Vacuum Stability in Dynamical Seesaw models

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Talk is based on arXiv: 2009.10116, 2103.02670

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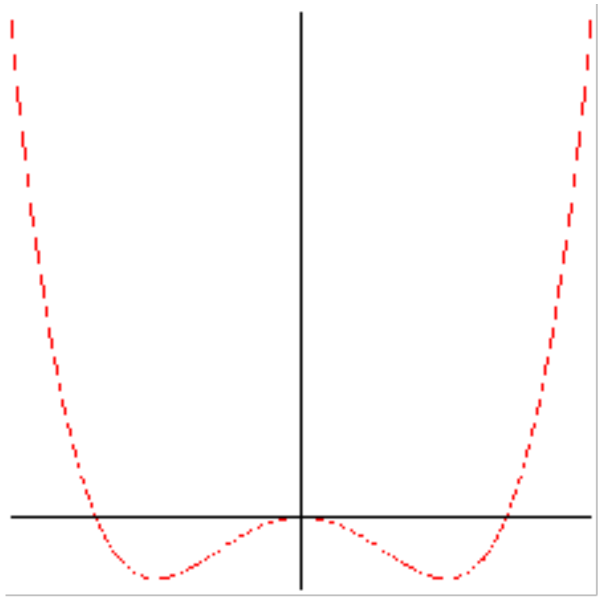
Phone 2021, University of Pittsburgh
25 th May, 2021

SM vacuum stability…..

Higgs Potential: $V = -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2$ At classical level minimum exist if

$\mu_\Phi^2 > 0$: broken symmetry (Mexican hat)



1. Vacuum is stable at large field values if: $\lambda > 0$
2. Perturbativity constraint: $|\lambda| < 4\pi$

→ 1 and 2 must be satisfied at each energy scale

After the Higgs mass measurements quartic coupling is known at EW scale: $m_h^2 = 2\lambda v^2$

High energy behaviour of λ can change the shape of the potential

One need to derive RG running of λ

Strumia et al, 1307.3536

gauge bosons loop

ext. leg corrections

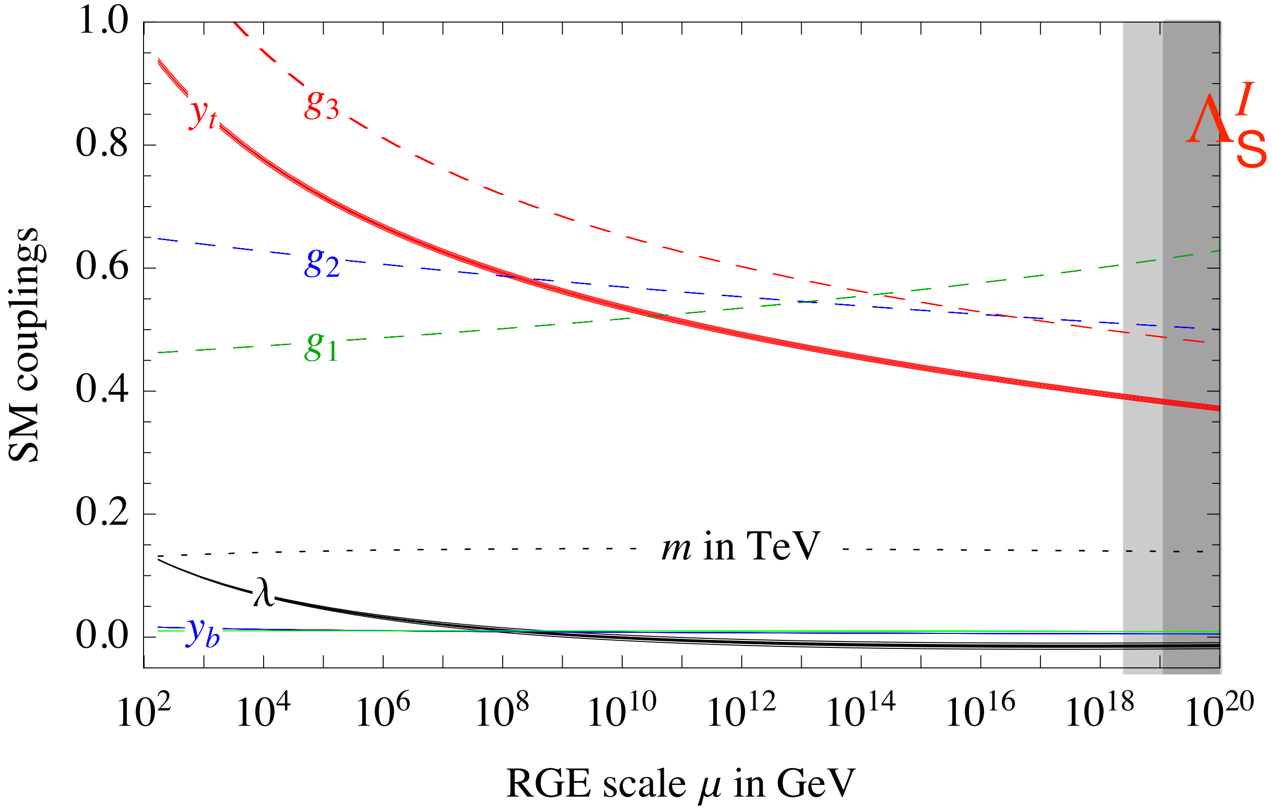
$$16\pi^2 \frac{d\lambda}{d\ln\mu} = +24\lambda^2 \quad +\lambda(12y_t^2 - 3g_1^2 - 9g_2^2) \quad -6y_t^4 \quad \underline{+\frac{3}{8}g_1^4 + \frac{3}{4}g_1^2 g_2^2 + \frac{9}{8}g_2^4}$$

scalar loop

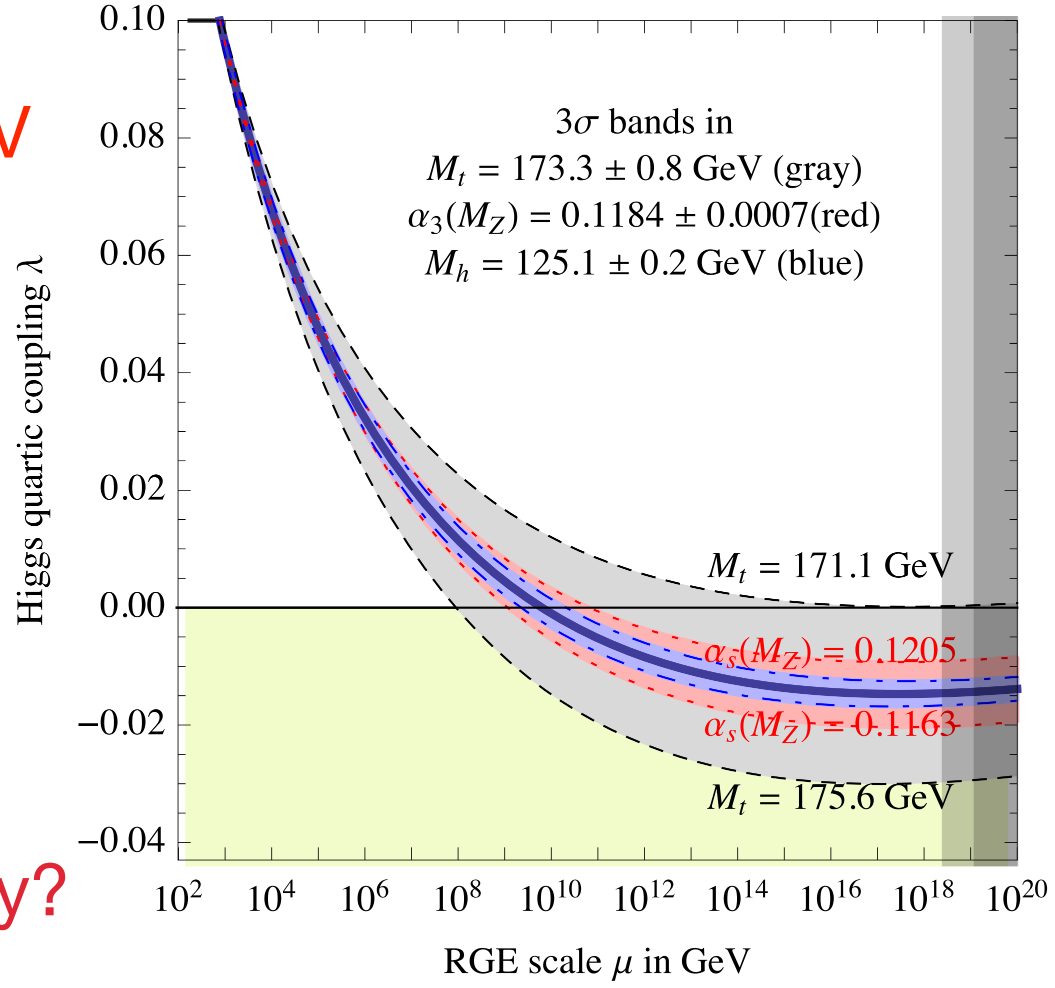
fermion loop

A <0 at EW scale

SM,Valle, Srivastava, arXiv:1903.03631

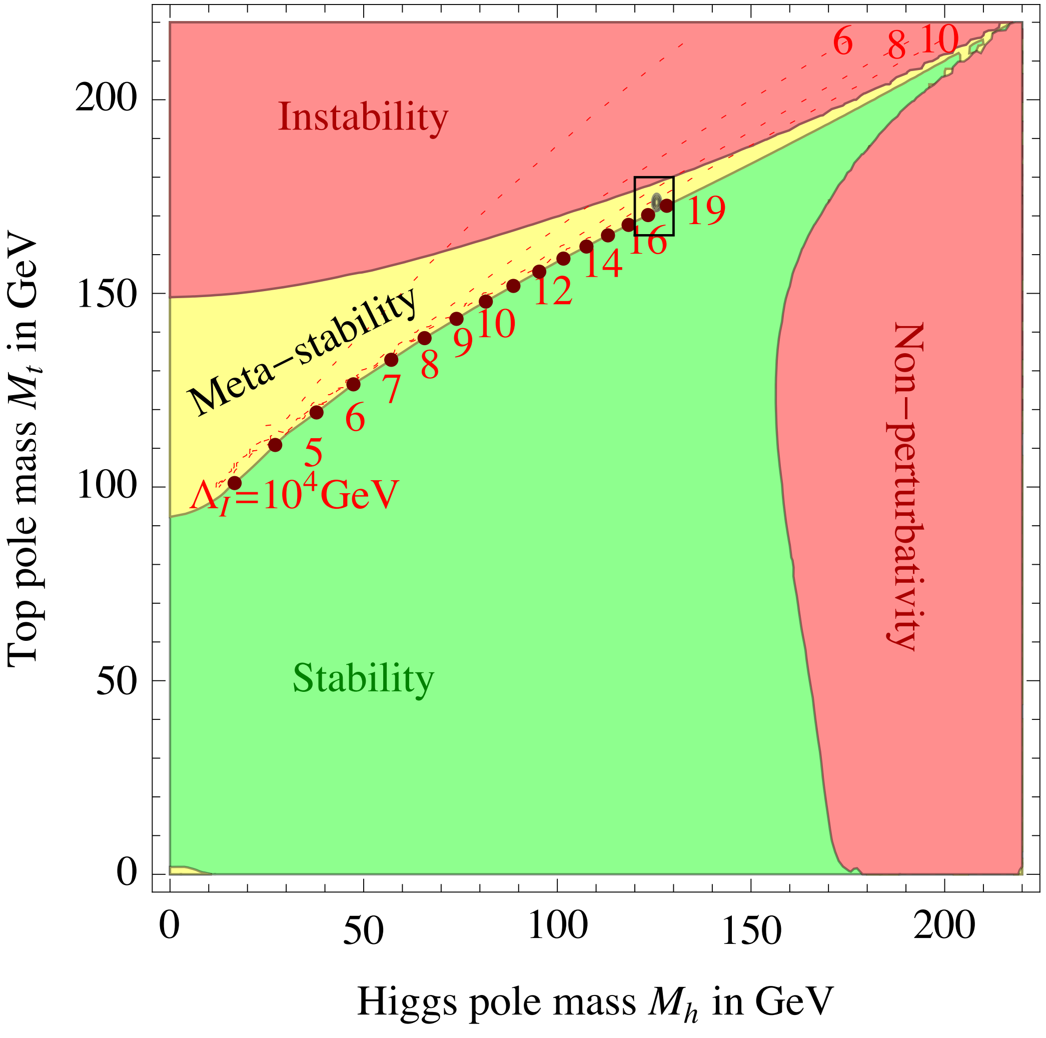


$\Lambda^I_{\text{SM}} \approx 10^9$ GeV

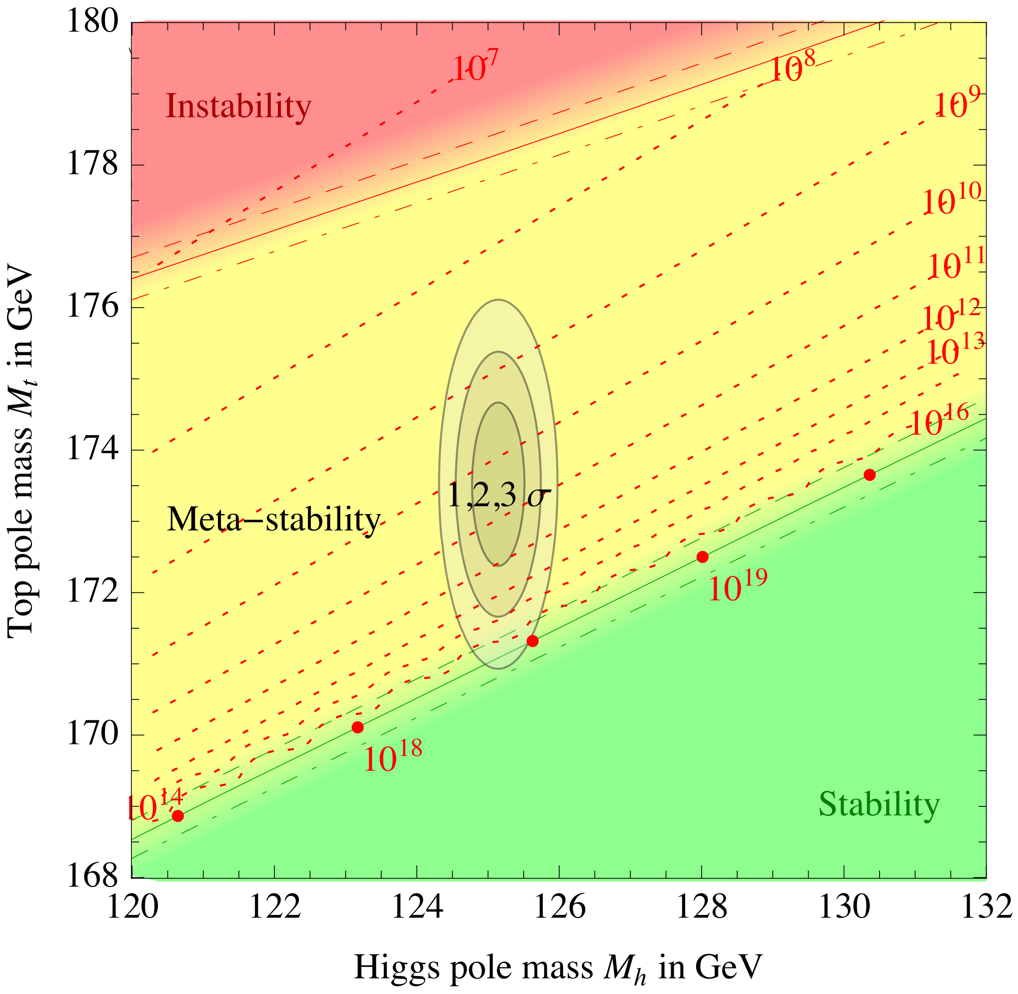


Strumia et al, 1307.3536

Is SM final theory?



3-loop RGEs



Vacuum stability in low scale seesaw

$$-\mathscr{L} = \sum_{ij} Y_\nu^{ij} L_i \tilde{\Phi} \nu_j^c + M^{ij} \nu_i^c S_j + \frac{1}{2}\mu_S^{ij} S_i S_j + \text{H.c.}$$

$$V = -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

$$m_\nu \approx m_D M^{-1} \mu_S (M^T)^{-1} m_D^T = \frac{v^2}{2} Y_\nu M^{-1} \mu_S (M^T)^{-1} Y_\nu^T$$

Valle et al, 1404.3752

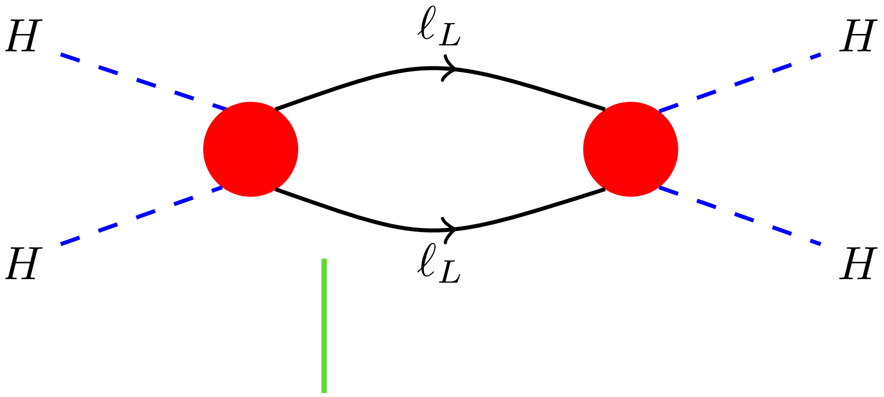
One can allow large $Y_\nu \sim \mathcal{O}(1)$, even for $M_N \sim \mathcal{O}(1)$ TeV

Schechter, Valle: PRD 22(1980) 2227

Effective Theory : Below $\mu \approx M$, ν^c and S are integrated out

mass is generated : $-\mathscr{L}_\nu^{d=5} = \frac{1}{2}\left(\bar{\ell}_L \Phi\right) . \kappa . \left(\Phi^T \ell_L^c\right) + \text{h.c.}$

$\kappa = Y_\nu M^{-1} \mu_S M^{-1T} Y_\nu^T$ is very small as $m_\nu = \kappa v_\Phi^2 / 2$



Running parameters : SM + κ,

corrections to λ but $\mathcal{O}(v^2\kappa^2)$

$16\pi^2 \beta_\kappa = 6y_t^2 \kappa - 3g_2^2 \kappa + \lambda_\kappa \kappa$ slowly increase with μ due to large y_t

Lindner et al, hep-ph/0203233

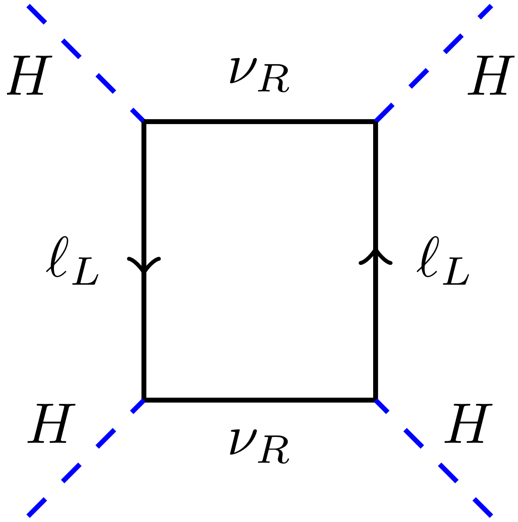
T. Ohlsson et al, 1009.2762

Conclusion: the running of λ below the scale $\mu \approx M$ will be almost the same as SM.

Full Theory: Y_ν now runs

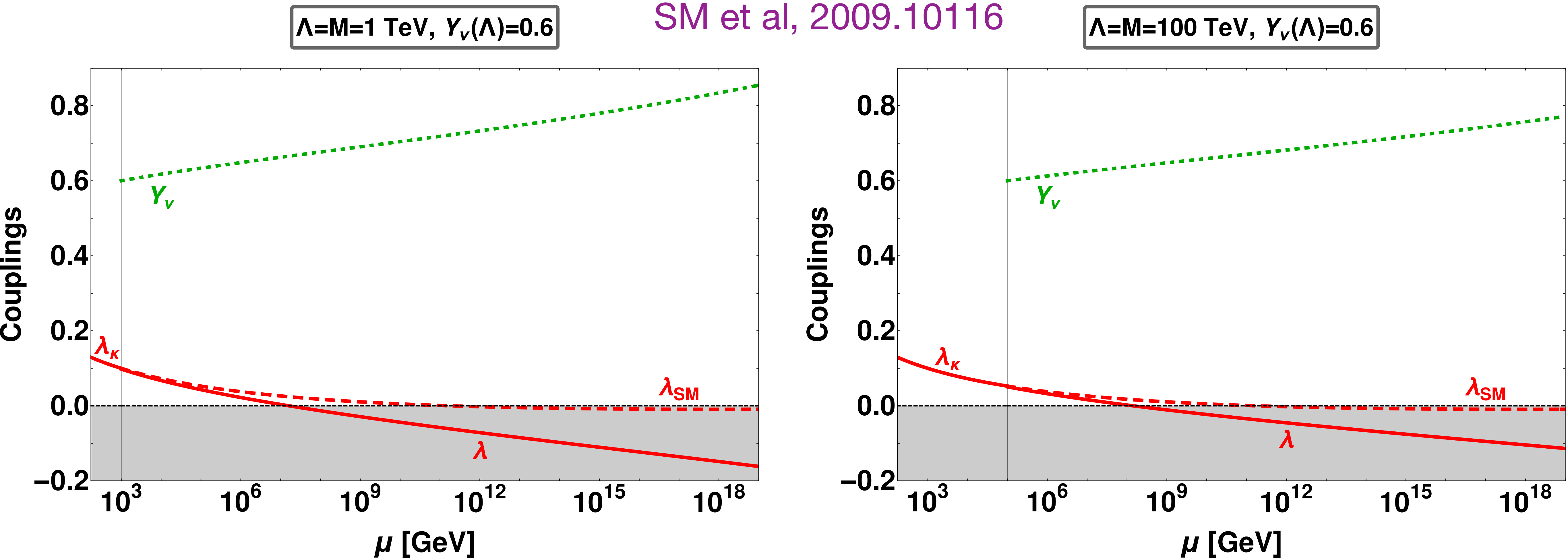
$$\beta_\lambda = \text{SM} + 4\lambda \text{Tr}\left(Y_\nu^\dagger Y_\nu\right) - 2\text{Tr}\left(Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu\right)$$

$$\beta_{Y_\nu} = \frac{3}{2} Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu\left(3y_t^2 - \frac{5}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y_\nu Y_\nu^\dagger\right)\right)$$



gives correction to λ in full seesaw theory

Important point : Y_ν runs for much longer time in low-scale seesaw for $M_N \sim 1$ TeV



SM et al, 2009.10116

Conclusion: Stability properties are even worse compare to High-Scale type-I seesaw

Dynamical low-scale seesaw: Lepton number is spontaneously broken by the vev of σ

SM et al, 2009.10116, 2103.02670

$$-\mathscr{L} = \sum_{i,j}^{3} Y_\nu^{ij} L_i \tilde{\Phi} \nu_j^c + M^{ij} \nu_i^c S_j + \underset{\displaystyle\downarrow}{Y_S^{ij} \sigma S_i S_j} + \text{H.c.}$$

$L[\nu^c] = 1,\ L[S] = 1,\ L[\sigma] = -2$

$$m_\nu \simeq \frac{v_\Phi^2}{\sqrt{2}} Y_\nu M^{-1} Y_S v_\sigma M^{-1T} Y_\nu^T$$

$\mu_s = Y_S v_\sigma / \sqrt{2}$, hence can take Y_S very small and $v_\sigma \sim \mathcal{O}$(TeV)

For $m_\nu \sim 0.1$ eV, we can have Yukawa couplings Y_ν of order one, for TeV scale v_σ and M.

Large Y_ν is needed to produce heavy neutrinos at collider

Potential: $V = \mu_\Phi^2 \Phi^\dagger \Phi + \mu_\sigma^2 \sigma^\dagger \sigma + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_{\Phi\sigma} (\Phi^\dagger \Phi)(\sigma^\dagger \sigma)$

CP-even scalars: H_1 and H_2

Imaginary part of the σ corresponds to the physical majoron $J =$ Im σ

Boundedness : $\lambda_\Phi(\mu) > 0,\ \lambda_\sigma(\mu) > 0,\ \lambda_{\Phi\sigma}(\mu) + 2\sqrt{\lambda_\Phi(\mu)\lambda_\sigma(\mu)} > 0,$

Perturbativity: $\lambda_\Phi(\mu) \leq 4\pi,\ \lambda_\sigma(\mu) \leq 4\pi$ and $|\lambda_{\Phi\sigma}(\mu)| \leq 4\pi$

Aside: KeV scale majoron is Warm dark matter candidate. $\Gamma_J \propto m_\nu$, Long lived

$$\lambda_\Phi = \frac{m_{H_1}^2 \cos^2\theta + m_{H_2}^2 \sin^2\theta}{2v_\Phi^2}$$

$$\lambda_\sigma = \frac{m_{H_1}^2 \sin^2\theta + m_{H_2}^2 \cos^2\theta}{2v_\sigma^2}$$

$$\lambda_{\Phi\sigma} = \frac{\sin 2\theta (m_{H_1}^2 - m_{H_2}^2)}{2v_\Phi v_\sigma}$$

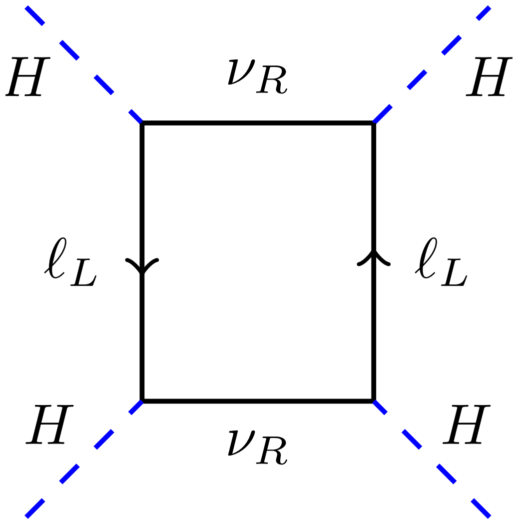
Two possibilities:

Case I: $m_{H_1} < 125$ GeV with $H_2 \equiv H_{125}$ i.e. $m_{H_2} = 125$ GeV

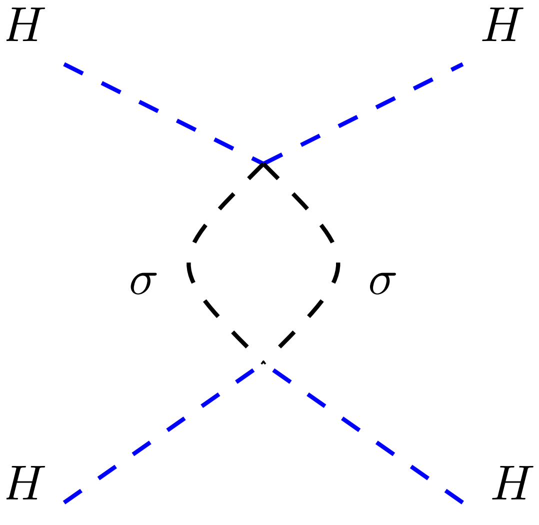
Case II: $m_{H_2} > 125$ GeV with $H_1 \equiv H_{125}$ i.e. $m_{H_1} = 125$ GeV

Effective theory $\mu < M$: SM+dim-5 operators are running

Full theory $\mu > M$: All the new couplings are running



Negative term



Positive term

$$16\pi^2\beta_{\lambda_\Phi} = \text{SM} \underset{\substack{\downarrow \\ \text{2nd diagram}}}{+\lambda_{\Phi\sigma}^2} + 4\lambda_\Phi \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \underset{\substack{\downarrow \\ \text{1st diagram}}}{-2\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)}$$

Reminder: $-6y_t^4$ term in SM

$$16\pi^2\beta_{\lambda_{\Phi\sigma}} = \frac{1}{10}\lambda_{\Phi\sigma}\left(+ 40\lambda_{\Phi\sigma} + 80\lambda_\sigma + 120\lambda_\Phi + 40\mathrm{Tr}\left(Y_S Y_S^\dagger\right) + 60y_t^2 + 20\mathrm{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 15g_1^2 - 45g_2^2\right)$$

Running of $\lambda_{\Phi\sigma}$ depends on the sign

$$16\pi^2\beta_{\lambda_\sigma} = 2\left(10\lambda_\sigma^2 + \lambda_{\Phi\sigma}^2 + 4\lambda_\sigma\mathrm{Tr}\left(Y_S Y_S^\dagger\right) - 8\mathrm{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right)\right)$$

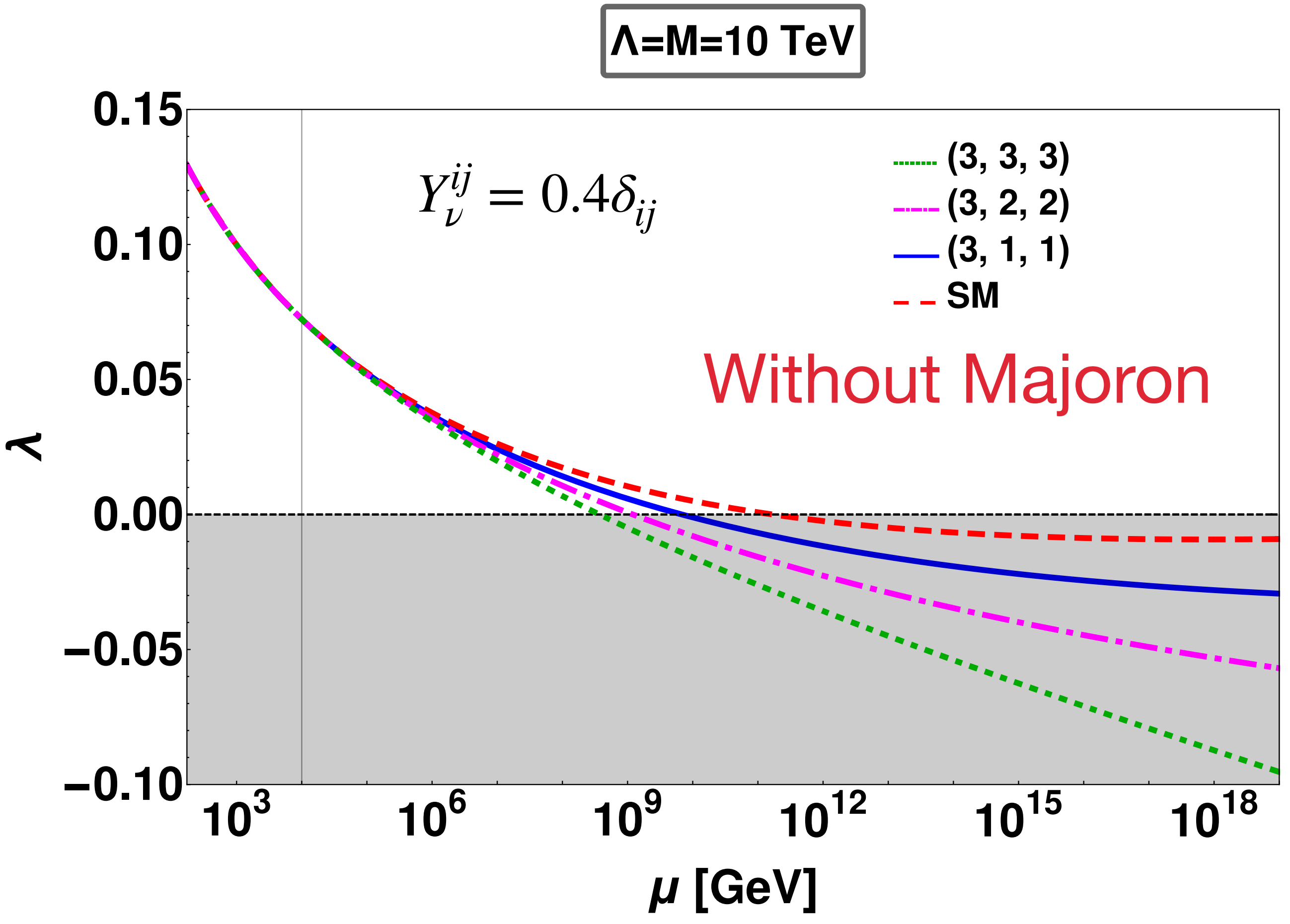
With $Y_S = 0, \lambda_\sigma$ increase

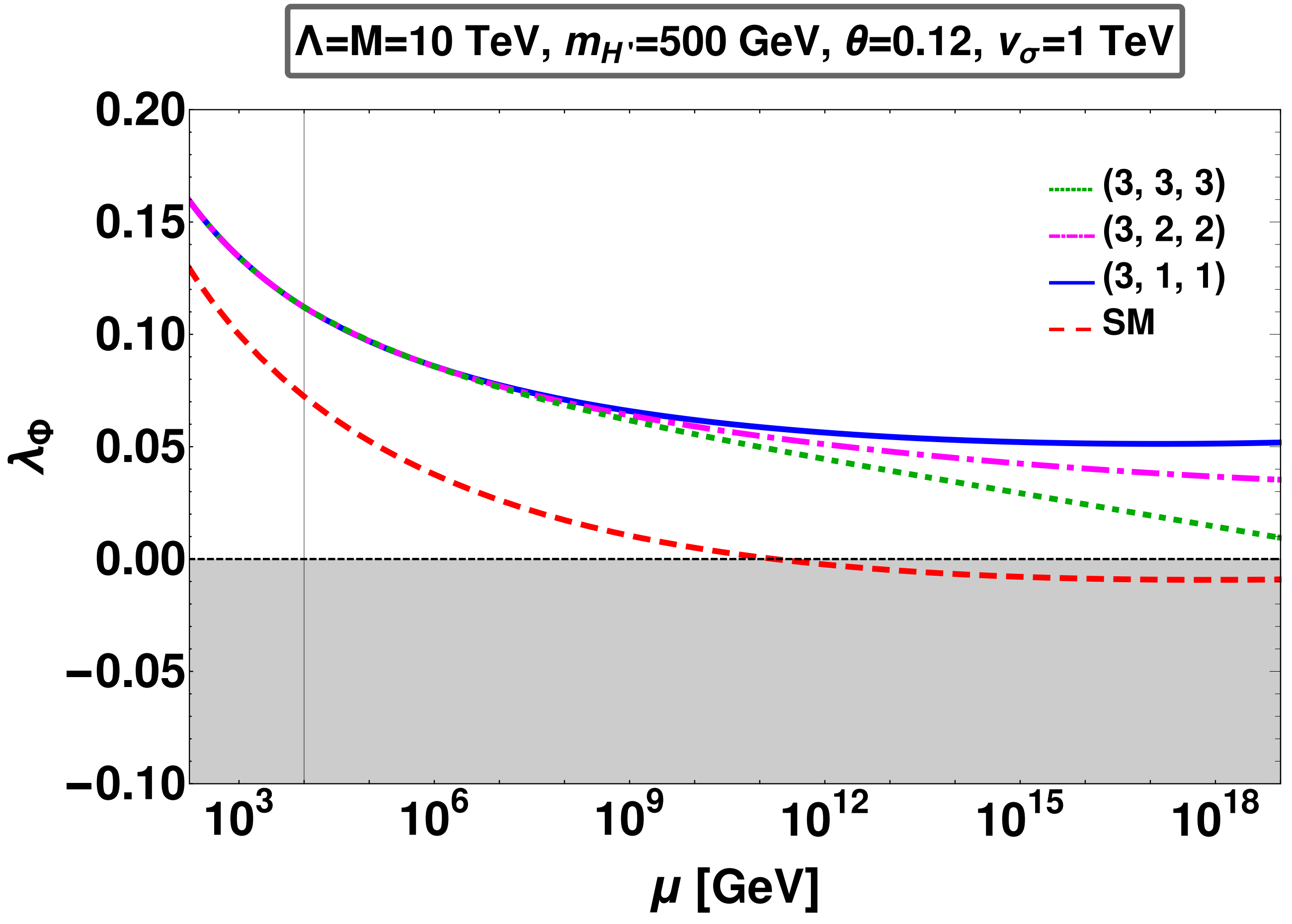
If v_σ is small λ_σ hits Landau pole

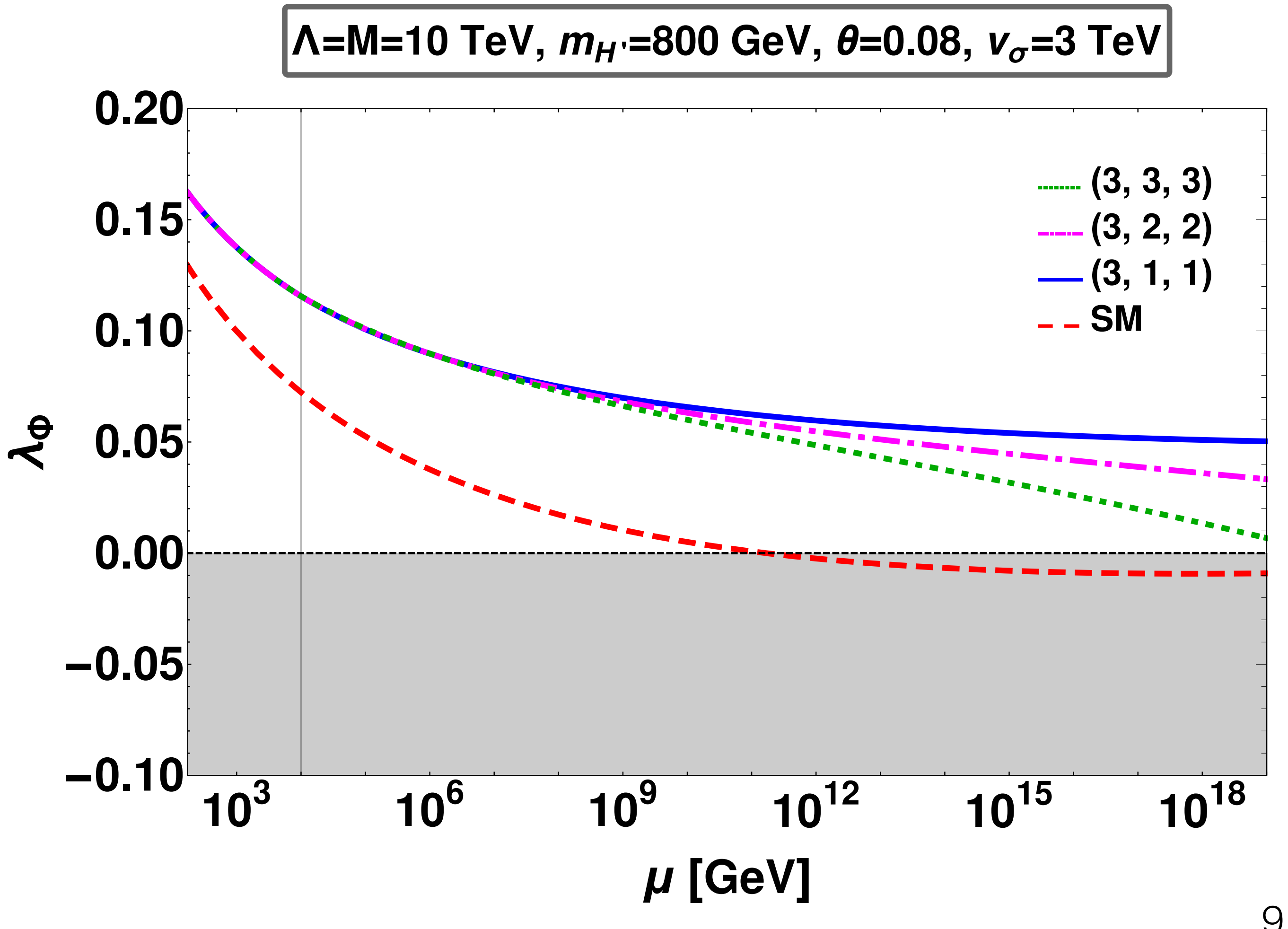
Yukawa coupling:

$$\beta_{Y_\nu} = \frac{3}{2}Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu\left(3y_t^2 - \frac{5}{4}g_1^2 - \frac{9}{4}g_2^2 + \mathrm{Tr}\left(Y_\nu Y_\nu^\dagger\right)\right)$$

$$\beta_{Y_S} = 2Y_S\mathrm{Tr}(Y_S^\dagger Y_S) + 4Y_S(Y_S^\dagger Y_S)$$


Λ=M=10 TeV


Λ=M=10 TeV, $m_{H'}$=500 GeV, θ=0.12, v_σ=1 TeV


Λ=M=10 TeV, $m_{H'}$=800 GeV, θ=0.08, v_σ=3 TeV

Heavy neutrino scale is fixed at $\Lambda \approx M = 10$ TeV. Below is the Eft

Heavy scalar $H_2 \equiv H'$ close to EW scale, hence run all $\lambda's$ from the EW scale

Stability: need large $\lambda_{\Phi\sigma}$ for large Y_ν

LHC: constraints exist on $\lambda_{\Phi\sigma}$ or θ

Goal: find optimal range consistent with LHC, stability and perturbativity

Collider constraints: Signal strength parameter and Invisible Higgs decay

$$h_{\text{SM}} \to \cos\theta\ H_{125} - \sin\theta\ H'$$

Trilinear coupling: $\mathscr{L}_{H_iJJ} = \sum g_{H_iJJ} H_i J^2,\ \mathscr{L}_{H_2H_1H_1} = g_{H_2H_1H_1} H_2 H_1^2$

$$g_{H_iJJ} = \frac{m_{H_i}^2}{2v_\sigma} O_{Ri2}, \qquad O_R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$g_{H_2H_1H_1} = \frac{\sin 2\theta}{4v_\sigma}(2m_{H_1}^2 + m_{H_2}^2)(-\cot\beta\cos\theta + \sin\theta),\ \tan\beta = \frac{v_\Phi}{v_\sigma}$$

Invisible Higgs Decay: $\Gamma^{\text{inv}}(H_1) = \Gamma(H_1 \to JJ),\ \ \Gamma^{\text{inv}}(H_2) = \Gamma(H_2 \to JJ) + \Gamma(H_2 \to H_1H_1 \to 4J)$

Large invisible Higgs decay for $v_\sigma \sim \mathscr{O}(\text{TeV})$

Visible sector: $\text{BR}_f(H_1) = \dfrac{\cos^2\theta\Gamma_f^{\text{SM}}(H_1)}{\cos^2\theta\Gamma^{\text{SM}}(H_1) + \Gamma^{\text{inv}}(H_1)},\ \text{BR}_f(H_2) = \dfrac{\sin^2\theta\Gamma_f^{\text{SM}}(H_2)}{\sin^2\theta\Gamma^{\text{SM}}(H_2) + \Gamma(H_2 \to JJ) + \Gamma(H_2 \to H_1H_1)}$

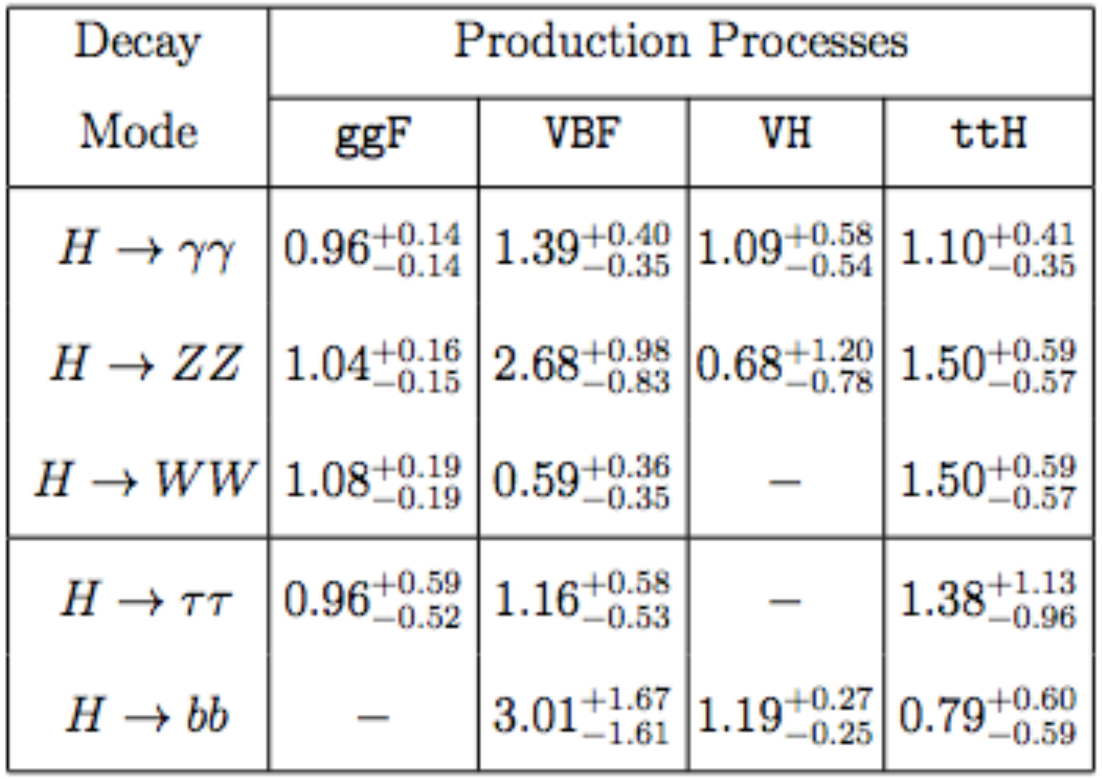
Invisible sector: $\text{BR}^{\text{inv}}(H_1) = \dfrac{\Gamma^{\text{inv}}(H_1)}{\cos^2\theta\Gamma^{\text{SM}}(H_1) + \Gamma^{\text{inv}}(H_1)},\ \text{BR}^{\text{inv}}(H_2) = \dfrac{\Gamma^{\text{inv}}(H_2)}{\sin^2\theta\Gamma^{\text{SM}}(H_2) + \Gamma(H_2 \to JJ) + \Gamma(H_2 \to H_1H_1)}$

Modified Higgs Production: $\sigma(pp \to H_1) = \cos^2\theta\sigma^{\text{SM}}(pp \to H_1),\ \sigma(pp \to H_2) = \sin^2\theta\sigma^{\text{SM}}(pp \to H_2)$

Signal strength: $\mu_f = \dfrac{\sigma^{\text{NP}}(pp \to h)}{\sigma^{\text{SM}}(pp \to h)} \dfrac{\text{BR}^{\text{NP}}(h \to f)}{\text{BR}^{\text{SM}}(h \to f)}$

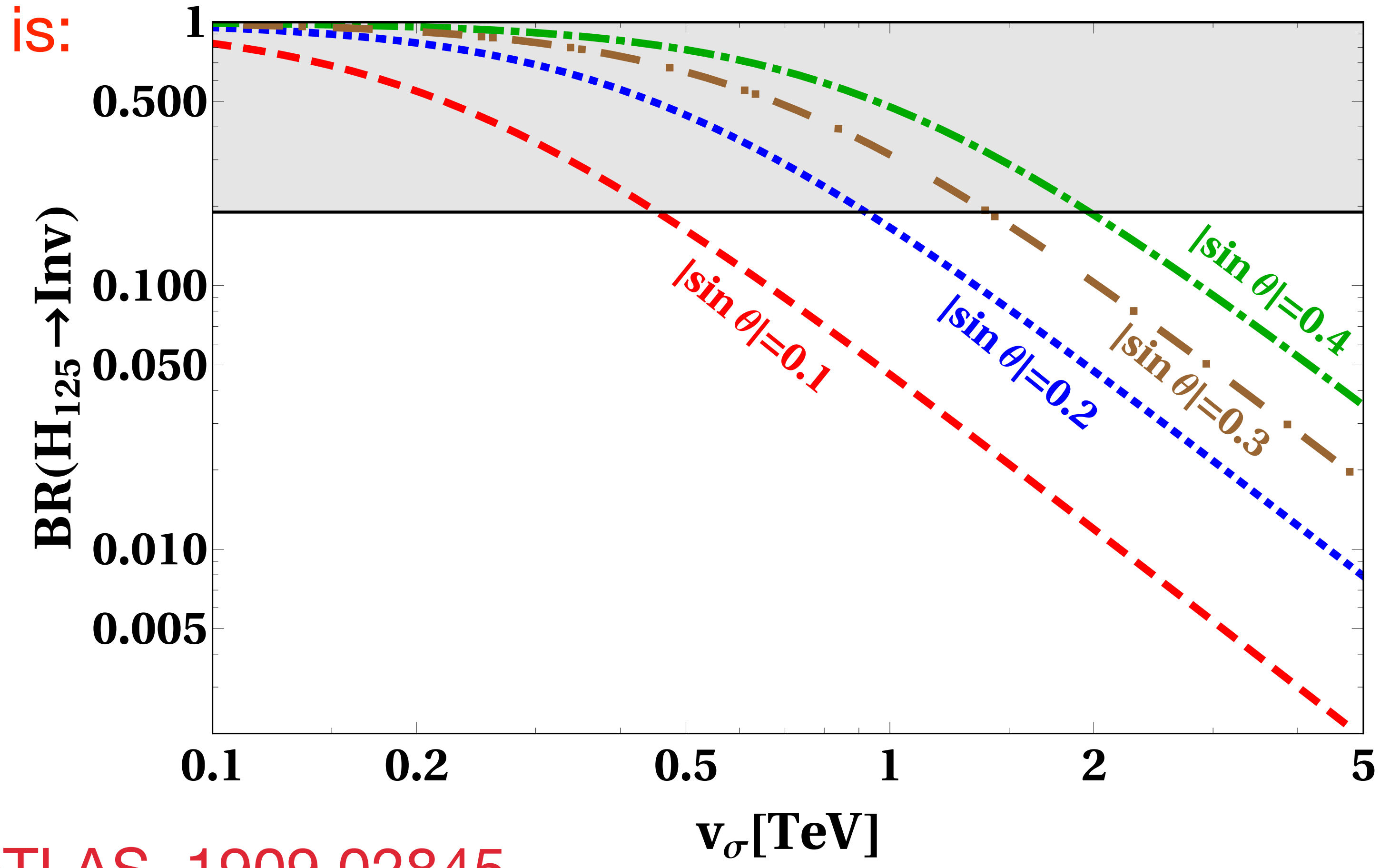
Present bound on Invisible Higgs decay is:

BR$(H_{125} \to$ Inv$) \leq 19\,\%$ CMS, 1809.05937

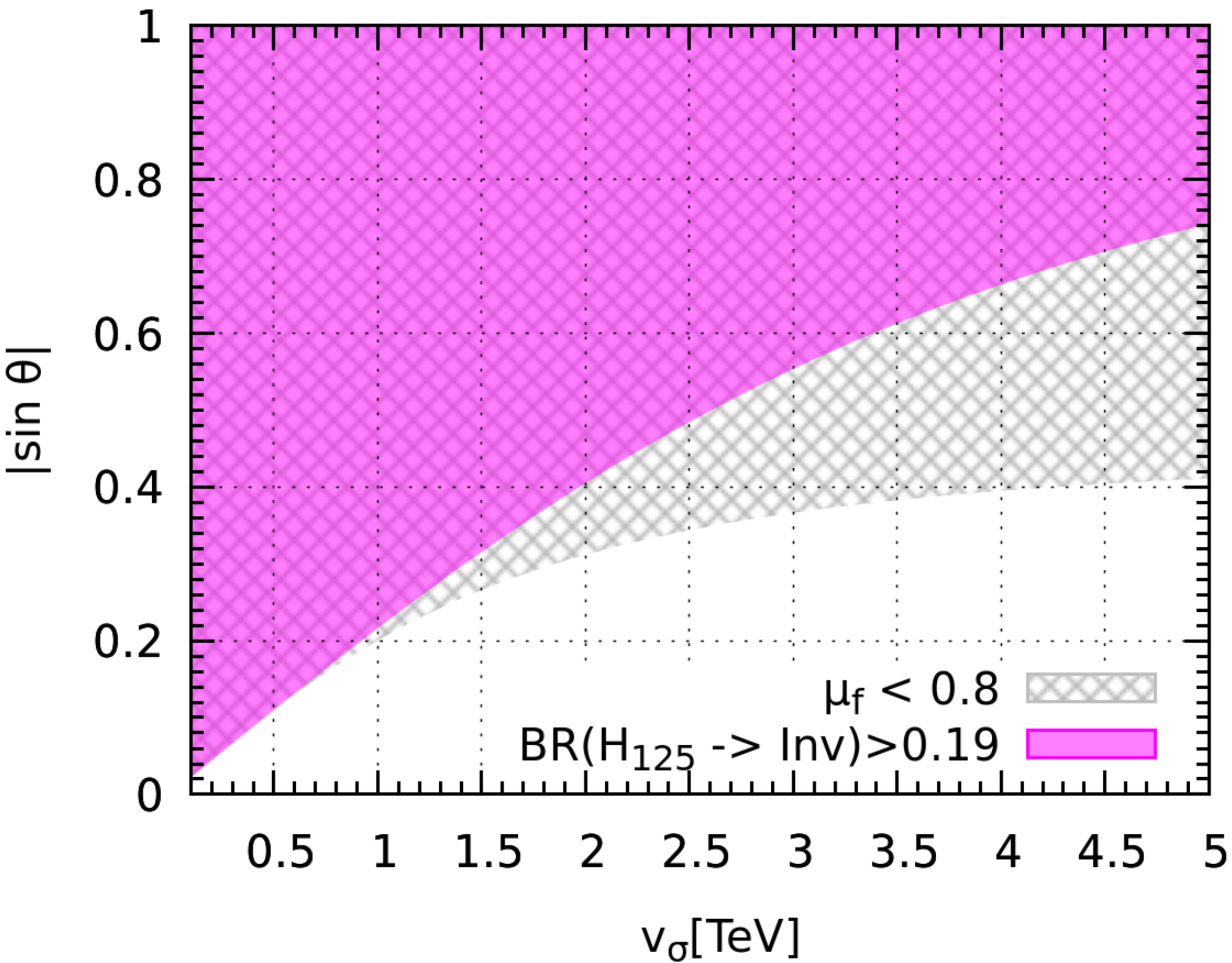


Decay Mode	Production Processes			
	ggF	VBF	VH	ttH
$H \to \gamma\gamma$	$0.96^{+0.14}_{-0.14}$	$1.39^{+0.40}_{-0.35}$	$1.09^{+0.58}_{-0.54}$	$1.10^{+0.41}_{-0.35}$
$H \to ZZ$	$1.04^{+0.16}_{-0.15}$	$2.68^{+0.98}_{-0.83}$	$0.68^{+1.20}_{-0.78}$	$1.50^{+0.59}_{-0.57}$
$H \to WW$	$1.08^{+0.19}_{-0.19}$	$0.59^{+0.36}_{-0.35}$	–	$1.50^{+0.59}_{-0.57}$
$H \to \tau\tau$	$0.96^{+0.59}_{-0.52}$	$1.16^{+0.58}_{-0.53}$	–	$1.38^{+1.13}_{-0.96}$
$H \to bb$	–	$3.01^{+1.67}_{-1.61}$	$1.19^{+0.27}_{-0.25}$	$0.79^{+0.60}_{-0.59}$

ATLAS, 1909.02845



v_σ	Upper limit on $\lvert\sin\theta\rvert$ from μ_f	Upper limit on $\lvert\sin\theta\rvert$ from $\mathrm{BR}^{\mathrm{Inv}}_{H_{125}} \leq 19\%$
700 GeV	0.150	0.154
1 TeV	0.201	0.218
2 TeV	0.317	0.417
3 TeV	0.375	0.586



For large v_σ, constraint is tighter from visible Higgs decay

Without invisible Higgs decay, constraint is very tight

Meaning of colour codes in subsequent Figures:

Green Region: This is the region where we can have stable vacuum all the way up to the Planck scale, and all the couplings are within their perturbative regime.

$$0 < \lambda_\Phi(\mu) < \sqrt{4\pi},\ 0 < \lambda_\sigma(\mu) < \sqrt{4\pi},\ \lambda_{\Phi\sigma}(\mu) + 2\sqrt{\lambda_\Phi(\mu)\lambda_\sigma(\mu)} > 0 \text{ and } |\lambda_{\Phi\sigma}(\mu)| < \sqrt{4\pi}$$

Red Region: In this region the vacuum is unstable, this means that any one or more than one of these conditions are realised:

$$\lambda_\Phi(\mu) \leq 0,\ \lambda_\sigma(\mu) \leq 0,\ \lambda_{\Phi\sigma}(\mu) + 2\sqrt{\lambda_\Phi(\mu)\lambda_\sigma(\mu)} \leq 0$$

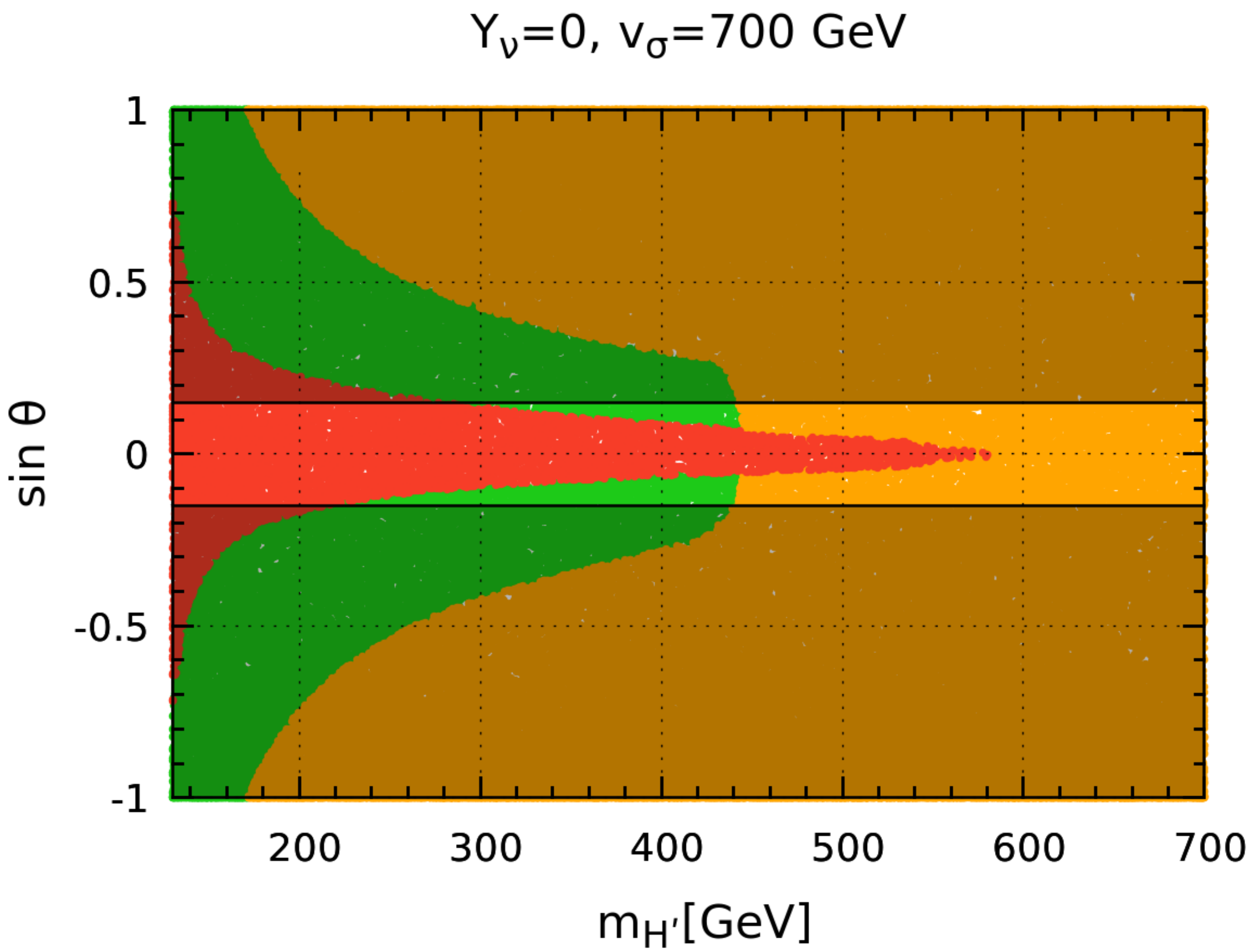
Landau poles are excluded

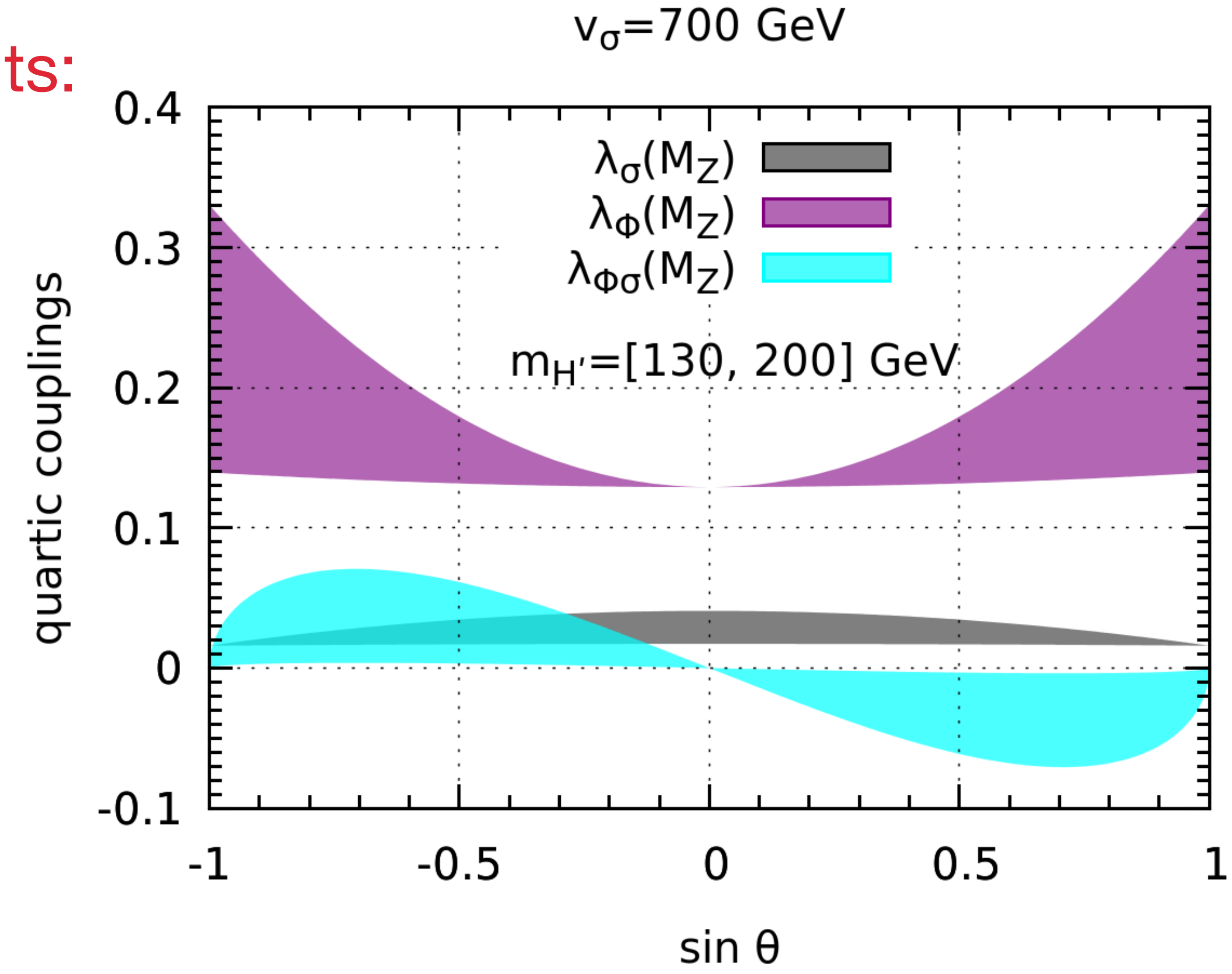
Orange Region: This region implies the existence of non-perturbative couplings at some energy scale before the Planck scale.

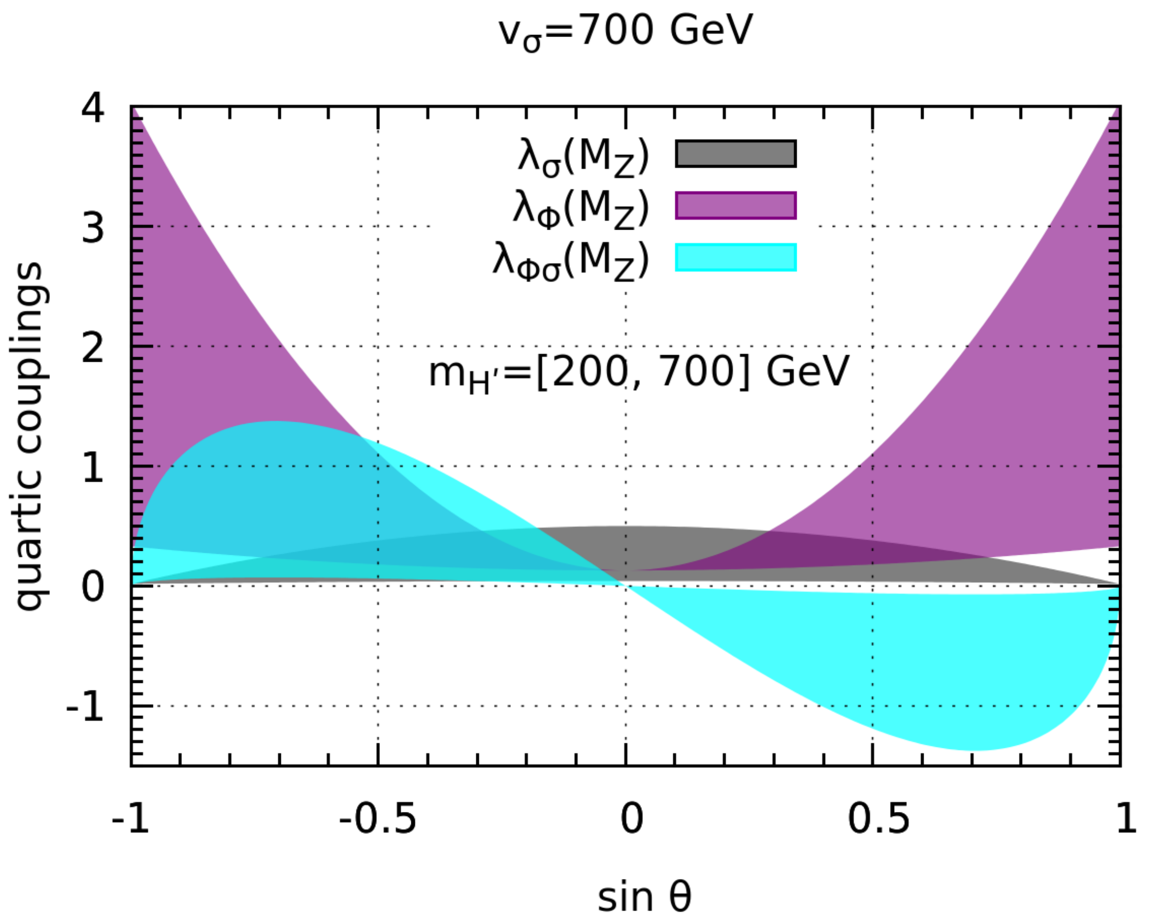
$$|\lambda_\Phi(\mu)| \geq 4\pi,\ |\lambda_\sigma(\mu)| \geq 4\pi,\ |\lambda_{\Phi\sigma}(\mu)| \geq 4\pi, |Y_\nu(\mu)| \geq 4\pi$$

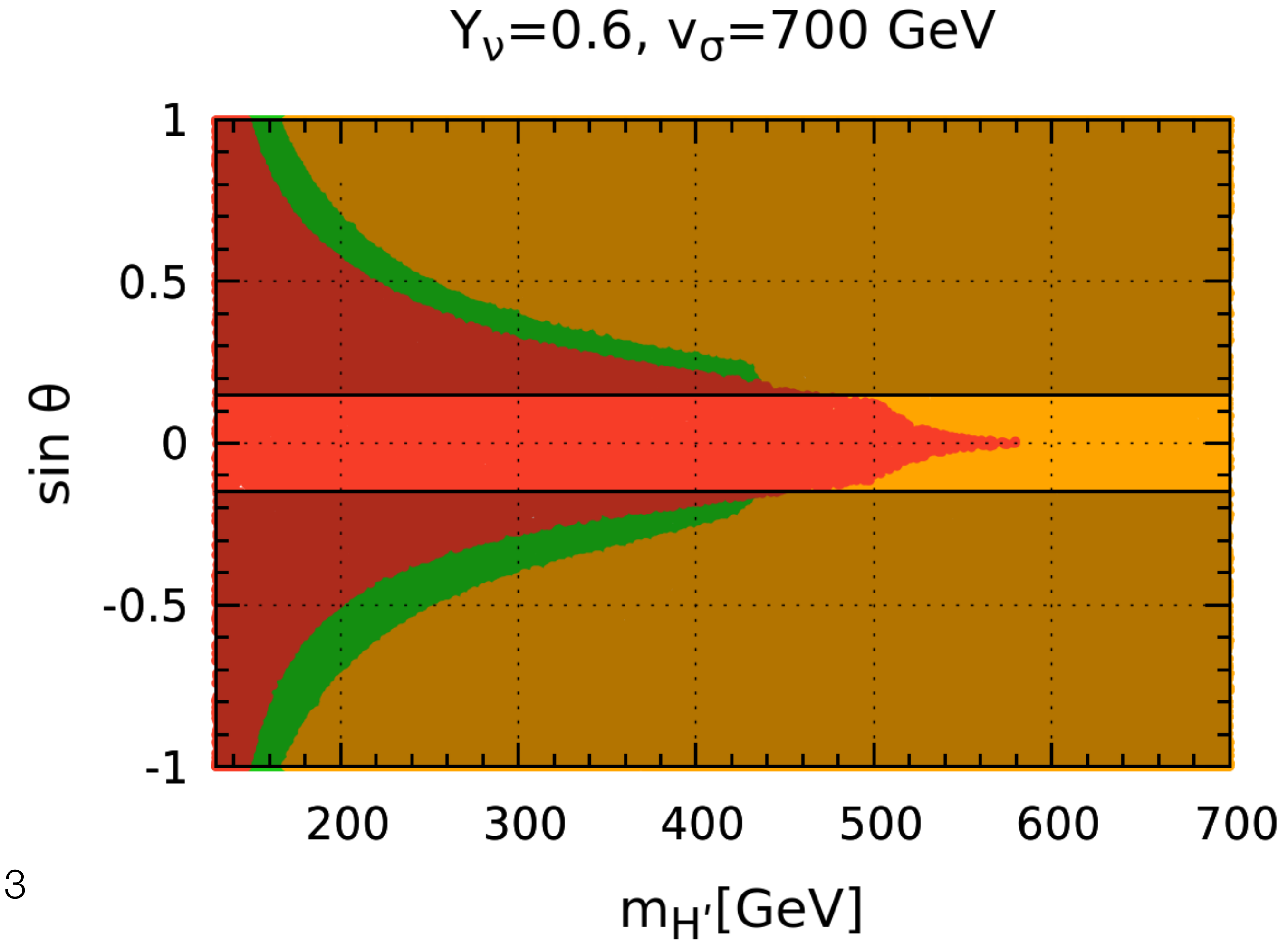
Landau poles are included

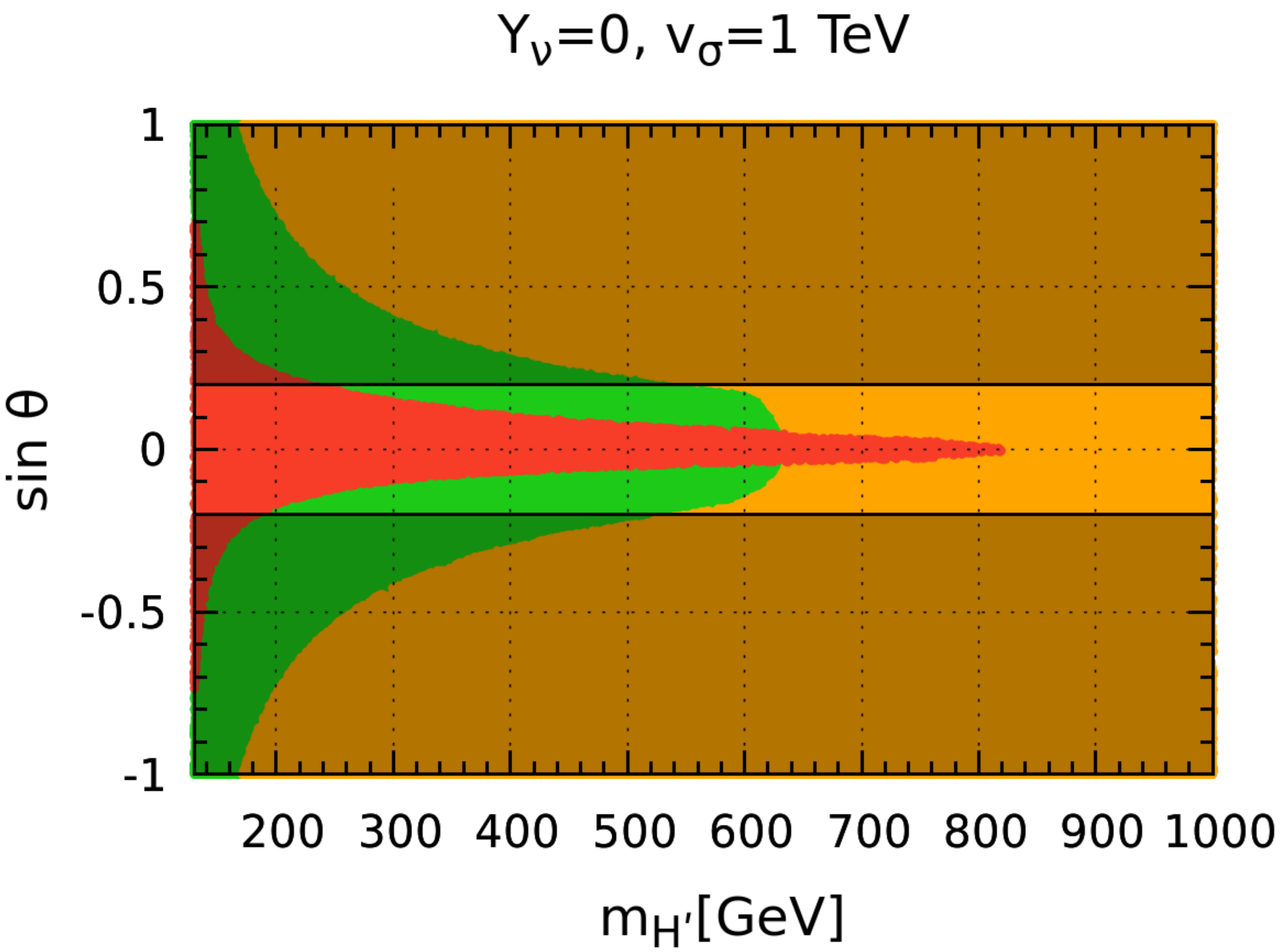
Interplay of stability and Collider constraints:

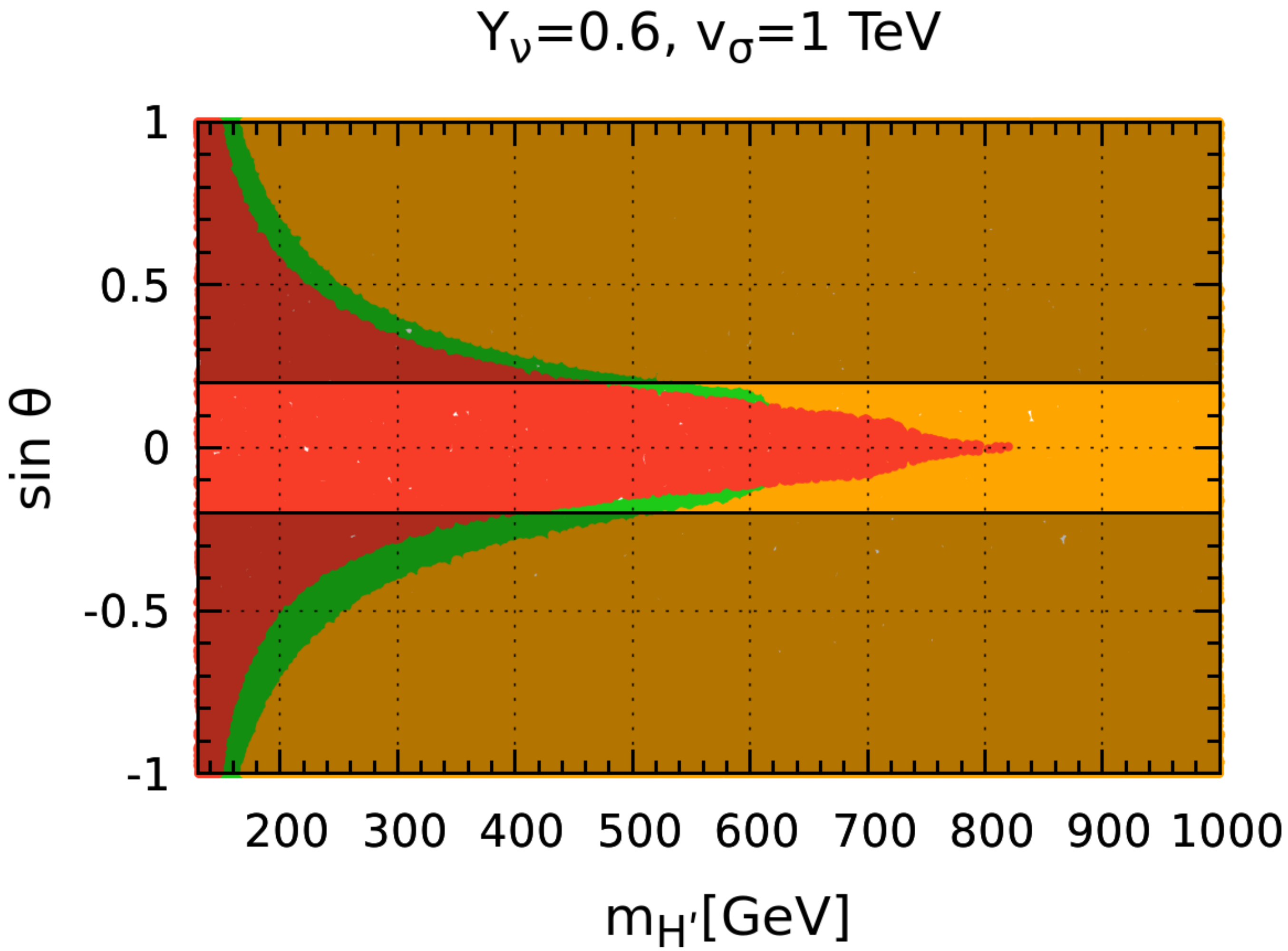


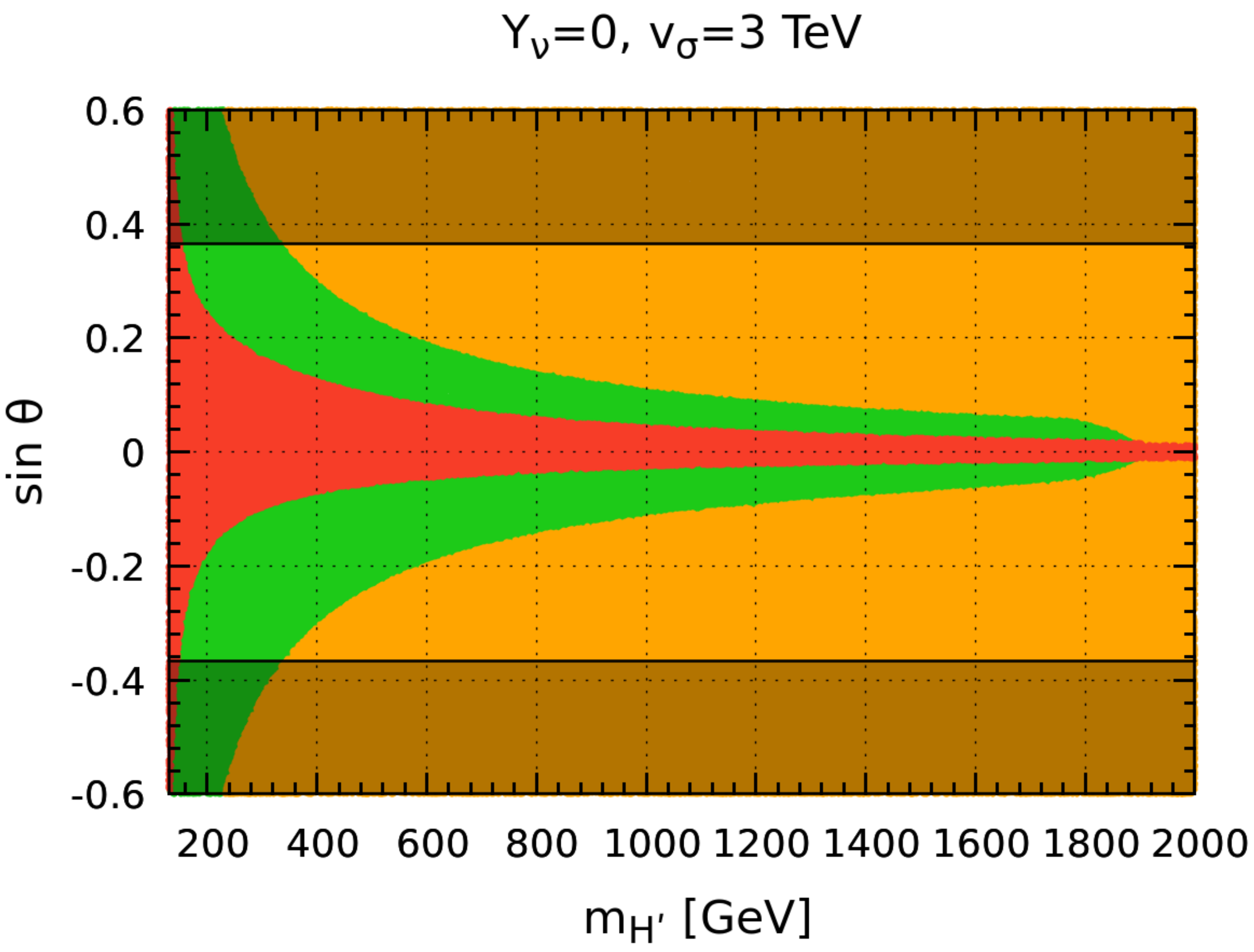


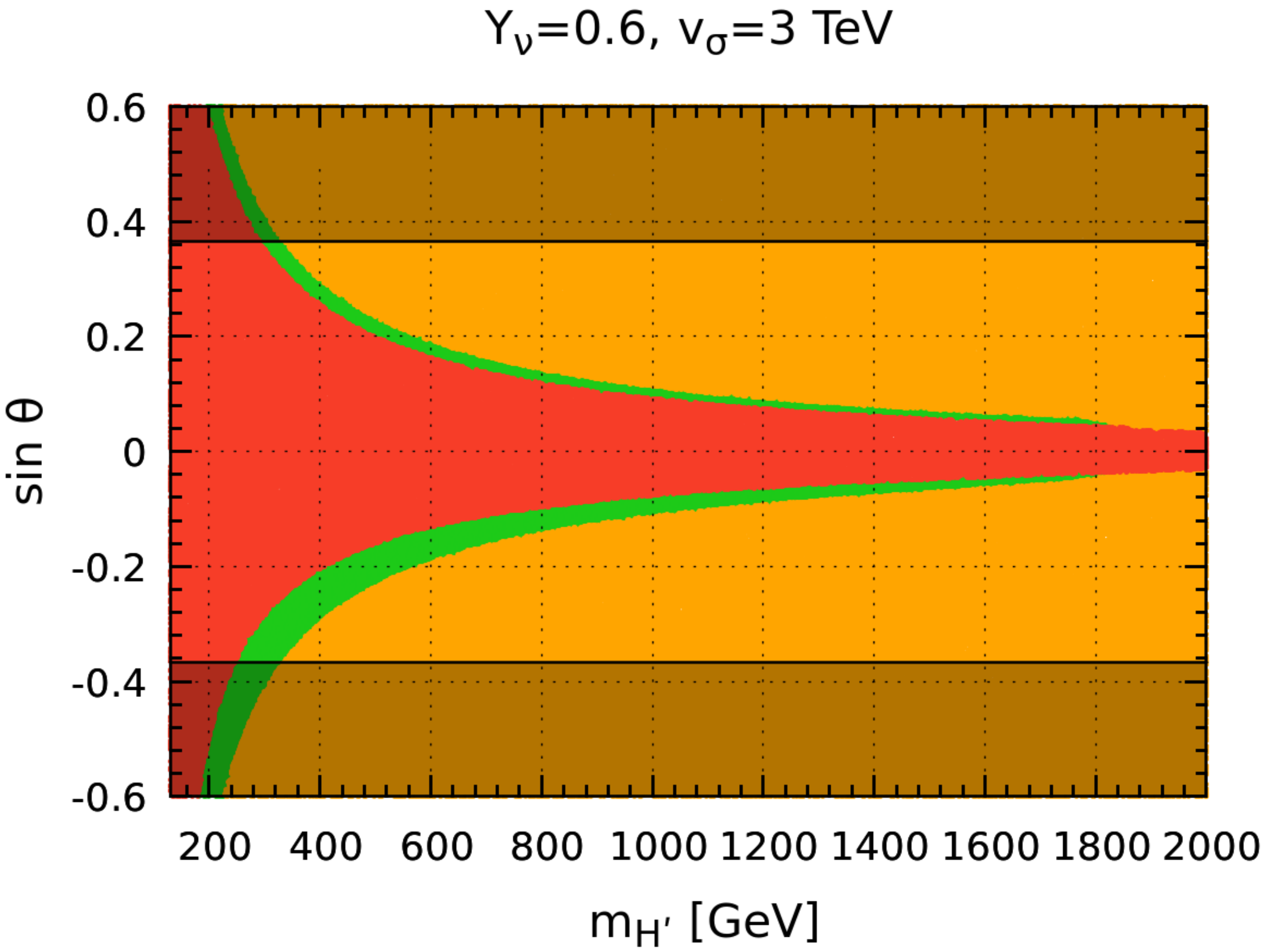




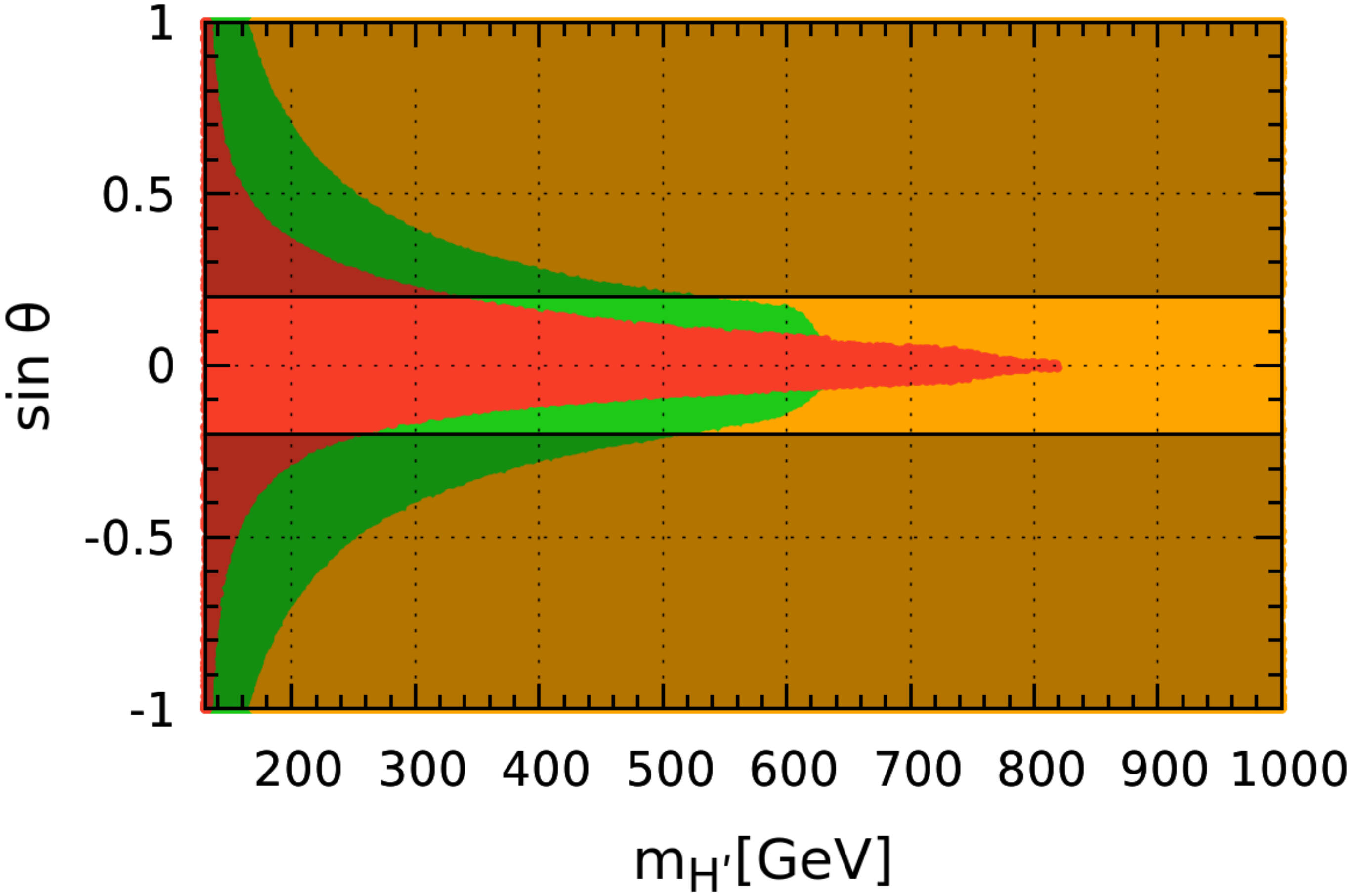

$Y_\nu=0$, $v_\sigma=1$ TeV


$Y_\nu=0.6$, $v_\sigma=1$ TeV

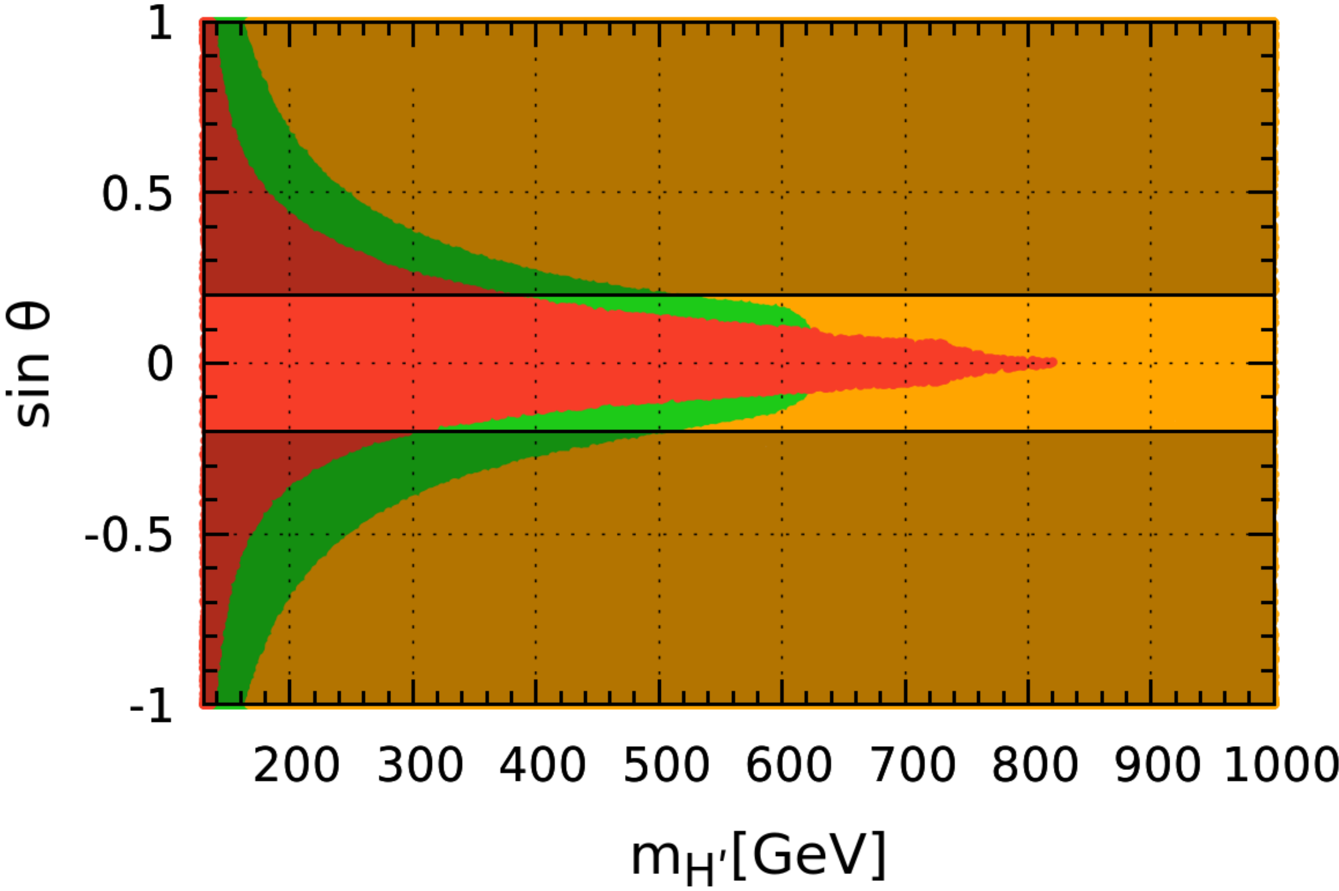

$Y_\nu=0$, $v_\sigma=3$ TeV


$Y_\nu=0.6$, $v_\sigma=3$ TeV

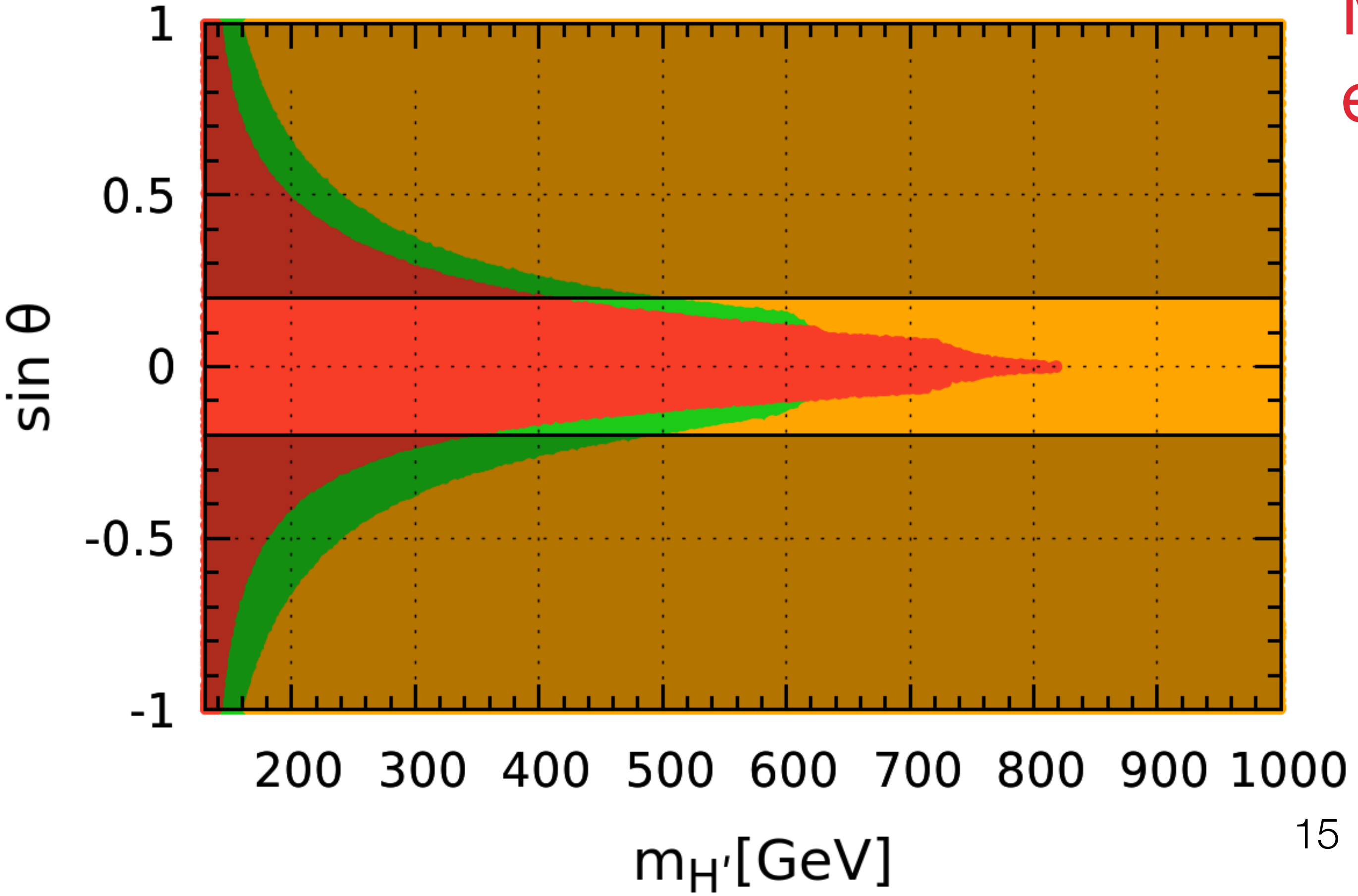
$Y_\nu=0.4,\ v_\sigma=1$ TeV



$Y_\nu=(0.4,0.4),\ v_\sigma=1$ TeV



$Y_\nu=(0.4,0.4,0.4),\ v_\sigma=1$ TeV



More generations implies more negative effect thats why green region shrinks

Conclusions: In presence of invisible Higgs decay constraint on the mixing angle between two CP-even Higgs is weaker and this helps to have stable vacuum

Summary:

- SM vacuum is metastable but very sensitive to top mass
- In neutrino mass models additional fermions has destabilising effect on vacuum

In low-scale seesaw with explicit lepton number breaking,
λ becomes negative much before the SM instability scale
for large Yukawa coupling

- In seesaw models with dynamical lepton number breaking, stability can be restored

- Advantage with dynamical low-scale seesaw model: neutrino mass, large heavy-light neutrino mixing, dark matter candidate, consistent vacuum, TeV scale heavy neutrino

Thank You for your attention

Back Up

Non-perturbative dynamics……..

Landau Pole: If $\beta_c = Ac^2$ one has

$$\mu \frac{dc(\mu)}{d\mu} = Ac^2(\mu) \Rightarrow \int_{M_Z}^{\mu} \frac{dc(\mu)}{Ac^2(\mu)} = \int_{M_Z}^{\mu} \frac{d\mu}{\mu} \Rightarrow c(\mu) = \frac{c(M_Z)}{1 - Ac(M_Z)\log\frac{\mu}{M_Z}}$$

Generalized: $\mu \frac{dc(\mu)}{d\mu} = Ac^n(\mu) \Rightarrow c(\mu) = \frac{c(M_Z)}{\left(1-(n-1)Ac(M_Z)^{(n-1)}\log\frac{\mu}{M_Z}\right)^{\frac{1}{n-1}}}$ $n > 1$ gives Landau pole

Continuous growth:

For $n \leq 1$, we will not have pole but $c(\mu)$ grows continuously. For example with $n = \frac{1}{2}$

$$c(\mu) = c(M_Z)\left(1 + \frac{A}{2\sqrt{c(M_Z)}}\log\frac{\mu}{M_Z}\right)^2,$$

and with $n = 0$: $\mu \frac{dc(\mu)}{d\mu} = A \Rightarrow c(\mu) = c(M_Z) + A\log\frac{\mu}{M_Z}$

Saturation:

If β_c has a zero at the finite value $c(\mu_*)$ then the growth of c will be saturated at $c(\mu_*)$ for $\mu \to \infty$.

Example: $\beta_c = (A - Bc(\mu))$. Lets say this has a zero at $c(\mu_*) = \frac{A}{B}$.

$$\mu \frac{dc(\mu)}{d\mu} = (A - Bc(\mu)) \Rightarrow \int_{\mu_*}^{\mu} \frac{dc(\mu)}{\left(A - Bc(\mu)\right)} = \int_{\mu_*}^{\mu} \frac{d\mu}{\mu} \Rightarrow c(\mu) = \frac{1}{B\mu^B}\left(-A\mu_*^B + B\mu_*^B c(\mu_*) + A\mu^B\right)$$

$$\Rightarrow c(\mu) = \frac{1}{B\mu^B}\left(-A\mu_*^B + B\mu_*^B c(\mu_*) + A\mu^B\right)$$

Hence $c(\mu) = \frac{A}{B}$ for any μ .

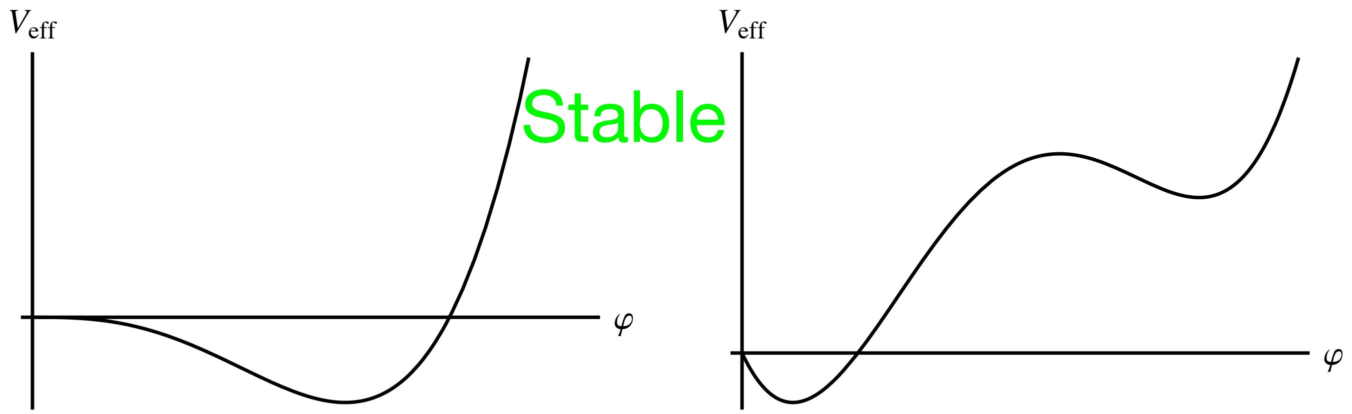
Effective potential at large field value: $V_{\text{eff}} \approx \lambda(\mu)\phi(\mu)^4$

V_{eff} depends on the running of λ

Few Possibilities: $1.$ If $A \approx 0$ and m_h is large : Landau Pole → $\lambda(\Lambda) = \dfrac{\lambda(v)}{1 - \frac{24}{(4\pi)^2}\lambda(v)\ln\frac{\Lambda}{v}}$

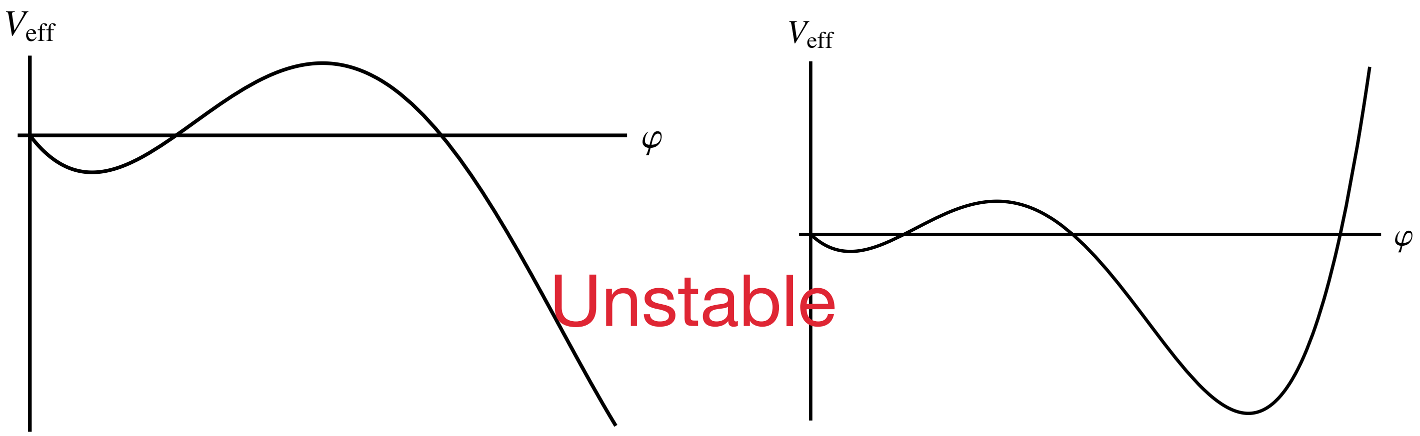
$2.$ $A < 0$ at weak scale but if not run negative enough at large ϕ :

V_{eff} is bounded from below



$3.$ $A < 0$ at weak scale and run negative enough at large ϕ :

V_{eff} is unbounded from below



$4.$ $A < 0$ at weak scale but change sign at large ϕ : V_{eff} develops another minimum