Phenomenology 2021 Symposium

Renormalization of scalar EFTs at higher orders

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Outline

- Motivations
- Background
- EFT operator basis construction
- Method of calculation
- Results
- Conclusions

Motivations

- Why EFT?
  - Widely used, e.g. SMEFT
- Why scalar EFT?
  - Toy model, easier to get higher orders and show structures clearly
  - ϕ^4 theory at Wilson-Fisher fixed point
  - Development of calculation tools
- Why higher mass dimension?
  - In SMEFT, $|\Delta B| = 1$ in dim 6 & $|\Delta B| = 2$ in dim 9
- Why higher loop?
  - To get higher precision to compare with data from LHC

Motivations

- Difficulty in counting the EFT operator basis
  - Redundancies between operators — Henning, Lu, Melia & Murayama [1512.03433]
  - Large numbers: 2, 84, 30, 993, 560, 15456, 11962, 261485, …
- Hilbert series method
- New method - on-shell method
- Structures in anomalous dimension matrix
  - Non-renormalization — zeros

  Alonso, Jenkins & Manohar [1409.0868]
  Cheung & Shen [1505.01844]
  Bern, Parra-Martinez & Sawyer [1910.05831]
- Structural/calculational approach to EFT

  Bern, Parra-Martinez & Sawyer [2005.12917]
  Caron-Huot & Wilhelm [1607.06448]
  Craig, Jiang, Li & Sutherland [2001.00017]
  Baratella, Fernandez, Harling & Pomarol [2010.13809]
  Elias Miro, Ingoldby & Riembau [2005.06983]
  Jiang, Shu, Xiao & Zheng [2001.04481]

Background

- Massless scalar EFTs

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{4!}\phi^4 + \sum_{d>4}\frac{\sum_i c_i^d \mathcal{O}_i^d}{\Lambda^{d-4}}$$

- EFT operators

$$\phi^6, \quad \phi^3\partial_\mu\partial^\mu\phi, \quad \phi^2\partial_\mu\phi\partial^\mu\phi, \ldots$$

IBP & EOM redundancies

- Hilbert series

Henning, Lu, Melia & Murayama [1706.08520]

$$H^{\text{real}}_{S\text{-matrix}}(\Delta) = \Delta^6 + 2\Delta^8 + 3\Delta^{10} + 5\Delta^{12} + 9\Delta^{14} + 16\Delta^{16} + 32\Delta^{18} + 65\Delta^{20} + \mathcal{O}(\Delta^{22})$$

$$\Delta^6\,\phi^6 + \Delta^8\left(\phi^4 t^4 + \phi^8\right) + \Delta^{10}\left(\phi^4 t^6 + \phi^6 t^4 + \phi^{10}\right) + \cdots$$

Operator construction

- Operator $\leftrightarrow$ states
- EOM $\leftrightarrow$ null states, IBP $\leftrightarrow$ descendant states
- Counting physical operators $\leftrightarrow$ counting primaries $\leftarrow$ Hilbert series
- Primary operators as physical basis

- In calculation, we will work with off-shell correlation function which is independent under IBP. They will also appear in the counterterms.
- Hilbert series can also count them!

$$H^{\text{real}}_{\text{C.F.}}(\Delta) = 3\Delta^6+6\Delta^8+12\Delta^{10}+25\Delta^{12}+53\Delta^{14}+120\Delta^{16}+279\Delta^{18}+680\Delta^{20}+\mathcal{O}(\Delta^{22})$$

$$H^{\text{real}}_{\text{F.F.}}(\Delta) = 3\Delta^6+6\Delta^8+12\Delta^{10}+25\Delta^{12}+53\Delta^{14}+119\Delta^{16}+275\Delta^{18}+664\Delta^{20}+\mathcal{O}(\Delta^{22})$$

Calculation method - Renormalization

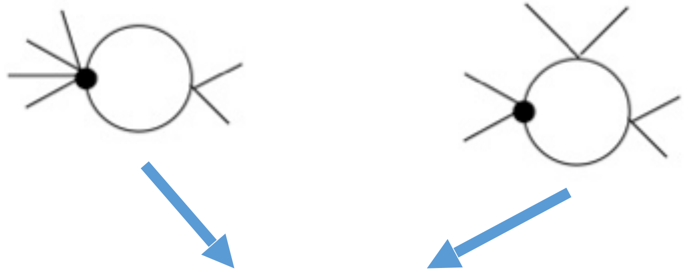
Work only in mixing with same mass dimension

$$\mu \frac{d}{d\mu} c_i^{(n)} = \sum_j \gamma_{ij}^{(n)} c_j^{(n)}$$

For example

Six point
Length six

$$\frac{1}{\Lambda^2}\left(c_6 \mathcal{O}_6 + c_4 \mathcal{O}_4\right) = \frac{1}{\Lambda^2}\left(c_6 \phi^6 + c_4 \phi^2 \partial_\mu \phi \partial^\mu \phi\right)$$



$$\begin{pmatrix} c_6(\mu) \\ c_4(\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{66} & \gamma_{46} \\ \gamma_{64} & \gamma_{44} \end{pmatrix} \begin{pmatrix} c_6(0) \\ c_4(0) \end{pmatrix}$$

$$c_i^{b(n)} = \sum_j Z_{ij}^{(n)} c_j^{(n)}$$

Counterterms in MS scheme of dimensional regularizations

Calculation method – R* method

Original: Chetyrkin & Tkachov 1982
Chetyrkin & Smirnov 1984
Smirnov & Chetyrkin 1985
Recent: Herzog & Ruijl [1703.03776]
Beekveldt, Borinsky & Herzog [2003.04301]

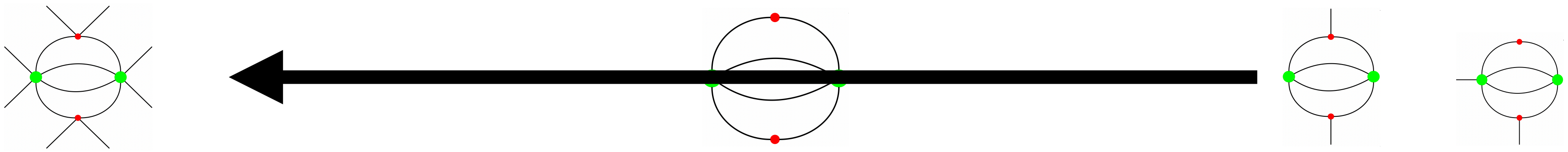
Recursively subtracts both UV and IR subdivergences

Additional IR subdivergences

Taylor expansion of the diagram

Infrared rearrangment

Diagram with degree of divergences ω → Logarithmic scaleless vacuum diagrams → Self-energy diagram or vacuum diagram with massive edges



Same counterterms but simpler diagrams

Calculation method – R* method

Original: Chetyrkin & Tkachov 1982
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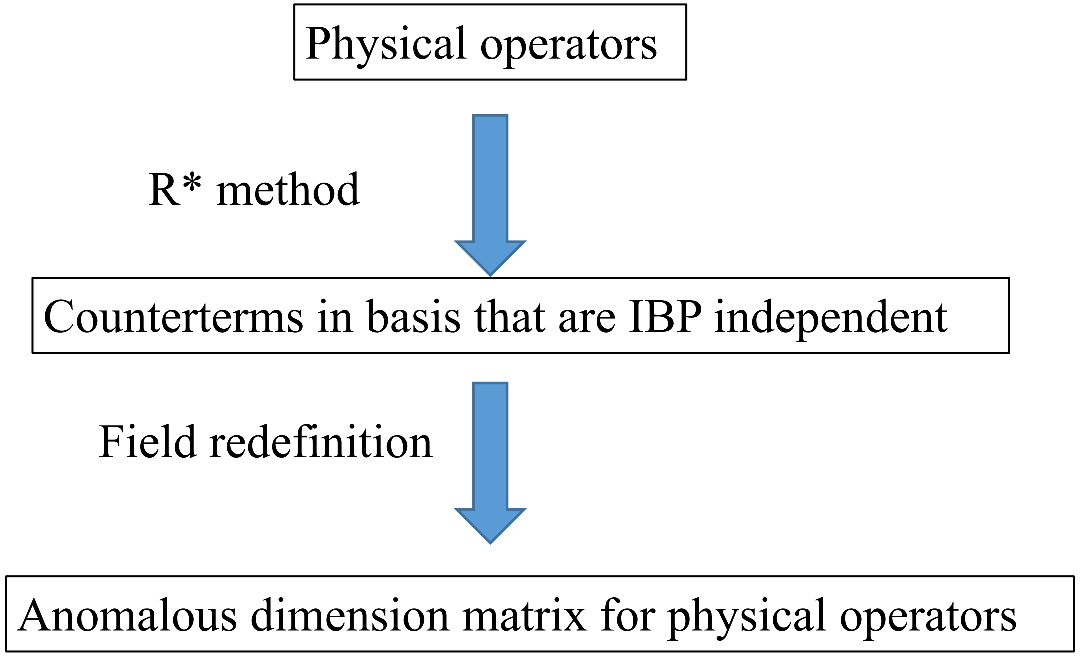
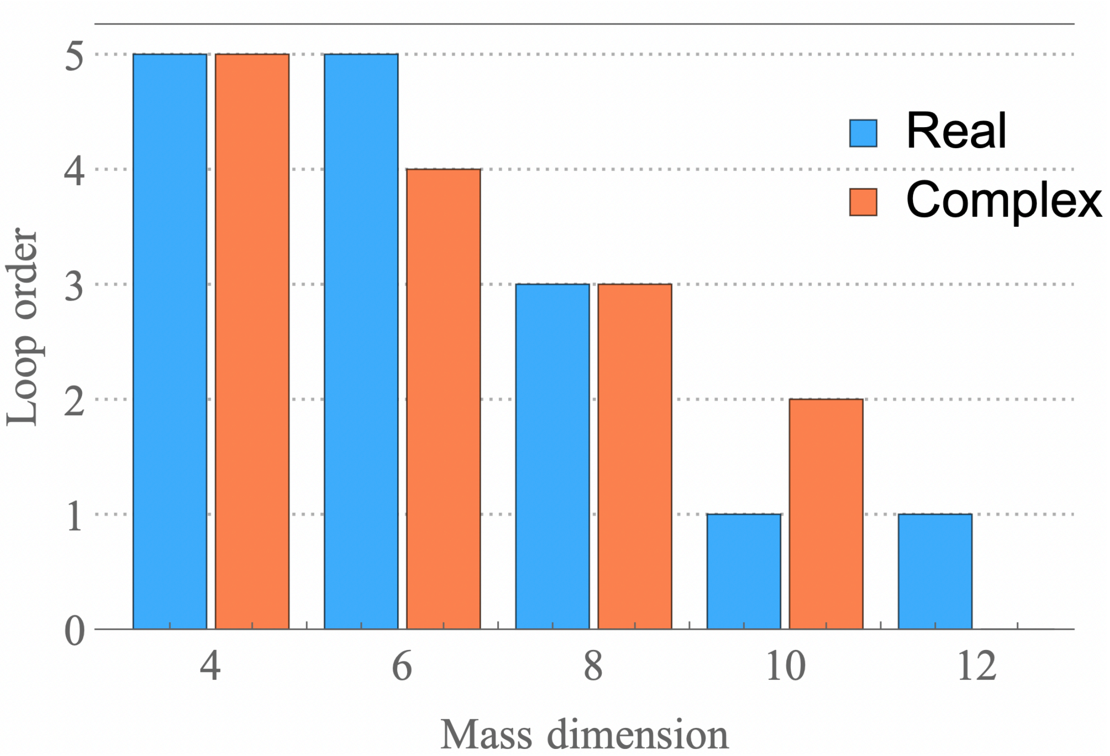


Results - Overview

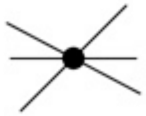
Basis of primary operators + R* method

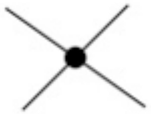
Higher-order results including
Five-loop for dim 6
one-loop for dim 12

Zeros in anomalous dimension matrix



Results – Dim 6





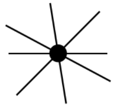
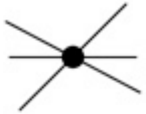
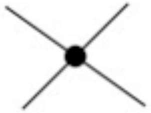
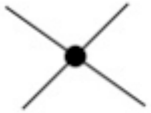
$$\gamma_{\mathrm{C}}^{(6)} = \begin{pmatrix} 14g - \frac{297g^2}{2} + \left(816\zeta_3 + \frac{14981}{8}\right)g^3 \\ -\left(\frac{32087\zeta_3}{2} - 2892\zeta_4 + 23320\zeta_5 + \frac{888983}{32}\right)g^4 & 0g - \frac{45g^2}{2} + \left(216\zeta_3 + \frac{6153}{16}\right)g^3 \\ & -\left(783\zeta_3 + 8100\zeta_5 + \frac{74079}{16}\right)g^4 \\ & -g + \frac{13g^2}{2} - \left(36\zeta_3 + \frac{383}{12}\right)g^3 \\ 0g + 0g^2 + 0g^3 + \frac{5g^4}{6} & +\left(\frac{769\zeta_3}{2} - 123\zeta_4 + 560\zeta_5 + \frac{7893}{32}\right)g^4 \end{pmatrix}$$

1. $n-2$ point mixing into n point in dim n only in basis of primary operators at one-loop

2. $\mathcal{O}_h$ mixing into $\mathcal{O}_l$ only for loop number $L > l(\mathcal{O}_h) - l(\mathcal{O}_l)$, see Bern, Parra-Martinez, Sawyer [1910.05831]

3. n mixing into $n-2$ point at three-loop in any basis of physical operators

Results – Dim 8

$$\gamma_{\mathrm{C}}^{(8)} = \begin{pmatrix} 29g - 409g^2 + \left(2352\zeta_3 + \frac{57765}{8}\right)g^3 & 0g - 240g^2 + \left(2304\zeta_3 + 5882\right)g^3 & \frac{54216g}{5} - \frac{958314g^2}{5} + \left(\frac{4083264\zeta_3}{5} + \frac{55731313}{15}\right)g^3 & -\frac{8856g}{5} + \frac{159894g^2}{5} - \left(\frac{713664\zeta_3}{5} + \frac{9281093}{15}\right)g^3 \\ 0g+0g^2+0g^3 & 4g - \frac{122g^2}{3} + \left(216\zeta_3 + \frac{4559}{12}\right)g^3 & -\frac{679g}{5} + \frac{14209g^2}{12} - \left(\frac{29216\zeta_3}{5} + \frac{3248605}{324}\right)g^3 & \frac{164g}{5} - \frac{9233g^2}{36} + \left(\frac{6056\zeta_3}{5} + \frac{2740291}{1296}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g+0g^2 + \frac{5g^3}{108} & \frac{11g}{3} - \frac{29g^2}{180} - \left(\frac{32\zeta_3}{3} + \frac{97091}{9720}\right)g^3 & -\frac{4g}{3} + \frac{1057g^2}{540} - \left(\frac{16\zeta_3}{3} + \frac{10868}{1215}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g+0g^2 + \frac{115g^3}{324} & \frac{46g}{3} - \frac{14557g^2}{540} + \left(96\zeta_3 + \frac{198001}{2430}\right)g^3 & -\frac{19g}{3} + \frac{2809g^2}{180} - \left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{pmatrix}$$

Blue square: same structures as in dim 6

Red square: new structures in same-length mixing sub-matrix

Results – Dim 8

See Craigie, Dobrev, Todorov. Annals Phys. (1985)

$$\begin{array}{cc} g\,\mathcal{O}_4^{(8)c}(1,0) & g\,\mathcal{O}_4^{(8)c}(0,1) \end{array}$$

$$\begin{pmatrix} \frac{11g}{3} & -\frac{4g}{3} \\ \frac{46g}{3} & -\frac{19g}{3} \end{pmatrix}$$

↓

Orthonormal choice

$$\mathcal{O}_4^{(8)c}\,(a,\,b) = \mathcal{O}_4^{(8)c}\left(3\sin\theta,\ 13\sin\theta + 2\sqrt{5}\cos\theta\right)$$

$$\mathcal{O}_4^{(8)c}\,(a',\,b') = \mathcal{O}_4^{(8)c}\left(3\cos\theta,\ 13\cos\theta - 2\sqrt{5}\sin\theta\right)$$

$$\begin{array}{cc} g\,\mathcal{O}_4^{(8)c}(a,b) & g\,\mathcal{O}_4^{(8)c}(a',b') \end{array}$$

$$\begin{pmatrix} -\frac{g}{9}(12 - 7\cos 2\theta + 8\sqrt{5}\sin 2\theta) & -\frac{g}{9}(8\sqrt{5}\cos 2\theta + 7\sin 2\theta) \\ -\frac{g}{9}(8\sqrt{5}\cos 2\theta + 7\sin 2\theta) & -\frac{g}{9}(12 + 7\cos 2\theta - 8\sqrt{5}\sin 2\theta) \end{pmatrix}$$

Results – Dim 10

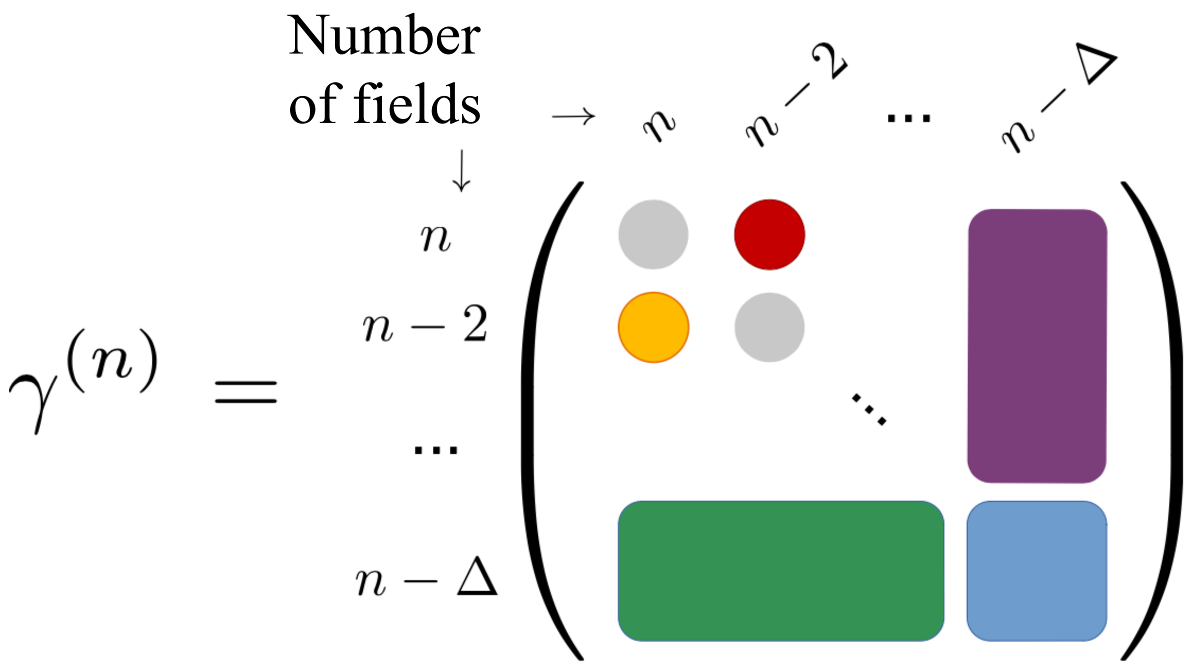
Blue square: same structures as in dim 6 and 8

Red square: Expected zeros because the C-odd sector does not mix into C-even sector

$$\gamma_C^{(10)} = \left(\begin{matrix}
50g - \frac{1725g^2}{2} & 0g + 1125g^2 & \frac{412500g}{7} - \frac{61613665g^2}{42} & \frac{3300g}{7} - \frac{19655g^2}{6} \\
0g+0g^2 & 15g - \frac{385g^2}{2} & -\frac{1474g}{7} + \frac{83689g^2}{45} & -\frac{234g}{7} + \frac{386467g^2}{315} \\
0g+0g^2 & 0g+0g^2 & \frac{35g}{6} - \frac{678389g^2}{15120} & -\frac{g}{6} + \frac{24379g^2}{15120} \\
0g+0g^2 & 0g+0g^2 & \frac{4g}{3} - \frac{331g^2}{630} & -\frac{2g}{3} - \frac{5401g^2}{945} \\
0g+0g^2 & 0g+0g^2 & \frac{15g}{2} - \frac{247109g^2}{5040} & -\frac{g}{2} - \frac{4127g^2}{1680} \\
0 & 0 & 0 & 0 \\
0g+0g^2 & 0g+0g^2 & 0g+0g^2 & 0g+0g^2 \\
0g+0g^2 & 0g+0g^2 & 0g+0g^2 & 0g+0g^2
\end{matrix}\right. (\dots)$$

$$(\dots) \left.\begin{matrix}
\frac{141900g}{7} - \frac{6734025g^2}{14} & 0 & -\frac{500460g}{7} + \frac{31376515g^2}{21} & -64740g + \frac{16819700g^2}{21} \\
\frac{326g}{7} - \frac{4577g^2}{7} & 0 & -\frac{43732g}{35} + \frac{5863532g^2}{315} & \frac{14272g}{5} - \frac{15354406g^2}{315} \\
\frac{7g}{6} - \frac{28265g^2}{3024} & 0 & \frac{1006g}{105} + \frac{1249571g^2}{15120} & -\frac{967g}{30} + \frac{1903001g^2}{3780} \\
2g - \frac{14965g^2}{756} & 0 & -\frac{4159g}{105} + \frac{3146033g^2}{7560} & \frac{914g}{15} - \frac{103037g^2}{315} \\
\frac{7g}{2} - \frac{31333g^2}{1008} & 0 & -\frac{1769g}{35} + \frac{243011g^2}{336} & \frac{573g}{10} + \frac{37049g^2}{1260} \\
0 & \frac{10g}{3} - \frac{770g^2}{27} & 0 & 0 \\
0g+0g^2 & 0 & -2g + \frac{223g^2}{24} & -g + \frac{7g^2}{24} \\
0g+0g^2 & 0 & -g + \frac{7g^2}{12} & -3g + \frac{251g^2}{24}
\end{matrix}\right)$$

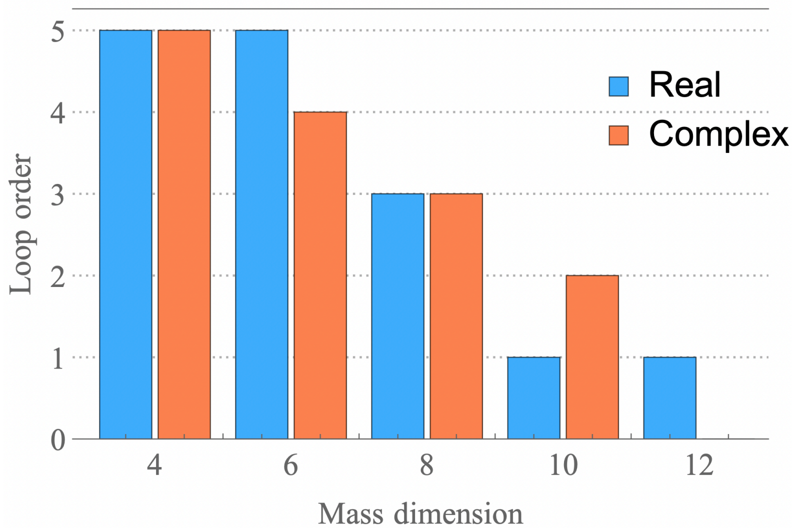
Results – Summary



- (red circle) 1-loop zero only when $\mathcal{O}^{(n)}_{n-2}$ is primary
- (purple square) Generally non-zero in a basis of primaries
- (green square) Zeros up to $L = \Delta$ loops
- (yellow circle) Unexpected 3-loop zero in any bases
- (blue square) Symmetric at 1-loop when the primary operators orthonormal

By Jasper RN

Conclusion



1. We used Hilbert series to classify the EFT operators
2. First application of R* method in EFT operator mixing

These two methods can be generalized to particles with spin

3. We got higher results.
4. The general structures of anomalous dimension matrix was analyzed.
   - Confirmation of known structures in literature
   - Zeros beyond literature, mixing between n and $n-2$ point in mass dimension n

5. We also have other results that not covered in this talk
   - systematic enumeration of evanescent operators in scalar EFT using Hilbert series
   - the proof of ring isomorphism between off-shell and on-shell basis

Thank you!

Backup slides

Hilbert series for real theory

$$
\begin{aligned}
H^{\text{real}}_{S\text{-matrix}}(\Delta, t, \phi) = {} & \Delta^6\,\phi^6 + \Delta^8\left(\phi^4 t^4 + \phi^8\right) + \Delta^{10}\left(\phi^4 t^6 + \phi^6 t^4 + \phi^{10}\right) \\
& + \Delta^{12}\left(\phi^4 t^8 + 2\phi^6 t^6 + \phi^8 t^4 + \phi^{12}\right) \\
& + \Delta^{14}\left(\phi^4 t^{10} + 4\phi^6 t^8 + 2\phi^8 t^6 + \phi^{10} t^4 + \phi^{14}\right) \\
& + \Delta^{16}\left(2\phi^4 t^{12} + 5\phi^6 t^{10} + 5\phi^8 t^8 + 2\phi^{10} t^6 + \phi^{12} t^4 + \phi^{16}\right) \\
& + \Delta^{18}\left(\phi^4 t^{14} + 13\phi^6 t^{12} + 9\phi^8 t^{10} + 5\phi^{10} t^8 + 2\phi^{12} t^6 + \phi^{14} t^4 + \phi^{18}\right) \\
& + \Delta^{20}\left(2\phi^4 t^{16} + 18\phi^6 t^{14} + 26\phi^8 t^{12} + 10\phi^{10} t^{10} + 5\phi^{12} t^8 + 2\phi^{14} t^6 + \phi^{16} t^4 + \phi^{20}\right) \\
& + \mathcal{O}(\Delta^{22}) \qquad \text{(A.20)}
\end{aligned}
$$

Hilbert series for complex theory

$$
\begin{aligned}
H^{\text{complex}}_{S\text{-matrix}}(\Delta, t, \phi, \phi^\dagger) = {} & \Delta^6 \left(\phi^2 \phi^{\dagger\,2} t^2 + \phi^3 \phi^{\dagger\,3}\right) + \Delta^8 \left(2\phi^2 \phi^{\dagger\,2} t^4 + \phi^3 \phi^{\dagger\,3} t^2 + \phi^4 \phi^{\dagger\,4}\right) \\
& + \Delta^{10} \left(2\phi^2 \phi^{\dagger\,2} t^6 + 4\phi^3 \phi^{\dagger\,3} t^4 + \phi^4 \phi^{\dagger\,4} t^2 + \phi^5 \phi^{\dagger\,5}\right) \\
& + \Delta^{12} \left(3\phi^2 \phi^{\dagger\,2} t^8 + 10\phi^3 \phi^{\dagger\,3} t^6 + 6\phi^4 \phi^{\dagger\,4} t^4 + \phi^5 \phi^{\dagger\,5} t^2 + \phi^6 \phi^{\dagger\,6}\right) \\
& + \Delta^{14} \left(3\phi^2 \phi^{\dagger\,2} t^{10} + 24\phi^3 \phi^{\dagger\,3} t^8 + 18\phi^4 \phi^{\dagger\,4} t^6 + 6\phi^5 \phi^{\dagger\,5} t^4 + \phi^6 \phi^{\dagger\,6} t^2 + \phi^7 \phi^{\dagger\,7}\right) \\
& + \Delta^{16} \left(4\phi^2 \phi^{\dagger\,2} t^{12} + 50\phi^3 \phi^{\dagger\,3} t^{10} + 61\phi^4 \phi^{\dagger\,4} t^8 + 20\phi^5 \phi^{\dagger\,5} t^6 + 6\phi^6 \phi^{\dagger\,6} t^4 \right. \\
& \qquad \left. + \phi^7 \phi^{\dagger\,7} t^2 + \phi^8 \phi^{\dagger\,8}\right) \\
& + \Delta^{18} \left(4\phi^2 \phi^{\dagger\,2} t^{14} + 133\phi^3 \phi^{\dagger\,3} t^{12} + 187\phi^4 \phi^{\dagger\,4} t^{10} + 81\phi^5 \phi^{\dagger\,5} t^8 + 22\phi^6 \phi^{\dagger\,6} t^6 \right. \\
& \qquad \left. + 6\phi^7 \phi^{\dagger\,7} t^4 + \phi^8 \phi^{\dagger\,8} t^2 + \phi^9 \phi^{\dagger\,9}\right) \\
& + \Delta^{20} \left(5\phi^2 \phi^{\dagger\,2} t^{16} + 215\phi^3 \phi^{\dagger\,3} t^{14} + 604\phi^4 \phi^{\dagger\,4} t^{12} + 296\phi^5 \phi^{\dagger\,5} t^{10} \right. \\
& \qquad \left. + 91\phi^6 \phi^{\dagger\,6} t^8 + 22\phi^7 \phi^{\dagger\,7} t^6 + 6\phi^8 \phi^{\dagger\,8} t^4 + \phi^9 \phi^{\dagger\,9} + \phi^{10} \phi^{\dagger\,10}\right) \\
& + \mathcal{O}(\Delta^{22}) \qquad \text{(A.21)}
\end{aligned}
$$

- Results for real theory

$$\gamma_C^{(6)} = 9g - \frac{359g^2}{6} + \left(216\zeta_3 + \frac{5773}{12}\right)g^3 - \left(\frac{5283\zeta_3}{2} - 459\zeta_4 + 3960\zeta_5 + \frac{1312907}{288}\right)g^4 + \left(1755\zeta_3^2 + \frac{510193}{16}\zeta_3 - \frac{25483}{4}\zeta_4 + 54621\zeta_5 - \frac{29025}{2}\zeta_6 + 73584\zeta_7 + \frac{333811365}{6912}\right)g^5$$

$$\gamma_C^{(8)} = \begin{array}{cc} g^3\,\mathcal{O}_8^{(8)c} & g\,\mathcal{O}_4^{(8)c} \end{array}$$
$$\gamma_C^{(8)} = \begin{pmatrix} 19g - 169g^2 + \left(636\zeta_3 + \frac{46255}{24}\right)g^3 & 2352g - \frac{76720g^2}{3} + \left(65856\zeta_3 + \frac{8676248}{27}\right)g^3 \\ 0g + 0g^2 + 0g^3 & \frac{g}{3} + \frac{49g^2}{27} - \left(\frac{12017}{1944} + \frac{20\zeta_3}{3}\right)g^3 \end{pmatrix}$$

$$\begin{array}{ccc} g^4\,\mathcal{O}_{10}^{(10)c} & g^2\,\mathcal{O}_6^{(10)c} & g\mathcal{O}_4^{(10)c} \end{array}$$
$$\gamma_C^{(10)} = \begin{pmatrix} 33g & 23100g & 233520g \\ 0g & 5g & \frac{112g}{5} \\ 0g & 0g & -2g \end{pmatrix}$$

$$\begin{array}{ccccc} g^5\,\mathcal{O}_{12}^{(12)c} & g^3\,\mathcal{O}_8^{(12)c} & g^2\,\mathcal{O}_6^{(12)c}{}_{(1,0)} & g^2\,\mathcal{O}_6^{(12)c}{}_{(0,1)} & g\,\mathcal{O}_4^{(12)c} \end{array}$$
$$\gamma_C^{(12)} = \begin{pmatrix} 51g & \frac{1121120g}{9} & \frac{477400g}{3} & \frac{284900g}{3} & \frac{7037295100g}{147} \\ 0g & \frac{41g}{3} & -\frac{4675g}{2} & \frac{395g}{4} & \frac{45981307g}{1372} \\ 0g & 0g & 2g & 0g & -\frac{26779g}{490} \\ 0g & 0g & 0g & 2g & -\frac{445574g}{245} \\ 0g & 0g & 0g & 0g & -\frac{g}{5} \end{pmatrix}$$