
Flavored Gauge-Mediated Supersymmetry Breaking Models with Discrete Non-Abelian Symmetry

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Based on: Everett, Garon 1610.09024
Everett, Garon, Rock 1812.10811
Everett, Garon, Rock 1912.12938
Eu, Everett, Leonard to appear

Flavored Gauge Mediation (FGM)

Motivation:

Minimal Gauge mediation : Higgs mass of ~ 125 GeV requires heavy stops/ maximal mixing

$$\Delta m_h^2 = \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left(\ln \frac{\tilde{m}_{t1}\tilde{m}_{t2}}{m_t^2} + \frac{X_t^2}{\tilde{m}_{t1}\tilde{m}_{t2}} \left(1 - \frac{X_t^2}{12\tilde{m}_{t1}\tilde{m}_{t2}} \right) \right)$$

$$X_t = A_t - \mu \cot \beta$$

A terms are zero, **stops must have masses** $> \mathcal{O}(10 \text{ TeV})$

One possible extension: Flavored gauge mediation

- Idea: $SU(2)_L$ doublet messengers mix with MSSM Higgs $H_{u,d}$
- New messenger Yukawa superpotential coupling terms eg. $Y_u Q\bar{u}H_u + Y'_u Q\bar{u}M_u$
- This Higgs-messenger mixing is governed by an imposed symmetry eg. $U(1)$ benchmark model by Ierushalmi et al. (2016)

FGM with discrete non-Abelian symmetry \mathcal{S}_3

- More constraining and thus more predictive
- \mathcal{S}_3 is often used in generation of fermion masses

Perez, Ramond, Zhang (2012)

Extend PRZ'12 work for 2-family scenario to 3 families:

- \mathcal{S}_3 : Higgs-messenger symmetry + part of family symmetry

	$\mathcal{H}_u^{(2)}$	$\mathcal{H}_u^{(1)}$	$\mathcal{H}_d^{(2)}$	$\mathcal{H}_d^{(1)}$	Q_2	Q_1	\bar{u}_2	\bar{u}_1	\bar{d}_2	\bar{d}_1	L_2	L_1	\bar{e}_2	\bar{e}_1	X_H
\mathcal{S}_3	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

- Extension of Higgs-messenger sector: μ and $B\mu$ can be tuned separately
- In the basis

$$Q = (Q_2, Q_1)^T = ((Q_2)_1, (Q_2)_2, Q_1)^T, \quad \bar{u} = (\bar{u}_2, \bar{u}_1)^T = ((\bar{u}_2)_1, (\bar{u}_2)_2, \bar{u}_1)^T$$

Superpotential (eg. up quarks)

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u} \mathcal{H}_u^{(1)} & \beta_{2u} \mathcal{H}_{u2}^{(2)} \\ \beta_{1u} \mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u} \mathcal{H}_{u1}^{(2)} \\ \beta_{3u} \mathcal{H}_{u2}^{(2)} & \beta_{3u} \mathcal{H}_{u1}^{(2)} & \beta_{4u} \mathcal{H}_u^{(1)} \end{pmatrix} \bar{u}$$

- Advantage: Possible sizable stop mixing

FGM with \mathcal{S}_3

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u} \mathcal{H}_u^{(1)} & \beta_{2u} \mathcal{H}_{u2}^{(2)} \\ \beta_{1u} \mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u} \mathcal{H}_{u1}^{(2)} \\ \beta_{3u} \mathcal{H}_{u2}^{(2)} & \beta_{3u} \mathcal{H}_{u1}^{(2)} & \beta_{4u} \mathcal{H}_u^{(1)} \end{pmatrix} \bar{u} \quad Y_u = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & \beta_{1u} & \beta_{2u} \\ \beta_{1u} & 1 & \beta_{2u} \\ \beta_{3u} & \beta_{3u} & \beta_{4u} \end{pmatrix}$$

- Our goal: Achieve realistic quark mass hierarchy at leading order



Need extra structures—relations among β_{iu}



Classification: Different paths to hierarchy

Case 1:

- Singlet-dominated limit

$$\beta_{1u} = 1, \quad \beta_{2u} \beta_{3u} = \beta_{4u}$$

- Democratic limit

$$\text{All } \beta_{iu} = 1$$

Case 2:

- Doublet-dominated limit

$$|\beta_{1u}| \gg \beta_{2u,3u} \gg \beta_{4u} = 0$$

Two different orderings:

$$\beta_{3u} > \beta_{2u}$$

$$\beta_{2u} > \beta_{3u}$$

Case 1: Democratic limit

- All coefficients are equal: $\beta_{1i} = \beta_{2i} = \beta_{3i} = \beta_{4i} = 1$

- MSSM Yukawa matrix:

$$Y_i = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Everett, Garon (2018)

- At leading order: 2 vanishing eigenvalues + an $\mathcal{O}(1)$ eigenvalue (3rd gen.)
- Flavor democratic mass matrix with $\mathcal{S}_{3L} \times \mathcal{S}_{3R}$ symmetry

Eu, Everett, Leonard (2021)

Generate non-zero 1st and 2nd gen. fermion masses:

$$Y_i^{(\text{corr})} = \frac{\tilde{y}_i \epsilon_i}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\tilde{y}_i \sigma_i}{\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

Xing (1996)

Fritzsch, Xing (2000)

$$\mathcal{S}_{3L} \times \mathcal{S}_{3R} \Rightarrow \mathcal{S}_{2L} \times \mathcal{S}_{2R} \Rightarrow \mathcal{S}_{1L} \times \mathcal{S}_{1R}$$

3 non-vanishing eigenvalues

These terms can be generated via renormalizable & non-renormalizable superpotential couplings

Estimation of relative strength of $\epsilon_{u,d,e}$ and $\sigma_{u,d,e}$

- Diagonalizing MSSM Yukawa using biunitary diagonalization:

$$(Y_u)^2 = \begin{pmatrix} y_u^2 & 0 & 0 \\ 0 & y_c^2 & 0 \\ 0 & 0 & y_t^2 \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 & 0 \\ 0 & -\frac{2\epsilon_u}{3\sqrt{3}} - \frac{8\epsilon_u^2}{27\sqrt{3}} + \frac{56\epsilon_u^3}{243\sqrt{3}} - \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 \\ 0 & 0 & \sqrt{3} + \frac{5\epsilon_u}{3\sqrt{3}} + \frac{8\epsilon_u^2}{27\sqrt{3}} - \frac{56\epsilon_u^3}{243\sqrt{3}} \end{pmatrix}$$

Keep to appropriate subleading orders

- Quark masses: Yukawa couplings multiplied by the appropriate Higgs VEV

$$m_t = \frac{y_t v_u}{\sqrt{2}} = \frac{y_t v \sin \beta}{\sqrt{2}} \quad m_b = \frac{y_b v_d}{\sqrt{2}} = \frac{y_b v \cos \beta}{\sqrt{2}}$$

where $v_u^2 + v_d^2 = v^2 = (246 \text{ GeV})^2$, $\tan \beta = \frac{v_u}{v_d}$

- Use the known fermion masses to find the relative strength of the parameters and examine their effects on sparticle spectra

Estimation of CKM matrix elements

$$\epsilon_u \approx 3 \times 10^{-2}, \sigma_u \approx 10^{-3}, \epsilon_d \approx 0.1, \sigma_d \approx 9 \times 10^{-3}, \epsilon_e \approx 0.3, \sigma_e \approx 8 \times 10^{-3}$$

- Using the approximation of unitary matrices up to order $\epsilon^4 \sigma^2$

$$|U_{\text{CKM}}| \approx \begin{pmatrix} 0.99 & 0.17 & 0 \\ 0.17 & 0.99 & 0.02 \\ 0.01 & 0.02 & 1 \end{pmatrix}$$

- Reasonable estimate compared to experimental data:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{bmatrix}$$

PDG (2020)

Effects of perturbations

$$\sigma_{u,d,e} : \Delta m_{\tilde{d}_{1,2}} = 0.01 \rightarrow 1\text{GeV}$$

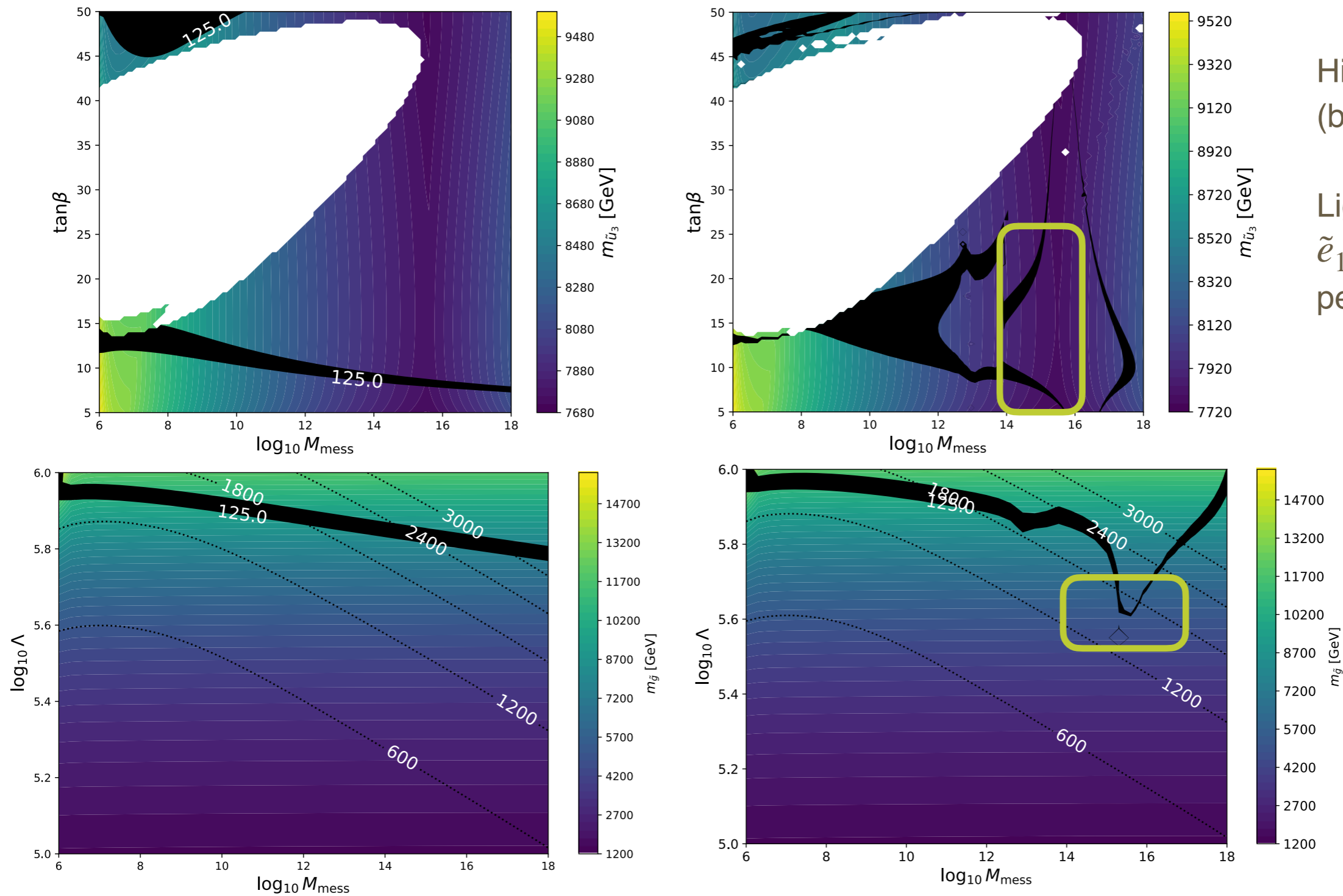
$$\sigma_{u,d,e} : \Delta m_{\tilde{u}_{4,5}} = 0.01 \rightarrow 1\text{GeV}$$

$$\epsilon_u : \Delta m_{\tilde{u}_{1,2}} = 0.02 \rightarrow 70\text{GeV}$$

$$\epsilon_u : \Delta m_{\tilde{d}_{1,2}} = 0.02 \rightarrow 25\text{GeV}$$

[GeV]	No perturbation	Nonzero $\sigma_{u,d,e}$	Nonzero $\sigma_{u,d,e}, \epsilon_u$	Nonzero $\sigma_{u,d,e}, \epsilon_{u,d,e}$
M_{mess}	10^{12}	10^{12}	10^{12}	10^{12}
Λ	7.7×10^5	7.7×10^5	7.7×10^5	7.7×10^5
$\tan \beta$	10	10	10	10
h^0	125.14	125.14	125.20	124.97
H^0	7175.6	7175.6	7176.6	7176.2
A^0	7175.6	7175.5	7176.6	7176.1
g	9334.0	9334.0	9334.0	9334.0
$\tilde{\chi}_1^0$	2120.9	2120.9	2120.9	2120.9
$\tilde{\chi}_2^0$	3914.7	3914.7	3914.7	3914.7
$\tilde{\chi}_3^0$	-5353.3	-5353.3	-5354.8	-5354.8
$\tilde{\chi}_4^0$	5356.0	5355.8	5357.3	5357.3
$\tilde{\chi}_1^\pm$	3914.9	3914.9	3914.9	3914.9
$\tilde{\chi}_2^\pm$	5356.1	5356.0	5357.6	5357.6
\tilde{e}_1	1876.8	1873.9	1873.9	1858.0
\tilde{e}_2	1876.9	1879.8	1879.7	1893.7
\tilde{e}_3	1985.7	1985.8	1985.7	1986.7
\tilde{e}_4	4799.3	4798.8	4798.8	4795.1
\tilde{e}_5	4799.3	4799.9	4799.9	4802.6
\tilde{e}_6	4812.0	4812.0	4812.0	4812.7
$\tilde{\nu}_1$	4798.3	4797.8	4797.8	4794.7
$\tilde{\nu}_2$	4798.4	4798.9	4798.9	4801.6
$\tilde{\nu}_3$	4820.8	4820.8	4820.8	4820.9
\tilde{u}_1	7334.8	7334.8	7299.9	7299.9
\tilde{u}_2	7334.8	7334.8	7365.9	7365.9
\tilde{u}_3	8164.4	8164.4	8166.5	8166.5
\tilde{u}_4	9338.3	9337.8	9324.3	9323.0
\tilde{u}_5	9338.3	9338.8	9350.2	9351.2
\tilde{u}_6	9601.3	9601.3	9602.8	9602.8
\tilde{d}_1	9338.5	9338.0	9324.8	9323.4
\tilde{d}_2	9338.5	9339.0	9350.5	9351.4
\tilde{d}_3	9456.6	9445.5	9445.4	9453.2
\tilde{d}_4	9456.6	9457.6	9457.5	9458.6
\tilde{d}_5	9466.4	9466.5	9466.4	9466.7
\tilde{d}_6	9641.4	9641.4	9642.5	9642.5

Full Parameter Space Scans



Higgs mass
(black shading)

Lighter gluino &
 \tilde{e}_1 masses with
perturbations

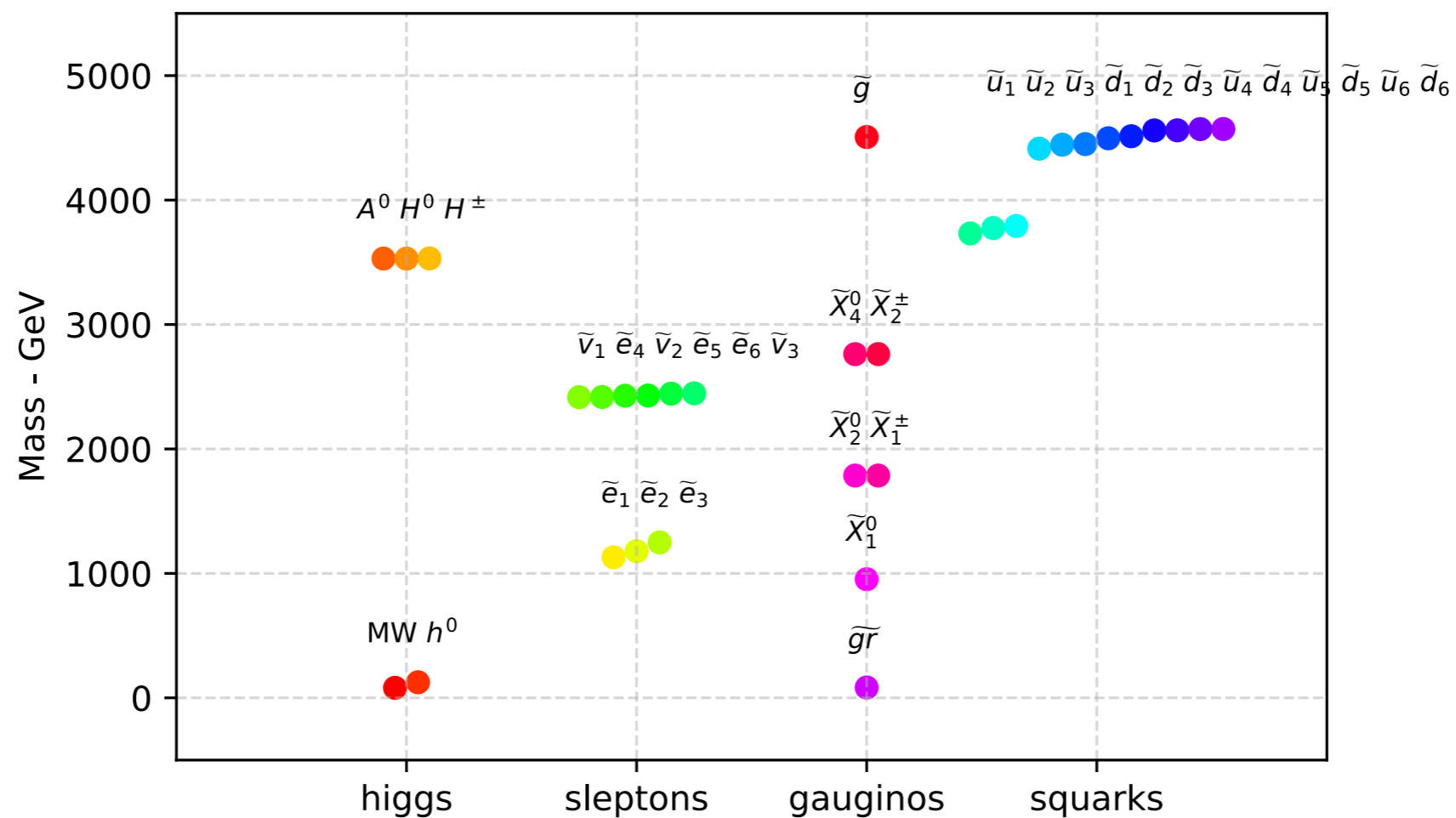
Example mass spectrum

$$M_{\text{Mess}} = 10^{15} \text{GeV}, \Lambda = 3.5 \times 10^5 \text{GeV}, \tan \beta = 20$$

$$\epsilon_u = 0.033, \epsilon_d = 0.108, \epsilon_e = 0.281$$

$$\sigma_u = 0.001, \sigma_d = 0.009, \sigma_e = 0.008$$

$$M_{\text{mess}} = 10^{15} \text{GeV}, \Lambda = 3.5 \times 10^5 \text{GeV}, \tan \beta = 20, \epsilon_u = 0.033, \epsilon_d = 0.108, \epsilon_e = 0.281$$

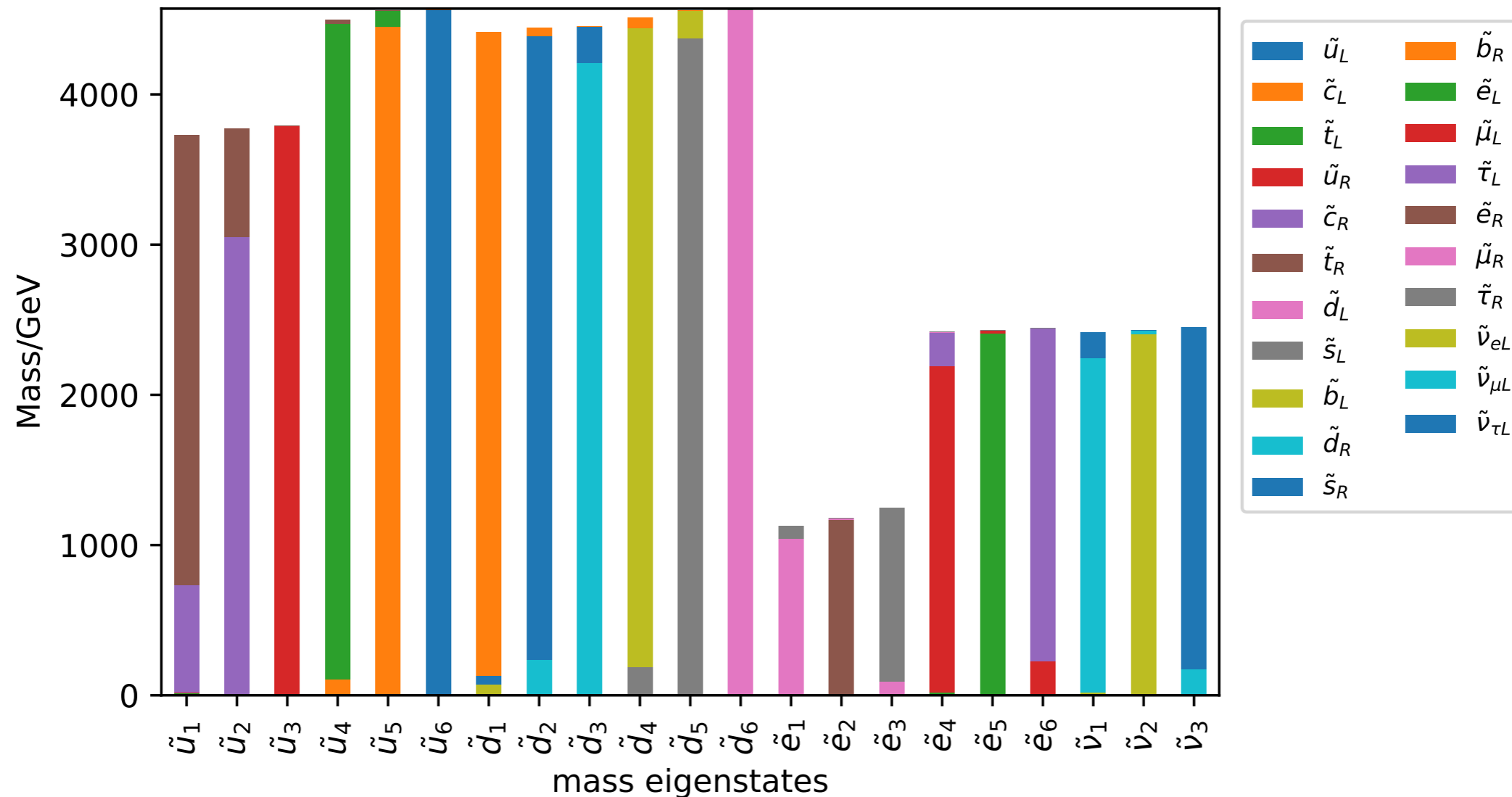


Example mass spectrum (continued)

$$M_{\text{Mess}} = 10^{15} \text{GeV}, \Lambda = 3.5 \times 10^5 \text{GeV}, \tan \beta = 20$$

$$\epsilon_u = 0.033, \epsilon_d = 0.108, \epsilon_e = 0.281$$

$$\sigma_u = 0.001, \sigma_d = 0.009, \sigma_e = 0.008$$



- No mixing of flavor eigenstates when there is no perturbation added
- Significant mixing between the second and the third generation

Flavor Constraints (SUSY Flavor problem)

- Flavor changing mixing in sfermion mass matrices \Rightarrow FCNC

Mass Insertion Approximation (MIA):

$$(\delta_Q^{IJ})_{XY} = \frac{(\Delta_Q^{IJ})_{XY}}{(m_{QI})_{XX}(m_{QJ})_{YY}}$$

eg. Super-CKM squark mass squared matrix

$$(M_{\tilde{U}}^2)_{LL} = \begin{pmatrix} (m_{U1}^2)_{LL} & (\Delta_U^{12})_{LL} & (\Delta_U^{13})_{LL} \\ (\Delta_U^{21})_{LL} & (m_{U2}^2)_{LL} & (\Delta_U^{23})_{LL} \\ (\Delta_U^{31})_{LL} & (\Delta_U^{32})_{LL} & (m_{U3}^2)_{LL} \end{pmatrix}$$

I, J : quark flavor

Q : up/ down quark superfield sector

X, Y : superfield chirality

- Non-degenerate squark masses but not strongly hierarchical \Rightarrow MIA 

- $|(\delta_Q^{IJ})_{XY}|$ predicted in our models are well bounded

Loose bounds since the constraints scale with squark masses (heavy squarks)

Mass insertion is proportional to mass difference between squarks which are small

Summary

So far we have ...

- Built models with **3 massive quarks** consistent with SM quark masses.
- Achieved reasonable **estimation of CKM** in Case 1 democratic model
- Explored SUSY parameter space in Case 1 democratic model
- Related SUSY breaking and flavor symmetry breaking with the same symmetry group \mathcal{S}_3
- Shown that our models with flavor mixing **satisfy FCNC constraints**
- Predicted sparticles mass spectra with stop mass lower than 10 TeV (MGM), in region not yet ruled out by experiments.

An aerial photograph of a city waterfront at sunset. The sun is low on the horizon, casting a golden glow over the scene. The water is dark blue with many sailboats scattered across it. The city buildings are visible on the left side, and a forested hillside is in the background.

Thank you!

Questions?