
- 2 Locality (causal relationship between distant events)
- 3 Unitarity

There is an additional symmetry under Lorentz boosts.

Fundamental, or derivable?

Otherwise, are there theories satisfying 1 – 3 that are not Lorentz invariant?

Outline

- 1 Overview
- 2 Spin 1
- 3 Spin 2 (Gravity)

General theory of spin 1 without assuming Lorentz invariance

Consistent picture of the spin 1 case:

- The Standard Model does not require Lorentz boost invariance. E.g. the speed c could be different in every sector: c_γ, c_e, \dots
(e.g. Colladay, Kostelecky [9809521])

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- Additionally, requiring locality in leading order interactions without Lorentz invariance \implies
 - ① The photon sector separately must be fully Lorentz invariant, i.e. is standard.
 - ② The matter sector need not be Lorentz invariant, though must have a conserved charge: $\dot{Q} = \partial_t \int d^3x J_0 = 0$.

(JL, Hertzberg, PRD **102**, 025022)

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More interesting with spin 2?

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Standard Lorentz invariant theory

$$\begin{aligned}\mathcal{L}_{\text{GR}} = & \frac{1}{2} \left(\eta^{\alpha\beta} \partial_\alpha h^{\mu\nu} \partial_\beta h_{\mu\nu} - \eta^{\mu\nu} \partial_\mu h^{(4)} \partial_\nu h^{(4)} \right) \\ & + \partial_\mu h^{\mu\nu} \partial_\nu h^{(4)} - \partial_\mu h^{\mu\alpha} \partial_\nu h_\alpha^\nu - \kappa h_{\mu\nu} T^{\mu\nu} + \dots\end{aligned}$$

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The leading order interaction is manifestly local:



$$\frac{\mathcal{L}_{\text{ex}}}{\kappa^2} = T_{\mu\nu} \frac{T^{\mu\nu}}{\square} - \frac{T^{(4)} T^{(4)}}{2 \square} \\ = T_{ij} \frac{T_{ij}}{\square} - \frac{T T}{2 \square} + \frac{T_{00} T_{00}}{2 \square} - 2 T_{0i} \frac{T_{0i}}{\square} + T_{00} \frac{T}{\square}$$

Here $T^{(4)} = \eta^{\mu\nu} T_{\mu\nu}$ and $T = \delta_{ij} T_{ij}$.

Non-Lorentz-invariant theory

Construction:

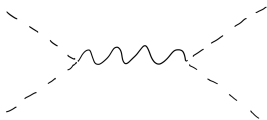
- Assume rotation invariance in order to *define* spin.
- Couple to some (not necessarily conserved) matter sector, $\mathcal{T}^{\mu\nu}[\Psi_m]$.
- Write down all the leading order operators:

$$\mathcal{L} = \left[-A \dot{h}_{0i} \partial_j h_{ij} + B \dot{h} \partial_i h_{0i} - \frac{1}{2} \left(C \partial_i h_{00} \partial_i h - D \partial_i h_{00} \partial_j h_{ij} \right. \right. \\ \left. \left. + E \partial_i h \partial_j h_{ij} + F (\partial_i h_{0i})^2 - G \partial_j h_{ij} \partial_k h_{ik} - H \partial_j h_{0i} \partial_j h_{0i} \right) \right. \\ \left. - \frac{1}{4} \left(I \dot{h}^2 - J \partial_i h \partial_i h - K \dot{h}_{ij} \dot{h}_{ij} + L \partial_k h_{ij} \partial_k h_{ij} \right) \right] - \kappa h_{\mu\nu} \mathcal{T}^{\mu\nu}$$

where $h = \delta_{ij} h_{ij}$.

$A = B = \dots = L = 1$ is GR, while otherwise \mathcal{L} breaks Lorentz invariance.

Tree-level scattering



E.g. the simplest 2 d.o.f. theory is immediately non-local:

$$\begin{aligned} \frac{\mathcal{L}_{\text{ex}}}{\kappa^2} = & \mathcal{T}_{ij} \frac{\mathcal{T}_{ij}}{\square} - \frac{\mathcal{T} \mathcal{T}}{2 \square} + \frac{\mathcal{T}_{00}}{2} \left[2a_5 \nabla^2 \partial_t^2 + a_6 \nabla^4 + a_7 \partial_t^4 \right] \frac{\mathcal{T}_{00}}{\square \nabla^4} \\ & - 2\mathcal{T}_{0i} \left[\frac{a_3 \nabla^2 + a_4 \partial_t^2}{\square \nabla^2} \right] \mathcal{T}_{0i} + \mathcal{T}_{00} \left[\frac{a_1 \nabla^2 + a_2 \partial_t^2}{\square \nabla^2} \right] \mathcal{T} \end{aligned}$$

where $a_{1,\dots,7}$ are functions of the coefficients A, \dots, L . E.g.

$$a_1 \equiv (E - J + L)/D,$$

$$a_5 \equiv (2JK + E(3I - K) - 3IJ + L(2I - K - 2B^2/H))/D^2.$$

Constraints from locality

Enforcing no-instantaneous-signaling in leading order gravitational interactions \implies

- 1 For 2 degrees of freedom all (5) graviton mass terms must vanish.
- 2 A, B, \dots, L organize such that the graviton sector is fully Lorentz invariant.
- 3 The matter sector is also Lorentz invariant, i.e. $\mathcal{T}^{\mu\nu} \equiv T^{\mu\nu}$ is the conserved energy-momentum tensor.

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Locality requires:

Spin 1 (single photon) \rightarrow internal $U(1)$.

Spin 2 (gravity) \rightarrow Lorentz symmetry.

Lorentz invariance is thus *required* when coupling to gravity, as compared to spin 1 where only the photon needed to be Lorentz invariant.

Time translations

If we also break time translation symmetry:

$$\{A, B, \dots, L\} \rightarrow \{A(t), B(t), \dots, L(t)\}$$

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- Recover some additional terms that prevent restoration of time-translation invariance:

$$\mathcal{L} = \mathcal{L}_{GR} + \alpha t \left(\frac{1}{2} \partial_i h \partial_i h - \frac{1}{2} \partial_k h_{ij} \partial_k h_{ij} + \partial_j h_{ij} \partial_k h_{ik} - \partial_i h \partial_j h_{ij} \right)$$

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At the leading order to which we are working, this is the Gauss-Bonnet term

$$\Delta \mathcal{L}_{GB} = f(t) \sqrt{-g} (R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})$$

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Including dimension 3 terms ($\sim h \partial h$) allows one to possibly recover cosmological backgrounds when requiring locality.

- Computationally challenging, and on non-trivial backgrounds causality itself may become more subtle.