# Anomaly-free $U(1)_R$ and the proton charge radius*

**Carlos Alvarado**
Dual CP Institute of High Energy Physics (Mexico)

May 25th
**Pheno 2021 @ U PITT**

***2105.xxxxx**

with Alfredo Aranda and Cesar Bonilla

# Mini outline

**Why?**

- Muons seem to be weird
- Current discrepancy in proton charge radius

- The proton charge radius (status)
- An anomaly-free $U(1)_R$ model (+ scalars)
- Spectrum & constraints; fitting discrepancies

**Concluding remarks**

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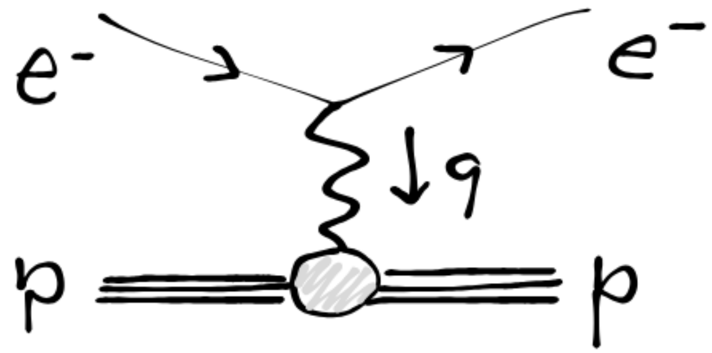
Probing of the proton's charge distribution: **scattering**




e-p scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}[e^- p \to e^- p]$$ with $p$ pointlike & spinless

Electric and magnetic form factors $J_p^\mu = \gamma^\mu F_1^D(q^2) + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2^D(q^2)$

$$F_1^D(q^2),\ F_2^D(q^2) \quad \leftrightarrow \quad G^{\text{elec.}}(q^2),\ G^{\text{magn.}}(q^2)$$

$$G^{\text{elec.}}(q^2) \approx 1 - \tfrac{1}{6}\langle r^2\rangle q^2 + \ldots$$

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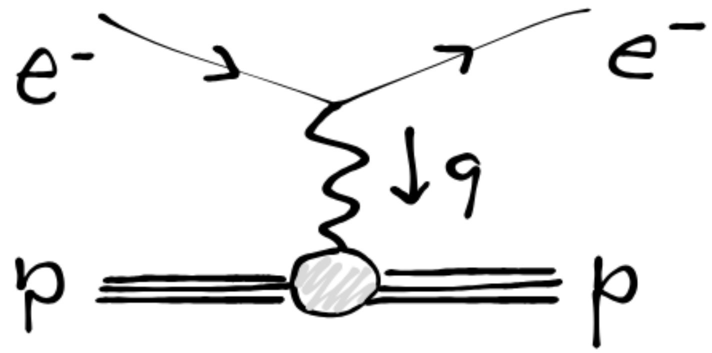


e-p scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}[e^- p \to e^- p]$$ with $p$ as extended distribution

Electric and magnetic form factors $J_p^\mu = \gamma^\mu F_1^D(q^2) + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2^D(q^2)$

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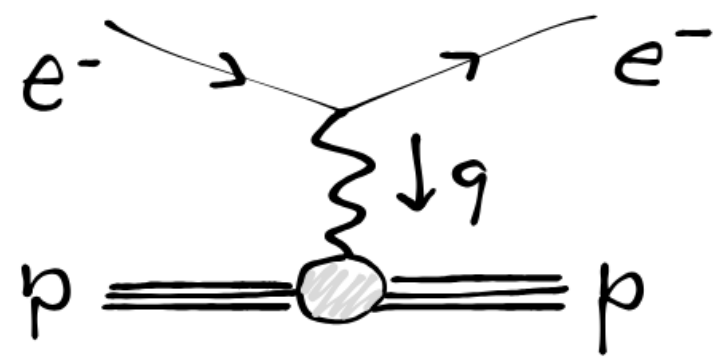

e-p scattering

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}[e^- p \to e^- p]$ with $p$ pointlike with magnetic moment

Electric and magnetic form factors $J_p^\mu = \gamma^\mu F_1^D(q^2) + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2^D(q^2)$

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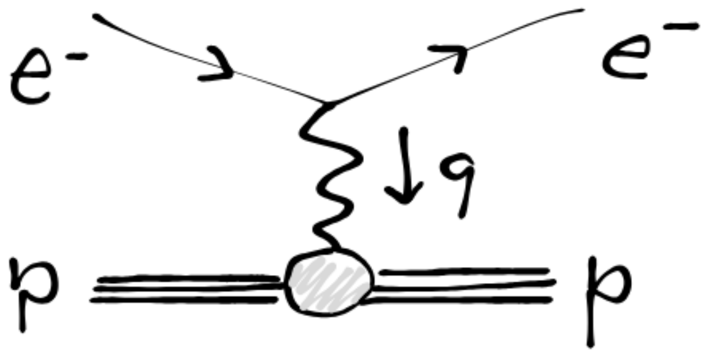


e-p scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}[e^- p \to e^- p]$$

with $p$ relativistic Dirac fermion with structure

Electric and magnetic form factors $J_p^\mu = \gamma^\mu F_1^D(q^2) + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2^D(q^2)$

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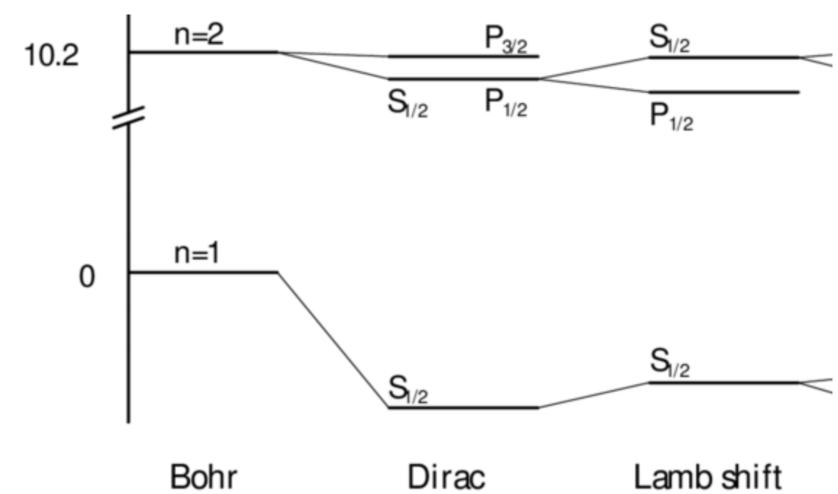
Also through **H-spectroscopy**: Lamb shift depends on $\langle r_p^2 \rangle$

$$E_{\text{Lamb}}(r) = 206.0336(15) - 5.2275(10)r^2 + \delta E_{2\gamma} \ [\text{meV}]$$




H spectroscopy

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Every 4 years electronic measurements of $\langle r_p^2 \rangle$ organized/combined by the CODATA committee

The NIST Reference on
Constants, Units, and Uncertainty

Fundamental Physical Constants

**proton rms charge radius**
$r_{\rm p}$
Concise form `8.414(19) x 10`$^{-16}$ `m`

***CODATA 2018**

| | | | |
|---|---|---|---|
| $0.833 \pm 0.010$ [fm] | BEZGINOV | 2019 | 2S-2P transition in H |
| $0.831 \pm 0.007 \pm 0.012$ | XIONG | 2019 | $e\,p \to ep$ form factor |

***PDG 2020**

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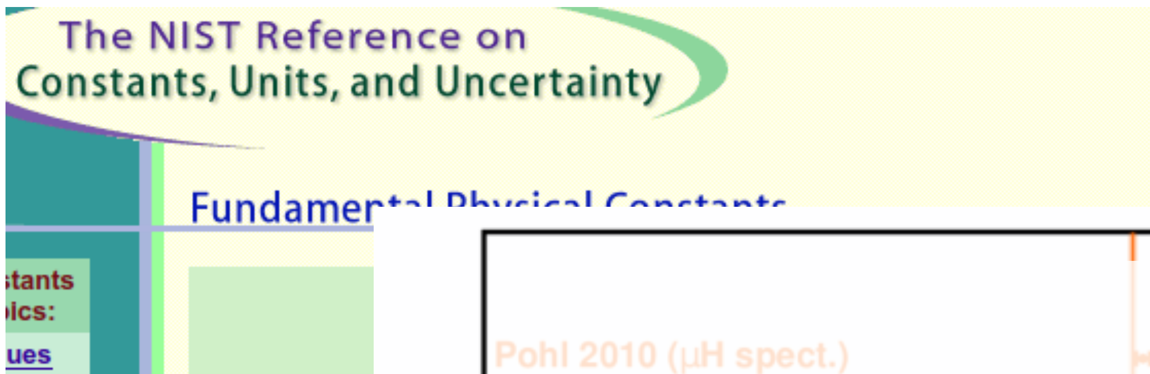


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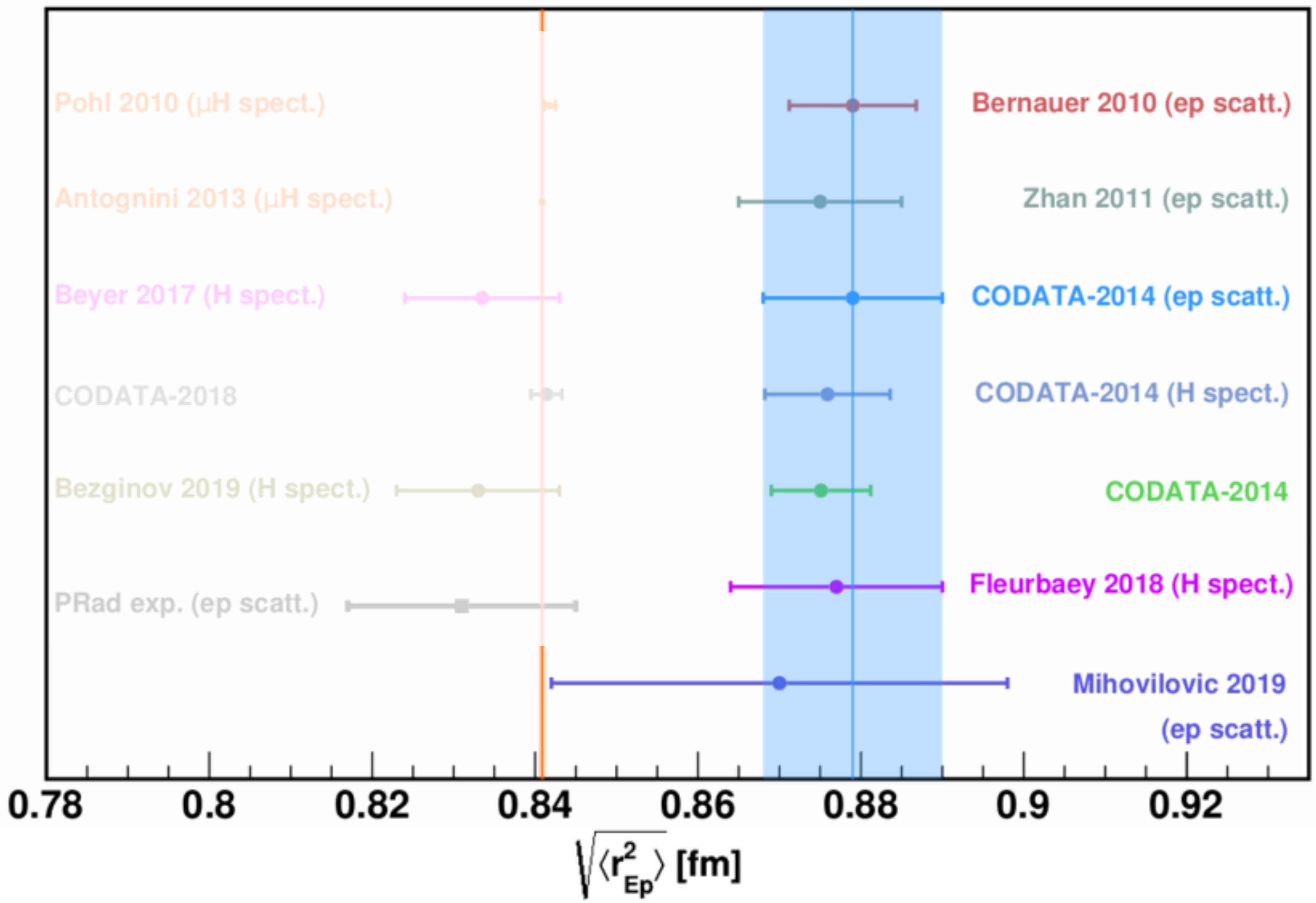

# The proton charge radius

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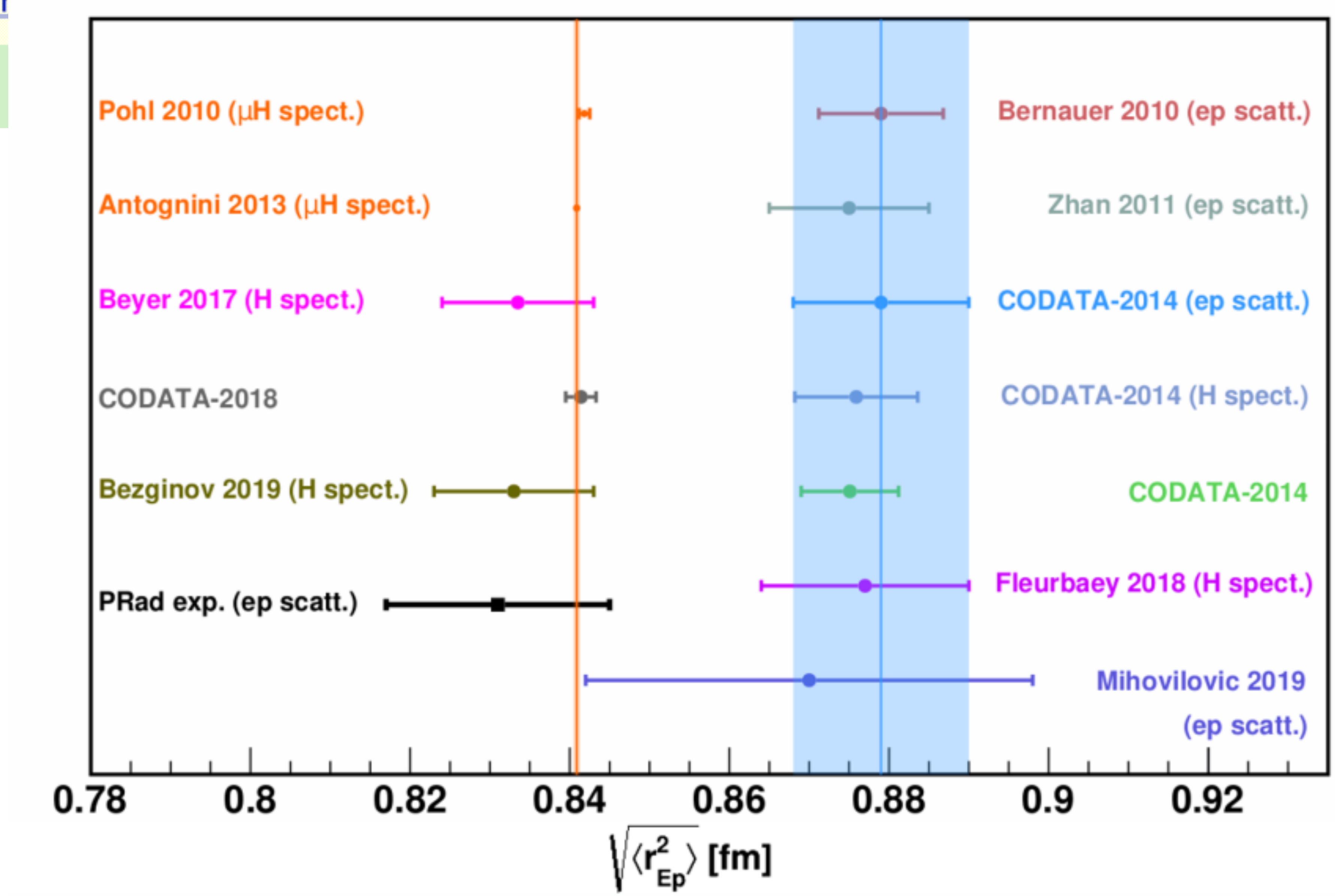

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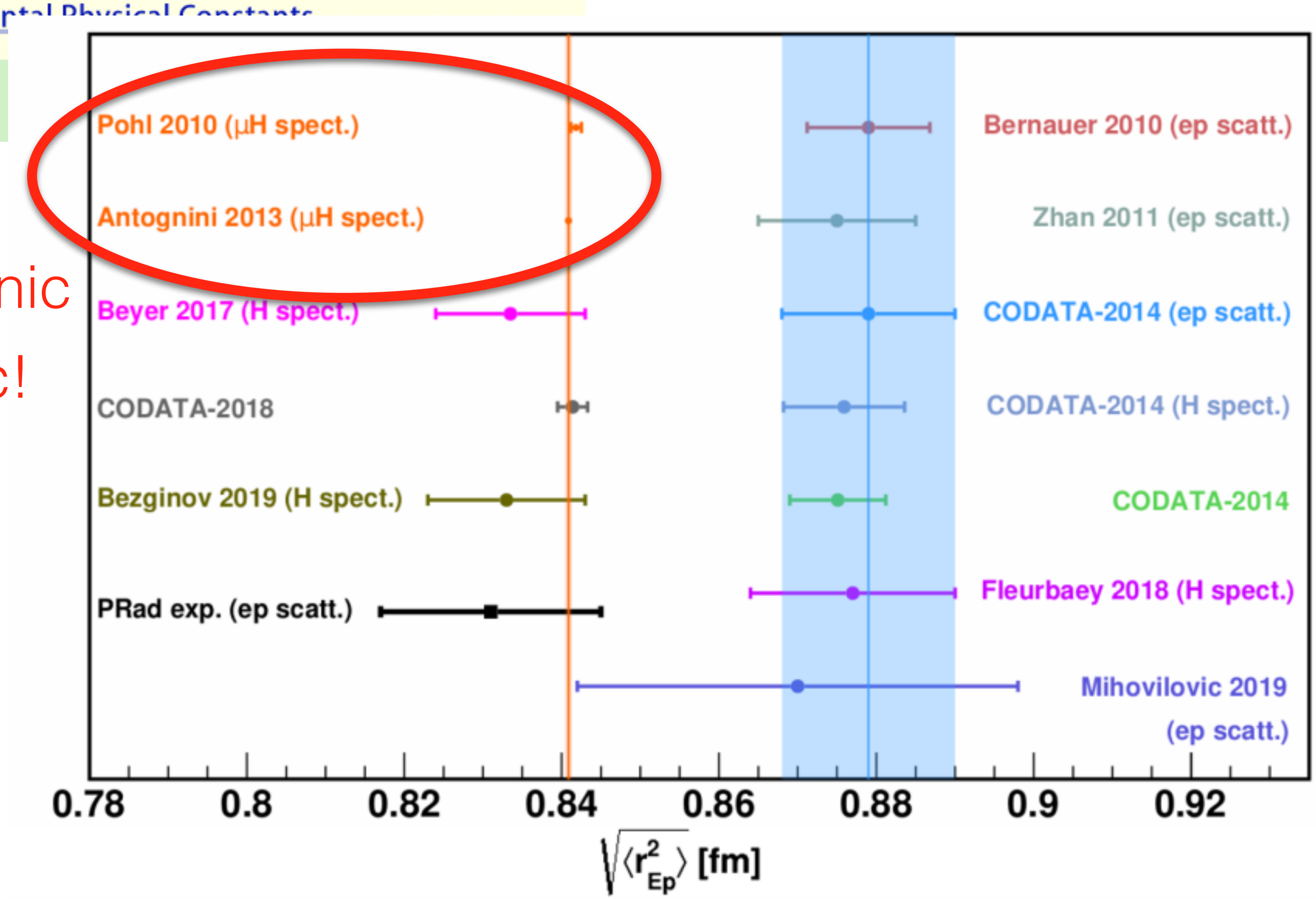
*PDG 2020

not electronic

but muonic!



## The proton charge radius

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Muonic hydrogen spectroscopy however found a highly discrepant result with e-measurements back in 2010 and 2013

$$r^{(\mu)} = 0.84087(39) \text{ fm}$$

***Antognini et al 2013**

$$r^{(e)} = 0.8775(51) \text{ fm}$$

***CODATA 2010**

"Charge radius puzzle" ~ 7$\sigma$

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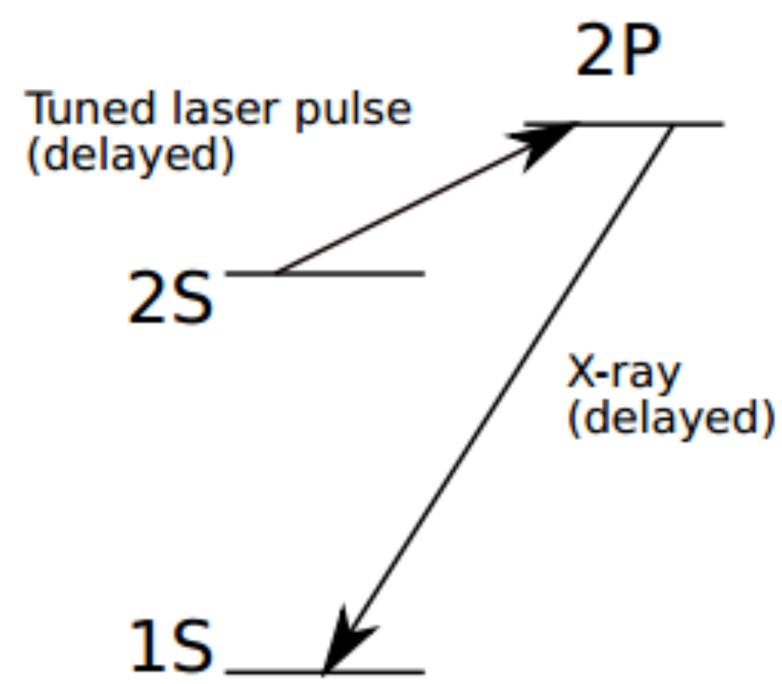
muonic Lamb shift




**(from C. Carlson 2015)**

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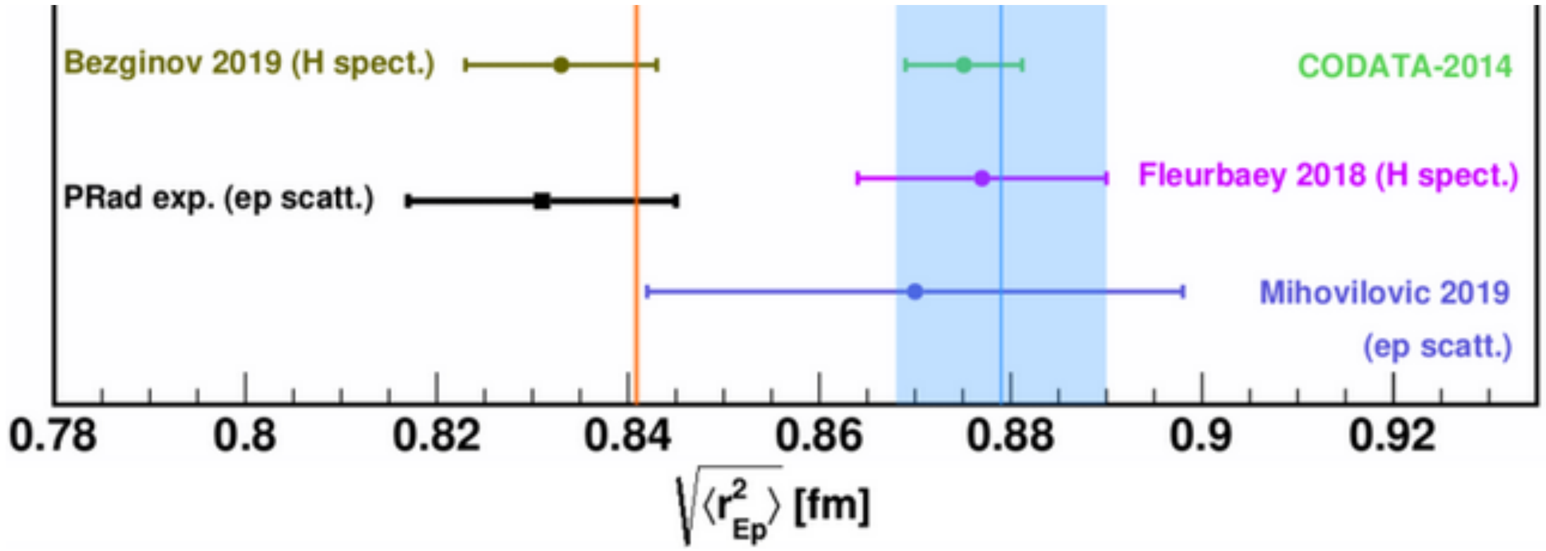
Discrepancy alive? Some recent e-measurements agree with muonic ones, but other ones (2019) still >2$\sigma$ off



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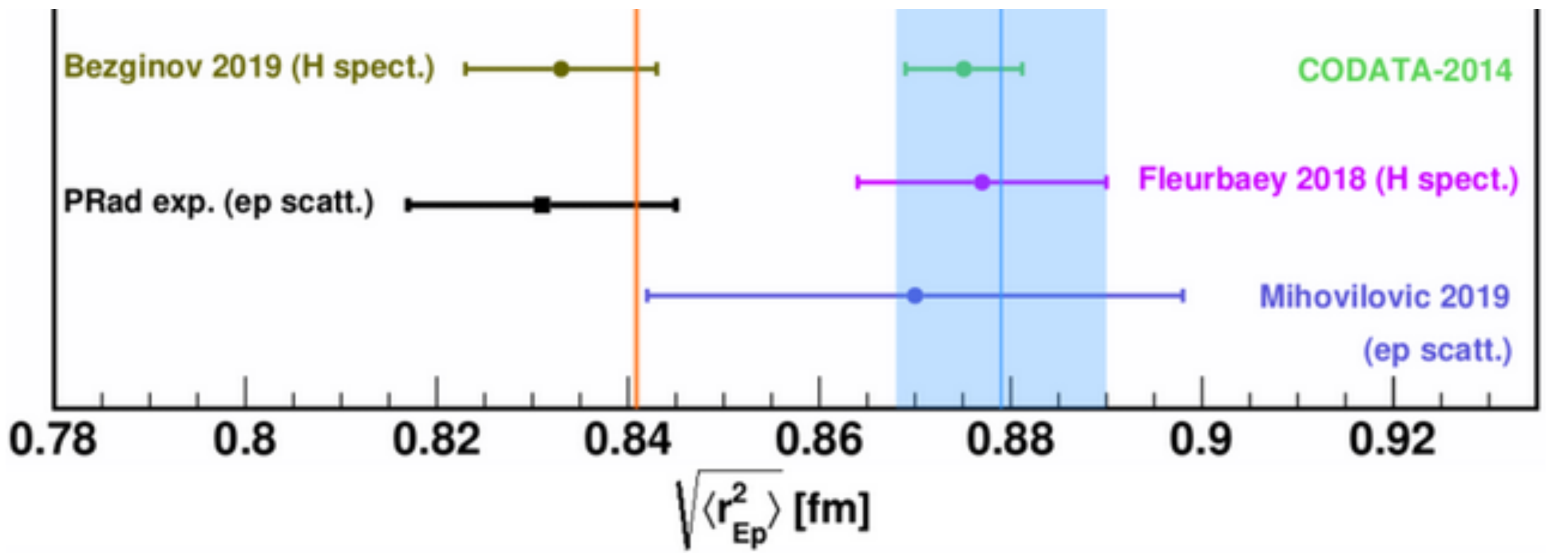
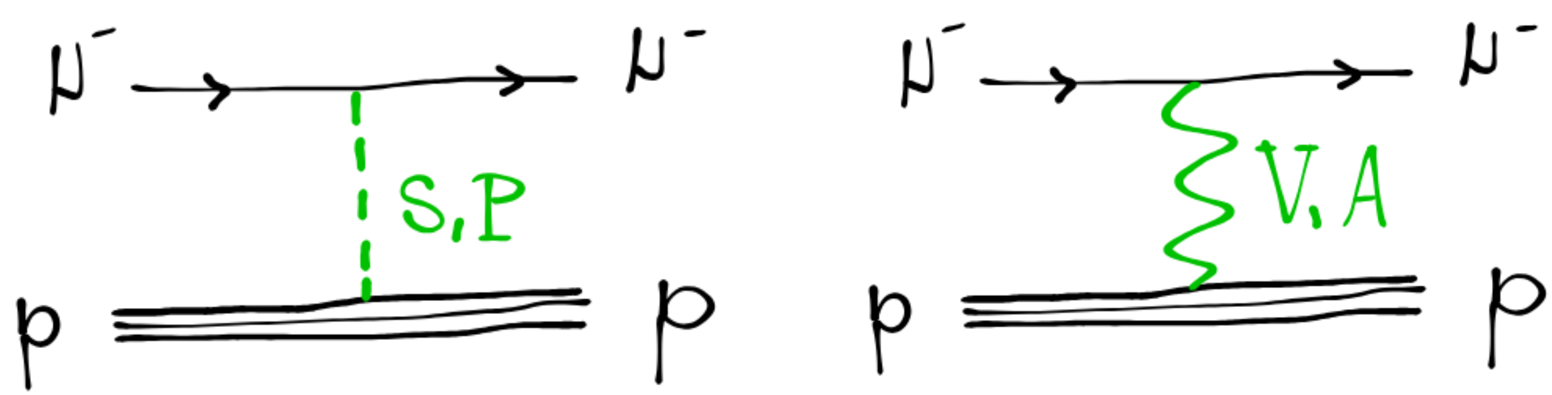
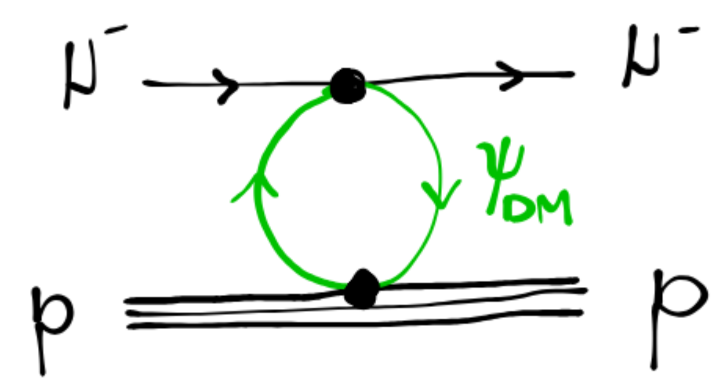



Possibilities: theoretical corrections to scattering & spectroscopy

...or new physics, **muonphilic interactions**




*Batell et al (2011), Carlson et al (2012), Pospelov et al (2014)




*Chuen San et al (2020)

# The proton charge radius

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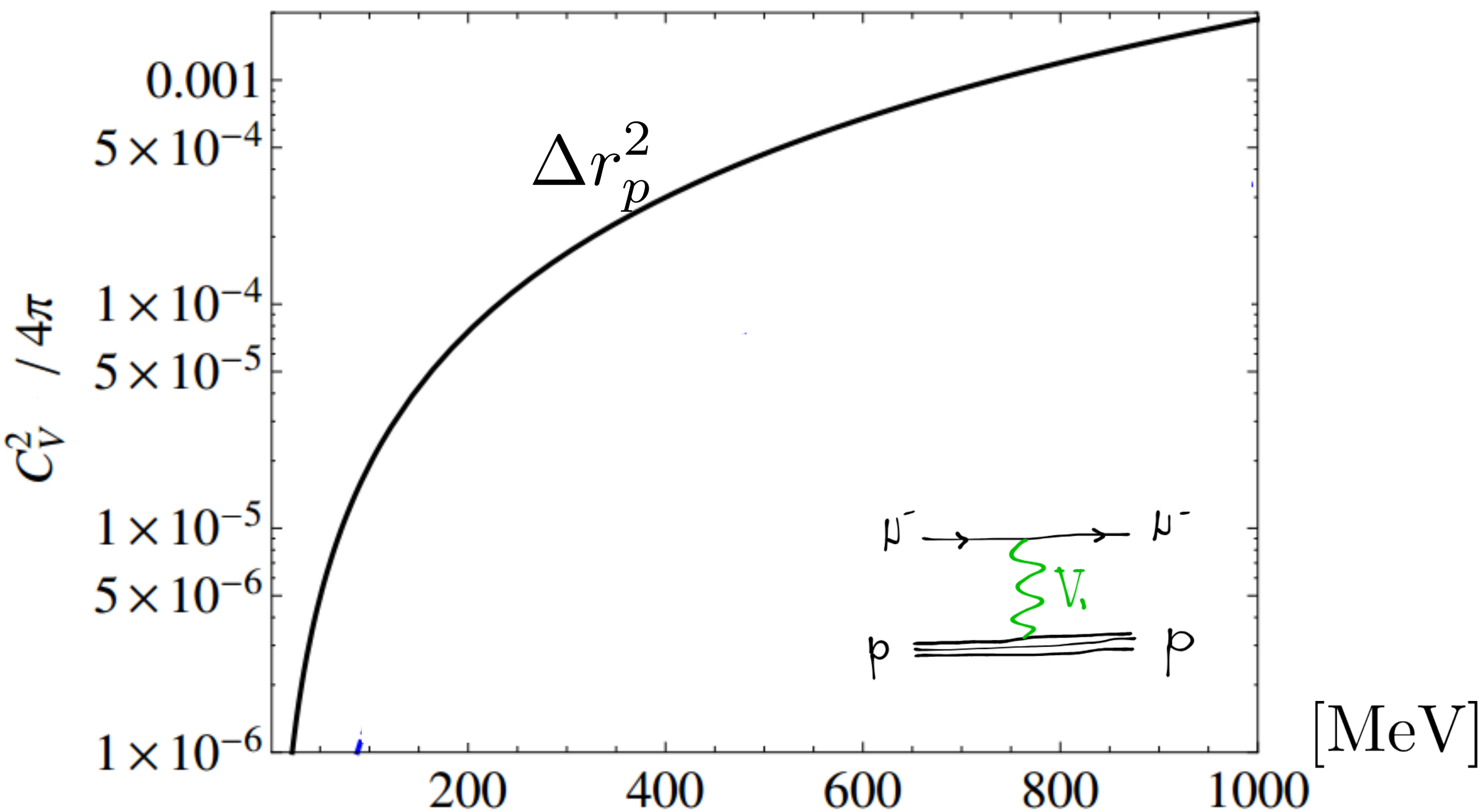
Given tree-level V couplings, how to fit $\Delta r_p^2$ ?




***adapted from Carlson et al (2012)**

# An anomaly-free U(1) model

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Rough requirements*: 1) no active neutrinos with large Fermi interactions, 2) No charged states below 100 GeV, 3) UV complete

***Batell et al 2011**

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So we tried a non-universal $U(1)$ gauge, coupled to RH fermions only

| | $\mu_R$ | $u_R$ | $d_R$ | $\nu_R$ | $\Phi_2$ | $s$ |
|---|---|---|---|---|---|---|
| $SU(2)_L$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| $U(1)_R$ | $-q_\nu$ | $q_\nu$ | $-q_\nu$ | $q_\nu$ | $q_\nu$ | $q_s$ |

(more solutions for i-th quark and j-th lepton generations)

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(more solutions for i-th quark and j-th lepton generations)

Give a SSB mass to $Z'$ with a singlet $s$

***Batell et al 2011**

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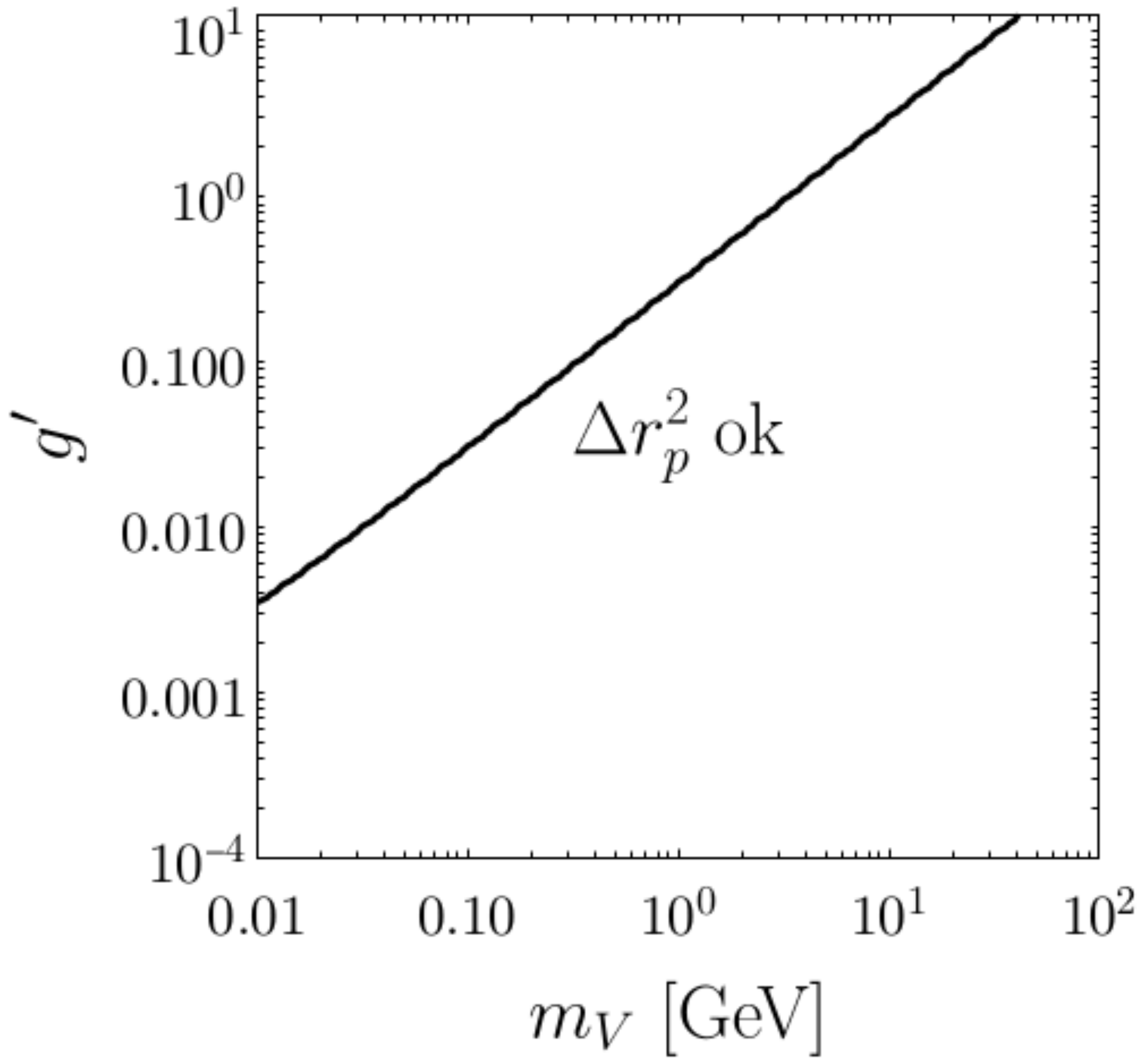
Assume $C_V^{(\mu)} = C_V^{(p)}$




RH-couplings only: $C_V = C_A = q_\nu g'/2$

# An anomaly-free U(1) model

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Usual Yukawa $\overline{L_\mu}\Phi_1\mu_R$ forbidden. Instead, generate muon mass with

$$y_{i\mu}\overline{L_i}\Phi_2\mu_R$$

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O(1) muon Yukawas if $v_2 \equiv \langle\Phi_2^0\rangle/\sqrt{2} \sim m_\mu$

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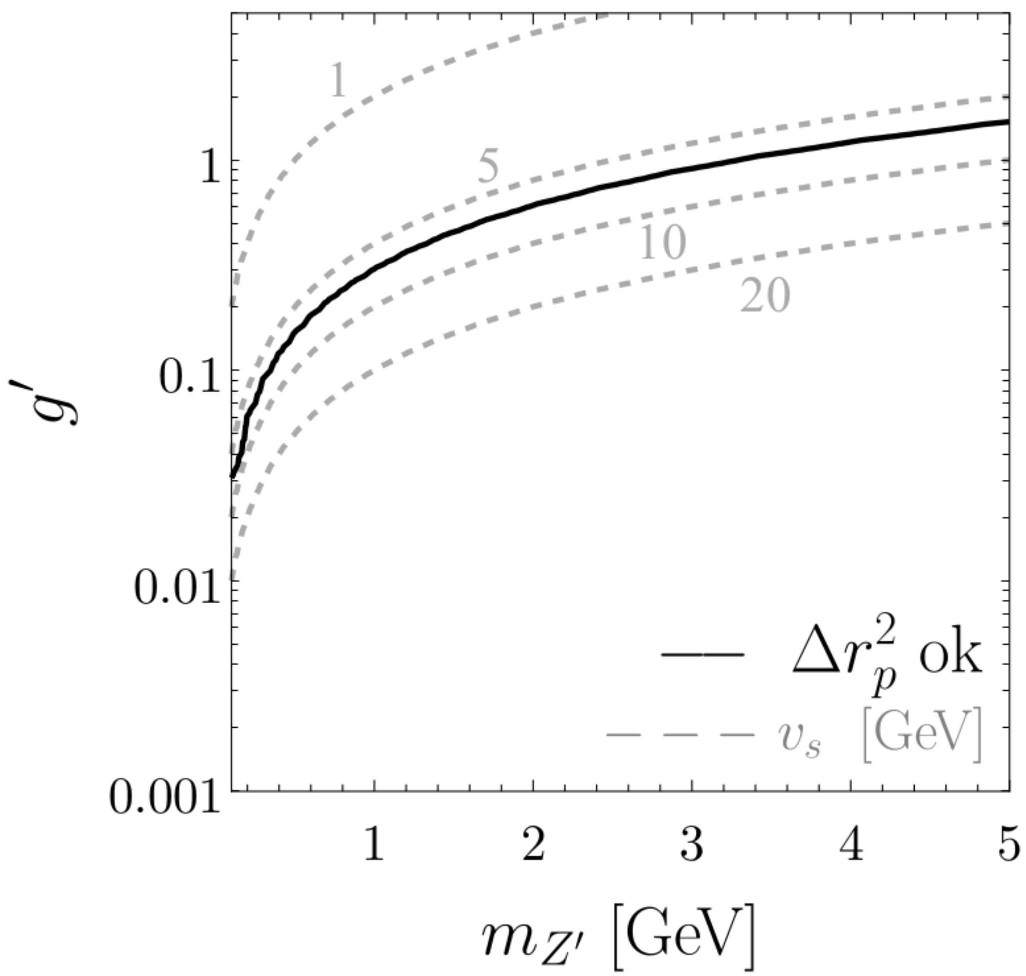
| $SU(2)_L$ | **1** | **1** | **1** | **1** | **2** | **1** |
| $U(1)_R$ | $-q_\nu$ | $q_\nu$ | $-q_\nu$ | $q_\nu$ | $q_\nu$ | $q_s$ |

O(1) muon Yukawas if $v_2 \equiv \langle\Phi_2^0\rangle/\sqrt{2} \sim m_\mu$

Also, $v_2 \ll v_s$ consistent with $\Delta r_p^2$

and $m_{Z'}^2 \approx g'^2 v_s^2/2$



# Spectrum & constraints

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- Spectrum

$h_{SM}, H^0, s_{\rm like}$ **CP-even**

$A^0$ **pseudoscalar**

$H^+$ **charged scalar**

- Avoid massless pseudoscalar $A^0$ through

$$\Delta V(\Phi_1, \Phi_2, s) \supset \kappa(s^2 \Phi_1^\dagger \Phi_2 + \text{ H.c.})$$

Previously invoked in the context of Dirac neutrino masses*

- $H^+$ pushed to** $m_{H+} \gtrsim 650 - 700 \text{ GeV}$ (muonphilic 2HDM)

***Bonilla & Valle (2016)**

****Yagyu el al (2017)**

# Spectrum & constraints

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$Z'$ not consistent with **both** $(g-2)_\mu$ and $\Delta r_p^2$

$\Delta a_\mu \equiv (g-2)_\mu/2$ too negative!

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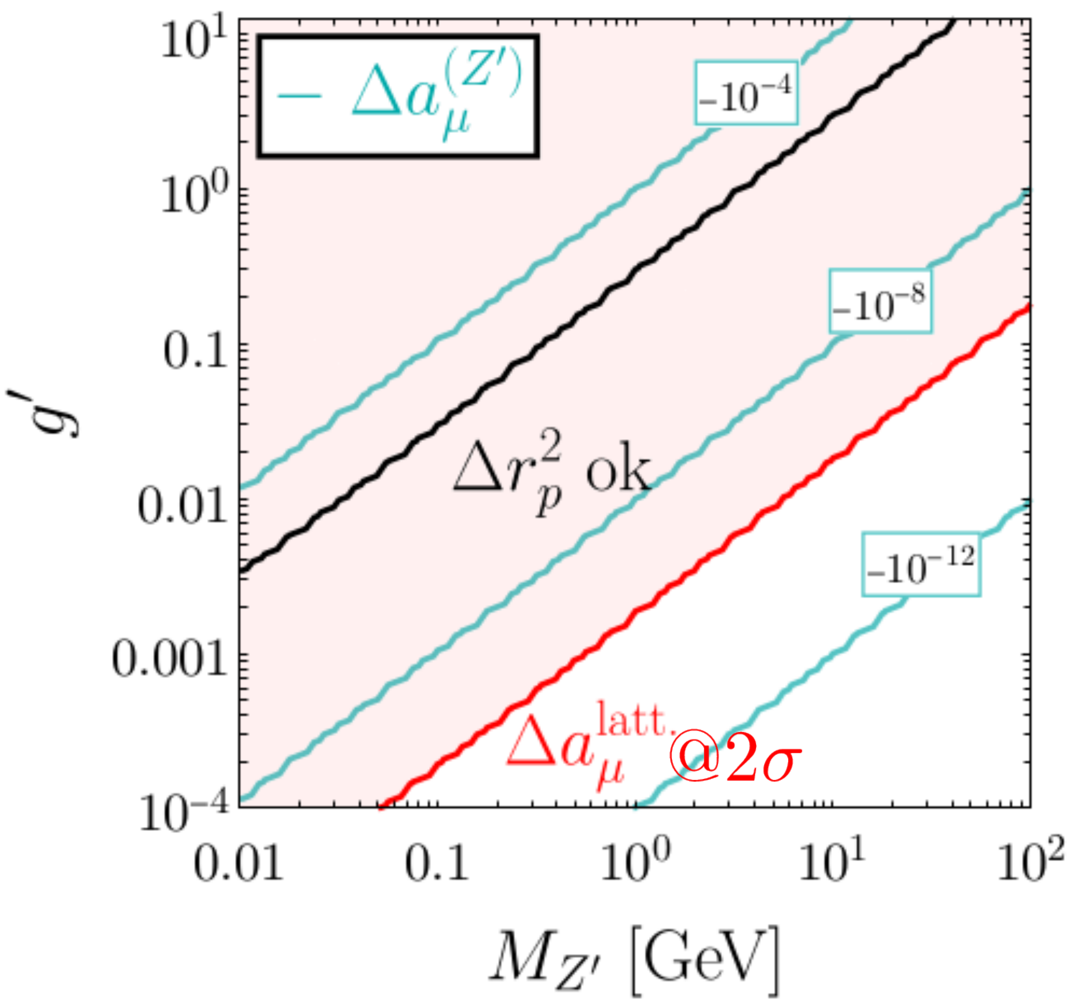


At $C_V = C_A$ the $\Delta a_\mu$ by $C_A$ negative, much larger than $\Delta a_\mu$ by $C_V$

$$\Delta a_\mu^{\rm latt.} = 109(71) \times 10^{-11}$$

***Borsanyi et al (Apr 2012)**

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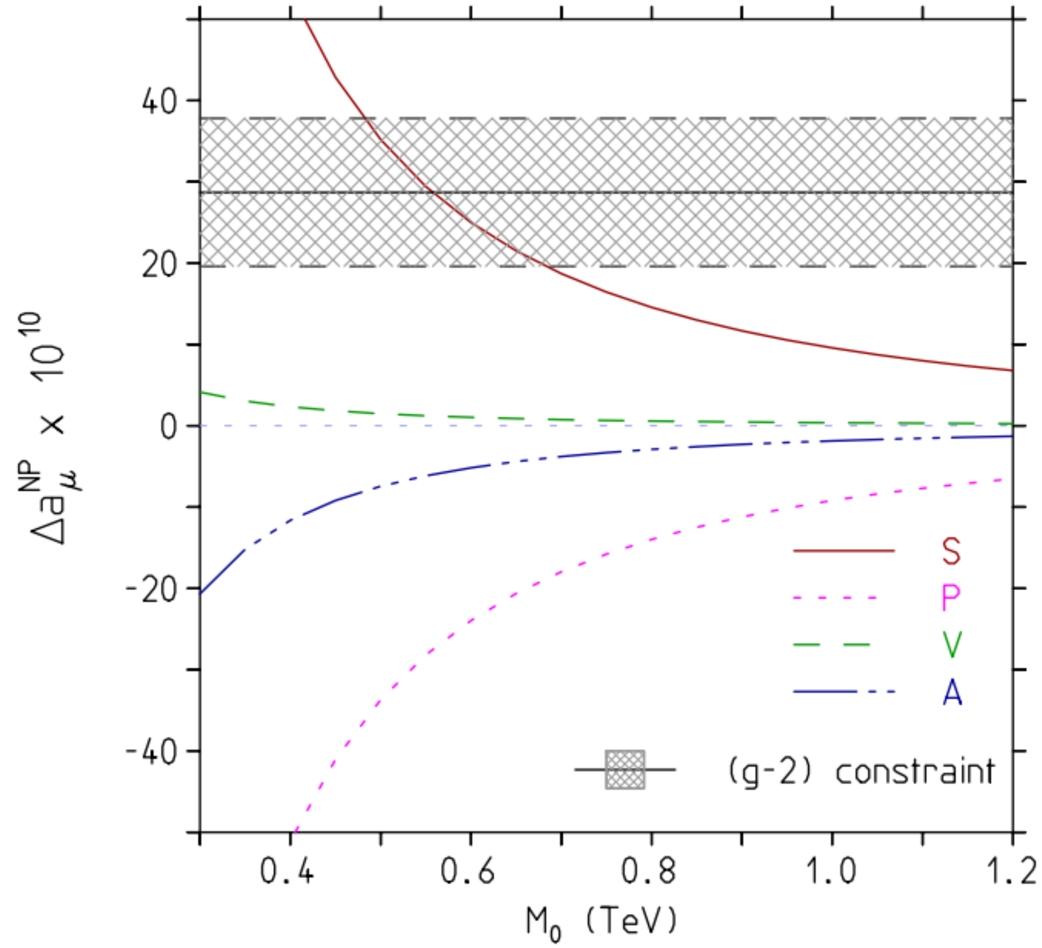
- $\Delta a_\mu$ large and positive for $s_{\text{like}}$ if $m_s$ small enough and Yukawa sufficiently large

- When $m_s$ sub-GeV the $H^0$, $A^0$ ,$H^+$ nearly degenerate, $m_{H^+}$ ~ TeV for all

- Partial $\Delta a_\mu$ cancellation between $H^0$, $A^0$ and $H^+$ if nearly degenerate & heavy




***Jegerlehner et al 2009**

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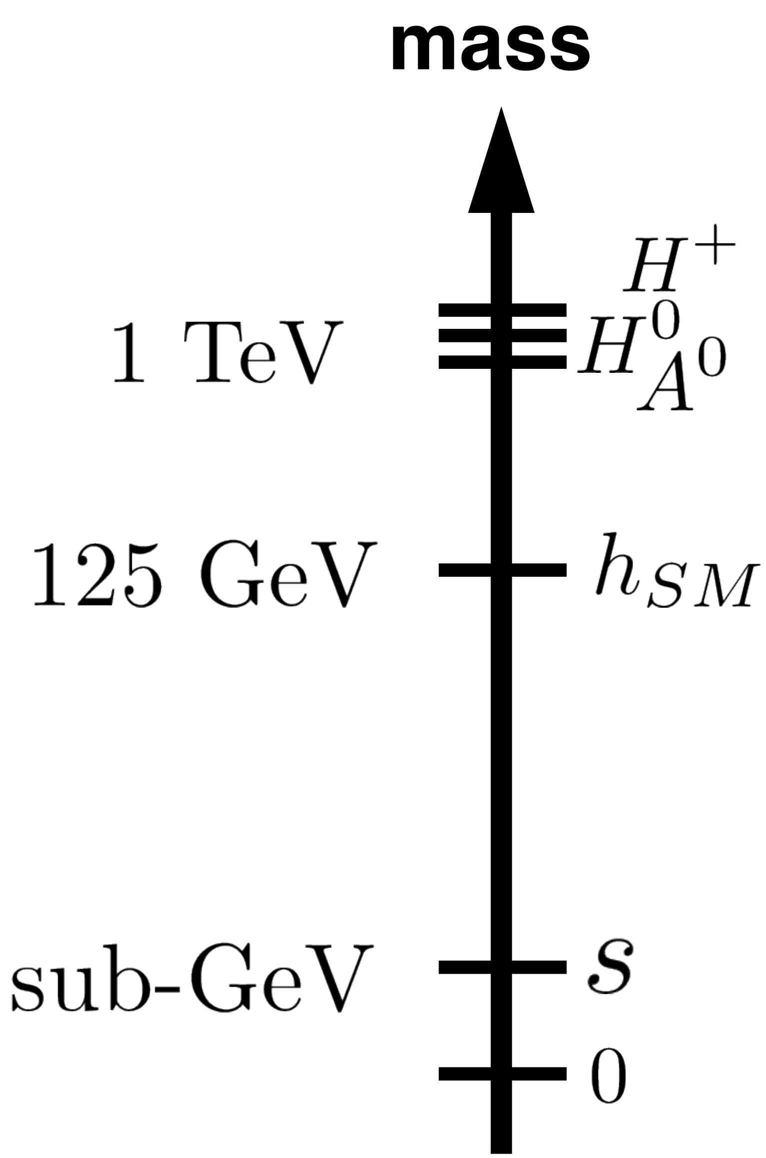
Try




(spectrum not typical, yet possible. States barely mixed)

# Spectrum & constraints

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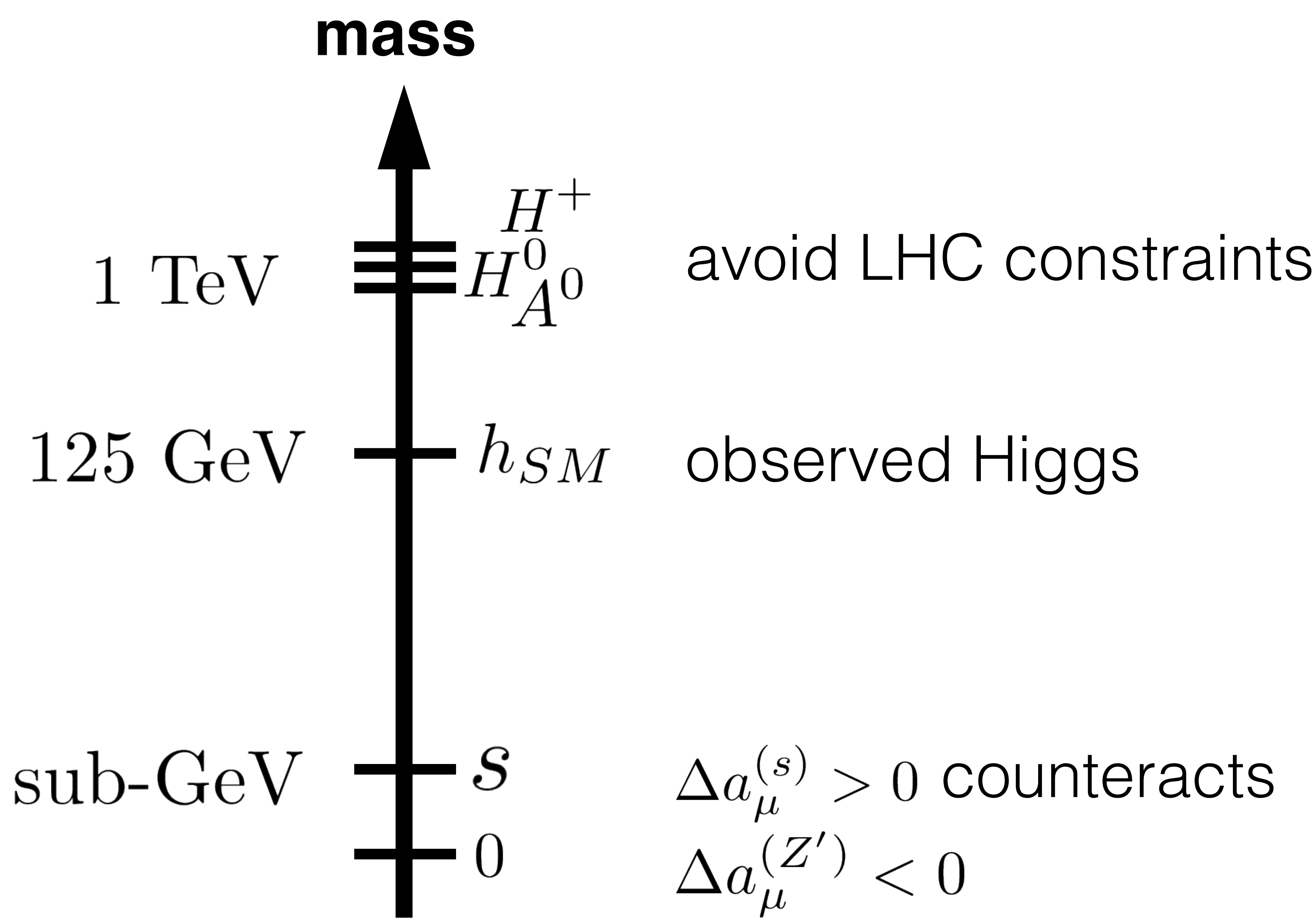


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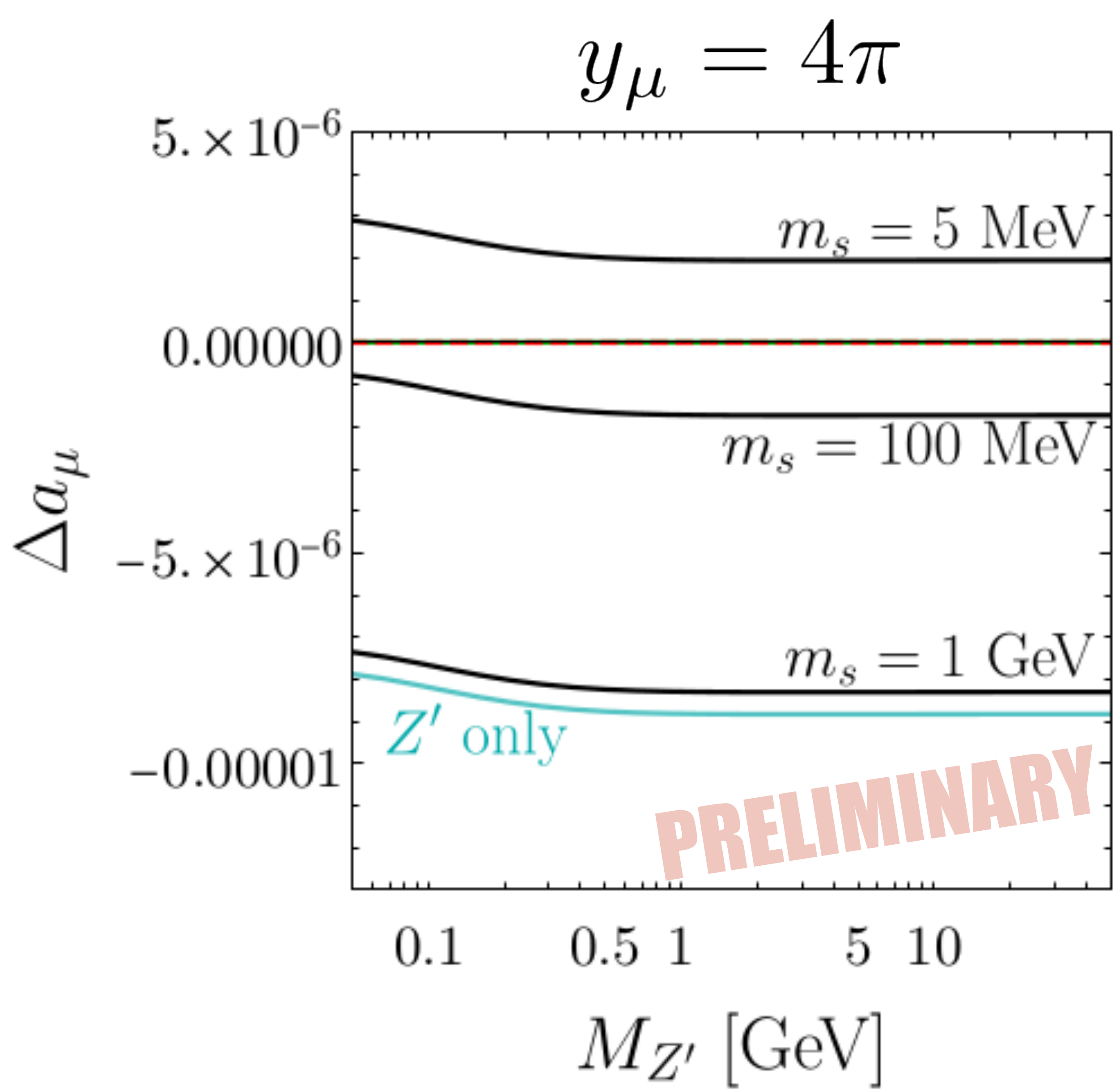
$\Delta a_\mu^{(s)} + \Delta a_\mu^{(Z')}$ together, **while fitting** $\Delta r_p^2$




$\Delta a_\mu^{(H^0, A, H^+)}$ highly subleading

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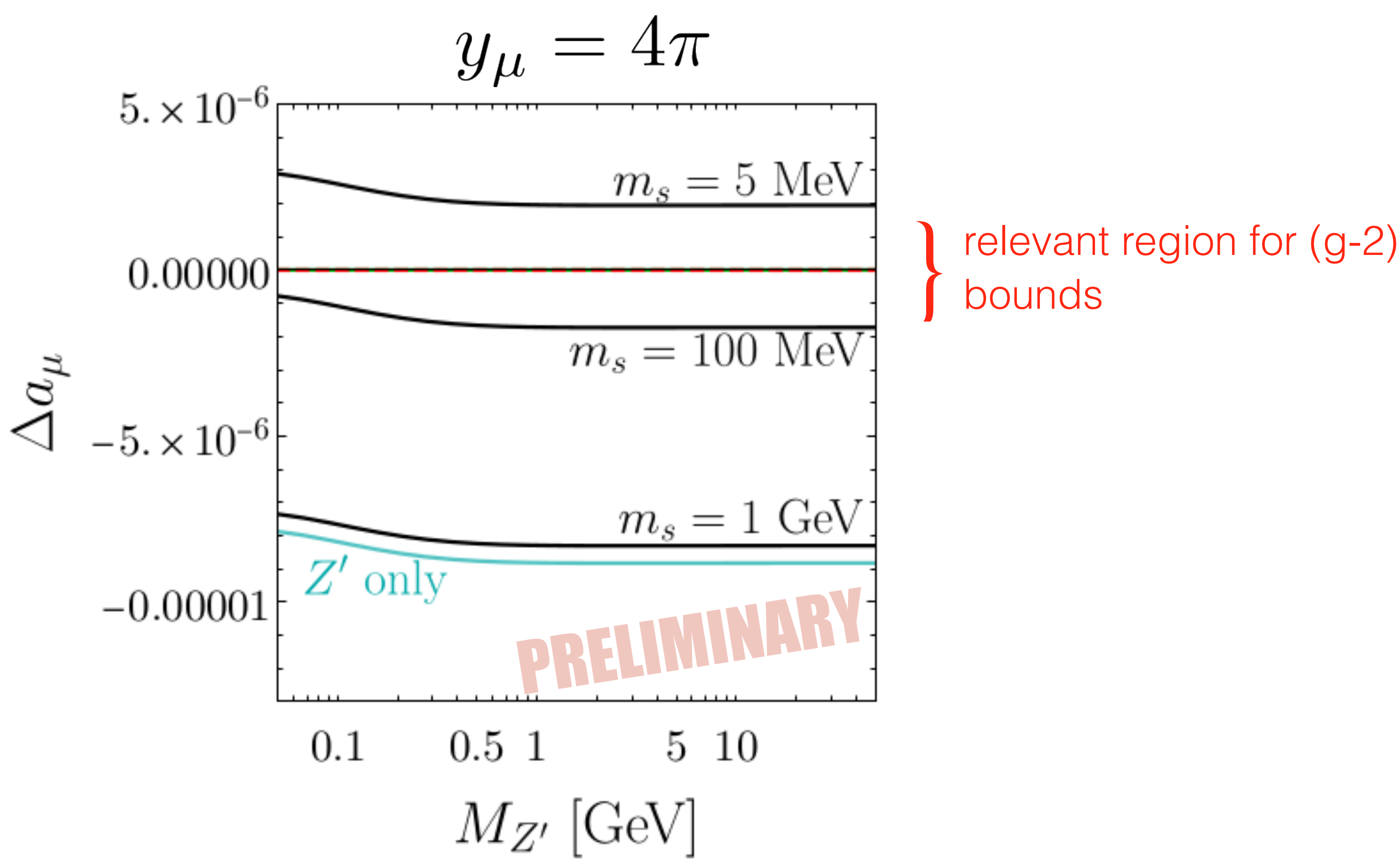


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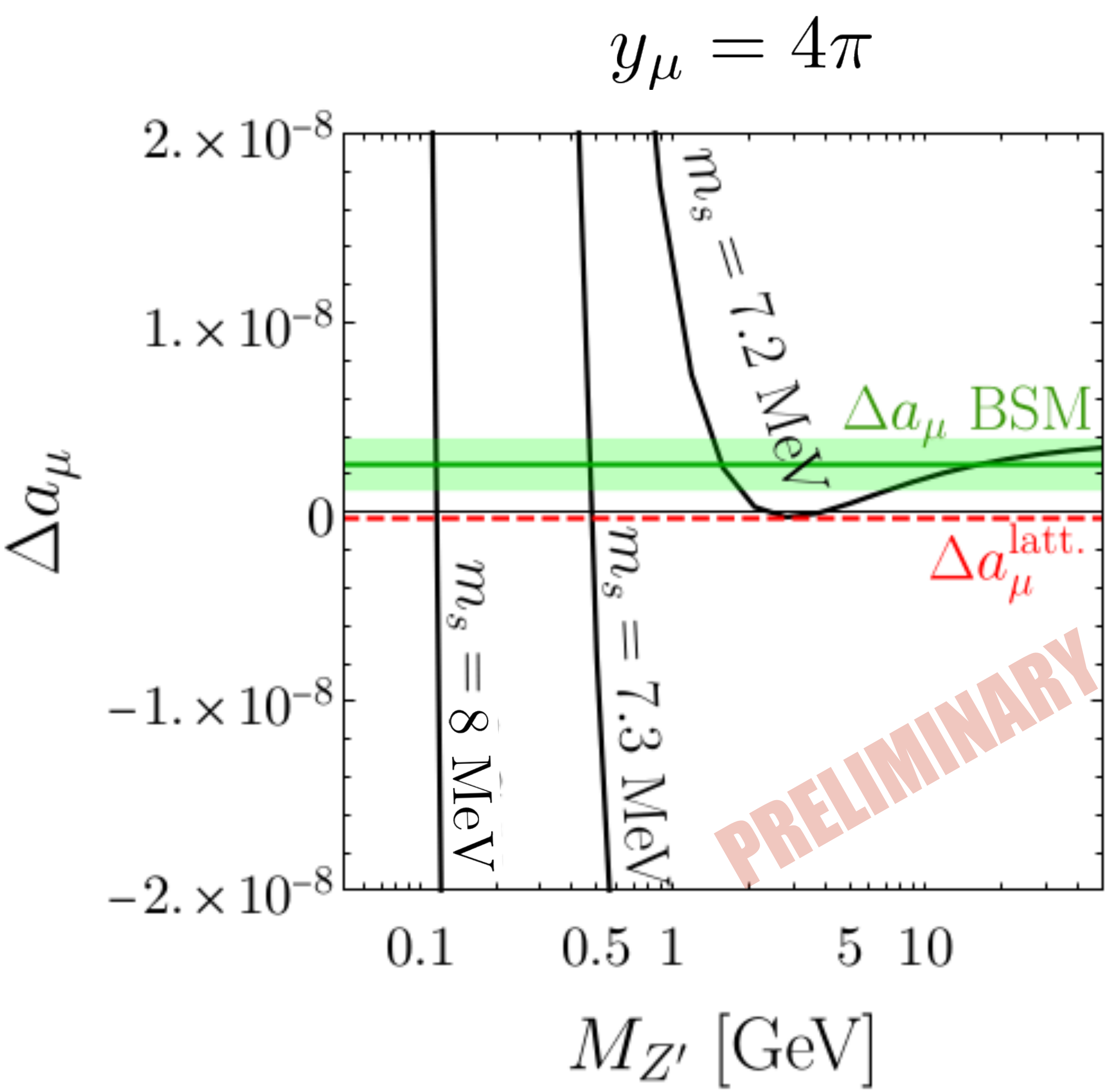
Zooming in the region relevant for $(g-2)_\mu$

$$\Delta a_\mu^{\rm BSM} = 251(59) \times 10^{-11}$$

***MUON G-2 FNAL (Apr 2021)**




What is next?

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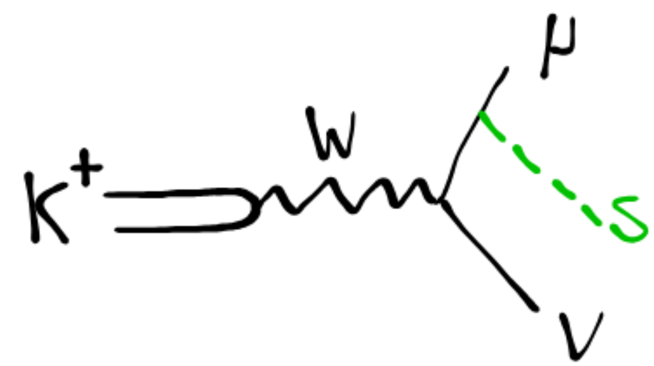
Ongoing

- Quark mixing ($V_{CKM}$) under control?

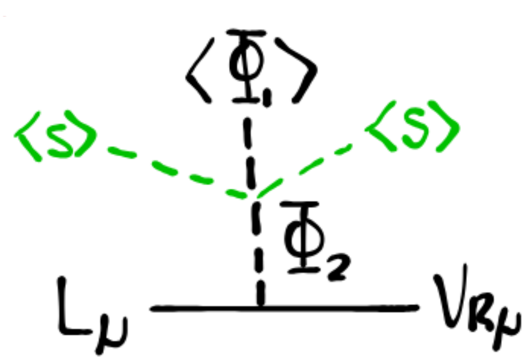
$$\mathcal{M}_{u,d} = \begin{pmatrix} v_2 & v_1 & v_1 \\ v_2 & v_1 & v_1 \\ v_2 & v_1 & v_1 \end{pmatrix}$$

- Kaon decay constraints




- Cosmological bounds on $Z'$ (i.e. $N_{\text{eff}}$)

- Neutrino mass



# Concluding remarks

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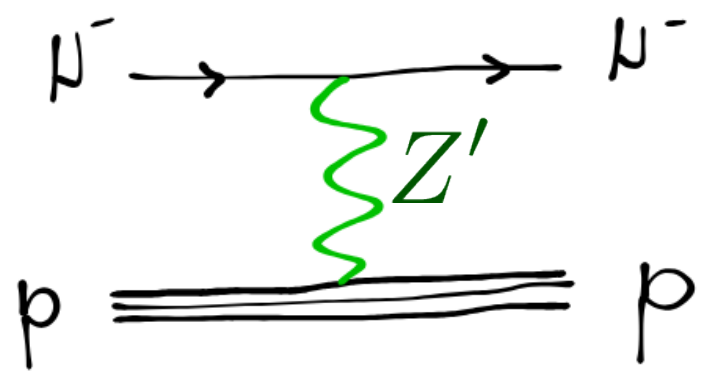
- The once many-$\sigma$ discrepancy between $e^-$ and $\mu^-$ measurements of the proton charge radius may seem to have gone away, but not conclusively.

- Anomaly-free U(1) gauge extension (+ scalars) addressing the $\Delta r_p^2$ discrepancy




- Scenario consistent with latest muon (g-2) results thanks to a few MeV light singlet-like scalar.

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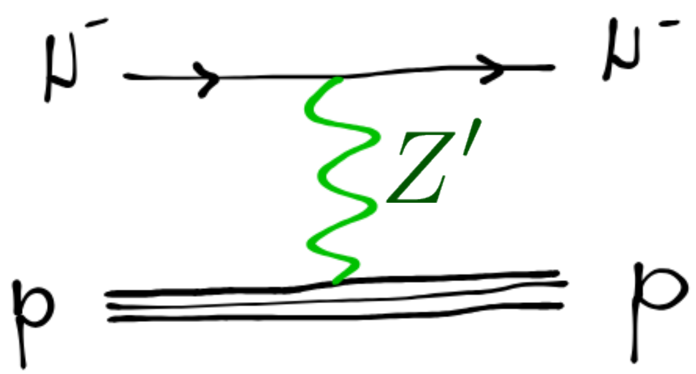


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**THANKS!**

# Backup

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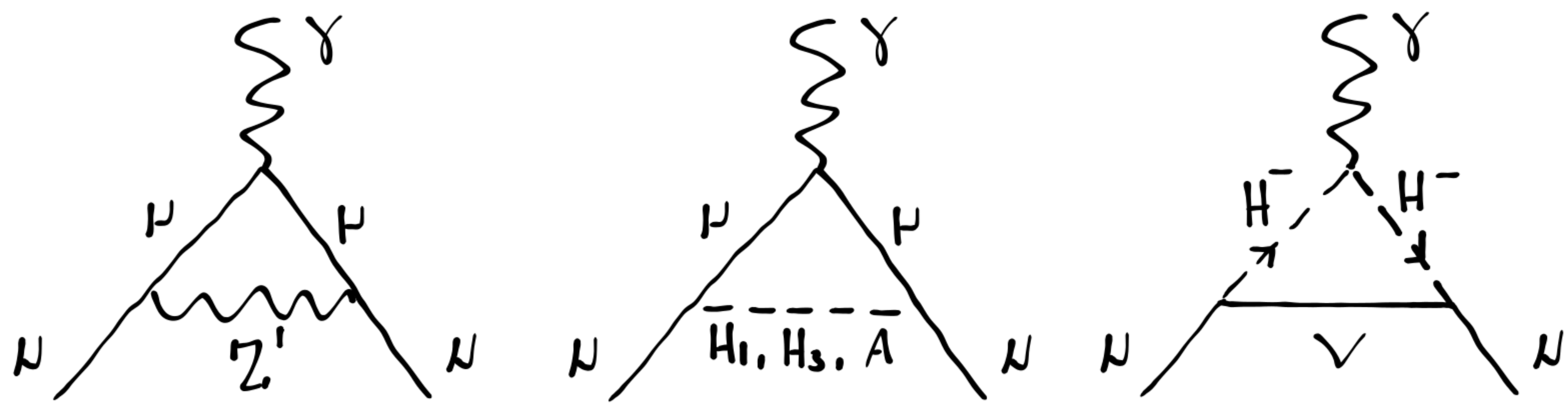


$$\Delta a_\mu^{(Z')} = \frac{1}{8\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\int_0^1 \mathrm{d}x \; \frac{\left|C_V[Z']\right|^2 P_V(x) + \left|C_A[Z']\right|^2 P_A(x)}{(1-x)(1-m_\mu^2/m_{Z'}^2) + x(m_\mu^2/m_{Z'}^2)}$$

$$P_{V,A}(x) \equiv 2x(1-x)(x-2\pm 2) + (m_\mu^2/m_{Z'}^2)x^2(1\mp 1)^2(1-x\pm 1) \; .$$

Similar expressions for scalars exchange.

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Sub-GeV singlet-like scalar