Institute of High Energy Physics

Positivity in Multi-Field EFTs

Xu Li
Institute of High Energy Physics

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base on 2101.01191 with C. Yang, H. Xu, C. Zhang, and S.-Y. Zhou

Motivation: Positivity Bounds

All possible
Ultraviolet(UV) physics

EFT:

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

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$$\begin{cases} \sum_i a_i C_i \geq 0 \\ \vdots \end{cases}$$

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Positivity Bounds

2-to-2 forward amplitude (spin-0): $\mathcal{M}(s,0) = c_0 + c_2 s^2 + c_4 s^4 + \cdots$

To extract dim-8 effect, we consider:

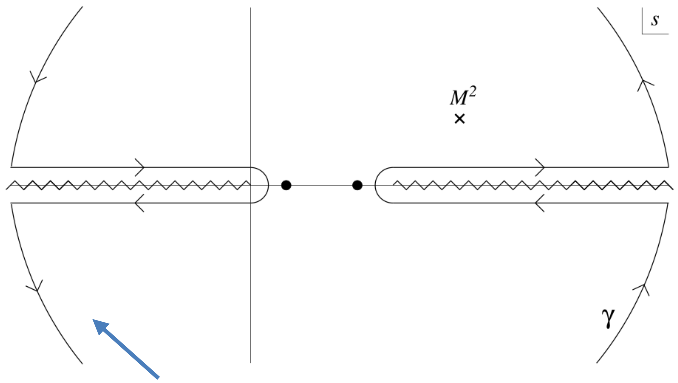
Dim-8 have leading energy dependence only, s^2.

$$c_2 = \frac{d^2}{ds^2}\mathcal{M}(s,0)$$

For elastic scattering $ij \to ij$

[A. Adams et al., JHEP 06]

Analyticity in s (causality)



Froissart bound (unitarity)

$$I = \oint_\gamma \frac{ds}{2\pi i}\frac{\mathcal{M}(s,0)}{(s-M^2)^3}$$

$$\frac{d^2}{ds^2}\mathcal{M}(s,0) = \frac{4}{\pi}\int ds \frac{s\sigma(s)}{s^3} + \mathcal{O}\left(\frac{M^2, m^2}{\Lambda^2}\right)$$

$= $ positive up to power suppressed corrections

$$c_2 > 0$$

Elastic Positivity Bounds

Superposition elastic scattering:

with $|u\rangle = u^i|i\rangle, |v\rangle = v^j|j\rangle$

General elastic bounds

$$\frac{d^2}{ds^2}\mathcal{M}_{uv\to uv}(s,0) = u^i v^j u^{k*} v^{l*} M^{ijkl} \geq 0$$

Arbitrary vectors

EFTs can involve more than one particles

define $M^{ijkl} \equiv \frac{d^2}{ds^2}\mathcal{M}_{ij\to kl}(s,0)$

However, the elastic bounds are not the optimal !

When numbers of the fields n larger than 2

Relevant developments

General 2-to-2 scattering of massless scalars:

$$\hat{\mathcal{M}}(\hat{s},t) = \sum_{n+m>0} c_{n,m}\hat{s}^n t^m$$

- $t=0$
- $c_n, n \geq 2$
- One-field

- $t \neq 0$

- $t=0$
- c_2
- Multi-field

Higher-dimension coef.

[N. Arkani-Hamed, et al. 2012.15849], [B. Bellazzini. et al. 2011.00037] [L-Y Chiang, et al, 2105.02862]

Beyond the forward limit

[A. Tolley et al., 2011.02400], [S. C-Huot. et al. 2011.02957]

Bounds for multi-field

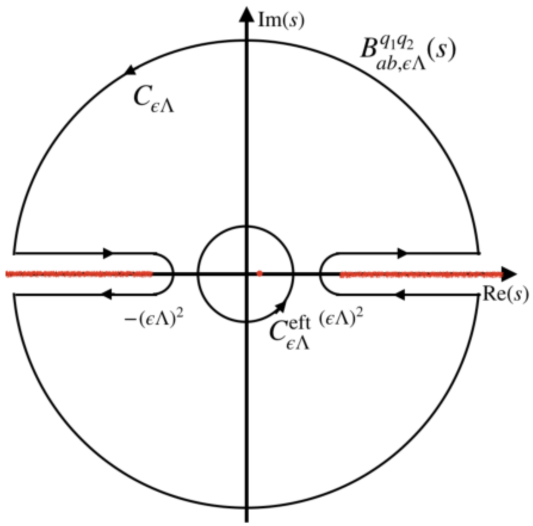
[T. Trott, 2011.10058], [CZ and S.-Y. Zhou PRL 125, 201601] [X. Li, et al, 2101.01191]

This talk will focus on this direction

Master formula

[more details see 1702.06134, 1902.08977,2011.10058]

2-to-2 forward scattering($t \approx 0$) for $ij \to kl$: $f = \frac{1}{2\pi i}\oint ds \frac{\mathcal{M}(s,0)}{(s-M^2)^3}$

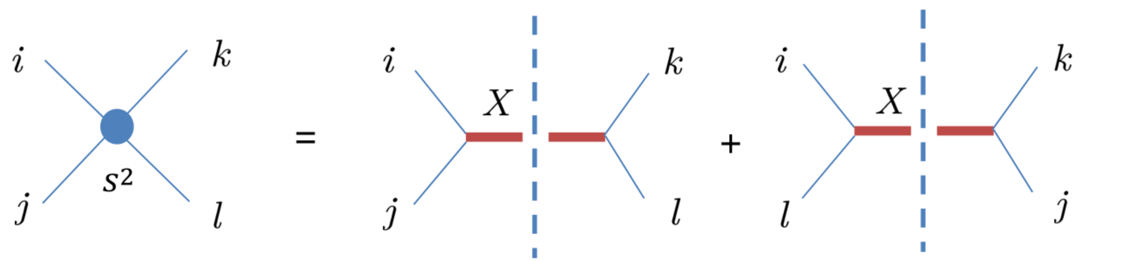


$$M^{ijkl} \equiv \frac{d^2}{ds^2}\mathcal{M}_{ij\to kl}(s,0)$$

$$= \frac{1}{2\pi i}\int_{(\epsilon\Lambda)^2}^{\infty} ds' \left(\frac{\mathrm{Disc}\mathcal{M}_{ij\to kl}(s')}{(s'-M^2/2)^3} - \frac{\mathrm{Disc}\mathcal{M}_{ij\to kl}(s')}{(s'-M^2/2)^3}\right)$$

$$+ \text{ residues at poles}$$

Optical theorem: $\mathrm{Disc}\mathcal{M}^{ijkl}(s) = \sum_X \mathcal{M}^{ij\to X}(s)\left(\mathcal{M}^{kl\to X}(s)\right)^*$



$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{{m_X}^{ij}{m_X}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

$m_X^{ij} \equiv \mathcal{M}^{ij\to X}$

Convex cone nature

[CZ and S.-Y. Zhou PRL 125, 201601]

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{m_X{}^{ij} m_X{}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

→ M^{ijkl} is **positive** linear combination of $m_X{}^{ij} m_X{}^{kl} + m_X{}^{il} m_X{}^{kj}$

→ 1. M^{ijkl} is a convex cone

We defined the cone as $\mathbf{C}^{n^4}$

Positivity bounds arise as boundary of cone!

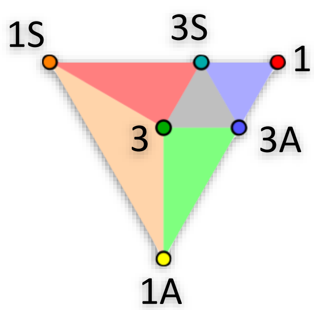
4-Higgs operators

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi],$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi],$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi].$$

→

$$F_{S,0} \geq 0\,,$$
$$F_{S,0} + F_{S,2} \geq 0\,,$$
$$F_{S,0} + F_{S,1} + F_{S,2} \geq 0\,.$$

Triangular cone



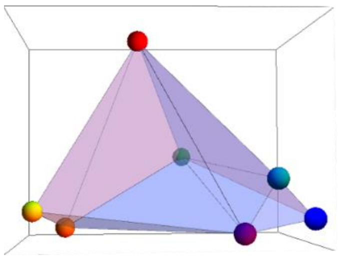
4-W operators

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

→

$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 0,$$
$$F_{T,2} + 8F_{T,10} \geq 0,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.$$

6-facet 4D cone



Xu Li

EFT without symmetry?

Solution : use the dual property of cone [X. Li, et al, 2101.01191]

Dual cone is defined as

$$\mathbf{C}^{n^4 *} = \{\mathcal{Q} | \mathcal{Q} \cdot \mathcal{M} \geq 0, \forall \mathcal{M} \in \mathbf{C}^{n^4}\}$$

1. C* is a cone.
2. C* is a set that contain all possible linear bounds
3. Hyperplane separation theorem→(C*)*=C
   ——it is enough to carve out exactly the C

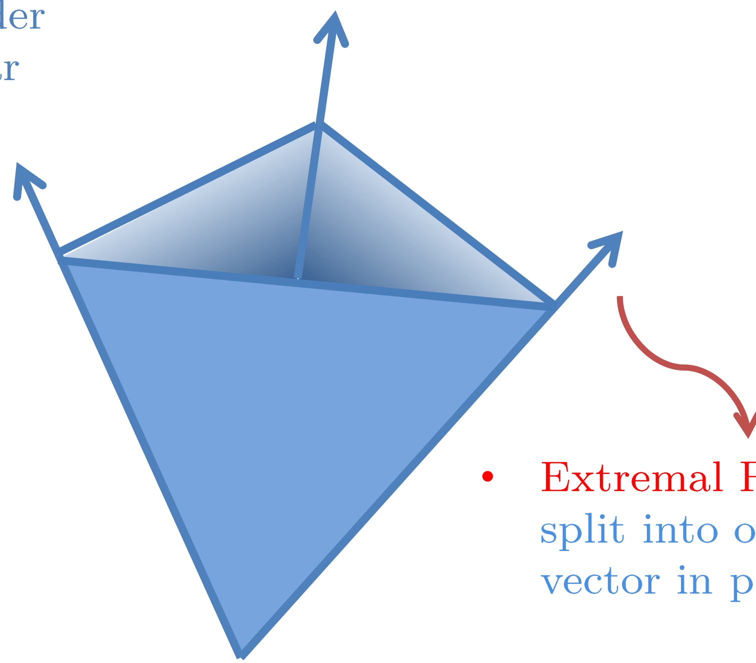
These properties make sure our bounds are complete

Dual cone

posi. bounds
= vectors in dual cone

Independent possible bounds ⟷ **Extremal Rays of cone C***

- A convex cone is closed under additions and positive scalar multiplications



- Extremal Ray (ER) : $\vec{n}_i^{ex}$ cannot be split into other vectors (like an edge vector in polyhedral cone)

dual cone: C*

Dual cone

How to find the ERs in dual cone? ...

Index symmetries of M^{ijkl}

$i \leftrightarrow k$ or $j \leftrightarrow l$ ⟷ Crossing symmetry: $s \leftrightarrow u$

$i \leftrightarrow j$ + $k \leftrightarrow l$ ⟷ Rotation symmetry (Pi around y-axis)

Defined a subspace of M : $\mathcal{M} \in \vec{\mathbf{S}}^{n^4}$ $(\mathcal{M}^{ijkl} = \mathcal{M}^{jilk} = \mathcal{M}^{klij} = \mathcal{M}^{ilkj})$

Dual cone: $\mathcal{Q} \in \vec{\mathbf{S}}^{n^4}$

cross-antisymmetric one will vanish

$$\mathcal{Q} \cdot \mathcal{M} \geq 0$$

$$\Rightarrow \mathcal{Q}^{ijkl} \sum_\alpha (m_\alpha^{ij} m_\alpha^{kl} + m_\alpha^{il} m_\alpha^{kj}) = 2 \sum_\alpha m_\alpha^{ij} \mathcal{Q}^{ijkl} m_\alpha^{kl}$$

$$\Rightarrow \mathcal{Q}^{(ij),(kl)} \succcurlyeq 0 \Rightarrow \mathcal{Q} \in \mathbf{S}_+^{n^2 \times n^2}$$

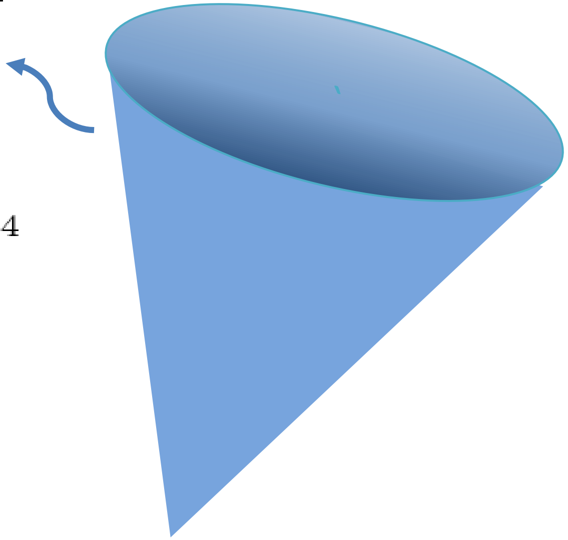
➡ $$\mathbf{Q}^{n^4} = \mathbf{S}_+^{n^2 \times n^2} \cap \vec{\mathbf{S}}^{n^4}$$

Semi-definite matrices

Dual cone---Spectrahedron

$\mathbf{S}_{+}^{n^2\times n^2}$: the set of n × n positive semi-definite matrices forms a convex cone

$\mathbf{S}_{+}^{n^2\times n^2}$



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Spectrahedron: the intersection of a cone with a linear (affine) subspace is well-defined in math

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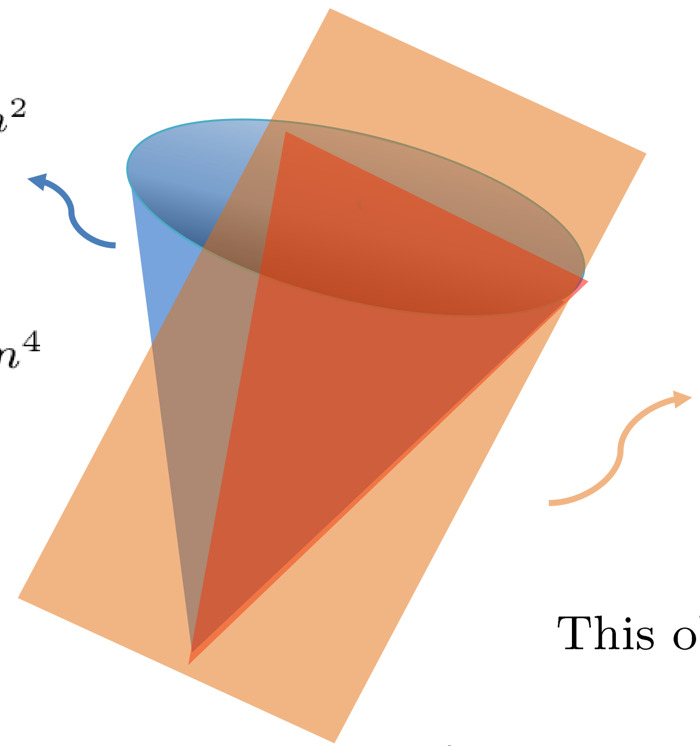

$\vec{\mathbf{S}}^{n^4}$

This object is a spectrahedron !

Spectrahedron: the intersection of a cone with a linear (affine) subspace

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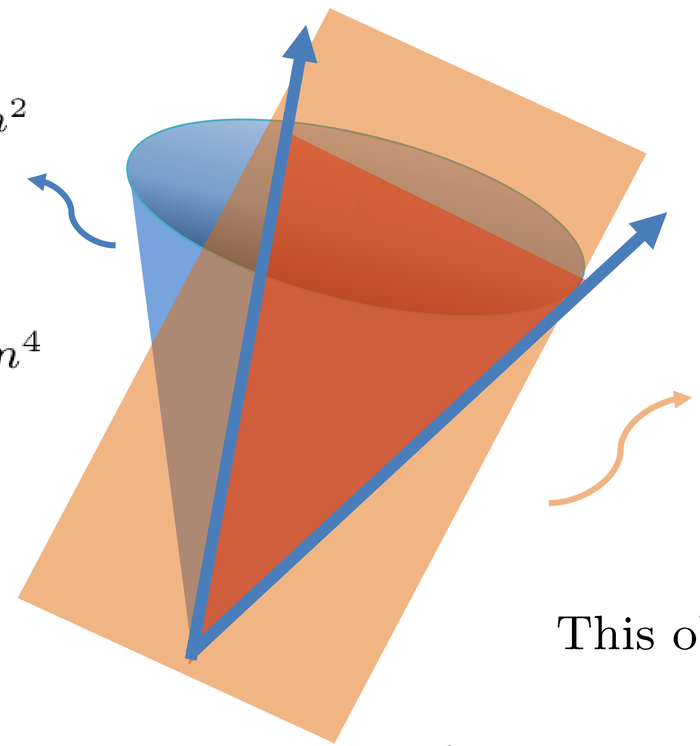

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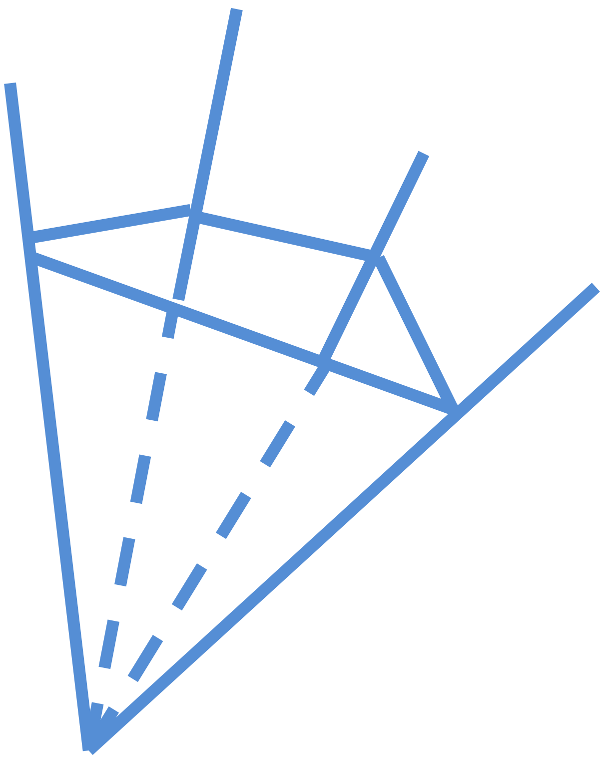
Ultimate goal : **finding ERs of Spectrahedron!**

The “MC” approach

Randomly search ERs



The "MC" approach

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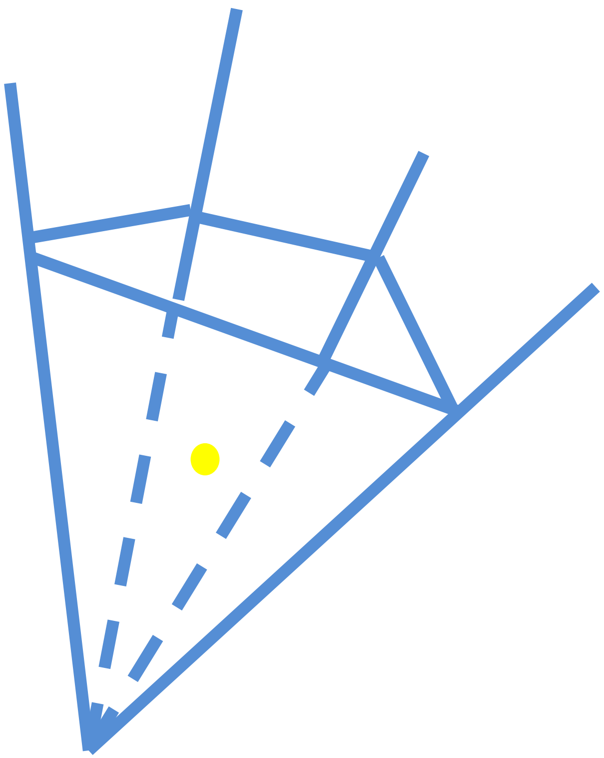
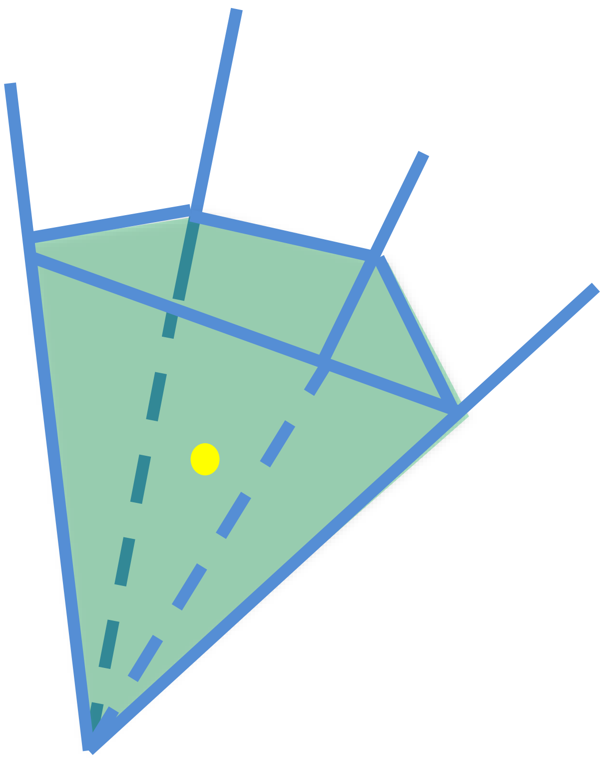
☛ Start with a **random point x**



The “MC” approach



Randomly search ERs

- Start with a **random point x**
- Find the (k-)face F(x)
  See [Ramana & Goldman 1995]

The "MC" approach

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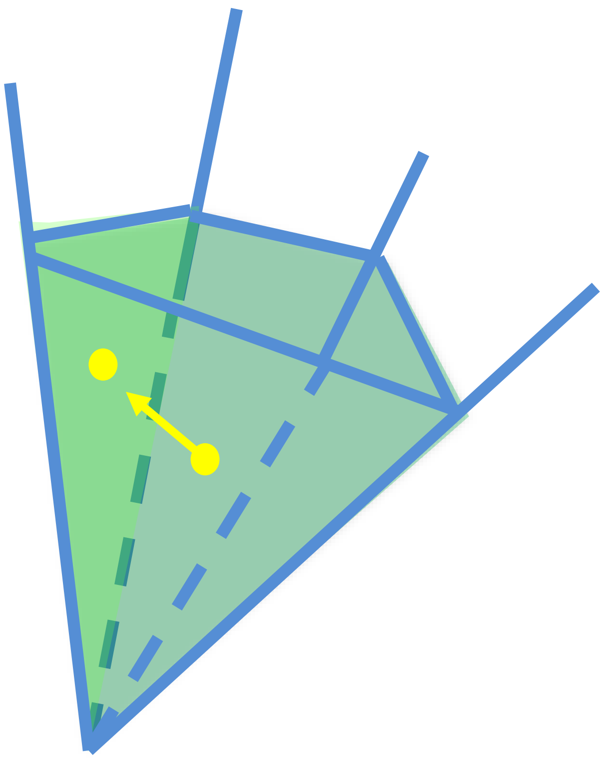

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- Take a random straight-line in F(x) that crosses x. Find its **intersection** with the boundary of the cone (this is a SDP).

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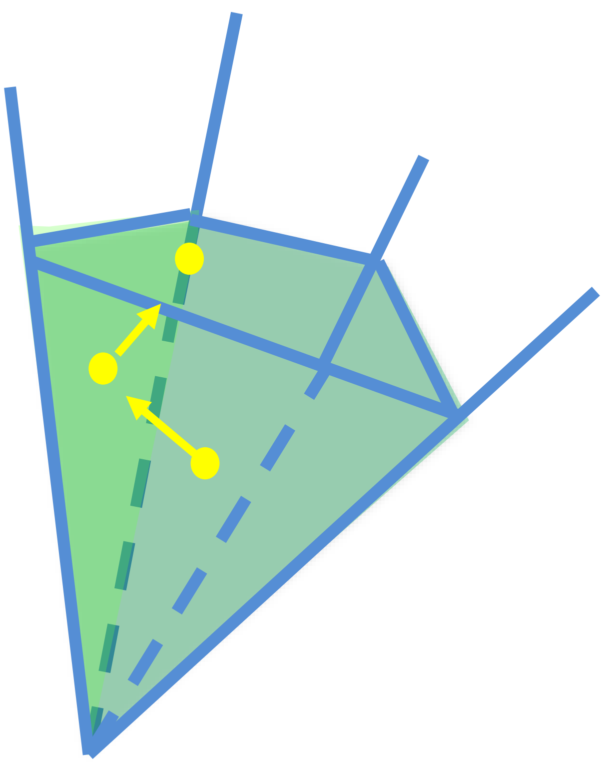

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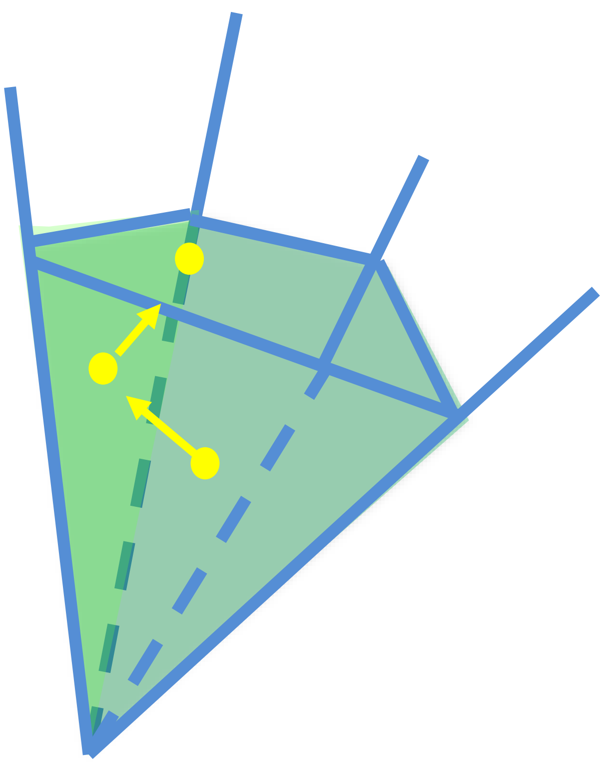

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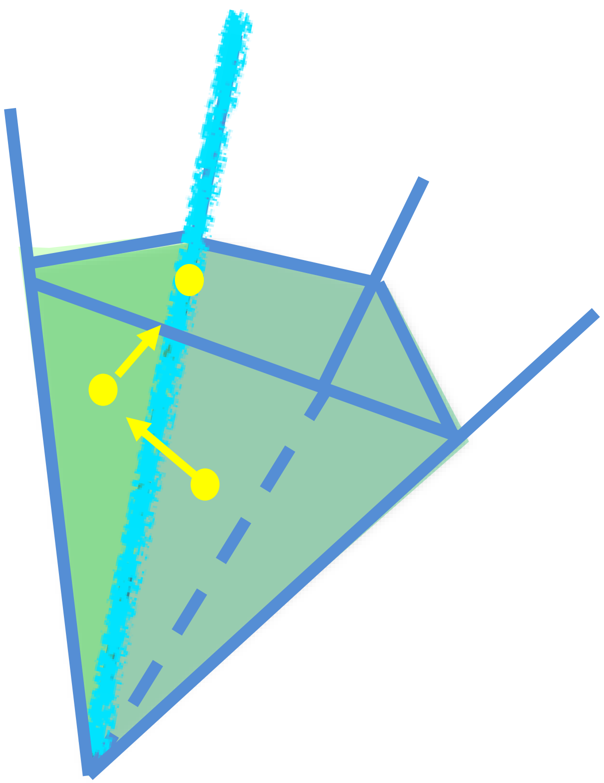

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- Take x to be the intersection point and **iterate**, if F(x) is not **dimension 1**
- If F(x) is **dimension 1**, An ER is found.

The SDP approach

If ERs are finite: The "MC" approach is enough

If ERs are infinite: We need the SDP approach also

The semi-definite programming (SDP) approach:

Given a M

$$\min \quad \mathcal{Q} \cdot \mathcal{M}$$

$$\text{subject to} \quad \mathcal{Q} \in \text{spectrahedron}$$

If the minimum is not negative, then M is allowed by positivity.

Applications

General 2-scalar case

$$\mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$$

$i,\ j,\ k,\ l = 1{,}2$

Using operators we can calculate M^{ijkl} , which is a function of Ci's

Crossing symmetry S^{n^4} will restrict the form

$$\mathbf{Q}^{2^4} \ni \mathcal{Q} = \begin{pmatrix} a & b & e & e \\ b & c & f & f \\ e & f & d & b \\ e & f & b & d \end{pmatrix}$$

1. They are extremal
2. They are complete

ERs Q_{ex} ->

$$\begin{matrix} kl = & 11 & 22 & 12 & 21 & \\ & \begin{bmatrix} a^2 & ab & ac & ac \\ ab & b^2 & bc & bc \\ ac & bc & 2c^2 - ab & ab \\ ac & bc & ab & 2c^2 - ab \end{bmatrix} & & & & \begin{matrix} ij = 11 \\ 22 \\ 12 \\ 21 \end{matrix} \end{matrix}$$

With $c^2 \geq ab$

$\mathcal{Q}_{\text{ex}} \cdot M \geq 0$ will give the bounds on Ci's !

Positivity bounds for 2-scalar EFTs

Finally get bounds !

$$
\begin{aligned}
& C_{1111} \geq 0 \quad \text{and} \quad 4C_{1111}C_{1212} - C_{1112}^2 \geq 0 \\
& \text{and} \quad \Big\{ C_{1112}C_{1122}C_{1222} - C_{1111}C_{1222}^2 - C_{1112}^2 C_{2222} + C_{1212}\left(-C_{1122}^2 + 4C_{1111}C_{2222}\right) \geq 0 \\
& \text{or} \quad \Big[\Delta \equiv 3\left(4C_{1111}C_{2222} - C_{1112}C_{1222}\right) + \left(C_{1122} + C_{1212}\right)^2 \geq 0 \\
& \text{and} \quad \frac{3C_{1112}^2}{4C_{1111}} - 2\left(C_{1122} + C_{1212}\right) \leq \sqrt{\Delta} \leq C_{1212} - 2C_{1122} \\
& \text{and} \quad 2\Delta^{3/2} \geq 27\left(C_{1111}C_{1222}^2 + C_{1112}^2 C_{2222}\right) - 9\left(C_{1122} + C_{1212}\right)\left(8C_{1111}C_{2222} + C_{1112}C_{1222}\right) \\
& \qquad + 2\left(C_{1122} + C_{1212}\right)^3 \Big] \Big\}
\end{aligned}
$$

What if n > 2 ?

—— **resort to the numerical approach**

4-gluon case $i, j, k, l = g$ (n=16 fields)

EFT operators:

$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$

$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$

Plus a $(\mathrm{D6})^2$ term: $f^{abc}G^{a\nu}_{\mu}G^{b\rho}_{\nu}G^{c\mu}_{\rho}$

$\vec{n}\cdot\vec{C} \geq 0 \;\rightarrow\;$ n given by

[0, 0, 0, 1, 0, 0, 0]	[0, 0, 6, 3, 7, 2, 0]	[24, 0, 12, 21, 15, 14, 0]	[0, 0, 96, 24, 64, 40, −81]
[0, 0, 1, 1, 1, 0, 0]	[8, 6, 1, 6, 0, 2, 0]	[24, 32, 24, 4, 8, 0, −27]	[40, 32, 80, 4, 0, 0, −189]
[2, 0, 1, 0, 0, 0, 0]	[0, 6, 3, 12, 5, 0, 0]	[48, 36, 21, 27, 25, 0, 0]	[0, 0, 24, 120, 40, 104, −81]
[0, 2, 0, 1, 0, 0, 0]	[8, 6, 1, 12, 0, 0, 0]	[32, 40, 4, 80, 0, 0, −27]	[0, 0, 120, 24, 104, 40, −81]
[0, 0, 3, 0, 2, 0, 0]	[0, 6, 6, 9, 10, 4, 0]	[0, 48, 0, 48, 0, 40, −81]	[96, 0, 144, 24, 64, 40, −81]
[0, 0, 0, 3, 0, 2, 0]	[0, 12, 0, 14, 0, 0, −9]	[24, 0, 36, 24, 16, 40, −81]	[48, 0, 96, 24, 0, 40, −243]
[1, 1, 2, 2, 0, 0, 0]	[0, 0, 8, 8, 0, 8, −27]	[0, 0, 48, 24, 32, 40, −81]	[0, 192, 168, 96, 112, 120, −405]
[6, 0, 3, 0, 2, 0, 0]	[12, 0, 14, 0, 0, 0, −27]	[0, 0, 24, 48, 16, 56, −81]	[168, 480, 168, 156, 56, 160, −729]
[4, 2, 2, 1, 2, 0, 0]	[6, 8, 12, 1, 0, 0, −27]	[88, 32, 56, 4, 40, 0, −27]	[264, 384, 156, 168, 16, 200, −729]
[0, 0, 4, 0, 0, 0, −9]	[8, 16, 4, 8, 0, 8, −27]	[96, 42, 27, 84, 25, 0, 0]	[288, 384, 216, 168, 0, 200, −891]
[6, 0, 6, 0, 5, 0, 0]	[0, 24, 0, 12, 0, 8, −27]	[96, 66, 42, 39, 50, 4, 0]	[480, 384, 480, 168, 160, 200, −729]
[0, 0, 3, 6, 5, 4, 0]	[8, 22, 1, 14, 0, 10, −27]	[120, 42, 39, 42, 40, 14, 0]	[336, 768, 672, 216, 0, 200, −2187]

4-gluon case

$i, j, k, l = g$ (n=16 fields)

EFT operators:

$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu} G^{A\mu\nu})(G^B_{\rho\sigma} G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu} \widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma} \widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu} G^{B\mu\nu})(G^A_{\rho\sigma} G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu} \widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma} \widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(7)}$	$d^{ABE} d^{CDE} (G^A_{\mu\nu} G^{B\mu\nu})(G^C_{\rho\sigma} G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE} d^{CDE} (G^A_{\mu\nu} \widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma} \widetilde{G}^{D\rho\sigma})$

Plus a $(D6)^2$ term: $f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}$

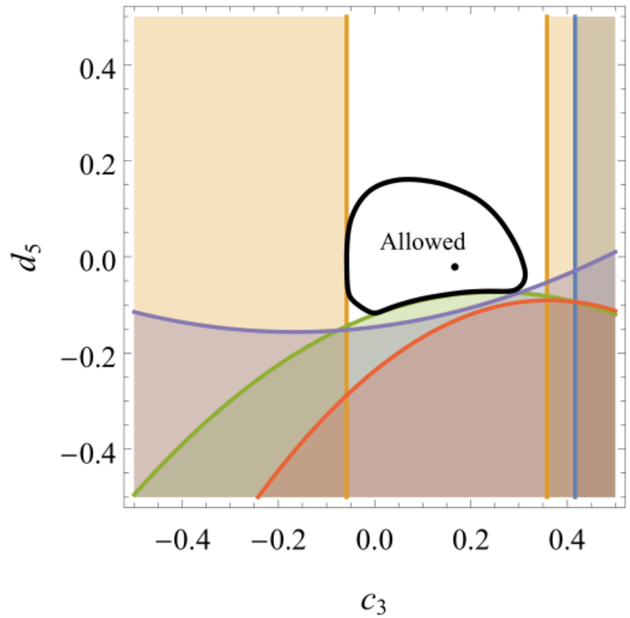
$\vec{n} \cdot \vec{C} \geq 0 \;\rightarrow\;$ n given by **7D polyhedral cone with 48 facets!**

$[0,0,0,1,0,0,0]$	$[0,0,6,3,7,2,0]$	$[24,0,12,21,15,14,0]$	$[0,0,96,24,64,40,-81]$
$[0,0,1,1,1,0,0]$	$[8,6,1,6,0,2,0]$	$[24,32,24,4,8,0,-27]$	$[40,32,80,4,0,0,-189]$
$[2,0,1,0,0,0,0]$	$[0,6,3,12,5,0,0]$	$[48,36,21,27,25,0,0]$	$[0,0,24,120,40,104,-81]$
$[0,2,0,1,0,0,0]$	$[8,6,1,12,0,0,0]$	$[32,40,4,80,0,0,-27]$	$[0,0,120,24,104,40,-81]$
$[0,0,3,0,2,0,0]$	$[0,6,6,9,10,4,0]$	$[0,48,0,48,0,40,-81]$	$[96,0,144,24,64,40,-81]$
$[0,0,0,3,0,2,0]$	$[0,12,0,14,0,0,-9]$	$[24,0,36,24,16,40,-81]$	$[48,0,96,24,0,40,-243]$
$[1,1,2,2,0,0,0]$	$[0,0,8,8,0,8,-27]$	$[0,0,48,24,32,40,-81]$	$[0,192,168,96,112,120,-405]$
$[6,0,3,0,2,0,0]$	$[12,0,14,0,0,0,-27]$	$[0,0,24,48,16,56,-81]$	$[168,480,168,156,56,160,-729]$
$[4,2,2,1,2,0,0]$	$[6,8,12,1,0,0,-27]$	$[88,32,56,4,40,0,-27]$	$[264,384,156,168,16,200,-729]$
$[0,0,4,0,0,0,-9]$	$[8,16,4,8,0,8,-27]$	$[96,42,27,84,25,0,0]$	$[288,384,216,168,0,200,-891]$
$[6,0,6,0,5,0,0]$	$[0,24,0,12,0,8,-27]$	$[96,66,42,39,50,4,0]$	$[480,384,480,168,160,200,-729]$
$[0,0,3,6,5,4,0]$	$[8,22,1,14,0,10,-27]$	$[120,42,39,42,40,14,0]$	$[336,768,672,216,0,200,-2187]$

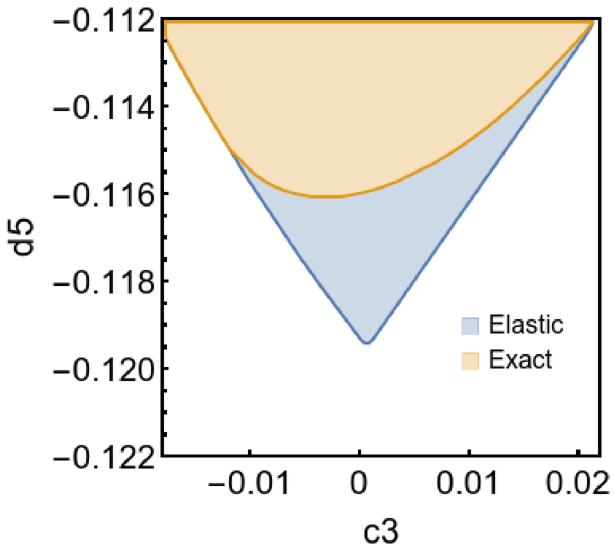
Example: Spin-2 EFT

- dRGT massive gravity (n = 5) —— (c3, d5):

  [PRL.106(2011) 231101, C. de Rham, et, al]



Elastic: elastic approach(superposed)
[JHEP 04 (2016) 002. C. Cheung and G. Remmen]



- Exact: SDP approach:

  improves slightly the minimum value of d5.

Example: Spin-2 EFT

- dRGT massive gravity (n = 5) —— (c3, d5):

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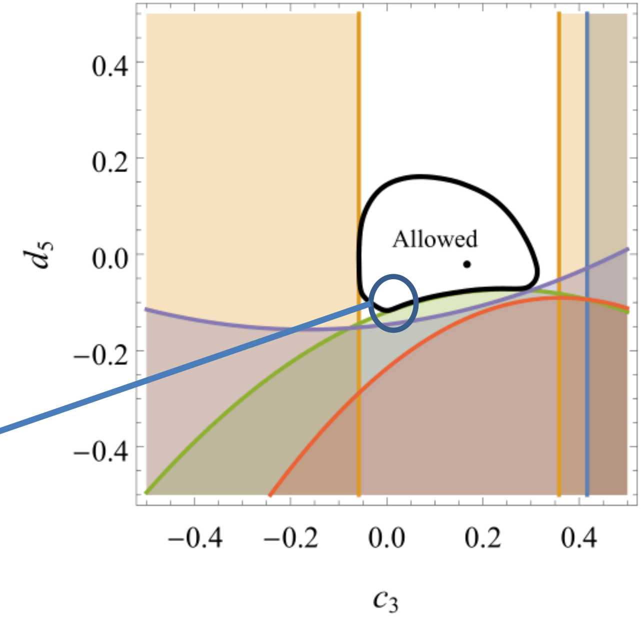
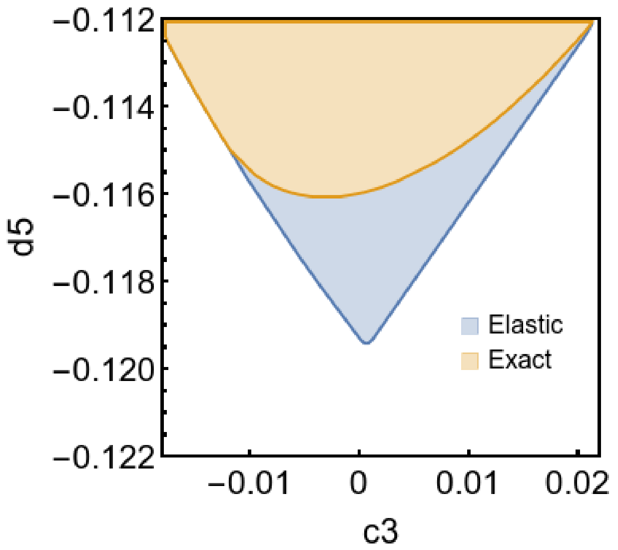


Elastic: elastic approach(superposed)
[JHEP 04 (2016) 002. C. Cheung and G. Remmen]



- Exact: SDP approach:

  improves slightly the minimum value of d5.

Summary

- Positive structures arise at the dim-8 level in EFT coefficient space, as a consequence of **axiomatic QFT principles**.

- We convert the problem of finding bounds to a geometric problem: finding the ERs of a spectrahedron.

  – can be solved using the **semi-definite programming**

- Improved previous results, and gave some new results.

Institute of High Energy Physics

Thank You!

Institute of High Energy Physics

Backup

Non elastic bounds for n=3

$$\mathcal{Q}_{ex} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 2 & 3 & 0 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 3 \end{bmatrix}$$

Which can be apply in SM flavor sector (n=3 fields)

This is a rank-4 matrix, so it cannot be written as uvuv, which is at most rank-2 by definition

General 2-scalar case

$$\mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$$

$$\mathcal{M}_{\text{scalar}} = \begin{bmatrix} 4C_{1111} & C'_{1122} & C_{1112} & C_{1112} \\ C'_{1122} & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122} \\ C_{1112} & C_{1222} & C'_{1122} & C_{1212} \end{bmatrix} \qquad \mathbf{Q}^{2^4} \ni \mathcal{Q} = \begin{pmatrix} a & b & e & e \\ b & c & f & f \\ e & f & d & b \\ e & f & b & d \end{pmatrix}$$

ERs Q_{ex} ->

$$\begin{array}{c} kl = \;\; 11 \quad 22 \quad\quad 12 \quad\quad\quad 21 \\ \begin{bmatrix} a^2 & ab & ac & ac \\ ab & b^2 & bc & bc \\ ac & bc & 2c^2 - ab & ab \\ ac & bc & ab & 2c^2 - ab \end{bmatrix} \begin{array}{l} ij = 11 \\ \quad\;\; 22 \\ \quad\;\; 12 \\ \quad\;\; 21 \end{array} \end{array}$$

With $c^2 \geq ab$

variable substitution

$$Q_{\text{ex}} \cdot \mathcal{M} \equiv \left[w^2 \quad \frac{rw+sw}{2} \quad rs \right] \cdot D \cdot \left[w^2 \quad \frac{rw+sw}{2} \quad rs \right]^T$$

$$\geq 0 \quad \forall r, s, w \in \mathbb{R},$$

It is quartic !

$$D = \begin{bmatrix} 2C_{1111} & C_{1112} & C_{1122} \\ C_{1112} & 2C_{1212} & C_{1222} \\ C_{1122} & C_{1222} & 2C_{2222} \end{bmatrix}$$

Example: EW Gauge Boson

$i, j, k, l = W, B$ (Hypercharge gauge boson)

- SMEFT VVVV (aQGC) operators (W+B, n=12 modes):
  (Anomalous quartic-gauge boson couplings)

- T-type:

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \quad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$

$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \quad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

$$O_{T,5} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$

$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \quad O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$

$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

- S-type:

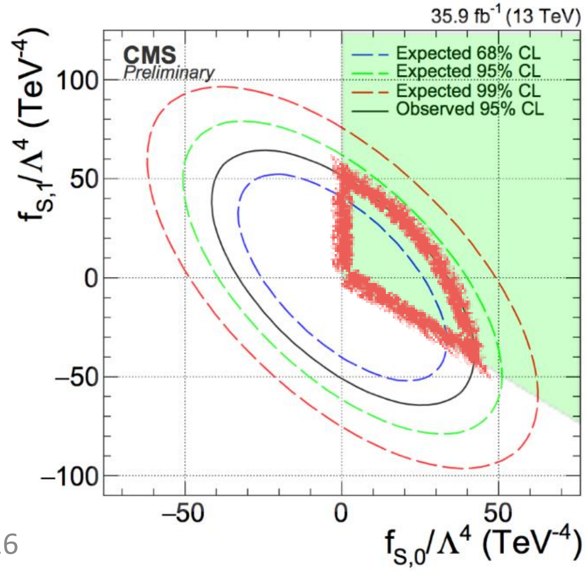
$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi],$$

$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi],$$

$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi].$$

- The aQGC couplings are used frequently to study VBS and tri-boson process at LHC
- Experimental limits are often given in term of aQGC operators at LHC

[Phys.Lett.B 798 (2019) 134985, CMS]



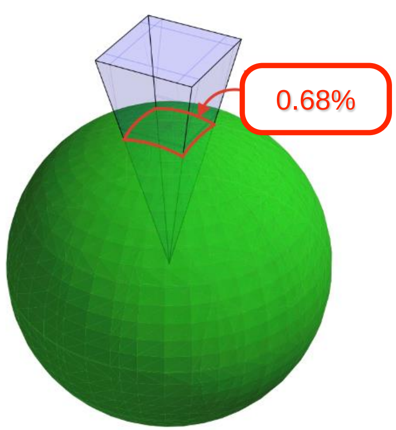
Example: EW Gauge Boson

$i, j, k, l = W, B$ (Hypercharge gauge boson)

We reproduced bounds by [2009.04490, K. Yamashita, CZ and S.-Y. Zhou]

Linear:

$$F_{T,2} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$
$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$
$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$



Quadratic:

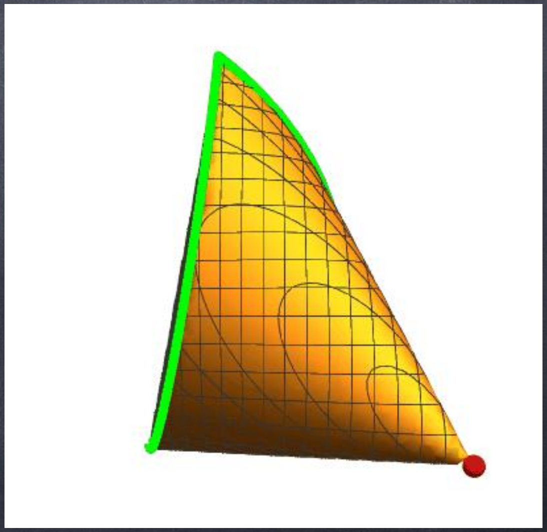
$$F_{T,9}\left(F_{T,2} + 4F_{T,10}\right) \geq F_{T,11}^2$$
$$16\left(2\left(F_{T,0} + F_{T,1}\right) + F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right) \geq \left(4F_{T,5} + F_{T,7}\right)^2$$
$$32\left(2F_{T,8} + F_{T,9}\right)\left(3F_{T,0} + F_{T,1} + 2F_{T,2} + 4F_{T,10}\right) \geq 3\left(4F_{T,5} + F_{T,7}\right)^2$$
$$2\sqrt{2}\sqrt{F_{T,9}\left(F_{T,2} + 8F_{T,10}\right)} \geq \max\left(-F_{T,7} - 4F_{T,11}, -4F_{T,6} - F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{\left(8F_{T,0} + 4F_{T,1} + 3F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right)} \geq \max\left(-8F_{T,5} - 4F_{T,6} - 3F_{T,7}, 8F_{T,5} + F_{T,7}\right)$$
$$4\sqrt{F_{T,9}\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(-F_{T,7} - 4F_{T,11}, -4F_{T,6} - F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{6}\sqrt{\left(2F_{T,8} + F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left[3\left(8F_{T,5} + F_{T,7}\right), -3\left(8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)\right]$$
$$\sqrt{6}\sqrt{\left(4F_{T,8} + 3F_{T,9}\right)\left(6F_{T,0} + 2F_{T,1} + 3F_{T,2} + 6F_{T,10}\right)} \geq \max\left(3\left(2F_{T,5} + F_{T,11}\right), -3\left(2F_{T,5} + F_{T,7} + F_{T,11}\right)\right)$$
$$2\sqrt{\left(12F_{T,8} + 7F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(-12F_{T,5} - 4F_{T,6} - 5F_{T,7} - 2F_{T,11}, 12F_{T,5} + F_{T,7} - 2F_{T,11}, 12F_{T,5} + F_{T,7} + 2F_{T,11}, 12F_{T,5} - 4F_{T,6} + F_{T,7} + 2F_{T,11}\right)$$

- At least for simple cases, the ext(G) can be found by inspection.
- E.g. simplest case:
  n=2, with some Z2 symmetry, e=f=0, T ->

$$\begin{pmatrix} a & b & 0 & 0 \\ b & c & 0 & 0 \\ 0 & 0 & d & b \\ 0 & 0 & b & d \end{pmatrix}$$

- There are two kinds of ERs
  - ER1: a=b=c=0, d=1
  - ER2: $ac=b^2$, d=|b|, a,c>0

A 3D cross section
of the 4D cone (a,b,c,d)

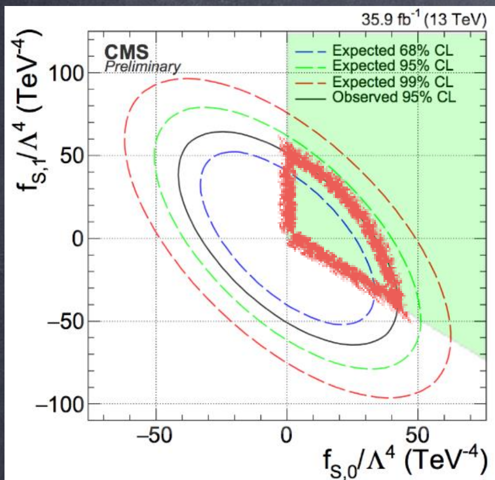


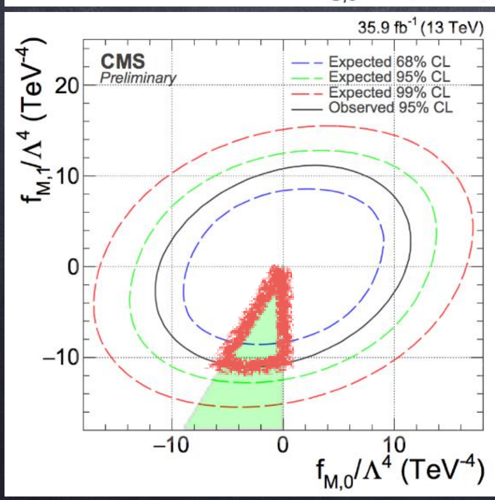
$$M^{ijkl} = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_2 & C_3 & 0 & 0 \\ 0 & 0 & C_4 & C_2 \\ 0 & 0 & C_2 & C_4 \end{pmatrix}$$

$$C_1, C_3, C_4 \geq 0 \text{ and } \sqrt{C_1 C_3} \geq \pm 2C_2 - C_4$$

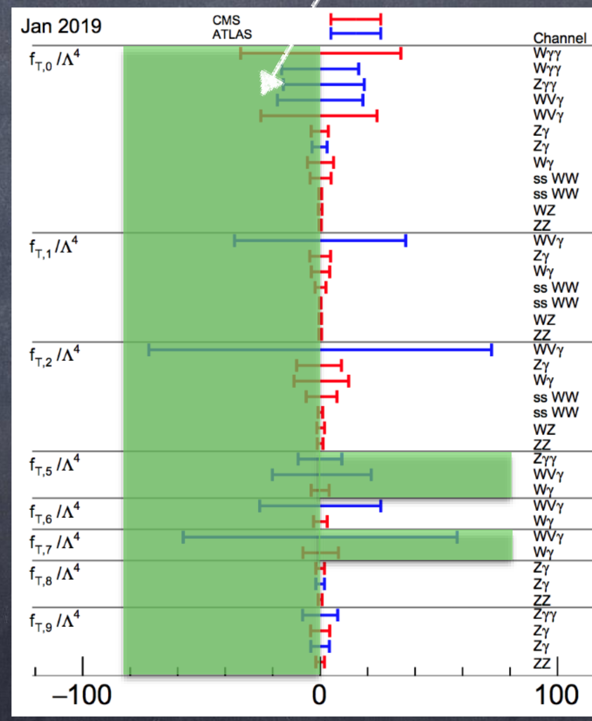
Pheno applications…

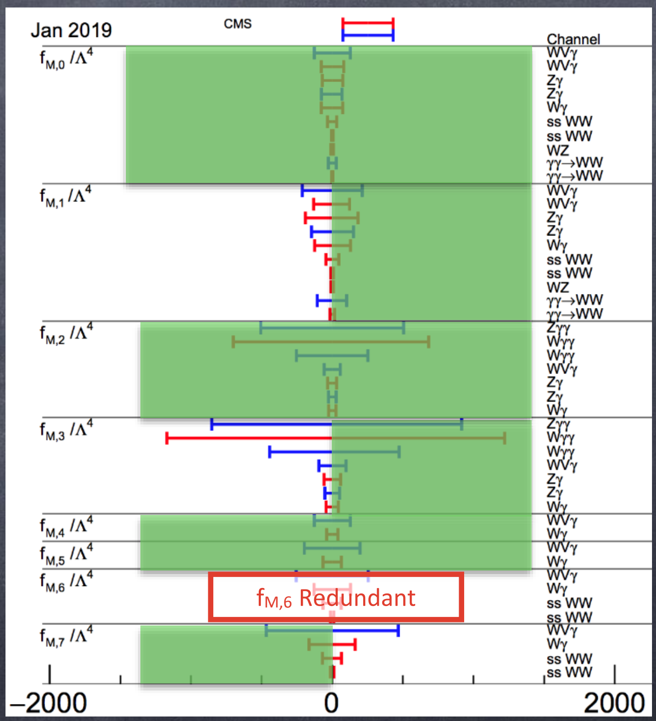
- May change the interpretation of measurements.





Excluded

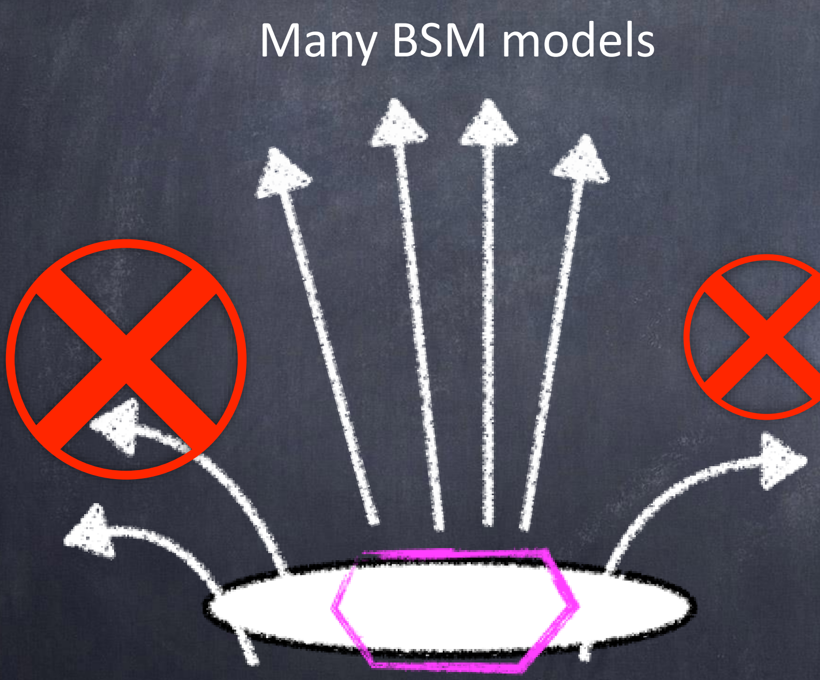


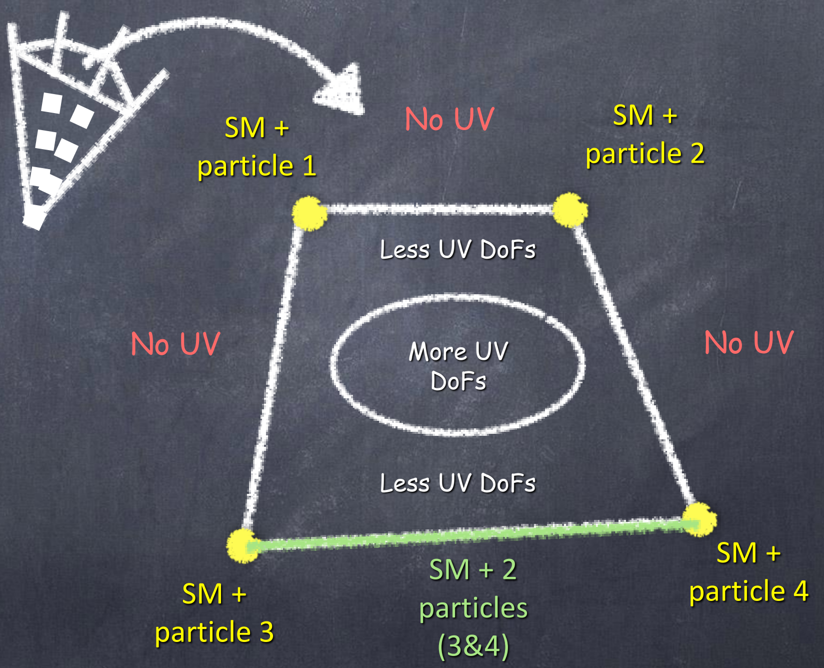


https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResults SMPaTGC#aQGC_Results

- Infer UV model from EFT measurements

Inverse problem: Given the measured values of the operator coefficients around the electroweak scale, to what extend can we possibly determine the nature of the new physics beyond the SM? [Gu, Wang, 2008.07551]
see also [S. Dawson et al. 2007.01296] [N. Arkani-Hamed et al. hep-ph/0512190]





[CZ and S.-Y. Zhou 2005.03047]
[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]