First Order Electroweak Phase Transitions in the SM with a Singlet Extension

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in collaboration with
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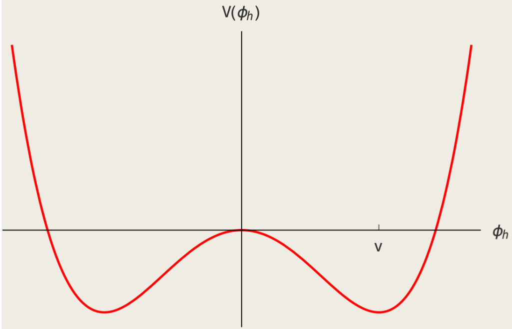
What we know about the Higgs from measurements



$$V^{SM}_{Higgs} = \frac{1}{4}\lambda_h \left(\phi_h^\dagger \phi_h\right)^2 - \frac{1}{2}\left|\mu^2\right| \left(\phi_h^\dagger \phi_h\right)$$

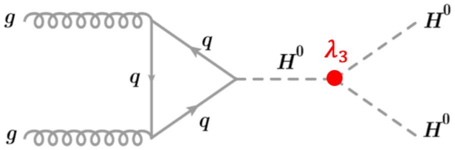
$\Downarrow$

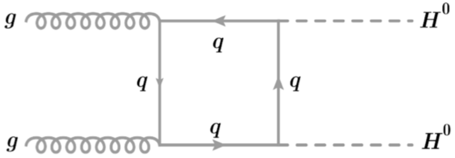
Vacuum expectation value (vev)

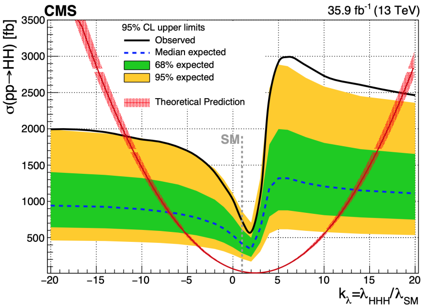
$\Downarrow$

Higgs boson mass

What we don't know from measurements...







$$\sigma_{gg\to H^0H^0} \propto \int \mathrm{d}\hat{t} \left| C_\triangle F_\triangle + C_\square F_\square \right|^2$$

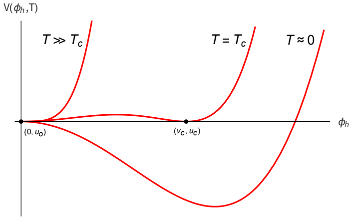
Confidence interval of 95% CL:
$-11.8 < \lambda_3 < 18.8$

Room for new physics

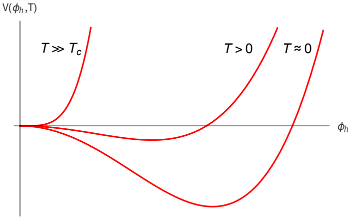
[1] arXiv:1811.09689

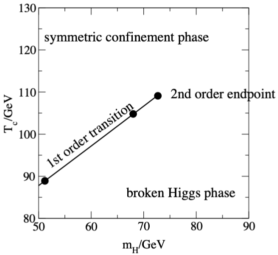
Possible early universe phase transitions

First Order



Smooth Crossover





Supports electroweak baryogenesis

sets the energy scale to levels we can measure

[2] arXiv:0010275

Outline

1. Introduction
    - What we know and don't know...
    - Early Universe Phase Transitions
2. The Model and Constraints
    - Higgs+Singlet Potential
    - Reparametrizing
    - Higgs Trilinear Coupling
    - Conditions Imposed at Critical Temperature
    - Equations to Solve
3. Numerical Study
    - Monte-Carlo Scan Code Structure
    - Calculating Nucleation Temperature
    - Results
4. Conclusions

Higgs+Singlet Potential

$$V_o = \frac{1}{2}\mu_h^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + t_s\phi_s + a_{hs}\phi_h^2\phi_s + \frac{1}{2}\mu_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \frac{1}{3}a_s\phi_s^3 + \frac{1}{4}\lambda_s\phi_s^4$$

At finite T, the one-loop thermal potential leading terms in the high temperature expansion

$$V_{1-loop}^{T\neq 0} = \left(\frac{1}{2}c_h\phi_h^2 + \frac{1}{2}c_s\phi_s^2 + m_3\phi_s\right)T^2,$$

where[3]

$c_h = \frac{1}{48}(9g^2 + 3g'^2 + 2(y_t^2 + 12\lambda_h + 2\lambda_{hs}))$

$c_s = \frac{1}{12}(4\lambda_{hs} + 3\lambda_s),$

$m_3 = \frac{1}{3}a_{hs}.$

$$V = V_o + V_{1-loop}^{T\neq 0}$$

[3] arXiv:1107.5441v1

Reparametrizing

$$V_o = \frac{1}{2}\mu_h^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + t_s\phi_s + a_{hs}\phi_h^2\phi_s + \frac{1}{2}\mu_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \frac{1}{4}\lambda_s\phi_s^4$$

Minimum Equations: $\left.\frac{\mathrm{d}V_o}{\mathrm{d}\phi_h}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} = 0$ $\left.\frac{\mathrm{d}V_o}{\mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} = 0$

In the basis (ϕ_h, ϕ_s), the mass squared matrix is

$$\mathcal{M}^2 = \begin{pmatrix} \left.\frac{\mathrm{d}^2V_o}{\mathrm{d}\phi_h^2}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} & \left.\frac{\mathrm{d}^2V_o}{\mathrm{d}\phi_h\mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} \\ \left.\frac{\mathrm{d}^2V_o}{\mathrm{d}\phi_h\mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} & \left.\frac{\mathrm{d}^2V_o}{\mathrm{d}\phi_s^2}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} \end{pmatrix} = \begin{pmatrix} 2v^2\lambda_h & 2a_{hs}v + 2vv_s\lambda_{hs} \\ 2vv_s\lambda_{hs} & \mu_s^2 + v^2\lambda_{hs} + 3v_s^2\lambda_s \end{pmatrix}$$

$\text{Diag}\left[\mathcal{M}^2\right] = \begin{pmatrix} m_h^2 & 0 \\ 0 & m_s^2 \end{pmatrix}$, where m_h is the Higgs mass and m_s is the singlet mass

Similarity invariance of the trace: $tr\left(\mathcal{M}^2\right) = tr\left(Diag\left[\mathcal{M}^2\right]\right)$

Determinant properties of rotational matrices: $det\left(\mathcal{M}^2\right) = det\left(Diag\left[\mathcal{M}^2\right]\right)$

Reparametrizing

Solve for μ_h^2, μ_s^2, a_{hs}, and a_s in terms of λ_h, λ_{hs}, λ_s, m_s, v_s, m_h, v; yields two sets of solutions:

$$\mu_h^2 = -v^2\lambda_h \pm \frac{v_s}{v}\Delta + v_s^2\lambda_{hs}$$
$$\mu_s^2 = m_h^2 + m_s^2 - 2v^2\lambda_h - v^2\lambda_{hs} - 3v_s^2\lambda_s$$
$$a_{hs} = \mp\frac{1}{2v}\Delta - v_s\lambda_{hs}$$
$$t_s = -v_s(m_h^2 + m_s^2 - 2v^2\lambda_h - v^2\lambda_{hs} - 2v_s^2\lambda_s) \pm \frac{v\Delta}{2}$$

where $\Delta = \sqrt{(m_h^2 - 2v^2\lambda_h)(2v^2\lambda_h - m_s^2)}$

Ranges of the new parameters

$$\begin{aligned} \text{Stability conditions}: &\quad \lambda_h\lambda_s \in [0, \sqrt{4\pi}], \quad \lambda_{hs} \in [-\sqrt{\lambda_h\lambda_s}, \sqrt{4\pi}] \\ \text{singlet mass}: &\quad m_s \lesssim 6\,\text{TeV} \\ \text{singlet vev}: &\quad v_s \text{ scales as } m_s \end{aligned}$$

Higgs Trilinear Coupling

Gauge to mass eigenstate basis: $(\phi_h, \phi_s) \to (h_1, h_2)$

$$\phi_h = h_1 \cos\theta - h_2 \sin\theta + v$$
$$\phi_s = h_1 \sin\theta + h_2 \cos\theta + v_s$$

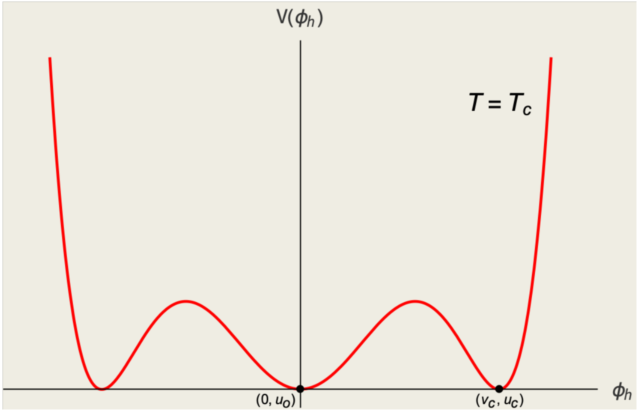
where θ is the mixing angle and is found from $\mathcal{M}^2$ with $\tan 2\theta = \frac{m_{12}+m_{21}}{m_{11}-m_{22}}$

Higgs Trilinear Coupling: Let $h_1 < h_2$, then $\lambda_3 = \frac{\mathrm{d}^3 V_o(h_1,h_2)}{\mathrm{d}h_1^3}$ $\wedge$ $\lambda_3^{SM} = \frac{3\lambda_h v^2}{m_h}$

$$\implies \kappa = \frac{2v^2\lambda_h}{m_h^2}\cos^3\theta\left(1 + \frac{\lambda_{hs}v_s + a_{hs}}{\lambda_h v}\tan\theta + \frac{\lambda_{hs}}{\lambda_h}\tan^2\theta + \frac{\lambda_s v_s}{\lambda_h v}\tan^3\theta\right)$$

$$= \cos^3\theta\left(1 + \frac{2v^2}{m_h^2}\left(\lambda_{hs} + \frac{v_s}{v}\lambda_s\tan\theta\right)\tan^2\theta\right)$$

Conditions Imposed at Critical Temperature



degenerate requirement:

$$V(0, u_o, T_c) = V(v_c, u_c, T_c)$$

minimization requirement:

$$\phi_h = 0 : \quad \frac{\mathrm{d}V(0, u_o, T_c)}{\mathrm{d}\phi_s} = 0$$

$$\phi_h = v_c : \quad \frac{\mathrm{d}V(v_c, u_c, T_c)}{\mathrm{d}\phi_h} = 0$$

$$\frac{\mathrm{d}V(v_c, u_c, T_c)}{\mathrm{d}\phi_s} = 0$$

and $\frac{\mathrm{d}^2 V}{\mathrm{d}\phi_h^2} > 0$ at critical points

Equations to Solve

degenerate requirement:

$$0 = (u_o - u_c)\left(4m_3 T_c^2 + 4t_s\right) + (u_o^2 - u_c^2)\left(2\mu_s^2 + 2c_s T_c^2\right) + (u_o^4 - u_c^4)\lambda_s$$

$$- v_c^2\left(2\mu_h^2 + 2c_h T_c^2 + 4a_{hs} u_c + v_c^2\lambda_h + 2u_c^2\lambda_{hs}\right)$$

minimization requirements:

$$0 = (m_3 + c_s u_o)\, T_c^2 + t_s + u_o \mu_s^2 + u_o^3 \lambda_s$$

$$0 = \mu_h^2 + c_h T_c^2 + 2a_{hs} u_c + v_c^2 \lambda_h + u_c^2 \lambda_{hs}$$

$$0 = (m_3 + c_s u_c)\, T_c^2 + t_s + \mu_s^2 u_c + (a_{hs} + u_c \lambda_{hs})\, v_c^2 + u_c^3 \lambda_s$$

Monte-Carlo Scan Code Structure

Generates Random Parameters → Impose FOEPT Constraints

- Generates N lists of random free parameters: $\left\{ \{m_\phi, \lambda_h, \lambda_{hs}, \lambda_s, v_s\}, \dots \right\}$
- Higg's coupling strength requires mixing angle to satisfy[4]

$$\sin^2\theta < 0.12$$

- Require resonance DiHiggs production to be within experimental constraints.
- Range for free parameters

$$m_s \in [0.25, 6]\ \mathrm{TeV}, \qquad v_s \in [-10, 10]\ \mathrm{TeV}$$

$$\lambda_h, \lambda_s \in \left[0, \sqrt{4\pi}\right], \qquad \lambda_{hs} \in \left[-\sqrt{\lambda_h\lambda_s}, \sqrt{4\pi}\right]$$

- Takes generated parameters, imposes constraints, solves four equations simultaneously.
- Checks if $0 < v_c < v$.
- Checks if solutions are global minimums at $V(v, v_s, 0)$, $V(0, u_o, T_c)$, and $V(v_c, u_c, T_c)$.
- Checks if phase transition is strong by requiring[5]

$$\frac{v_c}{T_c} > 1.3$$

[4] arXiv:1509.00672
[5] arXiv:1711.11541

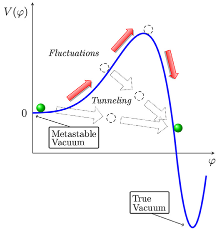
Calculating Nucleation Temperature

FindBounce - a Mathematica package to calculate the bounce. (Assuming thin wall bubbles)[6]

$$\Gamma \simeq Ae^{-B}(1+\mathcal{O}(\hbar))$$

where B is the "bounce".

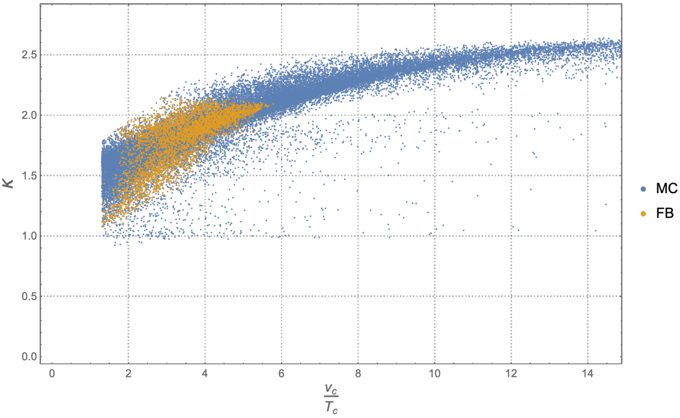
If the barrier is low enough, then thermal fluctuations can drive tunneling to occur during the nucleation of bubbles at the PT.

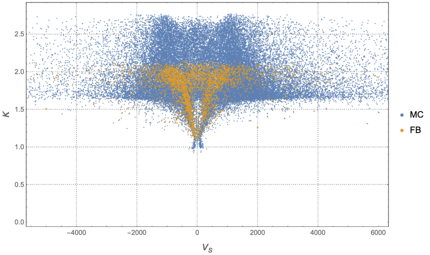


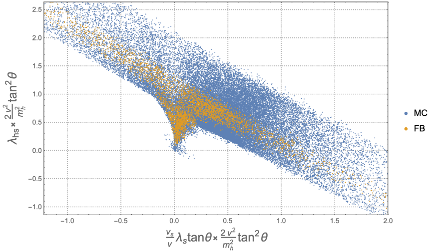
[6] arXiv:2002.00881

[7] arXiv:1809.06923

Results

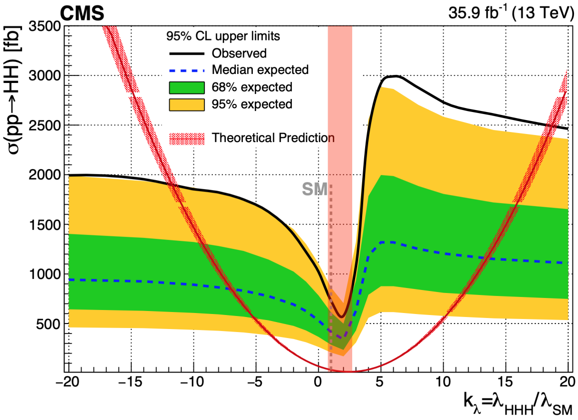


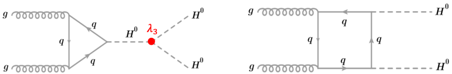




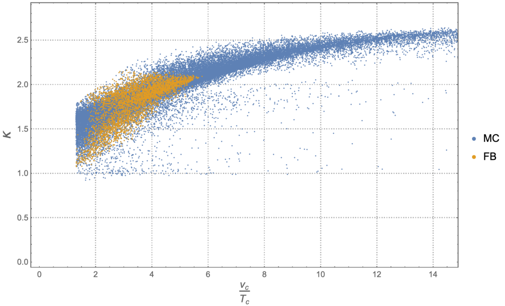
$$\kappa = \cos^3\theta \left(1 + \frac{2v^2}{m_h^2}\left(\lambda_{hs} + \frac{v_s}{v}\lambda_s \tan\theta\right)\tan^2\theta\right)$$

Conclusions





$$\sigma_{gg\to H^0H^0} \propto \int d\hat{t} \left| C_\triangle F_\triangle + C_\square F_\square \right|^2$$



Thank you!