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# Quark and Gluon Contents of a Lepton at High Energies

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[T. Han, Y. Ma, K.Xie 2007.14300]

[T. Han, Y. Ma, K.Xie 2103.09844]

# What is the PDF of a lepton?

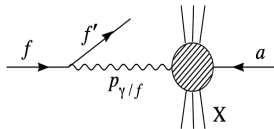
## “Equivalent photon approximation (EPA)”

[ C. F. von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the electron [E. J. Williams, Phys. Rev. 45, 729 (1934)]

$$\sigma(\ell^- + a \rightarrow \ell^- + X) = \int dx f_{\gamma/\ell} \hat{\sigma}(\gamma a \rightarrow X)$$

$$f_{\gamma/\ell, \text{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi} \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q^2}{m_\ell^2}$$



Extra terms:

[Frixione, Mangano, Nason, Ridolfi 2103.09844]

[Budnev, Ginzburg, Meledin, Serbo, Phys. Rept.(1975)]

## Applications at muon collider

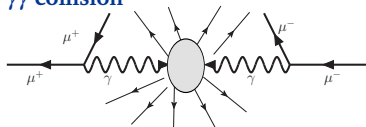
- Production cross sections

$$\sigma(\ell^+ \ell^- \rightarrow F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(ij \rightarrow F), \quad \tau = \hat{s}/s$$

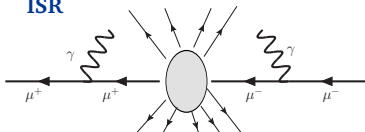
- Partonic luminosities

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{d\xi}{\xi} \left[ f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$

$\gamma\gamma$  collision



ISR



# A possible high-energy lepton collider: Why?

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## Why lepton colliders?

- **Leptons** are the ideal probes of short-distance physics
  - Cleaner background comparing to hadron colliders
  - High-energy physics probed with much smaller collider energy
- **ee colliders**
  - A glorious past: discovery of charm,  $\tau$ , and gluon
  - Important future: Precision EW constraints on BSM physics, Higgs physics
- **Muon colliders**
  - A *s*-channel Higgs factory: Higgs production enhanced by  $m_\mu^2/m_e^2 \sim 40000$ 
    - Direct measurements on  $y_\mu$  and  $\Gamma_H$
  - **Multi-TeV muon colliders**: Less radiations than electron
    - Center of mass energy 3–15 TeV and the more speculative  $E_{\text{cm}} = 30$  TeV
    - New particle mass coverage  $M \sim (0.5 - 1)E_{\text{cm}}$
    - Great accuracies for  $WWH$ ,  $WWHH$ ,  $H^3$ ,  $H^4$
    - ...

## Muon Collider Physics Potential Pillars

Direct search of heavy particles

SUSY-inspired, WIMP, VBF production,  $2 \rightarrow 1$

High rate indirect probes

Higgs single and self-couplings, rare Higgs decays, exotic decays

High energy probes

difermion, diboson, EFT, Higgs compositeness

# A high-energy muon collider at first glance

## What are the dominant processes at a high-energy muon collider?

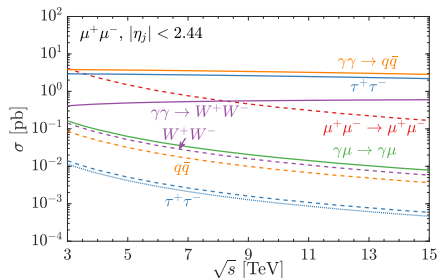
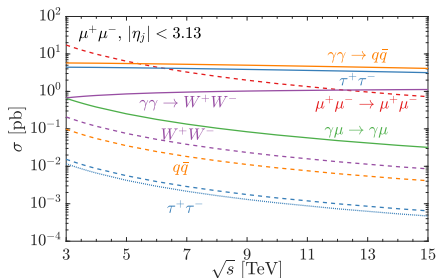
- Leading-order:  $\mu^+\mu^- \rightarrow \mu^+\mu^-$ ,  $\tau^+\tau^-$ ,  $q\bar{q}$ ,  $W^+W^-$ , and  $\gamma\mu \rightarrow \gamma\mu$
- $\gamma\gamma$  scatterings:  $\gamma\gamma \rightarrow \tau^+\tau^-$ ,  $q\bar{q}$ ,  $W^+W^-$

## Need some cuts:

- Detector angle:  $\theta > 5^\circ$  ( $10^\circ$ )  $\iff |\eta| < 3.13$  (2.44)
- Threshold:  $m_{ij} > 20$  GeV
- Need a  $p_T$  cut to separate from the nonperturbative hadronic production

[Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189, T. Barklow, etal, LCD-2011-020]

$$p_T > (4 + \sqrt{s}/3 \text{ TeV}) \text{ GeV}$$



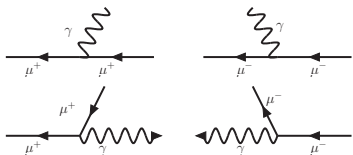
# Go beyond the EPA at a high-energy muon collider

## We have been doing:

- $\ell^+\ell^-$  annihilation



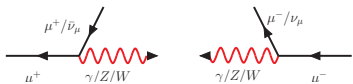
- EPA and ISR



- “Effective W Approx.” (EWA)

[G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

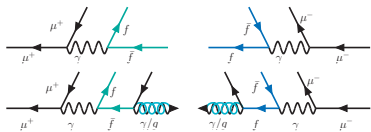
[S. Dawson, NPB 249 (1985) 42]



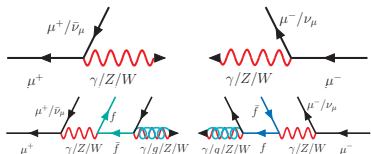
## We will add:

[T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

- Above  $\mu_{\text{QCD}}$ :  $\text{QED} \otimes \text{QCD}$   
 $q/g$  emerge



- Above  $\mu_{\text{EW}} = M_Z$ :  $\text{EW} \otimes \text{QCD}$   
EW partons emerge



In the end, everything is parton, i.e. **the full SM PDFs.**

# The PDFs for a muon collider

## ■ QED $\otimes$ QCD PDFs:

$$f_{\mu_{\text{val}}}, f_{\gamma}, f_{\ell_{\text{sea}}}, f_q, f_g$$

- Scale uncertainty: 20% for  $f_g/\mu$

- The averaged momentum fractions

$$\langle x_i \rangle = \int x f_i(x) dx$$

$Q(\mu^\pm)$	$\mu_{\text{val}}$	$\gamma$	$\ell_{\text{sea}}$	$q$	$g$
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
$M_Z$	97.9	2.06	0.028	0.035	0.0062

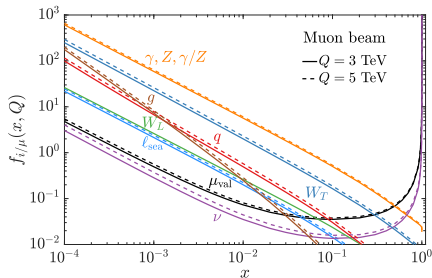
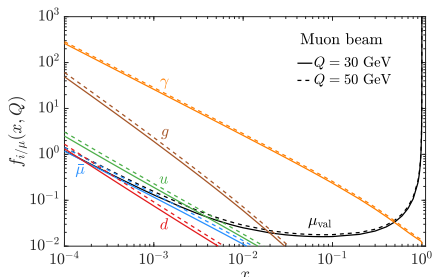
## ■ EW PDFs: All SM particles

$Q$	$\mu$	$\gamma, Z, \gamma/Z$	$W^\pm$	$\nu$	$\ell_{\text{sea}}$	$q$	$g$
$M_Z$	97.9	2.06	0	0	0.028	0.035	0.0062
3 TeV	91.5	3.61	1.10	3.59	0.069	0.13	0.019
5 TeV	89.9	3.82	1.24	4.82	0.077	0.16	0.022

- Scale uncertainty:  $\sim 20\%$  between  $Q = 3$  TeV and  $Q = 5$  TeV

- The EW correction is not small:  $\sim 100\%$  for  $f_d/\mu$  due to **relatively large SU(2) gauge coupling**.

- $W_L$  does not evolve

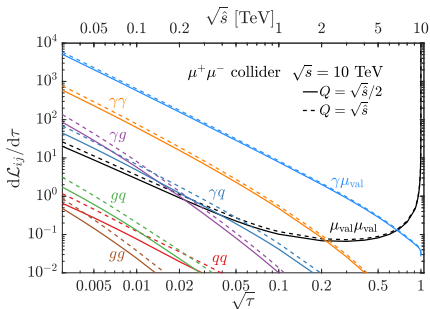
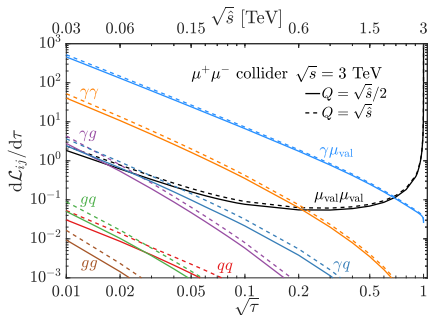


# Parton luminosities at a possible muon collider

Consider a 3 TeV and a 10 TeV machine

## ■ Partonic luminosities for

$\mu^+\mu^-$ ,  $\gamma\mu$ ,  $\gamma\gamma$ ,  $qq$ ,  $\gamma q$ ,  $\gamma g$ ,  $gq$ , and  $gg$



- The partonic luminosity of  $\gamma g + \gamma q$  is  $\sim 20\%$  of the  $\gamma\gamma$  one
- The partonic luminosities of  $qq$ ,  $gq$ , and  $gg$  are  $\sim 0.5\%$  of the  $\gamma\gamma$  one
- Given the stronger QCD coupling, **sizable QCD cross sections are expected.**
- Scale uncertainty is  $\sim 20\%$  ( $\sim 50\%$ ) for photon (gluon) initiated processes.

# Jet production of possible lepton colliders (I)

- Large photon induced non-perturbative hadronic production

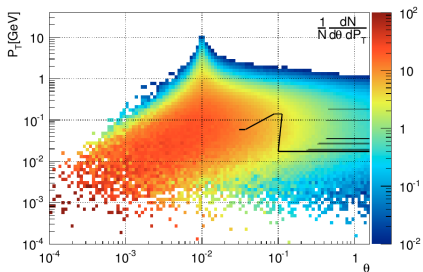
[ Drees and Godbole, PRL 67 1189, hep-ph/9203219]

[Chen, Barklow, and Peskin, hep-ph/9305247; Godbole, Grau, Mohan, Pancheri, SrivastavaNuovo Cim. C 034S1 ]

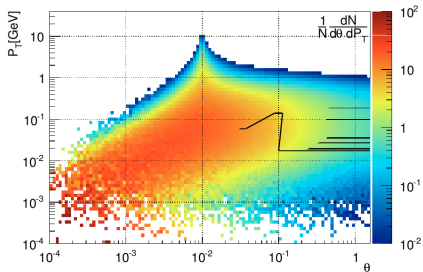
- $\sigma_{\gamma\gamma}$  may reach micro-barns level at TeV c.m. energies
- $\sigma_{\ell\ell}$  may reach nano-barns, after folding in the  $\gamma\gamma$  luminosity

- The events populate at low  $p_T$  regime

So we can separate from this non-perturbative range via a  $p_T$  cut.



(a) Pythia sample



(b) SLAC sample

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

## Jet production at a possible lepton collider (II)

- Low- $p_T$  range: photon induced non-perturbative hadronic production

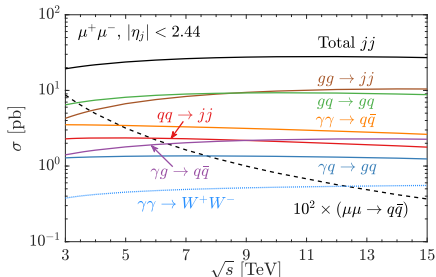
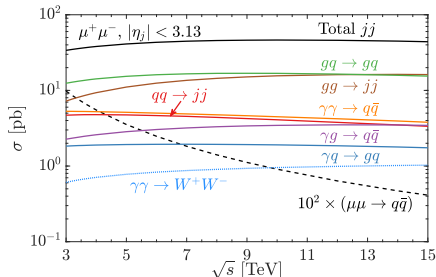
[Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189, T. Barklow, et al, LCD-2011-020]

- High- $p_T$  range [ $p_T > (4 + \sqrt{s}/3 \text{ TeV}) \text{ GeV}$ ]: perturbatively computable

$$\gamma\gamma \rightarrow q\bar{q}, \gamma g \rightarrow q\bar{q}, \gamma q \rightarrow gq,$$

$$qq \rightarrow qq (gg), gq \rightarrow gq \text{ and } gg \rightarrow gg (q\bar{q}).$$

- $Q = \sqrt{\hat{s}}/2$ , due to large  $\alpha_s \ln(Q^2)$ , a 30 ~ 40% enhancement if  $Q = \sqrt{\hat{s}}$



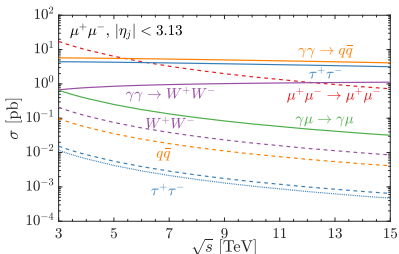
- Including the QCD contribution leads to much larger total cross section.
- $gg$  initiated cross sections are large for its large multiplicity;
- $gg$  initiated cross sections are large for its large luminosity.
- $\gamma\gamma$  initiated cross sections here are smaller than the EPA results.

# Refresh the picture of high-energy muon colliders

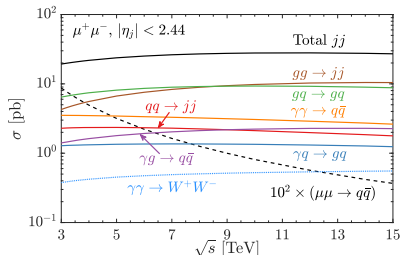
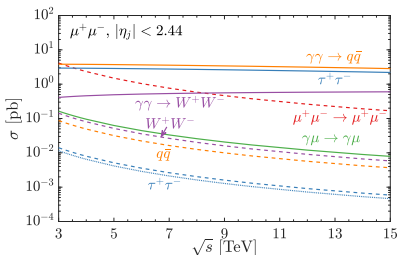
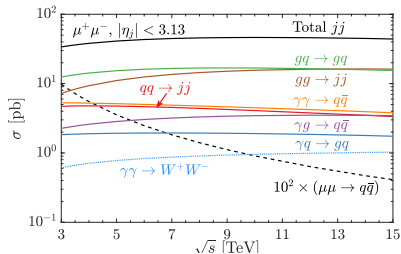
What is the dominant process at a high-energy muon collider?

- Quark/gluon initiated jet production dominates

Before:



After:



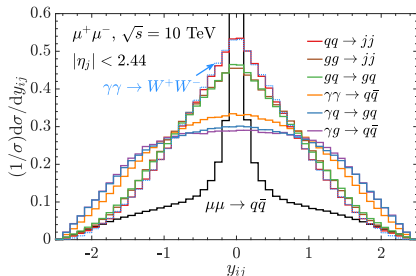
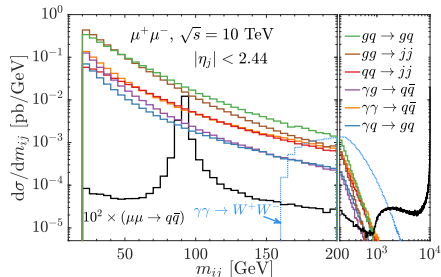
# Di-jet distributions at a muon collider

Rather a conservative set up:  $\theta = 10^\circ$

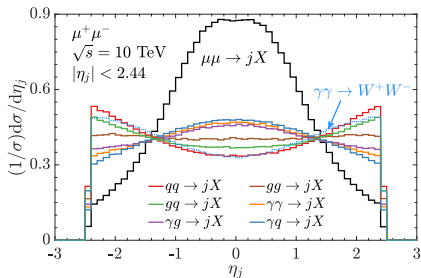
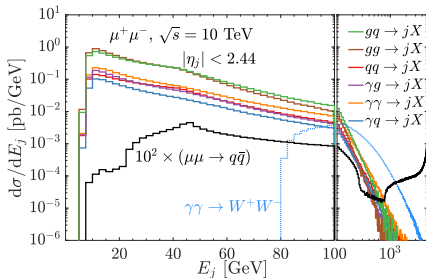
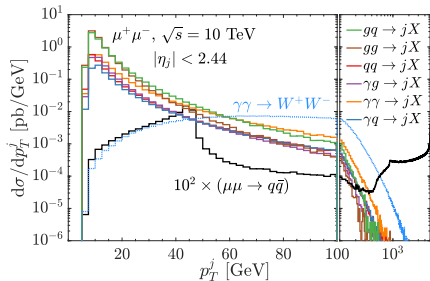
## Some physics:

Two different mechanisms:  $\mu^+\mu^-$  annihilation VS Fusion processes

- Annihilation is more than 2 orders of magnitude smaller than fusion process.
- Annihilation peaks at  $m_{ij} \sim \sqrt{s}$ ;
- Fusion processes peak near  $m_{ij}$  threshold.
- Annihilation is very central, spread out due to ISR;
- Fusion processes spread out, especially for  $\gamma q$  and  $\gamma g$  initiated ones.



# Inclusive jet distributions at a muon collider

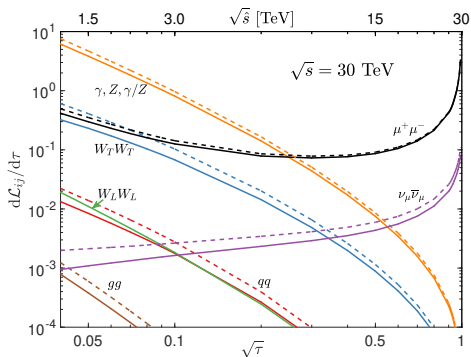


- Jet production dominates over  $WW$  production until  $p_T > 60$  GeV;
- $WW$  production takes over around energy  $\sim 200$  GeV.
- QCD contributions are mostly forward-backward;  $\gamma\gamma$ ,  $\gamma q$ , and  $\gamma g$  initiated processes are more isotropic.

# An EW version of HE LHC

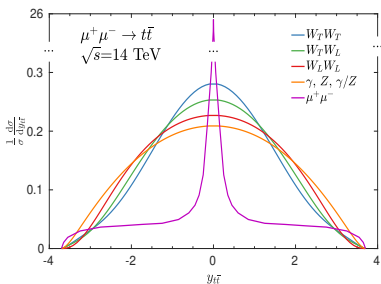
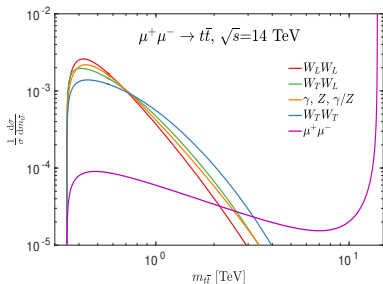
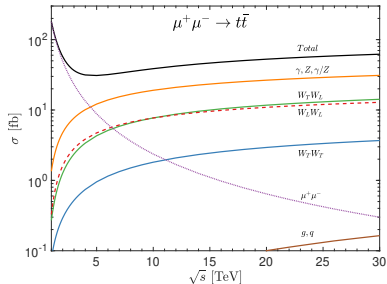
- All SM particles are partons when the machine energy is high
- We are able to determine the partons with their different polarizations

## The EW parton luminosities of a 30 TeV muon collider

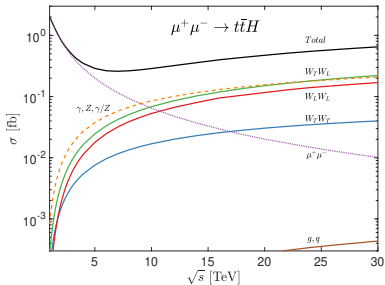
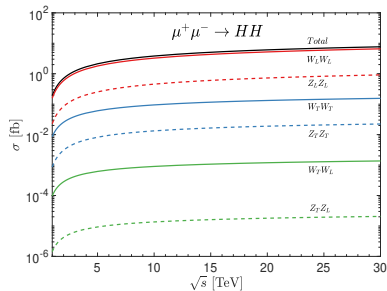
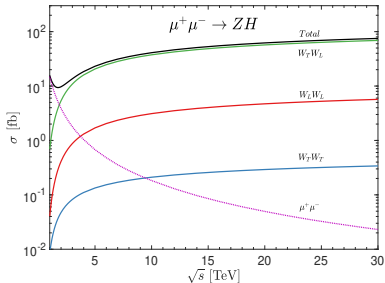
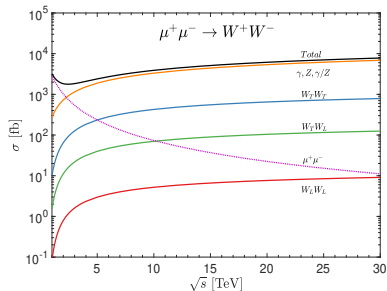


[T. Han, Y. Ma, K.Xie 2007.14300]

# One example: $t\bar{t}$ production at a muon collider

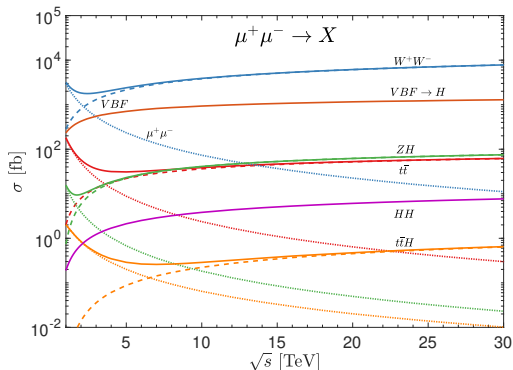


# Other processes: $W^+W^-$ , $ZH$ , $HH$ , $t\bar{t}H$



# The full picture: Semi-inclusive processes

Just like in hadronic collisions:  $\mu^+\mu^- \rightarrow$  exclusive particles + remnants



[T. Han, Y. Ma, K.Xie 2007.14300]

## Some observations:

- The annihilations decrease as  $1/s$ .
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as  $\ln^p(s)$ , which take over at high energies.
- The large collinear logarithm  $\ln(s/m_\mu^2)$  needs to be resummed, set  $Q = \sqrt{\hat{s}}/2$ .

# Summary and prospects

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## EWPDF is important and necessary:

- At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by proper PDFs.**
  - The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the **QCD partons (quarks and gluons) emerge.**
  - When  $Q > M_Z$ , the EW splittings are activated: the EW partons appear, and the existing QED $\otimes$ QCD PDFs may receive big corrections.

## A high-energy muon collider is an EW version of HE LHC

- There are many things to work on: SUSY, DM, Higgs, etc.
- Two classes of processes:  $\mu^+\mu^-$  annihilation VS fusions

[T. Han, Y. Ma, K.Xie 2007.14300]

### ■ The main background of is the jet production:

- Low  $p_T$  range: non-perturbative  $\gamma\gamma$  initiated hadronic production dominates

[Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189, T. Barklow, et al, LCD-2011-020]

- High  $p_T$  range,  $q$  and  $g$  initiated jet production dominates

[T. Han, Y. Ma, K.Xie 2103.09844]

- EWPDF allows to determine the contributions from different partons and their different polarizations.