Analytic continuations of two loop off-shell diboson amplitudes

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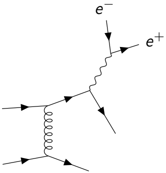
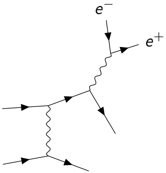
Motivation

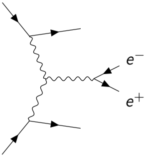
- Vector boson production in association with 2 jets is dominated by strong processes at $\mathcal{O}(\alpha_{EW}^2 \alpha_s^2)$ at tree level
- EW Z+2j at tree level corresponds to a smaller contributions coming in at order $\mathcal{O}(\alpha_{EW}^4)$. However, these have distinct signatures with a large M_{jj} along with two high energy jets.
- The EW Z+2j differential cross-sections is useful to constrain anomalous weak-boson self-interactions



Strong Z+2j



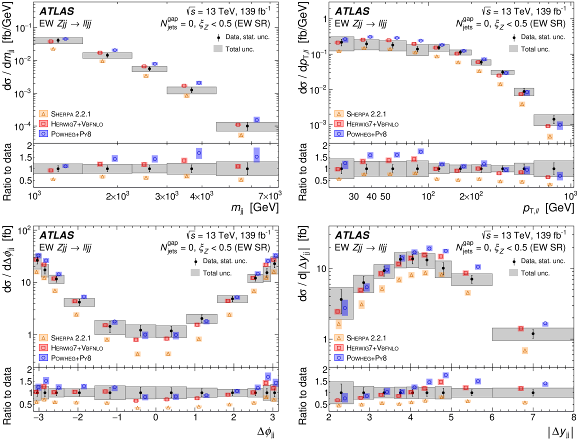
EW Z+2j



VBF

Experimental Results

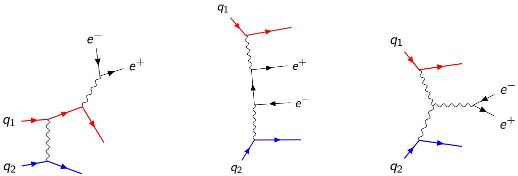
- Most recent ATLAS measurement at arxiv:2006.15458 for $\sqrt{s} = 13$ Tev at 139 fb^{-1}



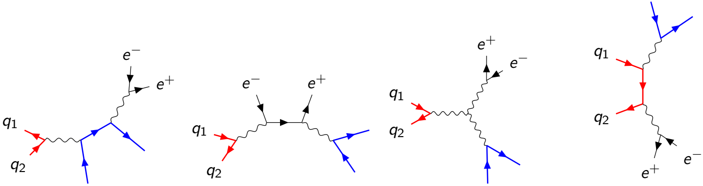
Tree level Diagrams at $\mathcal{O}(\alpha_{EW}^4)$

At LO, contributions come from:

- t-channel:

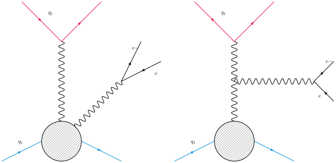


- others ($\sim 3\%$ at $\sqrt{s} = 13$ Tev, with cuts of minimum $m_{jj} = 120$ Gev ,$m_{ll} = 50$ Gev and $p_{t,j} = 25$ Gev):



QCD corrections $\mathcal{O}(\alpha_s^2\alpha_{EW}^4)$

- The two loop QCD contributions to the t-channel EW Z+2j can be grouped into diagrams with diboson form factors (Gehrmann et al. 2015, Caola et al 2015) and quark form factors (Gehrmann et al 2005) respectively upto crossings.



- Diagrams were generated with QGRAF and processed with FORM.
- The grouping of these diagrams were done in a semi-automated fashion using Mathematica to generate expressions for amplitudes and their interferences to tree level that can be use with MCFM.

Diboson - Master integrals

- For the diboson process all the master integrals have been identified and solved .
- Solutions to the master integrals have been analytically evaluated in the physical region.
- Henn et. al, 2014 provided the integrals in the form of GPLs upto weight 4.
- Gehrmann et al 2015, provided an optimized version of master integrals in a basis made up of classical logs, polylogs Li_n and the $Li_{2,2}$ function which are convenient for phenomenological computations. We are interested in analytically continuing these results for our computation.
- This has also been solved by the method simplified differential equations by Papdopolous et al., 2014.

Master integrals

- With a set of master integrals chosen, we can construct differential equations of the form

$$\delta_x \vec{f}(x, \epsilon) = A(x, \epsilon)\vec{f}(x, \epsilon)$$

where x are kinematic invariants.

- This system of equations can be converted to canonical form with some effort and a choice of basis masses which would look like,

$$\delta_x \vec{g}(x, \epsilon) = \epsilon A\tilde{(x)}\vec{g}(x, \epsilon)$$

- We have the full basis and results to compare to in literature.
- IBPs and differential equations were generated with Kira and Reduze.

Master Integrals

- We choose the parameterization

$$\frac{s}{m_a^2} = (1+x)(1+x\ y), \frac{t}{m_a^2} = -x\ z, \frac{m_b^2}{m_a^2} = x^2\ y$$

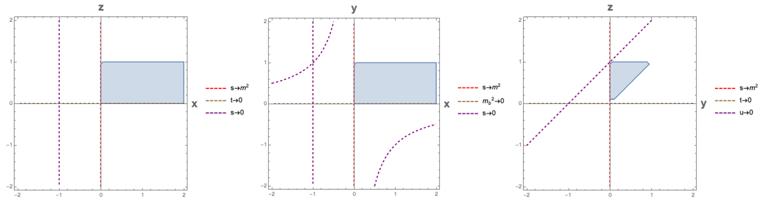
  as suggested in Henn et. al 2014 which rationalizes the Kallen function.
- We can solve the canonical basis order by order in ϵ
- The solutions can be easily integrated expressed in the form of in the form of GPLs depending, keeping in mind the order of integration for the three variables.
- While setting boundaries are generally challenging, comparing them to the existing results trivially fixes the boundaries.

Kinematics - Diboson

Diboson kinematic region:

$$s > 0,\ \ t < 0,\ \ u < 0,\ \ m_a^2 > 0,\ \ m_b^2 > 0$$

This corresponds to $x > 0,\ \ y > 0,\ \ y < z < 1$ in the physical region.



Kinematics - EW Z+2j

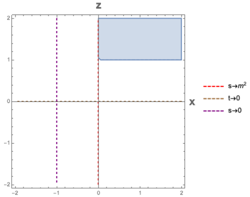
EW Z+2j region:

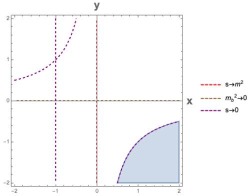
$$s < 0, t < 0, u > 0, m_a^2 > 0, m_b^2 < 0$$

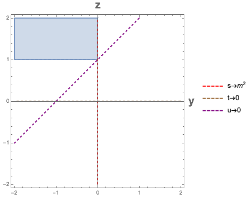
Using the same parameterization ,

$$x > 0,\ \ y < 0,\ \ z > 1,\ \ xy < -1$$

in the physical region,







Analytic Continuation

- With the feynman $i\epsilon$ prescription we need to specify for the changes in the corresponding signs in our region,

$$\begin{aligned} s &\to s - i\delta_1 \\ m_b^2 &\to m_b^2 - i\delta_2 \\ u &\to u + i\delta_3 \end{aligned}$$

- However this cannot be a priori matched an $i\epsilon$ prescription in the integration variables x,y,z making the problem somewhat non-trivial.

$$\frac{s}{m_a^2} = (1+x)(1+x\ y), \frac{t}{m_a^2} = -x\ z, \frac{m_b^2}{m_a^2} = x^2\ y$$

Analytic Continuation

- There are 6 GPLs that contribute to the innermost integration,

$$G_0(x), G_0(y), G_0(z), G_{-1}(x), G_{-1/x}(y), G_1(y-z)$$

- These will correspond to a physical thresholds as shown in the plots above
- To analytically continue the solutions we need to keep track of the region and which side of the threshold we are integrating in.
- The boundary constants can acquire additional factors of $2\pi i$ which need to be continued to all the Polylogs of higher weights as well.
- The other weight-1 logs which do not correspond to physical thresholds should not contribute in anyway.

Analytic Continuation: Example

- Consider the simple master integral,

$$Int[P23,1] \sim \int \frac{d^{4-2\epsilon}k_1 d^{4-2\epsilon}k_2}{(k_1+p_2)^2(k_1-k_2)^4(k_2+p_3)^4}$$

- In the physical region for diboson production, this corresponds to

$$\begin{aligned} Int[P23,1] = \quad & -1+\epsilon(2\log(x)+2\log(1+y-z)) \\ & +\epsilon^2(-2\log^2(x)-4\log(x)\log(1+y-z) \\ & -2\log^2(1+y-z))+\mathcal{O}(\epsilon^3) \end{aligned}$$

- In the physical region for EW+2j however this is off because this integral corresponds to $u=-m^2(x)(1+y-z)$, which is now evaluated on the side with $u>0$.

Analytic Continuation: Example

- So from $u = -m^2(x)(1 + y - z)$, we can use the rule $\log(1 + y - z) + \log(x) \to \log(1 + y - z) + \log(x) - 2\pi i$ to get the right form.
- This can also be realized as a change in the boundary conditions of the integral evaluted in the new region.
- The integrated form of $Int[P23, 1]$ without fixing the boundary constants looks like,

$$\begin{aligned} Int[P23,1] \sim g_{1,0} + \epsilon(g_{1,1} - 2g_{1,0}(\log(x) + \log(1+y-z))) \\ +\epsilon^2(g_{1,2} + 4g_{1,0}(\log^2(x) + \log(x)\log(1+y-z) \\ +\log^2(1+y-z)) - 2g_{1,1}(log(x) + log(1+y-z))) \\ +\mathcal{O}(\epsilon^3) \end{aligned}$$

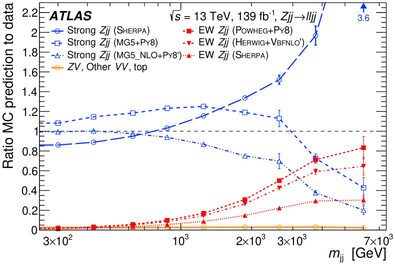
Analytic Continuation: Example

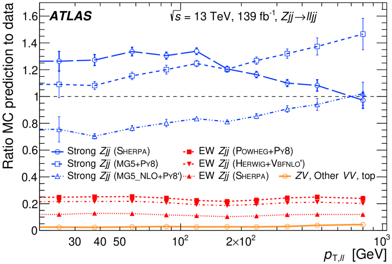
- So the boundary constants for the new physical region need to be shifted as $g'_{1,1} \to g_{1,1} - 4\pi i$, $g'_{1,2} \to g_{1,2} + 8\pi^2$.
- Unlike in this simple example, continuing the polylogs of higher weights can get complicated especially when have different ratios of these variables appear in the arguments.
- We therefore chose to fix this by numerically analyzing them to fix the boundary conditions upto rational factors of $(i\pi)^n$ depending on the weights.
- We have been able to analytically continue master integrals to the physical region for the EW Z+2j process using this strategy. These master integrals can now be used in the evaluation of the helicity amplitudes.

Conclusions and Future Directions

- We are able to compute the helicity amplitudes for the two loop QCD corrections to the EW Z+2j process and their interferences with tree level amplitudes.
- The last piece needed are the NLO+jet amplitudes.
- We can then implement these into a monte carlo generator (MCFM) to generate the full NNLO differential cross-sections for the EW Z+2j process.

Experimental Results





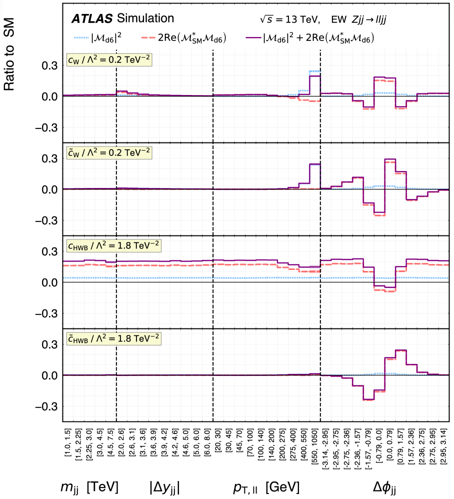
Experimental Results - SMEFT

- The EW Z+2j differential cross-sections is useful to constrain anomalous weak-boson self-interactions which can be introduced with an EFT like

$$\mathcal{L}_{eff} = \mathcal{L}_{\mathcal{SM}} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i$$

  where $\mathcal{O}_i$ are dimension 6 operators
- This EW Z+2j process is particularly sensitive to the dim 6 operators at high $p_{t,ll}$ and ϕ_{jj}

Experimental Results - SMEFT



QCD Corrections to EW Z+2j

To calculate the full QCD corrections at NNLO, we will need the amplitudes for the following,

- LO $\mathcal{O}(\alpha_{EW}^2)$:
  $qq \rightarrow Z(\rightarrow e^+e^-)qq$
- NLO $\mathcal{O}(\alpha_s \alpha_{EW}^2)$:
  $qq \rightarrow Z(\rightarrow e^+e^-)qq$ (1-loop),
  $qq \rightarrow Z(\rightarrow e^+e^-)qqj$,
  $gq \rightarrow Z(\rightarrow e^+e^-)qqq$
- NNLO $\mathcal{O}(\alpha_s^2 \alpha_{EW}^2)$:
  $qq \rightarrow Z(\rightarrow e^+e^-)qq$ (2-loop),
  $qq \rightarrow Z(\rightarrow e^+e^-)qqj$ (1-loop + jet)
  $pp \rightarrow Z(\rightarrow e^+e^-) + 4j$