Minimal SU(5) Unification

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arXiv:2105.01678 (Ilja Doršner, Emina Džaferović-Mašić, Saad)

Outline

- Georgi-Glashow model
- Towards Realistic $SU(5)$
- Minimal $SU(5)$ Unification (proposal)
- Summary

- Fermions

$$\overline{5}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \quad 10_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_2^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}.$$

- Scalars

$$24_H : \quad SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$5_H : \quad SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

Georgi, Glashow 1974

Georgi-Glashow Model: Drawbacks

- GG model: $\overline{5}_{F\,i} + 10_{F\,i} + 5_H + 24_H$

✗ $M_d = M_e^T$ Georgi, Jarlskog 1979

✗ $M_\nu = 0$

✗ Gauge coupling unification

Towards Realistic Model

✓ Renormalizable models

- $\overline{5}_{F\,i} + 10_{F\,i} + 5_H + 24_H + 45_H$ Georgi, Jarlskog 1979

- Fermion masses: Dorsner, Perez 2006

$$M_d = v_5\, Y_1 + v_{45}\, Y_2$$
$$M_e^T = v_5\, Y_1 - 3 v_{45}\, Y_2$$
$$M_u = v_5 \left(Y_3 + {Y_3}^T \right) + v_{45} \left(Y_4 - {Y_4}^T \right)$$

Four arbitrary 3×3 matrices.

Towards Realistic Model

- $\overline{5}_{F\,i} + 10_{F\,i} + 5_H + 24_H + 45_H$

✓ $M_d \neq M_e^T$

✓ Gauge coupling unification

✓ Proton decay (safe)

✗ $M_\nu = 0$

Towards Minimal Model

SU(5) Irreps
Dimension (name)
5
10
15
24
35
40
45
50
70
70′
75
105
126

- why large dim 45?

✗ $M_\nu = 0$?

SU(5) Irreps
Dimension (name)
5
10
15
24
35
40
45
50
70
70′
75
105
126

- Minimal possibility! (smaller dim)

✓ $M_\nu \neq 0$?

Dorsner, Saad 2019

A Novel $SU(5)$ Proposal

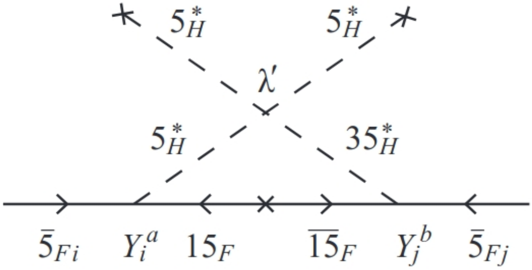
- $\overline{5}_{\boldsymbol{F}\,i} + 10_{\boldsymbol{F}\,i} + 5_{\boldsymbol{H}} + 24_{\boldsymbol{H}} + 35_{\boldsymbol{H}} + 15_{\boldsymbol{F}} + \overline{15}_{\boldsymbol{F}}$

- Decompositions

$$
\begin{aligned}
35_{\boldsymbol{H}} \equiv \Phi &= \Phi_1(1,4,-3/2) + \Phi_3(\overline{3},3,-2/3) + \Phi_6(\overline{6},2,1/6) \\
&\quad + \Phi_{10}(\overline{10},1,1) \\
15_{\boldsymbol{F}} \equiv \Sigma &= \Sigma_1(1,3,1) + \Sigma_3(3,2,1/6) + \Sigma_6(6,1,-2/3)
\end{aligned}
$$

Minimality

- ✓ least number of parameters
- ✓ lowest dimensional representations
- ✓ highly predictive theory
- ✓ $M_\nu \neq 0$



Fermion masses

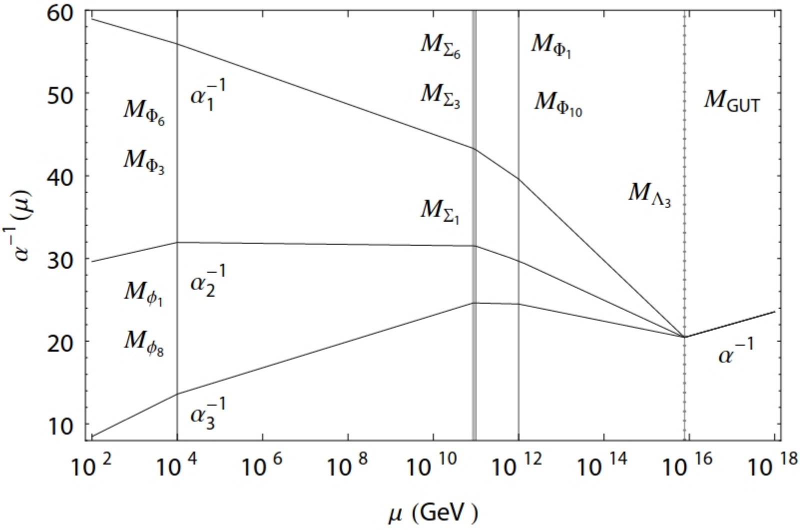
- Quarks, charged leptons, neutrino masses- all connected !
- $M_u = \left(\mathbb{I}_{3\times 3} + \delta^2 \; Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H Y^u$
- $M_d = \left(\mathbb{I}_{3\times 3} + \delta^2 \; Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H \left(Y^d + \delta \; Y^c Y^a\right)$
- $M_e = v_H {Y^d}^T$
- $(\mathcal{M}_\nu)_{ij} = \frac{\lambda' v_5^2}{8\pi^2} \left(Y_i^a Y_j^b + Y_j^b Y_i^a\right) \left[\frac{M_{\Sigma_1}}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \ln\left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2}\right)\right]$
- Minimum parameters:
  Y^a, Y^b, Y^c: vectors of length 3; Y^d_{diag}; Y^u: 3×3 matrix

Gauge Coupling Unification

Highly non-trivial:

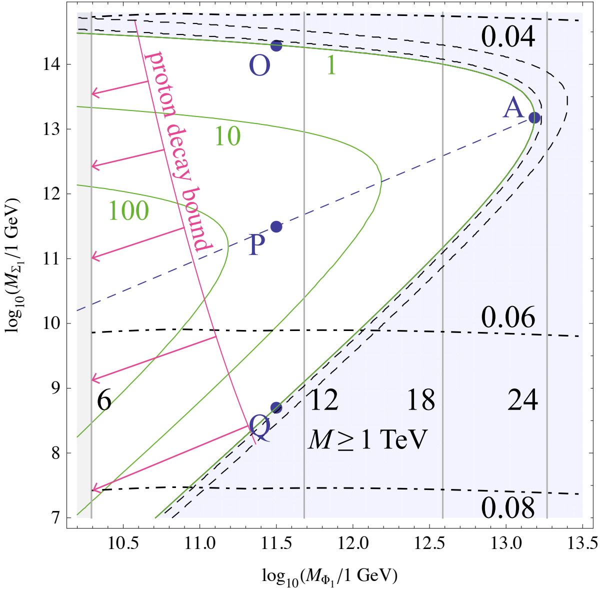
- $M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1}$
- $M^2_{\Phi_{10}} = M^2_{\Phi_1} - 3M^2_{\Phi_3} + 3M^2_{\Phi_6}$
- $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV (proton decay)
- $M_{\mathrm{GUT}} \geq 5 \times 10^{15}$ GeV (proton decay)
- $M_k \geq 1$ TeV ($k =$ any BSM state)
- ν-mass requires specific M_{Σ_1} and M_{Φ_1}
  (rules out most of the parameter space consistent with unification)

Gauge Coupling Unification

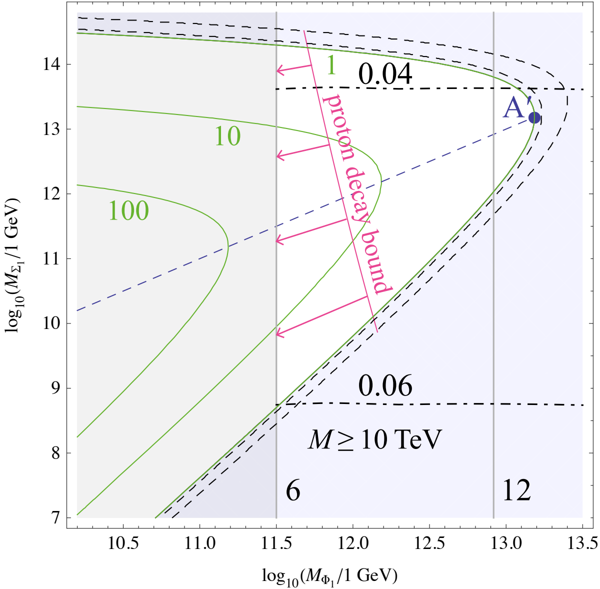


roughly $\sim$ three scales in the theory

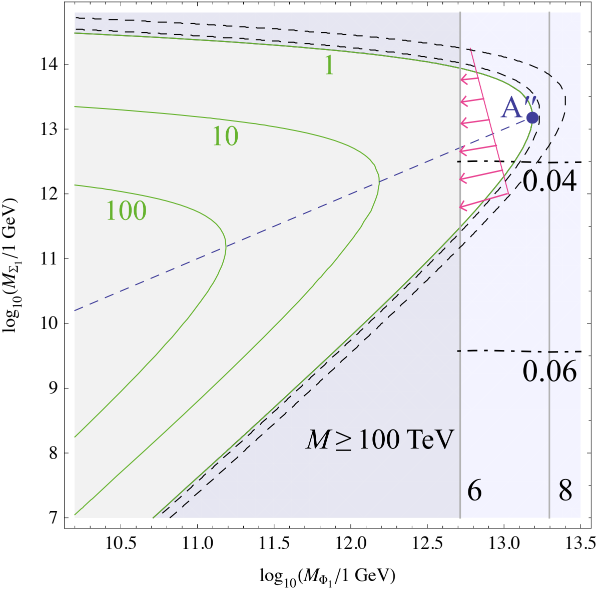
Full Parameter Space Exploration



Full Parameter Space Exploration



Full Parameter Space Exploration



Proton decay

- Dominant mode: $p \to \pi^0 e^+$
- Partial decay width: $\Gamma_p \sim \alpha_{GUT}^2 m_p^5 / M_V^4$
- Super-Kamiokande: $\tau^{\mathrm{exp}}_{p\to\pi^0 e^+} > 2.4 \times 10^{34}$ years
- An improvement by a factor of 2, 15, and 96 would completely rule out the $M \geq 100$ TeV, $M \geq 10$ TeV, and $M \geq 1$ TeV scenarios, respectively
- Hyper-Kamiokande: 10 years (20 years) of operation is 7.8×10^{34} (1.3×10^{35}) years

Minimal $SU(5)$ Unification: Summary

- ✓ Only lowest dimensional representations
- ✓ Least number of Yukawa parameters
- ✓ $M_d \neq M_e$
- ✓ $M_\nu \neq 0$ (via 1-loop diagram)
- ✓ All fermion masses & mixings are correlated
- ✓ Gauge coupling unification (restricted parameter space: ν-mass)
- ✓ Rest of the allowed parameter space fixed by Proton Decay
- ✓ $(1,3,0), (8,1,0), (3,3,\frac{2}{3}), (6,2,\frac{-1}{6}) \sim$ 1-100 TeV (scalars)

THANK YOU!