The Standard Model prediction for the muon $g-2$

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CERN and PRISMA+ Cluster of Excellence, Johannes Gutenberg-Universität Mainz

Phenomenology 2021 Symposium
University of Pittburgh
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PRISMA+

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The Standard Model and its limits

Highly successful theory, but no explanation for

- dark matter and dark energy
- matter-antimatter asymmetry
- hierarchy among particle masses:

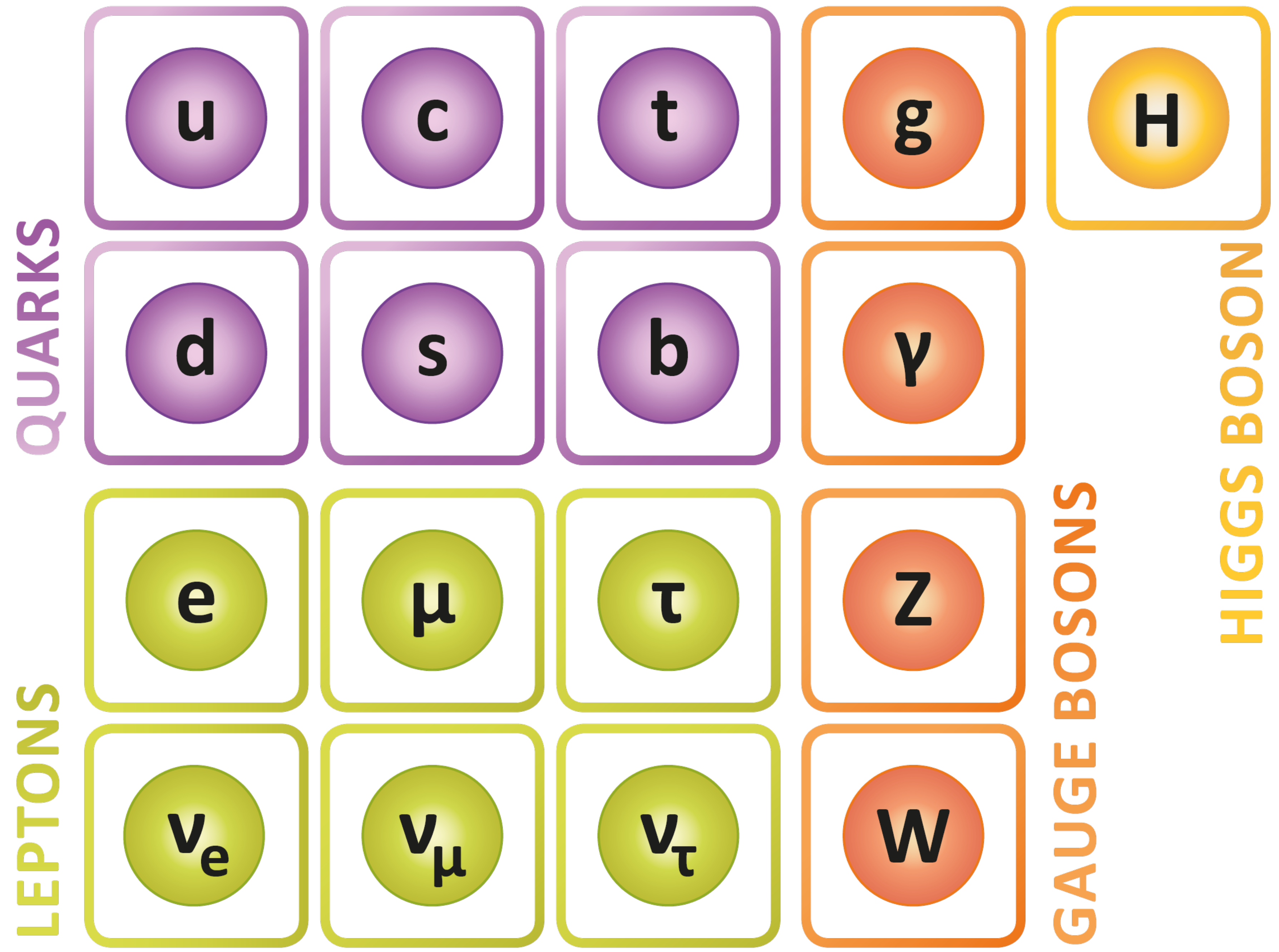
$$m_{\nu_e} \ll m_{\text{Higgs}} \ll m_{\text{Planck}}$$

- number of particle generations

Standard Model does not provide a complete description of Nature

Explore the limits of the Standard Model

- search for new particles and forces at high energies
- search for enhancement of rare phenomena
- confront precision measurements with SM predictions



Plenary Session [TUE]

James Mott

"Muon $g-2$ and muon physics on Fermilab campus"

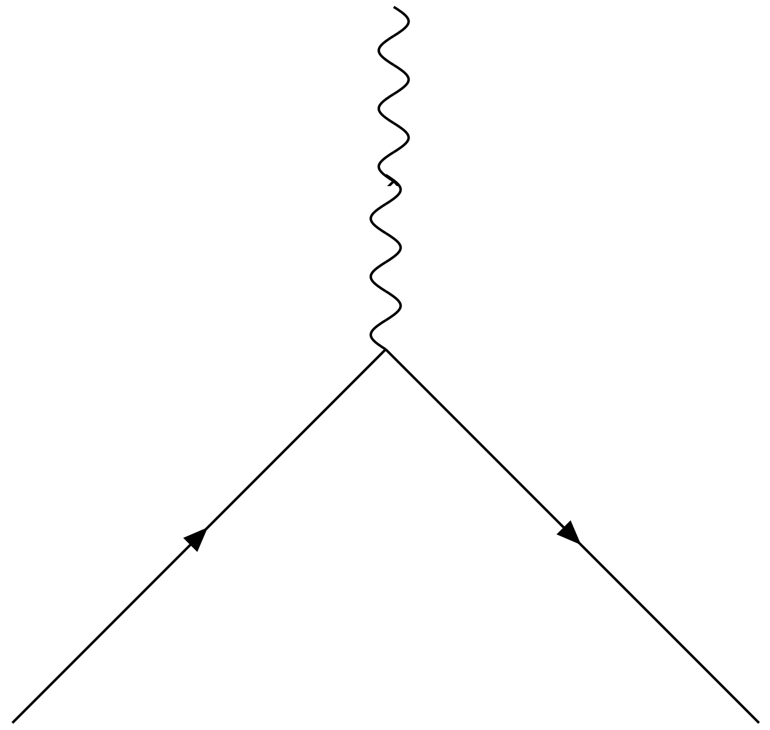
Bhupal Dev

"Anomalies and their implications"

Anomalous magnetic moment

Particle with mass m and charge e :

$$\boldsymbol{\mu} = g\,\frac{e\hbar}{2m}\,\boldsymbol{S}, \qquad \boldsymbol{S} = \frac{\boldsymbol{\sigma}}{2}$$



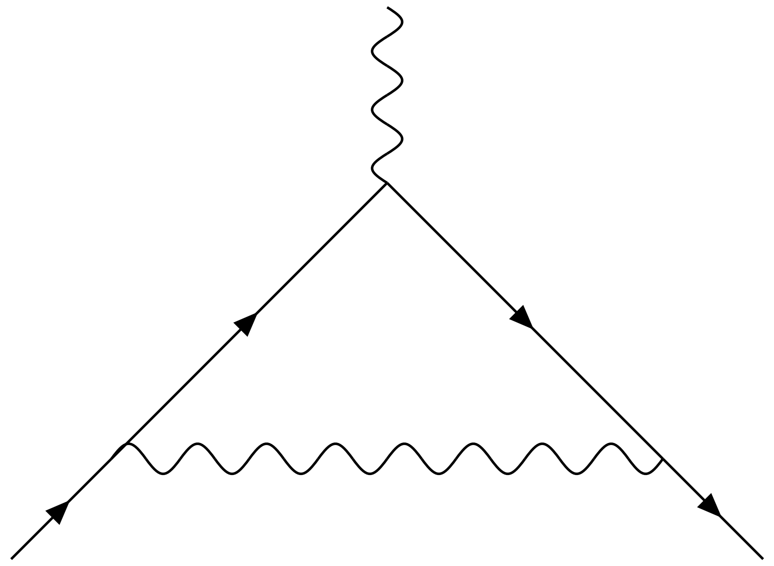
Pauli equation: $g = 2$

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = \left\{\frac{1}{2m}\,(\boldsymbol{p} - e\boldsymbol{A})^2 + e\Phi - \frac{e\hbar}{2m}\,\boldsymbol{\sigma}\cdot\boldsymbol{B}\right\}\psi(\boldsymbol{x},t)$$

Quantum corrections: $g = 2(1+a),\quad a:$ anomalous magnetic moment

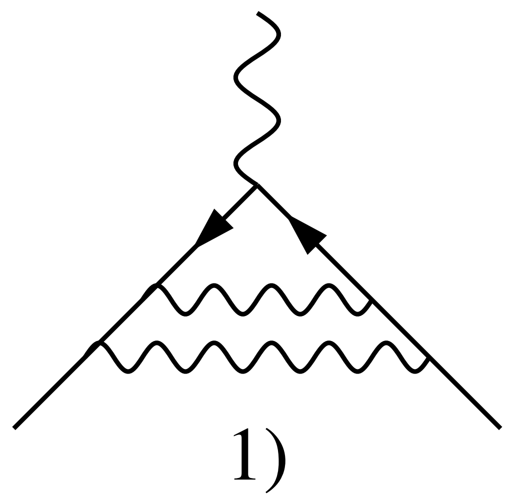
First-order QED correction calculated by Schwinger:

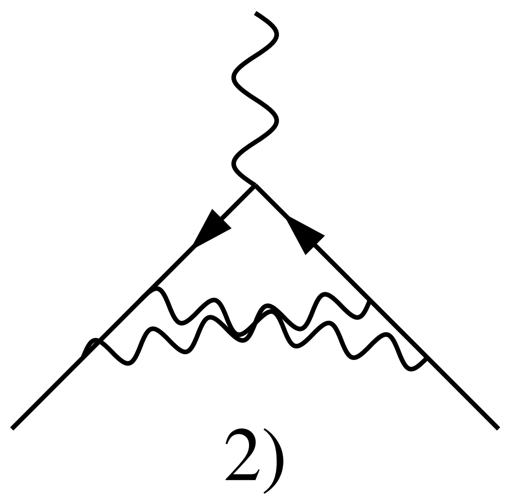
$$g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$

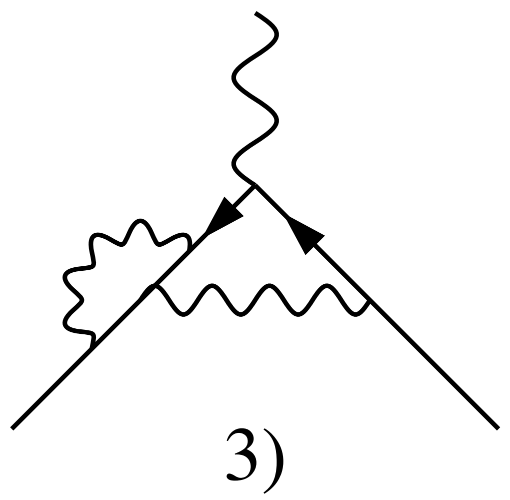


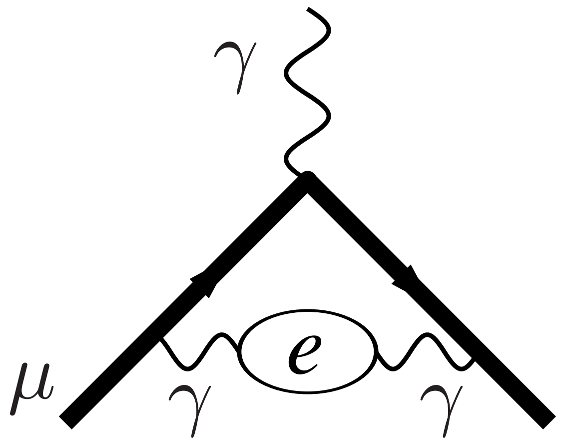
Higher-order corrections

QED corrections:









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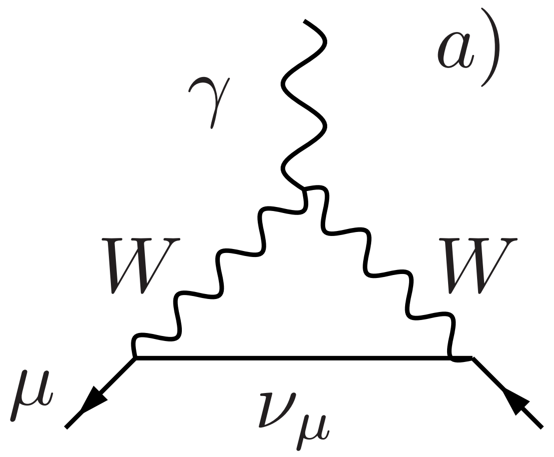
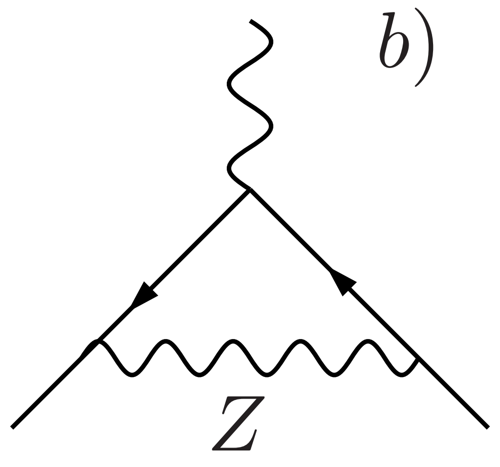
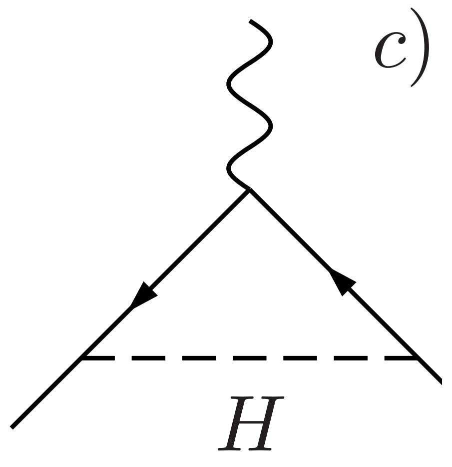
Weak corrections:







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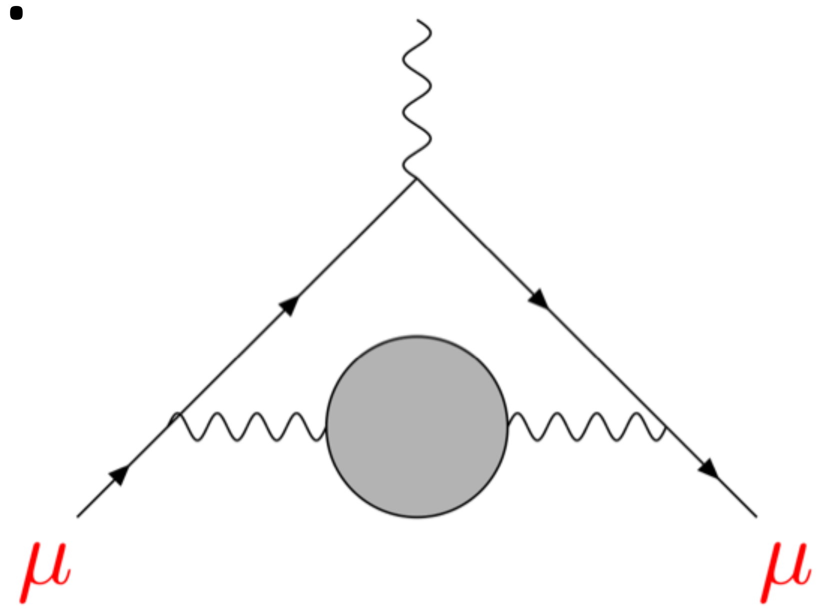
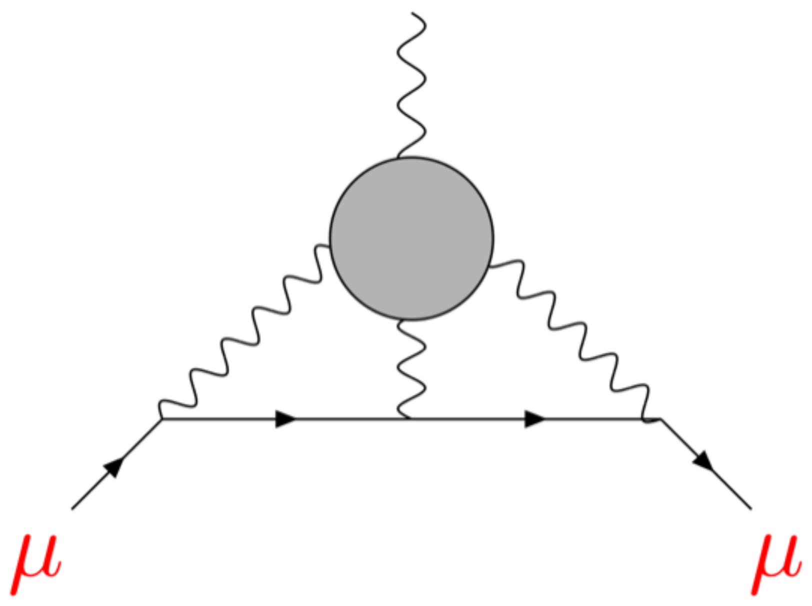
Hadronic corrections:





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The muon anomalous magnetic moment as a probe for new physics

Persistent tension between SM prediction for a_μ and E821 experiment at BNL

Can effects from BSM physics account for the shortfall of the SM prediction?

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm strong}$$

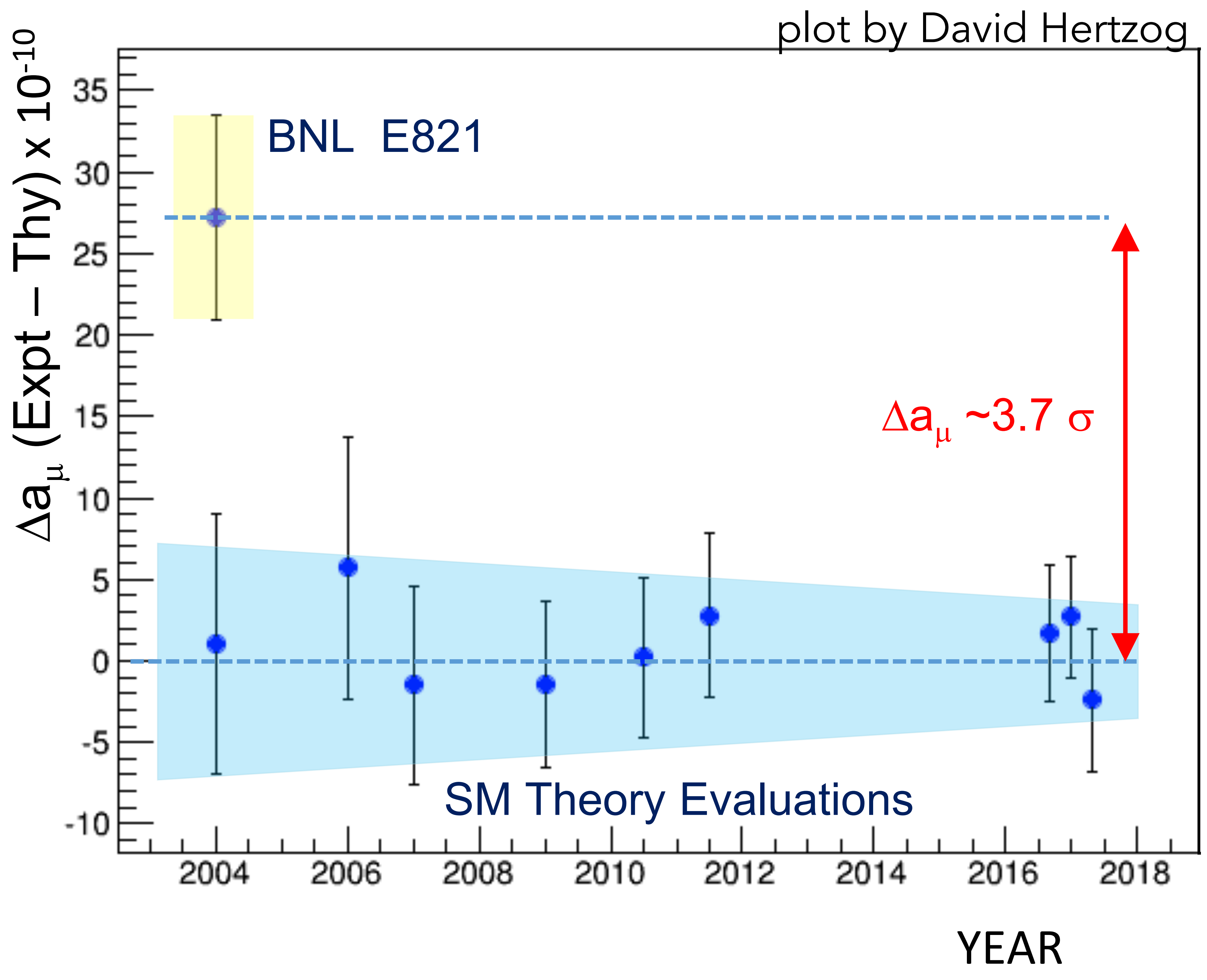
$$a_\mu = a_\mu^{\rm SM} + a_\mu^{\rm NP} \quad ?$$

Why the muon?

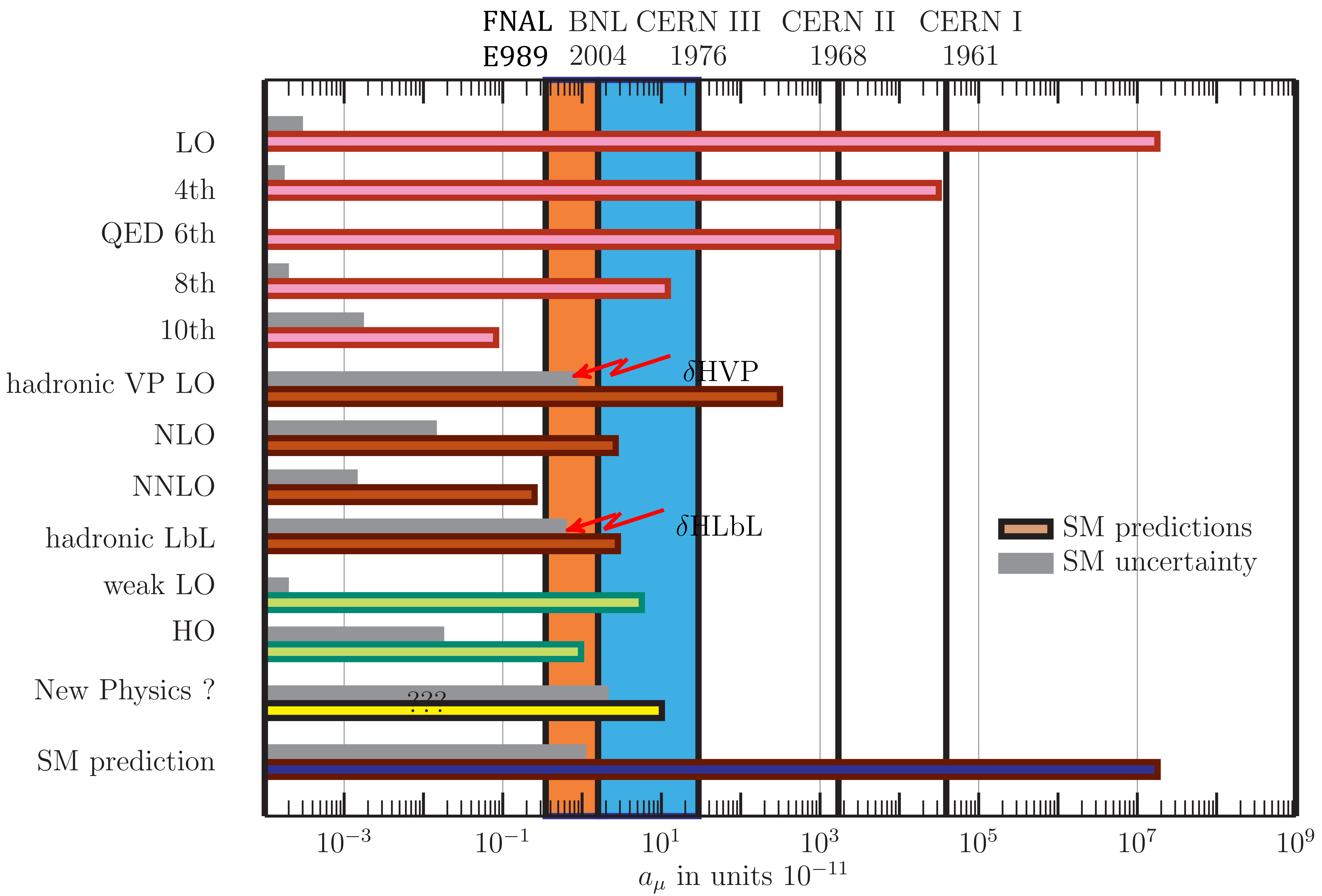
$$a_\ell^{\rm NP} \propto m_\ell^2/M_{\rm NP}^2, \quad \ell = e, \mu, \tau$$

→ sensitivity of a_μ enhanced by $(m_\mu/m_e)^2 \approx 4.3 \times 10^4$

While a_τ would be even more sensitive, τ's are difficult to handle experimentally

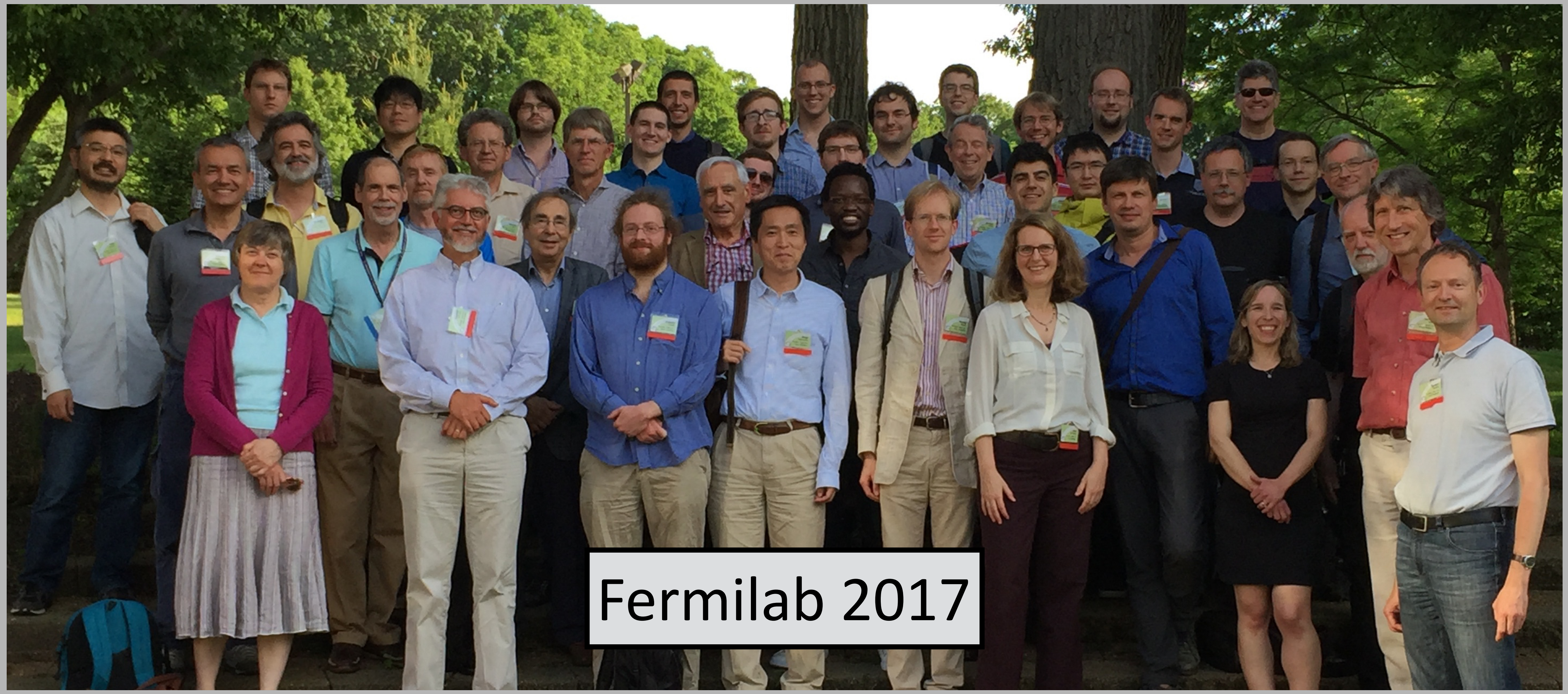


Theory confronts experiment



Plot by
Fred Jegerlehner

Muon $g-2$ Theory Initiative



Agree on common SM prediction

Focus on hadronic contributions

Prospects for increased precision

White Paper:

T. Aoyama et al., Phys Rep 887 (2020) 1

Muon $g-2$ Theory Initiative

Next plenary workshop:

28 June — 2 July 2021 @ KEK (virtual)

https://www-conf.kek.jp/muong-2theory/

White Paper:

T. Aoyama et al., Phys Rep 887 (2020) 1

Mainz 2018

Outline

Outline

QED and electroweak contributions to a_μ

Hadronic contributions:

- Leading-order vacuum polarisation
- Light-by-light scattering

Synthesis — Discussion — Outlook

QED contributions to a_μ

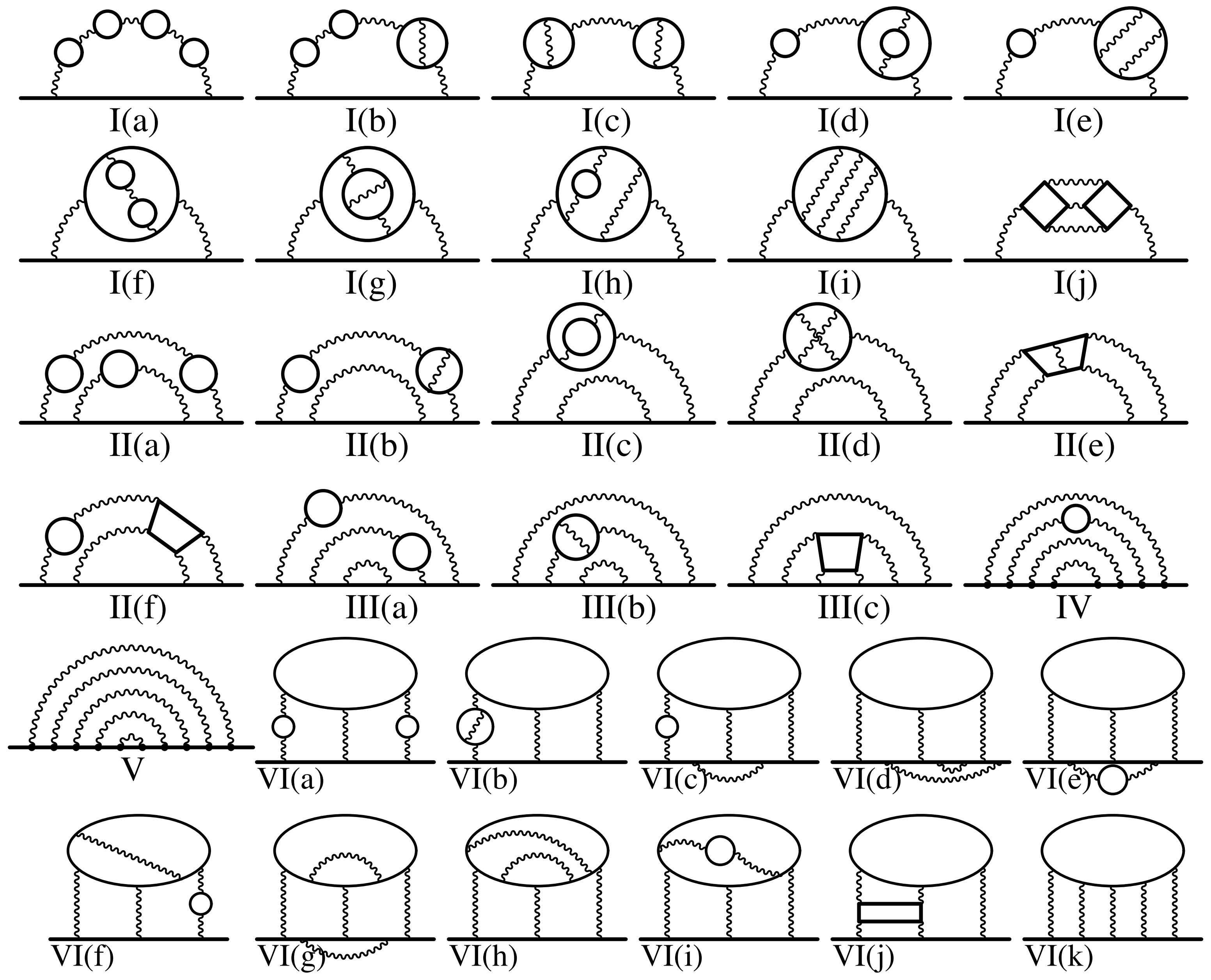
QED contribution has been worked out to in perturbation theory to 5-loop order:

PRL **109,** 111808 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

Complete Tenth-Order QED Contribution to the Muon $g - 2$

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]
[1]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[2]*Nishina Center, RIKEN, Wako, Japan 351-0198*
[3]*Department of Physics, Nagoya University, Nagoya, Japan 464-8602*
[4]*Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York 14853, USA*
(Received 24 May 2012; published 13 September 2012)

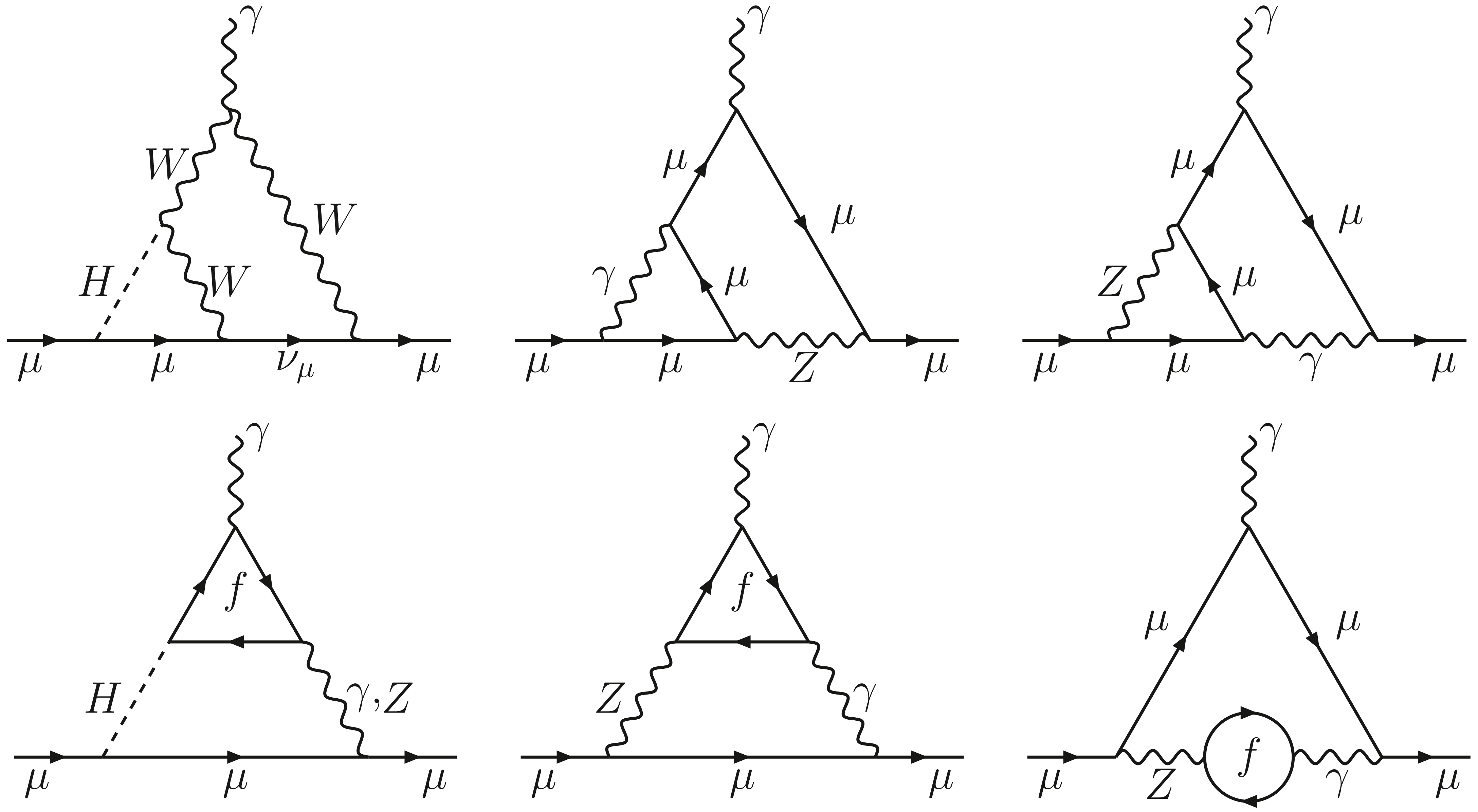
SM	116 591 810	100 %	#diagrams
QED(tot)	116 584 718.931	99,9939 %	
2	116 140 973.321	99,6133 %	1
4	413 217.626	0,3544 %	9
6	30 141.902	0,0259 %	72
8	381.004	0,0003 %	891
10	5.078	$4\cdot10^{-6}$ %	12672



Electroweak contributions to a_μ

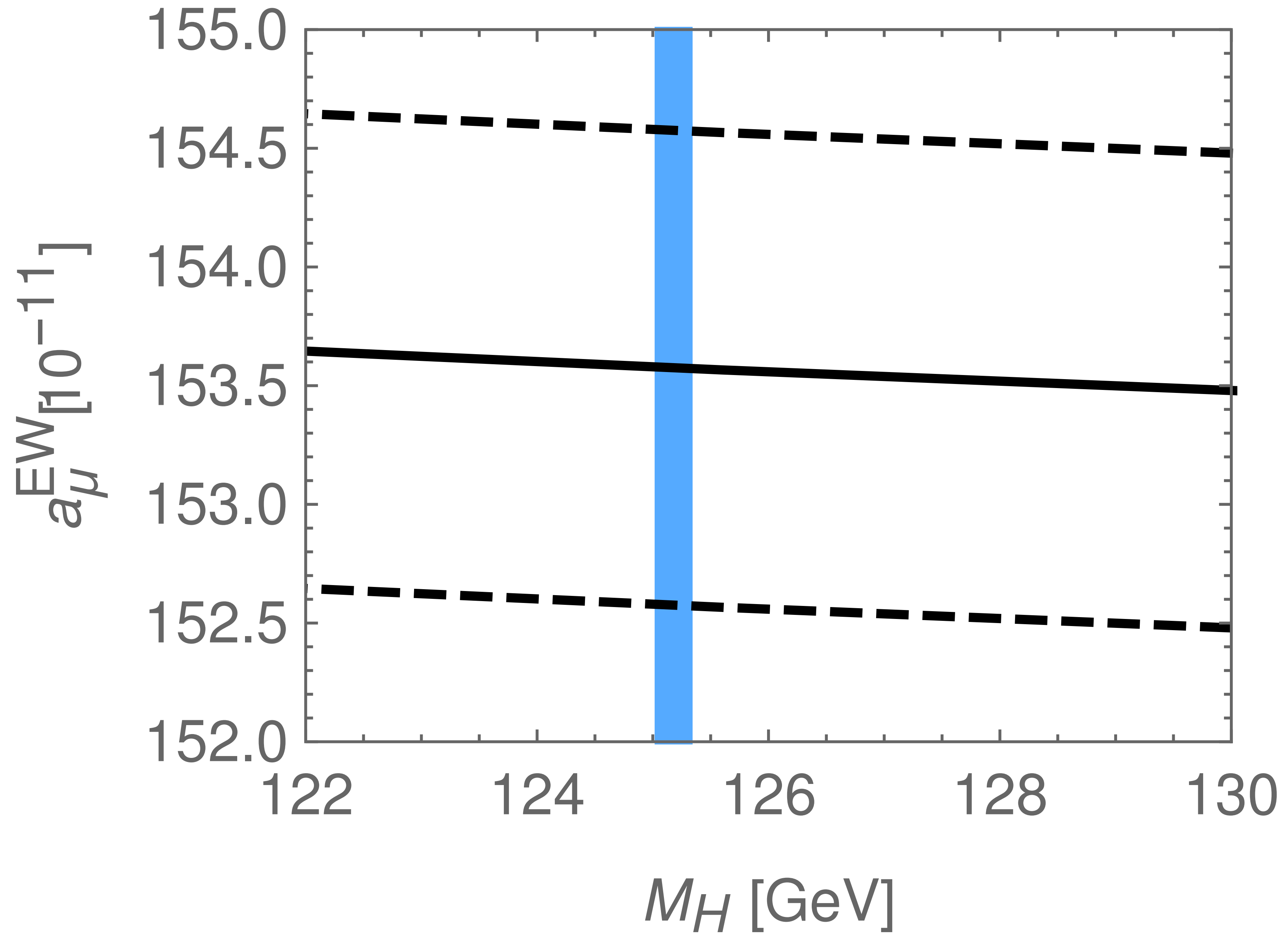
Weak contributions known to leading three-loop order

Sample two-loop diagrams:



$$a_\mu^{\text{weak}} = (153.6 \pm 1.0) \times 10^{-11}$$

Dependence on the Higgs mass

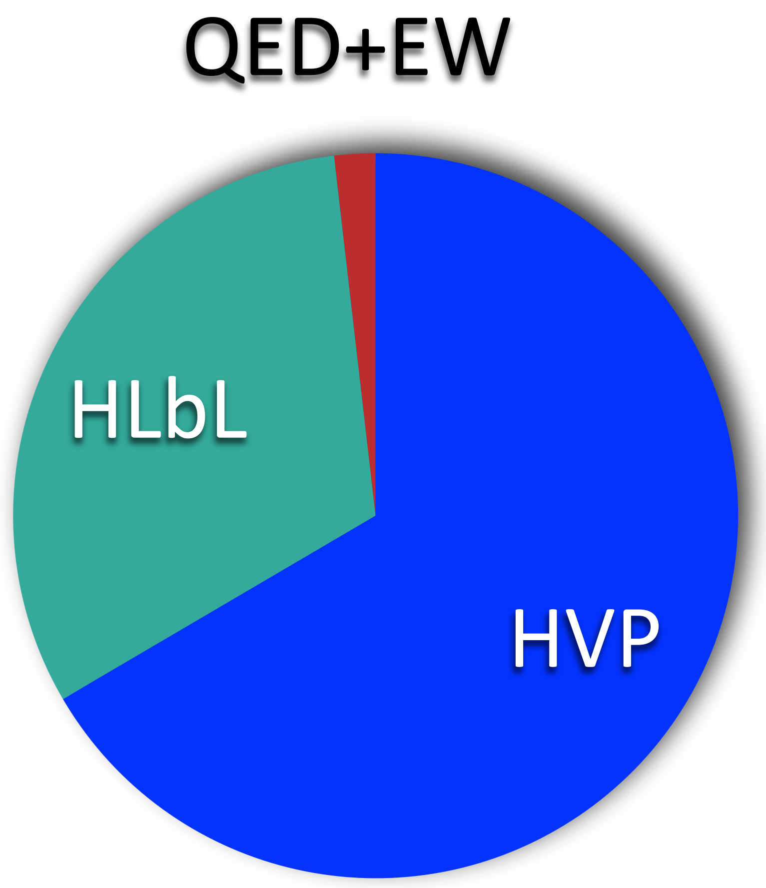


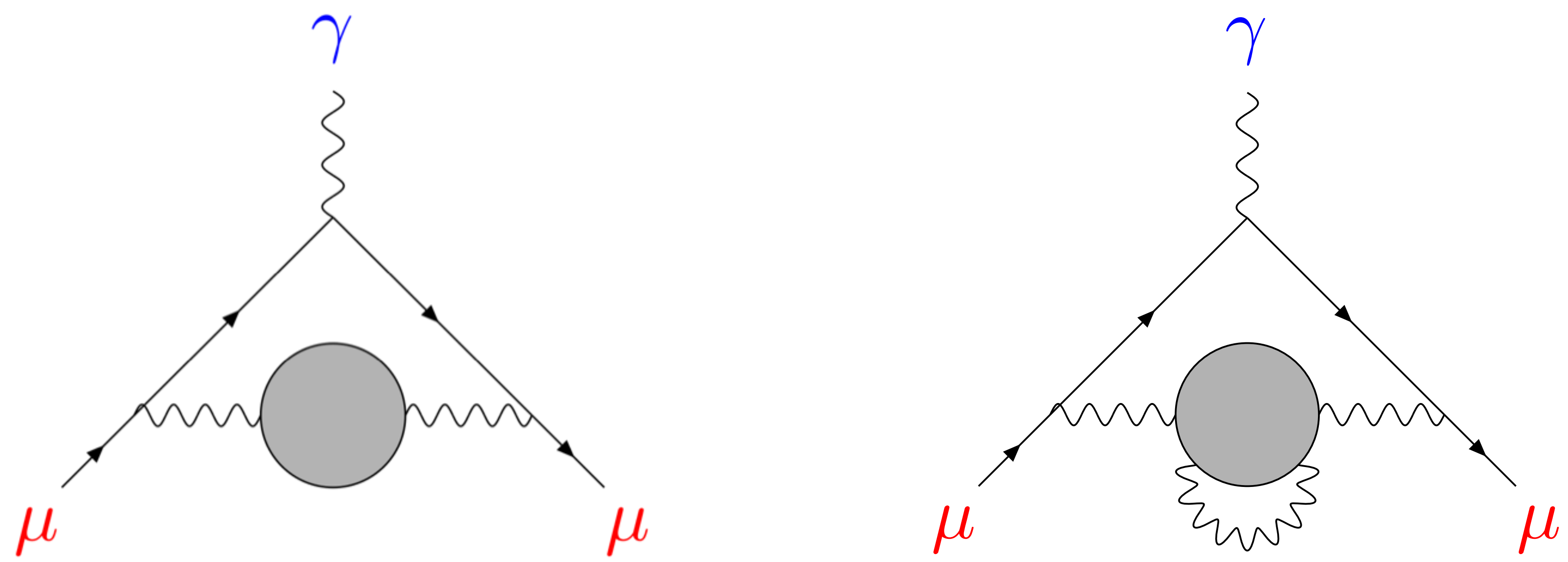
[Gnendiger et al., arXiv:1306.5546]

Hadronic contributions to a_μ

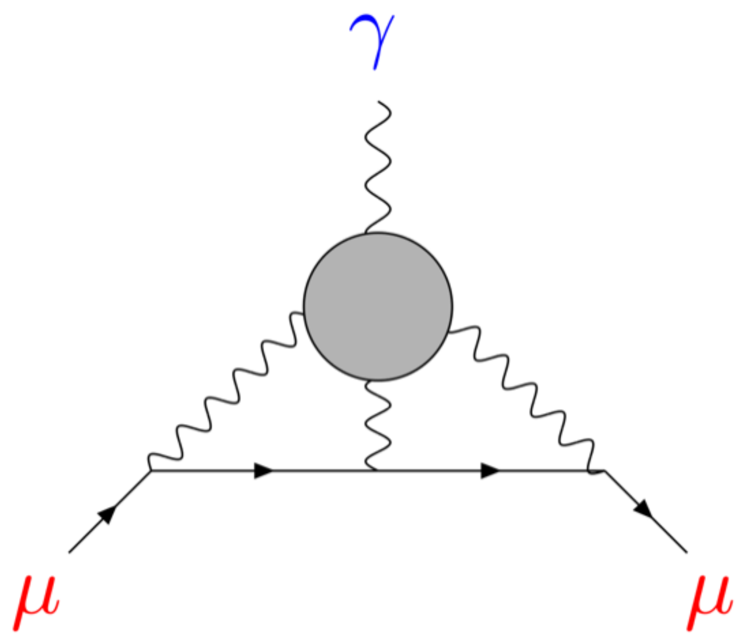
QED and electroweak contributions account for 99.994% of the SM prediction for a_μ

Error is dominated by strong interaction effects





Hadronic vacuum polarisation (HVP)

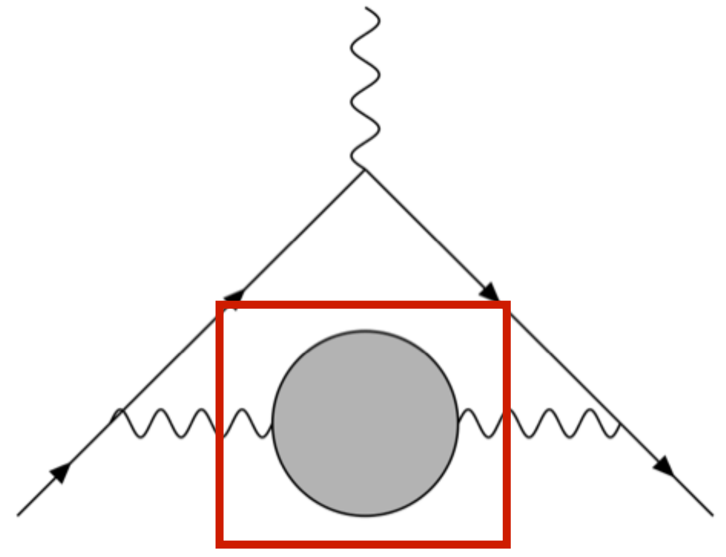


Hadronic light-by-light scattering (HLbL)

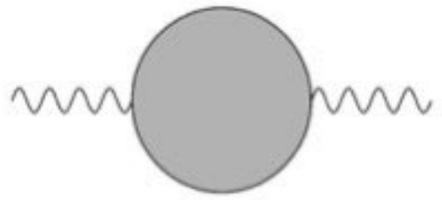
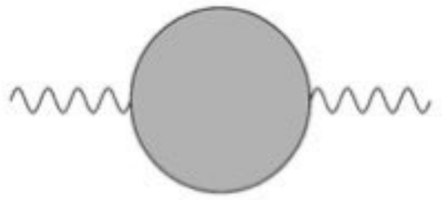
Two main approaches:

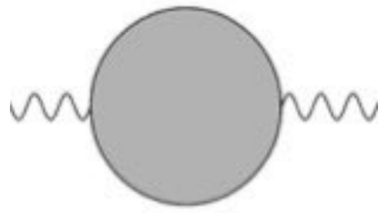
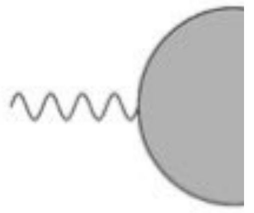
- Dispersion theory using experimentally determined cross sections ("data-driven")
- Lattice QCD calculations ("ab initio")

Hadronic vacuum polarisation from dispersion theory



Analyticity, unitarity & optical theorem imply:

 $= \int \frac{ds}{\pi(s-q^2)}\,\mathrm{Im}$ 

$2\,\mathrm{Im}$  $= \sum_{\rm had} \int d\Phi \,\Big|$  $\Big|^2$

$\propto \sigma(e^+e^- \to \mathrm{hadrons})$

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}, \qquad R_{\rm had}(s) = \frac{3s}{4\pi\,\alpha(s)}\,\sigma(e^+e^- \to \mathrm{hadrons})$$

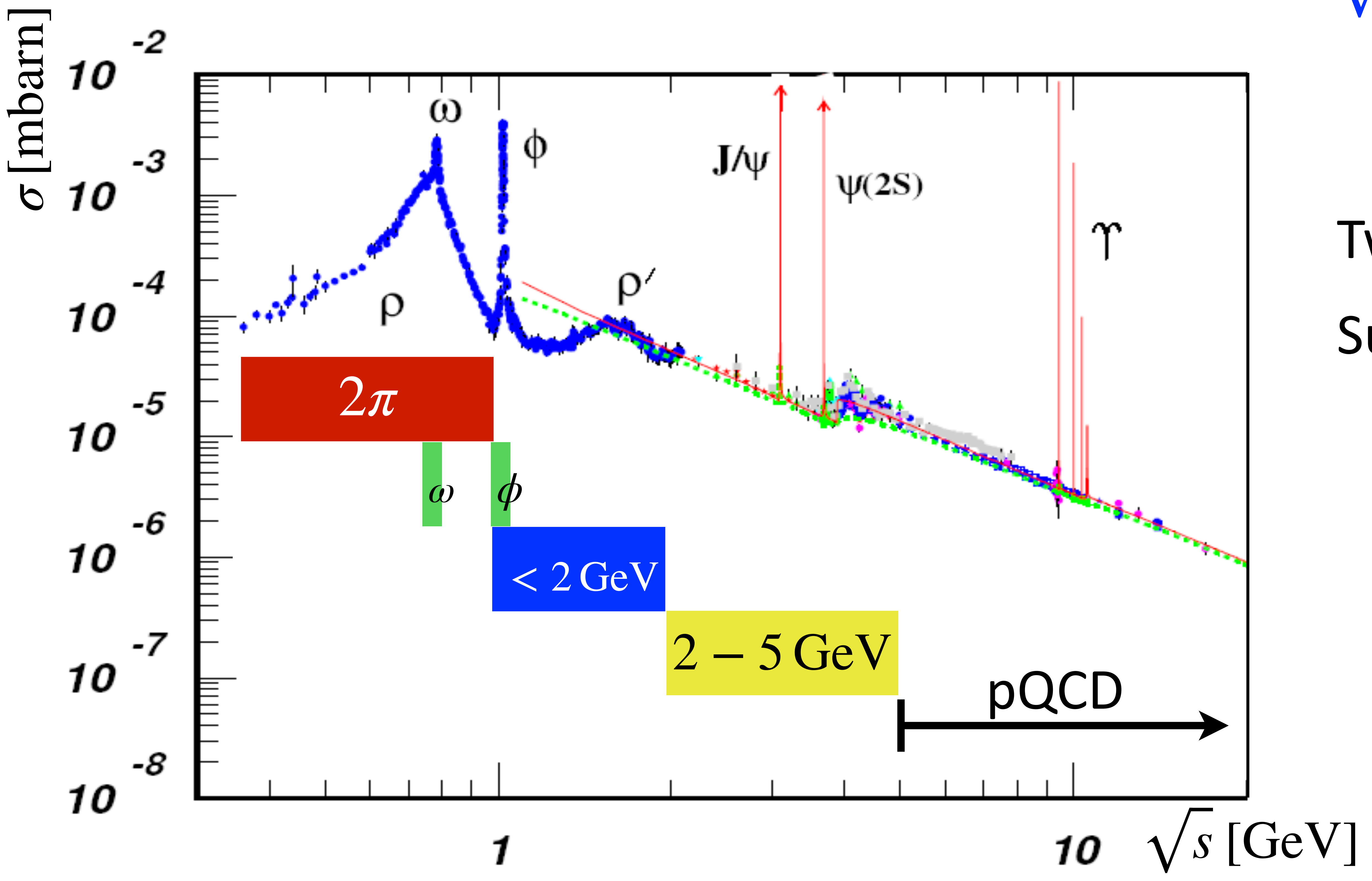
“R-ratio”

Hadronic effects cannot be treated in perturbation theory

- Use experimental data for $R_{\rm had}(s)$ in the low-energy regime (“data-driven approach”)
- Standard Model prediction is subject to experimental uncertainties

Data-driven approach: Hadronic cross sections

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$



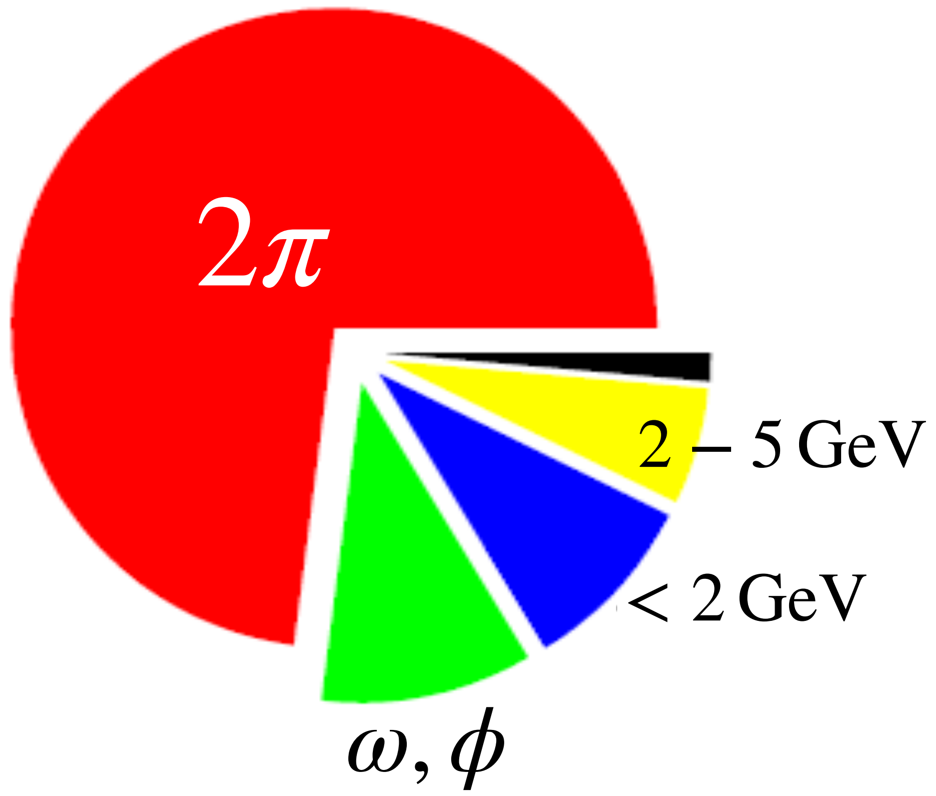
Decade-long effort to measure e^+e^- cross sections

$\sqrt{s} \lesssim 2\,\mathrm{GeV}$: sum of exclusive channels

$\sqrt{s} > 2\,\mathrm{GeV}$: inclusive channels, narrow resonances, perturbative QCD

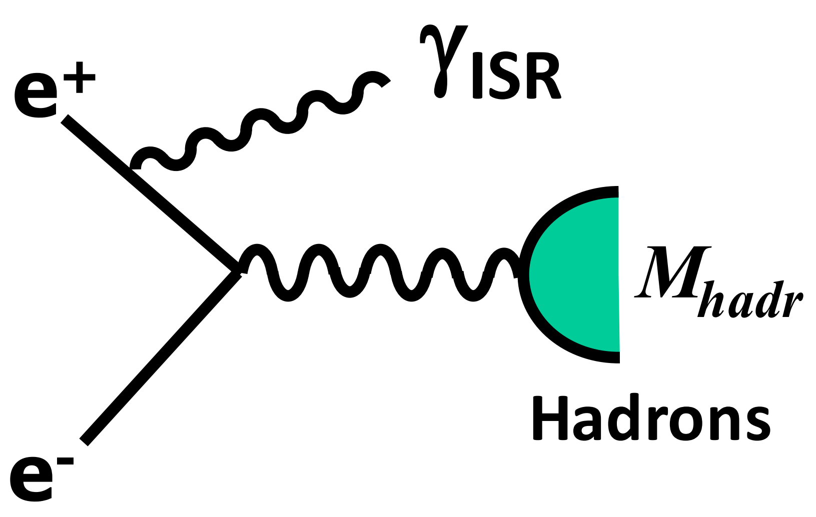
Two-pion channel accounts for $\approx 70\,\%$ of LO-HVP

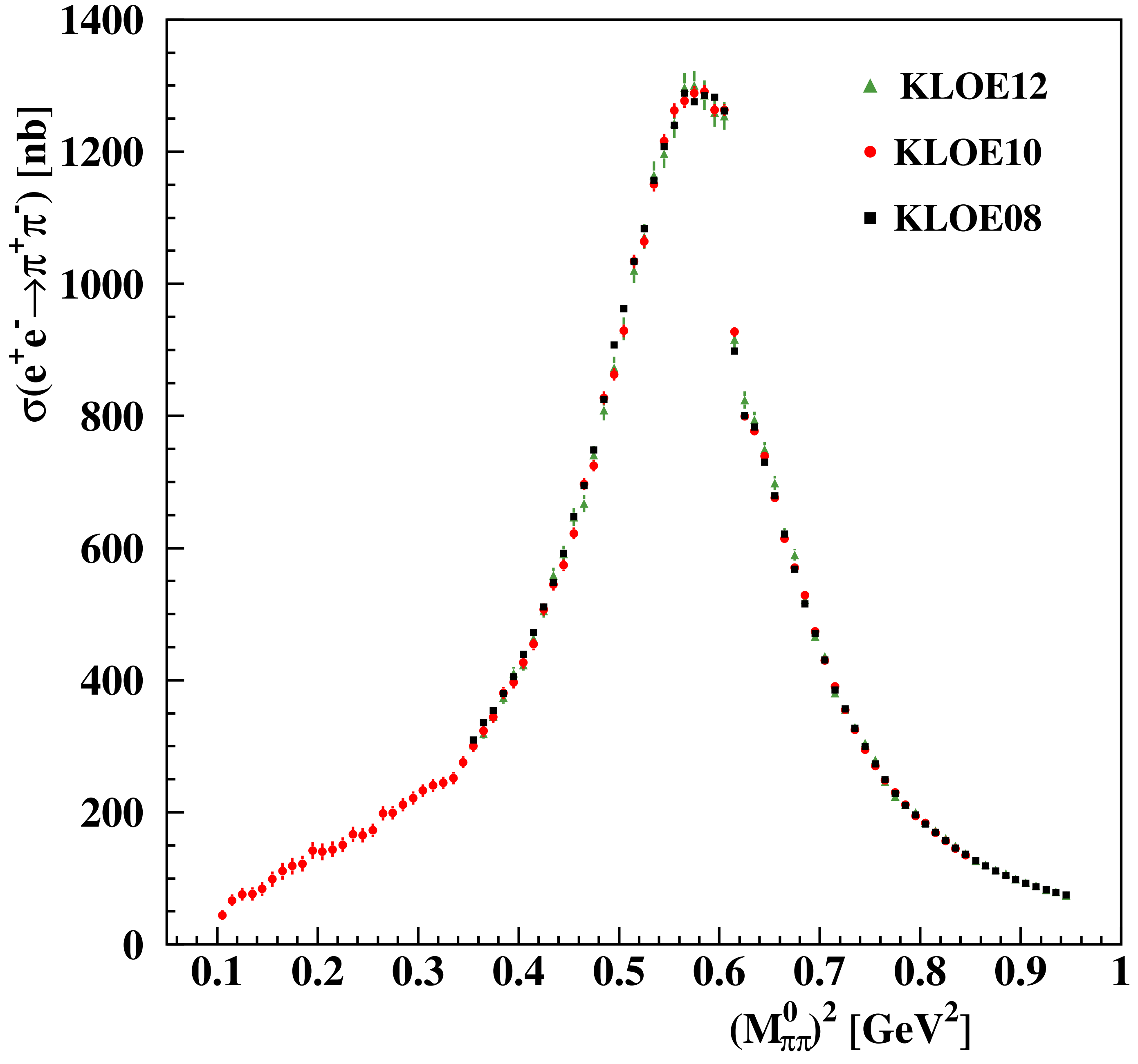
Subleading channels: ω, ϕ decays, final states with 3 pions, 2 kaons, 4 pions,...

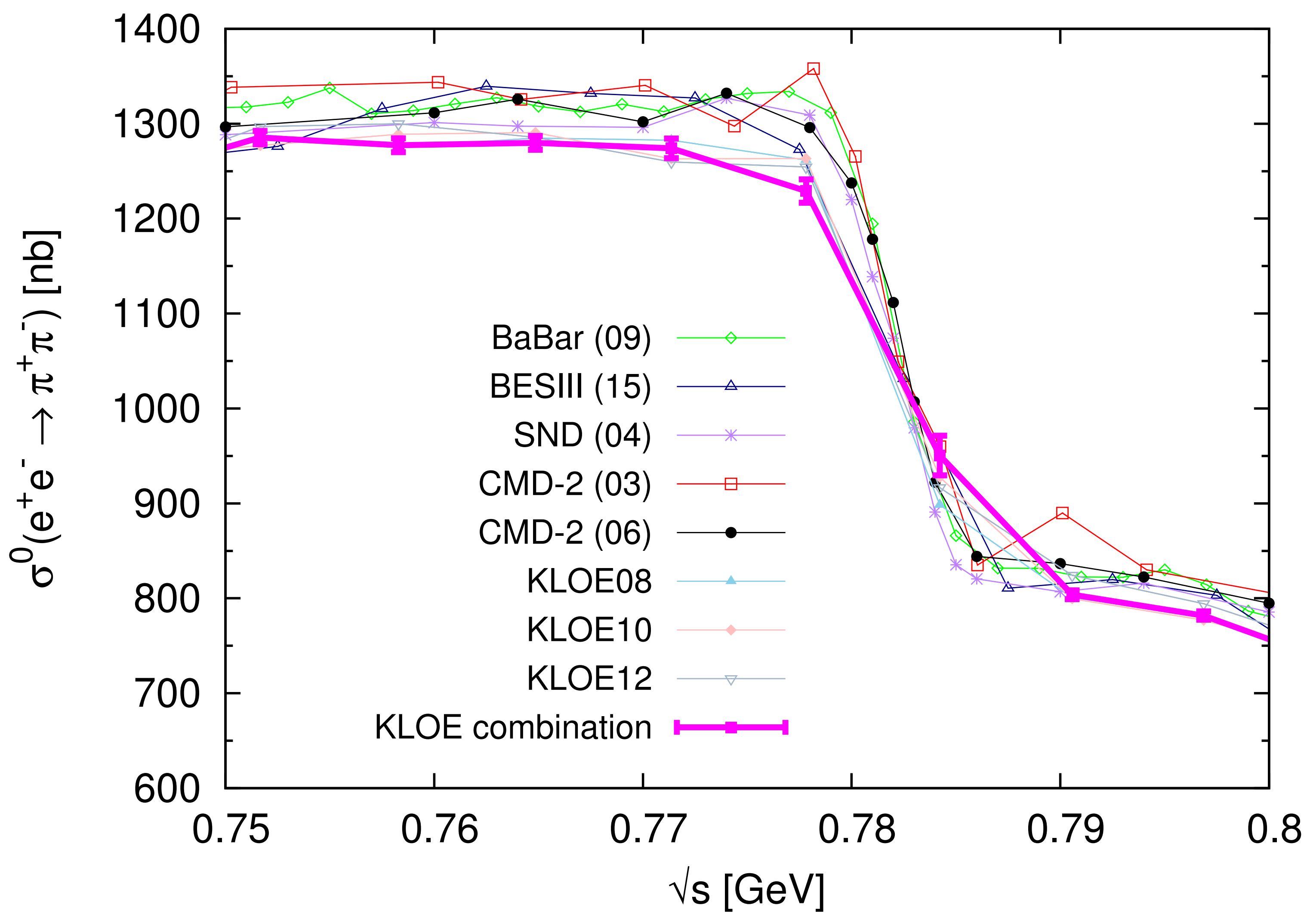


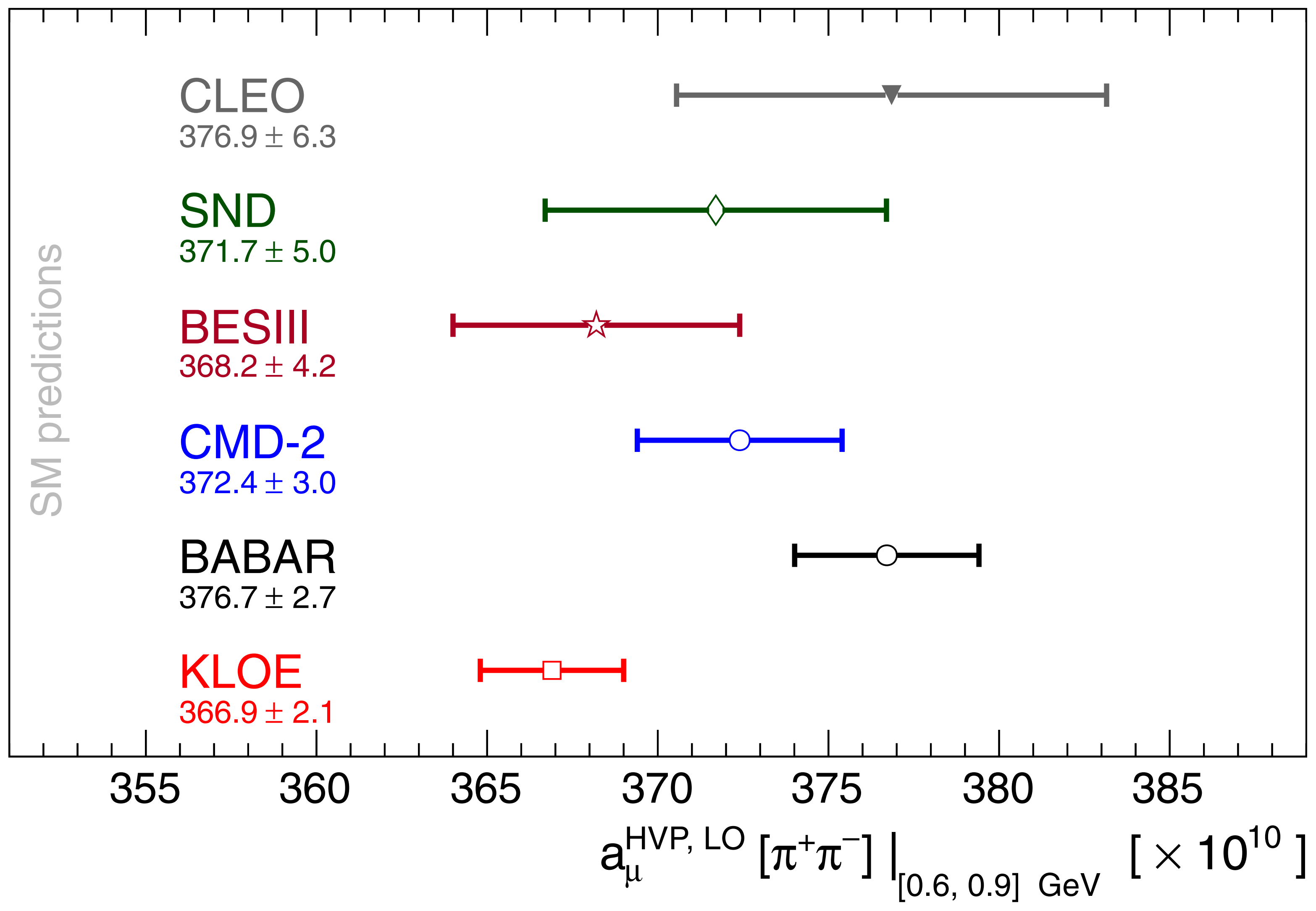
Two-pion channel

Initial State Radiation technique: energy scan at fixed collider energy (BaBar, KLOE, BESIII)









- Tension in the ISR data for $e^+e^- \to \pi^+\pi^-$ between BaBar and KLOE
- Extended analysis of BaBar data in progress
- New data: SND-3 (published) and CMD-3 (expected)
- Future prospects at BESIII, Belle II

Evaluation of the dispersion integral

Many different groups and analyses (DHMZ, KNT, FJ, CHHKS, BHLS,…)

Disagreement for some exclusive channels

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\mathrm{HVP,\ LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\mathrm{DV+QCD}}$	692.8(2.4)	1.2

Merging procedure: average of individual results + theoretical constraints + conservative error estimate (reflecting tensions in the data, differences in procedures)

$$a_\mu^{\mathrm{hvp,LO}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{syst}}(0.7)_{\mathrm{DV+QCD}} \times 10^{-10} = 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$

Hadronic vacuum polarisation from Lattice QCD

No reliance on experimental data (except for simple hadronic quantities, e.g. $m_{\rm nucl}, m_K, \ldots$)

Vacuum polarisation function depends smoothly on Euclidean momentum Q^2

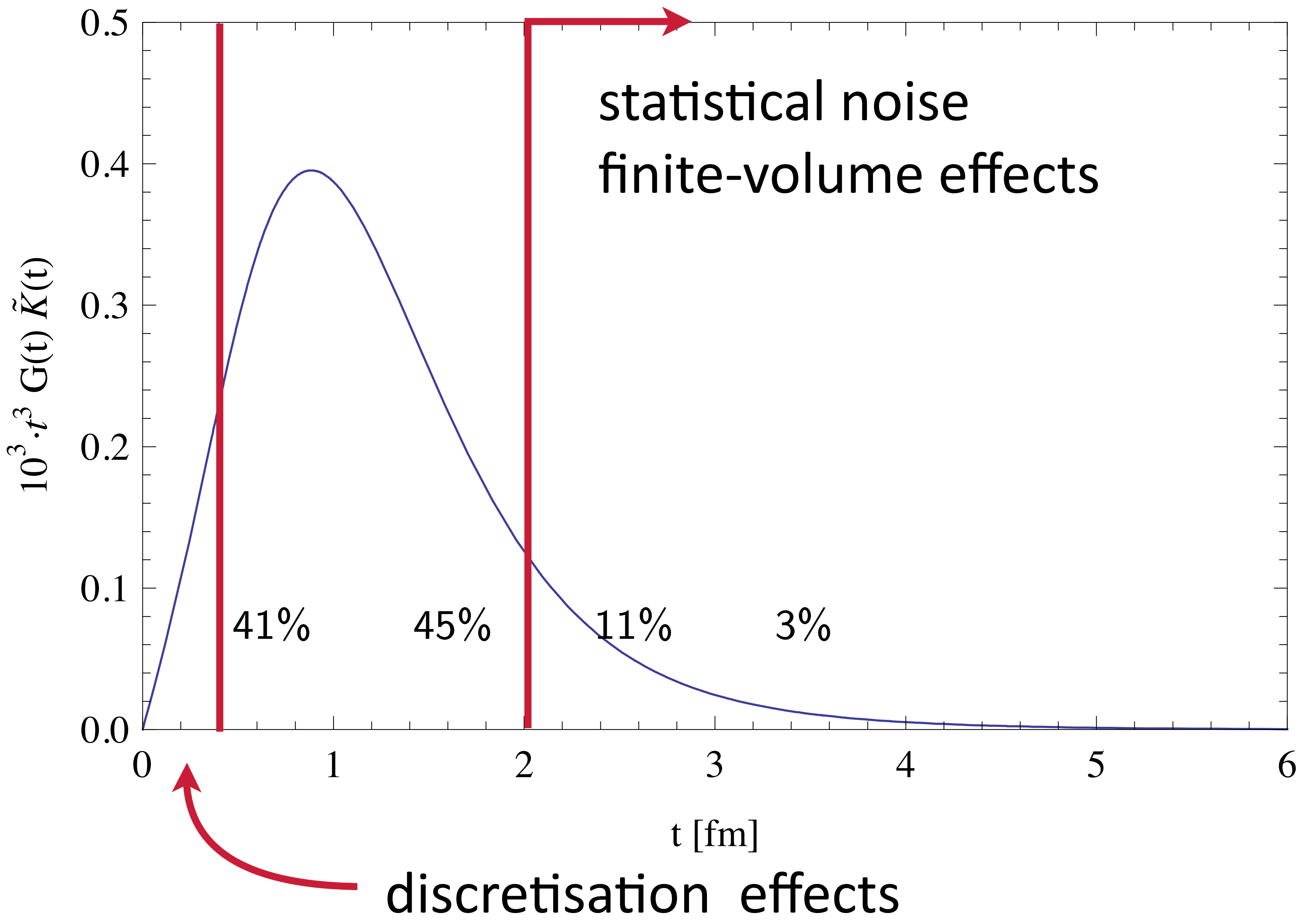
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, \tilde{K}(x_0)\, G(x_0), \qquad G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0)\rangle$$

[Bernecker & Meyer 2011]

$$J_\mu = \tfrac{2}{3}\bar{u}\gamma_k u - \tfrac{1}{3}\bar{d}\gamma_k d - \tfrac{1}{3}\bar{s}\gamma_k s + \tfrac{2}{3}\bar{c}\gamma_k c + \ldots$$

Challenges:

- Sub-percent statistical precision
- Finite-volume corrections
- Control over discretisation effects
- Quark-disconnected diagrams
- Isospin-breaking effects relevant

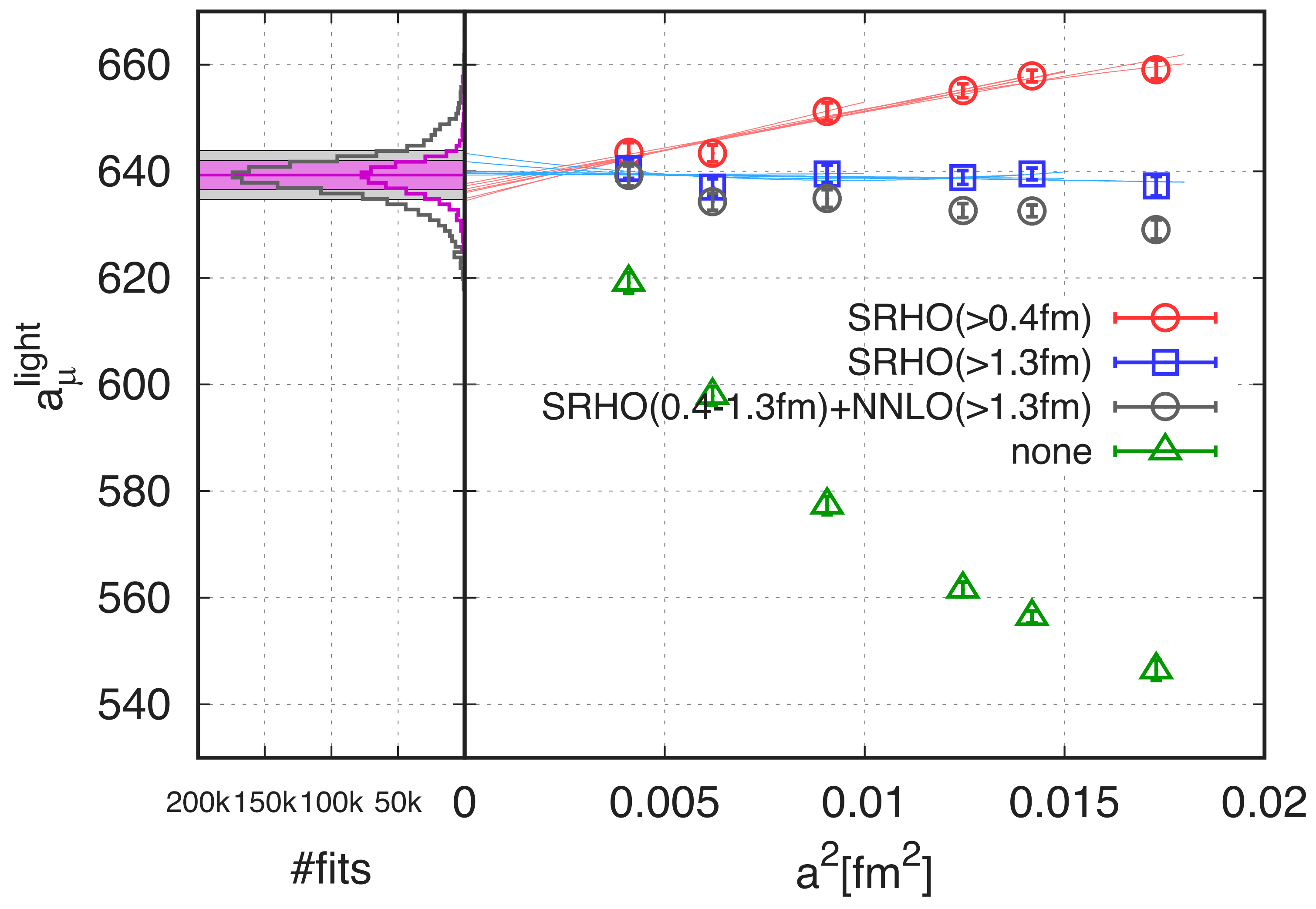


Hadronic vacuum polarisation from Lattice QCD

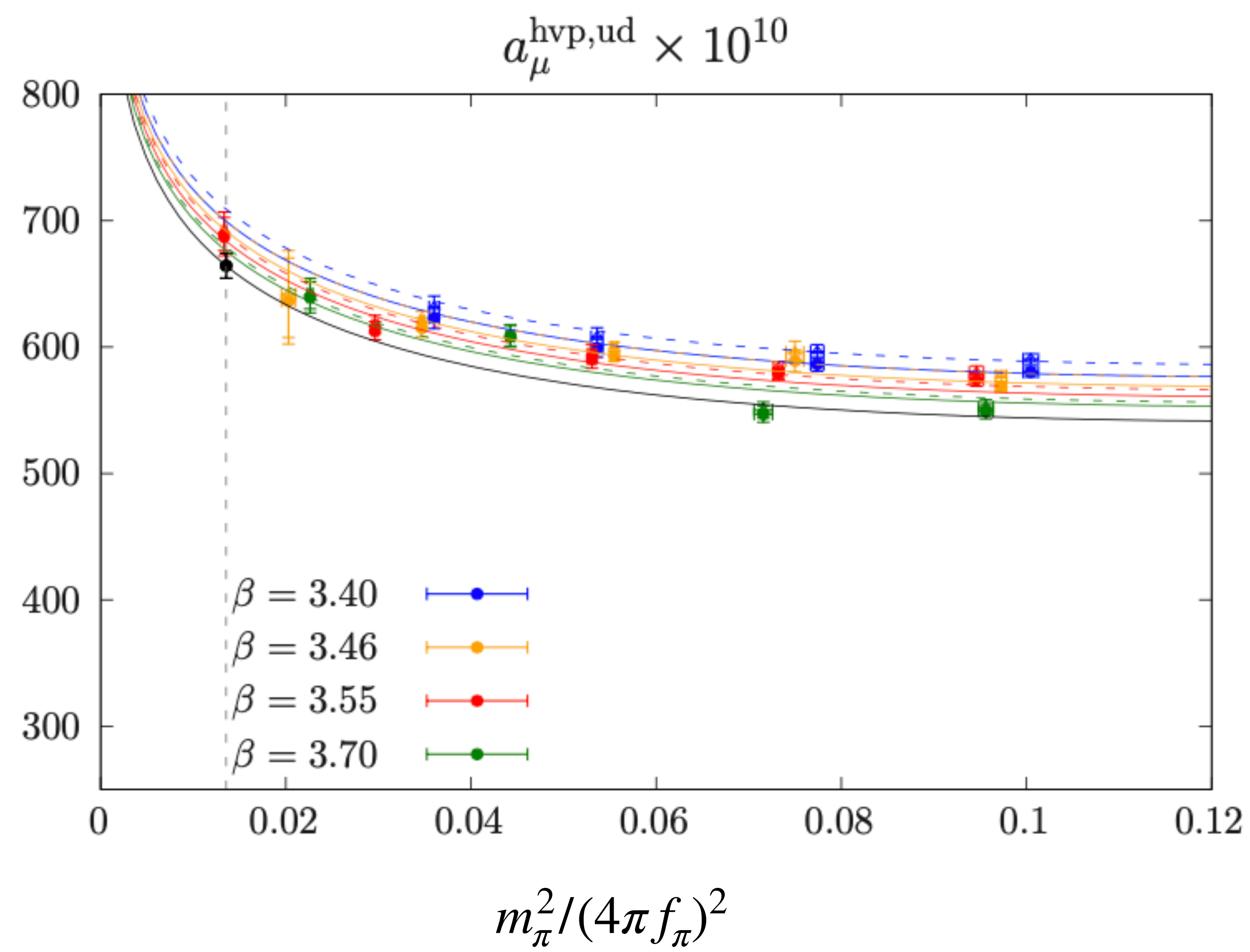
Range of discretisations of the QCD action probed by different groups

Finite-volume effects significant but well controlled

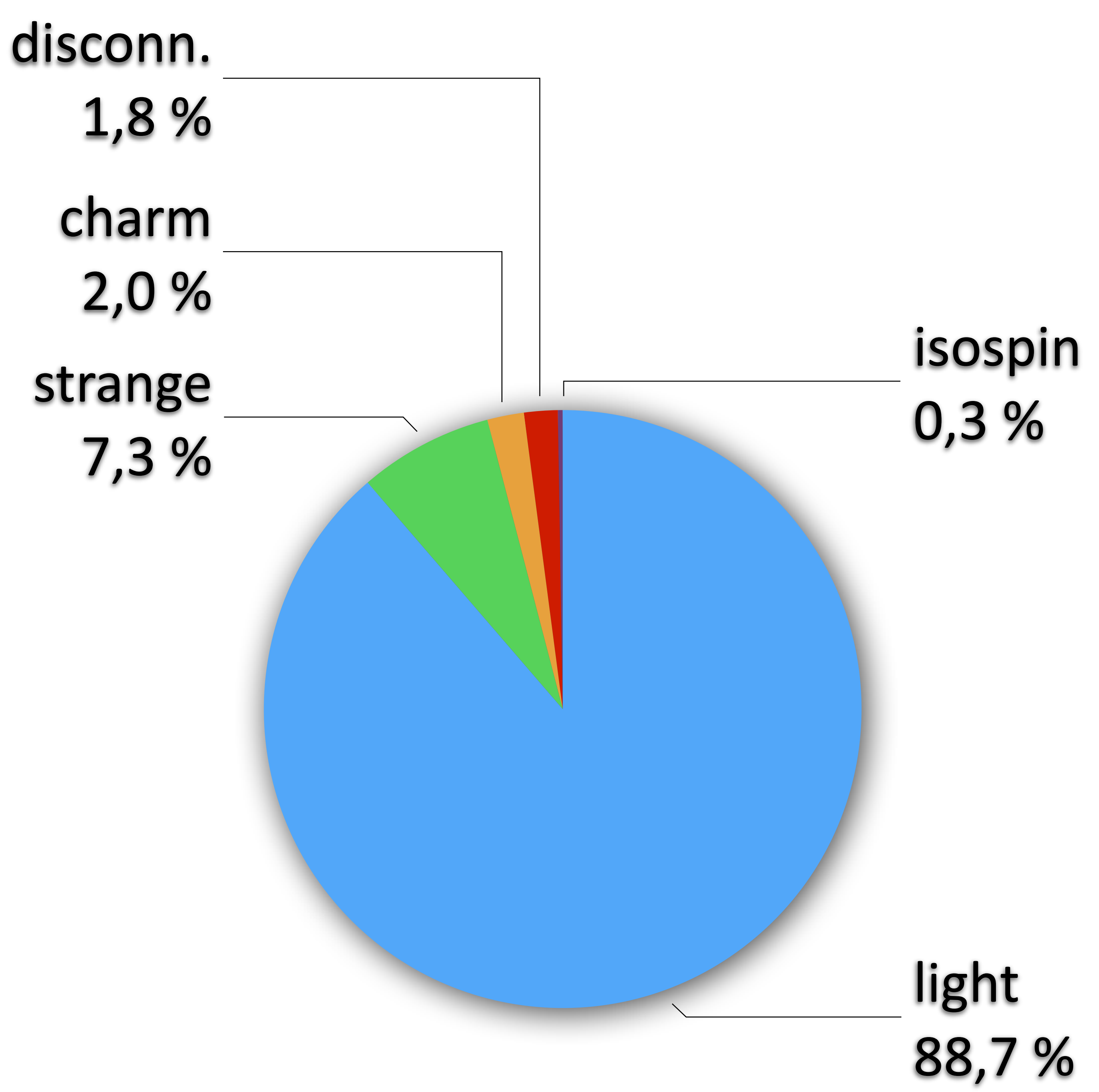
Extrapolation to the physical point



[BMW Collab. (Borsányi et al.), 2002.12347]

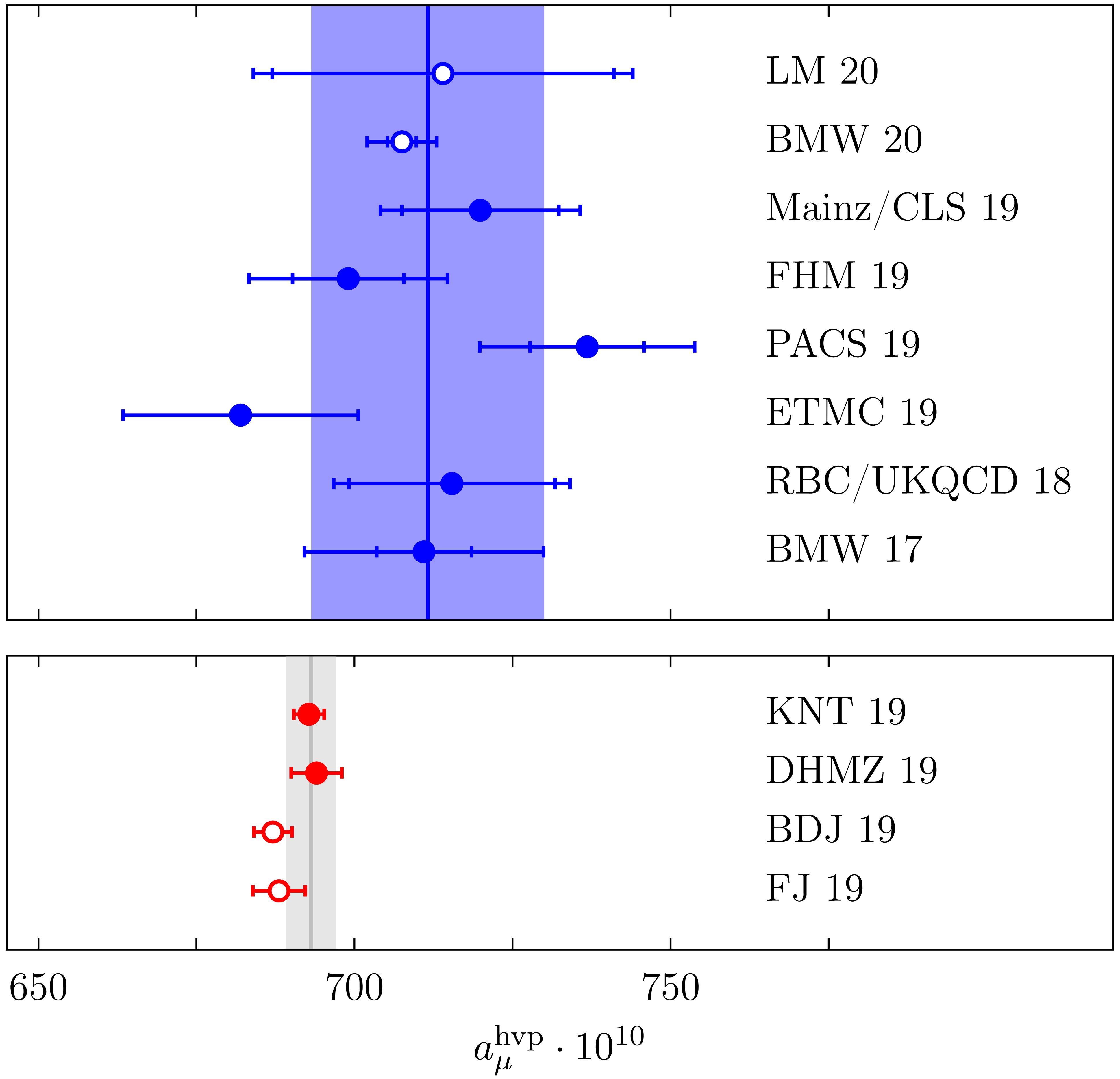


[Mainz/CLS (Gérardin et al.), 1904.03120]



Light-quark connected contribution dominates

Hadronic vacuum polarisation: R-ratio versus lattice QCD



White Paper, Muon $g-2$ Theory Initiative:

R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

Recent Lattice QCD result by BMW Collab.:

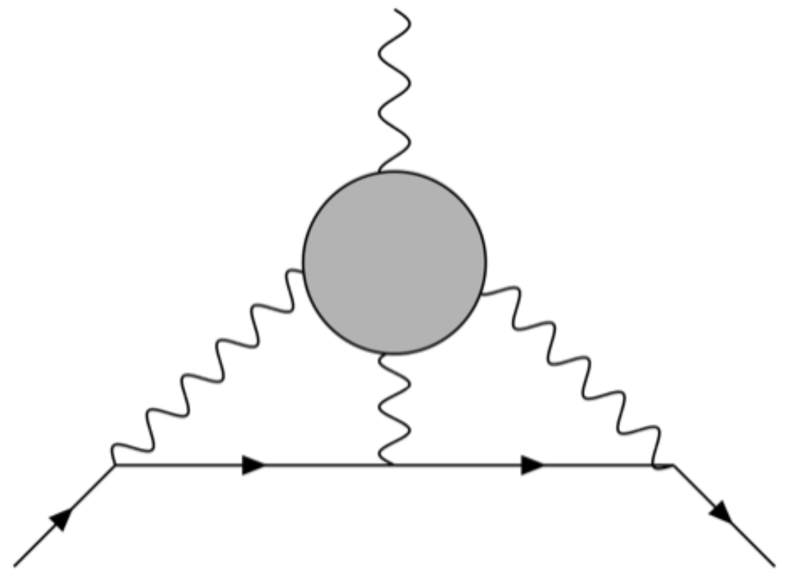
$$a_\mu^{\rm hvp,\,LO} = (707.5 \pm 2.3 \pm 5.0) \cdot 10^{-10} \quad [0.8\%]$$

[Borsányi et al., Nature 593 (2021) 7857, arXiv:2002.12347]

(2.1σ tension with R-ratio)

Requires independent confirmation

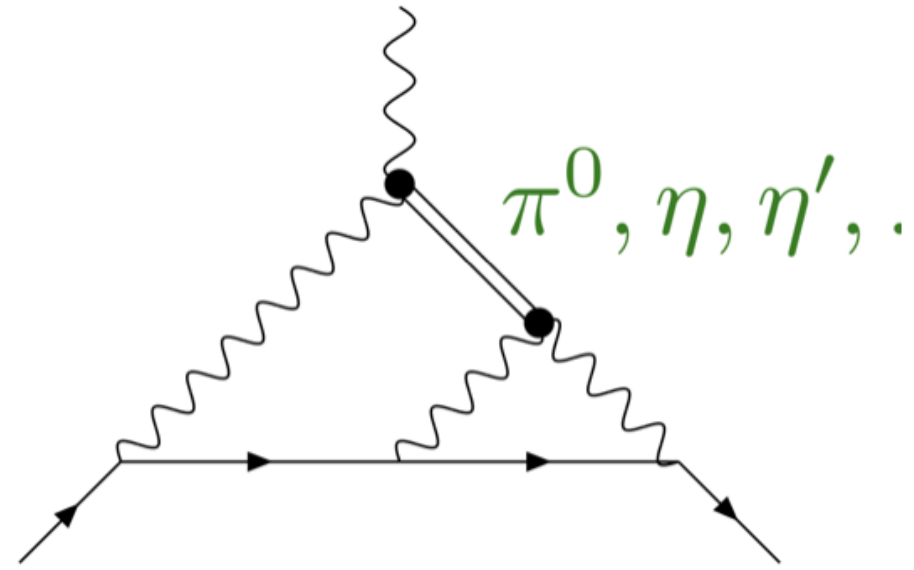
Hadronic light-by-light scattering



Dominant contribution from pseudoscalar meson exchange

→ transition form factor $\pi^0 \to \gamma^*\gamma^{(*)}$

Results for π^0-contribution:



$$(a_\mu^{\rm hlbl})_{\pi^0} = \begin{cases} (59.7 \pm 3.6) \cdot 10^{-11} & \text{Lattice QCD} \\ (62.6\,^{+3.0}_{-2.5}) \cdot 10^{-11} & \text{Disp. theory} \end{cases}$$

Lattice QCD *[Gérardin et al., 1903.09471]*

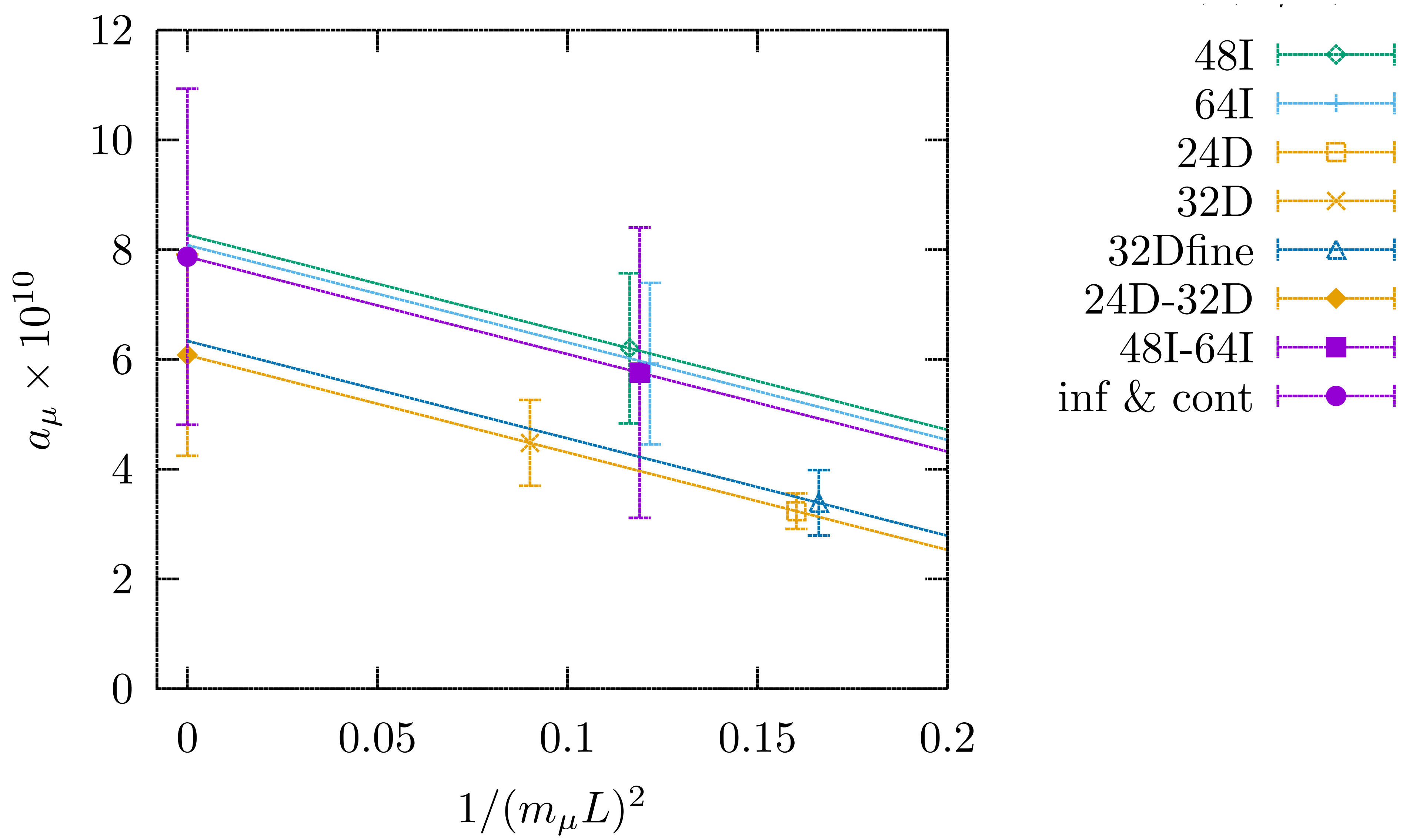
Disp. theory *[Hoferichter et al., 1808.04823]*

Direct lattice calculations:

$$a_\mu^{\rm hlbl} = \begin{cases} (\;\;78.7 \pm 35.4) \cdot 10^{-11} \\ (106.8 \pm 14.7) \cdot 10^{-11} \end{cases}$$

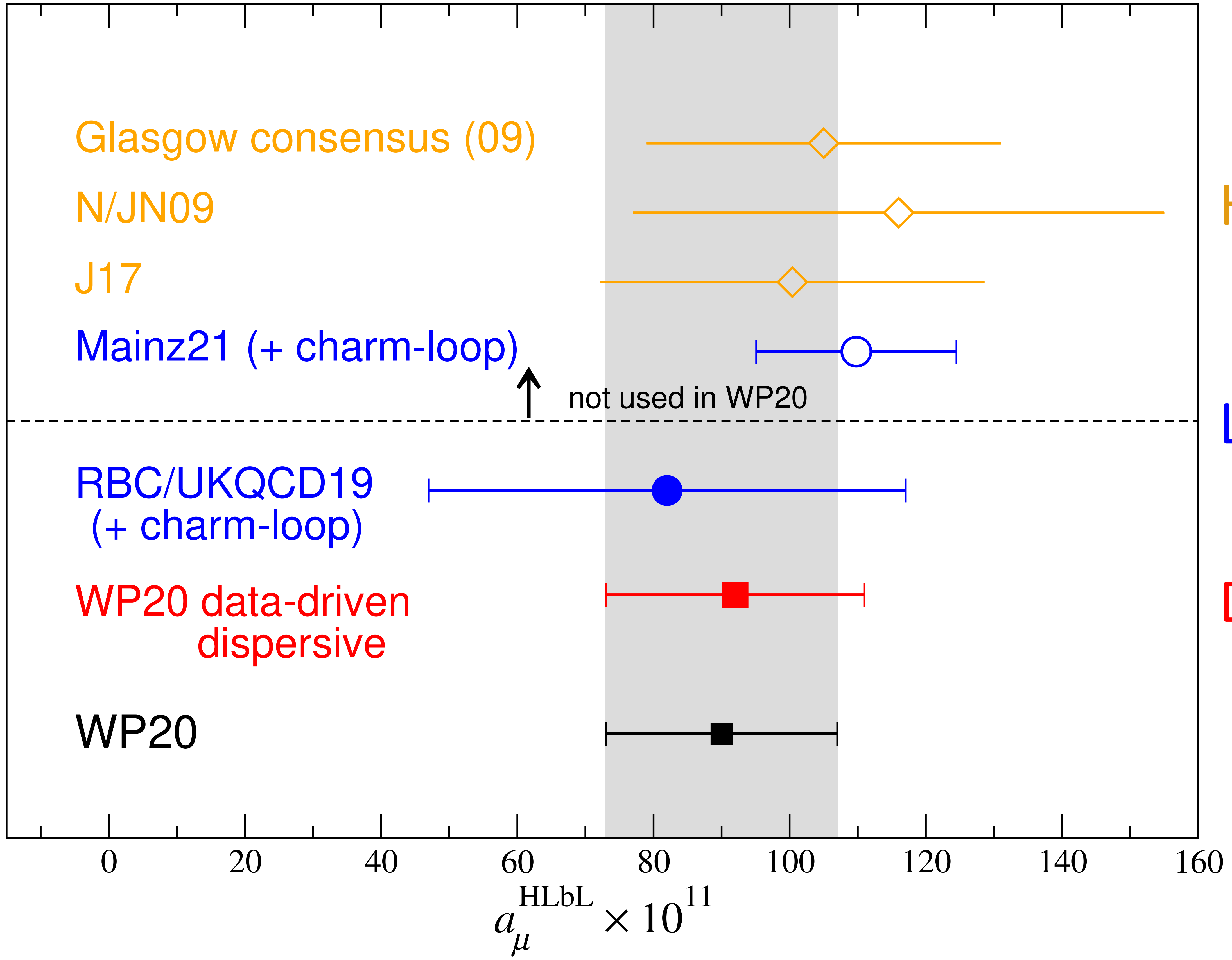
[Blum et al., 1911.08123]

[Chao et al., 2104.02632]



Hadronic light-by-light scattering

Current status



Hadronic models + pQCD

Lattice QCD (+ QED)

Data-driven

WP 20: $a_\mu^{\rm hlbl} = (92 \pm 18) \times 10^{-11}$

Synthesis

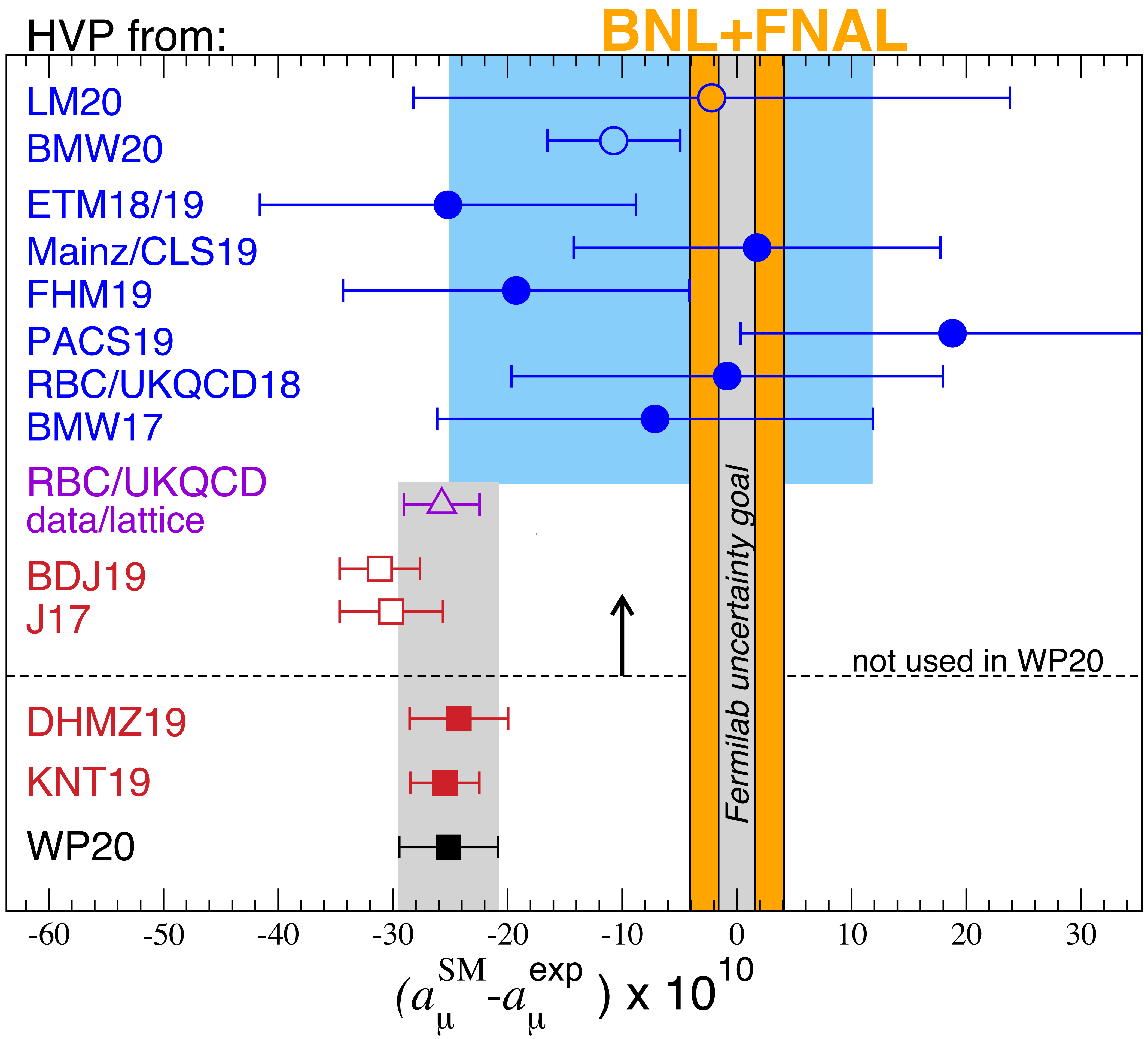
Contributions to the muon $g-2$ from electromagnetism, weak and strong interactions:

QED:	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm	
Weak:	$153.6(1.0) \times 10^{-11}$	0.01 ppm	
Hadronic vacuum polarisation:	$6845(40) \times 10^{-11}$	0.34 ppm	[0.6%]
Hadronic light-by-light scattering:	$92(18) \times 10^{-11}$	0.15 ppm	[20%]

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} = 116\,591\,810(43) \times 10^{-11} \qquad 0.37\ \text{ppm}$$

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

Standard Model prediction versus experiment



SM prediction:

$$a_\mu^{\rm SM} = 116\,591\,810(43) \times 10^{-11}$$

FNAL E989 (2021):

$$a_\mu^{\rm E989} = 116\,592\,040(54) \times 10^{-11}$$

Combined with BNL E821 (2004):

$$a_\mu^{\rm exp} = 116\,592\,061(41) \times 10^{-11}$$

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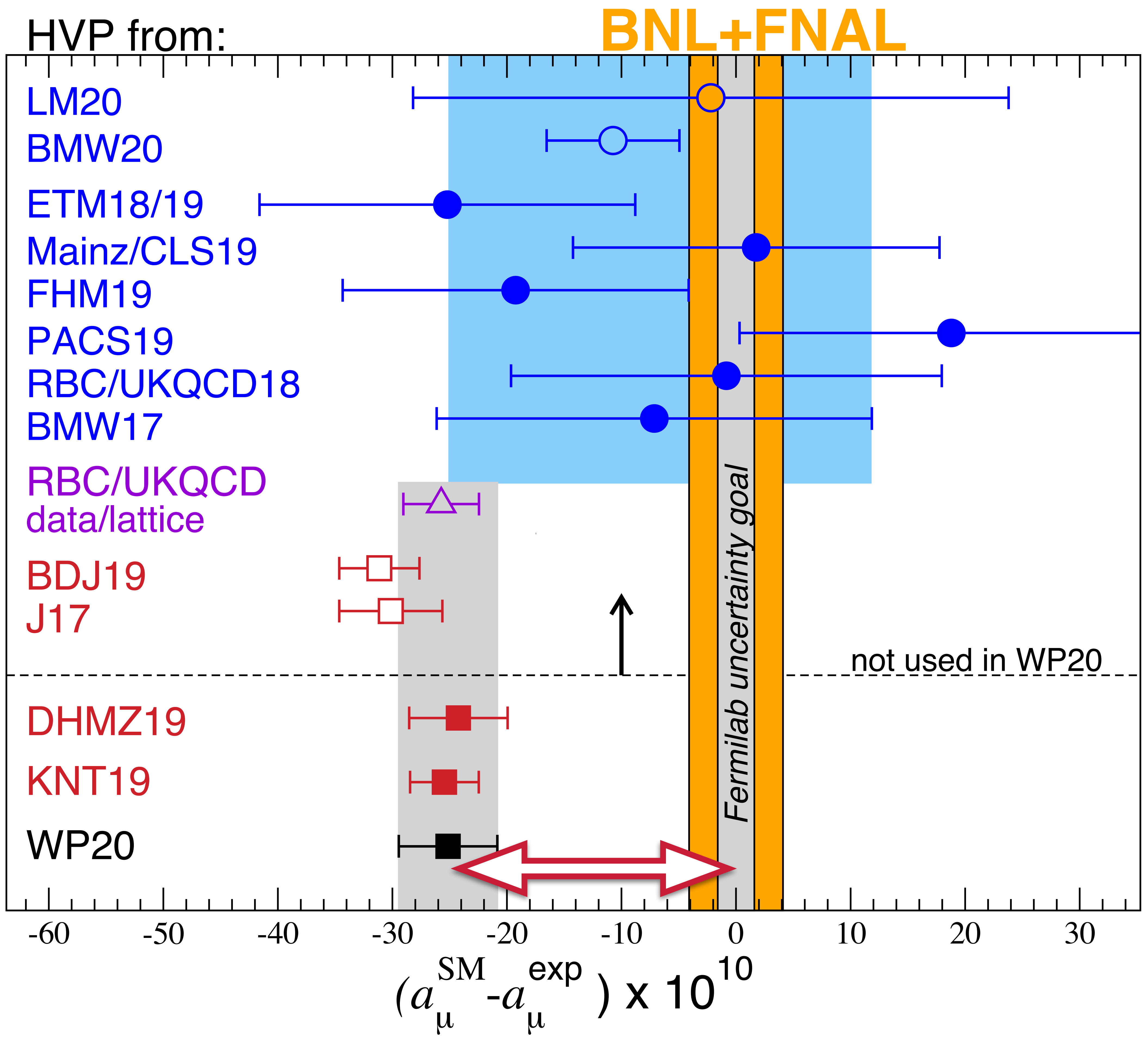

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$$\Rightarrow \quad a_\mu^{\rm SM} - a_\mu^{\rm exp} = 251(59) \times 10^{-10} \quad (4.2\,\sigma)$$

Discussion

Correlation between a_μ and the hadronic running of α :

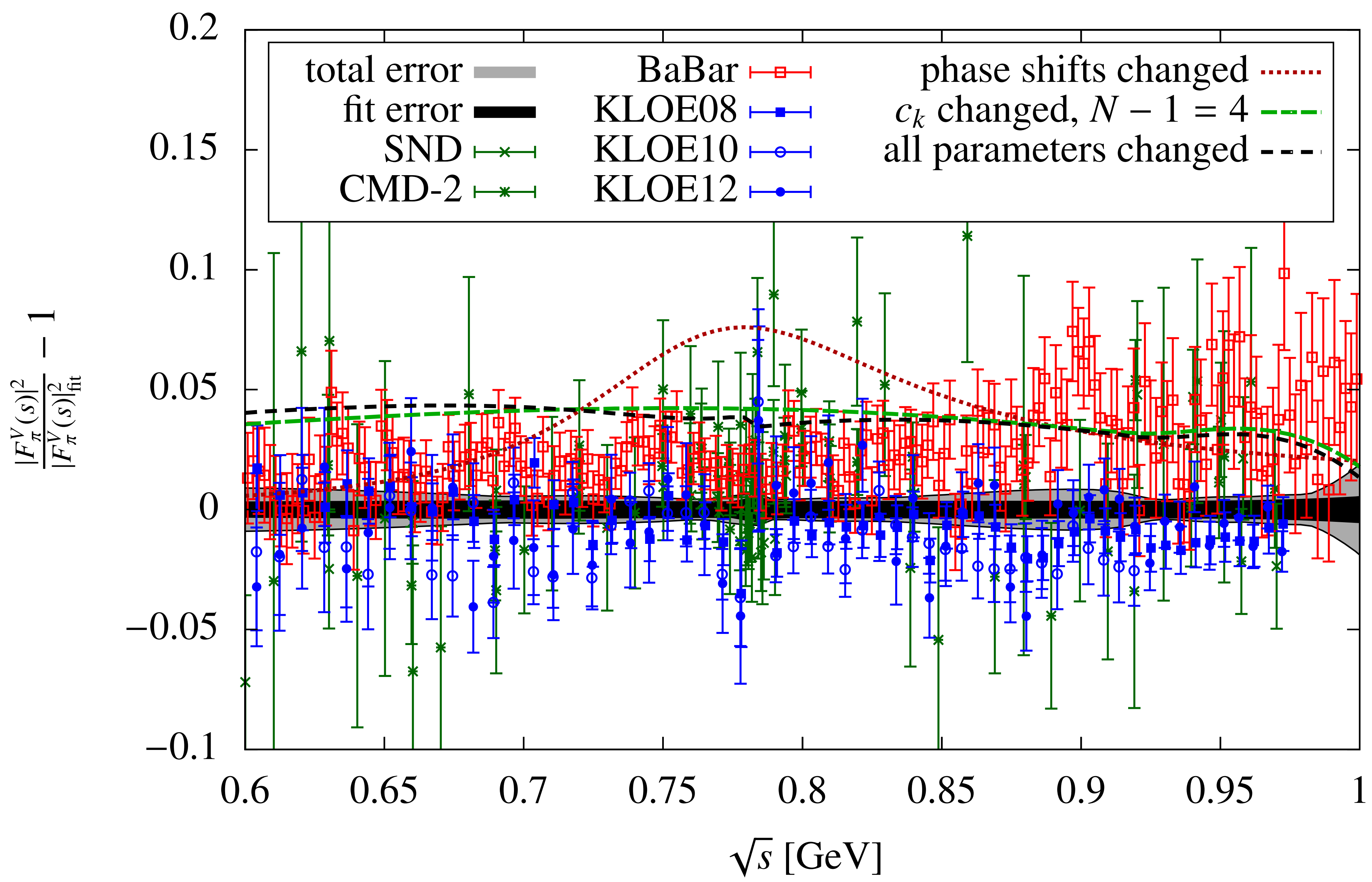
$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \frac{\alpha\, M_Z^2}{3\pi}\, \mathrm{P}\int_{m^2_{\pi^0}}^{\infty} ds\, \frac{R_{\rm had}(s)}{s(M_Z^2 - s)}$$

→ Input quantity for global electroweak fit

Can the SM accommodate a higher value for a_μ without increasing the tension in the global EW fit?

- Large changes in $R_{\rm had}(s)$ must occur for $\sqrt{s} \lesssim 2\,\mathrm{GeV}$
- Resulting scenarios differ substantially from data

[Crivellin et al., 2020; Keshavarzi et al., 2020; Malaescu & Schott, 2020; Colangelo, Hoferichter, Stoffer 2020]



Summary & Outlook

Increased tension of $4.2\,\sigma$ between Standard Model prediction of a_μ and experiment

Bulk of SM uncertainty due to the strong interaction

Hadronic vacuum polarisation:

- Decade-long experience with data-driven approach: $0.6\,\%$ precision
- Recent lattice calculation with $0.8\,\%$ error requires independent confirmation

Hadronic light-by-light scattering:

- Data-driven approach with almost fully quantified errors
- Good agreement with recent lattice QCD calculations: $\sim 15\,\%$

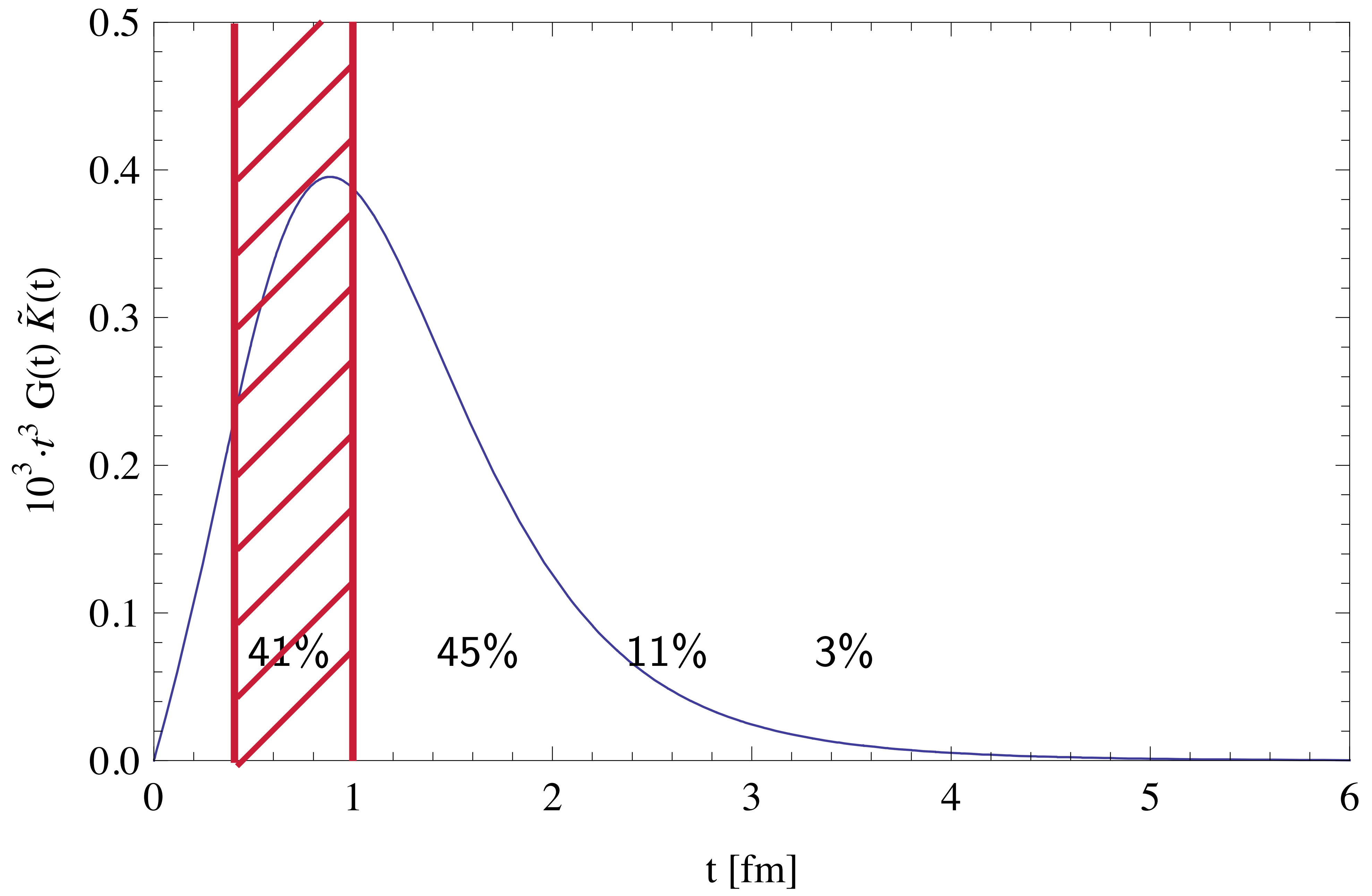
Future improvements:

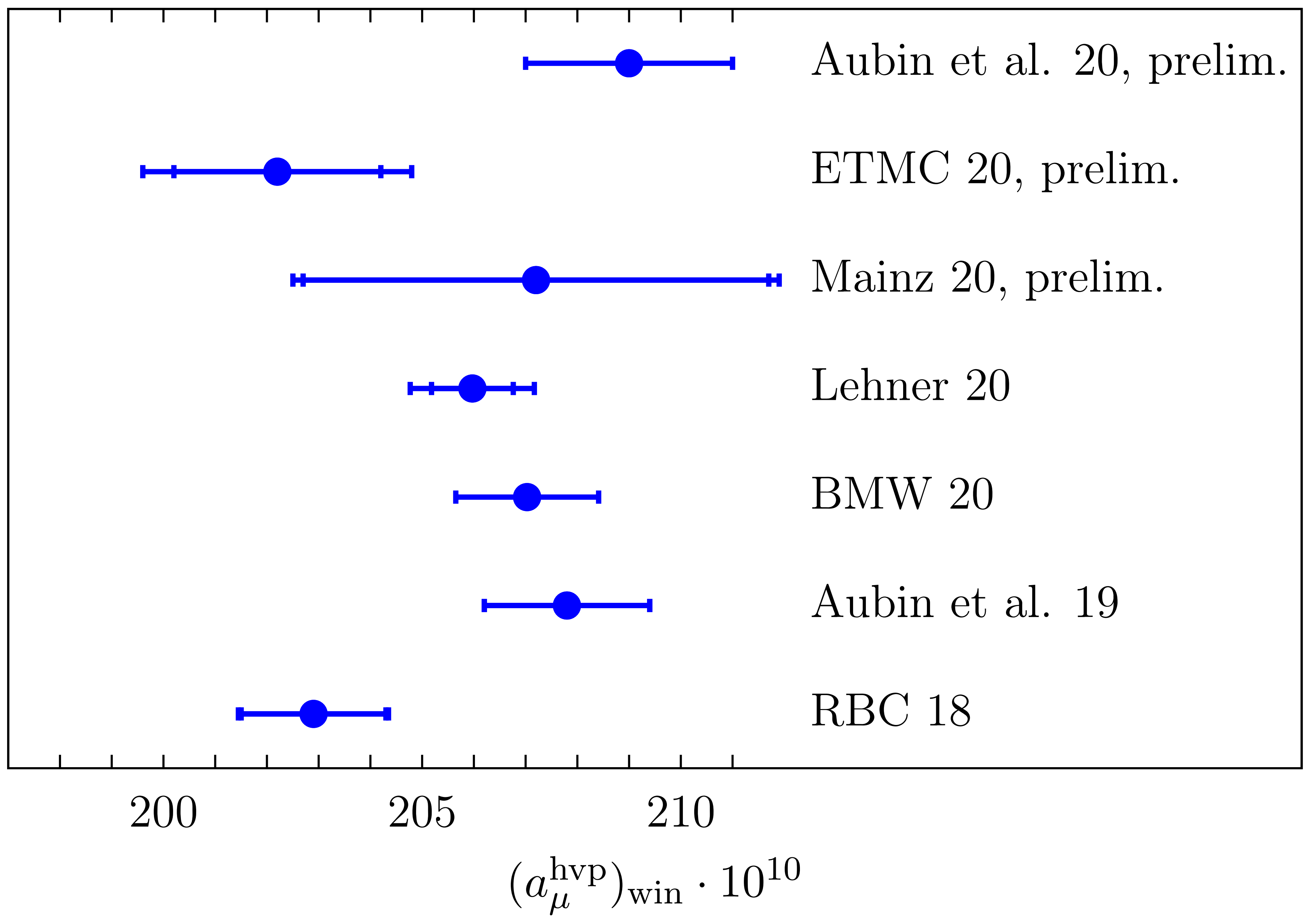
- Resolve / clarify the tension in the hadronic cross section data
- Check consistency of lattice calculations and dispersive approach

Backup

HVP in Lattice QCD: crosschecks

"Window" quantities: Designed to reduce / enhance sensitivity on certain systematics





Restrict convolution integral to sub-intervals sensitive to different systematic effects

Test consistency of different lattice discretisations

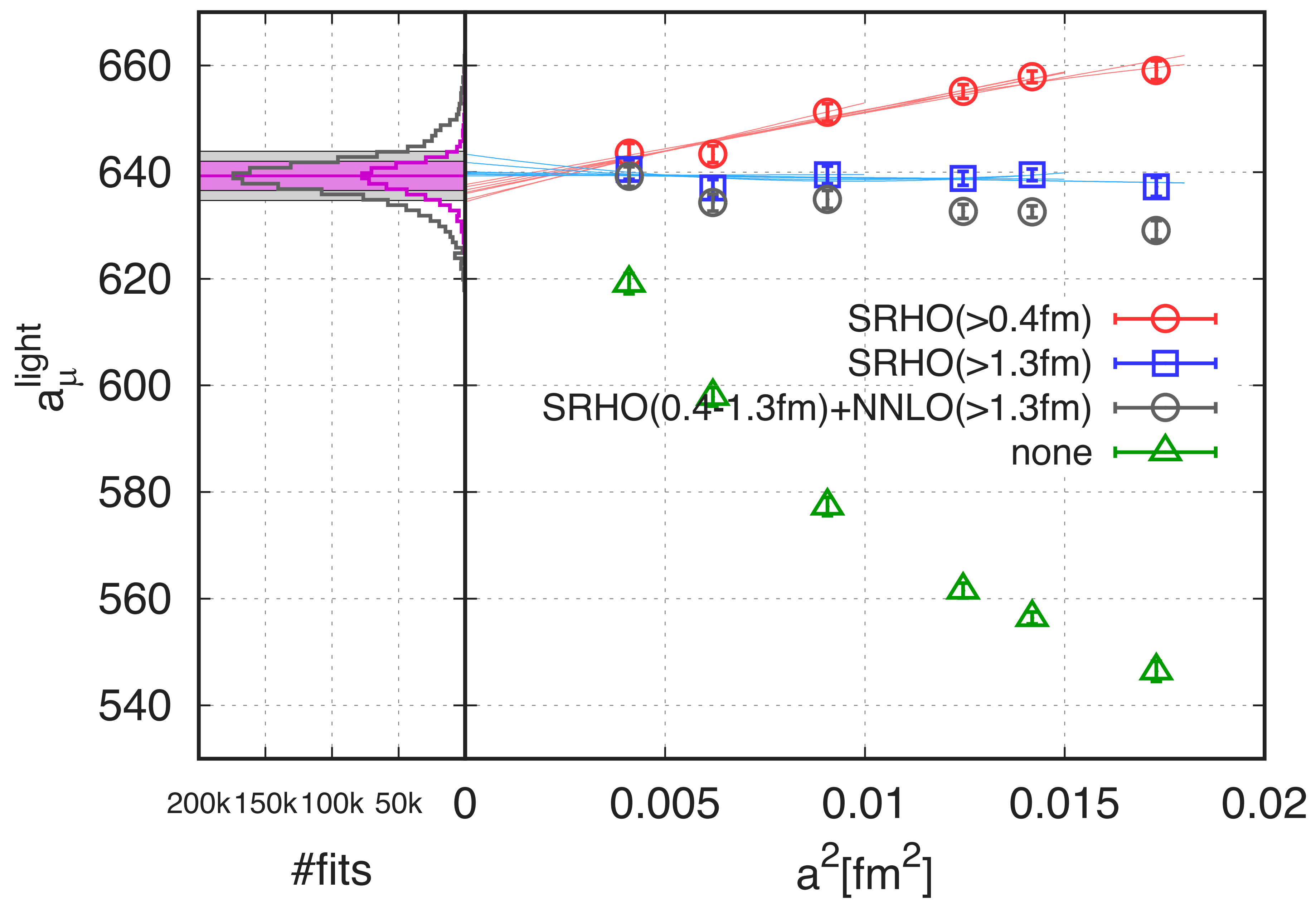
Comparison with corresponding result based on R-ratio

Calculation by BMW Collaboration

Staggered quarks: use EFT to correct for “taste-breaking” effects

Statistical analysis of different variants of continuum extrapolation yields systematic error

Full result



Window quantity